

Nonlinear dynamics and persistence in PPP relation: Does controlling for nonlinearity solve the PPP puzzle?

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This version: November 2007

Abstract

This paper uses half-life measures conditional on various purchasing power parity (PPP) regimes, to examine persistence in deviations from PPP within the context of nonlinear exponential smooth transition autoregressive (ESTAR) models. Sampling uncertainty of regime-dependent half-lives is quantified through a delta method approximation and by means of simulations. Small sample properties of proposed measure investigated by Monte-Carlo experiments. Regime-dependent half-life estimates and confidence intervals reveal possibility of both short-lived and long-lived deviations from PPP as well as noticeably different persistence dynamics across Euro and nonEuro zone currencies. Although typical point estimates are around one to fewer than two years, confidence bounds suggest notable uncertainty and persistence in several quarterly US Dollar PPP deviations over the floating period. Results show that controlling for nonlinear dynamics in PPP relationship may not necessarily resolve the PPP puzzle as argued in some recent nonlinear empirical PPP studies.

JEL Classification: F31, F41, C22.

Keywords: PPP, nonlinearity, regime-dependent half-life.

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1 Introduction

The purchasing power parity (PPP) puzzle involves the difficulty of reconciling high short-term volatility of PPP deviations with slow rates of reversion to a long run level (Rogoff 1996). The PPP puzzle has inspired two independent lines of research with the objective to “solve” the puzzle by reducing the half-lives of deviations from PPP. The first line of research relies on panel data methods while the second line incorporates nonlinearities into the PPP condition. Both lines of research have generated series of papers that suggest shorter half-lives than Rogoff’s (1996) consensus of 3 to 5 years range. Panel data studies that report shorter half-lives of 2 to 2.5 years by using mostly quarterly post-Breton-Woods data, include Wu (1996), Papell (1997, 2002), Fleissig and Strauss (2000), and Papell and Theodoridis (2001) among others. Murray and Papell (2005) challenge the findings from the panel data studies by extending median-unbiased methods to panel data models. They argue that while panel regressions provide more information on the persistence of real exchange shocks than univariate regressions, they do not help solve the puzzle.

Studies that incorporate nonlinearity into PPP relation are motivated by the exchange rate models with transportation costs in trading (see, Dumas (1992), and Sercu et al. (1995)). Among several others, Michael et al. (1997), Taylor et al. (2001), and Baum et al. (2001) show that exponential smooth transition autoregressive (ESTAR) models can characterize the non-linear adjustment in PPP deviations well over the floating period. Taylor (2001) argues that ignoring the nonlinear effects could cause an upward bias in half-life estimates when a linear model is incorrectly estimated. Baum et al. (2001) and Taylor et al. (2001) study the persistence of PPP deviations in ESTAR models. They use generalized impulse response functions (GIRF) conditional on an average history of the deviations and a set of initial shocks. They estimate summary measures of half-life estimates without confidence intervals. Their estimates are shorter than the consensus estimates as well as estimates from the panel data studies. Shintani (2006) suggests a semi-parametric approach (which does not require a specific nonlinear parametric model) and reports estimates that are mostly within the one year range. An

excellent survey of nonlinear exchange rate literature is provided by Sarno (2005).

This paper aims to contribute to the empirical nonlinear exchange rate literature by examining persistence dynamics in nonlinear ESTAR models. Under the ESTAR model, deviations from PPP follows a linear autoregressive process conditional on the regime at date t . In other words, given the value of the transition function at a date t , PPP deviations follow a linear model within that particular regime. The particular regime at any given date is dictated by the value of the transition variable as well as parameters of the model. By exploiting this property of ESTAR models, we propose to study the persistence dynamics of PPP deviations by using half-lives and confidence intervals across various regimes. The regime-dependent half-lives should provide useful insights into our understanding of the PPP puzzle as well as nonlinear persistence dynamics in PPP relation. The regime-dependent half-lives and confidence intervals estimated easily by standard asymptotic delta methods as well as through simulations. The simulated estimates obtained by generating artificial data that is calibrated on the estimated ESTAR models with errors drawn from the residuals. We examine the small sample properties of proposed persistence measure by Monte Carlo experiments.

Our findings show that half-life estimates and confidence intervals may vary depending on the degree of deviation of exchange rates from the prices ratios. Similar to the findings of the nonlinear literature (for example, Baum et al. 2001 and Taylor et al. 2001), point estimates for half-lives suggest fast nonlinear mean reversion. On the other hand, confidence intervals reveal there might be notable uncertainty about point estimates. Depending on the size of the deviations from the PPP, both persistent as well as short-lived deviations can characterize the PPP. This suggests that summary half-life measures obtained from nonlinear models without confidence bands may not reveal the persistence dynamics in PPP deviations accurately. Therefore, findings of studies that claim resolution of the PPP puzzle by incorporating nonlinearities should be interpreted cautiously. Our results also agrees with the findings of Kılıç (2007a) who shows that estimated ESTAR models can produce autocorrelations that can be consistent with persistent PPP deviations and therefore incorporating nonlinearity doesn't necessarily suggest less persistence in PPP. Our analysis suggests that modeling nonlinearity in PPP relation is

useful in providing insights into the persistence. However, controlling for nonlinearity may not resolve the puzzle as argued in the recent exchange rate studies. Despite differences in methods, our results are similar to the arguments raised by Murray and Papell (2005) against panel data studies which claim faster mean reversions in PPP deviations.

Our analysis reveals that the PPP deviations were more persistent during 1980s than 1990s. This might possibly due to the evolution of US Dollar against major currencies during early 1980s. Similar observations are also made in linear univariate and panel data studies which show evidence of breaks in the PPP deviations during early 1980s (see for example, Papell 2002 and Gadea et al .2004). Lastly, findings suggest that typically “nonlinear” PPP deviations are less persistent for US Dollar vis a vis the Euro-zone currencies than the nonEuro zone currencies. The rest of the paper organized as follows; section 2 presents the empirical model, section 3 provides and discusses the regime-dependent half-life estimates and confidence intervals. The last section provides a discussion of our findings.

2 Econometric Methodology

2.1 ESTAR Model

Following the nonlinear exchange rate literature, suppose that deviation of the logarithm of the exchange rate, $q_t = \log S_t - (\log P_t - \log P_t^*)$ (where S_t is the nominal exchange rate between two countries, P_t is the price level in home country, and P_t^* is the price level in foreign country) from its long run value q_0 which is a constant under PPP, follows the *ESTAR*(1) process,¹

$$q_t - q_0 = \phi(q_{t-1} - q_0) + \phi^*(q_{t-1} - q_0)F(\gamma, \mu, q_{t-d}) + u_t \quad (1)$$

where u_t is a stationary process, ϕ , ϕ^* , μ , and $\gamma > 0$ are unknown parameters, $d > 0$ is the delay parameter, q_{t-d} (i.e. d -period lagged q_t) is the transition variable. The transition function, $F(\gamma, \mu, q_{t-d}) = 1 - \exp(-\gamma(q_{t-d} - \mu)^2)$, governs the nonlinear behavior of the PPP deviations. The threshold parameter μ determines the equilibrium band for z_t . For large

¹A survey of recent developments in ESTAR modeling is given by van Dijk et al. (2002).

enough deviations from the equilibrium (in both positive and negative directions) the transition function takes values in the neighborhood of unity (outer regime) for any given value of γ . On the other hand, for deviations within the equilibrium band, the transition function takes values in the neighborhood of zero (inner regime). The larger the transition parameter the faster the transition between inner and outer regime is. One can also add lagged changes of PPP deviations in the linear part to allow for modeling serial correlation as in Kapetanios et al. (2003) and Park and Shintani (2005).

In the representation (1), critical parameters are ϕ and ϕ^* . Based on the implications of models in Dumas (1992) and Sercu et al. (1995), one can conjecture that while $\phi \geq 1$ is admissible, we must have $\phi^* < 1$, and $-1 \leq (\phi + \phi^*) < 1$ in (1). Intuitively, this says that for small deviations, q_t may be characterized by a unit root or even explosive behavior, but for large deviations the process is mean reverting. To our best knowledge, no formal proof of this conjecture exists in the econometrics literature. As pointed out in Park and Shintani (2005) and Kılıç (2007b), conditions for stationarity of nonlinear models such as the one in equation (1), are not well-known under more general error processes. However, Kapetanios et al. (2003) show that $-1 < (\phi + \phi^*) < 1$ is needed for the stationarity of model in (1), under the assumption that u_t is independently and independently distributed (i.i.d) with a nonnegative and continuous density and $\mu = 0$. Kılıç (2007a) provides simulation evidence on the stationarity of the model in (1) with errors drawn from the residuals of the empirically estimated ESTAR models for the PPP deviations. His findings show the model in (1) under the condition that $-1 < (\phi + \phi^*) < 1$ produces autocorrelations that do not change overtime systematically and decline with the distance in time.² Note the specification in (1), under $-1 \leq (\phi + \phi^*) < 1$, is consistent with exchange rate models with transport costs. These models predict that the deviation from PPP moves towards an attractor when it is sufficiently faraway from the attractor and shows some instability when it is in the neighborhood of the attractor.

