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## THE BACKUS-SMITH PUZZLE: THE ROLE OF EXPECTATIONS

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#### Resumen

Una eficiente diversificación de riesgo establece una relación positiva entre el tipo de cambio real y el consumo relativo entre países: el consumo debiera ser relativamente más alto cuando éste es comparativamente más barato. Sin embargo, contrario a la relación predicha por la mayoría de los modelos, la correlación empírica entre el tipo de cambio real y el consumo relativo es negativa (Backus y Smith, 1993). En este artículo extiendo un modelo estándar de dos países y dos bienes con mercados financieros internacionales incompletos introduciendo señales públicas de innovaciones futuras al factor de productividad total. En este contexto, una señal positiva aumenta el valor presente del flujo de ingresos doméstico, implicando que el consumo corriente puede aumentar más que el producto contemporáneo. Este aumento en la demanda genera una apreciación en el tipo de cambio real, sugiriendo así una potencial solución al puzzle de Backus-Smith. Cuando la economía es calibrada para los Estados Unidos y el resto de las economías industrializadas, las simulaciones numéricas muestran una correlación entre el tipo de cambio real y consumo relativo similar a la observada empíricamente.

#### Abstract

Efficient risk-sharing dictates a positive relationship between the real exchange rate and relative consumption across countries: consumption should be relatively high where consumption is relatively cheap. However, contrary to the positive relationship predicted by most models, the empirical correlation between bilateral real exchange rates and relative consumptions is typically negative (see Backus and Smith, 1993). In this paper I extend a standard two-country, two-good international business cycle model with internationally incomplete financial markets to incorporate public signals about future innovations to total factor productivity. In this environment, a positive signal increases the relative present value of domestic lifetime income, implying that current consumption can increase by more than current output. This increase in demand in turn generates an appreciation in the real exchange rate, suggesting a potential resolution to the Backus-Smith puzzle. When the economy is calibrated to the United States versus the rest of the industrialized world, numerical simulations deliver a correlation between the exchange rate and relative consumption that is similar to that observed empirically.

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## 1. Introduction and Motivation

Open-economy macroeconomic models predict a one-to-one relationship between the real exchange rate and the ratio of marginal utilities of consumption across countries when international financial markets are complete. This prediction follows directly from efficient risk-sharing considerations, and translates into a positive relationship between the real exchange rate and relative consumption. By contrast, the empirical correlation between these variables is actually negative for most country pairs. This is the so-called Backus-Smith puzzle.<sup>1</sup>

There are two natural extensions to the standard theory that one might consider, either separately or in various combinations, in order to address the Backus-Smith puzzle. One possibility is to introduce preference shocks or non-seprabilities in the utility function, either between consumption and leisure or between consumption at different dates. These extensions break the link between the ratio of marginal utilities of consumption, and the ratio of consumption itself. However, to date, introducing more general preferences has had little impact on the implied correlation between the real-exchange rate and relative consumption.<sup>2</sup> A second possibility is to relax the assumption of internationally complete financial markets. In this setting, there would no longer be a one-to-one link between the real exchange rate and the ratio of marginal utilities implied by perfect risk-sharing. When markets are incomplete, a variety of real frictions, such as imperfect competition, sticky prices, or transportation costs, may play a role in accounting for the puzzle<sup>3</sup>.

This paper assumes incomplete markets, but does not deviate from standard preferences or introduce any frictions in the goods market. Instead, it focuses on the role of changing expectations about future productivity growth in driving equilibrium consumption and real exchange rate dynamics. The context for the analysis is a standard international real

<sup>&</sup>lt;sup>1</sup> Backus and Smith (1993) were the first to illustrate this puzzle in a cross country study for the OECD countries.

<sup>&</sup>lt;sup>2</sup> See Chari, Kehoe and McGrattan (2001) for models with habit persistence and non-separability between consumption and leisure.

<sup>&</sup>lt;sup>3</sup> For example, Obstfeld and Rogoff (2000) emphasize the role of transportation costs and capital market imperfections in understanding the Backus-Smith puzzle.

business cycle macro model in which international asset trade is limited to a single noncontingent bond. The idea is that the agents form expectations about future productivity incorporating information contained in a public signal in addition to the information contained in the past history of productivity realizations. If agents observe a positive (negative) signal today, they develop more optimistic (pessimistic) expectations about the future. Then agents may increase (decrease) current consumption even if current productivity and output are little changed. These information shocks simultaneously generate an appreciation (depreciation) of the real exchange rate and higher (lower) relative consumption, consistent with the empirical evidence. The mechanism relies on internationally incomplete financial markets; since if markets were complete, changes in expectations about relative future productivity would have no implications for relative wealth or relative consumption.

The idea that new information can cause movements in real variables is not new and has been applied in several related contexts. Lucas (1972) developed a model to study the effects of unanticipated monetary shocks. In more recent work, Jermann and Quadrini (2003) explored the role of better prospects for future productivity growth in understanding the evolution of the firm-size distribution during the 1990s boom in the US. Danthine, Donaldson and Johnsen (1998) explored the relationship between expectations and growth in a general equilibrium model and concluded that fluctuations in expectations could help replicate some important stylized facts, particularly with respect to the volatility of consumption.

The model developed in this paper is compared to a benchmark model without signals. I conduct an extensive sensitivity analysis along three dimensions. First, I vary the persistence of shocks. This exercise is important because the magnitude of induced wealth effects in the model is directly related to the persistence of expected improvements in technology.<sup>4</sup> Second, I vary the elasticity of substitution between goods produced at home and abroad. This is motivated by the role of this elasticity in determining the extent of insurance against country-specific shocks provided by endogenous changes in the terms of

<sup>&</sup>lt;sup>4</sup> Baxter and Crucini (1995), among others, show that the gap between a bond economy model and a complete markets alternative depends crucially on the persistence of the shocks.

trade.<sup>5</sup> Finally, the paper presents numerical simulations for a taste shock model, a natural alternative explanation for the Backus-Smith puzzle.

This paper has two aims. The first is to provide empirical support for the proposed link between future productivity and public signals (or expectations). The second is to determine whether the expected wealth effects from shocks to expectations are big enough to account the Backus-Smith puzzle under an otherwise standard calibration.

The main findings are the following:

- 1. A calibrated version of the proposed model with signals offers a potential resolution to the Backus-Smith puzzle, and is also broadly consistent with the standard international business cycle facts pertaining to the relative volatility of various macroaggregates, and the cross-country correlations between them.
- 2. In order to generate a realistically low correlation between the real exchange rate and relative consumption in the model with signals, asset markets must be incomplete, productivity shocks must be highly persistent, and the elasticity of substitution between imports and domestically produced goods cannot be too close to unity. If these latter conditions are not satisfied, international risk-sharing remains too extensive even though markets are incomplete, and the Backus-Smith puzzle survives.
- 3. In an incomplete market model without signals, there is no value for the shock persistence that resolves the Backus Smith puzzle. Reducing the elasticity of substitution between foreign and domestic goods moves the correlation in the right direction, but replicating the correlation in the data requires an implausible degree of complementarity.
- 4. A standard model extended to incorporate country-specific taste shocks is a viable alternative to the signals story. However, to replicate the observed exchange rate-relative consumption correlation requires taste shocks of such a magnitude that the standard business cycle properties of the model become grossly counter-factual.

<sup>&</sup>lt;sup>5</sup> Cole and Obstfeld (1991) show that in an endowment economy, movements in the terms of trade provide perfect insurance when this elasticity is equal to unity.

The remainder of the paper is organized as follows. The next subsection discusses the related literature. Section 2 explains the theoretical relationship between the real exchange rate and relative consumption across countries. Section 3 summarizes the empirical evidence for the Backus-Smith puzzle. Section 4 describes the model and the calibration to the US and a hypothetical second economy that represents the rest of the world. The main results are in section 5. Finally, section 6 concludes.

## Related Literature

This is not the first paper to address the Backus-Smith puzzle. Chari, Kehoe and McGrattan (2002) develop a model with sticky prices and high risk aversion that replicates the volatility and persistence of the real exchange rate. However, this model generates a positive and near perfect correlation between relative consumption and the real exchange rate. This result holds even when financial markets are incomplete, in the sense that only a single non-contingent bond is traded internationally. In addition, Chari, Kehoe and McGrattan consider alternative utility functions, including preferences that are non-time-separable, but without much success.

