Market based CEO pay and stock price informativeness*

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Abstract

This paper examines how the information efficiency of the stock market affects the design of market based CEO pay. We show that the benefit and cost of linking CEO compensation to the stock price are inseparable. Due to the impossibility of informationally efficient markets, the stock price contains useful information for incentive contracting only if it contains noise that is unrelated to the CEO's actions. Contrary to the existing literature, we find that more noise trading leads to less market based CEO pay since it decreases the informativeness of the stock price. Despite the negative effect of noise traders, a more liquid company stock results in more market based pay. This is consistent with empirical evidence. Finally, we show that short-term trading weakens CEO incentives since it makes the stock market a worse aggregator of dispersed private information about the future value of the firm.

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1 Introduction

It is widely believed that giving managers stock-based incentive pay has the advantage of using the information content of stock prices.[1] The stock price contains information about the future value as decentralized trading aggregates the dispersed private information of a large number of speculators. According to Holmström (1979)'s "informativeness principle", the stock price therefore contains useful information for incentive contracting and should be included in CEO pay. An implication of this belief is that more informative stock prices should lead to more market-based pay.

But the argument is not entirely obvious due to the impossibility of informationally efficient capital markets (see Grossman and Stiglitz (1980)). It is indeed well known that information based trading is impossible without noise trading, e.g. stochastic life cycle motives, liquidity needs or the need to fulfill margin calls. This suggests that the stock price should not be included in a manager's incentive contract as it would expose him to shocks that are unrelated to the incentive problem.

The contribution of this paper is to explore the implications of the the impossibility of informationally efficient markets for the design of market based CEO pay. Although the issue of information aggregation in stock markets in itself has been widely studied (starting with Grossman (1976), Hellwig (1980) and Diamond and Verrecchia (1981)), to our knowledge it has not yet been applied to executive compensation. Holmström and Tirole (1993) study the role of the stock market as an indirect monitor of managerial performance. Contrary to them we find that more noise or liquidity trading always leads to less market-based compensation since it always reduces the information content of the stock price. Although more noise-trading reduces the adverse selection problem of trading on private information, this positive effect on the information content of the stock price for incentive contracting is outweighed by the negative effect of simply adding noise to the stock price.

[1] See for example Murphy (1999) for a survey of the literature on executive compensation.

Neither effect is present Holmström and Tirole (1993) who instead of examining inefficient information aggregation, show how noise trading motivates a single speculator to collect costly additional information about management's performance.[2] More noise trading allows him to make larger profits on his information that offset the cost of collecting it. When we expand our model of information aggregation to allow speculators to collect costly information, we confirm that more noise trading always leads to less market based CEO pay. While more noise trading indeed increases the marginal return on private information and hence leads to collecting more information, it is an indirect effect that is not strong enough to overturn the negative effect of noise trading.

outweighed by the direct consequence of adding noise to the stock price.

Our paper clarifies the role of liquidity, taken here to be the resilience of the stock price to shocks in the order flow (i.e. the inverse of Kyle (1985)'s lambda). More liquidity means that informed speculators trade more with each other leading to more market-based CEO pay *ceteris paribus*. But liquidity is an endogenous variable that depends on the amount of information vs. noise trading. While more noise trading leads to more liquidity, its overall effect on market based incentives is, as argued above, negative. To our knowledge, the explicit link of the optimal design of CEO pay with liquidity (Kyle's lambda) and stock price informativeness (the variance of the future value of the firm conditional on the present stock price) and the corresponding comparative statics are new.

The predictions of our analysis are consistent with recent empirical research and provide a theoretical background for it. Kang and Liu (2005) establish a significant positive cross-sectional link between the extent of market-based CEO pay and stock price informativeness for a sample of publicly traded US corporations. Garvey and Swan (2002) show a similar positive link between market-based pay and liquidity. At the same time Hartzell and Starks (2003)

[2]In an earlier paper without trading, Diamond and Verrecchia (1982) examine the use of stock prices in CEO pay to filter out the noise that garbles the impact of CEO actions on final firm value. In their model, all investors receive the same signal and the stock price perfectly reveals the common signal. Since this noise must be filtered out, optimal CEO pay in their model depends *negatively* on the company's stock price.

show that the link between the stock market and CEO compensation in the US is unlikely to be one envisaged by Holmström and Tirole (1993). They find that large dominant traders such as institutional investors act not as indirect monitors via speculative trading on their private information but appear to influence CEO pay directly, e.g. through shareholder activism. We instead view the stock market not as a monitor of CEOs but as a communication device where decentralized self-interested trading leads to the aggregation and dissemination of dispersed information about their actions.

Having explored the consequences of imperfect information aggregation via competitive stock trading for the design of CEO pay, we extend our model to analyze the role of speculators' trading horizons. It is often argued that a disadvantage of well developed stock markets, in particular the US one, is their encouragement of short-termism (see for example Froot et al. (1992)). There are different mechanisms through which short-termism in capital markets may adversely affect management and thus have real economic costs, e.g. take-over threats (Stein (1988)) or the mispricing of corporate assets (Shleifer and Vishny (1990)). Based on the tension between information and noise trading, this paper argues that short trading horizons reduce the information content of the stock price. Speculators with short horizons act less on their private information about the future value of the firm as the stock price reflects this information only imperfectly at the time they need to close their position. As a consequence market based incentives weaken leading to more managerial moral-hazard and worse firm performance on average.

The central message of our paper is that market based CEO pay is inevitably costly due to impossibility of informationally efficient markets. The strength of the stock market as an impartial aggregator of dispersed information about the value of the firm and hence management's performance is inextricably linked to the weakness of eliciting this information through self-interested decentralized trading.

Several papers have examined other costs of market based pay. One cost arises from the difference between information that is useful for trading and information that is useful for evaluating a CEO's actions (Paul (1992)). For example, a trader would care about future exogenous shocks to firm value, e.g. the possible entry of other firms, that are irrelevant to the evaluation of past managerial performance. A similar cost occurs when the CEO must be given incentives to perform several tasks. The stock prices only conveys information about the total value of the firm but not necessarily the value-added of the manager for each task.

Kim and Suh (1993) point to a measurement problem when examining market based CEO pay. They argue that using the "raw" price to construct market measures is problematic since the stock price impounds public information from earnings reports in addition to private information. As a result the information content of stock prices about management performance may be overstated.[3]

If one of management's activities can be the exaggeration of performance, then Goldman and Slezak (2006) show how stock based performance contracts induce CEOs to waste resources by manipulating the information transmitted to investors. Bolton et al. (2006) also take up the multi-tasking issue and ask: what if the market is inefficient so that the stock price no longer reflects the expected long-run fundamental value of a firm? In that case, a CEO has an incentive to wastefully increase the risk of his firm to play up the speculative component of the stock price.[4]

We show that even without any of these costs, in a setting where aggregate stock market information, i.e. the sum of all informed traders signals, perfectly reveals the future value of the firm and is sufficient statistic for CEO effort, his pay should not be entirely market based.

The organization of the paper is as follows. Section 2 presents the set-up of the model. Section 3 explains the information and incentive structure of the model. Section 4 introduces trading and section 5 relates the extent of market based pay to price informativeness, liquidity

[3]Bushman and Injejikian (1993) combine the arguments of Kim and Suh (1993) and Paul (1992) within a single framework.

[4]An alternative view of CEO pay is offered by Bertrand and Mullainathan (2001) and Bebchuk and Fried (2004) who see it as the outcome of a lack of board supervision and of abuses of managerial power.

and the fundamental parameters of the model. Section 6 examines the robustness of our results when speculators can collect costly information. Section 7 extends the model to allow for short-termism in the stock market. Section 8 discusses empirical implications and section 9 concludes. All formal proofs are in the appendix.