²Although it is possible to entertain different specifications, majority of the empirical research has focused on the specifications similar to the model in (1). See for example the estimated models in Taylor et al. (2001), and Baum et al. (2001) among others. We have also estimated $ESTAR(k)$ with $k > 1$ models. As discussed above, we know little about the stationarity properties of $ESTAR(k)$ model with $k > 1$. Besides, for all the series considered, the $ESTAR(1)$ model performed better than the alternatives in terms of several diagnostics and tests. Therefore, the discussion is cast in terms of an $ESTAR(1)$ specification.

2.2 Half-lives and confidence intervals for PPP deviations

A way of measuring the degree of persistence is to estimate half-life of the deviations from PPP. Assuming the deviations from PPP follows a linear $AR(1)$ model with parameter ϕ , at horizon h , the percentage deviation from equilibrium is $h\phi$. Then the half-life deviation from PPP defined as the smallest value h such that³

$$h = \max\left(\frac{\ln(1/2)}{\ln(\phi)}, 0\right), \quad (2)$$

for $\phi > 0$. Half-life of PPP deviations from linear AR model is constant and does not depend on the initial PPP deviations, the size of the shock or the history of the deviations. Koop et al. (1996), and van Dijk and Francis (2002) show half-life from nonlinear models may depend on the initial conditions, the history of the time series and the size and sign of the shocks. This on the other hand introduces difficulties in using half-lives as measures of persistence in nonlinear models (see the discussion in Koop et al. 1996, van Dijk and Francis 2002, and Shintani 2006). One approach is based on the GIRF as developed in a series of papers by Gallant et al. (1993), Potter (1995, 2000), and Koop et al. However, since the GIRFs depend on the history of the time series and the size of the shocks, half-life estimates typically will not be unique. In practice, summarizing all the information of many different half-lives is not an easy task since evaluation of each GIRF usually requires computer-intensive simulation methods. GIRFs are used by Taylor et al. (2001) and Baum et al., (2001), to measure the persistence of PPP deviations in ESTAR models. These studies report point estimates of half-lives conditional on the average history of the monthly real exchange rates as well as on various initial shocks without providing any confidence intervals. Shintani (2006) uses the largest Lyapunov exponent of the time series. His method relies on a semi-parametric estimation and does not specify an ESTAR or any other nonlinear model for the PPP deviations.

In this paper, we follow a simple approach that allows us to calculate both the point estimates and the confidence intervals. We calculate half-life estimates from the estimated ESTAR

³Typically higher order AR terms are ignored when computing the half-lives. Rossi (2006) shows that ignoring the higher order terms may under estimate the half-life of PPP deviations in linear models.

models conditional on the value of the function. One way to think about the ESTAR model is to imagine that at each given time t , the PPP deviation is in a regime which is characterized by the value of the transition function $F(q_t, \gamma, \mu)$. In other words, there exists a continuum of regimes and at each date t PPP deviations are characterized by the regime F_s (where $s = t - d$). When the deviations are small enough, we are in the inner regime where $F(\cdot) = 0$ for any given value of γ , μ and q_{t-d} . Whenever the deviations are sizable enough and hence d -lagged deviations exceed the threshold value μ in either directions, $F(\cdot) = 1$ and we are in the outer regime. In the outer regime, the speed of adjustment should be faster than all other regimes. Our proposed half-life measure uses this idea and conditioned on the regime that prevails at date t . In this framework, one can think that conditional on a regime, PPP deviation from its long run level follows an AR process with the AR coefficient given by $\phi + \phi^* F_s$ where dependence on q_{t-d} , γ , and μ is tacit in F_s . Therefore at horizon h , the half-life of deviation from PPP conditional on regime F_s is

$$(\phi + \phi_s^*)^{h_s} = \frac{1}{2}.$$

Then similar to a linear AR model, the half-life of PPP deviation conditional on regime F_s (that is regime-dependent half-life) can be defined to be the smallest h^4

$$h_s = \max \left(\frac{\ln(\frac{1}{2})}{\ln(\phi + \phi^* F_s)}, 0 \right), \quad (3)$$

Note that whenever $\gamma = 0$, then the model in Equation (1) becomes a linear $AR(1)$ model and therefore our half-life measure becomes exactly the half-life measure given in Equation (2). On the other hand, when $\gamma \rightarrow \infty$ or when the lagged PPP deviations become large enough so for a given γ , $F_s \rightarrow 1$, the h_s approaches to the half-life measure from a linear AR model with parameter $\phi + \phi^*$. Note also that whenever the process is exactly in the inner regime (that is $F_s = 0$), h_s is equal to the half-life measure from an AR process with autoregressive parameter

⁴Similar to half-lives from AR models, regime-dependent half-lives do not account for the potential effects of higher order augmentation terms. Given the findings of Rossi (2006), it is possible that the regime-dependent half lives will under estimate the persistence in nonlinear PPP deviations. This however should strengthen the main arguments raised in the paper about the reconciliation of the PPP puzzle.

given by ϕ .

Contrary to the GIRF approach, regime-dependent half-life measures do not rely on intensive simulations. To the extent that the time series process is linear in each given regime, the half-life as suggested above should approximately measure the degree of persistence in the process. In other words, the regime-dependent half-life measure assumes the PPP deviation in a particular regime can be approximated by a linear *AR* model where the speed of adjustment is constant within the regime but vary across regimes. The speed of adjustment changes with the deviations from PPP. For each regime, there is a unique half-life that measures the persistence in PPP deviations. An array of regime-dependent half-lives measure how the degree of persistence changes with each regime and with the degree of deviation from long run PPP. The regime-dependent half-lives should provide insights into our understanding of overall dynamics as well as persistence of PPP deviations in nonlinear models.⁵ Given the linear nature of the ESTAR model in each given regime F_s , the regime-dependent confidence intervals can be estimated easily in a similar fashion to those from linear *AR* models.

The conventional 95% confidence intervals based on the normal sampling assumption for \hat{h}_s , $(\hat{h}_s^L, \hat{h}_s^U)$ is

$$\hat{h}_s \pm 1.96 \times \hat{\sigma}_{\hat{h}_s}, \quad (4)$$

where $\hat{\sigma}_{\hat{h}_s}$ is the standard error of \hat{h}_s . The standard error, $\hat{\sigma}_{\hat{h}_s}$, is estimated by a delta method approximation. Since the standard asymptotic confidence intervals may perform poorly especially when the time series is persistent, we have also simulated distribution of regime-dependent half-lives and recorded the lower (2.5 percentile of simulated distribution) and upper bounds (97.5 percentile of simulated distribution). The data generating process (DGP) is calibrated according to the estimated ESTAR models reported in Table 6, with errors drawn from the residuals of the estimated models as well as from independent and identically distributed Gaussian innovations. We initialized the data at zero, and generated 10,000 samples of $T + 200$ observations for each series (where T is the sample size). In each simulation, we discarded the

⁵We do not claim the measure suggested here is the most proper way to measure the persistence in ESTAR models. Our approach should be thought as a step in the direction to understand what the persistence dynamics look like in PPP deviations when nonlinearity is modeled by the ESTAR model.

first 200 observations to minimize the impact of initialization. For each artificial sample, we estimated the *ESTAR* model as in (1), and computed and saved half-lives from (3).

2.3 Small sample properties of regime-dependent half-lives

To evaluate the performance of the methods used in the paper, we conduct Monte Carlo experiments. Experiments are used to examine two questions: (i) assuming the true data generating process is the *ESTAR* model in Equation (1), how does the regime-dependent half-life measure perform with the changes in the degree of nonlinearity and the persistence? (ii) assuming a true DGP of *ESTAR* process, how does the conventional half-life measure perform from a misspecified linear *AR* model? In both experiments the data is generated from the *ESTAR* model given in (1) with $\phi = 1$ and $\mu = 0$ and errors are drawn from independent standard Gaussian distribution. We have changed the other parameters of the model to follow the affect of changes in both the degree of persistence and nonlinear dynamics in the *ESTAR* model. Specifically, we choose ϕ^* and γ from the set, $-0.1, -0.25, -0.5, -1.0$ and $0.1, 1, 5$ respectively.⁶ With a given set of parameter values, we have generated 10,000 samples of sizes $100 + T$ with $T \in \{100, 200\}$. We have discarded the first 100 observations to minimize the possible impact of initial values.

In Table 1, first rows of each panel (that is, rows corresponding to h_s^0) report the actual regime-dependent half-lives for the regimes corresponding to median, lowest 10th, 25th, 75th, and 90th percentiles of the transition function $F(\cdot)$. In each panel the remaining four rows display mean, median and 95% confidence intervals of the simulated regime-dependent half-lives. For any given sample size and a value of γ , as the value of ϕ^* decreases (so the *ESTAR* process becomes less persistent) regime-dependent mean, and median half lives decrease with confidence intervals becoming narrower. Similarly, as the nonlinearity becomes acute (that is, as γ increases and moves away from the null of linearity), regime-dependent mean and median half lives decline with narrower confidence bands. This suggests that as the speed of transition between extreme regimes increases (so the nonlinear dynamics is more precise), the regime-dependent half-life measure should estimate the persistence in a given regime more accurately. As expected, we

⁶Since results when $\gamma = 5$ and $\mu \neq 0$ were qualitatively similar to the reported results, we did not display results from the entire experiments. These results can be obtained on request.

see the simulated mean and median half-lives with much larger confidence intervals whenever the transition function is closer to the inner regime (that is, $F = 0.1$). Reported results in Table 1, also reveal that as the sample size increases, the simulated mean and median half-lives becomes closer to one other as well as closer to the actual regime-dependent half-lives. Also, simulated 95% confidence bands become narrower with the increase in sample size. Overall, for any given sample size and a combination of parameters, half-lives are skewed with the degree of skewness decreases as the process moves away from the neighborhood of inner regime (that is, as $F(\cdot)$ takes on values near unity). This is intuitive as in the neighborhood of inner regime, the *ESTAR* process is persistent and in the inner regime, the half-life is infinity. Overall, the simulation evidence reported in Table 1 reveals that regime-dependent half-life measure might be useful in analyzing the degree of persistence as well as the nonlinear persistence dynamics in the *ESTAR* models.

