International equity and bond portfolio with firm entry*

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Abstract

This paper finds the international equity and bond portfolio, (which replicates the locally complete market), with investment in terms of variety/firm creation. The equity home bias appears because of negative conditional covariance between equity returns and non-financial income. The bond is mainly used to hedge the observed real exchange rate fluctuations with which it is perfectly correlated By specifying that the entry cost is paid in terms of capital goods as well as effective labor depending on an exogenous parameter, the optimal equity and bond position appears to be depending on this "redistributive" effect.

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1 Introduction

Documented as one of the six major puzzles in the international macroeconomics (Obstfeld and Rogoff (2000)), the home biased equity puzzle has received increasing attention of researchers.[1] Theoretically equity portfolios are considered as a hedge against the consumption risk. The consumption risk across countries arises from the real exchange rate fluctuation and the difference of non-financial income (labor income). In the world where the terms of trade don't provide a perfectly substitutable role for that purpose (Cole and Obstfeld (1991)) the international equity position should be used to realize stabilized consumption across countries.

From the above point of view, if we observe in reality a home biased equity position, equity returns should provide a positive (negative) income flow when real exchange rate appreciates (depreciates) (because of home bias in consumption). However as explored in Coeurdacier (2008) and Obstfeld (2007) under standard CRRA preference, this is possible only when there is sufficiently *low* elasticity of substitution between two countries' goods. Because only under low elasticity, dividends (equity returns) can decrease (increase) with terms of trade depreciation (appreciation).

This sensitivity of equity position with regard to the certain parameters has considered at odds and finally received a critical challenge from empirical side:.van-Wincoop and Warnock (2006) document that no significant correlation is found between equity returns and real exchange rate fluctuations Since then the essays to explain observed home biased equity by the hedging against the terms of trade risk has started to loose the interest. Indeed as pointed out in Coeurdacier, Kollmann and Martin (2007), Coeurdacier, Kollmann and Martin (2008), Coeurdacier and Gourinchas (2008), Engel and Matumoto (2006), the nominal bond or forward exchange position can be used to hedge the terms of trade risk perfectly leaving the equity to hedge other consumption destabilizing risks.

This paper introduces the bond other than equity taking into account the above recent development of the literature. The focus is on the steady state portfolio which replicates the locally complete market allocation so the first order dynamic is only relevant.[2] As it is noted the bond is used to hedge the real exchange rate risk. The home biased equity position arises from the *negative* correlation (conditional on bond returns) between equity returns and non-financial income. A home biased equity position is a good hedge if it provides positive financial flow when labor income decreases. Contrary to Baxter and Jermann (1997) which finds a positive correlation between them and the "wore than you think" foreign biased optimal equity position, a successful optimal home bias arises in this paper because of the one-time-to build investment whose mechanism is originally explored in Heathcote and Perri (2007) Because

[1] Empirical studies which point out the equity home bias are for example French and Poterba (1991) and Tesar and Werner (1995).

[2] For the portfolio dynamics using higher order approximation methode under imcomplete market see Devereux and Sutherland (2006) and Tille and van Wincoop (2007).

of the investment expenditure, equity returns decreases (increases) when the economy goes well (wrong) (labor income increases (decreases)).

The contribution of the paper is to rewrite and represent the above mechanism and intuitions in the world where the investment takes place in terms of new variety/firm creation. The model used in this paper is based on Ghironi and Melitz (2005), Bilbiie, Ghironi and Melitz (2007), (2007)a and (2008). The log linear version of the model is solved using labor market clearing conditions as Matumoto (2007). The attempt to find the equity portfolio with firm entry (without bond) has already taken in Arespa (2008) She finds the home biased equity position in the world where there is no terms of trade risk, based on Bergin and Corsetti (2008). A very nice feature of the paper is that by specifying that the entry cost is paid in terms of capital goods as well as effective labor depending on an exogenous parameter, the optimal equity position of Arespa (2008) and the optimal equity and bond portfolio found in Coeurdacier and Gourinchas (2008) are found as the extreme case of such redistributive effect.

The structure of the paper is as follows. In the next section the model is presented. The key aspect is the endogenous investment in terms of variety/firm creation. In section 3 the real exchange rate variation and relative dividend (operational profits) variations which are important for portfolio decisions are discussed. In section 4, the optimal equity and portfolio which satisfy the locally complete market are found. At the end a brief conclusion is given.

2 The model

There are two countries, Home and Foreign. The model is based on Ghironi and Melitz (2005) which contains the mechanism of endogenous firm entry (however without heterogeneity among firms in this paper). Also following Coeurdacier, Kollmann and Martin (2007) , Coeurdacier and Gourinchas (2008) and Coeurdacier et al. (2008), there are two types of financial asset (equity and bond) which are held internationally and two types of exogenous shocks (production cost and R&D shock). The investment takes place in the form of new firm creation which is financed by equity (mutual fund) holdings of each countries' household.

The large characters are nominal variables. The small characters are real variables denominated by local consumption basket (except consumption, real exchange rate and the number of varieties). Log-deviation from its steady state value is expressed with sans-serif font. Foreign variables are expressed with stars.

2.1 Households

In each country there is 1 unit mass of population where the representative household supplies inelastically 1 unit of labor. The utility depends only on consumption. The household maximizes the following utility at t-1.

$$E_{t-1}\sum\nolimits_{s=t}^{\infty}\beta^{s-t}U_t \tag{1}$$

$$U_t=\frac{C_t^{1-\gamma}}{1-\gamma} \tag{2}$$

where β is the discount factor. $\gamma \geq 1$ is the relative risk aversion. The consumption basket is defined as:

$$C_t=\left[\alpha^{\frac{1}{\omega}}C_{H.t}^{1-\frac{1}{\omega}}+(1-\alpha)^{\frac{1}{\omega}}C_{F.t}^{1-\frac{1}{\omega}}\right]^{\frac{1}{1-\frac{1}{\omega}}} \tag{3}$$

and

$$C_{H.t}=\left[\int\limits_0^{N_t}c_t\left(h\right)^{1-\frac{1}{\sigma}}dh\right]^{\frac{1}{1-\frac{1}{\sigma}}},\qquad C_{F.t}=\left[\int\limits_0^{N_t^*}c_t\left(f\right)^{1-\frac{1}{\sigma}}df\right]^{\frac{1}{1-\frac{1}{\sigma}}}. \tag{4}$$

α is home bias for consumption. ω is the elasticity of substitution between Home and Foreign goods. σ is the elasticity of substitution among varieties. With above specification the preference is Dixit-Stiglitz meaning the marginal utility from one additional variety is represented by $\frac{1}{\sigma-1}$. $c_t\left(h\right)$ $\left(c_t\left(f\right)\right)$ is the demand for individual Home (Foreign) variety. N_t $\left(N_t^*\right)$ is the number of Home (Foreign) varieties. The corresponding price indices are

$$P_t=\left[\alpha P_{H.t}^{1-\omega}+(1-\alpha)P_{F.t}^{1-\omega}\right]^{\frac{1}{1-\omega}} \tag{5}$$

and

$$P_{H.t}=\left[\int\limits_0^{N_t}p_t\left(h\right)^{1-\sigma}dh\right]^{\frac{1}{1-\sigma}},\qquad P_{F.t}=\left[\int\limits_0^{N_t^*}p_t\left(f\right)^{1-\sigma}df\right]^{\frac{1}{1-\sigma}}. \tag{6}$$

The similar expression holds for Foreign.

