Do Exchange Rate Regimes Matter for Inflation Persistence?

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Abstract

This paper presents an open economy extension of Barro-Gordon model, with degree of exchange rate flexibility and natural rate shocks to employment, to analyse the implications of different exchange rate regimes on inflation persistence. More flexible exchange rates result in more persistence in the inflation process, while constraint exchange rates result in lower persistence. Inflation persistence exhibits non-linear response to varying degrees of autocorrelation of shocks. A new result emerges in the model is that inflation persistence in home country is independent of that of foreign country, even if the peg is perfectly maintained. In the absence of asymmetric information, increased volatility in the transitory shocks would not result in more persistence. By contrast, asymmetric information makes inflation persistence responsive to past transitory shocks i.e., by an increase in its volatility would cause upward shift in persistence. Overall, the presence of asymmetric information brings about more persistence into the inflation process, because inflation expectations are 'contaminated' with the effects of past transitory shocks and policymaker partially accommodates current inflation expectations in setting optimal inflation rate.

Key words: Inflation persistence, Exchange rate regimes, Asymmetric information, the Lucas Critique

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1. Introduction

Recently, monetary policy research is increasingly focused on examining how inflation persistence affects conduct of monetary policy. For instance, the Inflation Persistence

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Network (IPN) in the European Central Bank is dedicated to carry out extensive research on the patterns of price setting and inflation persistence in the Euro area. Its main purpose is to examine the nature and degree of inflation persistence, which is part and parcel for a proper conduct of monetary policy.[1] Similarly, many authors in the recent literature highlight the importance of modelling and understanding the degree of inflation persistence. For example, Mishkin (2007), Sbordone (2007), Benati (2007) and Woodford (2006), are to name a few. Generally, researchers agree that policymakers may be more concerned about the degree of inflation persistence, in setting monetary policy.

The literature on inflation persistence reveals three main stylised facts. First, as described by Persson and Tabellini (1999), average inflation rates vary greatly across countries and time, though, with a common time pattern. Most OECD countries experience low inflation rates in the 1960s and very high inflation rates in the 1970s, and starting from the late 1980s, a convergence of inflation rates to lower levels. Second, the change in inflation rates occurs at different speeds and to different extents over time. Third, due to differences in inflation adjustment process, any disinflationary policy may lead to higher output costs [Fuhrer and Moore (1995)]. These stylised facts provide common grounds for researchers who attempt to explain inflation persistence over time. Particularly, several studies focus on the differences in speed and extent of inflation adjustment, in the context of changes in monetary/exchange rate regimes. Importantly, there is an ongoing debate on how changes in exchange rate regimes affect inflation persistence. However, only limited attempts have, so far, been made on theoretical grounds to explain the relation between inflation persistence and degree of exchange rate flexibility. The purpose of this paper is just that.

Inflation rates change across time mainly due to two reasons, a) change in the monetary policy framework (eg. shifting from fixed exchange rate regime to a floating exchange rate regime, or adopting an inflation targeting framework rather than focusing on exploiting short-run output gains etc.), and b) change in the 'inflation process' [Cecchetti and Debelle (2006)]. However, as claimed by Sargent (1999), these two reasons may be interrelated. Thus, several authors attempt to explain change in inflation process with reference to changes in exchange rate regimes. However, it appears to be no agreement among researchers on the issue that; changes in exchange rate regimes are associated with changes in inflation persistence. On one hand, several studies suggest that flexible exchange rate regimes result in higher persistence due to higher monetary accommodation [see Alogoskoufis and Smith (1991) and Alogoskoufis (1992)]. On the other hand, various researchers show that changes in inflation persistence are not caused by exchange rate regimes shifts, but due to some other factors such as oil price shocks, central bank reforms, outbreak of wars etc. [see Burdekin and Siklos, (1999), Bleaney, (2001)]. Recently, several authors provide alternative interpretations for changes in inflation persistence over time. For instance, Williams (2006) describes changes in inflation persistence in the light of changes in inflation expectations formation process. Further, Benati (2006) highlights the effects of changes in monetary policy framework such as moving on to an inflation targeting regime, on substantial changes in inflation

[1] See Angeloni *et al.*, (2004) for a preliminary findings of IPN.

persistence. Similarly, Angeloni *et al.*, (2004) show that significant persistence in aggregate levels is associated with changes in monetary policy regimes.

Despite a large volume of research on inflation persistence, there, still, exist some unexplored areas. As Ball (1995) reports "[t]he theoretical literature on monetary policy explains why inflation may be high: policymakers face a dynamic consistency problem […]. The models also explain why inflation may be low: policymakers care about their reputations[…]. The models are less helpful, however, in explaining why inflation varies between high and low levels over time" (p.330). Interestingly, even after more than a decade of Ball's comment, researchers have not been able to fill this gap, given the fact that there is only a few published articles devoted to theoretically examine inflation persistence over time. This paper, therefore, attempts to develop an analytical model that is capable of explaining inflation persistence over time.

The analytical models found in literature explaining inflation persistence broadly belong to three categories, namely, a) flexible price models, b) sticky price models, and c) sticky information models. One of the main candidates of flexible price models is the Barro and Gordon (1983) framework which is developed in a discretionary monetary policy framework where policymaker is free to adjust monetary policy strategies at any time. In this model, the conduct of monetary policy results in an 'inflationary bias', due to policymaker's desire to expand the economy above the natural rate of employment. Subsequently, several authors adopt the Barro-Gordon framework to study inflation persistence under alternative assumptions on behaviour of economic agents [For example, see Bleaney (2001), Reis (2003), and Cukierman (1992)].

Alternatively, following Taylor (1979, 1980), and Calvo (1983), sticky price models became quite common in the literature on inflation dynamics. These models are built on Taylor's standard wage contracting model or Calvo type price adjustment models. However, many authors later on find that the standard specifications of sticky-price models are incapable of explaining inflation persistence, for example, Fuhrer and Moore (1995). Fuhrer and Moore provide a wage contracting model with *relative* real wage that imparts significant inflation persistence, apart from the inherited persistence in the output gap process, as implied in the standard sticky price models. Subsequently, several researchers adopt sticky price models to explain inflation persistence, and specially, following pioneering work by Alogoskoufis and Smith (1991) and Alogoskoufis (1992), there has been an increasing trend to examine inflation persistence in the context of shifts in exchange rate regimes [see also Obstfeld (1995)].

Because of some disagreement among researchers on the sticky price assumption in the standard contracting models, Mankiw and Reis (2002) propose an alternative interpretation in which nominal rigidities exist due to *sticky information.* In their model, prices adjust slowly due to slower dispersal of information about macroeconomic conditions. As obtaining and processing information is costly, only a fraction of firms is able to adjust prices in response to new information, while rest of the firms set their prices based on 'outdated information'.

The model developed in this paper belongs to the first category i.e. Barro-Gordon framework, which mainly consists of two components namely, policy objective function and the expectations augmented Phillips curve. However, the model in this paper departs from existing literature in the way that these two components are specified. First, as the main purpose of this paper is to examine inflation persistence over different exchange rate regimes, the objective function is amended to reflect open economy characteristics. Thus, policymaker is concerned over deviations of domestic inflation rate from the foreign country, which issues reserve currency in the fixed exchange rate system. And also policy objective function includes an additional parameter to capture the effects of changes in exchange rate flexibility.

Even though the Barro-Gordon framework provides useful inputs to monetary policy analysis, some authors claim that it is less helpful to explain inflation persistence due to flexible price assumption.[2] However, attempts have been made to derive the basic Barro-Gordon framework from microfoundations with nominal rigidities [see Reis (2003)].

Another important aspect of the model is that inflation persistence is captured through a shock to the natural rate of employment, where the shock is assumed to be consisted of persistent and transitory components. The persistent component is assumed to follow an AR(1) process with an i.i.d. error term. Further, in solving the model, two key assumptions are made on the information structure. First, as described in section 4.1, the model is solved under the assumption of symmetric information where both policymaker and public share same information on the persistent component of the shock. Thus, public forms rational expectations on the policy responses in each period. Second, section 4.2 assumes asymmetric information on the part of public, where public observe shock to the natural rate with a two period lag, and they form forecasts of inflation rate, based on information available in the current period. However, we assume through out the paper that the type of policymaker in office and the prevailing exchange rate regime, as common knowledge in any given period of time.

The paper proceeds as follows. Section 2 presents the theoretical literature on inflation persistence and exchange rate regimes, specially focusing on Barro-Gordon framework. Section 3 describes the model. Section 4 analyses the solution of the model. Section 5 concludes.

2. Literature Review

In the Barro-Gordon framework, the short run trade-off between inflation and unemployment has been the central phenomenon. As described by Cukierman (1992) "central bank may be interested in both price stability and in maintaining employment above the natural level because it is concerned with social welfare and also it partially responds to political pressure" (p.27). However, due to uncertainty on policy variables, policymakers are not always able to derive the expected outcome. As described by

[2] However, Dittmar *et al.* (2004) report that the flexible price assumption is not a constraint to explain inflation persistence, when monetary authorities follow an interest rate rule. In a flexible-price model, they show that inflation generates more persistence, when the spread between real and nominal interest rates shows persistent changes.

Freidman and Phelps, a policy based on inflating the economy with the objective of increasing employment would lead to a situation where the average inflation rate is higher. Thus, policymakers face a dilemma as the overly ambitious employment target may produce an *inflationary bias* without a positive impact on employment.

Barro and Gordon describe that “[t]here is an apparent contradiction because the policymaker peruses an activist policy that ends up having no desirable effects – in fact, unemployment is unaltered but inflation ends up being excessive” (p.591). Therefore, a commitment to a rule has been considered to be optimal to get rid of inflation bias, along with the argument put forwarded by Kydland and Prescott (1977). Apparently, fixed exchange rate system is considered to be a commitment mechanism by many authors for the last couple of decades. Barro and Gordon also makes this point to say that “an exogenous shift from a regime that involved some commitment on nominal values – such as gold standard or possibly with fixed exchange rates – to one without such constraints would produce a rise in the average rates of inflation and momentary growth” (p.600). Thus, fixing the exchange rate against a low inflation reserve currency may seem as a better commitment tool for policymakers. For example Alogoskoufis and Smith (1991) and Persson and Tabellini (1999) show that fixed exchange rates are associated with lower average inflation rates. However, the credibility of fixed exchange rate as a commitment mechanism is subject to scrutiny in recent literature on optimal monetary policy commitment. Because our purpose is to evaluate inflation performance in the context of different exchange rate regimes, we abstract from such analysis in this paper.

Barro-Gordon framework provides simple but useful grounds to analyse policymaker’s behaviour. The basic model includes a standard specification of policy preferences and aggregate supply function in the form of an expectations augmented Phillips curve. The widely used policy objective function takes the form of a loss function which consists of employment and inflation fluctuations:

$$L_t = \frac{\lambda}{2}\left(\pi_t - \tilde{\pi}_t\right)^2 + \frac{1}{2}\left(y_t - \tilde{y}_t\right)^2, \tag{2.1}$$

where, π is domestic inflation rate and $\tilde{\pi}$ is target inflation rate, y is actual employment and $\tilde{y}$ is target level of employment, which is defined as a function of natural level of employment y^N plus a positive parameter κ, which relates to policymaker’s desire to expand the economy above the natural rate ($\tilde{y} = y^N + \kappa$). The parameter λ is the relative weight attached to inflation stabilization to employment stabilization.

The economy is characterised by an aggregate supply function of the following form:

$$y_t = y_t^N + \alpha\left(\pi_t - \pi_t^e\right), \tag{2.2}$$

where π^e is expected inflation, α is a positive parameter. Equations (2.2) can be used to explain the nominal wage setting process and the level of employment in the economy. For example, as described by Walsh (2003), if nominal wage contracts are set at the beginning of each period, an inflation surprise will result in reduction in real wage and subsequently more employment. On the other hand, if actual inflation is lower than the expected inflation, real wage would increase, which results in lower employment. Further, in this family of model, it is not uncommon to assume that the monetary policy instrument as money growth rate or policymaker directly chooses the inflation rate.

The literature on inflation persistence generally suggests two sources of inflation persistence, namely, a) serial correlation of money growth process and, b) serial correlation in inflation response to (serially uncorrelated) monetary policy shocks (Walsh, 2003). If the former is the only source of inflation persistence, it can explain persistence even without the assumption of price stickiness (flexible-price models), while in the latter case, inflation persistence is explained with sticky price models of the type Taylor or Calvo. However, Fuhrer (2006) adopts a different terminology in explaining sources of inflation persistence in the context of new Keynesian Phillips curve (NKPC) models where ; a) inflation exhibits persistence if the “driving process” is persistent (inherited inflation persistence) and, b) inflation exhibits persistence due to backward-looking terms of inflation process (intrinsic inflation persistence). The latter is consistent with the structural form of inflation persistence as explained by Gali and Gertler (1999). Further, intrinsic inflation persistence can be explained using an automatic indexation rule of changing prices, as discussed in Christiano *et al.* (2005), where firms change prices according to a degree of indexation based on past inflation.

On the issue of explaining sources of inflation persistence, Angeloni *et al.*, (2004) provide a broader description based on a structural inflation equation of hybrid NKPC type:

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t(\pi_{t+1}) - \lambda \hat{\mu}_t + \xi_t \tag{2.3}$$

where π is inflation, μ deviation of actual mark-up from the desired level, and ξ is exogenous mark-up shock. E is expectations operator. Accordingly, sources of inflation persistence correspond to each term in the right hand side of equation (2.3), namely, a) persistence in the mark-up gap (extrinsic persistence), b) dependence on past inflation due to price-setting mechanism (intrinsic persistence), c) persistence due to formation of inflation expectations (expectations-based persistence), and d) persistence in the stochastic error term (error-term persistence). However, it is noted that “these sources of persistence may be difficult to distinguish, in theory as well as empirically, since they interact in general equilibrium, and their relative importance will also very much depend on the monetary policy regime and the policy reaction function” (p.5).