To gain some insights into the performance of conventional linear half-life measure when a linear model is incorrectly specified, in Table 2, we report simulated mean, and median half-lives with the lower and upper percentiles from 10,000 replications. Clearly, misspecification of a linear model implies inconsistency of linear-half lives as an estimator of half-lives. Nevertheless, there may be some cases in which linear-half lives works well as an approximation. Reported results in Table 2 show that whenever the *ESTAR* model is persistent, the distribution of linear half-lives are skewed and the confidence intervals are typically wider. This holds true even we double the sample size a sign of inconsistency. On the other hand, when the process becomes less persistent with a given transition parameter, linear half-lives suggest short half-lives with narrower confidence intervals. Also, when γ increases, and $\phi^* \geq -0.25$, we see an increase in simulated mean and median linear half-lives and corresponding confidence intervals. On the other hand, when $\phi^* \leq -0.25$, simulated mean and median linear half-lives and corresponding confidence intervals become smaller with increases in γ . This may suggest that linear-half lives may both over and under estimate the half-lives depending on the degree of persistence in the true DGP for the nonlinear *ESTAR* model. The first part of this finding supports Taylor's (2001) claim that inappropriate linear specification may result in larger half-life estimates if

there is nonlinearity in the adjustment process. The second part of the finding however, suggests that overestimation of half-life may depend on the parameter values that characterize the nonlinearity and persistence dynamics in the *ESTAR* model.

3 Empirical Results

3.1 Data, integration and linearity properties of PPP deviations

Quarterly consumer price index (CPI), and US dollar denominated nominal exchange rate data gathered from IMF's CD-rooms. Our sample period is 1973:I-1998:IV for Euro-zone currencies and 1973:I-2007:I for nonEuro zone currencies. The Euro-zone currencies include Belgian Franc (BF), Dutch Guilder (DG), French Franc (FF), German Mark (GM), Italian Lira (IL), and Spanish Peseta (PS). The nonEuro zone currencies are Australian Dollar (AD), Canadian Dollar (CD), Danish Koruna (DK), Japanese Yen (JY), Swiss Franc (SF), and UK Pound (UKP). Deviations from PPP are the logarithmic deviations of nominal exchange rate from the log price differentials between home and foreign countries. Therefore, $(q_t - q_0)$ measures the percentage deviation of PPP from its long run value. Except for AS and UKP, all exchange rates are defined to be the national currencies per US Dollar. For AD and UKP, exchange rates are the US Dollar price of AD and UKP.

Before estimating *ESTAR* models and regime-dependent half-lives, we examine integration properties of the PPP deviations by using several unit-root statistics and a stationarity test. We also look at the evidence from several linearity tests. Testing, estimation and subsequent diagnostic tests for STAR models assume stationarity of the data. Therefore, it is desirable to verify integration properties of the time series under study before advancing with estimation and testing of *ESTAR* models. To evaluate stationarity and nonstationarity, we use augmented Dickey-Fuller (*ADF*) test of Said and Dickey (1984) and Generalized Least Square Dickey-Fuller (*DFGLS*) test of Elliot et al. (1996). We use tests of Kapetanios, et al. (2003) (*KSS*), and the recent statistic developed in Park and Shintani (2005) (*PS*-test) which tests (linear) unit-root within the *ESTAR* model. Lastly, we also use *KPSS*-test of Kwiatkowski, et al. (1992) which

tests the null of (trend-) stationarity against the alternative of nonstationarity.⁷ In Table 3, we report results of alternative unit root and stationarity tests. Specifically, results from *ADF* and *DF_{GLS}* test are reported based on two different lag-length selection criteria, namely the modified AIC (MAIC) and the sequential testing procedure as suggested in Ng and Perron (2001). Similarly *KPSS* test is reported for the delay parameter $d = 1, 2, 3, 4$. Since in majority of cases MAIC and sequential testing procedures suggested different lag numbers and test results differ occasionally, we report results from both lag selections for *ADF* and *DF_{GLS}* tests. Similarly, we also report results from the *KPSS* test for Bandwidths of size $l = 4$ and $l = 8$ with Newey-West variance-covariance estimator by using Bartlett kernel.

Table 3 displays the findings from unit root and stationarity tests. Based on the MAIC, the *ADF* test fails to reject the unit root null hypothesis for all the series. On the other hand, when one follows the sequential testing procedure, the same test rejects three out of six Euro zone currencies and one out of six nonEuro zone currencies at conventional significance levels. Based on MAIC, *DF_{GLS}* rejects the null of unit root for five out of six Euro-zone series usually at 10% level except for Italian Lira for which the rejection is at 5% level. The unit root null is rejected for only one out of six nonEuro area currencies by *DF_{GLS}* test based on MAIC. The sequential procedure produces stronger evidence against the unit root null by rejecting the null for all Euro-zone series at 5% or 1% levels and three out of six nonEuro zone series at 5% or 10% significance levels. *DF_{GLS}* fails to reject the unit root null for Australian Dollar, Canadian Dollar and Japanese Yen. *KPSS* test provides notable evidence on stationarity of Euro-area currencies. On the other hand, *KPSS* test rejects null of stationarity in PPP deviations for all nonEuro zone series except for Danish Krone. Except for Swiss Franc, KSS test fails to reject the null of a linear unit root against a stationary ESTAR process. Contrary to evidence from KSS, results from *PS* test is encouraging of stationary ESTAR alternative. Except for Australian and Canadian Dollar series, *PS* test rejects the null of unit root against the alternative of an ESTAR process (which is presumably stationary). The evidence is strongest for Danish Krone, Swiss Franc and UK Pound.

⁷To conserve space and since most of the tests used are well-known in the literature, we do not provide a discussion of unit root and linearity tests. Reader are referred to the cited papers for details.

We use three linearity tests that have been suggested in the time series literature for testing linearity against the nonlinear STAR models. The test suggested by Teräsvirta (1994), which we denote by LM_{E1} , tests the presence of linearity against ESTAR form of nonlinearity. Teräsvirta (1994), 's test is based on a first order approximation of the exponential function around $\gamma = 0$. The second test which is denoted by LM_L , is suggested by Luukonen et al. (1988), and the test is based on a third order approximation of the logistic function around the null that $\gamma = 0$. This test is designed specifically for the logistic STAR (LSTAR) model, but as argued by Granger and Teräsvirta (1993), this test can have power against ESTAR type nonlinearity. The last test, we denote by LM_{E2} is due to Escribano and Jordà(1999). LM_{E2} statistic is based on a second order Taylor series approximation of the exponential function around $\gamma = 0$. Escribano and Jordà(1999) argue that Teräsvirta (1994)'s approach may not be sufficient to capture certain characteristics of the exponential function, especially, the two inflection points of the function. They show through simulations that LM_{E2} test has higher power than LM_{E1} test. In Table 2, we report p -values from the linearity tests. Using a liberal significance level, say 15%, linearity tests only provide evidence of nonlinearity of STAR type in only five out of twelve series. The evidence on nonlinearity for majority of the series is weak. This finding may however be because of the power problems in linearity tests. Theoretical and simulation results in Kılıç (2004) and Sandberg (2006) suggest that linearity tests may either provide spurious results or may have problems in detecting nonlinearity when the time series are persistent. Findings from linearity tests and unit root tests (especially the results from PS test which suggests notable evidence on a stationary ESTAR process) suggest the need to be cautious in interpreting the results from such tests. We should also note that linearity tests and KSS test are based on approximating the nonlinearity around the neighborhood of linearity and hence may have power and size problems in small samples. Indeed, as will be discussed in the following section, we further test linearity (i.e. $\gamma = 0$) against the ESTAR alternative (i.e. $\gamma > 0$) through simulations.

3.2 Linear and nonlinear dynamics and half-lives

Given the evidence from unit and linearity tests in the previous subsection, we estimate linear *AR* and nonlinear *ESTAR* models and calculate half-lives from alternative models. Following the literature, we estimate half-lives from the linear model by running an *ADF* regression, $q_t - q_0 = \phi(q_t - q_0) + \sum_{i=1}^p \delta_i \Delta q_{t-i} + u_t$. In Table 5, we report summary of estimation results from the *ADF* regressions, with point estimates as well as the normal sampling asymptotic 95% confidence intervals for the half-lives (see also Rossi 2006).⁸ Estimated autoregressive parameters ($\hat{\phi}$) varies between 0.880 (for Swiss Franc) to 0.990 (Canadian Dollar). Similarly approximate half-life estimates range between about five (Swiss Franc) to seventy one quarters (Canadian Dollar) (i.e. a slightly more than one year to fewer than eighteen years). Half-life estimates from the *ADF* regressions typically will be biased because of high persistence in PPP deviations (see the discussions in Murray and Papell 2002, 2004, and Rossi 2006). Rossi (2006) also points out that ignoring the impact of augmentation terms in the *ADF* regression should underestimate the true half-life, therefore reported estimates considered to be approximate and should be interpreted cautiously. On the other hand, Taylor (2001) argues that half-life estimates from *ADF* regression should overestimate the persistence in PPP deviations if the true DGP is a nonlinear process. The simulations reported in section 2.3 however, show that the impact of nonlinearity depends on the parameter values in the *ESTAR* model. Nevertheless, except for the Canadian Dollar, based on normal sampling asymptotic theory, the 95% normal confidence intervals include 0.38 to about 24 (six years) quarters for most currencies. Half-lives and upper bounds of confidence intervals for Euro-area currencies are much lower than the nonEuro zone currencies, a result consistent with the evidence from unit root and stationarity tests reported in Table 3.