2.1.1 Budget constraint

The real budget constraint for the Home representative household is

$$\begin{aligned}&C_t+s_{h.t+1}x_{h.t}^s\left(N_t+N_{E.t}\right)+s_{f.t+1}x_{f.t}^s\left(N_t^*+N_{E.t}^*\right)+b_{h.t+1}x_{h.t}^b+b_{f.t+1}x_{f.t}^b\\&=w_t+s_{h.t}N_t\left(x_{h.t}^s+d_{h.t}\right)+s_{f.t}N_t^*\left(x_{f.t}^s+d_{f.t}\right)+b_{h.t}\left(x_{h.t}^b+1\right)+b_{f.t}\left(x_{f.t}^b+Q_t\right)\end{aligned} \tag{7}$$

The bonds and equities (that of mutual fund) are held internationally. One unit of Home (Foreign) real bond gives one unit of Home (Foreign) consumption goods at the next period. Without knowing which firm will die at the end of the period t, household finances all existing firms including new entrants, $N_{E.t}$ and $N_{E.t}^{*}$ by purchasing mutual fund. $s_{h.t+1}(s_{f.t+1})$ is the share holding by Home household into t+1 in the total capitalization of Home (Foreign) firms. $x_{h.t}^{s}\left(x_{f.t}^{s}\right)$ is the real share price. $d_{h.t}\left(d_{f.t}\right)$ is the real dividend for Home (Foreign) firm. $b_{h.t+1}(b_{f.t+1})$ is the real bond holding by Home household into t+1 which is denominated by Home consumption basket. $x_{h.t}^{b}\left(x_{f.t}^{b}\right)$ is the real bond price. w_t is the real wage. Q_t is the real exchange rate defined as:

$$Q_t = \frac{\varepsilon_t P_t^*}{P_t} \tag{8}$$

[3]

For the representative Foreign household the real budget constraint becomes:

$$\begin{aligned} &C_t^* + s_{f.t+1}^* x_{f.t}^{s*}\left(N_t^* + N_{E.t}^*\right) + s_{h.t+1}^* x_{h.t}^{s*}\left(N_t + N_{E.t}\right) + b_{f.t+1}^* x_{f.t}^{b*} + b_{h.t+1}^* x_{h.t}^{b*} \\ &= w_t^* + s_{f.t}^* N_t^*\left(x_{f.t}^{s*} + d_{f.t}^*\right) + s_{h.t}^* N_t\left(x_{h.t}^{s*} + d_{h.t}^*\right) + b_{f.t}^*\left(x_{f.t}^{b*} + 1\right) + b_{h.t}^*\left(x_{h.t}^{b*} + Q_t^{-1}\right) \end{aligned} \tag{9}$$

2.1.2 First order conditions

Optimal consumption of Home and Foreign goods by Home household is given by respectively

$$C_{H.t} = \alpha\left(\frac{P_{H.t}}{P_t}\right)^{-\omega} C_t, \qquad C_{F.t} = (1-\alpha)\left(\frac{P_{F.t}}{P_t}\right)^{-\omega} C_t \tag{10}$$

and using symmetry among varieties optimal consumption for each individual firm is given by

$$c_{h.t} = \left(\frac{p_{h.t}}{P_{H.t}}\right)^{-\sigma} C_{H.t}, \qquad c_{f.t} = \left(\frac{p_{f.t}}{P_{F.t}}\right)^{-\sigma} C_{F.t} \tag{11}$$

The optimization with respect to Home and Foreign mutual fund holding gives the following Euler share condition:

$$x_{h.t}^{s} = \beta\left(1-\delta\right) E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(x_{h.t+1}^{s} + d_{h.t+1}\right) \tag{12}$$

[3] This expression of budget constraint concerning the motion of firm is essentially the same as that of Bergin and Corsetti(2005) where the firm survive only two periods (one period for time to build and another for production). The difference is whether firms die "immediately or smoothly". Bergin and Corsetti's motion of firm is regarded as the special case where $\delta = 1$. In such case, contrary to full specification, the equity price depend only on t+1 discounted dividend.

$$x^s_{f.t} = \beta (1-\delta) E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(x^s_{f.t+1} + d_{f.t+1} \right) \tag{13}$$

Without transportation cost concerning asset trading LOP holds for real equity prices and dividend:

$$x^{s*}_{h.t} = Q_t^{-1} x_{h.t} \qquad x^s_{f.t} = Q_t x^{s*}_{f.t} \tag{14}$$

$$d^*_{h.t} = Q_t^{-1} d_{h.t}, \qquad d_{f.t} = Q_t d^*_{f.t} \tag{15}$$

The optimal Home and Foreign bond holding gives the following Euler bond conditions:

$$x^b_{h.t} = \beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(x^b_{h.t+1} + 1 \right) \tag{16}$$

$$x^b_{f.t} = \beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(x^b_{f.t+1} + Q_t \right) \tag{17}$$

Also for bonds LOP holds:

$$x^{b*}_{h.t} = Q_t^{-1} x^b_{h.t} \qquad x^b_{f.t} = Q_t x^{b*}_{f.t} \tag{18}$$

For notational convenience I define the relative price in terms of local consumption basket as follows:

$$\rho_{H.t} = \frac{P_{H.t}}{P_t}, \qquad \rho_{F.t} = \frac{P_{F.t}}{P_t}, \qquad \rho_{h.t} = \frac{p_{h.t}}{P_t}, \qquad \rho_{f.t} = \frac{p_{f.t}}{P_t}$$

$$\rho^*_{H.t} = \frac{P^*_{H.t}}{P^*_t}, \qquad \rho^*_{F.t} = \frac{P^*_{F.t}}{P^*_t}, \qquad \rho^*_{h.t} = \frac{p^*_{h.t}}{P^*_t}, \qquad \rho^*_{f.t} = \frac{p^*_{f.t}}{P^*_t}$$

2.2 Firms

2.2.1 Entry cost

Prior to entry each new entrants must pay a sunk entry cost. One firm / variety creation needs an amount of firm setting up goods, f_E. The production of such goods is done by the following technology using labor $l_{EM.t}$ and capital goods K_t.

$$f_E = z_{E:t} \left(\frac{l_{EM.t}}{\theta} \right)^{\theta} \left(\frac{K_t}{1-\theta} \right)^{1-\theta} \tag{19}$$

where $z_{E:t}$ is the TFP in firm setting up goods production. θ $(1-\theta)$ is the share of labor (capital) cost in total cost for firm setup. For simplicity I suppose capital goods K_t has the same composition as the consumption goods. The cost minimization problem yields the following factor demand,

$$l_{EM.t} = \mu_t \frac{\theta}{w_t} f_E, \qquad K_t = \mu_t (1-\theta) f_E, \tag{20}$$

where $\mu_t = w_t^{\theta}/z_{E.t}$ is the real cost of firm set up. Thus the following free entry condition determines the number of new entrants N_{Et} in each period.

$$x_{h.t}^{s} = \frac{f_E w_t^{\theta}}{z_{E.t}} \tag{21}$$

Note especially with $\theta = 1$ only labor is used for the creation of extensive margin and with $\theta = 0$ only capital is used. As we will see this has a very important implication for portfolio choice. Intuitively the new entry works as a "redistributive shock" between labor and financial income depending on θ.

