# Growth, Distribution and External Imbalances Sustainability in Emerging Economies* 

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#### Abstract

The paper investigates the relation between external imbalances, net foreign assetdebt accumulation, growth and distribution to allow for a description of actual alternative experiences of current account insustainabiliy and provide an alternative framework for the analysis of policy presecriptions. The framework emphasizes the role of effective demand, relative prices and distribution between broadly defined sectors and is tayloreed to the main structural features of emerging market economies. It can, therefore, help understanding the effects of alternative redistributional and expenditure reduction policies together with measures of capital restriction and of interest rate and capital flow shock not only on one country's solvency, but on a broader concept of sustainability.


JEL Classification Numbers: E12, E25, F34, F41, F43.
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## 1 Introduction: external imbalances sustainability and growth

The paper investigates the effect of foreign net assets and debt on growth, distribution and other main macro variables such as terms of trade, wealth, and capacity utilization. It aims at providing an alternative framework to analyze policy strategies such as expenditure contraction, wage cuts and nominal exchange rate depreciation to contain the adverse effect of external imbalances on the economy's external position and growth. Moreover it can provide insights into issues such as the desirability of debt reduction and concessional financing conditioned to the above mentioned domestic policy measures.

[^0]The literature, which has provided the theoretical background for the definition of debt sustainability and for the possible strategies to overcome a debt overhang, i.e. a situation in which the excessive inherited debt stock disincentives investment and debt repayment, has mostly favored an "incentive-compatible optimal-contract" approach: debt should either be rescheduled at concessional rates or reduced, or both, in a way to induce the debtor country to undertake the necessary actions that allow for debt repayment. Can a model based on aggregate economic and social relations provide useful and, from a certain perspective, richer insights to the debate?

Three assumptions that characterize standard models appear to be crucial in the present analysis: (i) Countries are treated as single unified optimizing entities; (ii) Consumption and investment decisions take into account all the possible macroeconomic interactions in a way that optimality, which includes full employment; (iii) Perfect information and rationality of market participants and certainty in the execution of obligations allow for all possible contracts: capital markets are perfect.

The optimal contract theory preserves assumption (i) and (ii) and allows for market imperfections due to information asymmetries. The existence of contract enforcement problems have constituted the departing point that allow to find adverse effects of debt on investment and growth in mainstream analysis. ${ }^{1}$

If the present value of the future expected resource transfers falls short the face value of debt, no creditor would spontaneously lend new funds, for the expected return of the lending would be smaller than the opportunity cost of funds; moreover, the borrowing country perceived as insolvent is likely to incur into a liquidity problem. The expectation of a failure in the full repayment can discourage debt roll over and voluntary lending and trigger a run of existing creditors. A disorderly withdrawal of creditors would however not only set off a financial and economic crisis in the debtor country but also penalize creditors that do not withdraw first by nullify the value of existing loans. Preventing such a free riding problem would make both creditors and debtor better off. Debt overhang models assume such situation as inherited from the past and investigate the alternatives for creditors countries to maximize the value of existing loans. While the rational debtor country can choose the utility maximizing amount of effort which improves its economic outcome but constitutes a direct cost for it, the creditor agent needs to choose before her loan-value maximizing interest rate given the expected reaction function of the debtor (Krugman, 1989). Investment is easily introduced in the picture as a form of costly effort for the debtor, while the loan value can be maximized by reducing the face value of existing debt (Obstfeld and Rogoff, 1996). A sort of Laffer curve characterizes the trade-off faced by creditors in their choice of either rescheduling or forgiving debt. A reduction of the face value of debt reduces the fraction of income appropriated by creditors in case of repayment and reduces the range of the possible economic outcomes for which the debtor would default. Investing becomes more convenient since the country can withhold a larger fraction of the increased production. At high levels of indebtedness this incentive effect of a debt reduction prevails on the creditors' hope that rescheduling the same amount of debt at the original rate could lead to higher debt repayments. When larger debt stocks have a depressing effect on growth by discouraging investment and reduce debt market value, rescheduling with partial debt forgiving is the most efficient policy.

Models of capital market's imperfection may help understanding how international lending can lead to inefficient outcomes and how coordinated actions can improve both parties' payoffs. However, while attractive in a two-agent setting, the optimal contract approach neglects much of the complexity affecting the debtor country's income generation and distribution and their relation with debt dynamics. The emphasis on the willingness to pay have serious limitations whenever the debtor party is not an homogeneous entity or the payment ability does not entirely depend on its will. The coordination failure of actors within an economic system under uncertainty typically leads to less than full employment resource utilization weakening the possibility of intertemporal allocations and giving larger role to effective demand and current outcomes.

Deeper insights into the debt sustainability problem can be obtained by removing also the assumptions (i) and (ii) and allowing for a representation of the economic system as the composition of different - at times contrasting - forces. Economic actors are grouped into institutional sectors which have a distinct and well specified behavior. Transactions between sectors are represented in an accounting system that provides a great deal of necessary conditions for the model solution; behavioral equations are not necessarily the outcome of fully maximizing behaviors since limited rationality and uncertainty are pervasive. Demand is at the center stage both in the short and in the long run, and interplays with financial markets and international capital flows.

Therefore, recognizing that countries are systems of interacting actors, the sustainability, illiquidity and insolvency issues become elements of a more complex macroeconomic scenario. ${ }^{2}$

Section 2 describes a model which includes all those sets of accounts such as aggregate demand and wealth decomposition, output shares, goods and financial transaction between sectors, and stock-flow consistent accumulation of foreign debt and productive capital. We start from the intuition that foreign debt servicing depresses capacity utilization, profitability and growth generating an effective demand leakage; if exogenous capital inflows have already extinguished their impact effect we can analyze the endogenous debt dynamics which affects demand, distribution and growth. More specifically distribution and international competitiveness is determined as the composition between nominal wage demand of workers and price inflation set by imperfectly competitive firms, and feed back on domestic and foreign demand of domestic goods and foreign debt dynamics. Then we look at the short-run equilibrium and stability which are also influenced by the changing debt levels.

Section 3 focus on the longer-run dynamics defining solvency conditions and investigating the effects of debt reduction and rescheduling under alternative policies. Section 4 draws some conclusions.

## 2 Model Description and short-run equilibrium

Production requires capital, labor, and imported intermediate inputs. Technology and production coefficients such as the capital/output capacity ratio so that the effective output/capital ratio can be used as an index of the capacity utilization.

There are three institutional sectors: households, firms, and the foreign sector; and three assets: productive capital, equities, and foreign debt.

Households hold only equities whose unit price is determined as the present value of firms net profits; they save a fraction of total income which comprises wages and dividends and buy new equities. A productive sector called firms includes enterprises and the domestic financial business. It finances investment with retained earnings by issuing equities and obtaining foreign loans in "dollarized" or exchange rate-indexed debt certificates. It invests a fraction of their net profits and distributes the rest to households. The "Foreign Sector" finances excess national expenditure over income. The real exchange rate affects domestic firms' competitiveness and the debt value, and hence aggregate demand, utilization and employment rates.

The balance sheets of the three sectors are reported in table A1 of appendix A, while table A2 presents the social accounting matrix (SAM) of the economy in real terms: most of our model relations are consistently derived from this accounting framework and will be explained in the following sections.

### 2.1 Relative prices and the distributive curve

Firms set prices charging a fixed mark-up $m$ over variable costs that include wages as well as imported intermediate inputs

$$
\begin{equation*}
P=(1+m)(w l+e \bar{P} a), \tag{1}
\end{equation*}
$$

where $w$ is the money-wage rate, $l$ the labor/output coefficient, $e$ the nominal exchange rate, $\bar{P}$ the foreign good price, and $a$ the imported intermediate goods/output coefficient.

The rate of profit on firms' accumulated capital is a residual obtained as a difference between the sales and the variable costs per unit of capital

$$
\begin{equation*}
r=\frac{P X-(w l+e \bar{P} a) X}{P K} \tag{2}
\end{equation*}
$$

where homogeneous capital and a uniform price for capital and consumption goods are assumed.

Let us define the wage share out of total output and the real exchange rate as $\psi \equiv w l / P$, and $\xi \equiv e \bar{P} / P$, respectively. ${ }^{3}$ The profit share is identically equal to the product of profit rate and the capital-output ratio, $\pi \equiv r / u$, where the output capital ratio, $u \equiv X / K$ is our measure of the level of capacity utilization. ${ }^{4}$

Substituting our newly defined real variables into (1) and using (2) we observe that the assumption of a fixed mark-up is equivalant to assuming a fixed profit share on total outputobtain and therefore we define $\pi=m /(1+m)$.

