Should one augment the Taylor Rule with an Exchange Rate Term?

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Abstract

Three specifications of a Taylor rule are embedded in a small open economy to evaluate whether one should include an exchange rate term in the rule to have a determinate REE that also is learnable in least squares sense. The answer is affirmative when it comes to contemporaneous data in the rule, but such a term is not necessary when lagged data are used in the rule, if the reaction to the CPI inflation rate is strong enough. Finally, the indeterminacy problem cannot be resolved when future expectations are used in the rule. We allow for interest rate inertia in the analysis.

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1 Introduction

It is argued in Taylor [7] that it is not necessary to include an exchange rate term in the interest rate rule to have a desirable equilibrium outcome. We investigate this issue further by embedding three specifications of a Taylor rule into a recently developed model by Galí and Monacelli [4] for a small open economy, where we include an exchange rate term in the rules, and allow for interest rate inertia in policy making.

Firstly, we search for regions in a rule's parameter space that give rise to a unique and stable REE. Secondly, such a REE should also be learnable in least squares sense, and this is because rational expectations is a rather strong assumption since it assumes that agents often have an outstanding capacity when it comes to deriving equilibrium outcomes of the variables in a model. We make use of the E-stability concept when doing the learnability analysis.[1]

The rest of the letter is organized as follows: The economy is outlined in Section 2, whereas determinacy and E-stability results are presented in Section 3. Section 4 concludes the letter with a short discussion.

2 A small open economy

2.1 Baseline model

A dynamic stochastic general equilibrium model with imperfect competition and nominal rigidities is presented in Galí and Monacelli [4] for a small open economy. After extensive derivations, their model can be reduced to a dynamic IS-type equation and a new Keynesian Phillips curve:

$$\left\{ \begin{array}{l} x_t = x^e_{t+1} - \alpha \left(r_t - \pi^e_{H,t+1} - \overline{rr}_t \right) \\ \pi_{H,t} = \beta \pi^e_{H,t+1} + \gamma x_t, \end{array} \right. \tag{1}$$

where x_t is the output gap, r_t is the nominal interest rate, $\pi_{H,t}$ is the domestic inflation rate, $\overline{rr}_t$ is the natural rate of interest, and the superscript e denotes rational expectations, where the dating of expectations is time period t.

Unfortunately, (1) is not in an appropriate form since there are no exchange rate terms in the equations. It is, however, possible to use the following equations that are derived in Galí and Monacelli [4] to rewrite (1) into a suitable form:

$$\left\{ \begin{array}{l} \pi_t = \pi_{H,t} + \delta \Delta s_t \\ s_t = e_t + p^*_t - p_{H,t}, \end{array} \right. \tag{2}$$

where π_t is the CPI inflation rate, s_t is the terms of trade, e_t is the nominal exchange rate that is the domestic price of the foreign currency, p^*_t is the index of foreign goods prices, $p_{H,t}$ is the index of domestic goods prices, and the asterisk denotes a foreign quantity. For interpretations of the structural parameters, we refer to Galí and Monacelli [4].

If we rewrite the equations in (1) with help of those in (2), we get the first two equations in the model investigated:[2]

[1] See Evans and Honkapohja [3] for an introduction to this literature.
[2] See the Appendix for a derivation of (3).

$$\begin{cases} x_t = x^e_{t+1} - \alpha\left(r_t - \frac{1}{1-\delta}\cdot\left(\pi^e_{t+1} - \delta\left(\Delta e^e_{t+1} + \pi^{e,*}_{t+1}\right)\right) - \overline{rr}_t\right) \\ \pi_t = \beta\pi^e_{t+1} + \gamma\left(1-\delta\right)x_t + \delta\left(\Delta e_t - \beta\Delta e^e_{t+1} + \pi^*_t - \beta\pi^{e,*}_{t+1}\right). \end{cases} \tag{3}$$

The third equation in the model, which also is derived in Galí and Monacelli [4], is the condition for uncovered interest rate parity:

$$r_t - r^*_t = \Delta e^e_{t+1}. \tag{4}$$

To complete the model in (3)-(4), we will augment it with a Taylor rule for the monetary authority.