2 The set-up of the model

The model assumes a standard moral-hazard problem between the owners and the management of a publicly traded firm as in Holmström and Tirole (1993). We introduce active trading of the firm's shares in a large competitive market where speculators have heterogenous, dispersed and imperfect information about the future value of the firm. Rational speculators' self-interested trading leads to an aggregation of information in the stock price that may be useful for incentivizing management.

A publicly traded firm is run by a risk-averse manager (the agent) whose unobservable effort drives the expected value of the firm. A collective of risk-neutral inside owners (the principal) owns the firm. They are value oriented investors in the sense that they hold the firm's shares until the firm is liquidated. The company stock is traded by a continuum of informed speculators, each of whom possesses different imperfect information about the future value of the firm. Moreover, there are traders who trade for reasons that are not related to any information about the firm, i.e. noise (or liquidity) traders. Finally, there is a market making sector that ensures that the stock price will be efficient and reflects all publicly available information.

The sequence of events is as follows. First, the principal hires a manager to run the firm and signs an incentive contract with him. Second, the manager exerts an unobservable effort e that determines the expected future value of the firm, $v = e + \theta$, where $\theta \sim N(0, \sigma_\theta^2)$. The shock θ garbles the impact of managerial effort on firm value. Better information about θ allows to give better incentives to management. Third, each speculator privately receives different imperfect information about the future value of the firm $s_i = v + \varepsilon_i$, where ε_i are i.i.d. random variables, $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$. Each speculator's information is costless and possibly only a very imprecise estimate of future value if ε_i is large. In our set-up there is no difference between information that is useful for trading and information that is useful for incentive contracting (see Paul (1992)). Speculators value information about shocks final asset value while incentive contracting values information about the shock that garbles the impact of effort on the value of the firm. Both shocks are identical here and are given by θ. Trading then results in a share price p. Fourth, the manager quits the firm and is paid according to his incentive contract. The assumption that the manager quits the firm before the final value v is realized captures the problem of rewarding him for decisions whose impact may take a long time to show results.

The manager's income contains a fixed wage, a market based element contingent on the stock price p and a non-market based element contingent on a non-price signal y. The signal y is available at the moment the manager quits the firm and contains unbiased but noisy information about the future value of the firm, e.g. accounting information: $y = v + \eta$, where $\eta \sim N(0, \sigma_\eta^2)$. The manager's total income I therefore is:[5]

$$I = a_0 + a_p p + a_y y \tag{1}$$

Finally, the value of the firm v realizes and the firm is liquidated at a net value π. The difference between the value of the firm v and its liquidation value π results from the cost of compensating the manager.

The manager's preferences are represented by a CARA utility function defined over income minus the (monetary) cost of effort: $U_m(e) = -\exp[-r_m(I - \frac{1}{2}e^2)]$, where r_m is the coefficient of constant absolute risk-aversion for the manager.

[5]We conform to the standard practice that the contract is linear in the signals.

Owners choose an incentive contract (a_0, a_p, a_y) that maximizes their expected wealth,

$$\max_{a_0,a_p,a_y} E[v - I] \tag{2}$$

subject to the manager acting in his own interest,

$$e = \arg\max_{e'} E[U_m(e')] \tag{3}$$

and subject to the manager's participation constraint:

$$E[U_m(e)] \geq 0 \tag{4}$$

where we have normalized the manager's outside opportunity to zero.

The aggregation of dispersed information via the trading of the firm's shares occurs in a standard noisy rational expectations market (see for example Hellwig (1980) or Diamond and Verrecchia (1981) - we follow the formulation in Vives (1995)). A speculator i maximizes the expected CARA utility of his return from buying x_i shares of the company stock at a price p:

$$U_i(x_i) = -\exp[-rx_i(\pi - p)] \tag{5}$$

where r is a speculator's coefficient of constant absolute risk aversion.

Speculators have rational expectations, i.e. they use all information available to them. Speculators realise that the stock price contains other speculators information about the future value of the firm and they therefore condition their trading not only on their private signal s_i but also on the publicly observable price p. A speculator's strategy therefore maps his private information s_i into a demand function $x_i(s_i, p)$.

In addition to speculators whose motive for trade is information, there are traders who trade the company stock for reasons that are unrelated to information about the asset. Examples are stochastic life cycle motives, margin calls or requirements for investors to hold certain assets in fixed proportions. Their demand u is assumed to be random according to $u \sim N(0, \sigma_u^2)$ and independent of all other random variables in the model. Such noise trading is necessary to solve the no-trade problem.[6]

The stock price is determined by a competitive risk neutral market making sector. It observes the aggregate limit order book, i.e. the joint demand caused by information and noise trading,

$$L(p) = \int_0^1 x_i(s_i, p)di + u \tag{6}$$

and sets the price efficiently conditional on public information:

$$p = E[\pi | L(p)] \tag{7}$$

The sequence of events is summarized in figure 1.

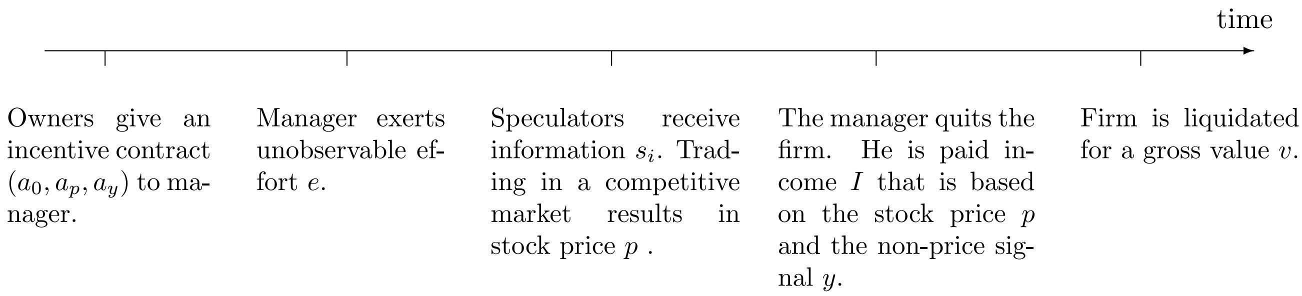

Figure 1: The timing of events

[6]See for example Dow and Gorton (2006) for a survey of the issues related to noise trading.

3 Incentives and information

This section explores the incentive and information structure of the model. The manager's problem in equation (3) is equivalent to:

$$e = \arg\max_{e'} E[I] - \frac{r_m}{2} Var[I] - \frac{1}{2} e'^2 \tag{8}$$

The first-order condition characterizing optimal managerial effort is:

$$e = a_p \frac{\partial E[p]}{\partial e} + a_y \tag{9}$$

For the moment we use the general notation $E[p]$ indicating that the market price reflects the speculators' inference process about the value of the firm v given their signals s_i, the information in the equilibrium price p and the amount of noise trading u.

The cheapest way to induce effort for the principal is to minimize the income risk borne by the risk-averse manager. An optimal contract must therefore choose a_p and a_y to minimize the variance of managerial income, $Var[I]$, subject to effort being optimal for the manager (equation (9)). The first-order condition of this optimization program can be written as:

$$\frac{a_p}{a_y} = \frac{Var[p] - Cov[p,y]\frac{\partial E[p]}{\partial e}}{Var[y]\frac{\partial E[p]}{\partial e} - Cov[p,y]} \tag{10}$$

Condition (10) characterizes the extent of market-based relative to non-market based pay, a_p/a_y, in terms of the information structure of the model only.