From the output cost decomposition of column 1 of the SAM (table A2) we obtain

$$
\begin{equation*}
\pi+\psi+\xi a=1 \tag{3}
\end{equation*}
$$

or, in units of capital,

$$
\begin{equation*}
u=r+\psi u+\xi a u ; \tag{4}
\end{equation*}
$$

output value is distributed between profits, wages, and intermediate inputs. For any constant profit share or mark-up, $1-\pi=\psi+\xi a$, there is a trade-off between the real exchange rate and wage share so that $\partial \xi / \partial \psi=-1 / a$. The real exchange rate is an actual "distributive variable" operating on the factor cost side.

Taking the growth rates of (1) keeping $m$ constant, yields

$$
\begin{equation*}
\hat{P}=(1-\lambda) \hat{w}+\lambda \hat{e} \tag{5}
\end{equation*}
$$

where the weight $\lambda=\lambda[\psi] \equiv \xi a /(\psi+\xi a)=1-\psi /(1-\pi)>0$, is the intermediate input share on total costs and is a function of the wage share. ${ }^{5}$

If firms have sufficient market power domestically and abroad, they can preserve profit margins by passing through factor price inflation according to the factors' shares on variable costs.

Nominal wage inflation, on the other hand, is determined by the workers' and firms' relative bargaining power: even neglecting a distinction between pure workers and rentiers (assuming that households can both work and hold firms shares), a natural hypothesis is that the households not only defend the real value of their wages by bargaining new nominal rates under the pressure of commodity price inflation, but also adjust it according to changed labor market conditions. Two plausible assumptions are that nominal wages respond positively to capacity utilization (increasing production is not accommodated by perfectly elastic labor supply) and negatively to the wage share level because of firms' rising resistance to workers' claims. A linear specification of this "money-wage Phillips curve" could be

$$
\begin{equation*}
\hat{w}=\theta_{0}+\theta_{u} u-\theta_{\psi} \psi, \tag{6}
\end{equation*}
$$

(see Taylor, 2004; and Barbosa-Filho and Taylor, 2005), where the parameters $\theta_{u}>0$ and $\theta_{\psi}>0$ give the wage increases required to expand output and the degree of firms' resistance to wage demands, respectively, while $\theta_{0}>0$ depends on the minimum wage. If labor productivity is assumed as constant, labor share and real wage growth rates coincide:

$$
\hat{\psi}=\hat{w}-\hat{P} .
$$

Using (5), (6) and rearranging we obtain

$$
\begin{equation*}
\hat{\psi}=\left(\theta_{e}+\theta_{u} u-\theta_{\psi} \psi\right) \lambda, \tag{7}
\end{equation*}
$$

where $\theta_{e}=\theta_{0}-\hat{e}$. Equation (7) defines the dynamics for the labor share as the result of workers and firms bargaining power and price and nominal wage inflation. Therefore, money-wages and commodity prices adjust to obtain the real wage and real exchange rate consistent with the exogenous change in the nominal exchange rate, with the desired markup of firms and with the labor market conditions represented by the wage share and by the level of capacity utilization as a proxy for employment. Recalling (7), we define

$$
\Psi \equiv \theta_{e}+\theta_{u} u-\theta_{\psi} \psi
$$

and observe that the equilibrium $\hat{\psi}=0$ corresponds to $\Psi=0$ and can be represented by a distributive curve

$$
\begin{equation*}
\theta_{e}+\theta_{u} u-\theta_{\psi} \psi=0 . \tag{8}
\end{equation*}
$$

Given $\psi>0$ and $0<\lambda<1$, and the assumption that larger real wages are associated with slower wage inflation, $\theta_{\psi}>0$, the distributive curve is both stable in $\psi$ and upward sloping in the $(u, \psi)$ space. Figure 1 upper panel shows the distributive curve and combines it with the product market equilibirum to be explained below.

We differentiate for the variables $u$ and $\psi$, the policy variable $\hat{e}$ and the parameter $\theta_{u}$ and obtain

$$
\begin{equation*}
\theta_{u} d u-\theta_{\psi} d \psi+u d \theta_{u}-d \hat{e}=0: \tag{9}
\end{equation*}
$$

Figure 1, lower panel, shows the effect of a larger depreciation and/or a reduction of the barganing power of workers A reduction of the sensitivity of money-wages to changes in employment levels, $d \theta_{u}<0$, or a faster nominal exchange depreciation, $d \hat{e}>0$, have, respectively, the effect of rotating and shifting downwards the distributive curve in the ( $u$, $\psi$ ) space, such as from $\Psi\left[\theta_{u H}, \hat{e}_{L}\right]=0$ to $\Psi\left[\theta_{u L}, \hat{e}_{H}\right]=0$, reducing the equilibrium wage shares and/or increasing the capacity utilization.

### 2.2 Investment and foreign interests flows

Financial and manufacture firms earn gross profits at a rate $r$ and pay foreign interests (receive revenues) $j \xi b$ on debt (net assets abroad), where $b \equiv B / K$ is the "foreign-good indexed" real debt in units of capital (real net assets if $b<0$ ). Its domestic real value changes with the terms of trade: a relative price increase in foreign goods raises the burden of debt repayment in the same goods. The exogenous interest rate, $j$, comprises a risk-free international interest rate, $\rho$, a LIBOR or a USD bond rate with the same maturity and characteristics of the debt certificate, and a country-specific risk premium $\varkappa$ :

$$
j=\rho+\varkappa .
$$

Decisions on investments are made by looking at their expected future profitability: the stream of net profits discounted by the risk free international interest rate is

$$
\begin{equation*}
q \equiv \frac{r-j \xi b}{\rho} \tag{10}
\end{equation*}
$$

assuming static expectations. ${ }^{6}$
This expected present value of the firm's profitability is the main argument the firms' investment function that can be specified as follows

$$
\begin{equation*}
g=\gamma+\vartheta q \tag{11}
\end{equation*}
$$

or

$$
\begin{equation*}
g=\gamma+\alpha \pi u-\alpha j \xi b \tag{12}
\end{equation*}
$$



Figure 1: Effective demand and distributive curves. The shape and position of the effective demand curve in the $(u, \psi)$ space depend on the saving propensities, export, import coefficients, and the debt ratio. The upper panel shows a phase diagram with profit-led (downward sloping) and a wage-led (upward sloping) effective demand curve determined by a low debt $b_{\mathrm{L}}$ and a high debt ratio, $b_{\mathrm{H}}$, respectively. The combination of the effective demand with the upward sloping distributive curve gives the short run equilibrium. Stability is assured by the relative slope of the two equilibrium schedules. The lower panel shows the comparative static effect of some parameter changes, where subscript H denotes higher and L lower values of the parameter.
where $g \equiv \dot{K} / K, \vartheta \equiv \alpha \rho$, and $\gamma$ is an autonomous component reflecting the entrepreneurial "animal spirits" on investment demand.

### 2.3 Savings, distribution and wealth effects

Households receive wages and profit income from distributed net profits. Their savings consist of a fraction of disposable income out of wages and distributed net profits and depend on the wealth effects on consumptions decisions. Since firms save also a fraction of their net profits in the form of retained earnings, is it possible to obtain a Kaldorian domestic saving function characterized by differentiated savings propensities out of wage and profit income.

$$
\begin{equation*}
\sigma=s_{p}(r-j \xi b)+s_{h} \psi u \tag{13}
\end{equation*}
$$

Equation 25 shows how domestic savings (per uit of capital) depend on the distribution between wages and exchange rate, the debt/capital ratio and the level of capacity utilization. ${ }^{7}$

### 2.4 External Balance

Home production and investment are financed both by domestic households and the foreign sector, that holds claims on domestic output in the form of foreign-currency denominated bonds. Foreign savings correspond to the domestic current account deficit (in units of capital)

$$
\begin{equation*}
z=\xi \dot{B} / K=\xi(a u-x+j b) \tag{14}
\end{equation*}
$$

where $\xi x$ are the "competitive" net exports simply specified as a linear function of the real exchange rate with a positive unitary elasticity. Trade balance $\xi(x-a u) K$ improves with a depreciation (appreciation) whenever $u<x / a(u>x / a)$, i.e. when the balance is in surplus (deficit). This extreme specification allows us to emphasize the role of capacity utilization in determining how export competitiveness and the real exchange rate affect one country's ability to improve its net external balance.