Among the four sources of persistence, the expectations-based persistence has several implications, depending on the assumptions made on the nature of expectations. For example, Roberts (1995, 1997) describes inflation persistence with the assumption of imperfectly rational expectations. Similarly, Ball (2000) proposes a ‘less-than-fully-rational expectations model with inflation persistence, where agents form ‘optimal univariate forecasts’. While his model is capable of accounting for inflation persistence across regimes, the model is based on some strong assumptions. Alternatively, Erceg and

Levin (2003) present a model that generates inflation persistence without imposing imperfect rational expectations or adding arbitrary lagged inflation terms. They highlight the importance of learning process of agents in distinguishing transitory shocks to monetary policy and persistent shifts in inflation target. Further, Milani (2005) presents a model with adaptive learning that generates inflation persistence without structural persistence of inflation and rational expectations assumption.

However, in the Barro-Gordon literature inflation persistence is often introduced through persistence of shocks. For example, Cukierman (1992) describes shocks as to have impact on the natural level of employment with persistent and transitory stochastic variables. Further, persistent component is assumed to follow a first order Markov process, while the transitory component follows a normally distributed white-noise process. In his model, inflation persistence is discussed under two alternative assumptions on information availability on the part of policymaker and the public. Under symmetric information, inflation exhibits persistence solely due to persistent shocks to the natural level of unemployment. Under asymmetric information, inflation exhibits more persistence as inflation becomes responsive to past transitory shocks. Because, public do not observe decomposition of shocks in the current period, inflation expectations respond to transitory shocks as well, and that will cause policymaker's response to be sensitive to transitory shocks, as the current inflation depends on inflation expectations of public. Eventually, current inflation responds to transitory shocks, even if shocks do not have real effects on current employment level.

However, some authors specify the structure of shocks slightly different manner. For an example, Reis (2003) introduces a dynamic general equilibrium model, in the context of Barro-Gordon framework and supply side shocks are characterised by zero mean and constant variance. As the model is explicitly derived from microfoundations, the shocks are identified as shocks to the mark-up of prices over marginal costs, and they result in deviations of unemployment from the equilibrium natural rate. The underlying source of inflation persistence in his model derives from the fact that the persistent changes in the natural rate of unemployment. More precisely, because the natural rate is time varying, even policymaker does not observe it, and, therefore, forms optimal forecasts. Because of imperfect information on the natural rate and supply shocks, policymaker's optimal forecasts may not be the same as the true value. Thus, as long as optimal forecast deviates from the natural level, actual inflation will deviate from the target level. Because forecast error is persistent, inflation tends to be higher than the target level until the error diminishes. Further, if the natural level of unemployment is underestimated (which is more likely a scenario), then the optimal response of policymaker would be to set actual inflation rate higher than the target level. According to Reis' specification, the process of updating forecasts has a geometric form and it would result in persistent deviations of inflation from the target.

Another important implication of the Barro-Gordon literature is that inflation persistence is often discussed using closed economy models. Only few studies are found to have used open economy extensions of Barro-Gordon model, for example, Bleaney (2001) provides some useful insights into research on inflation persistence across exchange rate regimes.

His model is a straightforward extension of the Barro-Gordon model, with a slightly modified policy objective function to represent home country is concerned with deviations of domestic inflation from the inflation rate of the foreign country. Bleaney assumes supply side shocks to follow an AR(1) process, through which persistence is accounted for. However, the policy objective function of his model seems to have contradictory objectives under a pegged exchange rate regime, as policymaker faces two inflation targets, at the same time, unless foreign and domestic inflation targets are assumed to be identical.

The following section describes the model used in the paper. Apparently, the building blocks of model are borrowed from the Barro-Gordon model. However, it can also be regarded as an open economy extension of Cukierman (1992).

3. The Model

Basically, the model consists of two components, namely, the policy objective function and a Phillips curve relationship. As described by several authors, specification of the economy in the Barro-Gordon model can well be supported by nominal rigidities. For example, Reis (2003) derives the expectations-augmented Phillips curve and the policy objective function with specific microfoundations in the form of a general equilibrium model with nominal rigidities.

3.1 Policy Objective Function

The model developed in this paper deviates from previous work mainly on the specification of policy objective function. The basic Barro-Gordon framework is a close economy model which implies that policymaker minimises expected loss due to deviations of actual inflation (π) and employment (y) from the desired levels. As the purpose of this paper is to explain inflation persistence over different exchange rate regimes, the model incorporates open economy characteristics. Thus, the objective function includes an additional term relating to deviations of domestic inflation rate from the inflation rate of the foreign country, which issues reserve currency in the fixed exchange rate system. Because, policymaker is entrusted with dual objectives, an additional parameter (d) is included in order to avoid conflicting implications of the objective function. This parameter indicates how committed the home country would be to maintain the peg, and is essentially defined as a continuum, (i.e., $d \in [0,1]$), of which the value is dependent upon the prevailing exchange rate regime. As the current exchange rate regime is expected to play a key role in optimal policy, *d* captures the effects of changes in exchange rate regimes on optimal inflation and its persistence.

Thus, policymaker minimises the present discounted value of expected losses:

$$\min \Pi = \sum_{i=0}^{\infty} \beta^{i} E_{t} L_{t+i} \ ,$$

where $\beta \in (0,1)$ is a discount factor and L_t is loss function which is quadratic in deviations of inflation and employment from target levels:

$$L_t = (1-d)\left[\frac{\lambda}{2}(\pi_t - \tilde{\pi}_t)^2 + \frac{1}{2}(y_t - \tilde{y}_t)^2\right] + \frac{d}{2}(\pi_t - \pi_t^f)^2 . \qquad (3.1)$$

Equation (3.1) is the open economy objective function where π is domestic inflation rate and $\tilde{\pi}$ is target inflation rate, y is actual employment and $\tilde{y}$ is target level of employment. π^f is foreign country's inflation rate, which is assumed to follow a fixed rule. The parameter λ is the relative weight attached to inflation stabilization to output stabilization. The parameter *d* is as defined above and it can accommodate for a range of exchange rate regimes within and including two extreme cases, namely, perfect fixity ($d \to 1$) and perfect flexibility ($d \to 0$).[3] For example, when $d \to 0$, the open economy objective function resembles to a closed economy model, as the fixed exchange rate system is completely abandoned.

The first term in brackets in the right hand side of equation (3.1) indicates costs associated with deviations of actual inflation from the target level. Similarly, the second term in brackets implies costs due to deviations of employment from the target level. The target level of employment is described as a function of natural level of employment y^N plus a positive parameter κ, which relates to policymaker's desire to expand the economy above the natural rate, i.e. $\tilde{y} = y^N + \kappa$. There are several interpretations for the existence of $\kappa > 0$, in policy objective function. Walsh (2003) describes two alternative interpretations, a) presence of labour market imperfections (such as wage tax, monopoly unions, monopolistic competition sectors etc.) which result in employment to be inefficiently low and, b) political pressure on central bank, because economic expansions would increase re-election prospects.[4] The term in parentheses in the extreme right of equation (3.1) relates to deviation of domestic inflation from inflation rate of the foreign country.

3.2 Specification of the economy

Following Barro and Gordon (1983), the short-run behaviour of the economy is described by an expectations-augmented Phillips curve, which implies that deviation of actual employment from the natural level is positively related to inflation surprises:

$$y_t = \bar{y}^N + \alpha(\pi_t - \pi_t^e), \qquad (3.2)$$

[3] Apart from the two extreme cases, the model may be used to account for some other related issues such as optimal exchange rate bands (see Cukierman *et al.*, 2004). In their model, exchange rate bands can also work as either of the two extreme cases depending on private agents' expectations on the reputation of policymaker. If public expects perfect reputation, exchange rate band would be seen as a perfect peg (*a zero band width*). On the contrary, if public expects no perfect reputation, then band would deviate within a certain width or perhaps it would go to the other extreme i.e., perfect flexibility (*a band of infinite width*). Thus, policymaker's emphasis over the trade off between flexibility of the exchange rate policy and cost of variability in the nominal exchange rate would really play a key role.

[4] See also Cukierman (1992) and Reis (2003).

where π^e is expected inflation, α is a positive parameter, and $\bar{y}^N$ is mean natural level of employment (a positive constant). Following Cukierman (1992), the natural rate of employment is defined as follows:

$$y_t^N = \bar{y}^N - u_t - \varepsilon_t , \qquad (3.3)$$

where u is the persistent component of the shocks and assumed to follow an AR(1) process and ε is a transitory stochastic variable with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \sigma_\varepsilon^2$. The persistent component is specified as:

$$u_t = \delta u_{t-1} + \upsilon_t , \qquad \upsilon_t \sim N(0, \sigma_\upsilon^2). \qquad (3.4)$$

where $\delta \in [0,1]$ which captures persistence in the natural rate, and υ is a normally distributed innovation term. This specification implies that natural level of employment exhibits stochastic fluctuations due to non-monetary factors, which is a widely expected phenomenon in empirical literature on the natural rate. Further, u can represent shocks due to changes in productivity, or coming from disutility of labour supply.

3.3 Policy Instrument

For simplicity, we assume that policymaker directly chooses the inflation rate, given current economic conditions. Initially, Barro and Gordon assume money growth as the policy instrument. However, most authors generally agree that money growth rate is closely linked to inflation rate. Cukierman (1992) defines the rate of inflation is equal to the money growth rate, abstracting from real shocks, growth and changes in velocity. Further, Walsh (2003) describes that distinction between policy instruments would be immaterial for the purpose of explaining determinants of average inflation rates, "[g]iven the focus on inflation, it will also be convenient at times simply to treat the inflation rate as the policy instrument" (p.370). However, it does make an impact in the discussion of stabilization policy.

4. Solving the Model and Analysis

The model presented in section 3 is solved under two key assumptions on information availability. First, the model assumes that policymaker and public share same information set. Policymaker solves for optimal rate of inflation in order to minimise the loss function, given current period's shocks and the constraint posed by the economy. Similarly, public form rational expectations independent of the past inflation rates, having obtained the same information on variables affecting the current policy choice. Therefore, the model reduces to the basic Barro-Gordon framework, in which policymaker and public solve a succession of 'one-shot' problems in each period.

Second, in the presence of asymmetric information, policymaker is assumed to possess up-to-date information over the value of natural rate of employment, and its decomposition into persistent and transitory components. However, public are assumed to obtain information over the shocks to natural rate after two periods. Nonetheless, in both cases, information over prevailing exchange rate regime (d) and the type of policymaker in office (λ) is assumed to be publicly available in each period.

4.1 Inflation Persistence under Symmetric Information

Due to symmetric information assumption, all variables in the objective function is in the policymaker's information set in period t, therefore, expectation operator is omitted from the objective function. [see Cukierman (1992)]. Similarly, as Walsh describes, even if policymaker aims at minimising the present discounted value of expected losses, the objective function of the basic Barro-Gordon framework does not imply a link between current decisions and future periods. By inverting the objective function to solve as a maximisation solution, and also using equations (3.2) and (3.3), the policy objective function is written as follows:

$$\Lambda = (1-d)\left[-\frac{\lambda}{2}(\pi_t)^2 - \frac{1}{2}\left(\alpha\left(\pi_t - \pi_t^e\right) + u_t + \varepsilon_t - \kappa\right)^2\right] - \frac{d}{2}\left(\pi_t - \pi_t^f\right)^2,$$

where domestic target inflation rate is normalised to zero for simplicity $(\tilde{\pi} = 0)$ and the definition of target output $\tilde{y} = y^N + \kappa$ is also used.

In what follows, a reduced form equilibrium inflation rate is derived given policymaker's optimal choice and rational expectations on the part of public. Further, based on the equilibrium inflation rate, the inflation persistence coefficient is obtained. The first-order condition of the maximisation problem implies semi-reduced form of optimal rule for setting inflation:

$$\pi_t = \frac{(1-d)\left[\alpha^2 \pi_t^e + \alpha\left(\kappa - u_t - \varepsilon_t\right)\right] + d\pi_t^f}{(1-d)\left(\lambda + \alpha^2\right) + d}. \tag{4.1}$$

Because the optimal policy depends on inflation expectations of public, taking unconditional expectation of equation (4.1), also using the fact that $E(u_t) = \delta u_{t-1}$:

$$\pi_t^e = \frac{(1-d)\alpha\left(\kappa - \delta u_{t-1}\right) + d\pi_t^f}{(1-d)\lambda + d} > 0, \tag{4.2}$$

The reduced form optimal inflation rate is derived by substituting equation (4.2) back into equation (4.1):

$$\pi_t = \frac{\alpha(1-d)\left(\kappa - \delta u_{t-1}\right) + d\pi^f}{(1-d)\lambda + d} - g\alpha(1-d)\left(\upsilon_t + \varepsilon_t\right), \tag{4.3}$$

where,

$$g = \left[(1-d)\left(\lambda + \alpha^2\right) + d\right]^{-1}.$$

Equation (4.3) implies that policymaker responds to shocks by accommodating more of the persistent component of the shock (δu_{t-1}), and less of the transitory component (ε_t). Further, because the persistent component is dependent upon the degree to which shocks are autocorrelated, the more persistent shocks are more strongly accommodated. This result is consistent with those in the literature [see, for example, Bleaney (2001)].