Following Teräsvirta (2004), for each given delay parameter $d \in \{1, 2, 3, 4\}$, we estimate an *ESTAR* model for each series. We report results from the models which performed the best in

⁸In both *AR* and *ESTAR* models, lagged difference of PPP deviations of order p are included to account for serial correlation in the data. We select lag lengths by the MAIC and the sequential testing procedure proposed by Ng and Perron (2001). Complete estimation results can be obtained on request. The reported results are based on the *ADF* regressions with lag lengths determined by sequential testing procedures.

several diagnostics tests in Table 6. All estimates of ϕ are around unity while estimates of ϕ^* are all negative and $\widehat{\phi} + \widehat{\phi}^* < 1$ suggesting the asymptotic stationarity of estimated models. In column corresponding to $t_{\phi+\phi^*=1}$, we report t -statistics for testing the null hypothesis that $\phi + \phi^* = 1$ against the alternative that $\phi + \phi^* < 1$. Since the distribution of $t_{\phi+\phi^*=1}$ test may depend on the parameters of the *ESTAR* model under the null (especially in small samples), we have also computed p -values throughout simulations. First, we estimated the model under the null hypothesis and saved the residuals. Then we generate data by calibrating on the parameters of the null model with errors drawn randomly from the residuals of the estimated null model. We run 10,000 simulations with sample size of $100 + T$ where T is the sample size for each PPP deviations. Finally, we estimate *ESTAR* models under the alternative and the and compute the corresponding t -statistic in each run. The reported p -values are the frequency of times the simulated t -statistic is smaller than the reported t -values in the Table.⁹ Reported t -statistics and the simulated marginal significance levels indicate rejection of the null in favor of the alternative hypothesis for all PPP deviations studied.

Since under the null of $\phi^* = 0$, γ and μ are not identified and estimates of ϕ are statistically indistinguishable from unity, we also report marginal significance levels for testing $\phi^* = 0$ against the one-sided alternative that it is negative. The reported marginal significance levels (displayed in brackets) are computed through 10,000 simulations. The data is generated under the null hypothesis with residuals drawn from the estimated null model (that is linear *AR* model) and estimation of the *ESTAR* model. The reported p -values are the frequency of times, the simulated t -statistic is smaller than the t -value computed from the reported *ESTAR* model. Reported p -values suggest that estimated ϕ^* are significantly smaller than zero at conventional significance levels for all except for Australian Dollar rates (for Australian Dollar the p -values is 0.103 which is slightly above 10% significance level).

Estimated transition parameters (normalized by the standard error of transition variables)

⁹We have also estimated *ESTAR* models with $\phi = 1$ as tests for $\phi = 1$ fail to reject this null at conventional significance levels. Since estimated parameters and diagnostics were similar, we report results from the unrestricted model. Full results can be obtained on request from the author. Although not reported, we reject the null $\phi^* = -1$ at 1% for all series except for Danish Krone for which rejection rate is 10% level. These results contrast with findings of Taylor et al. (2001) on monthly data with a different sampling period.

are statistically significantly greater than zero at conventional significance levels for all PPP deviations except for the UK Pound (the t -ratio for UK Pound is 1.385). However, since under the null $\gamma = 0$, ϕ^* and μ are not identified (and the time series becomes a linear AR model with a possible unit root) we also calculate p -values for testing $\gamma = 0$ versus $\gamma > 0$ via simulations. We generate data by calibrating on the parameter of the linear AR null model with errors drawn from the residuals. The reported marginal significance levels are the frequency of times in 10,000 simulations, the t -statistic for $\gamma = 0$ exceeds the t -ratio in the estimated $ESTAR$ models for each series. This test can also be thought to be a test for linearity against the alternative of nonlinear $ESTAR$ process. Results provide considerable evidence in favor of nonlinear $ESTAR$ model as the simulated p -values for $\gamma = 0$ are below 10% significance level for all but Australian and Canadian Dollars. These results are consistent with the results from unit root tests, especially with the findings from PS test reported in Table 3. Reported diagnostic statistics are all satisfactory for all series except Australian Dollar, Japanese Yen, Swiss Francs, and British Pounds for which p -values from Jerque-Bera statistics suggest some departure from normality. Reported residual sum of squares values when compared with the values from the linear models (reported in Table 4) also reveal notable relative gain from nonlinear models. Overall, estimation and extensive diagnostics tests provide considerable evidence in support of the $ESTAR$ model for majority of the series studied.

Estimated transition parameters, γ , and the plots of the estimated transition functions over the sample period displayed in Figure 1 (for Euro-zone currencies) and Figure 2 (for nonEuro zone currencies) show striking similarity in transition dynamics across Euro-zone currencies. Estimated threshold parameters also reveal notable similarity across Euro-zone currencies in contrast to nonEuro zone currencies.¹⁰ Careful inspection of the plots reveal that PPP deviations visits both of the extreme regimes (i.e. inner and outer regimes) during the sample period. All Euro-zone PPP deviations are in the neighborhood of inner regime during early 1980s (a period of US Dollar appreciation), and stay near outer regime most of 1970s, late 1980s and most of 1990s. The transition dynamics is more heterogenous for nonEuro area currencies.

¹⁰Not reported for space considerations, plots of the transition functions over transition variables have the usual U -shape as expected under the exponential function. These results can be obtained on request.

Except Canadian Dollar, all nonEuro deviations inclined to stay in the neighborhood of inner regime during early 1980s. Except for Japanese Yen and UK Pound, all other nonEuro deviations swings towards inner regime late 1990s (around 1997) and again in early 2000 (around 2001). Among all, Danish Krone and Swiss Franc show the most number of swings between the two extreme regimes. Papell (1997, 2002), Koedijk et al. (1998), Lothian (1988) and Gadea et al. (2004) argue that the failure of unit root tests and resulting findings of persistence in PPP deviations are because of the behavior of US dollar during 1980s. Our results also suggest that during 1980s PPP deviations were more persistent as they stay mostly in the neighborhood of inner regime.

In Table 7, we report regime-dependent half-life estimates and 95% asymptotic normal confidence intervals based on normal sampling (values in parentheses) and the 95% simulated confidence bands. We report half-lives and confidence intervals for different regimes with various degree of deviations from PPP. In particular, results are reported conditional on the highest (i.e. the maximum value of the transition function, essentially when the PPP deviations are in the outer regime), average, median, the 10th, 25th, 75th and 90th percentiles) of the transition function. Estimated half lives from PPP deviations show the nonlinear nature of the deviations from PPP. Smaller half-lives with narrower 95% confidence intervals (from both methods) are obtained for larger deviations (whenever the PPP deviations are near the outer regime). On the other hand, we observe large half-lives with wider 95% confidence bands for smaller deviations (whenever the PPP deviations are in the neighborhood of inner regime). The reported half-lives and asymptotic and simulated confidence bands point out that Australian Dollar and Canadian Dollar PPP deviations are the most persistent among all others. Even in the outer regime half-lives for these currencies are more than seven quarters with wider confidence intervals. The simulated upper bounds are about 40 and 59 quarters for Australian Dollar and Canadian Dollar deviations respectively. On the other hand, point estimates for most of the currencies are around 2 to 6.5 quarters with simulated upper bounds about 7 (for most of the Euro-zone series) to 24 quarters (for the UK Pound).

For all series, point estimates as well as confidence intervals from both methods are closer

to each other for average and median regimes. Conditional on the mean and median regimes average point half-live for the Euro-zone currencies is slightly longer than one year. This finding is consistent with the point half-life estimates reported in studies by Taylor, et al. (2001), Baum et al. (2001) and Shintani (2006). Indeed, mean and median regime point estimates for all series indicate fast mean reversions in all PPP deviations. Even for the Australian and Canadian Dollar, estimates are about 8 and about 11 quarters respectively. The confidence intervals for the mean and median regimes however, reveal much variation in persistence of PPP deviations across currencies. For example, asymptotic upper bounds based on the mean-regime range between about five quarters (Dutch Guilder) to around forty quarters (Danish Krone). On the other hand, simulated upper bounds are between slightly less than eight quarters (French Franc) to forty two and half quarters (Danish Krone). We note similar variations for the median regime confidence intervals as well as other regimes. Whenever PPP deviations are in the neighborhood of inner regime, point estimates show considerable persistence especially for nonEuro zone currencies.¹¹ Interestingly enough, point estimates for Danish Krone deviations suggest more persistence than Australian and Canadian Dollar deviations conditional on the lowest 10th quantile of the estimated transition function (i.e. $F_{10\%}$). Upper bound for the asymptotic confidence intervals for Krone deviations are significantly larger than those of Australian and Canadian Dollar deviations. However, corresponding simulated upper bounds for Krone is much lower not only than the latter two deviations but also several other deviations including Belgian Franc, Dutch Guilder, and German Mark. These observations reveal that in nonlinear models, the degree of persistence may also vary across currencies and regimes as such a currency that is more persistent in one regime may be less persistent in another regime.