From (14) we obtain the dynamic equation of the share of foreign currency denominated debt as a function of capacity utilization and growth rate:

$$
\begin{equation*}
\dot{b}=a u-x+(j-g) b \tag{15}
\end{equation*}
$$

### 2.5 Product market equilibrium

Stocks of assets and liabilities are given by past accumulation at any single point of time and can be considered as state variables. On the other hand, capacity utilization and distribution can adjust rapidly enough to obtain a series of temporary equilibria in the product and labor markets for any given level of the state variables.

Equilibrium in the product market is obtained when total savings, the sum of domestic (25) and foreign (14) savings supply equates investment demand (12), that is when excess demand is zero:

$$
U \equiv g-\sigma-z=0
$$

Substituting (12), (25), and (14) in the equilibrium condition and rearranging we obtain

$$
\begin{equation*}
U=-\left(\left(s_{p} \pi+s_{h} \psi+\xi a\right)-\alpha \pi\right) u+\xi x-\left(1-s_{p}\right) j \xi b+\gamma-\alpha j \xi b \tag{16}
\end{equation*}
$$

A first component of the excess demand is the product of capacity utilization and the difference between the total savings-output ratio, a sort of average propensity to save ( $s_{p} \pi+$ $s_{h} \psi+\xi a$ ), and the investment sensitiveness to output, $\alpha \pi$. The remaining part of the excess demand function collects the injections and leakages of savings and investment that depend on the debt-capital ratio $b$ and the distributional variable $\xi$, with $\xi=\xi[\psi]$ as from (4).

If output adjusts to clear the good market and responds positively to the excess demand for any level of wage share/real exchange rate, then adjustment stability requires a responsiveness of savings to any output changes larger than that of investments, that is $\left(s_{p}-\alpha\right) \pi+s_{h} \psi+\xi a>0$. We define

$$
M \equiv\left(s_{p}-\alpha\right) \pi+s_{h} \psi+\xi a,
$$

a sort of inverse of the the inverse of the traditional Keynesian "output multiplier," and

$$
N \equiv \gamma+\xi x-\left(1-s_{p}+\alpha\right) j \xi b,
$$

an "intercept" term of factors independent of the level of output. The equilibrium output is therefore $u=N / M$. We note that for $s_{p}-\alpha<1$ larger debt ratios reduce demand injections by reducing investment more than savings; the economy is therefore debt-burdened. Condition $M>0$ implies that positive level of capacity utilization is obtained for $N>0$, which sets an upper ceiling to the debt ratio, $b<(\gamma+\xi x) /\left(\left(1-s_{p}+\alpha\right) j \xi\right)$.

Bearing in mind (4) and that $\partial \xi / \partial \psi=-1 / a$, we can easily analyze the effect of a redistribution between $\psi$ and $\xi$ by choosing one of the two variables. It is rather convenient to differentiate (16) for $u$ and $\xi$ and obtain

$$
\begin{equation*}
\frac{d u}{d \xi}=\frac{x-\left(1-s_{p}+\alpha\right) j b-a\left(1-s_{h}\right) u}{M}, \tag{17}
\end{equation*}
$$

where $a\left(1-s_{h}\right) u$ is the variation of the average propensity to save in $M$ to distributional changes. Wage increases/exchange rate appreciations affect aggregate demand and output both by reducing the average propensity to save through larger consumption demand and lower import value and by reducing competitive exports and foreign debt interest payments.

Since capacity utilization rises with demand injections generated by consumption, investment, and the trade balance, a redistribution toward wages is expansionary whenever the consumption and investment increase induced by rising households' wage, rising net profit income and increasing firms' net profitability outweighs the effect of the loss of international competitiveness and the fall in export demand. ${ }^{8}$ In other words, the economy is "wage-led" if for a wage increase/real appreciation $d \xi<0$, the fall in net exports, $x d \xi<0$ is outweighted by the demand effect coming from the increase in the non-saved additional profits arising from debt service reduction, $-\left(1-s_{p}\right) j b d \xi>0$, the reduction of savings leakages through a
lower saving propensity, $-a\left(1-s_{h}\right) u d \xi>0$, and the larger investment demand due to the reduction of the debt burden on firms profitability, $\alpha j b d \xi<0$. The economy is otherwise "profit led".

Therefore after-interest-service profitability, consumption responsiveness, import dependence and export capacity affect together with the debt ratio the sensitivity of the capacity utilization to changes in the real wage. In the case of a wage-led system, any devaluation has a contractionary effect on aggregate demand regardless of their impact on the trade balance: the Marshall-Lerner condition can hold but the negative effect of redistribution on domestic demand is stronger than the positive effect of foreign demand. ${ }^{9}$

Using (16), (4) and (17) we observe that the system is wage-led if

$$
\begin{equation*}
b>\tilde{b} \equiv\left(x-\frac{a \gamma\left(1-s_{h}\right)}{s_{h}(1-\pi)+\left(s_{p}-\alpha\right) \pi}\right) \frac{1}{j\left(1-s_{p}+\alpha\right)} \tag{18}
\end{equation*}
$$

with $\tilde{b} \leq 0$ for $\left(s_{h}(1-\pi)+\left(s_{p}-\alpha\right) \pi\right) /\left(1-s_{h}\right) \leq a \gamma / x$, which implies that for plausible values of parameters (such as but not necessarily $0<s_{p}-\alpha<1$ ), the system may be wageled at high levels of debt and profit-led at low level of debt or if the economy accumulates net foreign assets. The dynamics of the external imbalances may play a critical role in switching from a regime to another.

Figure 1 above plots a family of effective demand curves, $U=0$, in the $(u, \psi)$ plane at different debt ratios. For increasing values of $b$, the whole demand schedule shifts leftward and the economy becomes wage-led for $b>\tilde{b}$, where

The latter considerations, for instance, imply that a low saving indebted country with large intermediate import over export share would be wage-led, while a creditor high saving country with a favorable import export ratio would be most likely profit-led, but many configurations of saving propensities, export capacity, and debt level can arise.

If the country is a net creditor $(b<0)$ an appreciation reduces the revenues from abroad in domestic currency. The overall effect of a wage increase/real appreciation on aggregate demand is negative if reduction of net exports and interest revenues and investment is stronger than the growth in domestic consumption. The larger the value of net foreign assets, the stronger is the effect of the reduction of foreign revenues on aggregate demand.

If the country is a net debtor, an appreciation reduces the cost of debt servicing which constraints investment and distributed profits. The effect of wage increase/real appreciation depends on the size of the reduction of the domestic value of foreign debt and of the expansion of domestic consumption relative to the contraction of exports. As before, the larger the debt size, the stronger will be the fall of debt servicing following a wage increase/real appreciation.

This would imply that high indebted countries are most likely wage-led if the import coefficient is large and the export coefficient small, and if the interest rate and the household propensity to consume are high. An indebted country can be profit-led if these conditions are reversed: an export-oriented country can improve its competitive position through a devaluation/real wage reduction even if foreign debt appreciates in local currency. ${ }^{10}$