To illustrate the condition, let us assume for a moment that instead of the stock price p there is second signal $l = mv + \zeta$, with $\zeta \sim N(0, \sigma_\zeta^2)$, in addition to y. The relative weight given to the two signals in the manager's incentive contract then is given by $a_l/a_y = (m/\sigma_\zeta^2)/(1/\sigma_\eta^2)$, i.e. it depends only on the signal-to-noise ratio of the two signals l and y.[7]

[7]See also Lambert and Larcker (1987) who point out that focusing on the relative weights placed on

In our set-up the market as a whole has perfect advance information about the future value of the firm and thus managerial performance since the speculators' individual errors ε_i cancel out on average, $\int s_i di = v$.[8] If the incentive contract could include aggregate market information, $\int s_i di$ then the contract should not use any other noisy information about future firm value, e.g. y representing for example accounting information, as this would only add extra noise to the manager's pay. Replacing p with $\int s_i di$ in equation (10) implies that $a_y = 0$, showing that aggregate market information is a sufficient statistics for effort (see Holmström (1979)). But an incentive contract cannot include the individual signals of all the speculators operating in the stock market. Instead, it can only include the stock price that is the outcome of decentralized self-interested trading, which we describe next.

4 Trading

A speculator's demand x_i that maximizes the expected CARA utility in (5) is given by the following standard condition:

$$x_i(s_i, p) = \frac{E[\pi - p|s_i, p]}{rVar[\pi - p|s_i, p]} \tag{11}$$

We follow Holmström and Tirole (1993) and normalize the price and the incentive contract in order to separate the trading and the incentive problem. The manager is paid $a_0 + a_p p$ in cash and the amount $a_y y$ is paid in shares transferred from long-term inside owners to the manager. This accounting convention leaves payoffs unchanged and the net liquidation value of the firm is $\pi = v - a_0 - a_p p$. The fraction of shares α that must be transferred is given by $a_y y = \alpha E[v - a_0 - a_p p|y, p]$ since this is the fair price of the firm's shares given public

performance measures has the advantage it is independent of confounding factors such as CEO risk aversion, their outside opportunities or disutility of effort, all of which are difficult to measure empirically.

[8] Note that this is not the case in Diamond and Verrecchia (1982) and Holmström and Tirole (1993) where the stock market provides *additional* information about the shock that garbles the impact of effort on firm value.

information at the moment the manager leaves.

Letting $\hat{p}$ be the normalized share price[9]

$$\hat{p} = a_0 + (1 + a_p)p \tag{12}$$

we can write the manager's income as follows:

Proposition 1 *Managerial income is linear in the normalized price $\hat{p}$ and the signal y.*

$$I = \hat{a}_0 + \hat{a}_{\hat{p}}\hat{p} + \hat{a}_y y \tag{13}$$

where $\hat{a}_0 = \frac{(1-\alpha)a_p}{1+a_p}a_0$, $\hat{a}_y = \alpha\frac{\tau_\eta}{\tau_\eta+\tau}$ *and* $\hat{a}_{\hat{p}} = 1 - \hat{a}_y$.

The efficient pricing of shares (7) becomes $\hat{p} = E[v|L(\hat{p})]$ and a speculator's demand (11) now is:

$$x_i(s_i, \hat{p}) = \frac{E[v|s_i, \hat{p}] - \hat{p}}{rVar[v|s_i, \hat{p}]} \tag{14}$$

Both pricing and speculators' trading are now in terms of the gross liquidation value v rather than the net value π.

The standard linear-normal framework admits linear equilibria so we write speculators' demand as:

$$x_i(s_i, \hat{p}) = \beta s_i + f(\hat{p}) \tag{15}$$

where β is the trading intensity of an informed trader on his private information and $f(\hat{p})$ is a linear function of the price.

The aggregate limit order book then is:

$$\begin{aligned} L(\hat{p}) = \int_0^1 x_i(s_i, \hat{p})di + u &= \beta(e + \theta) + u + f(\hat{p}) \\ &= z + f(\hat{p}) \end{aligned}$$

[9]The prices p and $\hat{p}$ are informationally equivalent.

where $z = \beta(e + \theta) + u$ is the part of the aggregate limit order book that is informative about the value of the firm v. The price setting condition $\hat{p} = E[v|L(\hat{p})]$ can therefore be written as $\hat{p} = E[v|z]$.

Optimal trading and efficient pricing yield the following proposition:

Proposition 2 *The stock price $\hat{p} = E[v|z]$ is given by:*

$$\hat{p} = (1 - \lambda\beta)e^* + \lambda\beta(e + \theta) + \lambda u \tag{16}$$

where e^* is the hypothesized equilibrium effort, e is the actual effort and

$$\lambda = \frac{\beta\tau_u}{\tau} \tag{17}$$

$$\beta = \frac{\tau_\varepsilon}{r} \tag{18}$$

where $\tau_j = 1/\sigma_j^2$ denotes the precision of random variable j and $\tau = Var[v|\hat{p}]^{-1} = \beta^2\tau_u + \tau_\theta$ is the informativeness of the equilibrium price.

In a rational expectations equilibrium the actual effort e that determines the posterior expectation of firm value via the speculators' information and the hypothesized equilibrium effort e^* that determines the prior expectation must coincide. The equilibrium price therefore is

$$\hat{p} = e^* + \lambda\beta\theta + \lambda u \tag{19}$$

The share price is affected by two random shocks, one that is useful for incentive contracting while the other is not. The first shock is due to information trading by speculators and provides information about θ, the noise that garbles the impact of managerial effort on firm value. The other shock is due to noise trading u which is unrelated to the moral hazard problem and should therefore not affect the manager's pay. But without noise trading, there cannot be any information trading as the willingness to sell by one informed trader signals unfavorable information about the asset to other informed traders deterring them from buying (this is a version of Milgrom and Stokey (1982)'s no-trade result).[10]

5 Market based compensation and price informativeness

Combining the condition on optimal incentives (equation (10)) with the stock price as an aggregator of dispersed private and imperfect information (proposition 2), we now state the following result:

Proposition 3 *The ratio of market based to non-market based compensation is given by:*

$$\frac{\hat{a}_{\hat{p}}}{\hat{a}_y} = \frac{\beta}{\lambda}\frac{\tau_u}{\tau_\eta} = \frac{\tau}{\tau_\eta} \tag{20}$$

The relative weight on market based pay increases if i) speculators have better information (lower σ_ε^2), ii) speculators are less risk averse (lower r), iii) the non-price signal y is less precise (higher σ_η^2), iv) future firm value is less volatile (lower σ_θ^2) and v) there is less noise (lower σ_u^2).

Proposition 3 shows that the ratio of market based compensation relative to non-market based compensation is given by the ratio of price informativeness $\tau = Var[v|\hat{p}]^{-1}$ to the precision of non-market information τ_η. This confirms that executive compensation contracts should emphasize stock prices when their information content is high (holding the information content of the non-price signal y, e.g. accounting information, constant).

The proposition also shows that the ratio of market-based to non-market-based pay can be written as the ratio of the precision of the shock u (due to noise trading) and the precision of

[10] If $\sigma_u^2 = 0$ then $\lambda\beta = 1$ and the price is $\hat{p} = v = e + \theta$. The price then provides more accurate information about the value of the firm than any individual speculator's signal s_i. Speculators would disregard their own signals and only use the information conveyed by the price. But this begs the question of how information can flow into the price in the first place.

the shock η (that affects the signal y), times a "market factor" β/λ. The market factor is the aggressiveness β with which speculators trade on their private information times the liquidity of the market λ^{-1}.[11] The extent of market-based pay is thus proportional to the liquidity of the market ceteris paribus. In line with Bagehot (1971)'s classic intuition that a liquid market is one in which informed traders trade more with each other, liquidity leads to a better aggregation of dispersed and heterogenous information about the future value of the firm and thus about the consequences of managerial actions.