Selaive and Tuesta (2003) study the relevance of imperfections in financial markets in a new open-economy macro model. They relax the uncovered interest parity condition by introducing a cost of holding bonds. If they set the cost of holding bonds to their preferred value, the correlation between relative consumption and real exchange rate decreases from 1 to 0.17. However, very high elasticities of substitution between home and foreign goods and/or implausibly large bond holding costs are required to generate such low correlations.<sup>6</sup>

In a line of research without price frictions, Corsetti, Debola and Leduc (2003) rely on very low elasticities to account for the Backus-Smith puzzle. The mechanism is that a positive productivity shock in the domestic tradable sector leads to a depreciation of the real exchange rate and, given the low elasticity of substitution, this depreciation is so large that

<sup>&</sup>lt;sup>6</sup> Sulevian and Tuesta (2001) do not discuss the implications of their model for a more complete set of business cycle moments.

relative domestic wealth and consumption decreases. However, this model has the awkward property that a positive productivity shock is immiserizing.<sup>7</sup>

In a similar study, Benigno and Thoenissen (2004) develop a flexible price model in which the production of the final good requires home and foreign tradable intermediate inputs, as well as a non-tradable domestic input. They obtain a correlation very close to the data. The logic of their model is based on the existence of a strong Balassa-Samuelson effect, where changes in the relative price of non-tradables drive the dynamics of the real exchange rate. However, Engel (1999) show that the relative prices of non-tradable goods account for almost none of the movement of US real exchange rates, and Egert, Drine, Lommatzsch and Rault (2002) show that the role of Balassa-Samuelson effect is limited for 9 CEE countries.

## 2. Theoretical Framework

This section shows the theoretical link between the real exchange rate and relative consumption under a standard RBC structure for different assumptions about the financial market. The model employed in this paper was developed by Backus, Kehoe and Kydland (1992) and it has been used by Heathcote and Perri (2002), among others. It is important to mention that this model is used frequently in the international macro literature. Thus, this choice allows assessment of the overall quantitative significance of the public signals as a complement to international macro models that focus on real exchange rate dynamics.

## 2.1 The Model

The world consists of two countries with infinitely lived households. In each period t, the economy faces a state vector,  $s_t \in S$ , where  $s^t$  denotes the history of events until date t and  $\pi(s^t)$  corresponds to the ex-ante probability of  $s^t$ .

<sup>&</sup>lt;sup>7</sup> Contrary to this implication, Easterly and Levine (2001) find that TFP rather than factor accumulation accounts for most of the income divergence across countries, where the relationship between these variables positive. Additionally, Acemoglu and Ventura (2001) estimate that a 1 percentage point faster growth is associated with a 0.6 percentage point deterioration in the terms of trade, which also goes against the low elasticity argument.

There are two intermediate goods: *a* and *b*, such that country 1 specializes in *a* and country 2 in *b*. The production technology of intermediate goods requires capital and labor, which are both rented from the households and are internationally immobile. The intermediate firm's production technology is characterized by  $F(z_i(s^t), k_i(s^t), n_i(s^t))$ , where  $z_i(s^t)$  is the technological shock,  $k_i(s^t)$  is the stock of capital in country *i* and  $n_i(s^t)$  is the labor in country *i*. The law of motion of the technological shock will be shown in section 4.<sup>8</sup> Given technological shock and history  $s^t$ , the intermediate firm of each country faces the following maximization problem:

$$\frac{Max}{k_i(s^t), n_i(s^t)} F(z_i(s^t), k_i(s^t), n_i(s^t)) - r_i(s^t)k_i(s^t) - w_i(s^t)n_i(s^t)$$
(1)

subject to  $k_i(s^t), n_i(s^t) \ge 0$ 

where  $r_i$  and  $w_i$  denote the payment to capital and labor, respectively.

The intermediate goods – a and b - are employed in the production of the final goods in each county according to  $G_i(a_i(s^t), b_i(s^t))$ . Therefore, the final goods firms face the following maximization problem:

$$\underset{a_{i}(s^{t}),b_{i}(s^{t})}{Max}G_{i}(a_{i}(s^{t}),b_{i}(s^{t})) - q_{i}^{a}(s^{t})a_{i}(s^{t}) - q_{i}^{b}(s^{t})b_{i}(s^{t})$$
(2)

subject to 
$$a_i(s^t), b_i(s^t) \ge 0$$

where  $q_i^a(s^t)$  and  $q_i^a(s^t)$  are the price of good *a* and *b* in country *i*, respectively. Notice that this notation implies a normalization, such that the final goods price is equal to 1.

The financial market structures to consider are the following: a) Complete Market, in which there is a complete array of Arrow securities denoted in units of good  $a^{9}$  and households in country *i* can trade any amount  $B_{i}(s^{t}, s_{t+1})$  of securities after history  $s^{t}$  that pays one unit of good *a* in period t+1 if the economy state is  $s_{t+1}$  (where the price of this

<sup>&</sup>lt;sup>8</sup> The law of motion of the technological shock is not crucial for the purposes of this section.

<sup>&</sup>lt;sup>9</sup> The results do not depend on the denomination of the payments.

bond is  $Q(s^t, s_{t+1})$ , and b) Bond Economy, in which there is only one non-contingent bond that pays one unit of good *a* in period t+1. This bond is denominated  $B_i(s^t)$  and its price is  $Q(s^t)$ .

The household of country 1 chooses a sequence of consumption,  $c_1(s^t)$ , labor,  $n_1(s^t)$ , capital investment,  $x_1(s^t)$ , and financial investments in order to solve the following problem:<sup>10</sup>

$$Max \quad \sum_{t=0}^{\infty} \sum_{s^{t}} \pi(s^{t}) \beta^{t} U(c_{1}(s^{t}), 1 - n_{1}(s^{t}))$$
(3)

subject to:

$$c_{1}(s^{t}) + x_{1}(s^{t}) + q_{1}^{a}(s^{t}) \sum_{s_{t+1}} Q(s^{t}, s_{t+1}) B_{1}(s^{t}, s_{t+1}) = q_{1}^{a}(s^{t}) \left( r_{1}(s^{t})k_{1}(s^{t}) + w_{1}(s^{t})n_{1}(s^{t}) \right) + q_{1}^{a}(s^{t}) B_{1}(s^{t-1}, s_{t}) \quad (4.1)$$

or

$$c_{1}(s^{t}) + x_{1}(s^{t}) + q_{1}^{a}(s^{t})Q(s^{t})B_{1}(s^{t}) = q_{1}^{a}(s^{t})(r_{1}(s^{t})k_{1}(s^{t}) + w_{1}(s^{t})n_{1}(s^{t})) + q_{1}^{a}(s^{t})B_{1}(s^{t-1})$$

$$(4.2)$$

and

$$k_1(s^{t+1}) = (1-\delta)k_1(s^t) + x_1(s^t)$$
(4.3)

$$c_1(s^{t+1}) \ge 0, \ n_1(s^t) \in [0,1]$$
  
(4.4)

where eqs. 4.1 and 4.2 correspond to the complete financial market and the bond economy, respectively; eq. 4.3 corresponds to the transition law of capital with a depreciation rate equal to  $\delta$ . The household problem takes as given the initial capital stocks, the initial productivities and the initial distribution of financial assets across countries.

## 2.2 Equilibrium

The equilibrium is given by a set of prices for all  $s^t$  and  $t \ge 0$  such that households and firms solve their respective maximization problem and markets clear. The market clearing conditions are:

#### i) Intermediate Goods

<sup>&</sup>lt;sup>10</sup> The household problem for country 2 is equivalent to the problem for country 1.

$$a_1(s^t) + a_2(s^t) = F(z_1(s^t), k_1(s^t), n_1(s^t))$$
(5)

$$b_1(s^t) + b_2(s^t) = F(z_2(s^t), k_2(s^t), n_2(s^t))$$
(6)

$$c_1(s^t) + x_1(s^t) = G_1(a_1(s^t), b_1(s^t))$$
(7)

$$c_2(s^t) + x_2(s^t) = G_2(a_2(s^t), b_2(s^t))$$
(8)

iii) Financial Markets

Complete Markets: 
$$B_1(s^t, s_{t+1}) + B_2(s^t, s_{t+1}) = 0 \quad \forall s_{t+1} \in S$$
 (9)

Incomplete Markets:  $B_1(s^t) + B_2(s^t) = 0$  (10)

## 2.3 Implications for Real Exchange Rate

The real exchange rate -  $e(s^t)$  - is defined as the relative price of the consumption goods from country 2 with respect to the consumption goods of country 1. This definition and the law of one price for the goods markets allow the real exchange rate to be expressed as:

$$e(s^{t}) = \frac{q_{1}^{a}(s^{t})}{q_{2}^{a}(s^{t})} = \frac{q_{1}^{b}(s^{t})}{q_{2}^{b}(s^{t})}$$
(11)

Under a complete financial market, the first order conditions from the household problem and the real exchange rate definition generate the following relationship among price and quantities<sup>11</sup>:

$$e(s^{t}) = \frac{U_{c}^{2}(s^{t})}{U_{c}^{1}(s^{t})}$$
(12)

where  $U_c^1(s^t)$  and  $U_c^2(s^t)$  correspond to the marginal utility of consumption in country 1 and 2, respectively.