### 2.6 Short-run macro equilibrium

Our system comprises four variables $u, \psi, g$, and $\xi$, and four equations (16), (8), (12), and (4). The short-run equilibrium values $u^{*}, \psi^{*}, g^{*}$, and $\xi^{*}$ are all functions of the state variable $b$, such as

$$
\begin{gathered}
u^{*}=u[b]=-\frac{1}{2 \theta_{u}}\left(\theta_{0}+\frac{\zeta_{1}[b]+\zeta_{2}[b]}{a\left(1-s_{h}\right)}\right), \\
\psi^{*}=\psi[b]=\frac{\theta_{0}+\theta_{u} u^{*}}{\theta_{\psi}}, \\
\xi^{*}=\xi[b]=\frac{1-\pi-\psi^{*}}{a}, \\
g^{*}=g[b]=\gamma+\alpha \pi u^{*}-\alpha j \xi^{*} b
\end{gathered}
$$

where

$$
\zeta_{1}[b]=j b \theta_{u}\left(1-s_{p}+\alpha\right)-x \theta_{u}-a \theta_{\psi}\left(1-\left(1-s_{p}+\alpha\right) \pi\right.
$$

and

$$
\begin{aligned}
\zeta_{2}[b] & =\left(4 a ( 1 - s _ { h } ) \theta _ { \psi } \left(j b\left(1-s_{p}+\alpha\right) \theta_{u}-x \theta_{u}(1-\pi)-a \gamma \theta_{u}-\theta_{0}\left(1-\left(1-s_{p}+\alpha\right) \pi\right)+\right.\right. \\
& \left.\left.j b\left(1-s_{p}+\alpha\right) \theta_{u}-x \theta_{u}-a\left(1-s_{h}\right) \theta_{0}-a \theta_{\psi}\left(1-s_{p}+\alpha\right) \pi\right)^{2}\right)^{\frac{1}{2}}
\end{aligned}
$$

are increasing functions of $b$. It is apparent that higher levels of debt reduce the wage share, the capacity utilization, and depreciates the real exchange rate. Both the resulting fall in the gross profit rate and the increase in the domestic value of debt depress the growth rate. On the other hand $u^{*}, \psi^{*}, g^{*}$, and $\xi^{*}$ determine the direction of change of the state variable feeding back to its law of motion (15). ${ }^{11}$

### 2.7 Stability and comparative statics

The stability and the comparative statics of the system are easily observed by differentiating the aggregate demand and distributive functions and by evaluating them at the equilibrium:

$$
\begin{aligned}
& {\left[\begin{array}{cc}
\left(1-s_{p}+\alpha\right) \pi+\left(1-s_{h}\right) \psi^{*}-1 & \left(1-s_{h}\right) u^{*}+j b\left(1-s_{p}+\alpha\right) / a-x / a \\
\theta_{u} & -\theta_{\psi}
\end{array}\right] \cdot\left[\begin{array}{l}
d u \\
d \psi
\end{array}\right]=} \\
& {\left[\begin{array}{c}
\left(1-s_{p}+\alpha\right) \xi^{*}(b d j+j d b)+\left(\pi u^{*}-j \xi^{*} b\right) d s_{p}-\xi^{*} d x \\
-u^{*} d \theta_{u}+d e
\end{array}\right] .}
\end{aligned}
$$

The partial derivatives $\partial U / \partial u \equiv\left(1-s_{p}+\alpha\right) \pi+\left(1-s_{h}\right) \psi^{*}-1, \partial U / \partial \psi \equiv\left(1-s_{h}\right) u^{*}+$ $j b\left(1-s_{p}+\alpha\right) / a-x / a, \partial \Psi / \partial u \equiv \theta_{u}$, and $\partial \Psi / \partial \psi \equiv-\theta_{\psi}$ determine the relative slope of the demand and distribution equilibrium schedules and the system stability. ${ }^{12}$

As discussed above $\partial U / \partial u$ is negative for reasonable values of investment sensitivity to profits and profit share, so that the trace of the matrix is negative. Given that $\partial U / \partial u<$ $0, \partial \Psi / \partial \psi<0$ and $\partial \Psi / \partial u>0$, a negative $\partial U / \partial \psi$ implies a positive determinant and a downward-sloping (profit-led) aggregate demand curve in the $(u, \psi)$ space. Also a positive $\partial U / \partial \psi$ implying an upward-sloping (wage-led) aggregate demand curve allows for stability if $(\partial U / \partial u) /(\partial U / \partial \psi)<(\partial \Psi / \partial u) /(\partial \Psi / \partial \psi)$, that is the aggregate demand curve is steeper than the distributive curve in the vicinity of the relevant equilibrium, that happens to be the case as shown in the previous sections.

The effect of changes of the state variable $b$, and of various parameters such as $s_{p}, j$, $x, \theta_{u}$, and $\hat{e}$ are captured in the last vector of the system. They can represents exogenous policy variables or shocks such as an expenditure reduction, $d s_{p}>0$, worsening of the external borrowing conditions, $d j>0$, a structural worsening in the export competitiveness, $d x<0$, a labor market reform leading to a generalized wage reduction for any level of output, $d \theta_{u}<0$, and a wage-reducing increase in the nominal devaluation rate, dê $>0$. As partly anticipated in the previous section, these adverse aggregate demand shocks, $d s_{p}>0, d j>0$, and $d x<0$, shift and rotate the aggregate demand schedule to the left reducing output for any given wage-exchange rate distribution. The bottom panel of figure 9 shows the effect of larger propensity to save, from $s_{p \mathrm{~L}}$ to $s_{p \mathrm{H}}$ in a profit- and wage-led system. From the differentiation above we note that debt and an interest rate increase have similar effects on the effective demand schedule by increasing interests payments: a negative interest rate shocks has a contractionary effect shifting the aggregate demand leftward. Similarly, $d \theta_{u}<0$ and $d \hat{e}>0$ respectively shifts and rotates the distributive curve downward reducing the wage share at any level of capacity utilization. While $d b>0, d s_{p}>0, d j>0$, and $d x<0$ have an unambiguous contractionary effect on both on output and the wage share, $d \theta_{u}<0$, and dê $>0$ can be contractionary or expansionary if the economy is wage- or profit-led, respectively.

The comparative statics of the system have some characteristic policy implications of most demand-constrained models. An expenditure reduction via a larger $d s_{p}$ have a shortrun contractionary effect on demand and output (a sort of paradox of thrift). ${ }^{13}$ On the other hand, wage cuts obtained by reforming the labor market or simply increasing the rate of nominal depreciation can have contractionary effects if the economy is wage-led (a sort of paradox of cost).

One specificity of the model is the role of debt both in the short-run and in the longer run. Moreover, the expansionary effect in a profit-led system of a wage cut rate largely depend on the elasticity of the aggregate demand to real depreciations, which is lower at low levels of the equilibrium wage share. That implies that wage cuts obtained at low wage share levels tend to have less significant effect on output, even in wage-led system. The implications of this varying output sensitivity to distribution becomes relevant in the longer-run dynamic system.

## 3 Longer run dynamics: foreign debt and steady state

The law of motion of foreign debt is obtained by the current account equation at the short-run equilibrium values

$$
\begin{equation*}
\dot{b}=a u^{*}-x-f+\left(j-g^{*}\right) b \tag{19}
\end{equation*}
$$

where $u^{*}=u[b], g^{*}=g[b]$, and $f$ is a transfer from abroad as a measure of the temporary flow effect of partial debt forgiveness.

The steady state growth path - debt, capital and the flow variables growing at the same rate $g$ - is obtained when the debt-foreign asset/capital ratio converges to an equilibrium at $\dot{b}=0$. If risk-averse foreign creditors have a country-specific opportunity cost equal to $j$, this interest rate is just enough to obtain debt rollover at the ongoing growth rate. We introduce time subscripts for notational clarity and suppose that the economy starts at $t$ and ends at $T$. Forward iterating the current account (19), integrating and rearranging we obtain the standard result that the present value of the cumulated debt at time $T$ is equal to the sum of the current debt $b_{t}$ and the present value of all future trade deficits, per units of capital, discounted at a variable growth-adjusted interest rate:

$$
\begin{equation*}
b_{T} \beta_{t, T}=b_{t}+\int_{t}^{T}\left(a u^{*}-x-f_{s}\right) \beta_{t, s} d s \tag{20}
\end{equation*}
$$

where

$$
\beta_{t, s}=\exp \left\{-\int_{t}^{s}\left(j-g^{*}\right) d \tau\right\}
$$

is the discount factor between periods $t$ and $s$, which is less than unity for an average growth rate smaller than the interest rate.

Over a finite time period solvency requires that all debt should be eventually repaid in full (and that country can turn into a net creditor), $b_{T} \leq 0$. This implies that no Ponzi debt rollover is allowed over the given time span and that the present value of all future resource transfers to the foreign sector (trade surpluses) is no smaller than the existing debt in units of capital

$$
\begin{equation*}
b_{t} \leq \int_{t}^{T}\left(x+f_{s}-a u^{*}\right) \beta_{t, s} d s \tag{21}
\end{equation*}
$$

Assuming an infinite time horizon both for creditors and debtors, $T \rightarrow \infty$, we find out that any convergent path of (19) leading to a steady state that satisfies $\lim _{T \rightarrow \infty} b_{T} \beta_{t, T}=0$ also satisfies the solvency constraint.