2.2.2 Motion of the firms

The motion of firms and the determination of the number of new entrants, N_{Et} are identical to Ghironi and Melitz (2005).

$$N_t = (1-\delta)(N_{t-1} + N_{Et-1}) \tag{22}$$

The production take place only one period after the entry. New entrants need "one time-to build". The number of firms is governed by an exogenous mortality rate δ embodied in the economy. Firms' "natural death" take place at the very end of the period after the investment has finished (financed by Home mutual fund). The similar conditions hold for Foreign.

2.2.3 Production

One period after the entry firms produce. For production only labor is used as input and its technology is liner:

$$y_{h.t} = z_t l_t \tag{23}$$

where z_t is labor productivity improving exogenous shock (production cost shock). The operational real profit (dividend) is expressed as:

$$d_{h.t} = \left(\rho_{h.t} - \frac{w_t}{z_t}\right) y_{h.t} \tag{24}$$

where using the goods market clearing condition,

$$y_{h.t} = c_{h.t} + c_{h.t}^{*} + N_{E.t} k_{h.t} + N_{E.t}^{*} k_{h.t}^{*} \tag{25}$$

$c_{h.t}$ ($c_{h.t}^{*}$) and $k_{h.t}$ ($k_{h.t}^{*}$) are the consumption and capital demand from Home (Foreign) households. Using optimal demand finds in the previous section,, $y_{h.t}$ can be rewritten as

$$y_{h.t} = \rho_{h.t}^{-\sigma} \rho_{H.t}^{\sigma-\omega} \left[\alpha M_t + (1-\alpha) Q_t^{\omega} M_t^{*}\right] \tag{26}$$

where M_t (M_t^*) is the consumption and investment goods demand in each county:

$$M_t = C_t + N_{E.t}K_t \text{ and } M_t^* = C_t^* + N_{E.t}^* K_t^* \tag{27}$$

Note that using free entry condition (21) and factor demand (20), it is verified that $1-\theta$ fraction of real investment cost is paid as capital goods: $K_t = (1-\theta)\, x_{h.t}^s$ and $K_t^* = (1-\theta)\, x_{f.t}^{s*}$. When $\theta = 0$ M_t (M_t^*) coincides to the aggregated demand in each country.

Profit maximization gives:

$$\rho_{h.t} = \frac{\sigma}{\sigma - 1}\frac{w_t}{z_t} \tag{28}$$

Individual real price is the real marginal cost over markup. Because there is no transportation cost, the LOP holds for exported goods, $\rho_{h.t}^*$ denominated in Foreign consumption basket:

$$\rho_{h.t}^* = Q_t^{-1}\rho_{h.t} \tag{29}$$

Finally using the above optimal pricing the real dividend is expressed as:

$$d_{h.t} = \frac{1}{\sigma}\rho_{h.t}^{1-\sigma}\rho_{H.t}^{\sigma-\omega}\left[\alpha M_t + (1-\alpha)\, Q_t^{\omega} M_t^*\right] \tag{30}$$

2.3 Labor market clearing condition

Exogenously supplied labor is used in the production (intensive margin) and in the firm creation. Home labor market clearing condition gives:

$$1 = N_t l_t + N_{E:t} l_{EM.t} \tag{31}$$

Noting $y_{h.t} = (\sigma - 1)\, \frac{d_{h.t}}{w_t} z_t$ and $l_{EM.t} = \theta \frac{x_{h.t}^s}{wt}$the above Home labor market clearing condition can be written as:

$$1 = (\sigma - 1)\, \frac{N_t d_{h.t}}{w_t} + \theta \frac{N_{E:t} x_{h.t}^s}{wt} \tag{32}$$

Again observe with $\theta = 0$ no labor income arises from firm creation activity. The similar expression holds for Foreign.

2.4 Perfect risk sharing condition

At the end I impose the complete market condition. Under complete market, the marginal utility which stems from an additional nominal wealth is the same across the countries. This gives:

$$Q_t = \left(\frac{C_t^*}{C_t}\right)^{-\gamma} \tag{33}$$

An increase of Home consumption relative to Foreign should be associated with a real depreciation. This condition closes the model and provides the planners' solution combined with above equations. In what follows I solve the log linearized version of the model about the relative variables between Home and Foreign. Then I search the steady state portfolios which replicates the *locally* complete market.

3 Real exchange rate, relative dividend and relative wage

Before going to the exact linearized solution about relative variables, I discuss the behavior of real exchange rate and relative dividend (operational profit) with endogenous entry. I can express the real exchange rate variation as:

$$\mathsf{Q}_t = (2\alpha - 1)\,\rho_t^R + \frac{2\alpha - 1}{\sigma - 1}\mathsf{N}_t^R \tag{34}$$

where ρ_t^R is the terms of trade defined as:

$$\rho_t^R = \left(\mathsf{Q}_t + \rho_{f.t}^*\right) - \rho_{h.t}$$

$$\mathsf{N}_t^R = \mathsf{N}_t - \mathsf{N}_t^*$$

As usual, without home bias there is no variation of real exchange rate ($\alpha = \frac{1}{2}$). With home bias terms of trade appreciation for Home (a decrease of ρ_t^R) makes real exchange rate appreciation. However here the variation of the real exchange rate moves with the variation of the relative number of varieties. An increase of the number of relative available varieties makes real exchange rate depreciated with the elasticity $(2\alpha - 1)/(\sigma - 1)$, home bias weighted by the term which represents the marginal utility from variety under Dixit-Stiglitz preference, $\frac{1}{\sigma - 1}$. Note contrary to Coeurdacier, Kollmann and Martin (2007)'s "i-pod shock" the variety effect doesn't appears on impact following the shock hitting the economy. This is because N_t^R behaves as the state variable in this model. As we will see this has a very important implication for steady state portfolio choice.