Therefore, under complete financial markets, according to eq. 12, the unconditional moments of the real exchange rate should equal the unconditional moments of the ratio of

<sup>&</sup>lt;sup>11</sup> This derivation assumes that the domestic and foreign countries are symmetric. If this is not the case, the true relationship is  $e(s^t) = k \frac{U_c^*(s^t)}{U_c(s^t)}, k > 0$ . However, the symmetry assumption is not relevant for the conclusions. Annex 1 contains the first order conditions that characterize the equilibrium in this economy.

marginal utilities. This relationship is the essence of the Backus-Smith puzzle, because it postulates a one to one relationship between the real exchange rate and the marginal utility consumption ratio, which in turn implies the existence of a strong and positive correlation between the real exchange rate and relative consumption across countries. This relationship is given by the ability of agents to perfectly insure against any shock, a situation that generates allocations of resources such that the value of an extra unit of consumption is the same across countries.

However, if the complete financial market assumption is relaxed and only the trading of a non-contingent bond is allowed, the previous relationship only holds in a weaker version. Namely, the link between the real exchange rate and consumption is given by<sup>12</sup>:

$$E_{t}\left[\hat{e}(s^{t+1}) - \hat{e}(s^{t})\right] \approx E_{t}\left[\left(\hat{U}_{c}^{2}(s^{t+1}) - \hat{U}_{c}^{2}(s^{t})\right) - \left(\hat{U}_{c}^{1}(s^{t+1}) - \hat{U}_{c}^{1}(s^{t})\right)\right]$$
(13)

where " $\hat{x}$ " means natural log of x.

Hence, the relationship between the real exchange rate and consumption only holds in expected first differences in this case. Therefore, in a stochastic environment with incomplete financial markets, the tight link between these variables is broken. This follows from the absence of an instrument to provide the households with perfect ex-ante insurance against country specific shocks. In other words, the non-contingent bond only allows reallocating wealth and partially smooth consumption over the time.

In summary, the relationship between the real exchange rate and the consumption ratio highly depends on the financial market structure. The complete financial market scenario implies a strong and positive relationship between these variables. Meanwhile, the case with only one non-contingent bond suggests a weaker relationship, because it only holds in expected first differences.

# 3. Empirical Evidence

<sup>&</sup>lt;sup>12</sup> The variables have been log-linearized. Annex 1 shows the derivation of this expression.

This section provides the evidence to evaluate to what extent the Backus-Smith puzzle holds. Graph 1 shows the evolution of the real exchange rate and the relative consumption of US with respect to the rest of the world<sup>13</sup>. It illustrates that the relationship between the real exchange rate and relative consumption is far from being positive in the case of the US. Moreover, the relationship is clearly negative for several periods. The clearest examples are from 1973 to 1975, 1985 to 1990 and, 1994 to 1998. Therefore, it is not surprising that the correlation for the entire period is -0.12.

To illustrate that the negative correlation is robust to the sample period, Graph 2 presents the correlation for different samples. Specifically, the correlation was computed for a sequential time period of 20 years. The correlation fluctuates between -0.30 and -0.10 since the early of the nineties, which reinforces the existence of a negative and imperfect correlation between the real exchange rate and relative consumption<sup>14</sup>.





Note: The series were logged and HP filtered using a parameter 1600.

 <sup>&</sup>lt;sup>13</sup> The Annex 2 contains a description of the sources and definitions employed to construct the data series.
 <sup>14</sup> The correlation also was computed sequentially since 1975.1 adding one observation in each new period. In this case, the correlation fluctuates between -0.15 and -0.10.



Note: the sequential correlation for 1992.4 corresponds to the correlation for the period 1973.1-1992.4; the sequential correlation for 1993.1 is the correlation for the period 1973.2-1993.1 and so on. In other words, the Graph shows a rolling correlation based on data of 20 years for each observation.

The lack of a positive correlation between the real exchange rate and relative consumption is not just present in the US. In fact, this phenomenon is relatively common across countries. Backus and Smith (1993) show that for a group of eight OECD countries the relationship between the real exchange rate and relative consumption was not the predicted by the standard macroeconomic models. They analyze the connection between the real exchange rate and relative consumption for different moments without find evidence of a positive and strong correlation for these variables<sup>15</sup>. It is also important to indicate that this evidence is robust to different measures of consumption. Concretely, Backus and Smith (1993) find similar results when they measure consumption only of nondurables and services and when they employ consumption in per capita terms.

Kollmann (1995) studies the relationship between the real exchange rate and relative consumption from a time series perspective. This study tests econometrically the behavior among the variables under analysis for OECD countries. The employed model is a traditional RBC model but allows the discount factor to vary across countries. The main

<sup>&</sup>lt;sup>15</sup> Remember that the complete market model predicts that the real exchange rate will be equal to the relative consumption across countries. This will imply not only a correlation equal to one, but also the same volatility for these variables, among other moments.

conclusion of this study is that neither the trend behavior nor high-frequency movements of consumption and real exchange rates are well explained by RBC models with complete markets. Therefore, Kollmann's (1995) results also provide a rejection of the implications of the international RBC models for consumption and real exchange rate. In another empirical study, Ravn (2001) concludes that the real exchange rate is rarely a determinant of the differences in marginal utilities of consumption.

Corsetti, Debola and Leduc (2003) compute the correlation between the real exchange rate and the relative consumption for a sample of 22 countries from 1973 to 2001 using annual data. The estimation is based on the real exchange and relative consumption of each country with respect to US and OECD group. When the OECD countries are used as a reference, the median of the correlation is -0.27. Similarly, in the case of the US, the estimated correlation is -0.30.

In conclusion, the evidence indicates that the consumption-real exchange rate anomaly is robust and its presence is common in a wide range of countries. Therefore, the puzzle under analysis is not a minor issue and its magnitude should urge us to devote more effort to understand the dynamics behind the real exchange rate.

# 4. Functional Forms and Parameters

## 4.1 Functional Forms

The functional forms are taken from Backus, Kehoe and Kydland (1995). Again, the criterion used to choose the functional forms is to select the most common specifications used in the literature. As was previously mentioned, the idea of this strategy allows us to focus on the impact of a different transition law for technological shocks.

For the preferences, the following Cobb-Douglas structure is used:

$$U(c_i(s^t), 1 - n_i(s^t)) = \frac{1}{\gamma} \left[ c_i^{\mu}(s^t)(1 - n_i(s^t))^{1 - \mu} \right]^{\gamma}$$
(14)

where  $\gamma$  measures the degree of risk aversion, and  $\mu$  determines the share of consumption and leisure in the consumption basket.

The intermediate and final firm's technology is defined as:

$$F(z_{i}(s^{t}), k_{i}(s^{t}), n_{i}(s^{t})) = e^{z_{i}(s^{t})}k_{i}^{\theta}(s^{t})n_{i}^{1-\theta}(s^{t})$$

$$G_{i}(a_{i}(s^{t}), b_{i}(s^{t})) = \begin{cases} \left[\omega_{l}a_{i}(s^{t})^{\frac{\sigma-1}{\sigma}} + (1-\omega_{l})b_{i}(s^{t})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma-1}{\sigma}} & i = 1 \\ \left[(1-\omega_{l})a_{i}(s^{t})^{\frac{\sigma-1}{\sigma}} + \omega_{l}b_{i}(s^{t})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma-1}{\sigma}} & i = 2 \end{cases}$$

$$(15)$$

where  $\omega_1$  measures the degree of substitution between domestic and foreign inputs, and  $\sigma$  corresponds to the elasticity of substitution between intermediate goods (i.e., *a* and *b*).

## 4.2 Parameters

Table 1 contains most of the parameters. They were obtained from Backus, Kehoe and Kydland (1995). These parameters are equivalent to the parameters employed by Heathcote and Perri (2002), where the exception is the elasticity of substitution between a and b. In fact, Heathcote and Perri (2002) use an elasticity of substitution between imports and exports equal to 0.9. The use of different values for this elasticity reflects the fact that there is a little evidence on an appropriate value for this parameter (Arvantis and Mikkola, 1996). In fact, a survey by McDaniel and Balistreri (2003) shows point estimations that go from 0.14 to  $13.0^{16}$ . Nevertheless, according to Chari, Kehoe and McGrattan (2002), the most reliable estimations lay between 1 and 2, where the most commonly used value is 1.5.

<sup>&</sup>lt;sup>16</sup> McDaniel and Balistreri (2003) point out that the perhaps the most robust findings are: a) the long-run estimates are higher than the short-run ones; b) more disaggregate the sample the higher the estimated elasticity, and c) cross-sectional estimations are higher than time series estimations.