Figure 2: Interest rate, expenditure and debt dynamics $c=.01 ; \rho=.03 ; \alpha=.1 ; \gamma=.03 ; a=.4$; $x=.1 ; \pi=1 / 7 ; j=.13 ; \theta_{e}=.01 ; \theta_{u}=.1 ; \theta_{\psi}=.1$ and $s_{h}=.1 ; s_{b}=.3$; for the low saving (left panel) and $s_{h}=.2 ; s_{b}=.3$; for the high saving (right panel). Interest rates are $j=.13$ solid line; $j=.10$ dashed; and $j=.03$ dashed-dotted line, respectively.

### 3.1 Debt dynamics and policy implications

Equilibrium and stability can be easily analyzed representing (19) in the ( $b, \dot{b}$ ) plane. We temporarily assume $f=0$. Figure 2 shows some benchmark examples in debt dynamics at different interest rate levels. Low- and high-saving countries may find an equilibrium (if any) at a high and low debt/capital ratio, respectively.

The left panel shows that, at high interest levels, a low-saving "prodigal" country may not reach a stable external balance and will supposedly be forced to change its saving and investment policy with or before defaulting. The same economy can converge to a stable steady state with an equilibrium debt/capital ratio which is larger the higher is the borrowing interest rate. A high saving "thrift" economy can either diverge from an unstable equilibrium debt/capital ratio, $b^{*}>0$, or reach a stable a steady state with $b^{*}<0$ for sufficiently low levels of $j$. ${ }^{14}$

We can regard as a desirable long-term outcome both a dynamic path converging to a low level of debt and a divergent path moving toward lower levels of debt (or larger positive assets) increasing output, appreciating exchange rate, and improving profits and wages. Conversely, we may regard as a undesirable long-term outcome both a dynamic path converging to a high level of debt and a divergent path moving toward increasing debt, contracting output, depreciating exchange rate, and falling profits and wages.

A series of scenarios may emerge from the debt dynamics depending on the structural characteristics of the economy and the relative weight of the mentioned short-run policy effects. One critical factor appears to be the inverse relation between debt output, growth and distribution via net profitability of investment. Since the law of motion of the debt-net foreign asset/capital ratio is not linear with

$$
\begin{equation*}
\frac{\partial \dot{b}}{\partial b}=\left(j-g^{*}\right)+a \frac{\partial u^{*}}{\partial b}-\frac{\partial g^{*}}{\partial b} b \tag{22}
\end{equation*}
$$

where

$$
\frac{\partial g^{*}}{\partial b}=\left(\alpha \pi-j b \frac{\partial \xi}{\partial \psi} \frac{\partial \psi}{\partial u^{*}}\right) \frac{\partial u^{*}}{\partial b}-j \xi,
$$

$\partial u^{*} / \partial b<0, \partial \xi / \partial \psi<0$, and therefore $\partial g^{*} / \partial b<0$, the existence, stability and unicity of the equilibrium is not warranted and there can be multiple equilibria and divergent dynamics.

Moreover, expenditure reduction and wage cut/real depreciation policies have a negative short-run impact effect on output. The debt-foreign asset/capital ratio dynamics (19) depends on how those policies that lead to short run output and growth rate fall combine at different levels of debt.

Stability and steady states. Trade account improvements due to output contraction may be not sufficient to stabilize debt if the effect of growth contraction prevails.

The function (19) can be non monotonic. This can imply the possibility of multiple equilibria or no steady state. A steady state may not be possible with $\dot{b}>0$ even if $\partial \dot{b} / \partial b<0$ at an initial debt ratio $b_{0}$, where the improvements of the trade balance are larger than the rise in the cost of debt (downward sloping curve in the $b, \dot{b}$ space, figure 2 left panel). In fact, the current account can improve up to a certain point because of the import contraction induced by the contractionary effect of debt accumulation, but then can be offset by the fall in the growth rate that can reverse the sign of $\partial \dot{b} / \partial b$ and speed up the debt growth toward unsustainable levels (upward sloping curve in the $b, \dot{b}$ space, before reaching $\dot{b}=0$, figure 2 left panel).

Solvency and Liquidity under uncertainty. Debt growth can be a source of "irrational" macroeconomic fragility both for high saving and low saving countries.

Under perfect foresight any country converging to a stable long run equilibrium would be perceived as solvent and therefore would not incur into liquidity problems. In this case, debt can be voluntarily rolled over at the same rate for the return of the new loans is just as large as the opportunity cost on that kind of risky loans $j$. Expectations, however, play a crucial role in determining the expected present value of future repayments and therefore they can be responsible of liquidity problems for otherwise solvent countries. Under the simple hypothesis of static expectations, for instance, a country is perceived as solvent if the present value of trade surpluses at the current capacity and growth rate (projected over an infinite horizon) is at least as large as the current debt, $b_{t} \leq\left(x+f-a u_{t}^{*}\right) /\left(j-g_{t}^{*}\right)$ which is satisfied if $\dot{b}_{t} \leq 0$. Myopic and impatient creditors may underestimate the effect of the growing debt on the trade balance and on the growth rate that may lead, under certain conditions, to a balance between $b_{t}$ and $\left(x+f-a u_{t}^{*}\right) /\left(j-g_{t}^{*}\right)$. Therefore an illiquidity and debt crisis can arise for increasing but nonetheless converging debt rates. The perception of insolvency leads to a cut off of new borrowing and potential illiquidity unless debt is rescheduled at a more concessional rate. As in the debt overhang theory, if the present value of potential resource transfer falls short of debt face value, its market value falls and can be only sold at a discount. However, incumbent creditors can be better off coordinating their rescheduling at a lower rate shifting the unstable equilibrium to the right of the existing debt stock and consequently inducing a recovery in terms of growth output and debt market value.

Unstable equilibria, expenditure cuts and debt servicing reductions. Expenditure reduction can lead to local unstable dynamics by reducing growth. The economy can be trapped in a falling growth, output, rising appreciation if the initial debt is on the wrong side of the new steady state.

An unstable steady state, $\partial \dot{b} / \partial b>0$ at $b^{*}$, can lead to a falling debt and rising output, growth and wage share, as well as the other way round depending on the relative position of the initial debt level and its steady state ( $b_{0}<b^{*}$ and $b_{0}>b^{*}$, respectively). A change in the propensity to save cuts output and improves the trade balance but reduces growth and can worsen the current account by raising $\partial \dot{b} / \partial b$. The mentioned paradox of thrift can be reversed in the longer run or can be augmented by the debt accumulation effect. A expenditure cut represented by larger saving propensities tend to tilt the function (19) counterclockwise (figure 2, comparison of the two panels). If the existing debt stock falls on the left side of the new equilibrium $b_{0}<b^{*}$, expenditure cuts can lead to the accumulation of net foreign assets and under certain conditions turn the country into a foreign creditor (figure 2, right panel).

However the opposite may well happen if the existing level of debt is so large that the improvement in the trade balance due to expenditure cuts is offset by the fall in growth due to output contraction. Indebted high-saving countries running trade surpluses can be trapped into a growing debt/current account deficit, falling growth, rate of profit, wage share and depreciation spiral. In this case output contraction is not enough to drive a trade surplus that can improve the current account at high debt values; if the positive equilibrium of $b$ is unstable and if the existing debt happens to be larger than it (right side of the equilibrium in figure 2, right panel), debt will become explosive and the country insolvent.

Wage-cut/depreciation in profit-Led economies. A wage rate cut by reducing labor bargaining power or by increasing the nominal depreciation rate in a profit led system leads to a short-run output expansion, worsening of the trade account, an ambiguous effect on growth due to the appreciation of debt. The $\dot{b}$ schedule shifts up with negative long-run effect on debt, output, growth and distribution.

A country wishing to reduce excess expenditure and free resources for debt repayment may expect a wage cut to be an effective policy with an immediate relief for firms, a deflationary effect on aggregate consumption and an improvement of competitiveness. Figure 3 and 4 show the effect of falling $\theta_{u}$ on the low- and high-saving countries, for a profit-led and wage-led system, respectively, providing a combination of the expenditure and cost reduction policies. From figure 1 and equation (17) we noted that a change in the wage share has a larger effect in the profit-led than in the wage-led system.