But liquidity λ^{-1} and stock price informativeness τ are endogenous and vary with the parameters of the model. The comparative statics of the ratio of market-based to non-market based pay with respect to the precision of speculators' private information ε, their risk aversion r and the accuracy of the signal y are straightforward.

Larger shocks θ that garble the impact of effort on firm value lead to less market-based pay. The variance of θ does not affect the relative weights given to signals in a setting without market-based pay since it does not change the signal-to-noise ratios (see the discussion following equation (10)). But with information aggregation via trading, the shock θ affects the volatility of the traded asset, makes it more difficult to trade on information (in the sense that it increases the adverse selection among speculators and market makers) and therefore lowers the liquidity of the market for the firm's shares.

The negative effect of noise trading u on market-based pay is noteworthy. Noise trading by itself has a negative effect on incentives as it adds noise to the stock price that is unrelated to the moral-hazard problem. There is also a positive effect as noise trading solves the no-trade problem, allows information trading and thus increases the liquidity of the market. But the positive effect is only indirect. More noise trading always reduces the informativeness of the stock price τ. This stands in contrast to the positive effect of noise trading on market-based pay shown in Holmström and Tirole (1993). We examine this difference in more detail in the

[11] Kyle (1985) introduced the inverse of the resilience of the price to order shocks, λ^{-1}, as an intuitive measure of market liquidity.

next section.

The following corollary confirms the real effect of trading conditions in the stock market on firm value via CEO effort.

Corollary 1 *CEO effort is given by:*

$$e = [1 + r(\tau_\theta^{-1} + (\tau_\eta + \tau - \tau_\theta)^{-1})]^{-1} \tag{21}$$

CEO effort increases when i) there is less noise trading (lower σ_u^2), ii) a more precise non-price signal (lower σ_η^2), iii) a lower volatility of final firm value (lower σ_θ^2), iv) speculators have better information (lower σ_ε^2) and v) they are less risk averse (lower r).

The manager exerts more effort, and thus increases expected firm value, when the stock price is more informative ceteris paribus.

6 Costly collection of information

Our negative result on the impact of noise trading on market-based incentives stands in contrast to the positive result of Holmström and Tirole (1993). In their model, noise trading has neither a direct effect on the noise of market-based pay nor does it solve the no-trade problem since there is only a single informed trader. Instead, noise trading allows to recoup the cost of collecting better information. Noise trading impacts on market-based incentives only indirectly via the improved quality of the informed trader's information, i.e. lower σ_ε^2 in our set-up. This indirect positive channel of noise trading is not present in our set-up so far since there is no cost of collecting better information - each of our speculators is endowed with a different imperfect piece of information about the future value of the firm.

This section explores the impact of Holmström and Tirole (1993)'s costly information collection channel in the context of our model. Suppose that before trading a speculator can improve the precision of his information about the future value of the firm τ_ε at a cost $c(\tau_\varepsilon) = k\tau_\varepsilon$. The speculator weighs the cost of better information against its benefit of leading to higher trading revenues. The "interim" revenue from trading conditional on observing the signal s_i and the stock price is:

$$\begin{aligned}
\Pi(s_i, \hat{p}) &= x_i(s_i, \hat{p}) E[v - \hat{p} | s_i, \hat{p}] \\
&= \frac{\tau_\varepsilon}{r}(s_i - \hat{p}) \left(\frac{\tau_\varepsilon s_i + \tau \hat{p}}{\tau_\varepsilon + \tau} \right) \\
&= \frac{\tau_\varepsilon^2}{r(\tau_\varepsilon + \tau)} (s_i - \hat{p})^2
\end{aligned}$$

The "ex-ante" revenue from trading then is:

$$\begin{aligned}
E[\Pi(s_i, \hat{p})] &= \frac{\tau_\varepsilon^2}{r(\tau_\varepsilon + \tau)} E[(s_i - \hat{p})^2] \\
&= \frac{\tau_\varepsilon^2}{r(\tau_\varepsilon + \tau)} E[(e^* + \theta + \varepsilon_i - (e^* + \lambda\beta\theta + \lambda u))^2] \\
&= \frac{\tau_\varepsilon^2}{r(\tau_\varepsilon + \tau)} \left(\frac{1}{\tau_\varepsilon} + \frac{1}{\tau} \right) \\
&= \frac{\tau_\varepsilon}{r\tau}
\end{aligned}$$

A speculator has higher expected revenues from trading when his private information is more precise and when the stock price is less informative, e.g. due to more noise trading. He can therefore invest in more precise information at cost $k\tau_\varepsilon$ when there is more noise trading. A speculator's choice of information solves

$$\max_{\tau_\varepsilon} E[\Pi(s_i, \hat{p})] - k\tau_\varepsilon$$

Although more noise trading increases the marginal return to private information, the effect is only an indirect one operating via a higher trading aggressiveness β. The next proposition shows that it is outweighed by the direct negative effect of simply adding exogenous noise to market-based pay.

Proposition 4 *Even when speculators can improve the precision of their signals* τ_ε *at a cost* $c(\tau_\varepsilon) = k\tau_\varepsilon$ *so that more noise trading leads to the collection of better information, the overall effect of more noise trading still is to reduce the extent of market-based pay.*

7 Speculators' short-termism

Having shown that the aggregation of dispersed private information about the future value of the firm via self-interested trading makes the extent of optimal market-based pay proportional to the informativeness of the stock price, we now present a simple extension to examine the impact of short-termism in the market on CEO incentives.

We extend the model by adding an extra round of trading before the manager quits the firm (see figure 2).

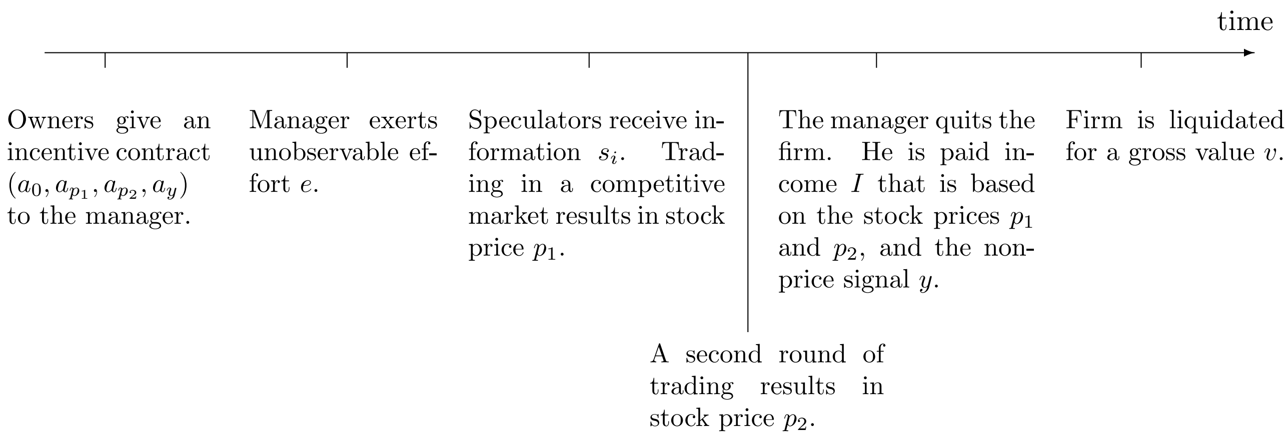

Figure 2: The timing of events

Managerial pay income is now given by

$$I = a_0 + a_1 p_1 + a_2 p_2 + a_y y$$

and noise trading will be i.i.d. across the two periods, $t = 1, 2$.

$$u_t \sim N(0, \sigma_u^2)$$

Let

$$L_1 = \int_0^1 x_{i1} di + u_1$$

be the order flow the market makers observe in the first trading round when an informed trader i takes the position x_{i1} in the first trading period. Analogously,

$$L_2 = \int_0^1 x_{i2} di - \int_0^1 x_{i1} di + u_2$$

is the net order flow in the second trading round.