Further, equation (4.3) can explain the relation between optimal inflation rate and degree of exchange rare flexibility. For example, the optimal inflation rate under a perfectly fixed exchange rate regime (i.e., when $d \to 1$) is given by,

$$\pi_t = \pi_t^f , \tag{4.4}$$

where policymaker is fully committed to maintain the inflation rate of the foreign country. On the other hand, under perfectly flexible exchange rate regime (i.e., when $d \to 0$), the optimal inflation rate yields,

$$\pi_t = \frac{\alpha(\kappa - \delta u_{t-1})}{\lambda} - \left(\frac{\alpha}{\lambda + \alpha^2}\right)(\upsilon_t - \varepsilon_t), \tag{4.5}$$

where policymaker optimally sets the inflation rate contingent with other parameters of the model. For instance, if policymaker pursues a overly ambitious employment target, i.e., a higher κ, inflation rate would be higher. On the other hand, if policymaker places higher weight on inflation stabilization (i.e., higher λ), equation (4.5) implies a lower inflation rate. More importantly, degree of autocorrelation of the persistent component of shocks would have grater impact on optimal inflation rate in the flexible exchange rate regime.

4.1.1 Optimal inflation rate when targeting the foreign country's inflation rate

In the similar fashion as described above, the model can be solved for targeting the foreign country's inflation rate. Then the optimal inflation rate reduces to:

$$\pi_t = \pi^f + \frac{\alpha(1-d)(\kappa - \delta u_{t-1})}{[(1-d)\lambda + d]} - g\alpha(1-d)(\upsilon_t + \varepsilon_t). \tag{4.3a}$$

where $g = [(1-d)(\lambda + \alpha^2) + d]^{-1}$.

It implies that, in the absence of shocks to natural rate, there is one-to-one relationship between domestic and foreign country's inflation rate, irrespective of the degree of exchange rate flexibility. This contradicts with previous results where home country pursues its own domestic inflation target, in which the relation between domestic and foreign inflation rate determined by the degree of exchange rate flexibility. However, optimal inflation rate can deviate from the foreign inflation rate due to persistent and transitory shocks to employment.

Further, when considering the role of *d*, under perfectly flexible exchange rate regime ($d \to 0$), optimal domestic inflation rate reduces to,

$$\pi_t = \pi^f - \frac{\alpha\kappa\delta}{\lambda}u_{t-1} - \frac{\alpha}{\left(\lambda + \alpha^2\right)}\left(\upsilon_t + \varepsilon_t\right),$$

which implies that when the domestic economy is hit by shocks, the domestic inflation rate can be larger than the foreign country's inflation rate to the extent that the shock is persistent. The transitory components of the shock also result in deviations in the domestic inflation rate. On the other hand, it implies an equality between the domestic and foreign inflation rate under perfectly pegged exchange rate regime ($d \to 1$), as shown in equation (4.4).

4.1.2 Optimal Depreciation Rate

Assuming purchasing power parity holds, equation (4.3a) can be used to express the optimal depreciation rate:

$$\dot{s}_t = \frac{\alpha(1-d)(\kappa - \delta u_{t-1})}{[(1-d)\lambda + d]} - g\alpha(1-d)(\upsilon_t + \varepsilon_t),$$

which refers to the effects of shocks on the optimal inflation rate. A negative productivity shock is associated with a higher optimal inflation rate which results in a depreciation of the domestic currency. More persistent shocks result in larger depreciation. Further, the impact of shocks is also determined also by the degree of exchange rate flexibility.

4.1.3 Equilibrium under Discretion and Commitment

Equation (4.3) implies that on average a discretionary policy (i.e., when $d \to 0$) would yield a positive inflation rate,

$$\pi_t = \frac{\alpha\kappa}{\lambda}. \tag{4.6}$$

which is increasing in unanticipated inflation (α), and the incentive of the policymaker to expand the economy (κ), and decreasing in weight on inflation stabilization. This gives an equilibrium inflation rate under a perfectly flexible exchange rate policy. However, this outcome is achieved at the expense of loss of credibility as rational agents expect policymaker's incentive to inflate the economy in absence of a commitment to maintain a perfectly fixed exchange rate policy.

On the contrary, if policymaker is committed to credibly maintain the fixed exchange rate (i.e., when $d \to 1$), average domestic inflation rate would be equal to foreign inflation rate, as shown in equation (4.4).

A comparison between equations (4.4) and (4.6) would yield important implications. The equilibrium inflation rate under discretion would definitely be positive, where as under commitment it could be either zero or closer to zero, depending on the inflation rate of the foreign country. If the foreign country is credibly committed to maintain a zero inflation rate, equilibrium inflation rate under commitment would be preferred to discretionary outcome. However, the choice between discretion and commitment to a rule becomes harder in this context, as by committing to a fixed exchange rate, policymaker loses the control of employment stabilization. However, as described by Persson and Tabellini (1990) "[s]imple rules means to abandon activist stabilization. And discretion means to accept a higher average equilibrium rate of inflation. Which of these costs is higher generally depends on the parameters in the economy" (p.25).

4.1.4 Inflation Persistence Coefficient under Symmetric Information

This section derives inflation persistence coefficient as the correlation coefficient of current and past inflation rates. Taking one period lag of equation (4.3) and also using the result $\delta u_{t-2} = u_{t-1} - \varepsilon_{t-1}$:

$$\pi_{t-1} = \frac{\alpha(1-d)(\kappa - u_{t-1} + \upsilon_{t-1}) + d\pi^{f}}{(1-d)\lambda + d} - g\alpha(1-d)(\upsilon_{t-1} + \varepsilon_{t-1}). \tag{4.7}$$

It follows that unconditional expectation of equation (4.3) yields:

$$E_t(\pi_t) = \frac{(1-d)\alpha\kappa + d\pi_t^{f}}{(1-d)\lambda + d}. \tag{4.8}$$

Using equations (4.3), (4.7) and (4.8), the covariance of current and past inflation is derived using the result; $Cov(\pi_t, \pi_{t-1}) = E[\pi_t - E(\pi_t)]\ [\pi_{t-1} - E(\pi_t)]$:

$$Cov(\pi_t, \pi_{t-1}) = \delta\left(\frac{\alpha(1-d)}{(1-d)\lambda + d}\right)^2 \sigma_u^2. \tag{4.9}$$

Further, using equation (4.3), the variance of the optimal inflation rate is derived as:

$$Var(\pi_t) = \alpha^2 \sigma_u^2 (1-d)^2 \left(\delta^2 \left(\frac{1}{(1-d)\lambda + d}\right)^2 + g^2\left((1-\delta^2) + \frac{\sigma_\varepsilon^2}{\sigma_u^2}\right)\right). \tag{4.10}$$

Equations (4.9) and (4.10) yield the correlation coefficient of current and past inflation:

$$\rho^{SI}_{(\pi_t, \pi_{t-1})} = \frac{\delta}{[(1-d)\lambda + d]^2 \left(\frac{\delta^2}{[(1-d)\lambda + d]^2} + \frac{(1-\delta^2) + \sigma}{[(1-d)(\alpha^2 + \lambda) + d]^2}\right)}, \tag{4.11}$$

where $\sigma = \sigma_\varepsilon^2 / \sigma_u^2$ (variance ratio).

This implies that when δ = 0, equation (4.11) becomes zero, i.e., $\rho^{SI}_{(\pi_t, \pi_{t-1})} = 0$, so the model explains no inflation persistence. When $\delta > 0$, inflation exhibits persistence depending on the parameters α , λ , d, and the variance ratio. The inflation persistence coefficient is expected to be positively related to changes in α , and negatively related to changes in λ and d. Further, it turns out that the persistence coefficient is negatively related to changes in the variance ratio. An increased volatility in the transitory component of the shock (σ_{ε}^{2}) would result in an increased variance ratio (having σ_{u}^{2} unchanged), and eventually less persistence in the inflation process. Intuitively, as ε_t is unanticipated by public, its increased volatility would not result in higher inflation expectations, and that would not exacerbate the pressure on persistent of the inflation process. By contrast, an increase in σ_{u}^{2} would result in more volatility of the persistent component , and having anticipated by public, it would result in more inflation persistence.

Further, the relation between parameters d and ρ has several implications. Equation (4.11) is turned out to be decreasing in d, implying more constraining exchange rate regimes may result in lower inflation persistence. Further, for extreme values of d i.e., when $d \rightarrow 0$ (i.e., perfect flexibility), persistence coefficient reduces to:

$$\rho_{(\pi_t,\pi_{t-1})}^{SI} = \frac{\delta}{\lambda^2\left(\left(\frac{\delta}{\lambda}\right)^2 + \frac{\left(1-\delta^2\right)+\sigma}{\left(\alpha^2+\lambda\right)^2}\right)}, \tag{4.12}$$

which implies a larger coefficient value than in equation (4.11). On the contrary, when $d \rightarrow 1$(i.e., perfect fixity), persistent coefficient is independent of other parameters of the model [$\rho_{(\pi_t,\pi_{t-1})}^{SI} = \delta \Big/ \left(1+\frac{\sigma_{\varepsilon}^{2}}{\sigma_{u}^{2}}\right)$]. Figure 1 depicts the response of inflation persistence coefficient to varying degrees of autocorrelation of socks and the degree of exchange rate flexibility. The graphs through out the paper (if not otherwise mentioned) are based on the parameter values; $\alpha = \lambda = 1$. And in the symmetric information case, the variance ratio is assumed to be 0.5. The value of α is more or less justified given the econometric evidence, which suggests that it takes values in the range of between 0.8 and 2. In case of λ , the model assumes that policymaker is equally concerned with inflation and employment stabilization. The values selected for variance ratio is consistent with the early work [see Reis (2003)]. However, it is noted that there are no established priors about the values of these components can take.

Result 1: *The degree of inflation persistence is positively correlated with both the degree of exchange rate flexibility and the degree of autocorrelation of shocks to natural rate. However, the response of inflation persistence to increased degree of exchange rate flexibility is lessoned for largely autocorrelated shocks.*

As shown in Figure 1, inflation persistence declines as $d \rightarrow 1$(i.e., moving towards more constraint exchange rates). Further, larger values of degree of autocorrelation (i.e., higher δ) result in higher inflation persistence.

Figure 1: Inflation persistence and degree of exchange rate flexibility: Symmetric information