Period=1 Quarter					
Туре	Parameter	Value			
Preferences	Discount Factor	$\beta = 0.99$			
	Consumption Share	$\mu = 0.34$			
	Risk Aversion	$1 - \gamma = 2$			
Technology	Capital Share	$\theta = 0.36$			
	Depreciation Rate	$\delta = 0.025$			
	Import Share of i-firms (for calibrating $\omega_1$ )	is = 0.15			
	Elasticity of Substitution between <i>a</i> and <i>b</i>	σ=1.5			

Table 1
Benchmark Parameters
Period=1 Ouarter

Note: Annex 3 shows how to compute  $\omega_1$  as a function of *is*.

To estimate the transition law for productivity shocks, this study constructed a series of productivity shocks for the US and the rest of the world. The estimation of this variable, as the Solow residual, requires information about the labor and capital stock. However, the latter variable is not available for all countries. Therefore, the productivity was estimated using only real GDP and total employment according to the following definition<sup>17</sup>:

$$z_{i}(s^{t}) = \ln(y_{i}(s^{t})) - (1 - \theta) \ln(n_{i}(s^{t}))$$
(17)

where  $y_i(s^t)$  is the real GDP in country i after history  $s^t$ .

The productivity shock process for international macro models is commonly estimated as an autoregressive vector with one lag. The main variant is the incorporation or not of spillover effects<sup>18</sup>. For example, Baxter and Crucini (1995), and Arvantis and Mikkola (1996) do not consider spillover effects in their computations. Meanwhile, Heathcote and Perri (2002) consider a symmetric spillover equal to 0.025, and Backus, Kehoe and Kydland (1995) consider a value of 0.088.

The third approach used to model the technological shocks considers a specification where the expected technological shocks depend on an extra variable<sup>19</sup>. Danthine, Donaldson and Johnsen (1998), for example, consider autonomous changes in growth expectations that they name as consumer confidence. On the other hand, Jermann and

<sup>&</sup>lt;sup>17</sup> According to Cooley and Prescott (1995), this approximation should be very close if we considered the level of capital. Because of the capital stock does not vary too much over the business cycle.

<sup>&</sup>lt;sup>18</sup> The spillover effect measures the impact on the home (foreign) productivity related to a change in the foreign (home) productivity.

<sup>&</sup>lt;sup>19</sup> These papers have been applied for closed economies.

Quadrini (2003) consider the implications of different prospects about future technological shocks. Basically, they consider a discrete markov process with two possible states for technology (high and low) and the technology's transition probabilities depend on an additional variable that also follows a discrete markov process with two states (old economy and new economy), such that the unconditional expected technological improvements are higher under the second state.

The starting point to model the technological shocks is an unrestricted model that considers an autoregressive component, spillover effects and the public signal<sup>20</sup>. To implement this model, the proxy for the signal was selected using several criteria, of which the following two are most important. The first one is the availability of the same data series for US and the rest of the world. This condition drastically narrows the set of possible candidates of series, because even though there are available series of consumer/business confidence or others similar ones for several countries, these measures are not necessarily equivalent across countries. Second, the data needed to meet a time length requirement. Based on these elements, the selected variable was the composite leading indicator (CL) published by the OECD<sup>21</sup>.

The number of lags for the signal variable was selected on the statistical significance of different specifications. The most successful proxy for the signal was the first difference of the composite leading indicator lagged 4 periods. It is important to make two comments with respect to this result. First, the selection of the change of the CL instead of its level could be indicating that agents are more concerned about the innovations in this signal than only on its level. Second, the selection of 4 lags seems consistent with the fact that according to OECD (2005) the CL provides aid for short short-term forecast from 6 to 12 months.

<sup>&</sup>lt;sup>20</sup> There is not a unique way to measure a variable such as expectations about the economic future. For example, Batchelor and Dua (1992) show that changes in consumption can be better explained by survey-based measures of expectations and uncertainties about income and real interest rates than traditional variables and Santero and Westerlund (1996) find that sentiment measures obtained from business surveys provide valuable information for the assessment of the economic situation and forecasting

<sup>&</sup>lt;sup>21</sup> The composite leading indicator is calculated by combining component series in order to predict the cycles of total industrial production, which is used as proxy measures for the aggregate economy (OECD, 2005).

More concretely, the estimated model corresponds to:

$$z_{t}^{USA} = \alpha_{1} z_{t-1}^{USA} + \beta_{1} z_{t-1}^{ROW} + \gamma_{1} \Delta C L_{t-4}^{USA} + \xi_{t}^{USA}$$

$$z_{t}^{ROW} = \alpha_{2} z_{t-1}^{ROW} + \beta_{2} z_{t-1}^{USA} + \gamma_{2} \Delta C L_{t-4}^{ROW} + \xi_{t}^{ROW}$$

$$\Delta C L_{t}^{USA} = \phi_{1} \Delta C L_{t-1}^{USA} + \mu_{t}^{USA}$$

$$\Delta C L_{t}^{ROW} = \phi_{2} \Delta C L_{t-1}^{ROW} + \mu_{t}^{ROW}$$
(18)

where the superscripts USA and ROW denotes USA and rest of the world, respectively;  $z_i^j$  corresponds to the productivity level in country j in period i;  $\Delta CL_i^j$  corresponds to the change in the composite leading indicator in country j in period i; and  $\xi_i^j, \mu_i^j$  are disturbances.

Table 2 presents the results of different hypotheses about the statistical significance of the spillover and signal effects. These results indicate that the spillover effects are not statistically significant when signals are introduced. This finding is consistent with Baxter and Crucini (1995), because they do not find a spillover effect between the US and Europe, they only find some evidence of spillover effects between the US and Canada. In addition, if symmetry is not imposed, as is the case in Heathcote and Perri (2002) estimations, then, the spillover effect for US economy is not statistically significant. As for the signal effect, Table 2 shows that this effect is statistically significant.

Therefore, the selected process for TFP considers the autoregressive component and the public signal lagged 4 periods. The results are shown in Tables 3 and 4. They suggest that the signals can play an important role in predicting future changes in productivity, because they have a significant impact on future productivity levels and they are also persistent. Despite these promising results, it is important to clarify that the purpose of these estimations is to validate whether there is room to consider a TFP specification with some kind of public signal. In this sense, the purposes of these estimations are: a) to provide evidence in favor of a TFP specification with signals, and b) to obtain a benchmark for simulations purposes. In other words, these estimations should be considered as evidence in favor of signals' role more than the unique and definitive estimation in this area. It is equally important to indicate that the variance-covariance matrix will play a crucial role in explaining the Backus-Smith puzzle. If the signal helps us to explain future productivity shocks (high  $\gamma's$  coefficients), and if  $\xi_i^j$  and  $\mu_i^j$  are positively and perfectly correlated, then it will be difficult to match the desired negative correlation between the real exchange rate and consumption ratio. The adjustment in the real exchange rate depends on the gap between current consumption and output across countries and this effect will be small if the current and expected productivity move together across countries. This consideration points out the fact that the Backus-Smith puzzle depends on the behavior of both domestic and foreign variables.

Finally, Tables 5 and 6 show the results for the restricted model – i.e., without signals-.

Testing Spillover and Signal Effect				
Null Hypotheses	P-Value			
Spillover Effect				
$\beta_1=0$	0.92			
$\beta_2=0$	0.60			
$\beta_1 = \beta_2 = 0$	0.86			
<u>Signal Effect</u>				
$\gamma_1 = 0$	0.07			
$\gamma_2=0$	0.00			
$\gamma_1 = \gamma_2 = 0$	0.00			

Table 2 Testing Spillover and Signal Effect

Productivity Process and Signals						
Equation/Variable	Coefficient	Std. Error	P-Value			
USA Z.t						
_USA						
$z_{t-1}$	0.998	0.0027	0.0000			
$\Delta CL_{t-4}^{USA}$	0.383	0.1237	0.0021			
R <sup>2</sup>	0.998918					
$z_t^{ROW}$						
7 ROW						
$\sim_{t-1}$	0.995	0.0020	0.0000			
$\Delta CL_{t-4}^{KOW}$	0.676	0.1892	0.0004			
R <sup>2</sup>	0.999497					
$\Delta CL_t^{USA}$						
$\Delta C L^{USA}$						
- 1-1	0.952	0.0244	0.0000			
R <sup>2</sup>	0.896211					
$\Delta CL_t^{ROW}$						
$\Delta CL_{k-1}^{ROW}$						
	0.927	0.0275	0.0000			
R <sup>2</sup>	0.875926					

Table 3 Productivity Process and Signals

Notes:  $z_t^i$  = productivity level of country i in period t.  $\Delta CL_t^i$  = first difference of natural log of composite leading indicator for country i in period t. The sample period for productivity equations for USA and ROW is 1960.1-2004.3 and 1972.4-2004.2. The sample period for the composite leading indicator is 1960:1-2004:3. The regressions were estimated by OLS. These results obtained by SURE method and weighted residuals are practically identical. The regressions were estimated with constants, but these results were omitted.