As in the static case, a competitive risk-neutral market making sector observing the aggregate limit order book ensures efficient pricing:

$$p_1 = E[\pi | L_1] \tag{22}$$

$$p_2 = E[\pi | L_1, L_2] \tag{23}$$

As before, we focus on linear symmetric equilibria in which a speculator's demand x_{it} is linear in prices p_t and his signal s_i, and we write the informative part of the order book as $z_1 = \beta_1 v + u_1$ and $z_2 = (\beta_2 - \beta_1)v + u_2$. We again normalize prices

$$\hat{p}_1 = a_0 + (1 + a_1 + a_2)p_1 \tag{24}$$

$$\hat{p}_2 = a_0 + a_1 p_1 + (1 + a_2)p_2 \tag{25}$$

to rewrite (22) and (23) as:[12]

$$\hat{p}_1 = E[v|z_1] \tag{26}$$

$$\hat{p}_2 = E[v|z_1, z_2] \tag{27}$$

Proposition 5 characterizes the pricing functions in the dynamic case.

Proposition 5 *The first and second period stock price are given by:*

$$\begin{aligned}\hat{p}_1 &= (1-\lambda_1\beta_1)e^* + \lambda_1\beta_1(e+\theta) + \lambda_1 u_1 \\ \hat{p}_2 &= (1-\frac{\tau_1}{\tau_2}\lambda_1\beta_1 - \lambda_2(\beta_2-\beta_1))e^* + (\frac{\tau_1}{\tau_2}\lambda_1\beta_1 + \lambda_2(\beta_2-\beta_1))(e+\theta) + \frac{\tau_1}{\tau_2}\lambda_1 u_1 + \lambda_2 u_2\end{aligned}$$

where e^* is the hypothesized equilibrium effort, e is the actual effort, $\tau_1 = Var[v|z_1]^{-1} = \tau_\theta + \beta_1^2\tau_u$, $\tau_2 = Var[v|z_1, z_2]^{-1} = \tau_1 + (\beta_2-\beta_1)^2\tau_u$ and

$$\begin{aligned}\lambda_1 &= \beta_1\frac{\tau_u}{\tau_1} \\ \lambda_2 &= (\beta_2-\beta_1)\frac{\tau_u}{\tau_2}\end{aligned}$$

The trading horizon of speculators determines their trading aggressiveness β_t, which in turn affects the liquidity of the market λ_t^{-1} and the informativeness of the stock price τ_t in each period. But before introducing speculators' short-termism, we confirm that without shortening trading horizons, without new information reaching the market between trading periods and without correlated noise trading across time, adding another trading round by itself is innocuous.

A speculator with a long investing horizon maximizes the expected utility of wealth from gains in both trading periods:

[12] As in the static case, the manager is paid his fixed and market based pay in cash while the remainder of his compensation is paid by transferring shares from inside owners. This accounting convention yields a net liquidation value of the firm $\pi = v - a_0 - a_{p_1}p_1 - a_{p_2}p_2$.

$$U_i(x_{i1}, x_{i2}) = -\exp[-r(x_{i1}(p_2 - p_1) + x_{i2}(\pi - p_2))]$$

The following proposition states that a competitive market with long investing horizons incorporates information into the stock price immediately (see Vives (1995)).

Proposition 6 *With long investing horizons, speculators' trading aggressiveness is constant and identical to the case with a single trading period:* $\beta_1 = \beta_2 = \frac{\tau_\varepsilon}{r}$.

There is no information trading in the second period so that speculators with long investment horizons pursue a buy-and-hold strategy. Consequently, any noise trading in the second period u_2 is absorbed by the competitive risk-neutral market making sector: the market is infinitely liquid in the second period, $\lambda_2 = (\beta_2 - \beta_1)\tau_u/\tau_2 = 0$, and the first and second period price are the same, $\hat{p}_1 = \hat{p}_2$. The manager's contract and effort are the same as in propositions 3 where $\tau = \tau_1 = \tau_2 = \beta^2\tau + \tau_\theta$ and $\beta = \beta_1 = \beta_2 = \tau_\varepsilon/r$.

Speculators with short trading horizons maximize

$$E[-\exp(-r(x_{i1}(p_2 - p_1)))|s_i, p_1] \tag{28}$$

in the first period and

$$E[-\exp(-r(x_{i2}(\pi - p_2)))|s_i, p_1, p_2] \tag{29}$$

in the second period.

We assume that speculators in the second period have access to all the information of the first period. The situation is either one where speculators live for two periods but undertake successive myopic one-period investments, or where a new generation of short-lived speculators enters the market in the second period inheriting the knowledge of the previous generation.

Proposition 7 *With short investing horizons, speculators' trading aggressiveness increases over time:* $\beta_1 = \frac{\tau_\varepsilon \tau_2}{r(\tau_\varepsilon + \tau_2)} < \beta_2 = \frac{\tau_\varepsilon}{r}$.

Speculators with a short investment horizon hold back in the first period because they have information about the final value of the firm v but cannot hold the asset until this value realizes. Instead they need to close their position early at a price $\hat{p}_2$, which is only an imperfect estimate of future firm value v. Speculators have therefore fewer incentives to trade aggressively on their information in the first period.

A direct consequence of less aggressive information trading in the first period is that the manager's optimal incentive contract is not contingent on the first period stock price as this would only expose him to unnecessary noise.[13]

Corollary 2 *Optimal CEO pay will not be based on the stock price in the first period.*

Since the optimal incentive contract for the CEO does not include the first period price $\hat{p}_1$ as a performance measure, the analysis of managerial incentives parallels the one carried out in the static case. The result in propositions 3 carries over with the informativeness of the stock price now being $\tau_2 = \tau_\theta + (\beta_1^2 + (\beta_2 - \beta_1)^2)\tau_u$, and speculators' trading aggressiveness being $\beta_1 = \tau_\varepsilon \tau_2 / (r(\tau_\varepsilon + \tau_2))$ and $\beta_2 = \tau_\varepsilon / r$ (propositions 5 and 7).

The next proposition summarizes the impact of short-termism in the stock market via market-based pay on CEO effort.

Proposition 8 *When speculators have short trading horizons then CEO pay is less contingent on the stock price and the CEO exerts less effort than when speculators have long trading horizons.*

Speculators with shorter trading horizons trade less aggressively on their information. This reduces the information content of the stock price and makes it more costly to provide market based incentives to management, which in turn leads to less managerial effort and ultimately to lower expected firm value.

[13]See also Froot et al. (1992) who argue in a different context that management's pay should not be linked to near-term stock price levels.

8 Discussion and empirical implications

Recent empirical research by Garvey and Swan (2002) and Kang and Liu (2005) establishes a significant cross-sectional link between the extent of stock-based CEO pay and stock market conditions for a sample of publicly traded US corporations that is consistent with our analysis. Kang and Liu (2005) find that CEO pay is more sensitive to changes in shareholder value when more information is impounded into stock prices. They measure the informativeness of the stock price using the probability of informed trading (PIN) of Easley et al. (1997) (see for example Chen et al. (2006)) for an application of the PIN as a measure of stock price informativeness in a different context) and the dispersion and error of analysts' forecasts. Similarly, Garvey and Swan (2002) find a negative link between both the bid-ask spread and the ratio of turnover to market capitalization and the extent of market-based CEO pay. They argue that the impact of these two measures of market liquidity on CEO pay is at least as large as the effect of traditional cross-sectional determinants such as size, risk or industry.[14]

Proposition 3 shows that a more informative stock price and a more liquid market lead to more market based pay ceteris paribus. The view of the stock market as an aggregator of dispersed private information, and thus as a communication device, provides a suitable theoretical background for these empirical results for the US. First, it is more difficult to reconcile them with the view of dominant investors trading on costly insider information and therefore acting as active indirect monitors of management (as in Holmström and Tirole (1993)). Hartzell and Starks (2003) find evidence against such indirect monitoring. Instead, large insiders such as institutional investors in the US appear to act as direct monitors and influence CEO compensation structures directly. They find that higher institutional investor concentration leads to subsequent changes in CEO pay but not vice versa as one would expect if unobserved insider trading was driving both measures simultaneously and endogenously. Second, Laffont and

[14]Schipper and Smith (1986) provide indirect evidence for the positive link between liquidity and market based CEO pay by examining carve-outs. After selling a subsidiary to the public equity market, management typically receives compensation contracts that include the new company's stock.