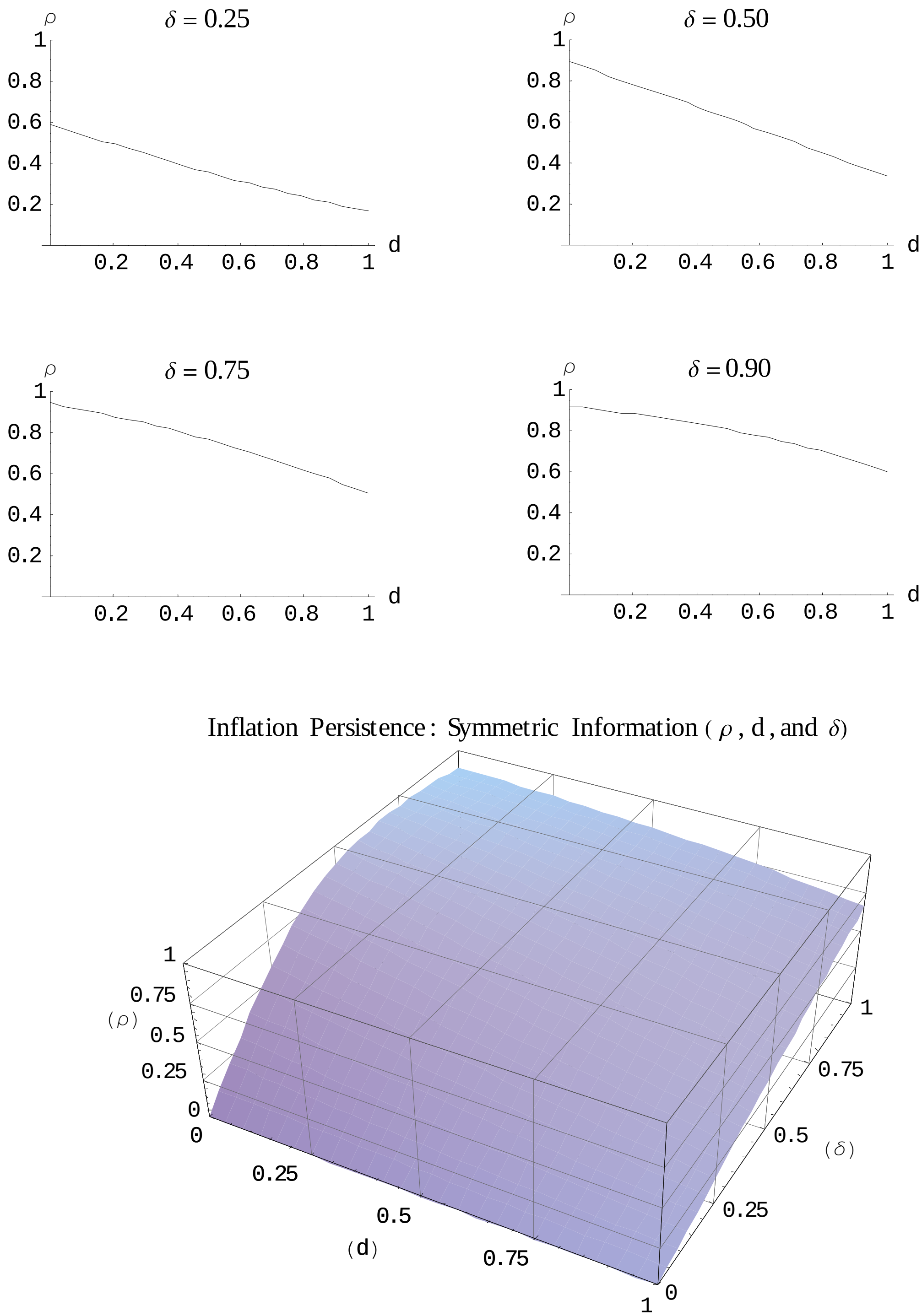
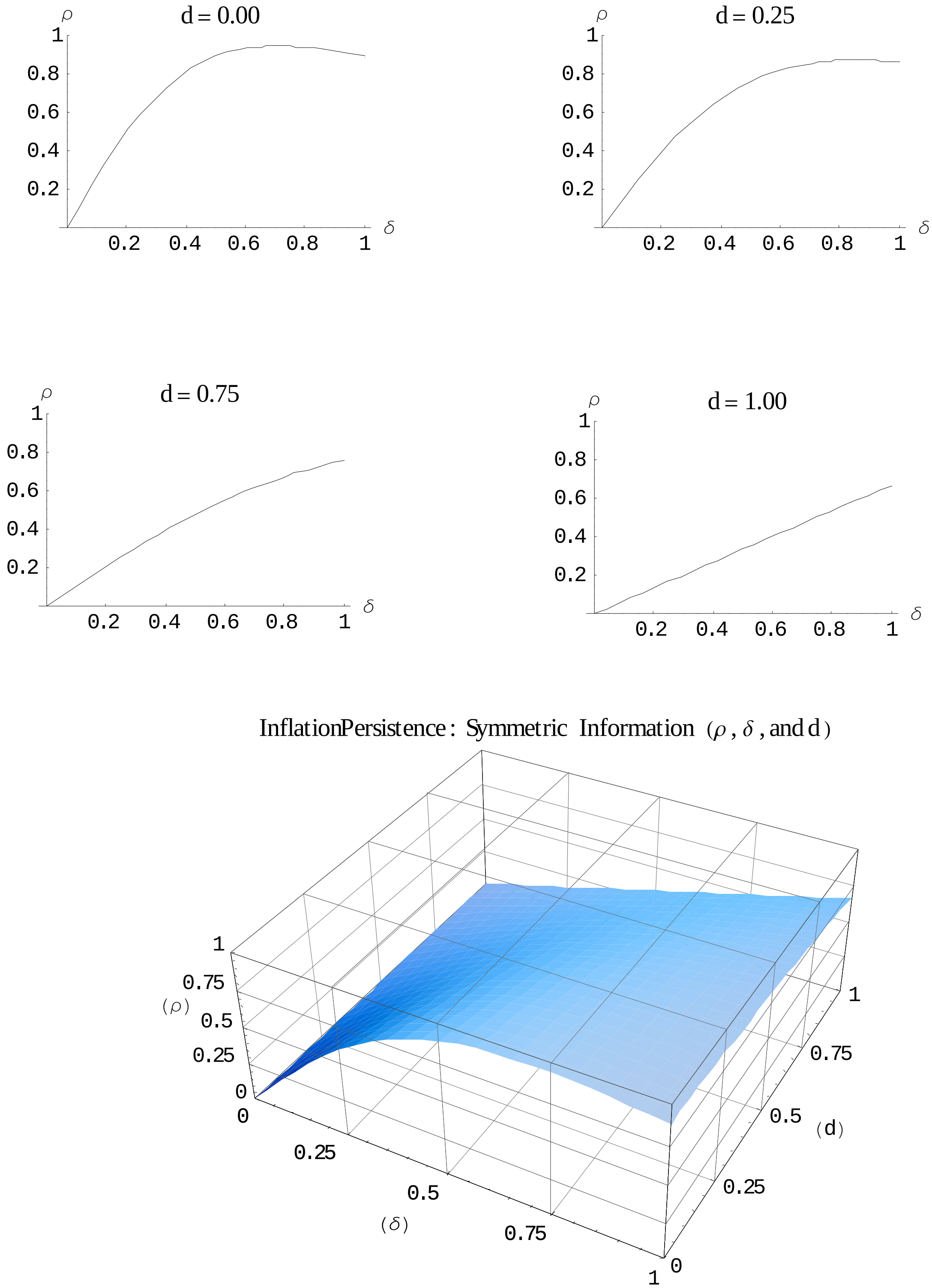


Figure 2: Inflation persistence and degree of autocorrelation of supply shocks: Symmetric information



However, the response of inflation persistence as $d \to 1$, is not linear for all values of δ. As can be seen in Figure 1, the path of inflation persistence is convex for values $\delta < 0.50$, where as it turns out be concave for values $\delta > 0.50$. Similarly, Figure 2 shows how inflation persistence responds to degree of autocorrelation of shocks for certain values of *d*. Again for lower values of d, persistence coefficient is more responsive, however, as $d \to 1$, the response declines. Overall, inflation persistence shows marked response for lower values of δ or *d* and the response declined as these values get close to one.

Further, inflation persistence shows expected response for other parameters of the model. For example, higher values of α are associated with more persistent inflation, and again persistence coefficient responds less to degree of exchange rate flexibility for largely autocorrelated shocks. On the other hand, for larger values of λ, inflation persistence tends to be more rigid, irrespective of the degree of exchange rate flexibility, and other parameters of the model. However, the degree of autocorrelation of shocks determines the level of persistence. Further, the model implies that when transitory component of shock is more volatile than the persistent component, inflation persistence tends to be lower.

Another important implication of equation (4.11) is that inflation persistence in home country is independent of that of foreign country, even if the peg is maintained perfectly. This is in stark contrast to the result derived in Bleaney (2001), as his model implies that home country would have lower inflation persistence only if the foreign country has lower persistence.

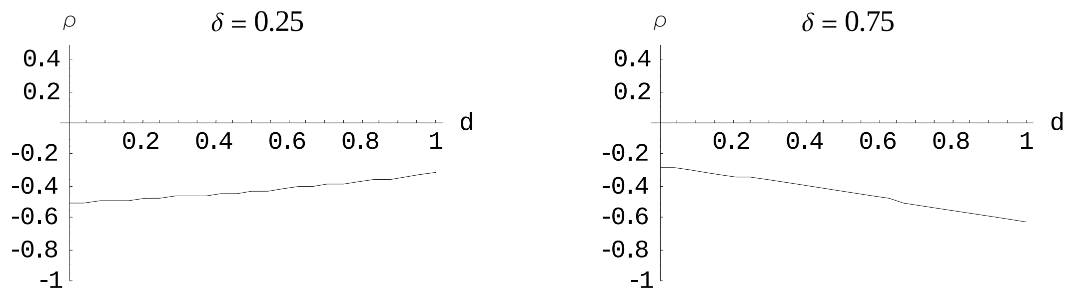
4.1.5 Comparative Statics under Symmetric Information

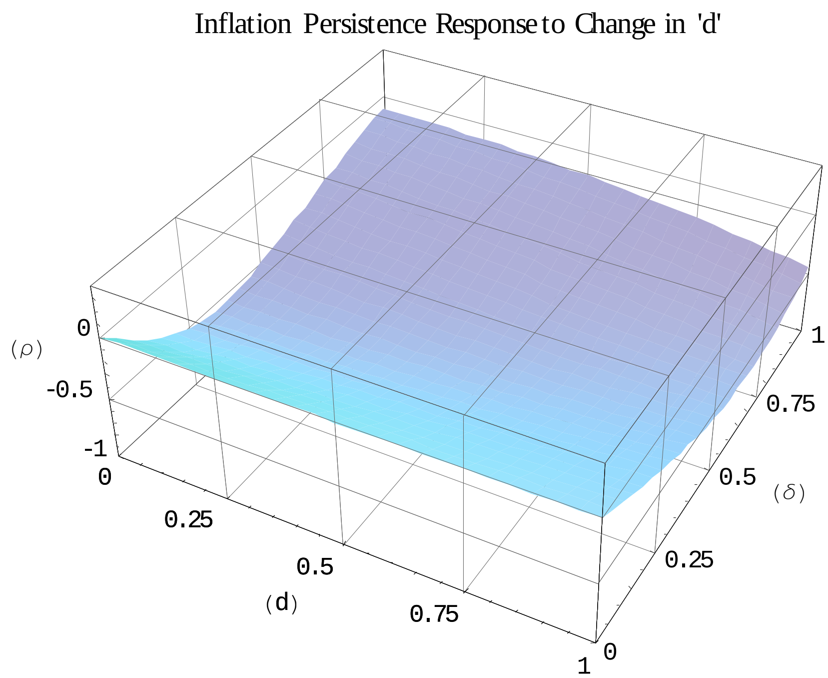
Figures 3 and 4 show the behaviour of inflation persistence to a change in the degree of exchange rate flexibility and the degree of autocorrelation of shocks, respectively.

Result 2: *The response of inflation persistence to a change in the degree of exchange rate flexibility is asymmetric given the value of* δ. *For lower values of* δ *(e.g.,* $\delta < 0.40$*) an increase of d results in more persistence, and for higher values of* δ *(e.g.,* $\delta > 0.60$*) an increase of d yields lower persistence.*

Figure 3 shows the path of inflation persistence coefficient to a change in the degree of exchange rate flexibility. When the shock to natural rate is less autocorrelated, any attempt to increase the degree of exchange rate flexibility would yield counter productive results. On the other hand, for highly autocorrelated shocks, increased constraint of the exchange rate would result in more persistence.

Figure 3: Inflation persistence response to a change in the degree of exchange rate flexibility: Symmetric Information

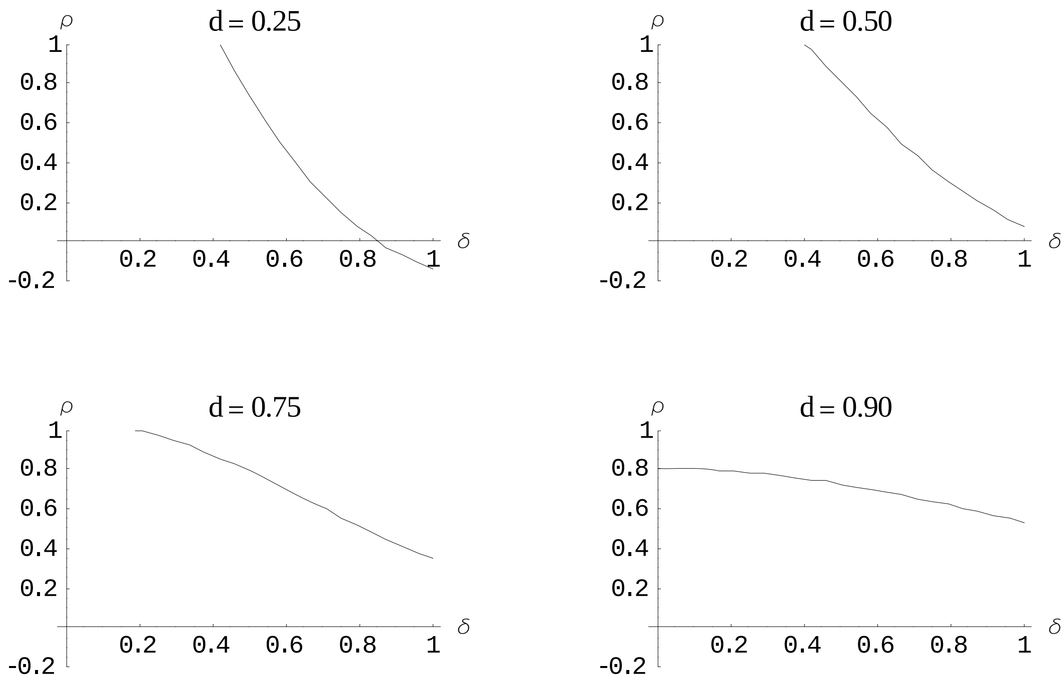




Result 3: *The response of inflation persistence to a change in the degree of autocorrelation of shocks yields inconclusive results. The path of inflation persistence is declining as* $\delta \to 1$, *and becomes less responsive for higher values of d.*

As shown in Figure 4, the initial positive response of inflation persistence tends to decrease at an decreasing rate, as the degree of exchange rate flexibility increases. At the perfectly pegged exchange rate, inflation persistence is constant irrespective of the degree of autocorrelation.

Figure 4: Inflation persistence response to a change in the degree of autocorrelation of supply shocks: Symmetric Information



4.2 Inflation Persistence under Asymmetric Information

Following Cukierman (1992), the information advantage of policymaker is characterized by the assumption that public observe actual employment and decomposition of transitory and stochastic components of shocks, with a two period lag. In contrast, policymaker possesses up-to-date information on the current state of the natural level of employment and its decomposition, enabling to forecast current and future natural levels of employment more precisely and thereby stabilize real fluctuations in employment.