A fall in $\theta_{u}$ raises temporarily capacity utilization in the profit-led system for any given debt level: imports increase more than growth and worsen the current account (equation, 19). Debt accumulates faster reducing output, growth and wages. Trade balance improvements due to output and growth rate reductions have, respectively, a positive and negative effect on the current account. At sufficient high level of debt the latter can prevail to former and debt accumulation can accelerate and take a non converging path. While the well known paradox of cost consists in an inverse relation between labor cost and output in wage-led systems,


Figure 3: Wage cuts, expenditure and debt dynamics in a profit-led economy. Weakening wage demand in a profit-led system. $c=.01 ; \rho=.03 ; \alpha=.1 ; \gamma=.01 ; a=.4 ; x=.1 ; \pi=1 / 7 ; j=.13 ; \theta_{0}=.01$; $\theta_{\psi}=.1$ and $s_{h}=.1 ; s_{b}=.3$; for the low saving (left panel) and $s_{h}=.2 ; s_{b}=.3$; for the high saving (right panel). Wage coefficients are: $\theta_{u}=.1$ solid line; $\theta_{u}=.07$ dashed; and $\theta_{u}=.03$ dashed-dotted line, respectively.
here a similar phenomenon applies even to profit-led system in the longer run affecting the output levels through debt dynamics.

Wage-cut/depreciation in wage-led economies. A wage rate cut by reducing labor bargaining power in a wage led system is rather ineffective.

Figure 4 shows a similar exercise for a wage led economy: low sensitivity of the aggregate demand function to wage shares weakens the negative effect of $\theta_{u}$ reduction on imports and growth, making this policy scarcely effective on the debt front and regressive on the distributional one.

### 3.2 Concessional rates and partial forgiving

Previous analysis showed that the convergence toward a high level of debt is both costly in terms of employment, growth and distribution and risky in terms of the likelihood of a debt crisis. Expenditure and labor cost reduction are indeed options a country can in principle implement with its own short run policy instruments, if able to overcome the implementation costs such as social unrest and a time preference problem arising if the consumption cut of today is expected to lead to a reduction of debt and an increase consumption in the longer run. ${ }^{15}$

Under certain conditions, however, those corrective measures may be neither necessary nor sufficient or even worsen the outcome.

Rescheduling at lower rates can be in the interest of both creditors and debtors by increasing the market value of existing loans. What is the effect of forgiving a part of the existing debt and/or reducing the interest rate in such scenarios? We observe that both lower interest rates and temporary transfers $(f>0)$ associated with debt forgiving shift the the current account curve downward when the country is a net debtor, reducing the speed of convergence to and level of the long run debt equilibrium for low saving countries and


Figure 4: Wage cuts, expenditure and debt dynamics in a wage-led economy $c=.01 ; \rho=.03 ; \alpha=.1 ; \gamma=.03 ; a=.4 ; x=.1 ; \pi=1 / 7 ; j=.13 ; \theta_{0}=.01 ; \theta_{\psi}=.1$ and $s_{h}=.1 ; s_{b}=.3$; for the low saving (left panel) and $s_{h}=.2 ; s_{b}=.3$; for the high saving (right panel). Wage coefficients are: $\theta_{u}=.1$ solid line; $\theta_{u}=.07$ dashed; and $\theta_{u}=.03$ dashed-dotted line, respectively.
increasing the speed of debt reduction for higher saving countries. While in the former case concessional rates can lead to lower debt equilibria and prevent the illiquidity due to myopic expectations, in the latter case it may prevent explosive debt paths scenarios. By making the existing debt smaller that its unstable equilibrium, a relative small debt or interest reduction can have larger effects on igniting a self-sustained recovery.

Under certain conditions more concessional rates and a partial forgiving may reduce the social cost of such contractionary transition, by increasing the speed of debt reduction at any level of the propensity to save (figure 2 , right panel), under other circumstances an expenditure contraction can lead toward debt repayment only if combined with a rescheduling at lower rate and a reduction of the debt burden.

## 4 Conclusions

The model provides an alternative and comprehensive framework to analyze the relation between macro dynamics and endogenous foreign debt accumulation and its effects on the main elements of one country's welfare, such as growth, capacity utilization cum employment, wealth, terms of trade and distribution.

Most of the current literature identifies a negative effect of high levels of debt on investment and growth and distinguishes between the solvency on existing and new loans and the liquidity problem of debtors who require new lending to stabilize or eventually pay out their debt. Mainstream literature, identifying debtor's behavior as the outcome of a rational intertemporal utility maximizing single entity, which has full control on output and payment decisions, has focused on the design of the proper contractual mechanism or domestic policy correction to enforce repayment. Debt relief and concessional lending, which may benefit existing creditors but require their coordination, are typically conditioned to adjustment policies whose effects are among the most controversial issues in policy analysis.

Previous analysis showed that the convergence toward a high level of debt is both costly in terms of employment, growth and distribution and is risky in terms of the likelihood of a debt
crisis. Investment are nonlinearly reduced at high debt levels because foreign debt servicing depresses profitability and therefore disincentives growth, but contrary to intertemporal full employment models debt output reduction derives from the endogenous lack of demand and not because the unwillingness to pay debt. Moreover, expectations of a repayment failure can be the cause of sudden stop of new lending which makes repayment an impossible event regardless of the potential solvency of the system.

We argued that the effectiveness of different standard policies differs from system to system and have radically different social costs which feed back to the ability to repay it. In fact, the concept of sustainability as the convergence to a steady growing state does not capture the real impact of debt burden on wealth, distribution and employment. Indeed an economy may incur into illiquidity and then a debt crisis even if converging to a steady state. Moreover, even if the system converges to a steady state debt ratio, the latter can be not "sustainable" for it can be socially unbearable if associated to low employment, regressive distribution and unfavorable terms of trade.

Expenditure contraction by cutting expenditure and wages can have both short-run and longer-run adverse effects. A weakening of the workers bargaining power (besides the immediate regressive distributional effect) have typically a positive impact effect on output in profit-led systems and then adverse feed backs on trade balance and growth and therefore on debt, growth, output and distribution. On the other hand higher savings do not always prevent explosive paths, although under certain condition can allow convergence toward lower debt ratios through a costly adjustment involving capacity utilization, employment and growth reduction at the onset.

Some policy tools such as partial debt relief and concessional rescheduling have gained wider international consensus in recent years under alternative theoretical framework. The present analysis can help understanding the desirability of internationally concerted policies over standard adjustment tools from both a strict judgment of effectiveness and an often neglected social perspective.

## A Stock-flow aspects of the model

The balance sheets of the three sectors are reported in table A1, while table A2 presents the social accounting matrix (SAM) of the economy in real terms: most of our model relations are consistently derived from this accounting framework and will be explained in the following sections.

TABLE A1
Balance Sheets

| Households (HH.) |  | Firms | Foreign (ROW) |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $p_{E} E$ | $\Omega_{h}$ | $K$ | $\xi B$ | $\xi B$ | $\Omega_{f}$ |
|  |  | $p_{E} E$ |  |  |  |
|  |  | $\Omega_{b}$ |  |  |  |

TABLE A2
Social Accounting Matrix (SAM)

|  | Output <br> Cost(1) | HH. <br> $(2)$ | Firms <br> $(3)$ | ROW <br> $(4)$ | Cap. <br> $(5)$ | Equit. <br> $(6)$ | Bonds <br> $(7)$ | Tot. <br> $(8)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (A) Output |  | $C_{h}$ |  | $\xi x K$ | $g K$ |  |  | $X$ |
| Incomes |  |  |  |  |  |  |  |  |
| (B) HH. | $\psi X$ |  | $D_{b}$ |  |  |  |  | $Y_{h}$ |
| (C) Firms | $\pi X$ |  |  |  |  |  |  | $Y_{b}$ |
| (D) ROW | $\xi a X$ |  | $j \xi B$ |  |  |  |  | $Y_{f}$ |
| Flows of Funds |  |  |  |  |  |  |  |  |
| (E) HH. |  | $S_{h}$ |  |  |  | $-p_{E} \dot{E}$ |  | 0 |
| (F) Firms |  |  | $S_{b}$ |  | $-g K$ | $p_{E} \dot{E}$ | $\xi \dot{B}$ | 0 |
| (G) ROW |  |  |  | $S_{f}$ |  |  | $-\xi \dot{B}$ | 0 |
| (H) Tot. | $X$ | $Y_{h}$ | $Y_{b}$ | $Y_{f}$ | 0 | 0 | 0 |  |

Line F of the SAM shows that investment, $g K$, is financed by new bonds, $\xi \dot{B}$, equities, $p_{E} \dot{E}$, and by retained earnings, $S_{b}$, the latter being equal to a fraction $s_{b}$ of net profits, $S_{b}=s_{b}(r-j \xi b) K$,

$$
\begin{equation*}
g K=S_{b}+p_{E} \dot{E}+\xi \dot{B} . \tag{23}
\end{equation*}
$$

Therefore, investment demand responds to net profitability of productive capital, it is stimulated by capacity utilization and depressed by the debt-servicing payments.