Using the log linearized perfect risk sharing condition(33), the variation of relative dividend is expressed as (see appendix)

$$\mathsf{d}_t^R = (\lambda - 1)\,\rho_t^R - \frac{\sigma - \lambda}{\sigma - 1}\mathsf{N}_t^R + (2\alpha - 1)(1 - \theta)\,S_I^M\left(\mathsf{N}_{E.t}^R + \mathsf{x}_t^{sR}\right) \tag{35}$$

where

$$\lambda \equiv \omega\left[1 - (2\alpha - 1)^2\right] + (2\alpha - 1)^2\left[S_C^M\frac{1}{\gamma} + (1 - \theta)\,S_I^M\right]$$

$$S_I^M \equiv \frac{N_E x}{M} = \frac{\beta\delta}{1 - \beta(1 - \delta)}\frac{1}{\sigma}, \text{ and } S_C^M \equiv \frac{C}{M} = \frac{\sigma - 1}{\sigma} - (1 - \theta)\frac{\beta\delta}{1 - \beta(1 - \delta)}\frac{1}{\sigma} \tag{36}$$

The operational profits include the term which comes from the investment demand under $\theta \neq 1$. $\lambda > 0$ as $1 > \alpha > 1/2$. I suppose $\sigma > \omega$ (the elasticity among varieties is larger than that of between Home and Foreign goods). Roughly this implies $\sigma > \lambda$. As we expected under home bias profit decreases with terms of trade appreciation. Under $\sigma > \lambda$ an increase of relative number of Home originated varieties decreases the relative profits. This is because Home firms competes more closely with Home originated firms with the elasticity σ than that of imported varieties. Different from the competition effect due to a rise of the number of varieties which is generally captured as a rise of individual real price, this competition effect arises from the fact that the elasticities between Home and Foreign and among varieties in that country are different.

4 International portfolio with firm entry

4.1 Relative budget constraint

Financial market clearing conditions give $s_{h.t} + s^*_{h.t} = 1$ and $s_{f.t} + s^*_{f.t} = 1$ and that of bond market, $b_{h.t} + b^*_{h.t} = 0$ and $b_{f.t} + b^*_{f.t} = 0$ $\forall t$. I suppose there exist a steady state symmetric portfolios such as $s = s_{h.t} = s^*_{f.t}$ and $b = b_{h.t} = b^*_{f.t}$. s is the share of equity holding and b is the bond position denominated in the steady state consumption C. Using these relations the relative budget constraint can be written as:

$$\begin{aligned} C_t - Q_t C^*_t = w_t - Q w^*_t \\ + (2s-1)\left[\left(N_t d_{h.t} - N^*_t Q_t d^*_{f.t}\right) - \left(N_{E.t} x^s_{h.t} - N^*_{E.t} Q_t x^{s*}_{f.t}\right)\right] \\ + 2b\left(1 - Q_t\right) \end{aligned} \quad (37)$$

Then I log linearize the above budget constraint:

$$\mathsf{C}^R_t - \mathsf{Q}_t = \mathsf{S}_W \mathsf{w}^R_t + (2s-1)\left[\mathsf{S}_D\left(\mathsf{N}^R_t + \mathsf{d}^R_t\right) - \mathsf{S}_I\left(\mathsf{N}^R_{E.t} + \mathsf{x}^{sR}_t\right)\right] - 2b\mathsf{Q}_t \quad (38)$$

where $\mathsf{C}^R_t = \mathsf{C}_t - \mathsf{C}^*_t$. S_W (S_I, and S_D) is a symmetric steady state share of real labor income (real investment and real dividend) in total consumption. They are respectively defined as

$$S_W \equiv \frac{w}{C},\ S_D \equiv \frac{N d_h}{C} \text{ and } S_I \equiv \frac{N_E x_h}{C}$$

For detailed expression see the appendix. The above log linearized budget constraint says that the consumption difference measured in Home consumption basket is equal to the difference of labor income, asset returns (net of investment) and the relative bond returns, all measured in Home consumption. This is equivalent to say that the nominal consumption expenditure difference is equal to the nominal income differences which arises from above 3 sources.

4.2 Steady state portfolios

I search the optimal equity and bond position which replicate the Pareto optimal complete market allocation. Under complete market it should be:

$$\mathsf{C}_t^R - \mathsf{Q}_t = \left(\frac{1}{\gamma} - 1\right)\mathsf{Q}_t \tag{39}$$

Plugging this in (38)

$$\left(\frac{1}{\gamma} - 1\right)\mathsf{Q}_t = \mathsf{S}_W \mathsf{w}_t^R + (2s-1)\left[\mathsf{S}_D\left(\mathsf{N}_t^R + \mathsf{d}_t^R\right) - \mathsf{S}_I\left(\mathsf{N}_{E.t}^R + \mathsf{x}_t^{sR}\right)\right] - 2b\mathsf{Q}_t \tag{40}$$

The above expression says that the optimal portfolio should be constructed so that it realizes the allocation under complete market in whatever shocks which hit the economy. These "shocks" will turned out to be the projected fluctuations of terms of trade and investment shocks.

From (34)

$$\mathsf{Q}_t = (2\alpha - 1)\,\rho_t^R + \frac{2\alpha - 1}{\sigma - 1}\mathsf{N}_t^R \tag{41}$$

Using the log-linearised labor market clearing condition for both Home and Foreign,

$$\mathsf{w}_t^R = (\sigma - 1)\frac{S_D^M}{S_W^M}\left(\mathsf{N}_t^R + \mathsf{d}_t^R\right) + \theta\frac{S_I^M}{S_W^M}\left(\mathsf{N}_{E.t}^R + \mathsf{x}_t^{sR}\right) \tag{42}$$

With the relative dividends (35),

$$\mathsf{N}_t^R + \mathsf{d}_t^R = (\lambda - 1)\,\rho_t^R + (2\alpha - 1)(1-\theta)\,S_I^M\left(\mathsf{N}_{E.t}^R + \mathsf{x}_t^{sR}\right) \tag{43}$$

Finally noting on impact of the shock,

$$\mathsf{N}_t^R = 0 \tag{44}$$

[4]

Plugging these expressions the log linearized relative budget constraint (**??**) is expressed with terms of trade ρ_t^R and investment $\mathsf{N}_{E.t}^R + \mathsf{x}_t^{sR}$. It is easy to solve the optimal equity and bond portfolio for whatever realization of terms of trade and investment. These are given by (see Appendix for detailed):

$$s = \frac{1}{2} + \frac{1}{2}\frac{S_D^M(\sigma - 1)(2\alpha - 1)(1-\theta) + \theta}{1 - S_D^M(2\alpha - 1)(1-\theta)} > \frac{1}{2} \tag{45}$$

[4] This has an important consequance because the variety effect arises in the specificantion here only one period after (not on imact of the shock), the welfare based and obserbed real exchange rate coincide. As a result for portfolio choice the housholds consider only terms of trade risk. However this dosen't mean there is no effect from entry. The effect of endogenous entry (or investment) appears in genreal equiliburium as a higher labor demand; hence as a higher terms of trade appreciation, which should be hedged by portofolio.