Substituting equations (3.2) and (3.3) into policy objective function:

$$E_t \sum_{i=0}^{\infty} \beta^i \left\{ (1-d)\left[\frac{\lambda}{2}(\pi_t)^2 + \frac{1}{2}\left(h_t - \alpha\left(\pi_t - \pi_t^e\right)\right)^2 \right] + \frac{d}{2}\left(\pi_t - \pi_t^f\right)^2 \right\}, \qquad (4.13)$$

where, $h_t \equiv \tilde{y}_t - \bar{y}^N + u_t + \varepsilon_t \equiv H + u_t + \varepsilon_t$. (4.14)

The parameter H is assumed to be positive because policymaker perceives the natural level of employment to be too low due to distortionary taxes, and also policymaker partially responds to political pressures so that desired level of employment is kept above the natural level. The equilibrium condition is characterised by strategic responses of policymaker and public. Thus, policymaker chooses the inflation rate, given expectations of public, in order to minimise the expected loss due to deviations of inflation and employment from target levels. Similarly, public form expectations, given their perception of the response of policymaker, in order to minimise *conditional mean forecast error*. Because of this strategic interaction between optimal policy of policymaker, and optimal inflation forecasts of public, solution of the model needs to be characterised simultaneously, which is achieved by using the method of undetermined coefficients.

4.2.1 Solving for equilibrium

Policy objective function implies that optimal inflation depends on *h*, π^e and π^f in the current period, and currently expected next period's values of *h*, π^e, π^f and π. The reason why only next period's values are considered, is because public observe values of u_t after two periods, and therefore current inflation is needed only for forecasting $t+1$ inflation rate. Thus, inflation expectations from $t+2$ and onwards are not influenced by the choice of π_t. Therefore, optimal value of π_t depends on h_t, π_t^e, π_t^f and period *t* expectations on h_{t+1}, π_{t+1}^e and π_{t+1} (assuming foreign country to follow a fixed rule). The solution of policymaker's decision strategy is described by the following linear function:

$$\pi_t = K_1 h_t + K_2 \pi_t^e + K_3 \pi_t^f + K_4 E_{G,t} h_{t+1} + K_5 E_{G,t}\left(\pi_{t+1} - \pi_{t+1}^e\right) \tag{4.15}$$

where $K_i, i = 1,...,5$ are coefficients to be determined. The subscript 'G' refers to expectations of policymaker. In what follows, we assume that in the beginning of each period, public enter into nominal wage contracts given their inflation expectations, based on information set I_t, which includes information on employment level and persistent component of natural level up to and including $t-2$, and past inflation rates up to and including π_{t-1}.

Accordingly, policymaker chooses current inflation rate, given public's expectations, after observing current level of employment and after observing persistent and transitory components of the natural level.

Taking equation (4.15) one period forward:

$$E\left[\pi_{t+1} \mid I_{t+1}\right] \equiv \pi_{t+1}^e = K_1 E\left[h_{t+1} \mid I_{t+1}\right] + K_2 \pi_{t+1}^e + K_3 \pi_{t+1}^f + K_4 E\left[E_{G,t+1} h_{t+2} \mid I_{t+1}\right]$$

$$+ K_5 E\left[E_{G,t+1}\left(\pi_{t+2} - \pi_{t+2}^e\right) \mid I_{t+1}\right]. \tag{4.16}$$

Using equation (4.14), expected value of the third term in the right-hand side of equation (4.16) is written as:

$$E\left[h_{t+2} \mid I_{t+1}\right] = H + \delta^3 u_{t-1} + \delta^2 E\left[\upsilon_t \mid I_{t+1}\right] \tag{4.17}$$

Also, given public information set in period $t+1$, expected value of the last term in equation (4.16) is equal to zero:

$$E\left[E_{G,t+1}\left(\pi_{t+2} - \pi_{t+2}^e\right) \mid I_{t+1}\right] = 0 \tag{4.18}$$

Substituting (3.4) into (4.15), using the result in (4.14):

$$\pi_t = K_1\left(H + \delta u_{t-1} + \upsilon_t + \varepsilon_t\right) + K_2\pi_t^e + K_3\pi_t^f + K_4 E_{G,t}\left(H + \delta^2 u_{t-1} + \delta\upsilon_t\right) + K_5 E_{G,t}\left(\pi_{t+1} - \pi_{t+1}^e\right).$$

Rearranging yields:

$$\pi_t - K_1\left(H + \delta u_{t-1}\right) - K_2\pi_t^e - K_3\pi_t^f - K_4\left(H + \delta^2 u_{t-1}\right) - K_5 E_{G,t}\left(\pi_{t+1} - \pi_{t+1}^e\right) = K_1\left(\upsilon_t + \varepsilon_t\right) + K_4\delta\upsilon_t\,. \tag{4.19}$$

Equation (4.19) reveals the basic informational problem of the public. They are interested in getting as accurate as possible an estimate of υ_t but observe only a mixture of this variable with other stochastic variable, as shown in the left hand side of the equation. However, according to the assumptions on information set, public know all the terms in the left-hand side of equation (4.19), except for $E_{G,t}\left(\pi_{t+1} - \pi_{t+1}^e\right)$. Following Cukierman, we assume, for simplicity, that public assume this expression is equal to zero. However, this assumption may restrict the rationality of public's expectations formation procedure.

From equation (4.19):

$$E\left[\upsilon_t \mid I_{t+1}\right] = \frac{\left(K_1 + \delta K_4\right)\sigma_\upsilon^2}{\left(K_1 + \delta K_4\right)^2 \sigma_\upsilon^2 + K_1^2 \sigma_\delta^2}\left[\pi_t - \omega(t)\right], \tag{4.20}$$

where $\omega(t) = K_1\left(H + \delta u_{t-1}\right) - K_2\pi_t^e - K_3\pi_t^f - K_4\left(H + \delta^2 u_{t-1}\right).$ (4.21)

Now, the problem of public, as implied in equation (4.20), is to obtain the best forecast of υ_t conditional on $\pi_t - \omega(t)$. This best forecast is equal to conditional expected value and is given by the right hand side of equation (4.20), where the term preceding $\pi_t - \omega(t)$ is the regression coefficient of υ_t on $\pi_t - \omega(t)$.

Using (3.4) in equation (4.14) with one period lead, i.e., $h_{t+1} = H + u_{t+1} + \varepsilon_{t+1}$,

$$E\left[h_{t+1} \mid I_{t+1}\right] = H + \delta^2 u_{t-1} + \delta E\left[\upsilon_t \mid I_{t+1}\right]. \tag{4.22}$$

Now substituting equations (4.17), (4.18), (4.20) and (4.22) into (4.16) yields:

$$E[\pi_{t+1} \mid I_{t+1}] \equiv \pi^e_{t+1} = K_1\left(H + \delta^2 u_{t-1} + \delta E[\upsilon_t \mid I_{t+1}]\right) + K_2 \pi^e_{t+1} + K_3 \pi^f_{t+1} + K_4\left(H + \delta^3 u_{t-1} + \delta^2 E[\upsilon_t \mid I_{t+1}]\right),$$

which implies, after rearranging, an expression for public's expectation formation process:

$$\pi^e_{t+1} = \frac{1}{1-K_2}\left\{(K_1 + K_4)H + K_3 \pi^f_{t+1} + \delta^2 (K_1 + \delta K_4) u_{t-1} + \delta\theta[\pi_t - \omega(t)]\right\}, \tag{4.23}$$

where, $$\theta \equiv \frac{(K_1 + \delta K_4)^2 \sigma^2_\upsilon}{(K_1 + \delta K_4)^2 \sigma^2_\upsilon + K_1^2 \sigma^2_\delta}. \tag{4.24}$$

Equation (4.23) implies that a unit increase in π_t increases inflation expectations in the following period by:

$$\frac{\partial \pi^e_{t+1}}{\partial \pi_t} = \frac{\delta\theta}{1-K_2} \tag{4.25}$$

Differentiating policy objective function with respect to π_0, using (4.25) and the fact that $\partial \pi^e_{t+i} / \partial \pi_t = 0$ for $i \geq 2$, the optimal inflation rate in the semi-reduced form can be derived:

$$\pi_0 = \frac{\alpha(1-d)}{[(1-d)(\lambda + \alpha^2) + d]} h_0 + \frac{\alpha^2(1-d)}{[(1-d)(\lambda + \alpha^2) + d]} \pi^e_0 + \frac{d}{[(1-d)(\lambda + \alpha^2) + d]} \pi^f_0$$
$$- \frac{\alpha\beta\delta^2(1-d)}{1-K_2} \theta\left[E_{G,0} h_1 - \alpha E_{G,0}\left(\pi_1 - \pi^e_1\right)\right]. \tag{4.26}$$

Because the structure of policymaker's decision problem is the same in each period as in the period 0, the decision strategy for any period is given by:

$$\pi_t = \frac{\alpha(1-d)}{[(1-d)(\lambda + \alpha^2) + d]} h_t + \frac{\alpha^2(1-d)}{[(1-d)(\lambda + \alpha^2) + d]} \pi^e_t + \frac{d}{[(1-d)(\lambda + \alpha^2) + d]} \pi^f_t$$
$$- \frac{\alpha\beta\delta^2(1-d)}{1-K_2} \theta\left[E_{G,t} h_{t+1} - \alpha E_{G,t}\left(\pi_{t+1} - \pi^e_{t+1}\right)\right]. \tag{4.27}$$

The coefficients of equation (4.27) provide the solutions to the undetermined coefficients in equation (4.15) such that:

$$K_1 = \frac{\alpha(1-d)}{[(1-d)(\lambda + \alpha^2) + d]}, \tag{4.29}$$

$$K_2 = \frac{\alpha^2(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}, \tag{4.30}$$

$$K_3 = \frac{d}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]} \tag{4.31}$$

$$K_4 = -\frac{\alpha\beta\delta^2(1-d)}{1-K_2}\left(\frac{\left(K_1+\delta K_4\right)^2\sigma_\upsilon^2}{\left(K_1+\delta K_4\right)^2\sigma_\upsilon^2+K_1^2\sigma_\delta^2}\right),$$

$$= -\frac{\alpha\beta\delta^2(1-d)\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}{(1-d)\lambda+d}\frac{\left[\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]+\delta K_4\right]^2\sigma_\upsilon^2}{\left[(1-d)\left(\lambda+\alpha^2\right)+d++\delta K_4\right]^2\sigma_\upsilon^2+\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]^2\sigma_\delta^2},$$

$$\equiv \Omega\left(K_4\right)^5, \tag{4.32}$$

$$K_5 = -\alpha K_4 \quad . \tag{4.33}$$

Using these results and also using the fact that $E_{G,t}h_{t+1} = H + \delta u_t$, optimal inflation rate follows:

$$\pi_t = \frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}\left(h_t + \alpha\pi_t^e + \frac{d}{\alpha(1-d)}\pi_t^f\right) - |K_3|\left[H + \delta u_t - \alpha E_{G,t}\left(\pi_{t+1} - \pi_{t+1}^e\right)\right]. \tag{4.34}$$

If inflation rate of foreign country is assumed to be zero, the first term in parentheses is reduced to $\left(h_t + \alpha\pi_t^e\right)$, which implies the difference between desired and actual employment levels when inflation rate in home country is set equal to zero in period t. The second term in the right hand side of equation (4.34) implies the same difference as expected by policymaker in period *t*, for the following period. The implications of this equation are straightforward. A positive deviation of actual employment from the desired level in the current period would result in higher optimal inflation rate. On the contrary, if policymaker expects a positive future deviation of actual employment, optimal inflation rate would be lower. As described by Cukierman, the behaviour of these two terms may well be explained proportionately to the marginal cost of low employment. An expansionary policy in the current period would increase next period's inflation expectations which results in lower employment, i.e., higher marginal cost of low employment in the next period. Because policymaker dislikes reduction in next period's employment, it may attempt to reduce higher inflation expectations by lowering current inflation rate. Thus, the inflation bias of the policymaker in the current period would be partly off set due to perceived reductions in employment in the next period.

[5] Following Cukierman (1992), it can be shown that K_4 has always a non-positive solution. (p.281).

However, the impact of current deviations of employment on the optimal inflation rate would also depend on the degree of flexibility of the nominal exchange rate. Equation (4.34) implies that when the exchange rate is more rigid (i.e., a higher *d*) the coefficient of the first term becomes smaller constraining the policymaker's temptation to pursue an activist policy. On the other hand, a more flexible exchange rate implies a strong incentive of policymaker to respond to current marginal cost of low employment. Thus, the open economy version of Cukierman model clearly implies an asymmetric response of policymaker to varying marginal cost of low employment under different exchange rate regimes, and the results may be generalised into an open economy version of Barro-Gordon model as well.

4.2.2 Persistence in Inflation Expectations

An important advantage of the assumption of information asymmetry is that it helps to model public's expectation formation process more realistically. In real world, public may not be informed about the persistent and transitory components of shocks, at the same time as policymaker. Therefore, as shown in the following result, their expectations formation process includes transitory shocks as well.

Using equations (4.19) and (4.21) to form:

$$\pi_t - g(t) = (K_1 + \delta K_4)\upsilon_t + K_1 \varepsilon_t \,,$$

as shown in Appendix A, using the solutions in equations (4.29), (4.30), and (4.23) with the above expression and taking one period lag yields the current periods inflation expectations:

$$\pi_t^e = \frac{(1-d)(\lambda+\alpha^2)+d}{(1-d)\lambda+d}\left[\left(\frac{\alpha(1-d)}{[(1-d)(\lambda+\alpha^2)+d]}+K_4\right)H + K_3\pi_t^f\right]$$

$$+\frac{[(1-d)(\lambda+\alpha^2)+d]}{(1-d)\lambda+d}\left(\frac{\alpha(1-d)}{[(1-d)(\lambda+\alpha^2)+d]}+\delta K_4\right)$$

$$.\left[\delta^2 u_{t-2} + \delta\theta\left(\upsilon_{t-1} + \frac{\alpha(1-d)}{\alpha(1-d)+\delta[(1-d)(\lambda+\alpha^2)+d]K_4}\varepsilon_{t-1}\right)\right] \qquad (4.35)$$

Equation (4.35) explains inflation persistence implied in the model under asymmetric information. Because public do not possess as much information as policymaker, they are unable to fully disentangle *previous* period innovation to persistent part of employment (υ_{t-1}), from the transitory part of employment in that period, (ε_{t-1}). Thus, period *t* expectations are affected by past transitory shocks, which results in persistence in inflation expectations. On the contrary, policymaker obtains up-to-date information on the decomposition of permanent and transitory shocks, so it does not directly react to transitory shocks. However, policymaker partly accommodates current inflation expectations as implied by α in equation (4.34). Because public expectations are affected by past transitory shocks, the current inflation is also then affected by transitory shocks. Thus, asymmetric information transforms transitory shocks to natural employment into persistent movements in actual inflation.