Assuming that domestic markets value equities according to their net profitability, their real price is obtained by capitalizing current net profit rate at the international discount rate - a measure of the opportunity cost of financing production:

$$
\begin{equation*}
p_{E}=\left(\frac{r-j \xi b}{\rho}\right) \frac{K}{E} \tag{24}
\end{equation*}
$$

which implies that $p_{E} E=q K . q$ has therefore the multiple role of representing the valuation of a unit of productive capital in the asset market, being the main determining factor in the investment decisions, in the asset pricing and in the households' and firms' wealth valuation.

The latter aspect emerges easily by observing table A1. The firm's balance sheet gives information on the composition of the sources of capital: if firm's productive capital is valued at the replacement costs $K$, the net worth of the firm - the difference between its assets and debt and equity capital - is a non-zero residual that absorbs the difference between the capital value and debt and equity valuation:

$$
\Omega_{b}=K-\xi B-p_{E} E . \text { Since }
$$

$\Omega_{b}=K-(r / \rho) K-\xi B+(j / \rho) \xi B$, the net worth equals zero whenever all the rates are equalized, $r=j=\rho$.

Combining all sectors' balance sheets we observe that the value domestic real assets is shared between the two domestic sectors, firms and households, and the foreign:

$$
K=\Omega_{f}+\Omega_{h}+\Omega_{b}
$$

As showed in (23), the excess investment over firms saving is financed by households and foreign investors. Since foreign loans are accumulated according to the current account equation (below), new equity issues are obtained as a residual.

Differentiating firms' balance sheet

$$
\dot{K}=\xi \dot{B}+\dot{\xi} B+p_{E} \dot{E}+\dot{p}_{E} E+\dot{\Omega}_{b}
$$

using (23) and rearranging we obtain

$$
\dot{\Omega}_{b}=S_{b}-\dot{\xi} B-\dot{p}_{E} E
$$

that is, net firms' worth responds to net profitability and real exchange rate changes.
The latter price and stock variation analysis gives us an anticipation of the depressing effect of a growing debt value on asset prices, households' wealth and accumulation. Large debt values reduce the market valuation of capital (10) and therefore household wealth (table 1 ); on the other hand, growing debt reduces capital accumulation (12) and firms' net worth and therefore the country's overall wealth.For simplicity sake and better focus on the object of the paper, the construction of the model is such as to restrict the effect of stock adjustment on flow variables to the endogenous determination of the debt/capital ratio, while leaving wealth ratios to be determined by flow variable adjustment.

Households receive wages and profit income from distributed net profits (line B, table A2). Savings consist of a fraction $s_{h}$ of disposable income out of wages and distributed net profits and depend on the wealth effects on consumptions decisions. Assuming that consumption demand depends also on a fraction $c$ of wealth $p_{E} E$ and recalling that $p_{E} E=q K$, households savings are reduced by an amount equal to $c q K$. Define $v \equiv c / \rho$. Household savings become

$$
S_{h}=s_{h}\left(\left(1-s_{b}\right)(r-j \xi b)+\psi u\right) K-v(r-j \xi b) K
$$

Total domestic savings, the sum of households' and firms' savings, is therefore

$$
\left.S_{h}+S_{b}=s_{h}\left(1-s_{b}\right)(r-j \xi b)+\psi u\right) K-v(r-j \xi b) K+s_{b}(r-j \xi b) K
$$

which, defining $s_{p}=s_{b}+s_{h}-s_{b} s_{h}-v$, becomes

$$
\begin{equation*}
S_{h}+S_{b}=s_{p}(r-j \xi b) K+s_{h} \psi u K . \tag{25}
\end{equation*}
$$

Equation 25 shows how domestic savings depend on the distribution between wages and exchange rate, the debt/capital ratio and the level of capacity utilization. ${ }^{16}$

Home production and investment are financed both by domestic households and the foreign sector, that holds claims on domestic output in the form of foreign-currency denominated bonds. Consolidating all its internal accounts, its net worth, $\Omega_{f}$, equals the home
foreign debt, $\xi B$, in domestic good units. Its savings correspond to the domestic current account deficit (G, recalling D and 4, table A2)

Equilibrium in the product market is obtained when total savings, the sum of domestic (25) and foreign (14) savings supply (E, F, and G, recalling D and line 4, table A2)

$$
\begin{equation*}
S=S_{h}+S_{b}+S_{f}=\left(s_{p} \pi+s_{h} \psi+\xi a\right) u K+\left(1-s_{p}\right) j \xi b K-\xi x K \tag{26}
\end{equation*}
$$

Differentiating (14) we observe how the current account deficit in unit of capital responds to changes in the capacity utilization, exchange rate devaluations and debt growth, as follows

$$
d\left(\frac{S_{f}}{K}\right)=\xi a d u+j \xi d b+(a u-x+j b) d \xi
$$

The differentiation shows that an output contraction, $d u<0$, leading to a fall in imports, coupled with an exchange rate depreciation $d \xi>0$ that improves the trade surplus, (au $x) d \xi<0$, may not suffice to reduce the debt/capital ratio and prevent a divergent debt dynamics. If the current account deficit falls short of output growth, the effect of debt/capital ratio growth $j \xi d b>0$ and of the debt appreciation $j b d \xi>0$, can dominate trade account improvements and lead to larger current account deficits. Indeed, high debt/low income countries are typically forced into costly trade surpluses and devaluations which nonetheless may not suffice to stabilize their debt at sustainable levels.

Sovereign debt is traded at par "foreign-good" value $b$ and rolled over as long as there is a perception of solvency and liquidity, this is consistent with the assumption of a constant interest rate and risk premium.

The extended system embracing all the entries of the SAM would include the determination of stock variations, $\dot{E}$ and $\dot{B}$ for instance, valuation $p_{E}$ and wealth. The latter two are however direct functions of net profitability and we leave them implicit to concentrate on the main flow dynamics in the short run and the long run stock ratio dynamics.

We note that the determination of the main flow and distributive variables $u^{*}, \psi^{*}, g^{*}$, and $\xi^{*}$ and the dynamics of the state variable $b$ capture all the relevant flow and stock dynamics of the system. Indeed, the other relevant changing stocks, the equity value, the sectoral and the aggregate wealth are functions of the debt-capital ratio. Recalling (10), (24) and that $p_{E} E=q K$, we observe that

$$
\begin{equation*}
q^{*}=q[b]=\frac{\pi u^{*}-j \xi^{*} b}{\rho} \tag{27}
\end{equation*}
$$

and therefore $\Omega_{h} / K=q^{*}, \Omega_{b} / K=1-q^{*}-\xi^{*} b$ and $\Omega_{f} / K=\xi^{*} b$.
Finally, we note that our use of per output shares $\pi$ and $\psi$ does not hide the more interesting dynamics of the profit $\pi /(1-\xi a)$ and wage share $\psi /(1-\xi a)$ in GDP units, respectively. Observing the distribution in GDP shares at the equilibrium,

$$
\frac{\pi}{\left(1-\xi^{*} a\right)}+\frac{\psi^{*}}{\left(1-\xi^{*} a\right)}=1
$$

we find again that a reduction of capacity utilization and in the output wage share is associated with a real exchange rate depreciation and net output $(1-\xi a)$ contraction and therefore a larger profit rate in GDP units; signs are preserved shifting from one to the other normalization while the profit share in GDP units shows the usual trade-off with the wage share. Moreover, the wage share and net output correlation - wage recipients get less of a smaller pie - reduce the relative fall of the wage share to GDP.