$$b=\frac{1}{2}\left(1-\frac{1}{\gamma}\right)+\frac{1}{2}\left(\frac{\lambda-1}{2\alpha-1}\right)\frac{S_D\left(\sigma-1+\theta\right)}{1-S_D^M\left(2\alpha-1\right)\left(1-\theta\right)} \tag{46}$$

where

$$S_D^M=\frac{1}{\sigma},\text{ and } S_D=S_D^M\left[1+\left(1-\theta\right)\frac{S_I^M}{S_C^M}\right]$$

The optimal equity portfolio is home biased as it is found in Heathcote and Perri (2007), Coeurdacier, Kollmann and Martin (2007), Coeurdacier et al. (2008) and Coeurdacier and Gourinchas (2008) and Arespa (2008).The bond position is positive (long in local and short for Foreign bond) or negative (short in local bond and long in Foreign bond) depending on λ. And this is the same result as Coeurdacier, Kollmann and Martin (2008) and Coeurdacier and Gourinchas (2008)

Combined with Lucas diversification term, 1/2,.the home biased equity position appears because of the negative conditional covariance between relative labor income and relative mutual fund returns (net of investment). The mechanism is originally presented in Heathcote and Perri (2008) and developed in detail in Coeurdacier and Gourinchas (2008). With endogenous investment (here expressed in terms of new variety creation) relative returns net of investment from mutual fund are positive when relative investment fluctuations are negative, which decreases the relative wage. In such case home biased mutual fund position becomes a good hedge because it gives a positive income flow and stabilize the consumption. .

As relative bond returns are *perfectly* correlated with the real exchange rate in this setting, (no variety shock because the new entry takes place only one period after.) the real exchange rate (terms of trade) risk is totally hedged using bond (the first term for bond position). As a result no hedging motivation arises for equity from real exchange rate risk. To put another way there is no Baxter and Jermann (1997) terms which potentially would make foreign biased equity position. As pointed out in van-Wincoop and Warnock (2006), this is consistent with the empirical finding. For risk averse household ($\gamma>1$) facing such real exchange rate risk, it is optimal to be long for her local bond and short for Foreign because when her consumption basket becomes expensive such bond position generates positive income flow and vise versa. The second term in the bond portfolio is the hedge against the labor income risk arising from the terms of trade fluctuations. Suppose Home is hit by a shock which makes appreciated the terms of trade. When the elasticity of substitution between Home and Foreign goods, ω is sufficiently high ($\lambda>1$) relative operational profits decrease ($\mathsf{d}_t^R<0$) and relative wage decreases ($\mathsf{w}_t^R<0$) (See (42) and (43)). In such case knowing the terms of trade and nominal bond return is correlated perfectly being long in local bond is a good hedge because it gives a positive income flow induced by the appreciation of real exchange rate.. Contrary when Home is hit by a shock which makes depreciated the terms of trade being short in her local real bond is good hedge against this labor income risk.

Other than the home bias in consumption captured in α which makes changed the equity position, the originality of the optimal equity and bond position found in this paper is the redistributive parameter θ. Higher θ means higher fraction of labor is used in the firm setting up giving higher wage fluctuations for the same magnitude of terms of trade and investment shocks. As a result stronger home biased position for equity is required to stabilize the consumption. The sensitivity of bond position is always about the elasticity of substitution λ but bond position also changes with θ. I report the sensitivity of the portfolio to this parameter in Figure 1 and Figure 2 for a high and a low elasticity of substitution ($\omega = 2$ and $\omega = 0.5$).

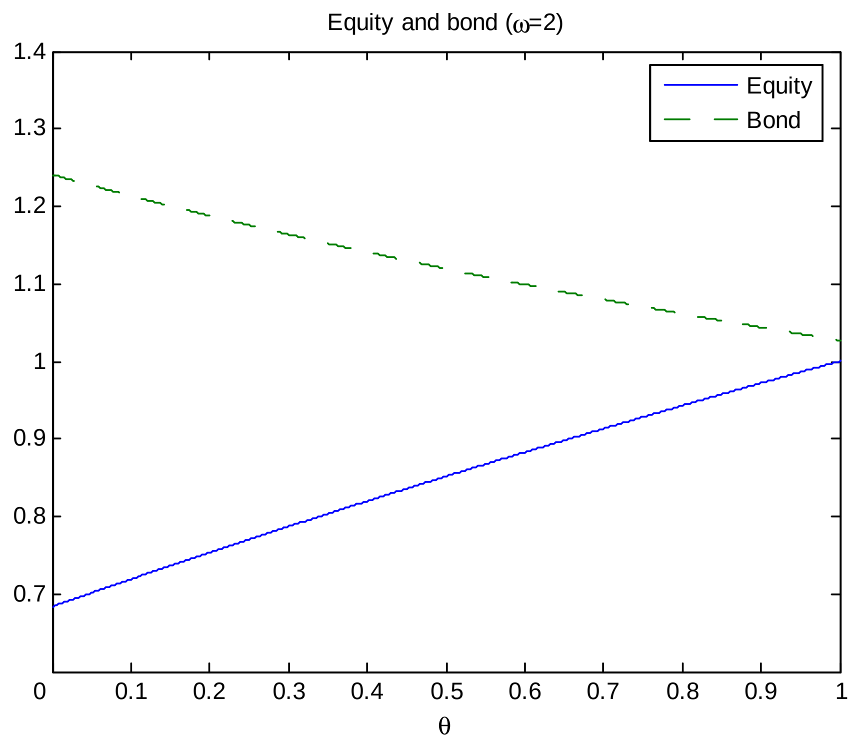

Figure 1

(47)

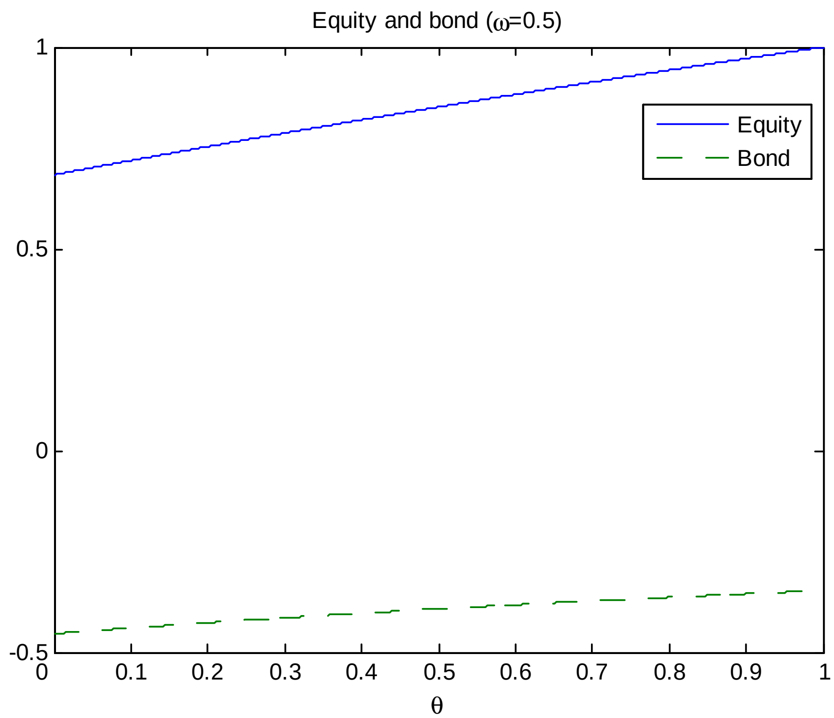

Figure 2. These are calibrated with $\gamma = 2$, $\beta = 0.99$, $\sigma = 3.8$, $\alpha = 0.72$ and $\delta = 0.1$.