However, equation (4.35) is not a reduced form solution to the policymaker's optimization problem. Therefore, as shown in Appendix B, a reduced form expression is derived for optimal inflation rate chosen by policymaker under asymmetric information. In the same token of inflation persistence implied in equation (4.35), the reduced form optimal inflation implies the inflation persistence due to sluggishness in inflation expectations in terms of various components of the natural rate employment. Importantly, the role played by the parameter relating to constraint of the nominal exchange rate, and the sensitivity to the foreign inflation rate explicitly modeled.

$$\pi_t = \frac{\left(\alpha(1-d)+\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]K_4\right)}{(1-d)\lambda+d}H+\left(\frac{d}{(1-d)\lambda+d}\right)\pi_t^f$$

$$+\frac{\left(\alpha(1-d)+\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]\delta K_4\right)}{(1-d)\lambda+d}\delta^2 u_{t-2}$$

$$+\left(\frac{(1-d)\left(\lambda+\alpha^2\theta\right)+d}{(1-d)\lambda+d}\right)\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}+\delta K_4\right)\delta\upsilon_{t-1}$$

$$+(1-\alpha\delta(1-\theta)K_4)\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}+\delta K_4\right)\upsilon_t+\frac{(1+K_4\alpha\delta\theta)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}\alpha(1-d)\varepsilon_t$$

$$+\left(\frac{\alpha^3(1-d)^2}{((1-d)\lambda+d)\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}\right)\delta\theta\varepsilon_{t-1} \qquad (4.36)$$

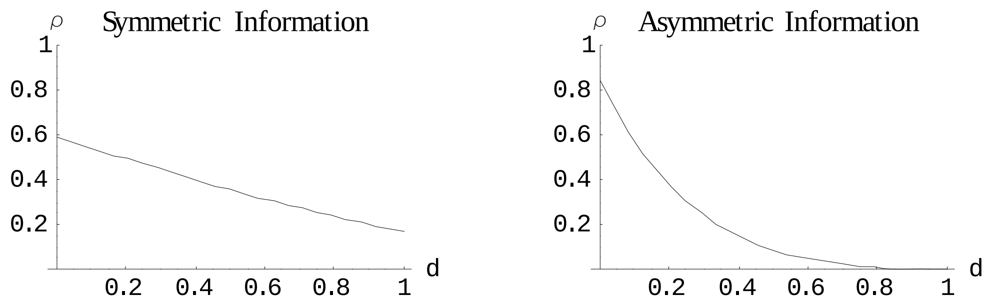
In this result, the impact of exchange rate flexibility and the foreign country's inflation rate provide more insight into policymaker's optimization solution. More importantly, equation (4.36) can explain inflation persistence given the assumption of asymmetric information. The key implication of inflation persistence derives from the fact that optimal inflation rate in current period responds to past transitory shocks to natural rate of employment, despite they do not affect the natural rate. Because our assumption allows public to obtain information about the components of shocks to natural rate after two periods, public do not observe persistence and transitory components of previous period's shocks (υ_{t-1}), in the current period. Therefore, they take π_{t-1} alternatively, into expectations formation process. However, π_{t-1} are also affected by transitory shocks ε_{t-1} because of lack of information, and therefore inflation expectations always carry some element of transitory shocks to natural rate. Since, policymaker in each period responds to inflation expectations, current inflation responds to past transitory shocks.

In order to explain the behaviour of inflation persistence under asymmetric information, an expression is derived for correlation between current and past inflation using the optimal inflation rate in equation (4.36). As shown in Appendix C, the inflation persistence coefficient takes the following form:

$$
\begin{aligned}
\rho^{AI}_{(\pi_t,\pi_{t-1})} = & \left\{ \frac{\left[(1-d)\left(\alpha^2+\lambda\right)+d\right]^2}{\left[(1-d)\lambda+d\right]} \left[(1-d)^3 \alpha^4 \delta\theta\left(1+\alpha\delta\theta K_4\right)\right]\sigma_\varepsilon^2 \right. \\
& \left. + \frac{\left[(1-d)\left(\alpha^2\theta+\lambda\right)+d\right]^2}{\left[(1-d)\lambda+d\right]} \delta\left(1+\alpha\delta K_4(1-\theta)\right)\left[\frac{(1-d)\alpha}{\left[(1-d)\left(\alpha^2+\lambda\right)+d\right]^2}+\delta K_4\right]^2 \sigma_\nu^2 \right\} \Bigg/ \\
& \left\{ \left(\frac{(1-d)\alpha\delta\theta}{\left[(1-d)\left(\alpha^2+\lambda\right)+d\right]}\right)^2 \left[(1+\alpha K_4)^2 + \left(\frac{(1-d)\alpha^2}{\left[(1-d)\lambda+d\right]}\right)^2\right]\sigma_\varepsilon^2 \right. \\
& + \frac{\delta^4}{\left[(1-d)\lambda+d\right]^2}\left[(1-d)\alpha+\left[(1-d)\left(\alpha^2+\lambda\right)+d\right]\delta K_4\right]^2 \sigma_u^2 \\
& \left. + \left(\left(1-\alpha\delta K_4(1-\theta)\right)^2 + \left(\frac{\delta^2\left[(1-d)\left(\alpha^2\theta+\lambda\right)+d\right]^2}{\left[(1-d)\lambda+d\right]^2}\right)\left[\frac{(1-d)\alpha}{\left[(1-d)\left(\alpha^2+\lambda\right)+d\right]^2}+\delta K_4\right]^2 \right)\sigma_\nu^2 \right\}
\end{aligned}
\tag{4.37}
$$

Equation (4.37) implies that the inflation persistence coefficient under asymmetric information responds to variance of various components of natural level of employment. Further, as implied in equation (4.11), persistence is introduced through nonzero values of δ, and it also depends on the parameters α, λ, *d*, and θ which relates to the *speed of learning*. As implied in equation (4.36), where current inflation also responds to previous transitory shocks, asymmetric information may result in more persistence in the inflation process. As shown in Figure 6, that may seem to be the case for lower variance values of the error term of the persistent component of shocks. Figure 6 compares inflation persistence under symmetric and asymmetric information, for $\delta = 0.25$ and $\sigma = 0.5$. The initial level of persistence is lower under symmetric information (about 0.6) and it is higher under asymmetric information (about 0.8). However, this result is true only for lower variance of the error term in the persistence component (e.g., $\sigma_\nu^2 = 0.01$). Further, the path of persistence is significantly different under two cases, as $d \to 1$, where persistence tends to be $\rho^{SI}_{(\pi_t,\pi_{t-1})} = \delta \Big/ \left(1+\frac{\sigma_\varepsilon^2}{\sigma_u^2}\right)$ under symmetric case, and it approaches zero under asymmetric information.

Figure 6: Inflation persistence under symmetric and asymmetric information



Moreover, the response of inflation persistence under asymmetric information differs from symmetric information on various counts.

Result 6: *Inflation persistence increases only for up to a certain levels of autocorrelation of shocks (e.g., $\delta < 0.50$), and beyond that it starts declining, for given values of variance of shocks to employment.*

Figure 7 shows the path of inflation persistence under different levels of autocorrelation of shocks, given variance of all components to be equal to 0.05. The initial level of persistence is increasing as $\delta \to 0.50$, and it tends to decrease afterwards as $\delta \to 1$. This response is much clearer in Figure 8, where inflation persistence coefficient has upward trend as $\delta \to 0.50$, and starts declining. Further, similar to the symmetric information case, a similar response can be observed for a higher variance σ_v^2 of persistent component, which leads to less persistence, having the relation between ρ and δ in tact, as shown in Figure 7.

Figure 7: Asymmetric Response of Inflation persistence for different values of degree of autocorrelation of supply shocks

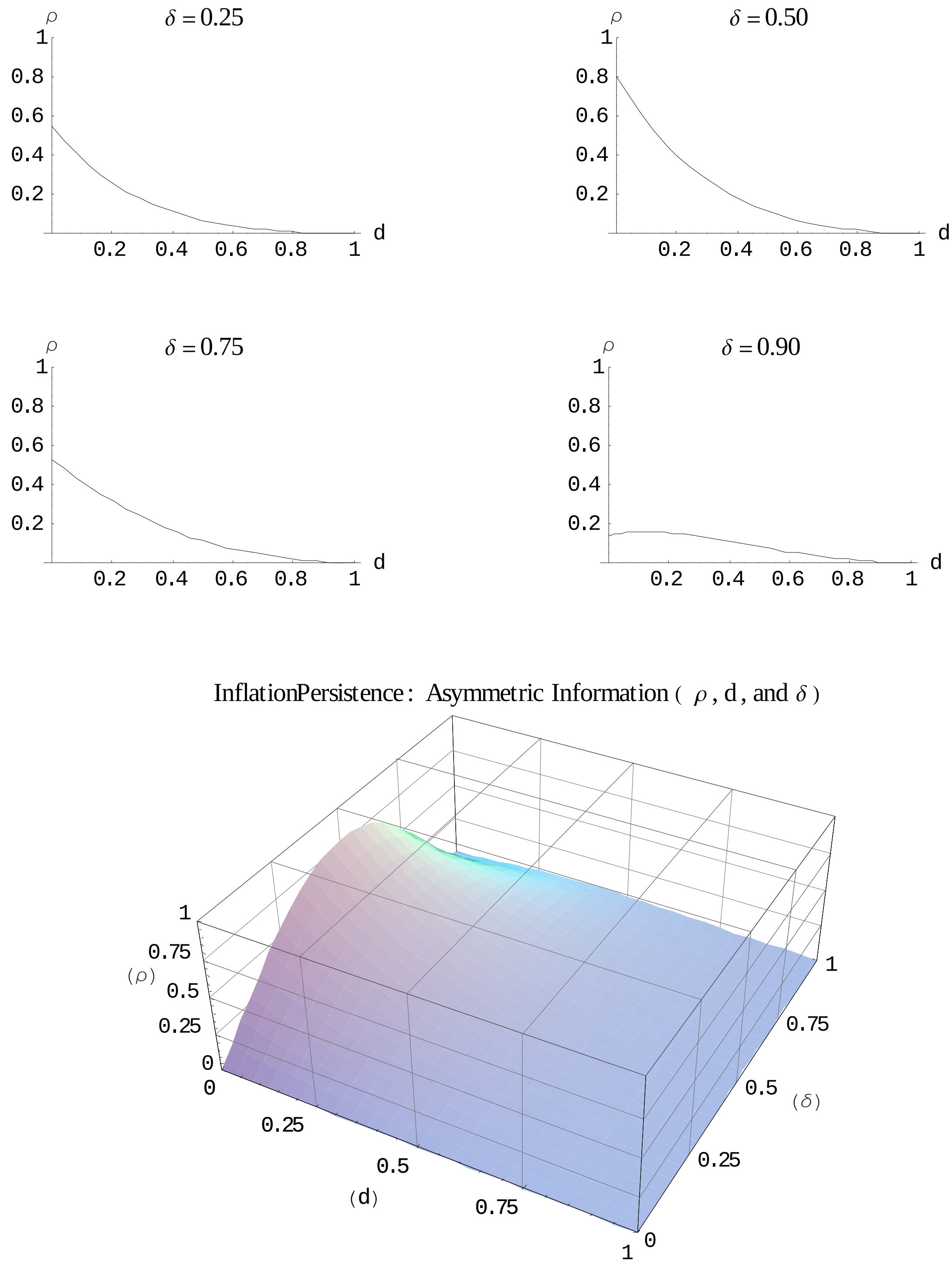
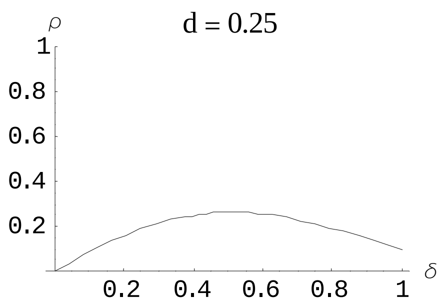
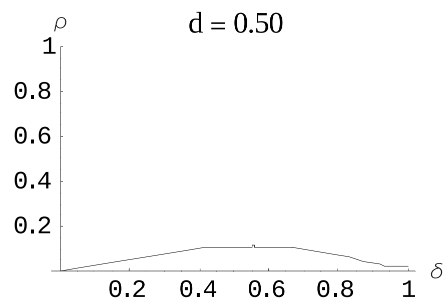
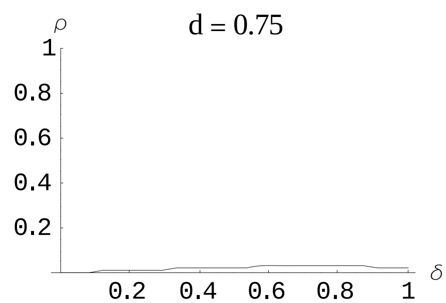
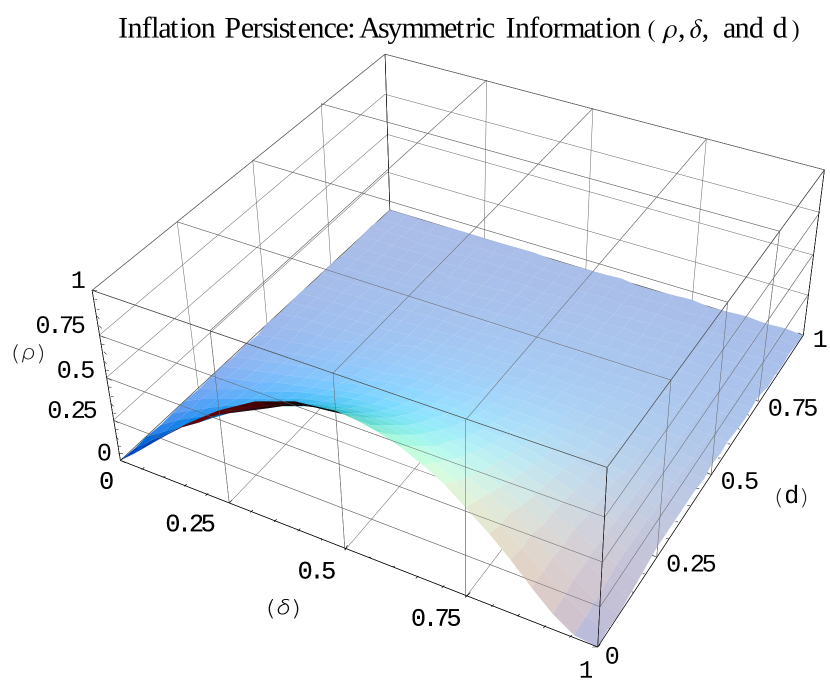