Table 4
Productivity Process and Signals
Residuals' Structure

Residuals Structure			
Variables	Variance		
ξ <sup>USA</sup>	5.24E-05		
ξκοι	1.66E-05		
$\mu^{USA}$	2.62E-06		
$\mu^{ROW}$	1.71E-06		
Variables	Correlation		
$\xi_t^{USA}, \xi_t^{ROW}$	0.1181		
$\mu_t^{USA}, \mu_t^{ROW}$	0.0043		
$\xi_t^{USA}, \mu_t^{USA}$	0.0340		
$\xi_t^{USA}, \mu_t^{ROW}$	-0.1790		
${\xi_t^{ROW}}, {\mu_t^{ROW}}$	-0.0762		
$\xi_t^{ROW}, \mu_t^{USA}$	0.0246		

Estimation						
Equation/Variable	Coefficient	Std. Error	P-Value			
$z_t^{USA}$						
$Z_{t-1}^{USA}$	0.997	0.0025	0.0000			
$z_t^{ROW}$						
$Z_{t-1}^{ROW}$	0.995	0.0021	0.0000			

# Table 5 Productivity Process: Restricted Model

Notes:  $z_t^i$  = natural log of productivity level of country i in period t. The sample period for USA and ROW is 1960.1-2004.3 and 1972.4-2004.3. The regressions were estimated by OLS. These results obtained by SURE method and weighted residuals are practically identical.

Productivity Process and Signals Residuals' Structure			
Variables	Variance		
ξ <sup>USA</sup> ξ <sup>ROW</sup>	5.88E-05 1.83E-05		
Variables	Correlation		
$\xi_t^{USA}, \xi_t^{ROW}$	0.1692		

Table 6
Productivity Process and Signals
Residuals' Structure

#### 5. Results

#### 5.1 Simulation

The model is analyzed for two different financial market structures: complete market economy and a non-contingent bond economy. The complete financial market is included for completeness in order to understand the dynamics of the model. However, it is important to remember that the complete market scenario is not consistent with the Backus-Smith puzzle by construction (see equation 12). Therefore, the analysis will focus on the potential of signals to explain the Backus-Smith puzzle in the bond economy case.

The model is solved by linearizing the equations that characterize the equilibrium around the steady state. In the bond economy, the law of motion for bonds is not stationary. To deal with this problem, it is imposed a small cost on bond holdings<sup>22</sup>. Finally, to simulate

<sup>&</sup>lt;sup>22</sup> Schmitt-Grobe and Uribe (2003) show for several alternatives to induce stationarity that all models deliver practically identical dynamics at business cycle frequency, as measured by unconditional second moments and impulse response functions.

the model economy, the variances of  $\xi_t^{ROW}$  and  $\mu_t^{ROW}$  are set equal to the variances of  $\xi_t^{USA}$  and  $\mu_t^{USA}$ , respectively<sup>23</sup>. Table 6 presents the results for the benchmark case with and without signals.

The main result is that the correlation consumption-real exchange rate is strongly reduced. In a bond economy with signals, the simulated correlation between the real exchange rate and relative consumption is 0.06. In contrast, the obtained correlation for a complete market scenario is 0.99. This result differs from Chari, Kehoe and McGrattan (2002), because their simulations for complete and incomplete financial markets generate a correlation equal to one in both cases.

The model with signals also exhibits important improvements. In effect, the volatility of consumption and employment and, the correlation of output with exports and imports get closer to the empirical moments. Other moments, like the volatility of the terms of trade, real exchange rate, exports and imports exhibit a moderated improvement. In fact, the simulated volatility of these variables in the model with signals is just a little higher than the value for the model without signals. Therefore, the proposed model in this paper should be considered as one step toward a more comprehensive understanding of international business cycles.

The main drawback of this model with respect to traditional models is the simulated correlation between consumption and output. Concretely, the traditional model for a bond economy generates a value of 0.99, whereas the model with signal generates a correlation of 0.58 - the observed correlation is 0.84 -. This trade off in terms of the correlation consumption-output is inherent to the model's logic, because of signals can increase current consumption without necessarily observing a higher output today<sup>24</sup>.

<sup>&</sup>lt;sup>23</sup> This kind of assumption has been employed by Heathcote and Perri (2002), Kollmann (1995); Backus, Kehoe and Kydland (1995), among others.

<sup>&</sup>lt;sup>24</sup> Corsetti, Debola and Leduc (2003) do not report the correlation between consumption and output. However, their model also contains mechanisms that allow an increase in consumption without a simultaneously increase in domestic output. Concretely, it could be the case that an increase in foreign productivity can increase the domestic consumption – because of an increase of the domestic wealth in relation to the foreign wealth – without an increase in output.

Finally, the model does not exhibit any improvement with respect to correlations of output, labor and investment between countries. However, these moments have been matched by other studies only if financial autarky is assumed (Heathcote and Perri, 2002). The model with signals can also replicate those moments under financial autarky. Therefore, the degree of international finance intermediation seems to play an important role in determining those moments. However, it's hard to justify empirically and theoretically the assumption of financial autarky.

		Ber	iciinark	Paramete	ers			
Volatilities								
Economy	% std. dev	% std. dev./%std. dev. of y		% std. Dev				
	У	с	х	n	ex	im	р	e
US Data	1.59	0.78	2.91	0.65	4.00	5.22	2.80	3.66
With Signal								
Complete Markets	1.24	0.52	3.32	0.49	1.58	1.36	0.91	0.64
Bond Economy	1.22	0.65	3.45	0.54	1.30	1.22	0.64	0.46
Without Signal								
Complete Markets	1.24	0.47	2.62	0.35	0.98	1.01	0.76	0.53
Bond Economy	1.20	0.54	2.67	0.31	1.03	1.05	0.49	0.35
Economy	Correlation			Corr	elation with	Output		
	$c_1/c_2$ ,rer	c,y	x,y	n,y	ex,y	im,y	p,y	e,y
US Data	-0.12	0.84	0.95	0.86	0.39	0.82	-0.20	0.16
With Signal								
Complete Markets	0.98	0.67	0.87	0.84	0.44	0.44	0.55	0.57
Bond Economy	0.06	0.58	0.90	0.75	0.33	0.75	0.47	0.47
Without Signal								
Complete Markets	0.99	0.95	0.98	0.98	0.75	0.60	0.64	0.64
Bond Economy	0.94	0.99	0.97	0.99	0.52	0.87	0.57	0.57
Economy		Correlation Between						
	y1,y2		c	c1,c2	X1	<b>,</b> X <sub>2</sub>	n <sub>1</sub>	<b>,</b> n <sub>2</sub>
US Data	0.58		0	).35	0.	36	0.	44
With Signal								
Complete Markets	0.02		0.63		-0.14		0.21	
Bond Economy	0.08		0.08		-0.11		0.04	
Without Signal								
Complete Markets	0.08		0	).59	-0.	.18	-0.	.18
Bond Economy	0.15		0	).32	-0.	.16	0.	11

Table 6 Model Results Benchmark Parameters

Notes: a) the data statistics are calculated from US and rest or the world series for the period 1973.1-2004.3 (see Appendix A for details). b) variables have been logged and Hodrick-Prescott filtered with a parameter of 1600. c) y=gross domestic product, c=consumption, x=investment, n=labor, ex=exports ( $a_2$ ), im=imports ( $b_1$ ), p= terms of trade, and e= real exchange rate. The statistics from the model are the averages of 200 simulations each 100 periods long.

## 5.2 Impulse-Response

This section shows the dynamics generated by the introduction of a shock equal to 1 standard deviation of  $\mu_t^{USA}$ . This exercise illustrates several points; the most important is that the signal's wealth effect is big enough to generate different paths under complete markets vis-à-vis a bond economy. This result relies heavily on the persistence of the productivity shock (Graph 3) and the value elasticity of substitution between intermediate goods. This point is developed latter.

In the bond economy, the dynamic of a positive signal in US starts with a higher expected productivity, which generates a higher consumption of final goods and leisure because households expect to be richer. Consequently, the lower labor supply has a negative impact on domestic output and more specifically on the production of a (i.e., the intermediate goods produced in the US), which pushes up the price of these goods or, equivalently, reduces the terms of trade.

With respect to the foreign country, the lower terms of trade has a negative wealth effect, which in turn implies a lower consumption of final goods and leisure in this country. The effects of these changes are: a) higher relative consumption in US with respect to the foreigner country and, b) higher supply of labor in the foreign country will push down the terms of trade further. The real exchange rate's path will be determined by the terms of trade, because both variables move together in this model. In consequence, the consumption in the US will be higher than it is in the rest of the world and, simultaneously, it will be observed a lower real exchange rate, which is consistent with the Backus-Smith puzzle.