Regime dependent half-lives and confidence intervals illustrate that mostly, simulated confidence intervals are much wider than the normal sampling intervals. The difference between asymptotic and simulated confidence intervals increases as the PPP deviations become more persistent (i.e. as the PPP deviations are near the inner regime). This may suggest that normal

¹¹Since whenever the PPP deviations are exactly in the inner regime, i.e. $F(.) = 0$, half-life estimates tend to infinity as the estimated *ESTAR* model parameters imply the deviations may behave nearly a unit root process or even a process with explosive roots. Therefore, we do not analyze the half-lives when $F(.) = 0$.

sampling confidence intervals may under estimate the uncertainty in the degree of persistence. This finding is also consistent with the findings from linear half-life literature (see Rossi 2006 who shows normal sampling method underestimate especially the upper bounds in linear models). Careful inspection of the reported results show that for most PPP deviations, regime dependent half-lives and upper bounds decline as the PPP deviations approach to outer regime where the speed of adjustment to long-run PPP is much faster. Results also tell that on average Euro-zone currencies and the currencies that are closer to the zone have less persistent deviations than the other currencies. This finding agrees with the results from unit root tests (Table 3) as well as parameter estimates from linear and *ESTAR* models reported in tables 5 and 6 respectively.

4 Conclusions

In this paper, we study persistence of PPP deviations within the context of estimated *ESTAR* models by using regime-dependent half-lives as a measure of persistence. Regime-dependent confidence intervals for the point estimates calculated through normal sampling approach and through simulations. We analyze the small sample performance of regime-dependent half-life estimates via simulations. Results show that the proposed method is useful in analyzing the nonlinear persistence dynamics of PPP deviations. Monte Carlo results show that 95% confidence intervals may vary across regimes and with the parameter values that characterize the persistence in the *ESTAR* model and the nonlinear transition dynamics. The experiments also show that half-life estimates based on linear *AR* models when the true DGP is a nonlinear *ESTAR* process may both over and under estimate the true half-lives depending on the parameter values in the *ESTAR* model.

Extensive diagnostic statistics, tests and estimation of linear and nonlinear models show that *ESTAR* models characterize the nonlinear dynamics in quarterly US Dollar PPP deviations well over the floating period. Our findings point out differences in transition dynamics and persistence across Euro-zone and nonEuro zone currencies. Findings from regime-dependent

half-lives reveal that point estimates and especially the upper bounds of 95% confidence intervals vary significantly with the size of the deviations. The larger are the deviations the smaller are the point estimates for half-lives and the narrower are the confidence intervals. The smaller PPP deviations are more persistent than the large deviations with sizable upper bounds of confidence intervals. Our findings illustrate the usefulness of obtaining confidence intervals in understanding the persistence dynamics of PPP deviations in nonlinear models. Although, point estimates conditional on an average history of PPP deviations may imply fast mean reversion, confidence intervals provide more information on how persistence changes with the size of deviations from PPP. It turns out that regime-dependent confidence intervals provide more accurate information about the sampling uncertainty and hence the persistence of PPP deviations. Findings in the paper show that upper bounds vary significantly across regimes and currencies. This suggests the need for exercising caution in interpreting the results from the recent nonlinear empirical literature on PPP which argue that accounting for nonlinearities may resolve the PPP puzzle.

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Table 1: Small sample performance of conditional half-lives

		$T = 100$					$T = 200$				
		F_{med}	$F_{10\%}$	$F_{25\%}$	$F_{75\%}$	$F_{90\%}$	F_{med}	$F_{10\%}$	$F_{25\%}$	$F_{75\%}$	$F_{90\%}$
		$\phi = 1.0, \phi^* = -0.1, \gamma = 0.1 \mu = 0.0$									
h_s^0		13.51	68.97	27.38	8.89	7.35	13.51	68.97	27.38	8.89	7.3
\widehat{h}_s	mean	9.76	52.12	20.81	7.60	6.61	7.81	48.47	17.07	6.65	6.10
	med	6.62	57.86	12.08	6.58	6.58	5.19	35.94	9.26	4.34	4.93
	2.5%	0.42	0.32	0.37	0.24	0.00	0.53	0.34	0.43	0.42	0.22
	97.5%	51.81	175.06	102.46	50.36	39.08	32.31	174.92	78.34	34.65	25.24
		$\phi = 1.0, \phi^* = -0.25, \gamma = 0.1 \mu = 0.0$									
h_s^0		5.19	27.38	10.74	3.34	2.72	5.19	27.38	10.74	3.34	2.72
\widehat{h}_s	mean	6.73	42.56	14.18	3.84	3.32	5.00	37.36	11.58	3.18	2.88
	med	3.33	27.38	8.90	2.47	2.41	2.98	25.03	6.76	2.41	2.31
	2.5%	0.39	0.29	0.30	0.44	0.34	0.48	0.30	0.32	0.48	0.35
	97.5%	46.47	169.27	73.20	22.79	16.61	25.64	157.97	58.92	14.21	10.61
		$\phi = 1.0, \phi^* = -0.5, \gamma = 0.1 \mu = 0.0$									
h_s^0		2.41	13.51	5.19	1.48	1.16	2.41	13.51	5.19	1.48	1.16
\widehat{h}_s	mean	4.00	35.14	10.98	2.25	2.01	3.75	32.21	9.99	2.33	2.05
	med	2.02	13.51	5.45	1.24	1.06	1.94	13.51	5.19	1.24	1.09
	2.5%	0.22	0.24	0.23	0.00	0.00	0.24	0.23	0.23	0.34	0.16
	97.5%	24.87	159.42	63.36	12.01	8.94	20.59	153.44	59.32	11.52	8.50
		$\phi = 1.0, \phi^* = -1.0, \gamma = 0.1 \mu = 0.0$									
h_s^0		1.00	6.58	2.41	0.50	0.30	1.00	6.58	2.41	0.50	0.30
\widehat{h}_s	mean	2.81	25.86	8.78	1.44	1.11	2.61	23.68	8.42	1.35	1.09
	med	1.00	7.67	3.17	0.53	0.41	0.96	6.58	2.68	0.59	0.54
	2.5%	0.00	0.22	0.20	0.00	0.00	0.00	0.20	0.19	0.00	0.00
	97.5%	19.33	140.12	63.61	7.16	4.74	14.62	129.02	57.91	6.47	4.74
		$\phi = 1.0, \phi^* = -0.1, \gamma = 1.0 \mu = 0.0$									
h_s^0		13.51	68.97	27.38	8.89	7.35	13.51	68.97	27.38	8.89	7.35
\widehat{h}_s	mean	8.15	50.00	19.33	6.20	5.79	7.67	47.07	17.10	6.51	6.69
	med	6.58	43.97	10.52	5.81	6.26	5.97	34.05	8.92	5.17	5.48
	2.5%	0.42	0.35	0.39	0.00	0.00	0.55	0.38	0.48	0.42	0.32
	97.5%	30.99	177.34	92.66	24.25	22.18	28.33	175.25	85.18	26.28	20.61
		$\phi = 1.0, \phi^* = -0.25, \gamma = 1.0 \mu = 0.0$									
h_s^0		5.19	27.38	10.74	3.34	2.72	5.19	27.38	10.74	3.34	2.72
\widehat{h}_s	mean	5.07	40.82	12.70	3.54	3.09	4.93	37.49	11.39	3.49	3.18
	med	3.17	27.38	7.78	2.46	2.41	3.07	25.53	6.83	2.43	2.41
	2.5%	0.47	0.30	0.31	0.48	0.36	0.59	0.35	0.39	0.44	0.33
	97.5%	21.12	165.15	61.77	13.23	10.59	14.69	138.19	43.89	10.34	8.69
		$\phi = 1.0, \phi^* = -0.5, \gamma = 1.0 \mu = 0.0$									
h_s^0		2.41	13.51	5.19	1.48	1.16	2.41	13.51	5.19	1.48	1.16
\widehat{h}_s	mean	3.66	22.05	9.07	2.06	1.82	3.37	20.59	6.37	2.07	1.72
	med	2.01	13.51	5.34	1.18	1.05	1.98	13.51	5.19	1.14	1.04
	2.5%	0.15	0.22	0.20	0.00	0.00	0.00	0.21	0.19	0.00	0.00
	97.5%	18.09	141.10	43.66	9.55	7.42	13.01	112.70	26.55	8.51	6.87
		$\phi = 1.0, \phi^* = -1.0, \gamma = 1.0 \mu = 0.0$									
h_s^0		1.00	6.58	2.41	0.50	0.30	1.00	6.58	2.41	0.50	0.30
\widehat{h}_s	mean	2.52	20.84	9.05	1.20	1.00	2.59	13.17	8.12	1.32	0.96
	med	1.06	8.16	3.65	0.50	0.35	1.02	6.58	2.31	0.51	0.37
	2.5%	0.00	0.27	0.23	0.00	0.00	0.00	0.26	0.21	0.00	0.00
	97.5%	13.40	115.17	37.57	6.04	4.04	10.42	90.97	20.38	4.28	3.12

Key: Mean, median, 95% quantiles and mean square error of simulated half-lives from 10000 simulations.