$$ \tag{48} $$

In particular when $\theta = 0$ (only capital goods are needed as sunk entry cost)

$$s = \frac{1}{2} + \frac{1}{2}\frac{S_W^M (2\alpha - 1)}{1 - S_D^M (2\alpha - 1)} > \frac{1}{2} \tag{49}$$

$$b = \frac{1}{2}\left(1 - \frac{1}{\gamma}\right) + \frac{1}{2}\left(\frac{\lambda - 1}{2\alpha - 1}\right)\frac{S_W}{1 - S_D^M (2\alpha - 1)} \tag{50}$$

where I used $S_D^M (\sigma - 1) = S_W^M$ and $S_D (\sigma - 1) = S_W$ under $\theta = 0$. The above equity position is identical to Arespa (2008) where the investment takes place with capital goods.

When $\theta = 1$ (only labor is needed as sunk entry cost).

$$s = 1 \tag{51}$$

$$b = \frac{1}{2}\left(1 - \frac{1}{\gamma}\right) + \frac{1}{2}\left(\frac{\lambda - 1}{2\alpha - 1}\right) \tag{52}$$

where I used $S_D = 1/\sigma$. The complete home bias equity position appears. Long or short bond position depending on λ is amplified at the extreme. This is the same set of equity and bond portfolio found in Coeurdacier et Gourinchas (2008) for the case of redistributive shocks.

5 Conclusion

This paper seeks the international equity and bond portfolio in the environment where the investment takes place in terms of variety/firm creation. The stable equity home bias appears successfully because of negative conditional covariance between equity returns and labor income. The bond is used to hedge the observed real exchange rate fluctuations with which it is perfectly correlated and the labor income fluctuations which arises from the terms of trade fluctuations. The bond position could be long or short depending on the preference parameter, especially the elasticity of substitution between Home and Foreign goods. In addition the portfolio found in this paper incorporates the redistributive effect of income in the form of parameter.

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A The system

Price indices (or variety effect)

$$\alpha\rho_{H.t}^{1-\omega}+(1-\alpha)\,\rho_{F.t}^{1-\omega}=1 \tag{53}$$

$$\rho_{H.t}=N_t^{\frac{1}{1-\sigma}}\rho_{h.t}, \qquad \rho_{F.t}=N_t^{*\frac{1}{1-\sigma}}\rho_{f.t} \tag{54}$$

$$\alpha\rho_{F.t}^{*1-\omega}+(1-\alpha)\,\rho_{H.t}^{*1-\omega}=1 \tag{55}$$

$$\rho_{F.t}^{*}=N_t^{*\frac{1}{1-\sigma}}\rho_{f.t}^{*}, \qquad \rho_{H.t}^{*}=N_t^{\frac{1}{1-\sigma}}\rho_{h.t}^{*} \tag{56}$$

Pricing

$$\rho_{h.t}=\frac{\sigma}{\sigma-1}\frac{w_t}{z_t} \tag{57}$$

$$\rho_{f.t}^{*}=\frac{\sigma}{\sigma-1}\frac{w_t^{*}}{z_t^{*}} \tag{58}$$

Profits

$$d_{h.t}=\frac{1}{\sigma}\rho_{h.t}^{1-\sigma}\rho_{H.t}^{\sigma-\omega}\left[\alpha M_t+(1-\alpha)\,Q_t^{\omega}M_t^{*}\right] \tag{59}$$

$$d_{f.t}^{*}=\frac{1}{\sigma}\rho_{f.t}^{*1-\sigma}\rho_{F.t}^{*\sigma-\omega}\left[\alpha M_t^{*}+(1-\alpha)\,Q_t^{-\omega}M_t\right] \tag{60}$$

Aggregated demand

$$M_t=C_t+N_{E.t}K_t \tag{61}$$

$$M_t^{*}=C_t^{*}+N_{E.t}^{*}K_t^{*} \tag{62}$$

Capital share in the entry cost

$$K_t=(1-\theta)\,x_{h.t}^{s} \tag{63}$$

$$K_t^{*}=(1-\theta)\,x_{f.t}^{s*} \tag{64}$$

Free entry

$$x_{h.t}^{s}=\frac{f_E w_t^{\theta}}{z_{E.t}} \tag{65}$$

$$x_{f.t}^{s*}=\frac{f_E^{*}w_t^{*\theta}}{z_{E.t}^{*}} \tag{66}$$

Number of firms

$$N_t = (1-\delta)\left(N_{t-1} + N_{E.t-1}\right) \tag{67}$$

$$N_t^* = (1-\delta)\left(N_{t-1}^* + N_{E.t-1}^*\right) \tag{68}$$

Euler bonds

$$x_{h.t}^b = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(x_{h.t+1}^b + 1\right) \tag{69}$$

$$x_{f.t}^b = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(x_{f.t+1}^b + Q_t\right) \tag{70}$$

$$x_{h.t}^{b*} = \beta E_t \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}\left(x_{h.t+1}^{b*} + Q_t^{-1}\right) \tag{71}$$

$$x_{f.t}^{b*} = \beta E_t \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}\left(x_{f.t+1}^{b*} + 1\right) \tag{72}$$

Euler shares

$$x_{h.t}^s = \beta(1-\delta) E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(x_{h.t+1}^s + d_{h.t+1}\right) \tag{73}$$

$$x_{f.t}^s = \beta(1-\delta) E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(x_{f.t+1}^s + d_{f.t+1}\right) \tag{74}$$

$$x_{h.t}^{s*} = \beta(1-\delta) E_t \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}\left(x_{h.t+1}^{s*} + d_{h.t+1}^*\right) \tag{75}$$

$$x_{f.t}^{s*} = \beta(1-\delta) E_t \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}\left(x_{f.t+1}^{s*} + d_{f.t+1}^*\right) \tag{76}$$

LOP of asset, dividend and goods

$$x_{h.t}^{s*} = Q_t^{-1} x_{h.t}^s \qquad x_{f.t}^s = Q_t x_{f.t}^{s*} \tag{77}$$

$$x_{h.t}^{b*} = Q_t^{-1} x_{h.t}^b \qquad x_{f.t}^b = Q_t x_{f.t}^{b*} \tag{78}$$

$$d_{h.t}^* = Q_t^{-1} d_{h.t}, \qquad d_{f.t} = Q_t d_{f.t}^* \tag{79}$$

$$\rho_{h.t}^* = Q_t^{-1} \rho_{h.t}, \qquad \rho_{f.t} = Q_t \rho_{f.t}^* \tag{80}$$