Notice that the higher consumption and lower GDP in the US will be conciliated trough a trade deficit and a lower level of domestic investment. With respect to the crowding out of the domestic investment in the short run, this effect is also present in similar environments for closed economies (Danthine, Donaldson and Johnsen, 1998). The difference is that in an open economy, the country with an expected higher productivity will have a higher investment rate in the long run.

The dynamics of the complete market scenario are different in several aspects, the most relevant of which is the feature that consumption and the real exchange rate move together. To understand this scenario more clearly, it is necessary to recall that this case is equivalent to a central planner scenario with perfect risk sharing. In this case, the positive signal in the US implies a higher level of expected wealth and the central planner will allocate a higher level of consumption and leisure in both countries. However, the consumption will increase more in the US than in the foreign country, where the extra-consumption in the US compensates the lower level of leisure in the US with respect to the foreign country - it is more efficient to allocate more work in the more productive country.

The previous allocation of labor determines the main departure from the complete market scenario with respect to the bond economy. In effect, the lower (higher) level of labor in the foreigner (domestic) country will have a positive effect in the terms of trade, which in turn will compensate the initial deterioration of this variable and the net effect on the terms of trade will be positive. Therefore, the complete market scenario will be characterized by a dynamic of the relative consumption and real exchange rate that is not consistent with the Backus-Smith puzzle.

Why does the bond economy not replicate the complete market economy? In other words, why is a non-contingent bond not a close substitute to a complete array of Arrow securities? To answer this, it is important to consider at least two elements: a) the shocks' persistence, and b) the relatively high elasticity of substitution.

The persistence of the shocks implies that innovations in their level have big effects on the expected consumer's wealth. Meanwhile, an elasticity of substitution higher than 1 allows agents to substitute the intermediate goods and, as a consequence, diminishes the gains of the domestic shock obtained by the foreign country in the form of favorable terms of trade<sup>25</sup>. Both elements involve a higher fluctuation in the rate of the relative wealth among countries and, therefore, it is plausible to state that a non-contingent bond does not insure the households enough against country specific risks under this setting.

<sup>&</sup>lt;sup>25</sup> Heathcote and Perri (2002) develop this point more extensively.

In summary, the mechanism introduced through the signal effect will work in opposite directions in the financial market structures under analysis. Clearly, this conclusion is highly dependent on the magnitude of the wealth effect related to the signals, where the main determinants of this wealth effect are the persistence of the shocks and the elasticity of substitution between intermediate goods.



Graph 3 Impulse Responses for 1 standard deviation innovation in US' signal



Graph 3 Impulse Responses for 1 standard deviation innovation in US' signal (cont.)

## 5.3 Sensitivity Analysis

## 5.3.1 Elasticity of Substitution and Shocks' Persistence

The correlation between the real exchange rate and relative consumption is estimated for elasticities of substitution from 0.5 to 2.0 (Graph 4). The selection of this range of values is based on many studies of US which indicate elasticities between 1 and 2 (Chari, Kehoe and McGrattan, 2002). Additionally this selection will help to check how low the implicit elasticity is to explain the Backus Smith puzzle under the Corsetti, Debola and Leduc (2003)'s mechanism. There are three main conclusions from this exercise. Firstly, negative correlations are consistent with elasticities higher than 1.5. For instance, if the elasticity of substitution is 1.60, then, the computed correlation is -0.10, which is practically equal to its empirical value. Secondly, negative correlations are also consistent with elasticities below 0.6, but these values are outside of the range of plausible estimates in the literature. Therefore, using low elasticities of substitution can be considered an incomplete solution to the Backus-Smith puzzle. Finally, if the elasticity of substitution varies between 0.9 and 1.0, then, a non-contingent bond brings enough insurance to households against country specific risks. This result is consistent with Cole and Obstfeld (1991)'s findings, because the terms of trade will be a efficient mechanism to achieve perfect risk sharing for elasticities close to 1.

The sensitivity analysis for persistence considers three exercises. The first exercise modifies the persistence of the autoregressive components in the productivity shock process, setting the same value for both countries ( $\alpha_1 = \alpha_2$ ). The second exercise performs a similar sensitivity analysis for the signal process ( $\beta_3 = \beta_4$ ). The third exercise considers modify the impact of the direct effect of the signal upon future productivity shocks ( $\beta_1 = \beta_2$ ). The results are shown in Graphs 5 to 7, respectively.

The main conclusion is that there exists a low degree of flexibility in terms of varying the persistence of the productivity shocks. In fact, if the degree of persistence exhibits a small deviation from values close to one, then, the bond economy immediately generates correlations close to one. However, it is important to point out that the persistence of the shocks alone is not enough to generate negative correlations, because the model without signals also employs a productivity shock process that is highly persistent and the correlation still is close to  $1^{26}$ .

 $<sup>^{26}</sup>$  Concretely, if the persistence is 0.99, then, the correlation is 0.94. Moreover, if the persistence goes to 0.999, the correlation still is high (0.81).









Graph 7 Sensitivity of Direct Effect of Signal Shocks



## 5.3.2 Taste Shock Model

The taste shock model is a natural candidate to explain the Backus-Smith puzzle, because taste shocks can change the level of consumption without a simultaneous change in the marginal utility of consumption<sup>27</sup>. Therefore, in order to asses the quantitative

<sup>&</sup>lt;sup>27</sup> Recall that the theoretical relationship between the real exchange rate and the relative consumption is given in terms of marginal utilities of consumption.

implications of this approach, the bond economy without signals was adapted to incorporate taste shocks in a similar way to Stockman and Tesar (1995).

The new specification for the preferences is the following<sup>28</sup>:

$$U(c_i(s^t), 1 - n_i(s^t)) = \frac{1}{\gamma} \left[ (\tau \cdot c_i(s^t))^{\mu} (1 - n_i(s^t))^{1 - \mu} \right]^{\gamma}$$
(20)

where  $\tau$  is the taste shock distributed normally with mean 1 and standard deviation  $\sigma_{\tau}$ .

To compute the model, the variance of taste shocks is set to be 2.5 times the variance of the productivity shocks. It is also assumed that these shocks are orthogonal to the other shocks. These assumptions are equivalent to the setting employed by Stockman and Tesar  $(1995)^{29}$ . The correlation between relative consumption and real exchange rate under this specification is 0.78.

The factor of expansion of the taste shocks' variance was modified from 1 to 10 times the productivity shocks' variance (Graph 8). However, the correlation under study is positive even for a variance 10 times higher than the variance of the productivity shocks. In fact, the required factor to match a correlation equal to -0.10 is 200. Therefore, the needed specification to explain the Backus-Smith puzzle using taste shocks is quantitatively challenging. In addition, the required taste shocks' volatility generates a correlation between consumption and output equal to -0.54.

The lack of responsiveness of the correlation between relative consumption and real exchange rate is explained by the fact that the taste shocks tend to compensate among them. To clarify this point, the relationship between the real exchange rate and relative consumption can be written as follow:

<sup>&</sup>lt;sup>28</sup> This specification is also equivalent to  $\frac{1}{\gamma}\tau_1[(c_i(s^t))^{\mu}(1-n_i(s^t))^{1-\mu}]^{\gamma}$ , because of the non-separability between consumption and leisure. Therefore, the specification proposed is equivalent to propose taste shocks that modify directly the discount term.

<sup>&</sup>lt;sup>29</sup> Stockman and Tesar (1995) compute the taste shock volatility under several scenarios and the coefficient of 2.5 times the TFP variance is the highest.

$$E_{t}\left[\hat{e}(s^{t+1}) - \hat{e}(s^{t})\right] \approx \underbrace{E_{t}\left[\left(\hat{U}_{c}^{2}(s^{t+1}) - \hat{U}_{c}^{2}(s^{t})\right) - \left(\hat{U}_{c}^{1}(s^{t+1}) - \hat{U}_{c}^{1}(s^{t})\right)\right]}_{Direct \ Effect \ on \ Consumption} + \underbrace{\mu \cdot \gamma E_{t}\left[\left(\hat{\tau}_{t+1}^{2} - \hat{\tau}_{t}^{2}\right) - \left(\hat{\tau}_{t+1}^{1} - \hat{\tau}_{t}^{1}\right)\right]}_{Direct \ Effect \ of \ Taste \ shocks}$$

$$(21)$$

where  $\hat{U}_c^i$  is functionally equivalent to the marginal utility of consumption without taste shocks, "^" means natural log,  $\mu > 0$  and  $\gamma < 0$ .