Maskin (1990) show that a single large trader with private information typically finds it more profitable to conceal the information and to trade in such a way that the price does not reflect his private information at all.[15] In that case the stock price does not incorporate additional information that could be exploited for incentive contracting. And third, a concern with indirect CEO monitoring by a single informed trader is its robustness to the threat of collusion between the CEO and the trader. This issue does not arise in our analysis since information is highly dispersed across a competitive market.

This discussion suggests that the Holmström and Tirole (1993) channel of indirect CEO monitoring applies to situations where information is not highly dispersed among many traders in a competitive market, where insider trading legislation is weak or difficult to enforce and where direct shareholder activism is not viable. While this situation may not be an accurate description of developed market economies in general, and of the US in particular, it could capture the conditions in developing economies. Existing empirical research on comparing financial systems provides evidence that stock markets affect firms' corporate governance and that this link is due to information provision rather than the exertion of control. But the evidence does not yet allow to disentangle the issue further. Using industry level data across 38 countries, Tadesse (2004) finds that liquid stock markets promote economic performance via market based governance of which information aggregation and incentive contracting is one possible channel. Other possible governance channels are direct control by dominant shareholders or takeover activity. Gupta (2005) however identifies the positive role of financial markets as information producers on firm performance. She studies partial privatization programs in which government sells only non-controlling shares to the public and shows that it has a positive impact on firms' profitability, productivity and investment. Her approach allows to eliminate the confounding effect of direct shareholder control on the relationship between stock market trading and firm

[15] An information monopolist trading strategically in the US stock market may also run the danger of violating section 10(b) of the Securities Exchange Act. Courts have interpreted this section in conjunction with Rule 10b-5 to prohibit insider trading by a corporate "outsider" (see http://www.sec.gov/answers/insider.htm for more information.)

performance.

According to proposition 8, shorter investment horizons of traders should lead to less market based CEO pay and lower CEO effort in equilibrium. To our knowledge, this prediction has not yet been tested directly. Kang and Liu (2005) however show a sharp increase in the positive link between measures of stock price informativeness and the sensitivity of market based CEO pay to shareholder value after the stock market bubble burst in 2000. If traders acted more myopically in the run up to the stock market bubble then our model provides a possible rationale for the surprising increase. Short-termism in the market made stock prices a worse aggregator of dispersed information in the years prior to 2000. After the bubble burst, traders' short-termism subsided, stock prices became more informative about firm value and thus CEO performance increasing the extent of market based pay in equilibrium.[16]

Finally, our analysis may also provide a new perspective on the debate on the relationship between risk and incentives (see Prendergast (2002)). Core et al. (2003) for example find that counter to the standard predictions of agency models, the variation in the relative weights on price and non-price measures in total CEO compensation is an increasing function of their relative variances. One possible explanation is based on the observation that a more informative stock price is also more volatile.[17] Adding information aggregation via trading to a standard agency problem as in our paper, could generate a positive relationship between relative incentives and the volatility of performance measures based on stock prices when the informativeness of the stock price increases.

9 Conclusion

This paper presents a model where the benefit of market based CEO pay is that the stock market aggregates useful but dispersed private information about past managerial performance

[16]Note that our analysis and the cited evidence examine the composition of CEO pay but not its level.

[17]This can be seen from $Var[\hat{p}] = Var[E[v|\hat{p}]] = Var[v] - Var[v|\hat{p}]$.

via self-interested trading. The private information present in the market would allow, if aggregated costlessly, to perfectly observe the future value of the firm, a sufficient statistics for the managerial effort in the sense of Holmstrom (1979).

It is however well known that speculators only trade on their private information if there is noise trading, e.g. trade due to margin calls or life-cycle motives. Noise trading is unrelated to management's action and should therefore not affect their incentive schemes according to Holmström's "informativeness principle". But since such noise trading makes room for information trading, it is a necessary cost of market based pay.

This paper analyzes the impact of this link between information and noise trading on the design of market based pay. Contrary to Holmström and Tirole (1993) we find that more noise trading results in less market based pay because it simply adds noise to the stock price and thus reduces its informativeness. This negative role of noise trading is robust to allowing speculators to collect costly information.

Despite the negative effect of noise trading, a more liquid market for the company stock leads to more market based pay ceteris paribus. A more liquid market allows more information trading and is therefore better at aggregating dispersed information.

Short termism in the stock market reduces liquidity (Vives (1995)). It lowers the aggressiveness with which speculators trade on their private information since the stock price reflects their information only imperfectly at the time they need to close their positions. We show that short-termism makes the stock price less informative about management performance, weakens market based incentives and increases the cost of CEO moral hazard.

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A Appendix: Proofs

In order to calculate the conditional distributions, we use the following standard result for normally distributed variables:

Result 1 *Let Y_i be a $(n_i \times 1)$ vector with mean μ_i, i=1,2, and variance-covariance matrices Σ_{ij}, then*

$$Y_2|Y_1 = y_1 \sim N([\mu_2 + \Sigma_{21}\Sigma_{11}^{-1}(y_1 - \mu_1)], [\Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}])$$

Proof of proposition 1

It is easier to calculate the conditional expectation and variance using the following information equivalent of price $\hat{p}$,

$$\hat{\hat{p}} = \frac{\hat{p} - (1-\lambda\beta)e^*}{\lambda\beta} = e + \theta + \frac{1}{\beta}u \tag{30}$$

Using result 1 we have

$$\begin{aligned} E[v|y,\hat{\hat{p}}] &= e^* + (\sigma_\theta^2, \sigma_\theta^2) \begin{pmatrix} \sigma_\theta^2 + \sigma_\eta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \frac{1}{\beta^2\sigma_u^2} \end{pmatrix}^{-1} \left(\begin{pmatrix} y \\ \hat{\hat{p}} \end{pmatrix} - \begin{pmatrix} e^* \\ e^* \end{pmatrix} \right) \\ &= \frac{\beta^2\sigma_\eta^2\sigma_\theta^2\hat{\hat{p}} + \sigma_u^2\sigma_\theta^2 y + \sigma_u^2\sigma_\eta^2 e^*}{\beta^2\sigma_\eta^2\sigma_\theta^2 + \sigma_u^2(\sigma_\eta^2 + \sigma_\theta^2)} \end{aligned}$$

Substituting back and using $\tau = \beta^2\tau_u + \tau_\theta$ we obtain:

$$E[v|y,\hat{p}] = \frac{\tau_\eta y + \tau\hat{p}}{\tau_\eta + \tau}$$

We can therefore rewrite managerial income as

$$\begin{aligned} I &= a_0 + a_p p + a_y y \\ &= a_0 + a_p p + \alpha E[v - a_0 - a_p p|y,p] \\ &= (1-\alpha)a_0 + (1-\alpha)\frac{\hat{p} - a_0}{1+a_p} + \alpha E[v|y,\hat{p}] \\ &= \frac{(1-\alpha)a_p}{1+a_p}a_0 + \left(\frac{(1-\alpha)a_p}{1+a_p} + \frac{\alpha\tau}{\tau_\eta + \tau}\right)\hat{p} + \frac{\alpha\tau_\eta}{\tau_\eta+\tau}y \end{aligned}$$

Proof of proposition 2

It is immediate from result 1 that

$$E[v|z] = e^* + \frac{\beta\sigma_\theta^2}{\beta^2\sigma_\theta^2 + \sigma_u^2}(z - \beta e^*)$$

Let $\lambda = \frac{\beta\sigma_\theta^2}{\beta^2\sigma_\theta^2+\sigma_u^2}$; dividing both numerator and denominator by $\sigma_\theta^2\sigma_u^2$ and rewriting the expression in terms of precisions give us the result in λ. Substituting then for $z = \beta(e+\theta)+u$ gives the result for $\hat{p} = E[v|z]$.