Labor Market clear

$$1 = (\sigma - 1)\frac{N_t d_{h.t}}{w_t} + \theta \frac{N_{E:t} x_{h.t}^s}{w_t^*} \tag{81}$$

$$1 = (\sigma - 1)\frac{N_t^* d_{f.t}^*}{w_t^*} + \theta \frac{N_{E:t}^* x_{f.t}^{s*}}{w_t^*} \tag{82}$$

Perfect risk sharing condition

$$Q_t = \left(\frac{C_t^*}{C_t}\right)^{-\gamma} \tag{83}$$

B Steady state

The steady state values are expressed without time subscript. At the symmetric steady state, $Q = 1$, $p_h = p_h^* = p_f = p_f^*$, $N = N^*$ and $P_H = P_H^* = P_{F,} = P_F^*$.Then $\rho_H = \rho_H^* = \rho_F = \rho_F^* = 1$ and $\rho_h = \rho_h^* = \rho_f = \rho_f^* = N^{-1}$. Also $C = C^*$, $N_E = N_E^*$ and $K = K^*$. Then $M = M^*$. Also $d_h = d_f^*$ and $x_h = x_h^* = x_f = x_f^*$. First I discuss the steady state ratio relative to M. Using these conditions, the steady state share of real dividend relative to the demand addressed to each firm becomes:

$$S_D^M \equiv \frac{Nd_h}{M} = \frac{1}{\sigma}$$

With this condition and from the Euler share and the low of the motion of firm the steady state share of investment becomes:

$$S_I^M \equiv \frac{N_E x_h}{M} = \frac{\beta\delta}{1 - \beta(1 - \delta)}\frac{1}{\sigma}$$

Using the above 2 steady sate share, form the labor market clearing condition the steady state share of labor income relative to the aggregate demand becomes:

$$S_W^M \equiv \frac{w}{M} = S_D^M(\sigma - 1) + \theta S_I^M$$

Finally at the symmetric steady state, it must be $C + N_E x_h = w + Nd_h$ (aggregated demand=aggregated income), using this identity,

$$S_C^M \equiv \frac{C}{M} = S_W^M + S_D^M - S_I^M \tag{84}$$

Noting $M = C + (1 - \theta) N_E x_h$ using the above steady state ratios defined relative to M, the steady state ratios relative to the consumption becomes

$$S_I \equiv \frac{N_E x_h}{C} = \frac{S_I^M}{1 - (1 - \theta) S_I^M}$$

$$S_D \equiv \frac{Nd_h}{C} = S_D^M [1 + (1 - \theta) S_I] \tag{85}$$

and

$$S_W \equiv \frac{w}{C} = S_D(\sigma - 1) + \theta S_I$$

C Log-liner system

Price indices (or variety effect)

$$\alpha\rho_{H.t}+(1-\alpha)\,\rho_{F.t}=0 \tag{86}$$

$$\mathsf{N}_t+(1-\sigma)\,(\rho_{h.t}-\rho_{H.t})=0, \qquad \mathsf{N}_t^*+(1-\sigma)\left(\rho_{f.t}-\rho_{F.t}\right)=0, \tag{87}$$

$$\alpha\rho_{F.t}^*+(1-\alpha)\,\rho_{H.t}^*=0 \tag{88}$$

$$\mathsf{N}_t^*+(1-\sigma)\left(\rho_{f.t}^*-\rho_{F.t}^*\right)=0, \qquad \mathsf{N}_t+(1-\sigma)\,(\rho_{h.t}^*-\rho_{H.t}^*)=0, \tag{89}$$

Pricing

$$\rho_{h.t}=\mathsf{w}_t-\mathsf{z}_t \tag{90}$$

$$\rho_{f.t}^*=\mathsf{w}_t^*-\mathsf{z}_t^* \tag{91}$$

Dividends (operational profits)

$$\mathsf{d}_{h.t}=(1-\sigma)\,\rho_{h.t}+(\sigma-\omega)\,\rho_{H.t}+\alpha\mathsf{M}_t+(1-\alpha)\,(\omega\mathsf{Q}_t+\mathsf{M}_t^*) \tag{92}$$

$$\mathsf{d}_{f.t}^*=(1-\sigma)\,\rho_{f.t}^*+(\sigma-\omega)\,\rho_{F.t}^*+\alpha\mathsf{M}_t^*+(1-\alpha)\,(-\omega\mathsf{Q}_t+\mathsf{M}_t) \tag{93}$$

Aggregated demand

$$\mathsf{M}_t=\mathsf{S}_C^M\mathsf{C}_t+(1-\theta)\,\mathsf{S}_I^M\,(\mathsf{N}_{E.t}+\mathsf{x}_{h.t}^s) \tag{94}$$

$$M_t^*=S_C^M C_t^*+(1-\theta)\,S_I^M\left(\mathsf{N}_{E.t}^*+\mathsf{x}_{f.t}^{s*}\right) \tag{95}$$

Capital share in the entry cost

$$\mathsf{K}_t=\mathsf{x}_{h.t}^s \tag{96}$$

$$\mathsf{K}_t^*=\mathsf{x}_{f.t}^{s*} \tag{97}$$

Free entry

$$\mathsf{x}_{h.t}=\theta\mathsf{w}_t-\mathsf{z}_{E.t} \tag{98}$$

$$\mathsf{x}_{h.t}^*=\theta\mathsf{w}_t^*-\mathsf{z}_{E.t}^* \tag{99}$$

Number of firms

$$\mathsf{N}_{t+1} = (1-\delta)\,\mathsf{N}_t + \delta \mathsf{N}_{E.t} \tag{100}$$

$$\mathsf{N}^*_{t+1} = (1-\delta)\,\mathsf{N}^*_t + \delta \mathsf{N}^*_{E.t} \tag{101}$$

Euler bonds

$$\gamma\left(\mathsf{E}_t \mathsf{C}_{t+1} - \mathsf{C}_t\right) = \beta \mathsf{E}_t \mathsf{x}^b_{h.t+1} - \mathsf{x}^b_{h.t} \tag{102}$$

$$\gamma\left(\mathsf{E}_t \mathsf{C}^*_{t+1} - \mathsf{C}^*_t\right) = \beta \mathsf{E}_t \mathsf{x}^{b*}_{f.t+1} - \mathsf{x}^{b*}_{f.t} \tag{103}$$