It follows from eq. (22) that the magnitude of the wedge between the real exchange rate and relative consumption will depend on the persistence and cross country correlation of the taste shocks. If the shocks are permanent, then, the taste shocks' effect will be low. The intuition is that permanent shocks do not provide incentives to consume more today than tomorrow, because the current marginal valuation of consumption does not change with respect the future (i.e., low direct effect on consumption), and the expected change of taste shocks will be also equal to zero (i.e., low direct effect of taste shocks). Alternatively, if the shocks are completely transitory, the consumption will be very responsive (i.e., strong direct effect on consumption), but it will also have a big effect on the second component (direct effect of taste shocks), which can compensate for the first one and, therefore, the change in the real exchange will be low. In this sense, the completely transitory shock case could be very similar to the model without taste shocks. Finally, the second required element is the correlation of the taste shocks across countries, because if the shocks are perfectly correlated across countries, then, the relative marginal utility of consumption will be constant in this case.

Therefore, if the shocks are not permanent/totally transitory and they are negatively correlated across countries, then, the taste shocks model could have a chance to work. Based on these elements, the taste shock model was simulated imposing a correlation of - 0.95 for the taste shocks across countries with  $\rho = 0.95^{30}$ . In this case, the required volatility of the taste shocks to match the consumption-real exchange rate correlation decreases to nine times the volatility of the technological shocks. Consequently, the taste shocks model

<sup>&</sup>lt;sup>30</sup> The model was simulated with different degrees of persistency for the taste shocks and the value of 0.95 implied the best match in terms of consumption-real exchange rate correlation.

can only account for the Backus-Smith puzzle under a very stringent distribution of the taste shocks.



# 6. Conclusions

This paper shows that the Backus-Smith puzzle can be explained by a model economy with public signals about future productivity shocks. To obtain a negative correlation between the real exchange rate and relative consumption, the setting corresponds to a bond economy with productivity shocks that are highly persistent and an elasticity of substitution around 1.6. These elements are simultaneously required to achieve a negative correlation.

The model with signals can also improve several other moments. This is especially true with respect to the volatility of consumption and output and, the correlation of exports and imports with output. In fact, the empirical volatility of consumption is equal to 0.78 times the output volatility and the model with signals can generate volatility equal to 0.65, while the model without signals can only generate a volatility of 0.54. In the case of labor, the empirical volatility is 0.65 times the output volatility and the model with signals implies a

volatility of 0.54, which is compared with a volatility of only 0.31 for the model without signals.

Theoretically, the taste shock model can produce a similar outcome to a model with signals, because the taste shocks break the link between consumption and its marginal utility. However, the results indicate that the signal model outperforms the taste shock model in the following two dimensions: a) the volatility of the taste shocks is hard to justify empirically, and b) the big tradeoff in terms of the correlation between consumption and output. This situation contrasts with the signal model because the distribution for the signals can be supported empirically while the tradeoff is less important.

Despite these positive results, there still exist several moments that are not fully explained by this model, perhaps the most important of which is the volatility of the real exchange. The simulations imply a volatility around to 0.50-0.60, and its empirical counterpart is 3.66. Incorporating signals into RBC models, therefore, helps to obtain a more realistic dynamic of the variables of interest, but still provides a partial solution. In this sense, the signal model economy presented here should be seen as a step towards a more comprehensive understanding of the international macro models.

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### Annex 1

### The Real Exchange Rate and Consumption Ratio

This Annex summaries the model and presents the derivations of the relationship between the real exchange rate and relative consumption.

- 1. The Model and First Order Conditions
- Intermediate firms' problem (i=1,2):

$$\begin{array}{l}
\text{Max}_{k_{i}(s^{t}),n_{i}(s^{t})} F(z_{i}(s^{t}),k_{i}(s^{t}),n_{i}(s^{t})) - r_{i}(s^{t})k_{i}(s^{t}) - w_{i}(s^{t})n_{i}(s^{t}) \\
\text{subject to } k_{i}(s^{t}),n_{i}(s^{t}) \ge 0
\end{array}$$
(1)

where  $r_i$  and  $w_i$  denote the payment to the capital and labor, respectively.

• Final goods firms' problem:

$$\begin{array}{l}
\text{Max}_{a_{i}(s^{t}),b_{i}(s^{t})} G_{i}(a_{i}(s^{t}),b_{i}(s^{t})) - q_{i}^{a}(s^{t})a_{i}(s^{t}) - q_{i}^{b}(s^{t})b_{i}(s^{t}) \\
\text{subject to } a_{i}(s^{t}),b_{i}(s^{t}) \ge 0
\end{array}$$
(2)

where  $q_i^a(s^t)$  and  $q_i^a(s^t)$  are the price of good a and b in country i, respectively

• The household of country 1's problem:

$$Max \quad \sum_{t=0}^{\infty} \sum_{s^{t}} \pi(s^{t}) \beta^{t} U(c_{1}(s^{t}), 1 - n_{1}(s^{t}))$$
(3)

subject to:

$$c_{1}(s^{t}) + x_{1}(s^{t}) + q_{1}^{a}(s^{t}) \sum_{s_{t+1}} Q(s^{t}, s_{t+1}) B_{1}(s^{t}, s_{t+1}) = q_{1}^{a}(s^{t}) \left( r_{1}(s^{t})k_{1}(s^{t}) + w_{1}(s^{t})n_{1}(s^{t}) \right) + q_{1}^{a}(s^{t}) B_{1}(s^{t-1}, s_{t})$$

or

$$c_1(s^t) + x_1(s^t) + q_1^a(s^t)Q(s^t)B_1(s^t) = q_1^a(s^t)(r_1(s^t)k_1(s^t) + w_1(s^t)n_1(s^t)) + q_1^a(s^t)B_1(s^{t-1})$$

and

$$k_1(s^{t+1}) = (1-\delta)k_1(s^t) + x_1(s^t)$$
(4)

$$c_1(s^{t+1}) \ge 0, \ n_1(s^t) \in [0,1]$$
 (4)

$$k_1(s^0), k_2(s^0), B_1(s^0), B_2(s^0)$$

where eqs. 3 correspond to the complete financial market and incomplete market budget constraint; eq. 4 corresponds to the transition law for capital whit a depreciation rate equal to  $\delta$ .

The FOC's derived from this model are the following<sup>31</sup>:

$$k_i(s^t):$$
  $F_{k_i}(s^t)) - r_i(s^t) = 0$  (5)

$$n_i(s^t):$$
  $F_{n_i}(s^t) - w_i(s^t) = 0$  (6)

$$a_i(s^t):$$
  $G_{a_i}(s^t) - q_i^a(s^t) = 0$  (7)

$$b_i(s^t):$$
  $G_{b_i}(s^t) - q_i^b(s^t) = 0$  (8)

$$c_1(s^t):$$
  $U_{c_1}(s^t) - \lambda(s^t) = 0$  (9)

$$n_1(s^t): \qquad U_{n_1}(s^t) + \lambda(s^t)q_1^a(s^t)w_1(s^t) = 0 \tag{10}$$

$$k_1(s^{t+1}): \qquad \lambda(s^t)\pi(s^t) - \beta \sum_{s^{t+1}} \lambda(s^{t+1}) q_1^a(s^{t+1}) \Big[ 1 - \delta + r_1(s^{t+1}) \Big] \pi(s^{t+1}) = 0 \qquad (11)$$

$$B_1(s^t, s_{t+1}): \qquad \lambda(s^t)\pi(s^t)Q(s^t, s_{t+1})q_1^a(s^t) - \beta\lambda(s^{t+1})q_1^a(s^{t+1})\pi(s^{t+1}) = 0 \qquad (12)$$

$$B_{1}(s^{t+1}): \qquad \lambda(s^{t})\pi(s^{t})Q(s^{t})q_{1}^{a}(s^{t}) - \beta \sum_{s^{t+1}} \lambda(s^{t+1})q_{1}^{a}(s^{t+1})\pi(s^{t+1}) = 0 \qquad (13)$$

where  $F_{x_i}(s^t)$  corresponds to the derivative of  $F(\cdot)$  with respect to  $x_i$  in state  $s^t$ ,  $\lambda(s^t)$  is the Lagrange multiplier of the household's budget constraint in state  $s^t$ .

Eq. (12) is the relevant FOC for the complete financial market case, and eq(13) is the corresponding FOC for the bond economy. The model is closed with the respective budget constraint for each case and the market clearing conditions<sup>32</sup>.

### 2. The Real Exchange Rate and Relative Consumption

### 2.1 Complete Market Case

<sup>&</sup>lt;sup>31</sup> To simplify the notation, it is assumed that  $s^t$  is independent in the following sense  $\pi(s^{t+1} | s^t) = \pi(s^{t+1})$ .