Table 2: Simulated half-lives from the *AR* model. DGP:

$$y_t = \phi y_{t-1} + \phi^* y_{t-1} (1 - e^{\gamma(y_{t-1} - \mu)^2}) + u_t$$

	$T = 100$		$T = 200$	
	$\phi^* = -0.1, \gamma = 0.1$	$\phi^* = -0.1, \gamma = 1.0$	$\phi^* = -0.1, \gamma = 0.1$	$\phi^* = -0.1, \gamma = 1$
mean	22.27	29.09	29.19	33.12
med	4.72	5.28	7.51	9.24
2.5%	0.00	0.00	0.00	0.00
97.5%	88.52	102.27	112.14	141.69
	$\phi^* = -0.25, \gamma = 0.1$	$\phi^* = -0.25, \gamma = 1.0$	$\phi^* = -0.25, \gamma = 0.1$	$\phi^* = -0.25, \gamma = 1$
mean	5.68	5.28	3.85	4.59
med	2.75	3.24	2.72	3.33
2.5%	0.88	1.16	1.19	1.51
97.5%	22.96	20.75	12.56	14.34
	$\phi^* = -0.5, \gamma = 0.1$	$\phi^* = -0.5, \gamma = 1.0$	$\phi^* = -0.5, \gamma = 0.1$	$\phi^* = -0.5, \gamma = 1$
mean	2.46	2.39	1.70	2.39
med	1.74	1.80	1.33	1.20
2.5%	0.59	0.75	0.42	0.35
97.5%	8.25	6.90	4.80	4.59
	$\phi^* = -1.0, \gamma = 0.1$	$\phi^* = -1.0, \gamma = 1.0$	$\phi^* = -1.0, \gamma = 0.1$	$\phi^* = -1.0, \gamma = 1$
mean	1.56	0.85	1.56	0.83
med	1.20	0.64	1.25	0.64
2.5%	0.35	0.16	0.43	0.16
97.5%	4.59	2.45	4.12	2.23

Key: Mean, median, 95% quantiles and mean square error of simulated half-lives from 10000 simulations.

Table 3: Unit Root and Stationarity Tests for Quarterly US Dollar Real Exchange Rates

Series	<i>ADF</i>		<i>DF_{GLS}</i>		<i>KPSS</i>		<i>KSS</i>	<i>PS(d)</i>			
	<i>maic</i>	<i>seq</i>	<i>maic</i>	<i>seq</i>	l_4	l_8		$d = 1$	$d = 2$	$d = 3$	$d = 4$
BF(4)	-1.71(1)	-2.18(3)	-1.67*(1)	-2.17†(3)	0.28	0.17	-1.74	-2.73	-3.01	-3.08*	-3.22*
DG(4)	-1.92(1)	-2.46(3)	-1.78*(1)	-2.30†(3)	0.23	0.15	-1.77	-2.75	-3.00	-3.23*	-3.56†
FF(4)	-2.00(1)	-2.60*(3)	-1.93*(1)	-2.61‡(3)	0.21	0.14	-1.74	-3.26*	-3.25*	-3.40†	-1.40
GM(4)	-1.95(1)	-2.42(3)	-1.89*(1)	-2.42†(3)	0.26	0.17	-1.72	-3.00	-3.30†	-3.61†	-3.83†
IL(3)	-2.34(1)	-3.04†(3)	-2.31†(1)	-3.04‡(3)	0.32	0.22	-2.47	-2.92	-3.15*	-3.11*	-3.34†
SP(3)	-1.99(1)	-2.69*(3)	-1.52(1)	-2.16†(3)	0.36*	0.23	-2.10	-2.83	-3.06*	-3.04*	-3.16*
AD(3)	-1.90(1)	-1.90(1)	-1.39(1)	-1.39(1)	1.33‡	0.81‡	-0.94	-1.49	-1.69	-1.58	-1.95
CD(3)	-1.94(3)	-1.62(1)	-1.19(3)	-0.94(1)	1.38‡	0.81‡	-2.32	-2.52	-2.62	-2.73	-2.39
DK(3)	-2.56(3)	-2.56(3)	-2.28†(3)	-2.28†(3)	0.15	0.10	-2.07	-3.45†	-3.52†	-3.45†	-3.85†
JY(4)	-2.15(1)	-2.45(4)	-1.18(1)	-1.42(4)	1.12‡	0.69†	-2.49	-2.72	-3.06*	-2.83	-2.87
SF(4)	-2.54(1)	-3.15*(4)	-1.48(1)	-1.82*(4)	0.37*	0.24	-2.68*	-3.48†	-3.40†	-3.57†	-4.10‡
UKP(3)	-2.03(2)	-2.47(3)	-1.27(2)	-1.63*(3)	0.82‡	0.53†	-2.65	-3.39†	-3.87‡	-4.12‡	-3.62†

Key: The values in parentheses next the names of the series are the lag lengths selected by sequential testing procedure of Ng and Perron (2001) (starting a maximal lag of 8) in the ESTAR models and the reported *KSS* and *PS* tests are computed by using the indicated lag lengths. *maic* and *seq* refers to Modified AIC and sequential lag selection procedures suggested by Ng and Perron (2001) respectively. The 1%, 5%, and 10% critical values for ADF test with an intercept are -3.43, -2.86, and -2.57 respectively. The 1%, 5%, and 10% critical values for *DF_{GLS}* test without a trend (interpolated critical values from tables presented by Elliott, et al. 1996) are -2.60, -1.95, and -1.62 respectively. The 1%, 5% and 10% critical values for KPSS test with a constant are 0.739, 0.463 and 0.147 respectively. The 1%, 5%, and 10% critical values for KSS test with an intercept are -3.48, -2.93, and -2.66 respectively. KSS test is computed using demeaned data in all countries. The 1%, 5%, and 10% critical values for *PS* test with an intercept are -3.86, -3.30, and -3.03 respectively. In all cases ‡, † and * denote significance at 1%, 5%, and 10% levels respectively.

Table 4: p -values for linearity tests for quarterly US Dollar real exchange rates

p -values	q_{t-1}	q_{t-2}	q_{t-3}	q_{t-4}	q_{t-1}	q_{t-2}	q_{t-3}	q_{t-4}	q_{t-1}	q_{t-2}	q_{t-3}	q_{t-4}
	Australian Dollar				Belgian Franc				Canadian Dollar			
pLM_{E1}	0.730	0.595	0.634	0.608	0.658	0.694	0.895	0.855	0.373	0.207	0.374	0.809
pLM_L	0.375	0.356	0.524	0.587	0.519	0.483	0.603	0.731	0.325	0.202	0.437	0.254
pLM_{E2}	0.500	0.466	0.331	0.519	0.365	0.529	0.828	0.653	0.644	0.321	0.416	0.799
	Danish Krona				Dutch Guilder				French Franc			
pLM_{E1}	0.185	0.136	0.409	0.113	0.445	0.737	0.733	0.687	0.046	0.093	0.431	0.219
pLM_L	0.126	0.126	0.271	0.301	0.298	0.383	0.182	0.439	0.091	0.082	0.164	0.162
pLM_{E2}	0.060	0.082	0.189	0.060	0.181	0.515	0.599	0.409	0.022	0.102	0.209	0.246
	German Mark				Italian Lira				Japanese Yen			
pLM_{E1}	0.387	0.481	0.441	0.411	0.435	0.151	0.214	0.096	0.126	0.403	0.528	0.280
pLM_L	0.311	0.317	0.244	0.395	0.530	0.343	0.349	0.126	0.216	0.510	0.599	0.320
pLM_{E2}	0.142	0.321	0.279	0.227	0.344	0.108	0.274	0.241	0.166	0.300	0.331	0.341
	Spanish Peseta				Swiss Franc				UK Pound			
pLM_{E1}	0.538	0.396	0.654	0.554	0.579	0.742	0.680	0.435	0.198	0.120	0.090	0.244
pLM_L	0.513	0.553	0.736	0.778	0.433	0.621	0.612	0.461	0.388	0.028	0.179	0.113
pLM_{E2}	0.405	0.309	0.393	0.449	0.457	0.611	0.363	0.261	0.344	0.038	0.077	0.776

Key: LM_{E1} , LM_L and LM_{E2} are the linearity tests of Teräsvirta (1994), Luukonen et al. (1988), and Escribano and Jordà(1999) respectively as discussed in the text. The values in the first rows for each test correspond to the p - values from the F - distribution with the appropriate degrees of freedom.