## B Binding capacity constraints

There are some natural limits to capacity utilization such as non-production and the technologically given capacity-capital ratio, or some more stringent constraints induced by the institutional and economic context. We can, therefore, plausibly assume a fixed mark-up rule only between a given range of capacity utilization $u^{L} \leq u \leq u^{F}$, where $u^{L}$ is a nonnegative lower bound and an $u^{F}$ upper limit no larger than the full employment utilization rate. These boundaries provide additional constraints to the pricing mechanisms and, if binding, induce mark-up adjustment. (The alternative resulting short- and long-run system is sketched in the appendix.) However, as confirmed by some exemplificative simulations at the end of the chapter, indebted countries can afford a positive level of capacity below full employment so that these non-binding constraints can be safely neglected in the following analysis.

At the capacity upper $u^{F}$ or lower bound $u^{L}$, nominal wage and price dynamics combine to determine the mark-up rate. For debt ratios $b>b^{L}$, where $u^{*}\left[b^{L}\right]=u^{L}$ and $b<b^{F}$, where $u^{*}\left[b^{F}\right]=u^{F}$, we can suppose that excess demand (supply) leads to increasing (falling) mark-ups

$$
\dot{\pi}=\left(\left(1-s_{p}+\alpha\right) \pi+\left(1-s_{h}\right) \psi^{i}-1\right) u^{i}-\left(1-s_{p}+\alpha\right) j\left(1-\psi^{i}-\pi\right) \frac{b}{a}+\left(1-\psi^{i}-\pi\right) \frac{x}{a}+\gamma,
$$

where

$$
\psi^{i}=\frac{\theta_{e}+\theta_{u} u^{i}}{\theta_{\psi}}
$$

and $i=L, F$.
This leads to an equilibrium profit share function

$$
\pi^{i}[b]=\frac{\left(1-s_{p}+\alpha\right)\left(1-\psi^{i}\right) j b-\left(1-\psi^{i}\right) x+a u^{i}\left(1-\psi^{i}+s_{h} \psi^{i}\right)-a \gamma}{\left(1-s_{p}+\alpha\right)\left(j b+a u^{i}\right)-x} .
$$

which after rearranging,

$$
\pi^{i}[b]=\left(1-\psi^{i}\right) \frac{\left(1-s_{p}+\alpha\right) j b+a u^{i}-x}{\left(1-s_{p}+\alpha\right)\left(j b+a u^{i}\right)-x}+\frac{a u^{i} s_{h} \psi^{i}-a \gamma}{\left(1-s_{p}+\alpha\right)\left(j b+a u^{i}\right)-x},
$$

shows an inverse relation between debt and profit share.

The equilibrium exchange rate and growth rate and current account becomes

$$
\begin{gathered}
\xi^{i}[b]=1-\pi^{i}[b]-\psi^{i}, \\
g^{i}[b]=\gamma+\alpha \pi^{i}[b] u^{i}-\alpha j \xi^{i}[b] b,
\end{gathered}
$$

and

$$
\dot{b}=a u^{i}-x+\left(j-g^{i}[b]\right) b .
$$

Therefore, as in the endogenous capacity case, increasing debt depreciates the real exchange rate and reduces growth, but unequivocally worsens the current account and debt accumulation.

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## Notes

${ }^{1}$ Summaries of recent mainstream literature can be found in Obstfeld and Rogoff (1996, chp. 6) and Agénor and Montiel (1999, chp. 14). The lack of a supranational authority able to enforce the execution of contracted obligations between sovereign countries give rise to the "sovereign risk" problem. The debtor entity would have the interest of defaulting instead of paying back unless deterred with either a direct sanction proportional to the income or the threat of a cut off from international borrowing. Punishment, would make contracts self-enforcing by representing the cost able to counterbalance the obvious advantages of repudiating debt. However, the absence of a completely efficient sanction when international debt contracts are not honored affects crucially the possibility of borrowing.

The focus is naturally shifted from the ability to pay to the willingness to pay: even in a stochastic environment, sovereign debtor countries decide optimally their economic outcome and the possibility of defaulting given the credible threats of creditors. Investment decisions are affected by the calculation of how much of the resources not consumed today can be employed in a production of output which will be "implicitly taxed" by creditors tomorrow.
${ }^{2}$ The issue of internal debt sustainability in a growing system has been pragmatically examined in different ways by Blanchard et al. (1990), and Pasinetti (1998 and 2000); dynamic relations and the possible trade-off between external debt and growth tackled by Bhaduri (1987), financial fragility and foreign capital flows by Foley (2003), a synthesis of some macro mainstream views and empirical assessment of the nonlinear relation between growth and debt can be found Pattilo et al. (2002).
${ }^{3}$ Given our assumption of a single (composite) consumption/investment domestic good and of a single (composite) foreign good, the natural definition for the real exchange rate is the price of the foreign output per domestic output units: a devaluation consists of a rise of foreign good prices in terms of the domestic ones.
${ }^{4}$ As anticipated, we use the output capital ratio $u$ as proxy of the capacity utilization, $X / Q$. The latter is the ratio of effective, $X$, to potential output, $Q$, allowed by the existing capital for a fixed technologically given capacity-capital ratio $Q / K$. Since the effective to potential output ratio, $X / Q$, is the product of the output capital ratio, $u$, and the fixed coefficient, $Q / K$, the determination of the degree of capacity utilization can be simply described by the determination of $u$.
${ }^{5}$ Hereafter we conventionally use dotted variables for time derivatives, hat variables for rate of growth and square brackets to denote the argument of a function.
${ }^{6}$ Static expectations are a simplifying assumption as far as interest rate dynamics is concerned (present rates are good proxies of future rates under uncertainty). However, since Meese and Rogoff (1983) showed that a random walk outpredicts any fundamental based model in forecasting the exchange rate, static expectation may be regarded as an extremely rational response to nominal exchange rate unpredictability.
${ }^{7}$ For small values of $c$ the composite propensity to save out of profit income, $s_{p}$, is larger than the propensity to save out of wage income, $s_{p}>s_{h}$. This implies that, for a given capacity utilization and real exchange rate, a redistribution between factors of production would affect aggregate savings along Kaldorian lines (Kaldor, 1956). However, the wage-exchange rate trade-off and changes in capacity utilization allow for both a positive and a negative effect of wage increases on domestic savings as emphasized by the aggregate demand analysis below.
${ }^{8}$ Bhaduri and Marglin (1990) provide a neat description of the effect of redistribution on aggregate demand in both a closed and an open economy.
${ }^{9}$ See Krugman and Taylor (1978), Taylor (1991, Chp. 7) and La Marca (2004, 2005), for examples of possible contractionary devaluations.
${ }^{10}$ As emphasized in Bhaduri and Marglin (1990: 390) "...in its search for the export surplus, an open economy could be driven increasingly toward the conservative exhilarationist logic emphasizing the desirability of low labor costs." In the presence of foreign denominated debt the final outcome of a devaluation is rather ambiguous even for export oriented economies
${ }^{11}$ The extended system embracing all the entries of the SAM would include the determination of stock variations, $\dot{E}$ and $\dot{B}$ for instance, valuation $p_{E}$ and wealth. The complete system is dealt with in the appendix A
${ }^{12}$ This 2 x 2 matrix is a simplified Jacobian that abstract from the multiplicative term $\left(\psi^{*} \lambda^{*}\right)$ which, recalling that $\dot{\psi}=\Psi(\psi \lambda)$, cancels out in our analysis.
${ }^{13}$ Though not explicitly modeled, domestic aggregate saving may include government dissaving so that an expenditure reduction policy may have the same effect of a contractionary fiscal policy, excluding Ricardian equivalence effects.
${ }^{14}$ One can imagine that a country turning into a foreign creditor will accumulate also low interest bearing reserves and therefore enjoy a lower average interest rate. Assuming that the interest rate depends on the level of debt ratio would make the debt law of motion more concave and enhance some nonlinear effects described below.
${ }^{15}$ As mentioned before a rise in the national savings propensities can be interpreted as an effect of a budget deficit reduction and other contractionary policies which traditionally would be the first policy advices for a debt stricken country.
${ }^{16}$ For small values of $c$ the composite propensity to save out of profit income, $s_{p}$, is larger than the propensity to save out of wage income, $s_{p}>s_{h}$. This implies that, for a given capacity utilization and real exchange rate, a redistribution between factors of production would affect aggregate savings along Kaldorian lines (Kaldor, 1956). However, the wage-exchange rate trade-off and changes in capacity utilization allow for both a positive and a negative effect of wage increases on domestic savings as emphasized by the aggregate demand analysis below.


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