To solve for β we need to characterize the distribution of $v|s_i, \hat{p}$. Using again result 1,

$$\begin{aligned} E[v|s_i,\hat{\hat{p}}] &= e^* + (\sigma_\theta^2, \sigma_\theta^2)\begin{pmatrix} \sigma_\theta^2+\sigma_\varepsilon^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \frac{1}{\beta^2\sigma_u^2}\end{pmatrix}^{-1}\left(\begin{pmatrix} s_i \\ \hat{\hat{p}}\end{pmatrix} - \begin{pmatrix} e^* \\ e^*\end{pmatrix}\right) \\ &= \frac{\beta^2\sigma_\varepsilon^2\sigma_\theta^2\hat{\hat{p}} + \sigma_u^2\sigma_\theta^2 s_i + \sigma_u^2\sigma_\varepsilon^2 e^*}{\beta^2\sigma_\varepsilon^2\sigma_\theta^2 + \sigma_u^2(\sigma_\varepsilon^2+\sigma_\theta^2)} \end{aligned}$$

where we used again the information equivalent $\hat{\hat{p}}$ instead of $\hat{p}$. Substituting $\hat{p}$ for $\hat{\hat{p}}$ and writing the expression in terms of precision $\tau_j = 1/\sigma_j^2$, we obtain:

$$E[v|s_i,\hat{p}] = \frac{\tau_\varepsilon s_i + (\beta^2\tau_u + \tau_\theta)\hat{p}}{\tau_\varepsilon + (\beta^2\tau_u+\tau_\theta)}$$

Next we need to calculate

$$\begin{aligned} Var[v|s_i,\hat{p}] &= \sigma_\theta^2 - (\sigma_\theta^2,\sigma_\theta^2)\begin{pmatrix} \sigma_\theta^2+\sigma_\varepsilon^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2+\frac{1}{\beta^2\sigma_u^2}\end{pmatrix}^{-1}\begin{pmatrix}\sigma_\theta^2 \\ \sigma_\theta^2\end{pmatrix} \\ &= \frac{1}{\tau_\varepsilon + (\beta^2\tau_u+\tau_\theta)} \end{aligned}$$

Last, substituting $E[v|s_i,\hat{p}]$ and $Var[v|s_i,\hat{\hat{p}}] = Var[v|s_i,\hat{p}]$ into (11) yields

$$x_i(s_i,\hat{p}) = \frac{\tau_\varepsilon}{r}(s_i - \hat{p})$$

This means that $\beta = \tau_\varepsilon/r$.

Proof of propostion 3

Immediate from substituting (16) into (10).

Proof of corollary 1

We use proposition 1 and replace the contract (a_0, a_p, a_y) in (1) with $(\hat{a}_0, \hat{a}_p, \hat{a}_y)$ and the stock price p with $\hat{p}$ (equation (12)). The optimal contract still has to satisfy (10), but now with the normalized weights $\hat{a}_p$ and $\hat{a}_y$ and the normalized price $\hat{p}$:

$$\hat{a}_y[Var[y]\frac{\partial E[\hat{p}]}{\partial e} - Cov[\hat{p},y]] = \hat{a}_{\hat{p}}[Var[\hat{p}] - Cov[\hat{p},y]\frac{\partial E[\hat{p}]}{\partial e}]$$

Using proposition 2 to substitute for $\hat{p}$ and rearranging yields equation (20). To derive the expression for optimal CEO effort we need to calculate the absolute weights $(\hat{a}_{\hat{p}}, \hat{a}_y)$ the contract places on the stock price and the non-price signal.

Since the manager's participation constraint (4) will be binding at the optimum, the optimal managerial contract $(\hat{a}_0, \hat{a}_{\hat{p}}, \hat{a}_y)$ maximizes

$$E[v] - \frac{r_m}{2} Var[I] - \frac{1}{2} e^2$$

subject to managerial effort being optimal:

$$e = \hat{a}_{\hat{p}} \lambda \beta + \hat{a}_y \tag{31}$$

Substituting for e, v, I, taking first-order conditions with respect to $\hat{a}_{\hat{p}}$ and $\hat{a}_y$, and rearranging gives the absolute weights on the stock price $\hat{p}$ and the signal y.

$$\begin{aligned}
\hat{a}_y &= \frac{\sigma_u^2}{\sigma_u^2 + \beta^2 \sigma_\eta^2 + r((\sigma_u^2 + \beta^2 \sigma_\eta^2)\sigma_\theta^2 + \sigma_\eta^2 \sigma_u^2)} \\
\hat{a}_{\hat{p}} &= \frac{\sigma_\eta^2(\sigma_u^2 + \beta^2 \sigma_\theta^2)}{\sigma_\theta^2 \left[\sigma_u^2 + \beta^2 \sigma_\eta^2 + r((\sigma_u^2 + \beta^2 \sigma_\eta^2)\sigma_\theta^2 + \sigma_\eta^2 \sigma_u^2)\right]}
\end{aligned}$$

Now we substitute for the absolute weights in (31) and use the definition of λ in proposition 2 to write optimal effort as

$$e = \left[1 + r(\sigma_\theta^2 + \frac{\sigma_\eta^2 \sigma_u^2}{\beta^2 \sigma_\eta^2 + \sigma_u^2})\right]^{-1}$$

which is the expression in the proposition after writing variances as precisions and using the definition of τ in proposition 2.

Proof of proposition 4

Suppose for a moment only that the optimal precision of a speculator's information is increasing with the extent of noise trading, $\tau_\varepsilon = \tau_\varepsilon(\tau_u)$ with $\tau_\varepsilon' < 0$. The relative weight of market-based to non market-based pay (equation (20)) increases with the volatility of noise trading, i.e. $\frac{\partial(\hat{a}_p)/\hat{a}_y}{\partial \tau_u} < 0$, iff

$$\frac{\tau_\varepsilon'}{\tau_\varepsilon} < -\frac{1}{\tau_u} \tag{32}$$

A speculator chooses the precision of his signal τ_ε in order to maximize the expected revenue of trading net of the cost of collecting better information:

$$\max_{\tau_\varepsilon} \frac{\tau_\varepsilon}{r((\frac{\tau_\varepsilon}{r})^2 \tau_u + \tau_\theta)} - k\tau_\varepsilon$$

The first-order condition is

$$\frac{r(r^2\tau_\theta - \tau_u \tau_\varepsilon^2)}{(r^2 \tau_\theta + \tau_u \tau_\varepsilon^2)^2} = k$$

We assume that speculators' risk aversion satisfies $r > \frac{\sigma_\theta}{\sqrt{3}\sigma_\varepsilon^2 \sigma_u^2}$ so that the marginal benefit of better information is positive and the second-condition for solving for a maximum is satisfied. If this is not the case then a speculator would never want to collect better information.