Table 5: Results from the linear AR models and half-life estimates with 95% asymptotic confidence intervals:

Estimated Models: $q_t - \mu = \phi(q_{t-1} - \mu) + \sum_{i=1}^p \delta_i \Delta q_{t-i} + u_t$.

series	$\hat{\phi}$	SSR	p_{LM}	p_{JB}	ADF	\hat{h}	(c_L, c_U)
BF	0.919 (0.031)	0.310	0.103	0.609	-2.042	8.21	(1.62, 14.80)
DG	0.896 (0.037)	0.306	0.128	0.758	-2.683*	8.42	(1.25, 15.58)
FF	0.898 (0.037)	0.282	0.117	0.778	-2.629*	6.43	(1.53, 11.34)
GM	0.902 (0.037)	0.309	0.171	0.590	-2.607*	6.71	(1.56, 11.85)
IL	0.912 (0.039)	0.294	0.056	0.268	-2.613*	7.53	(0.72, 14.33)
SP	0.931 (0.029)	0.267	0.110	0.814	-2.690*	9.67	(1.42, 17.91)
AD	0.945 (0.026)	0.315	0.023	0.013	-1.901	12.18	(0.61, 23.75)
CD	0.990 (0.013)	0.083	0.039	0.812	-1.901	70.74	(0.00, 225.46)
DK	0.946 (0.027)	0.395	0.111	0.227	-2.735*	12.16	(0.38, 23.95)
JY	0.935 (0.026)	0.445	0.121	0.032	-2.428	10.53	(2.00, 19.03)
SF	0.880 (0.037)	0.500	0.119	0.316	-2.685*	5.43	(1.90, 8.98)
UKP	0.916 (0.034)	0.312	0.094	0.000	-2.759*	13.00	(1.37, 14.44)

Key: Table reports the LSE from the linear model. p_{LM} is the p -values from maximal Ljung-Box test statistics for up to one, two, three and fourth order serial correlations in residuals, p_{JB} is the p -value for the JB statistics for testing normality of residuals. \hat{h} is the half-life estimate that is based on the $AR(1)$ parameter and c_L and c_U reports the asymptotic 95% confidence intervals computed through Delta method.

Table 6: Estimated ESTAR models and diagnostics statistics:

$$q_t - \mu = \hat{\phi}(q_{t-1} - \mu) + \hat{\phi}^*(q_{t-1} - \mu) \left(\exp\left(-\frac{\hat{\gamma}}{se(q_{t-d})} (q_{t-d} - \mu)^2\right) \right) + \sum_{i=1}^p \delta_i \Delta q_{t-i} + u_t.$$

	\hat{d}	$\hat{\phi}$	$\hat{\phi}^*$	$\hat{\gamma}$	$\hat{\mu}$	$t_{\hat{\phi}+\hat{\phi}^*}$	p_{LM}	p_C	p_{NLE1}	p_{NLL}	p_{NLE2}	p_{JB}	SSR
BF	3	1.002 (0.066)	-0.154 (0.073)[0.006]	2.743 (1.685)[0.078]	0.304 (0.057)	-3.036 [0.027]	0.494	0.201	0.728	0.851	0.795	0.947	0.292
DG	4	1.084 (0.088)	-0.267 (0.091)[0.109]	2.387 (0.903)[0.044]	0.225 (0.030)	-3.893 [0.009]	0.623	0.170	0.571	0.370	0.468	0.587	0.281
FF	3	1.056 (0.067)	-0.274 (0.087)[0.084]	2.158 (1.196)[0.067]	0.254 (0.037)	-3.462 [0.014]	0.515	0.539	0.772	0.757	0.761	0.449	0.255
GM	4	1.107 (0.079)	-0.307 (0.047)[0.094]	2.727 (1.041)[0.038]	0.240 (0.022)	-4.671 [0.001]	0.105	0.608	0.299	0.658	0.468	0.757	0.270
IL	2	1.112 (0.078)	-0.243 (0.086)[0.053]	3.820 (1.037)[0.051]	0.179 (0.021)	-2.879 [0.021]	0.182	0.354	0.055	0.219	0.342	0.297	0.278
SP	2	1.105 (0.074)	-0.208 (0.072)[0.014]	2.857 (0.997)[0.061]	0.230 (0.028)	-2.626 [0.028]	0.518	0.283	0.570	0.850	0.316	0.741	0.250
AD	3	1.027 (0.099)	-0.112 (0.099)[0.103]	3.620 (2.376)[0.128]	-0.196 (0.041)	-3.029 [0.071]	0.482	0.234	0.251	0.173	0.120	0.079	0.291
CD	2	1.119 (0.067)	-0.183 (0.068)[0.055]	3.786 (0.911)[0.103]	0.129 (0.012)	-2.885 [0.082]	0.084	0.443	0.574	0.448	0.258	0.338	0.076
DK	1	1.387 (0.186)	-0.501 (0.187)[0.061]	2.646 (0.618)[0.047]	-0.041 (0.036)	-3.403 [0.012]	0.149	0.380	0.367	0.577	0.568	0.738	0.359
JY	1	1.034 (0.038)	-0.129 (0.051)[0.000]	4.658 (1.486)[0.062]	0.283 (0.018)	-2.950 [0.030]	0.397	0.188	0.204	0.319	0.422	0.077	0.430
SF	4	1.078 (0.087)	-0.266 (0.093)[0.089]	2.647 (1.386)[0.089]	0.147 (0.034)	-4.114 [0.004]	0.421	0.366	0.119	0.233	0.205	0.003	0.455
UKP3		1.016 (0.072)	-0.147 (0.084)[0.013]	2.597 (1.875)[0.093]	-0.174 (0.056)	-2.825 [0.036]	0.122	0.217	0.764	0.289	0.291	0.047	0.304

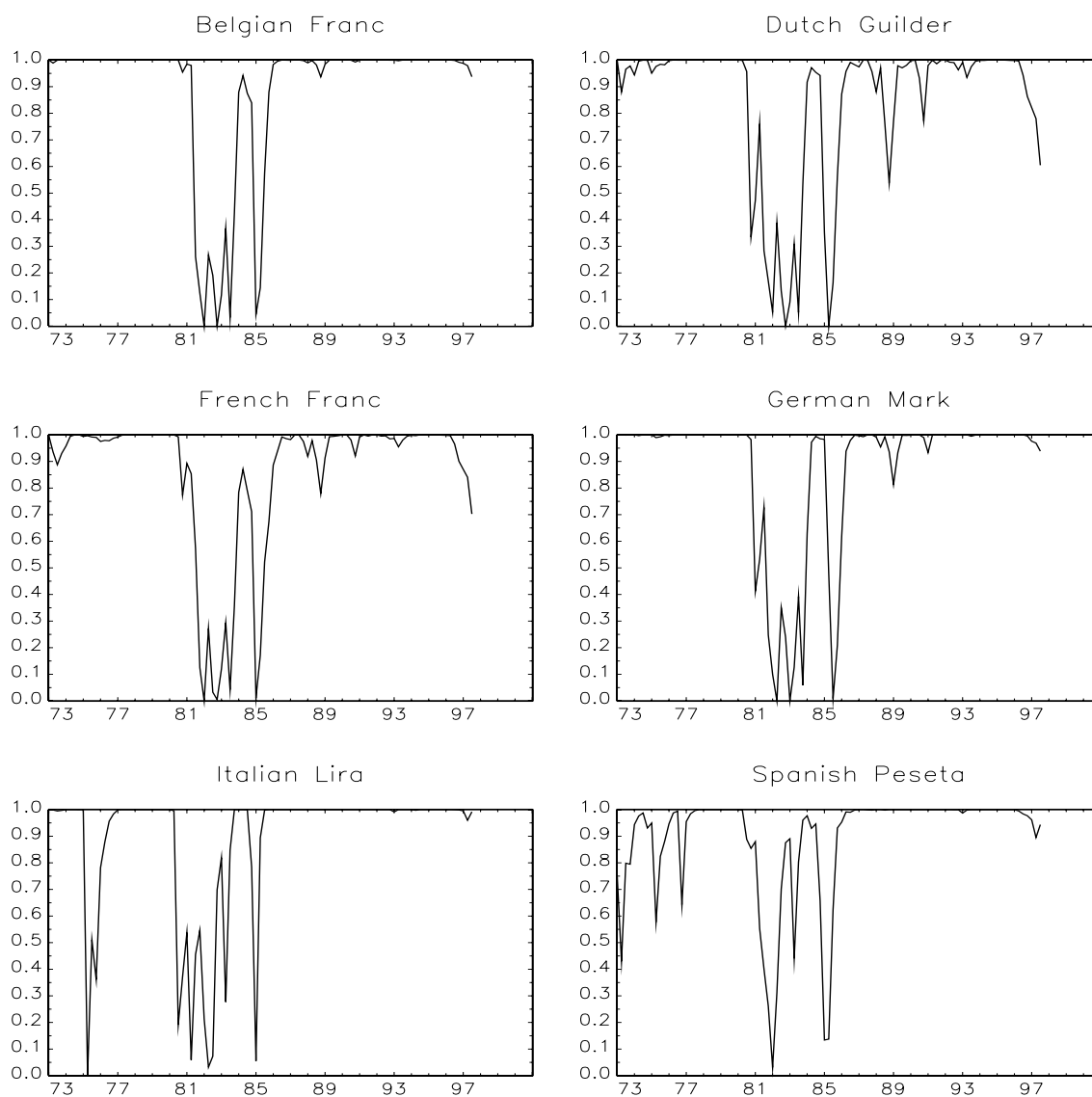
Key: For each country ESTAR models are estimated by using constrained MLE library in Gauss. The transition parameter in the ESTAR model (γ) is standardized by the standard error of the transition variable. The delay parameter d and reported models are selected by using the lowest standard error of the regression from the estimated ESTAR models together with the diagnostic tests reported in the table. Reported models also compared and contrasted with linear specifications in terms of similar diagnostic tools. p_{LM} is the p -values from LM test statistics for up to fourth order serial correlations in residuals, p_C is the p -value for testing parameter constancy in the estimated model, and p_{NLE} and p_{NLL} are the p -values corresponding to the *maximum* LM test statistic for no remaining nonlinearity of exponential (ESTAR) and logistic LSTAR form respectively with delay parameter in the range 1 to 4. (See Eitrheim and Teräsvirta, 1996).