Figure 8: Inflation persistence and degree of autocorrelation of supply shocks









4.2.3 Comparative Statics under Symmetric Information

The Figures from 9 and 10 show the behaviour of inflation persistence to a change in the degree of exchange rate flexibility and degree of autocorrelation.

Result 7: *The response of inflation persistence to a change in the degree of exchange rate flexibility is negative, and more flexibility results in more persistence, as expected. However, for highly autocorrelated shocks, the impact of change in d would be minimal.*

Figure 9 shows the response of inflation persistence to a change in *d*, under different degrees of autocorrelation of shocks. As $d \rightarrow 1,$ persistence coefficient declines through out, and it is highly noticeable for lower values of δ. For highly autocorrelated shocks, a change in *d* would have only minimal impact. However, results change remarkably for an increase in the variance of persistent component. For example, for higher variance of σ_{υ}^{2}, the path of the persistence component becomes highly volatile. All graphs in Figure 8 are based on $\sigma_{\upsilon}^{2} = 0.01$.

Result 8: *The initial response of inflation persistence to an increase in the degree of autocorrelation of shocks is positive. However, the path of persistence becomes negative for higher autocorrelated shocks (e.g.,* $\delta > 0.40$*).*

As shown in Figure 10, the initial positive impact due to an increase in autocorrelation dies out as $\delta \rightarrow 1$. During the process, more autocorrelated shocks results in lowering the degree of inflation persistence. Also the impact of a change in the degree of autocorrelation lessens for more constraint exchange rates.

Figure 9: Inflation persistence response to a change in the degree of exchange rate flexibility

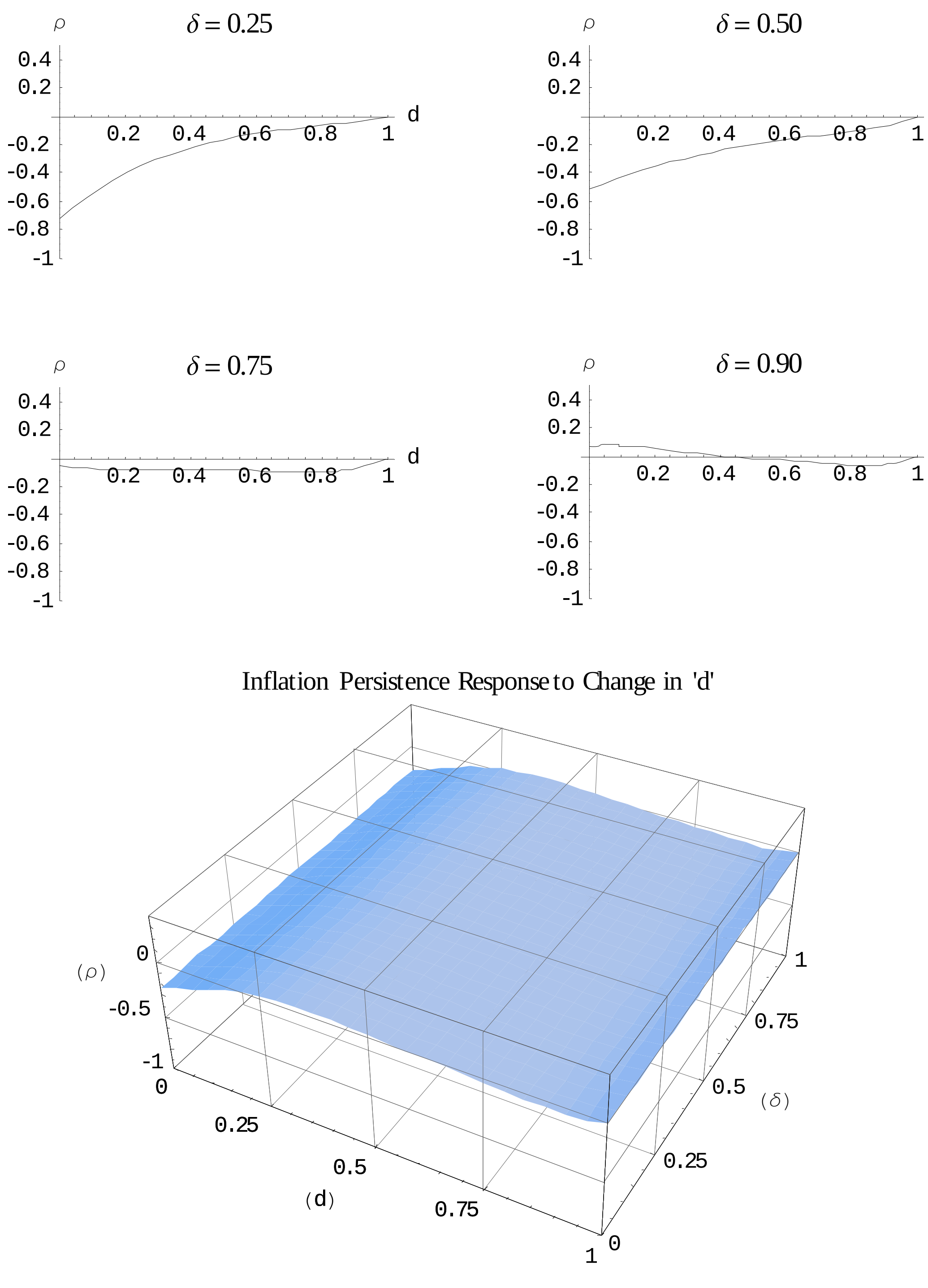
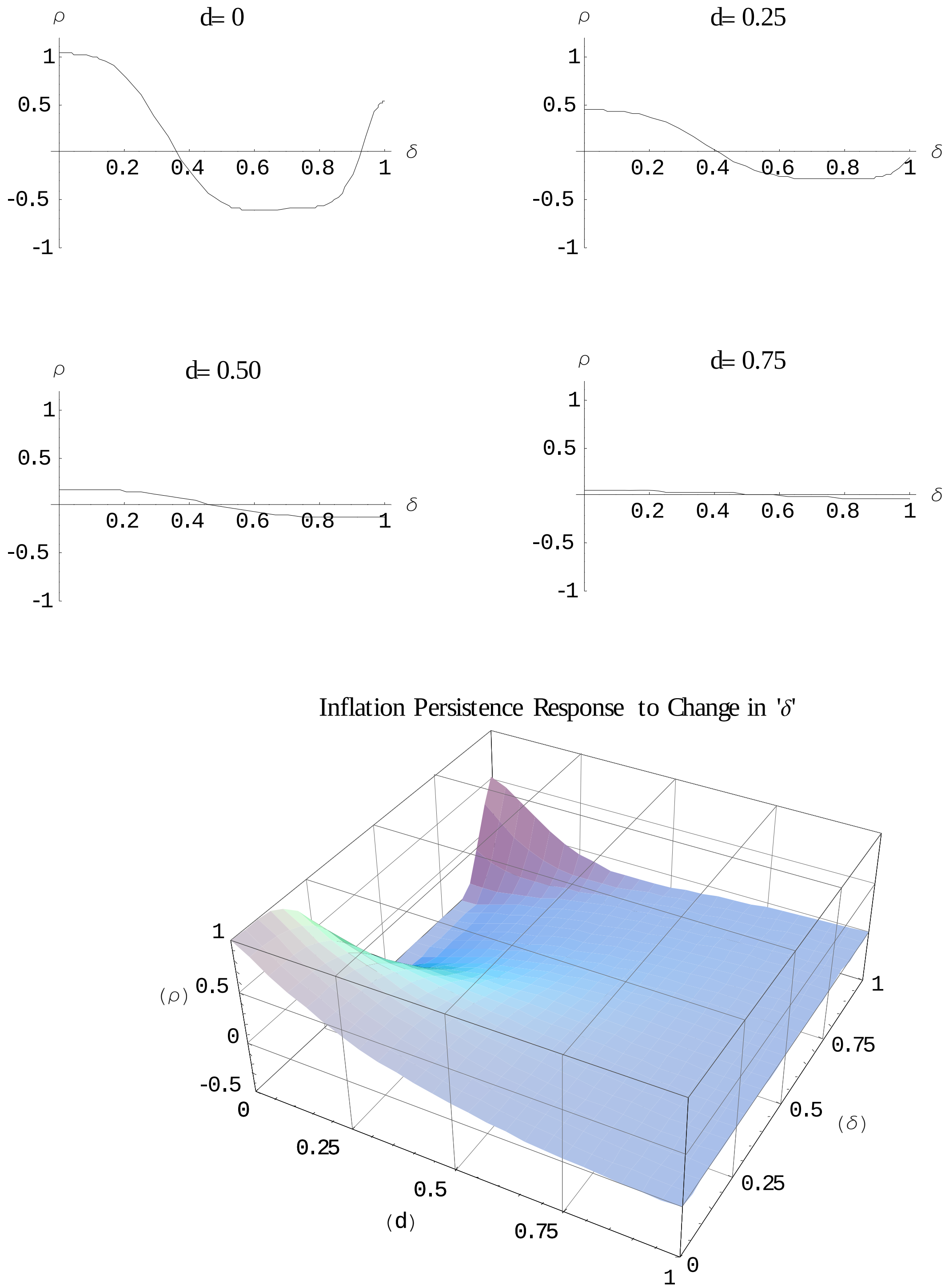


Figure 10: Inflation persistence response to a change in the degree of autocorrelation of natural rate shock



5. Conclusion

The model developed in this paper is an open economy extension of Barro-Gordon model, with degree of exchange rate flexibility and natural rate shocks to employment, to analyse the implications of different exchange rate regimes on inflation persistence. Despite the fact that Barro-Gordon model is originally developed under flexible price assumptions, the model can well be extended to explain inflation persistence. As described by Reis (2003), both key elements of Barro-Gordon framework i.e., policy objective function and the expectations-augmented Phillips curve can be derived as reduced form relations in a general equilibrium model with nominal rigidities.

Given the main purpose of the paper is to model inflation persistence across exchange rate regimes, the basic Barro-Gordon framework is extended to an open economy model. The parameter relating to nominal exchange rate flexibility is a modification introduced into the model along with the objective to target the inflation rate of the foreign country, against which the peg is maintained. Both of these new elements in the model play plausible roles in the reduced form solutions. Also, the specification of shocks to natural rate of employment as persistent and transitory components is central to model's implications on inflation persistence.

The model is solved under two alternative assumptions on information availability. Under symmetric information, the model implies inflation persistence due to persistent shocks to the natural rate of employment. The same implication is found in Cukierman (1992) model, however, Cukierman assumes that public can calculate optimal policy of the policymaker without error. Further, the degree of persistence is determined by the degree of flexibility of nominal exchange rate, i.e., more flexible a regime implies more persistence and vice versa. However, in the present model, the degree of inflation persistence is independent of the inflation rate of the foreign country, which is contrary to the findings of previous authors. The parameters relating to activist policy and inflation stabilization yield expected results while the former is positively related to inflation persistence and the latter is negatively related.

On the other hand, the model implies more plausible results on inflation persistence in presence of asymmetric information. The key implication of inflation persistence derives from the fact that optimal inflation rate in current period responds to past transitory shocks to the natural rate of employment. The reason why, as public do not update information as quickly as policymaker; they cannot fully disentangle previous period innovation to persistent component of employment from the transitory component of employment in that period. Thus, current expectations are affected by past transitory shocks. Because, policymaker partly accommodates current inflation expectations the current inflation rate is also then affected by transitory shocks. Thus, in the same line of argument of Cukierman, the model implies that asymmetric information results in transforming transitory shocks to natural rate of employment into persistent movements in optimal inflation rate. Consequently, calibration results show a higher inflation persistence coefficient under asymmetric information. However, the persistence coefficient declines at a faster rate under asymmetric information, as the exchange rate becomes more constraint.