Letting $\hat{\tau}_\varepsilon = \tau_\varepsilon^2$ the positive solution to the first-order condition is

$$\hat{\tau}_\varepsilon = \frac{r(\sqrt{1+8kr\tau_\theta} - 1 - 2kr\tau_\theta)}{2k\tau_u}$$

which is positive as long as the marginal cost of collecting better information is low, $k < (r\tau_\theta)^{-1}$. This means that $\tau_\varepsilon(\tau_u) = K\tau_u^{-1/2}$ where K collects the terms that do not depend on τ_u, and

$$\frac{\tau_\varepsilon'}{\tau_\varepsilon} = -\frac{1}{2\tau_u} > -\frac{1}{\tau_u}$$

which contradicts (32). Hence $\frac{\partial(\hat{a}_p)/\hat{a}_y}{\partial\tau_u} > 0$.

Proof of proposition 5

Applying result 1 and writing expression in terms of precisions $\tau_j = 1/\sigma_j^2$:

$$E[v|z_1] = e^*(1 - \frac{\beta_1^2\tau_u}{\beta_1^2\tau_u + \tau_\theta}) + \frac{\beta_1\tau_u}{\beta_1^2\tau_u + \tau_\theta}z_1$$

Letting $\lambda_1 = \frac{\beta_1\tau_u}{\beta_1^2\tau_u+\tau_\theta}$, substituting $z_1 = \beta_1(e+\theta)+u$ and denoting $\tau_1 = \beta_1^2\tau_u + \tau_\theta$ gives the result for the first period price.

Applying result 1 again and using notation from above:

$$E[v|z_1,z_2] = e^*(1 - \frac{\beta_1^2\tau_u + (\beta_2-\beta_1)^2\tau_u}{\beta_1^2\tau_u + (\beta_2-\beta_1)^2\tau_u + \tau_\theta}) + \frac{\beta_1^2\tau_u}{\hat{\beta}_1^2\tau_u + (\beta_2-\beta_1)^2\tau_u + \tau_\theta}z_1 + \frac{(\beta_2-\beta_1)^2\tau_u}{\beta_1^2\tau_u + (\beta_2-\beta_1)^2\tau_u + \tau_\theta}z_2$$

Letting $\lambda_2 = \frac{(\beta_2-\beta_1)^2\tau_u}{\hat{\beta}_1^2\tau_u+(\beta_2-\beta_1)^2\tau_u+\tau_\theta}$ and $\tau_2 = \tau_1 + (\beta_2-\beta_1)^2\tau_u$ gives the result for the second period price.

Proof of proposition 6

See proposition 4.1 in Vives (1995). A detailed proof is available from the authors upon request.

Proof of proposition 7

The situation in the second period with a short investing horizon is identical to the one with a long horizon. Thus we know from proposition 6 that $\beta_2 = \tau_\varepsilon/r$.

Using the normalizes prices $\hat{p}_1$ and $\hat{p}_2$ we can rewrite (29) as

$$E\left[-\exp\left(-rx_{i1}\frac{1}{1+a_2}(\hat{p}_2 - \hat{p}_1)\right)|s_i, \hat{p}_1\right]$$

Maximizing with respect to x_{i1} yields

$$\frac{1}{1+a_2}x_{i1} = \frac{E[\hat{p}_2 - \hat{p}_1|s_i,\hat{p}_1]}{Var[\hat{p}_2 - \hat{p}_1|s_i,\hat{p}_1]} \tag{33}$$

Using the pricing functions of proposition 5 we can write

$$\begin{aligned} E[\hat{p}_2 - \hat{p}_1 | s_i, \hat{p}_1] &= \lambda_2(\beta_2 - \beta_1) E[v - \hat{p}_1 | s_i, \hat{p}_1] \\ Var[\hat{p}_2 - \hat{p}_1 | s_i, \hat{p}_1] &= \lambda_2^2((\beta_2 - \beta_1)^2 Var[v - \hat{p}_1 | s_i, \hat{p}_1] + \sigma_u^2) \end{aligned}$$

Result 1 allows to calculate

$$\begin{aligned} E[v | s_i, \hat{p}_1] &= \frac{\tau_\varepsilon s_i + \tau_1 \hat{p}_1}{\tau_\varepsilon + \tau_1} \\ Var[v | s_i, \hat{p}_1] &= \frac{1}{\tau_\varepsilon + \tau_1} \end{aligned}$$

Substituting back into (34) and (34), and then into (33) using also the result for β_2 yields

$$\frac{1}{1 + a_2} x_{i1} = \frac{\tau_\varepsilon \tau_2}{r(\tau_\varepsilon + \tau_2)} (s_i + \hat{p}_1)$$

so that $\beta_1 = \frac{\tau_\varepsilon \tau_2}{r(\tau_\varepsilon + \tau_2)}$.

Proof of corollary 2

As in the static case, maximizing expected net firm value (2) subject to the incentive constraint (3) and the participation constraint (4) means that the dilution free contract $(\hat{a}_1, \hat{a}_2, \hat{a}_y)$ solves

$$\min_{\hat{a}_1, \hat{a}_2, \hat{a}_y} Var[I]$$

subject to

$$e = \hat{a}_1 \frac{\partial E[\hat{p}_1]}{\partial e} + \hat{a}_2 \frac{\partial E[\hat{p}_2]}{\partial e} + \hat{a}_y$$

From the first-order conditions, $\hat{a}_1$,$\hat{a}_2$ and $\hat{a}_y$ must satisfy

$$\begin{aligned} \hat{a}_1 \left[\frac{\partial E[\hat{p}_2]}{\partial e} Var[\hat{p}_1] - \frac{\partial E[\hat{p}_1]}{\partial e} Cov[\hat{p}_1, \hat{p}_2] \right] &+ \hat{a}_2 \left[\frac{\partial E[\hat{p}_2]}{\partial e} Cov[\hat{p}_1, \hat{p}_2] - \frac{\partial E[\hat{p}_1]}{\partial e} Var[\hat{p}_2] \right] \\ &+ \hat{a}_y \left[\frac{\partial E[\hat{p}_2]}{\partial e} Cov[\hat{p}_1, y] - \frac{\partial E[\hat{p}_1]}{\partial e} Cov[\hat{p}_2, y] \right] = 0 \end{aligned}$$

The first period stock price is therefore not included if

$$\frac{\partial E[\hat{p}_2]}{\partial e} Cov[\hat{p}_1, \hat{p}_2] = \frac{\partial E[\hat{p}_1]}{\partial e} Var[\hat{p}_2]$$

and

$$\frac{\partial E[\hat{p}_2]}{\partial e} Cov[\hat{p}_1, y] = \frac{\partial E[\hat{p}_1]}{\partial e} Cov[\hat{p}_2, y]$$

Some algebra (available on request from the authors) shows that after substituting β_1 and β_2 from proposition 7 into the pricing functions of proposition 5 the conditions hold.

Proof of proposition 8

We need to compare the informativeness of the stock price $Var[v|\hat{p}_1, \hat{p}_2]^{-1}$ with long and short investment horizons. With long investment horizons, the information content is

$$\tau_2 = \tau_\theta + \left(\frac{\tau_\varepsilon}{r}\right)^2 \tau_u$$

and with short investment horizons it is

$$\tau_2 = \tau_\theta + \left(\frac{\tau_\varepsilon}{r}\right)^2 \left[\left(\frac{\tau_2}{\tau_2 + \tau_\varepsilon}\right)^2 + \left(\frac{\tau_\varepsilon}{\tau_2 + \tau_\varepsilon}\right)^2\right] \tau_u$$

Thus, the information content of the stock price is lower when speculators have a short investment horizon. The conclusions on the relative weight of market based pay and on CEO effort then follow directly from proposition 3.