<sup>&</sup>lt;sup>32</sup> The market clearing conditions are specified in section 2.2.

From FOC's, and their homologous for the foreign country, it is obtained the following equation:

$$\frac{U_{c_2}(s^{t+1})}{U_{c_1}(s^{t+1})} \cdot \frac{q_2^a(s^{t+1})}{q_1^a(s^{t+1})} = \frac{U_{c_2}(s^t)}{U_{c_1}(s^t)} \cdot \frac{q_2^a(s^t)}{q_1^a(s^t)}$$
(14)

Iterating equation (14), the relationship between the real exchange rate and relative consumption is given by:

$$\frac{U_{c_2}(s^t)}{U_{c_1}(s^t)} = \kappa \cdot \frac{q_1^a(s^t)}{q_2^a(s^t)} = \kappa \cdot e(s^t)$$
(15)

where  $\kappa = \frac{U_{c_2}(s^0)}{U_{c_1}(s^0)} \cdot \frac{q_2^a(s^0)}{q_1^a(s^0)}$ . If the asymmetry assumption between country 1 and 2 is imposed at period 0, then,  $\kappa = 1$ .

Additionally, the ratio of marginal utilities can be approximated as the consumption ratio for a wide set of preferences employed in the literature. Therefore, equation (15) can be expressed as:

$$\frac{c_1(s^t)}{c_2(s^t)} = e(s^t) \tag{16}$$

In consequence, equation (16) and the asymmetry assumption imply a correlation equal to 1 between the real exchange rate and the relative consumption. Moreover, this relationship implies that the real exchange moments (variance, autocorrelation, etc.) are equal to the relative consumption's moments.

#### 2.2 Incomplete Market Case

Similarly to the previous case, the following conditions are obtained:

$$\sum_{s^{t+1}} U_{c1}(s^{t+1}) \cdot q_1^a(s^{t+1}) \cdot \pi(s^{t+1}) = U_{c1}(s^t) \cdot q_1^a(s^t) \cdot \pi(s^t)$$
(16)

$$\sum_{s^{t+1}} U_{c_2}(s^{t+1}) \cdot q_2^a(s^{t+1}) \cdot \pi(s^{t+1}) = U_{c_2}(s^t) \cdot q_2^a(s^t) \cdot \pi(s^t)$$
(17)

Equations (16) and (17) can be expressed as:

$$E_{t}\left[U_{c1}(s^{t+1}) \cdot q_{1}^{a}(s^{t+1})\right] = U_{c1}(s^{t}) \cdot q_{1}^{a}(s^{t}) \cdot \pi(s^{t})$$
(16)

$$E_{t}\left[U_{c_{2}}(s^{t+1})\cdot q_{2}^{a}(s^{t+1})\right] = U_{c_{2}}(s^{t})\cdot q_{2}^{a}(s^{t})\cdot \pi(s^{t})$$
(17)

Equation (16) can be log-linearized as follow:

$$E_{t}\left[\exp\left\{\log\left[U_{c1}(s^{t+1})\cdot q_{1}^{a}(s^{t+1})\right]\right\}\right] = U_{c1}(s^{t})\cdot q_{1}^{a}(s^{t})\cdot \pi(s^{t})$$
(18)

$$\exp\left[E_{t}\left\{\log U_{c1}(s^{t+1}) + \log q_{1}^{a}(s^{t+1})\right\} + \frac{1}{2}Var\left(\log U_{c1}(s^{t+1}) + \log q_{1}^{a}(s^{t+1})\right)\right] = U_{c1}(s^{t}) \cdot q_{1}^{a}(s^{t}) \cdot \pi\left(s^{t}\right)$$
(19)

$$E_{t}\left\{\log U_{c1}(s^{t+1}) + \log q_{1}^{a}(s^{t+1})\right\} + \chi_{0}^{1} = \log U_{c1}(s^{t}) + \log q_{1}^{a}(s^{t}) + \log \pi\left(s^{t}\right)$$
(20)

where  $\chi_0^1$  is a constant equal to  $\exp\left[\frac{1}{2}Var\left(\log U_{c1}(s^{t+1}) + \log q_1^a(s^{t+1})\right)\right]$ .

The equivalent log-linearized expression for equation (17) is:

$$E_{t}\left\{\log U_{c2}(s^{t+1}) + \log q_{2}^{a}(s^{t+1})\right\} + \chi_{0}^{2} = \log U_{c2}(s^{t}) + \log q_{2}^{a}(s^{t}) + \log \pi(s^{t})$$
(21)

From equations (20) and (21), it's obtained the following condition:

$$E\left[(\hat{U}_{c_2}(s^{t+1}) - \hat{U}_{c_2}(s^t)) - (\hat{U}_{c_1}(s^{t+1}) - \hat{U}_{c_1}(s^t))\right] \approx E\left[\hat{e}(s^{t+1}) - \hat{e}(s^t)\right]$$
(18)

where  $\hat{x}$  corresponds to the natural log of x.

The relationship obtained is much weaker than the obtained one for the complete markets. Specifically, the relationship between the real exchange rate and the relative consumption is only held in expected first differences.

## Annex 2

### Data

The series employed corresponds to an up date of the time series used by Heathcote and Perri (2002). The detail of such series is the following:

- a) GDP: Gross Domestic Product at constant prices. Source: OECD Main Economic Indicators (www.sourceoecd.org).
- b) Consumption: private consumption plus final Government consumption at constant prices. Source: OECD Main Economic Indicators (www.sourceoecd.org).
- c) Investment: gross fixed capital formation at constant prices. Source: OECD Main Economic Indicators (www.sourceoecd.org).
- d) Employment: index of civilian employment. Source: OECD Main Economic Indicators (www.sourceoecd.org).
- e) US Exports and Imports: Source: OECD Quarterly National Accounts (www.sourceoecd.org).
- f) Real Exchange Rate: Broad Index reported by the Board of Governors (research.stlouisfed.org/fred2/).
- g) Terms of Trade: ratio of import prices over export prices. The import and export prices were constructed as the ratio between the nominal and real imports and exports. Source: OECD Quarterly National Accounts (www.sourceoecd.org).
- h) Composite Leading Indicator: CLI Total industry excluding construction (long term trend). Source: OECD Main Economic Indicators (www.sourceoecd.org).

The series for the rest of the world correspond to an aggregate of the following countries: a) Canada, b) Japan, c) Austria, d) Belgium, e) Denmark., f) Finland, g) France, h) Germany, i) Greece, j) Ireland, k) Italy, l) Norway, m) Netherlands, n) Portugal, o) Spain, p) Sweden, and q) United Kingdom. The aggregation of GDP, consumption and investment were made employing the PPP exchange rates.

With respect to the employment series, there is only information to a) Canada, b) Japan, c) Austria, d) Finland, e) France, g) Germany, h) Italy, i) Norway, j) Spain, k) Sweden, and l) United Kingdom. In this case, the aggregation was made employing the 1995 population data.

Finally, the composite leading indicator employed for the rest of the world corresponds to the index for EU15.

#### Annex 3

#### Calibration Share Imports

To calibrate the import share (*is*), the following conditions were employed:

a) The terms of trade can be expressed as:

$$p = \frac{w_2}{w_1} \left(\frac{a_1}{b_1}\right)^{\frac{1}{\sigma}} \tag{1}$$

where  $w_1$  and  $w_2$  measures the home-bias in country 1 and 2, respectively.

b) A symmetric equilibrium is characterized by:

$$y_1 = y_2 \tag{2}$$

$$a_1 = b_2 \tag{3}$$

$$b_1 = a_2 \tag{4}$$

$$p = 1 \tag{5}$$

c) It is assumed:

$$w_1 = w \text{ and } w_2 = 1 - w \tag{6}$$

d) The import share corresponds to:

$$is = \frac{b_1}{y_1} \tag{7}$$

The ratio of domestic consumption of  $a_1$  with respect to imports of  $b_1$  can be expressed as:

$$\frac{a_1}{b_1} = \frac{y_1 - a_2}{y_1} \cdot \frac{y_1}{b_1} = \left[1 - \frac{b_1}{y_1}\right] \cdot \left[\frac{b_1}{y_1}\right]^{-1} = \left[1 - is\right] \cdot \left[is\right]^{-1}$$
(8)

From (1), (5), (6) and (8), the following relationship between is and w is obtained:

$$w = \left\{ \left[ 1 - is \right]^{\frac{1}{\sigma}} \cdot \left[ is \right]^{-\frac{1}{\sigma}} + 1 \right\}^{-1} \left\{ \left[ 1 - is \right]^{\frac{1}{\sigma}} \cdot \left[ is \right]^{-\frac{1}{\sigma}} \right\}$$
(9)

Then, the value of w is calibrated to replicate in steady state the value of *is* for a given value of  $\sigma$ .

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