Further, comparative statics of the model imply that the response of inflation persistence to changes in the degree of exchange rate flexibility is non-linear under both information assumptions. Inflation persistence is more responsive to lower values of exchange rate flexibility, than higher values. However, the response of persistence to changes in the variance of the transitory component of shocks seems to have opposing effects. Under symmetric information, more volatility of transitory shocks brings down inflation persistence while the contrast occurs under asymmetric information. Nonetheless, more volatility in the persistent shocks results in less persistence under asymmetric information. Overall, the persistence component is more responsive to variance parameters under asymmetric information.

The model described in this paper can well be extended on several dimensions. One plausible extension would be to model inflation persistence under overlapping wage contracts. Due to the impact of inflation expectations on future employment, policymaker confronts with contradicting outcomes when responding to current periods shocks to natural rate of employment. Therefore, one channel to explain inflation persistence over time would be through interaction of overlapping wage contracts with policymaker's objective of attaining high employment. Such work would contribute to yet unresolved question of whether to which persistence generating mechanisms i.e. persistence due to shocks to natural rate or persistence due to overlapping wage contracts would be more practically important. Further, the model could account for the effects of exchange rate shocks and costs of exchange rate fluctuations within and between exchange rate regimes. Moreover, it would be interesting to see implications of the model when the effects of exchange rate variability are fully endogenised.

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Appendix A

Derivation of equation (4.35)

Using equations (4.19) and (4.21) to form:

$$\pi_t - g(t) = (K_1 + \delta K_4)\upsilon_t + K_1 \varepsilon_t \,,$$

and substituting (4.29) and (4.30) into (4.23) with the above expression:

$$\pi^e_{t+1} = \frac{1}{1 - \frac{\alpha^2(1-d)}{[(1-d)(\lambda + \alpha^2) + d]}} \left\{ \left(\frac{\alpha(1-d)}{[(1-d)(\lambda + \alpha^2) + d]} + K_4 \right) H + K_3 \pi^f_{t+1} \right.$$

$$+\delta^2\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}+\delta K_4\right)u_{t-1}$$

$$+\delta\theta\left[\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}+\delta K_4\right)\upsilon_t+\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}\varepsilon_t\right].$$

Expanding terms:

$$\pi_{t+1}^e=\frac{(1-d)\left(\lambda+\alpha^2\right)+d}{(1-d)\lambda+d}\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}+K_4\right)H+\frac{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}{(1-d)\lambda+d}\left(K_3\pi_{t+1}^f\right)$$

$$+\frac{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}{(1-d)\lambda+d}\delta^2\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}+\delta K_4\right)u_{t-1}$$

$$+\frac{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}{(1-d)\lambda+d}\delta\theta\left[\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}+\delta K_4\right)\upsilon_t+\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}\varepsilon_t\right].$$

Rearranging terms and taking one period lag yields the equation (4.35):

$$\pi_t^e=\frac{(1-d)\left(\lambda+\alpha^2\right)+d}{(1-d)\lambda+d}\left[\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}+K_4\right)H+K_3\pi_t^f\right]$$

$$+\frac{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}{(1-d)\lambda+d}\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]}+\delta K_4\right)$$

$$\cdot\left[\delta^2u_{t-2}+\delta\theta\left(\upsilon_{t-1}+\frac{\alpha(1-d)}{\alpha(1-d)+\delta\left[(1-d)\left(\lambda+\alpha^2\right)+d\right]K_4}\varepsilon_{t-1}\right)\right].$$

Appendix B

Derivation of equation (4.36)

Substituting equations (4.29) and (4.30) into (4.23) and rearranging yields:

$$\pi_{t+1}^{e} = \frac{(1-d)(\lambda+\alpha^{2})+d}{(1-d)\lambda+d}\left\{\left(\frac{\alpha(1-d)}{[(1-d)(\lambda+\alpha^{2})+d]}+K_{4}\right)H+K_{3}\pi_{t+1}^{f}\right.$$

$$+\delta^{2}\left(\frac{\alpha(1-d)}{[(1-d)(\lambda+\alpha^{2})+d]}+\delta K_{4}\right)u_{t-1}$$

$$\left.+\left(\frac{\alpha(1-d)}{[(1-d)(\lambda+\alpha^{2})+d]}+\delta K_{4}\right)\delta\theta\left[\upsilon_{t}+\frac{\alpha(1-d)}{\alpha(1-d)+[(1-d)(\lambda+\alpha^{2})+d]\delta K_{4}}\varepsilon_{t}\right]\right\}. \tag{B.1}$$

Taking conditional expectations of equation (4.27) and subtracting the resulting equation from (4.27) yields:

$$\pi_{t}-\pi_{t}^{e}=\frac{\alpha(1-d)}{[(1-d)(\lambda+\alpha^{2})+d]}\left[\delta(\upsilon_{t-1}-E[\upsilon_{t-1}\mid I_{t}])+\upsilon_{t}+\varepsilon_{t}\right]$$

$$+K_{4}\left[\delta^{2}(\upsilon_{t-1}-E[\upsilon_{t-1}\mid I_{t}])+\delta\upsilon_{t}-\alpha E_{G,t}\left[\pi_{t+1}-\pi_{t+1}^{e}\right]\right] \tag{B.2}$$

Substituting (4.19) into (4.20) yields:

$$E[\upsilon_{t}\mid I_{t+1}]=\left(\frac{\alpha(1-d)}{[(1-d)(\lambda+\alpha^{2})+d]}+\delta K_{4}\right)\theta\left[\upsilon_{t}+\frac{\alpha(1-d)}{\alpha(1-d)+[(1-d)(\lambda+\alpha^{2})+d]\delta K_{4}}\varepsilon_{t}\right]. \tag{B.3}$$

Leading (B.2) by one period and substituting (B.3) into the resulting equation yields:

$$\pi_{t+1}-\pi_{t+1}^{e}=\left(\frac{\alpha(1-d)}{[(1-d)(\lambda+\alpha^{2})+d]}+\delta K_{4}\right)$$

$$\left[\delta(1-\theta)\upsilon_{t}-\delta\theta\left(\frac{\alpha(1-d)}{\alpha(1-d)+[(1-d)(\lambda+\alpha^{2})+d]\delta K_{4}}\right)\varepsilon_{t}+\upsilon_{t+1}\right]$$

$$+\frac{\alpha(1-d)}{[(1-d)(\lambda+\alpha^{2})+d]}\varepsilon_{t+1}-K_{4}\alpha E_{G,t}\left[\pi_{t+2}-\pi_{t+2}^{e}\right]. \tag{B.4}$$

Equation (B.4) implies that unexpected inflation in $t+1$ depends on realizations of shocks in periods $t, t+1$ and later periods, and not on earlier periods.

Also the last term in equation (B.4) implies:

$$E_{G,t}\left\{E_{G,t+1}\left[\pi_{t+2}-\pi_{t+2}^{e}\right]\right\}=E_{G,t+1}\left[\pi_{t+2}-\pi_{t+2}^{e}\right]=0 \qquad . \tag{B.5}$$

The first equality is a result of the law of iterated projections, and second equality is because policymaker's information does not include shocks to be realized from period $t+1$ onwards. Taking conditional expectation of (B.4), given information set of policymaker in period *t*:

$$E_{G,t}\left[\pi_{t+1}-\pi_{t+1}^{e}\right]=\delta\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^{2}\right)+d\right]}+\delta K_{4}\right)(1-\theta)\upsilon_{t}-\delta\theta\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^{2}\right)+d\right]}\right)\varepsilon_{t}\,. \tag{B.6}$$

Substituting (B.1) with a one period lag, and (B.6) into (4.27), using the results $h_t \equiv H+u_t+\varepsilon_t$ and $E_{G,t}h_{t+1}=H+\delta u_t$:

$$\begin{aligned}
\pi_{t} &= \frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^{2}\right)+d\right]}\left[H+u_{t}+\varepsilon_{t}\right] \\
&\quad+\frac{\alpha^{2}(1-d)}{\left[(1-d)\left(\lambda+\alpha^{2}\right)+d\right]}\left\{\frac{(1-d)\left(\lambda+\alpha^{2}\right)+d}{(1-d)\lambda+d}\left[\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^{2}\right)+d\right]}+K_{4}\right)H+K_{3}\pi_{t}^{f}\right.\right. \\
&\quad+\delta^{2}\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^{2}\right)+d\right]}+\delta K_{4}\right)u_{t-2} \\
&\quad\left.\left.+\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^{2}\right)+d\right]}+\delta K_{4}\right)\delta\theta\left(\upsilon_{t-1}+\frac{\alpha(1-d)}{\alpha(1-d)+\left[(1-d)\left(\lambda+\alpha^{2}\right)+d\right]\delta K_{4}}\right)\varepsilon_{t-1}\right]\right\} \\
&\quad+\frac{d}{\left[(1-d)\left(\lambda+\alpha^{2}\right)+d\right]}\pi_{t}^{f} \\
&\quad-\frac{\alpha\beta\delta^{2}(1-d)}{1-K_{2}}\theta\left(H+\delta u_{t}\right) \\
&\quad+\frac{\alpha^{2}\beta\delta^{2}(1-d)}{1-K_{2}}\theta\left[\delta\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^{2}\right)+d\right]}+\delta K_{4}\right)(1-\theta)\upsilon_{t}-\delta\theta\left(\frac{\alpha(1-d)}{\left[(1-d)\left(\lambda+\alpha^{2}\right)+d\right]}\right)\varepsilon_{t}\right] \\
&\quad.
\end{aligned} \tag{B.7}$$

Expanding equation (B.7) using the definition for $K_4 = -\frac{\alpha\beta\delta^2(1-d)}{1-K_2}\theta$, and after some rearrangements yields equation (4.36).

Appendix C
Derivation of equation (4.37)

Taking one period lag of equation (4.36):

$$\pi_{t-1} = \frac{\left(\alpha(1-d)+\left[(1-d)(\lambda+\alpha^2)+d\right]K_4\right)}{(1-d)\lambda+d}H + \left(\frac{d}{(1-d)\lambda+d}\right)\pi_t^f$$

$$+\frac{\left(\alpha(1-d)+\left[(1-d)(\lambda+\alpha^2)+d\right]\delta K_4\right)}{(1-d)\lambda+d}\delta^2 u_{t-3}$$

$$+\left(\frac{(1-d)(\lambda+\alpha^2\theta)+d}{(1-d)\lambda+d}\right)\left(\frac{\alpha(1-d)}{\left[(1-d)(\lambda+\alpha^2)+d\right]}+\delta K_4\right)\delta\upsilon_{t-2}$$

$$+(1-\alpha\delta(1-\theta)K_4)\left(\frac{\alpha(1-d)}{\left[(1-d)(\lambda+\alpha^2)+d\right]}+\delta K_4\right)\upsilon_{t-1}+\frac{(1+K_4\alpha\delta\theta)}{\left[(1-d)(\lambda+\alpha^2)+d\right]}\alpha(1-d)\varepsilon_{t-1}$$

$$+\left(\frac{\alpha^3(1-d)^2}{((1-d)\lambda+d)\left[(1-d)(\lambda+\alpha^2)+d\right]}\right)\delta\theta\varepsilon_{t-2}. \tag{C.1}$$

Taking unconditional expectations of equation (4.36);

$$E_t\pi_t = \frac{\left(\alpha(1-d)+\left[(1-d)(\lambda+\alpha^2)+d\right]K_4\right)}{(1-d)\lambda+d}H + \left(\frac{d}{(1-d)\lambda+d}\right)\pi_t^f. \tag{C.2}$$

Using equations (C.1) and (C.2) in the statistical result for covariance yields:

$$Cov(\pi_t,\pi_{t-1}) = E_t(\pi_t - E_t\pi_t)(\pi_{t-1}-E_t\pi_t) =$$

$$\left[\left(\frac{(1-d)(\lambda+\alpha^2\theta)+d}{(1-d)\lambda+d}\right)(1-\alpha\delta(1-\theta)K_4)\left(\frac{\alpha(1-d)}{[(1-d)(\lambda+\alpha^2)+d]}+\delta K_4\right)^2\delta\sigma_\upsilon^2\right]$$

$$+\left[\left(\frac{\alpha^4(1-d)^3(1+K_4\alpha\delta\theta)}{((1-d)\lambda+d)[(1-d)(\lambda+\alpha^2)+d]^2}\right)\delta\theta\sigma_\varepsilon^2\right]. \qquad \text{(C.3)}$$

Similarly using equation (4.36) and (C.2) in the statistical result for variance yields:

$$Var = E_t(\pi_t - E_t\pi_t)^2 =$$

$$\frac{(\alpha(1-d)+[(1-d)(\lambda+\alpha^2)+d]\delta K_4)^2}{[(1-d)\lambda+d]^2}\delta^4\sigma_u^2$$

$$+\left(\frac{\alpha(1-d)}{[(1-d)(\lambda+\alpha^2)+d]}+\delta K_4\right)^2\left[\left(\frac{(1-d)(\lambda+\alpha^2\theta)+d}{(1-d)\lambda+d}\right)^2\delta^2+(1-\alpha\delta(1-\theta)K_4)^2\right]\sigma_\upsilon^2$$

$$+\left(\frac{\alpha^2(1-d)^2\delta^2\theta^2}{[(1-d)(\lambda+\alpha^2)+d]^2}\right)\left[(1+K_4\alpha)^2+\frac{\alpha^4(1-d)^2}{[(1-d)\lambda+d]^2}\right]\sigma_\varepsilon^2. \qquad \text{(C.4)}$$

Dividing equation (C.3) by (C.4) and with some rearrangements yields the equation (4.37).