Table 7: Conditional Half-lives and 95% Confidence Intervals in quarters

	F_{max}	F_{mean}	F_{med}	$F_{10\%}$	$F_{25\%}$	$F_{75\%}$	$F_{90\%}$
BF	4.84 (0.00-12.82) [0.27-10.44]	5.23 (0.00-11.33) [0.48-10.81]	4.84 (0.00-10.76) [0.33-13.62]	5.70 (0.00-12.76) [0.00-33.56]	4.90 (0.00-10.92) [0.41-24.86]	4.84 (0.00-10.78) [0.30-11.49]	4.84 (0.00-10.92) [0.27-10.94]
DG	2.23 (0.00-4.47) [0.34-8.66]	2.73 (0.58-4.88) [0.63-11.47]	2.29 (0.44-4.13) [0.00-13.06]	8.40 (2.19-14.61) [0.00-84.52]	2.72 (0.59-4.85) [0.00-52.76]	2.24 (0.41-4.06) [0.34-9.30]	2.23 (0.20-4.27) [0.34-8.78]
FF	2.17 (0.29-4.04) [0.30-7.39]	2.60 (0.76-4.43) [0.61-7.72]	2.20 (0.60-3.80) [0.41-14.24]	8.60 (1.74-15.45) [0.74-28.54]	2.52 (0.72-4.32) [0.25-20.92]	2.17 (0.60-3.74) [0.30-9.70]	2.82 (0.26-5.38) [0.30-7.58]
GM	3.10 (0.00-6.70) [0.35-8.66]	3.40 (0.00-6.57) [0.62-13.66]	3.10 (0.02-6.19) [0.00-16.16]	4.49 (0.00-9.41) [0.00-36.53]	3.17 (0.00-6.33) [0.00-21.06]	3.10 (0.02-6.19) [0.35-9.21]	3.10 (0.00-6.26) [0.35-8.94]
IL	4.91 (0.00-17.75) [0.36-11.11]	5.24 (0.00-15.61) [0.00-20.21]	4.91 (0.00-15.29) [0.00-17.75]	6.69 (0.00-23.01) [0.00-49.09]	4.92 (0.00-15.35) [0.00-43.45]	4.91 (0.00-15.30) [0.34-11.96]	4.91 (0.00-15.78) [0.35-11.43]
SP	6.39 (0.00-24.88) [0.35-14.13]	7.22 (0.00-25.55) [0.49-20.67]	6.41 (0.00-22.73) [0.00-22.67]	10.94 (0.00-49.19) [0.00-38.88]	7.25 (0.00-26.79) [0.00-32.67]	6.39 (0.00-22.74) [0.34-14.61]	6.39 (0.00-23.46) [0.34-14.08]
AD	7.87 (0.0-29.89) [0.27-40.22]	8.73 (0.00-29.96) [0.42-36.79]	7.87 (0.13-27.81) [0.00-50.60]	15.06 (0.00-62.67) [0.00-58.87]	8.05 (0.00-28.54) [0.00-57.37]	7.87 (0.00-27.86) [0.24-41.69]	7.87 (0.00-27.21) [0.27-43.35]
CD	10.32 (0.00-39.47) [0.00-58.76]	11.22 (0.00-36.48) [0.00-36.08]	10.33 (0.00-34.90) [0.00-57.07]	15.72 (0.00-65.09) [0.00-48.30]	10.63 (0.00-36.36) [0.00-55.04]	10.33 (0.00-35.10) [0.00-54.35]	10.33 (0.00-38.52) [0.00-51.98]
DK	5.73 (0.05-37.49) [0.00-20.28]	8.05 (0.00-39.94) [0.00-42.50]	7.10 (0.00-37.04) [0.00-36.60]	22.02 (0.00-229.36) [0.00-30.32]	11.16 (0.00-79.17) [0.00-62.53]	5.89 (0.00-35.86) [0.00-27.26]	5.74 (0.00-36.68) [0.00-24.98]
JY	6.45 (0.00-22.57) [0.31-12.01]	7.51 (0.00-19.17) [0.50-13.11]	6.95 (0.0-18.37) [0.34-14.22]	8.83 (0.00-24.14) [0.00-196.50]	6.95 (0.00-18.37) [0.00-71.51]	6.95 (0.00-18.37) [0.31-11.61]	6.73 (0.00-18.66) [0.31-11.90]
SF	3.33 (0.00-8.15) [0.36-8.37]	4.18 (0.00-8.72) [0.61-10.31]	3.43 (0.00-7.51) [0.30-11.36]	11.06 (0.00-41.18) [0.00-66.62]	4.80 (0.00-21.35) [0.00-22.32]	3.34 (0.00-7.61) [0.35-8.80]	3.34 (0.00-8.00) [0.35-8.38]
UKP	4.96 (0.00-13.77) [0.00-23.93]	5.85 (0.00-13.22) [0.00-31.58]	5.07 (0.00-11.92) [0.00-31.22]	11.61 (0.00-39.21) [0.00-68.68]	6.09 (0.00-15.31) [0.00-55.84]	4.96 (0.00-11.85) [0.34-26.63]	4.96 (0.00-11.91) [0.35-23.97]

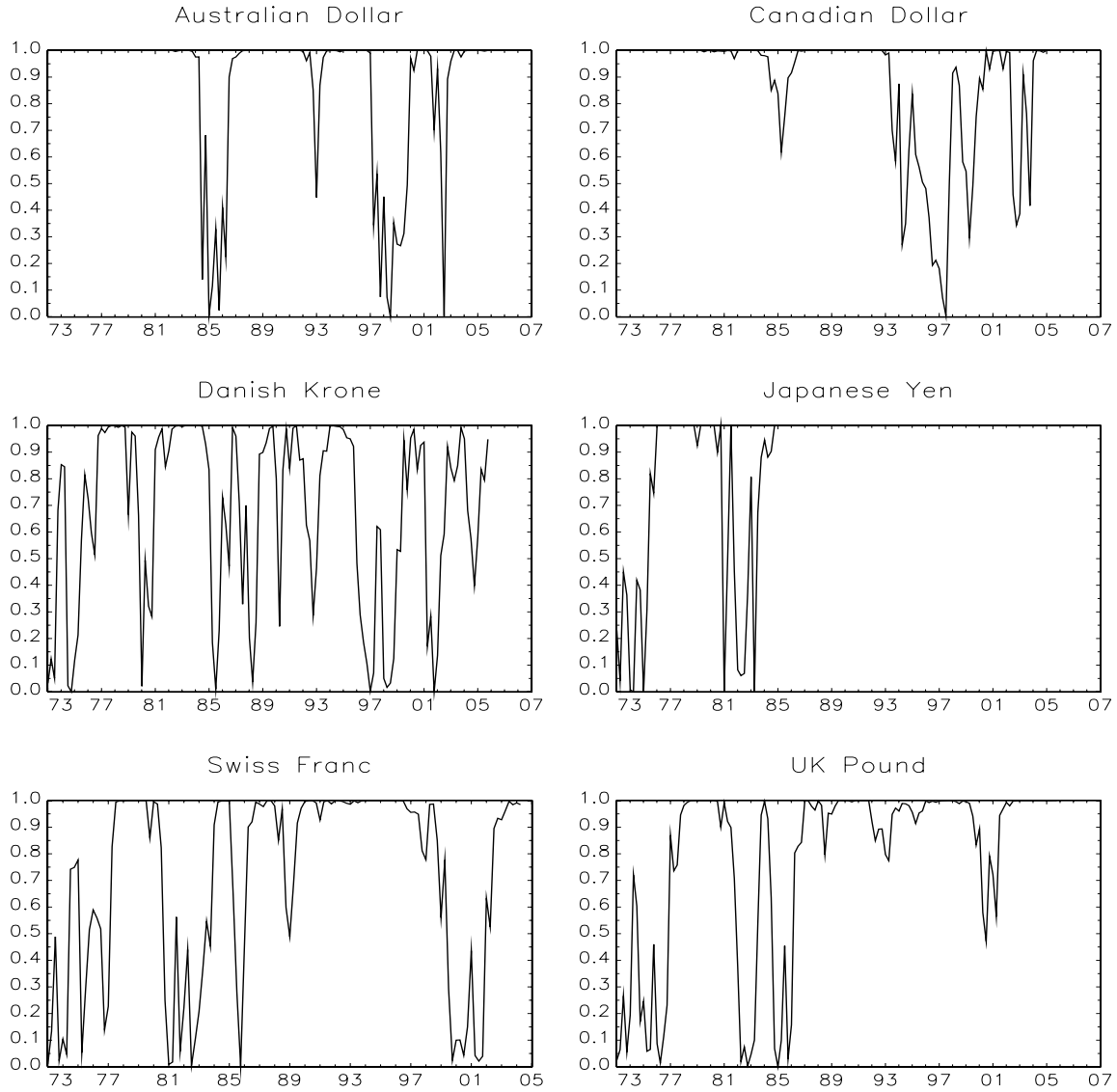
Key: The values in first rows for each currency are the point estimates for the conditional half-life. The values inside the parentheses are the lower and upper bounds of the 95% confidence intervals computed from a delta method approximation.

Figure 1: Estimated transition functions: Euro-zone currencies



Key: The Figure plots the estimated transition functions over time for Euro-zone PPP deviations.

Figure 2: Estimated transition functions: Non-Euro zone currencies



Key: The Figure plots the estimated transition functions against time for non-Euro zone PPP deviations.