Euler shares

$$\gamma\left(\mathsf{E}_t \mathsf{C}_{t+1} - \mathsf{C}_t\right) = \beta(1-\delta)\,\mathsf{E}_t \mathsf{x}^s_{h.t+1} - \mathsf{x}^s_{h.t} + \left[1 - \beta(1-\delta)\right] \mathsf{E}_t \mathsf{d}_{h.t+1} \tag{104}$$

$$\gamma\left(\mathsf{E}_t \mathsf{C}^*_{t+1} - \mathsf{C}^*_t\right) = \beta(1-\delta)\,\mathsf{E}_t \mathsf{x}^{s*}_{f.t+1} - \mathsf{x}^{s*}_{f.t} + \left[1 - \beta(1-\delta)\right] \mathsf{E}_t \mathsf{d}^*_{f.t+1} \tag{105}$$

LOP of asset, dividend and goods

$$\mathsf{x}^{s*}_{h.t} = \mathsf{x}^s_{h.t} - \mathsf{Q}_t \qquad \mathsf{x}^s_{f.t} = \mathsf{Q}_t + \mathsf{x}^{s*}_{f.t} \tag{106}$$

$$\mathsf{x}^{b*}_{h.t} = \mathsf{x}^b_{h.t} - \mathsf{Q}_t \qquad \mathsf{x}^b_{f.t} = \mathsf{Q}_t + \mathsf{x}^{b*}_{f.t} \tag{107}$$

$$\mathsf{d}^*_{h.t} = \mathsf{d}_{h.t} - \mathsf{Q}_t, \qquad \mathsf{d}_{f.t} = \mathsf{Q}_t + \mathsf{d}^*_{f.t} \tag{108}$$

$$\rho^*_{h.t} = \rho_{h.t} - \mathsf{Q}_t, \qquad \rho_{f.t} = \mathsf{Q}_t + \rho^*_{f.t} \tag{109}$$

Labor Market clear

$$S^M_W \mathsf{w}_t = (\sigma - 1)\, S^M_D \left(\mathsf{N}_t + \mathsf{d}_{h.t}\right) + \theta S^M_I \left(\mathsf{N}_{E.t} + \mathsf{x}^{s*}_{h.t}\right) \tag{110}$$

$$S^M_W \mathsf{w}^*_t = (\sigma - 1)\, S^M_D \left(\mathsf{N}_t + \mathsf{d}^*_{f.t}\right) + \theta S^M_I \left(\mathsf{N}^*_{E.t} + \mathsf{x}^{s*}_{f.t}\right) \tag{111}$$

Perfect risk sharing condition

$$\mathsf{Q}_t = \gamma\left(C_t - C^*_t\right) \tag{112}$$

D Relative variations

Definition of relative variables are following :

$$\mathsf{z}_t^R = \mathsf{z}_t - \mathsf{z}_t^*$$

$$\mathsf{C}_t^R = \mathsf{C}_t - \mathsf{C}_t^*$$

$$\mathsf{N}_t^R = \mathsf{N}_t - \mathsf{N}_t^*$$

$$\mathsf{N}_{E.t}^R = \mathsf{N}_{E.t} - \mathsf{N}_{E.t}^*$$

$$\mathsf{x}_t^{sR} = \mathsf{x}_{h.t}^s - \mathsf{x}_{f.t}^s = \mathsf{x}_{h.t}^s - \left(\mathsf{Q}_t + \mathsf{x}_{f.t}^{s*}\right)$$

$$\mathsf{d}_t^R = \mathsf{d}_{h.t} - \mathsf{d}_{f.t} = \mathsf{d}_{h.t} - \left(\mathsf{Q}_t + \mathsf{d}_{f.t}^*\right)$$

$$\mathsf{w}_t^R = \mathsf{w}_t - \left(\mathsf{Q}_t + \mathsf{w}_t^*\right)$$

Note in particular for relative dividends using (92) and (93) the relative dividends are expressed as:

$$\mathsf{d}_t^R = (\omega - 1)\,\rho_t^R - \frac{\sigma - \omega}{\sigma - 1}\mathsf{N}_t^R + (2\alpha - 1)\left(M_t - M_t^*\right) - (2\alpha - 1)(\omega - 1)\,Q_t \quad (113)$$

Plugging the variations of M_t and M_t^* this is further developed as

$$\begin{aligned}\mathsf{d}_t^R &= (\omega - 1)\,\rho_t^R - \frac{\sigma - \omega}{\sigma - 1}\mathsf{N}_t^R \\ &+ (2\alpha - 1)\left[S_C^M\left(\mathsf{C}_t^R - \mathsf{Q}_t\right) + (1-\theta)\,S_I^M\left(\mathsf{N}_{E.t}^R + \mathsf{x}_t^{sR}\right)\right] + (2\alpha - 1)\left[S_C^M + (1-\theta)\,S_I^M - \omega\right]Q_t\end{aligned} \quad (114)$$

Finally using the complete market condition (39) and the real exchange rate variation (34) it becomes (35).

E The optimal portfolio

Plugging the terms,

$$\begin{aligned}\left(\frac{1}{\gamma} - 1\right)(2\alpha - 1)\,\rho_t^R = \mathsf{S}_W &\left[(\sigma - 1)\,\frac{S_D^M}{S_W^M}\left(\mathsf{N}_t^R + \mathsf{d}_t^R\right) + \theta\,\frac{S_I^M}{S_W^M}\left(\mathsf{N}_{E.t}^R + \mathsf{x}_t^{sR}\right)\right] \\ &+ (2s - 1)\left[\mathsf{S}_D\left(\mathsf{N}_t^R + \mathsf{d}_t^R\right) - \mathsf{S}_I\left(\mathsf{N}_{E.t}^R + \mathsf{x}_t^{sR}\right)\right] - 2b\,(2\alpha - 1)\,\rho_t^R\end{aligned} \quad (115)$$

Noting that $\mathsf{N}_t^R = 0$ and also plugging the following relative dividends,

$$\mathsf{N}_t^R + \mathsf{d}_t^R = (\lambda - 1)\,\rho_t^R + (2\alpha - 1)\,(1 - \theta)\,S_I^M\left(\mathsf{N}_{E.t}^R + \mathsf{x}_t^{sR}\right) \tag{116}$$

The relative budget constraint becomes the function of terms of trade ρ_t^R and investment risk $\mathsf{N}_{E.t}^R + \mathsf{x}_t^{sR}$. Regrouping the terms then s and b are found by solving,

$$(2\alpha - 1)\,(1 - \theta)\,S_I^M\left[\mathsf{S}_W\,(\sigma - 1)\,\frac{S_D^M}{S_W^M} + (2s - 1)\,\mathsf{S}_D\right] + \theta\mathsf{S}_W\,\frac{S_I^M}{S_W^M} - (2s - 1)\,\mathsf{S}_I = 0 \tag{117}$$

$$\left(\frac{1}{\gamma} - 1\right)(2\alpha - 1) + 2b\,(2\alpha - 1) - (\lambda - 1)\left[\mathsf{S}_W\,(\sigma - 1)\,\frac{S_D^M}{S_W^M} + (2s - 1)\,\mathsf{S}_D\right] = 0 \tag{118}$$