2.2 Taylor rules investigated

The monetary authority is using a Taylor rule when setting the nominal interest rate, where three specifications of the rule are investigated: (i) lagged data in the rule:

$$r_t = \zeta_r r_{t-1} + \zeta_x x_{t-1} + \zeta_\pi \pi_{t-1} + \zeta_e \Delta e_{t-1}, \tag{5}$$

(ii) contemporaneous data in the rule:

$$r_t = \zeta_r r_{t-1} + \zeta_x x_t + \zeta_\pi \pi_t + \zeta_e \Delta e_t, \tag{6}$$

and (iii) forward expectations in the rule:

$$r_t = \zeta_r r_{t-1} + \zeta_x x^e_{t+1} + \zeta_\pi \pi^e_{t+1} + \zeta_e \Delta e^e_{t+1}. \tag{7}$$

We have also included the interest rate in the previous time period in the rules to allow for inertia in monetary policy.[3]

3 Properties of the economy

3.1 Determinacy

3.1.1 Lagged data in the Taylor rule

After substituting the Taylor rule in (5) into the baseline model in (3)-(4), the complete model is

$$\mathbf{\Gamma}\cdot\mathbf{y}_t = \mathbf{\Theta}\cdot\mathbf{y}^e_{t+1} + \mathbf{\Xi} + \mathbf{\Pi}\cdot\overline{rr}_t, \tag{8}$$

where the state of the economy is

$$\mathbf{y}_t = \left[x_t, \pi_t, \Delta e_t, r_t\right]', \tag{9}$$

and the relevant coefficient matrices are[4]

[3] The vigilant reader might object that Taylor [7] is referring to the real exchange rate in his discussion, whereas we have included a nominal exchange rate term in the rules. It is, however, an easy exercise to transform the rules in (5)-(7) to rules that are functions of the real exchange rate, q_t, via the following identity: $\Delta q_t \equiv \Delta e_t + \pi^*_t - \pi_t$.

[4] Before putting the model in matrix form, shift the rule in (5) one time period forward in time.

$$
\mathbf{\Gamma}=\left[\begin{array}{cccc}
1 & 0 & 0 & \alpha \\
\gamma(\delta-1) & 1 & -\delta & 0 \\
0 & 0 & 0 & 1 \\
-\zeta_x & -\zeta_\pi & -\zeta_e & -\zeta_r
\end{array}\right], \qquad (10)
$$

and

$$
\mathbf{\Theta}=\left[\begin{array}{cccc}
1 & \frac{\alpha}{1-\delta} & \frac{\alpha\delta}{\delta-1} & 0 \\
0 & \beta & -\beta\delta & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{array}\right]. \qquad (11)
$$

To have a unique and stable REE, exactly one eigenvalue of the coefficient matrix $\mathbf{\Theta}^{-1}\cdot\mathbf{\Gamma}$ must be inside the unit circle since x_t, π_t and Δe_t are free, and r_{t-1} is pre-determined (see, e.g., Blanchard and Kahn [1]). However, deriving conditions for determinacy is not meaningful since the expressions would be too large and cumbersome to interpret. Instead, we adopt the strategy in several other papers and illustrate our findings graphically using calibrated values of the structural parameters.[5] Specifically, the following parameter values, or range of values, are used in the analysis:

$$
\left\{\begin{array}{llll}
\alpha=\frac{1}{0.157}, & \beta=0.99, & \gamma=0.024, & \delta=0.2,\ 0.4, \\
\zeta_r=0,\ 1, & \zeta_x=0,\ 0.5, & 0\leq\zeta_\pi\leq 8, & -4\leq\zeta_e\leq 4.
\end{array}\right. \qquad (12)
$$

See Bullard and Mitra [2], and references therein, for the values of the parameters α, β and γ.

$\delta\in[0,1]$ is index of openness of the economy since it is the share of consumption allocated to imported goods. When $\delta=0.2$, the index is slightly larger than the import/GDP ratio in the U.S., and when $\delta=0.4$, which is the value used in Galí and Monacelli [4], the index corresponds roughly to the import/GDP ratio in Canada. We search for determinacy regions when the monetary authority neglects the interest rate in the previous time period ($\zeta_r=0$), when there is inertia in policy making ($\zeta_r=1$), when there is no output reaction ($\zeta_x=0$), and when the monetary authority slightly reacts to the output gap when setting the interest rate ($\zeta_x=0.5$).

First and foremost, $\zeta_e=0$ belongs to the determinacy region when ζ_π is large enough, meaning that the monetary authority does not have to care about the exchange rate when setting the interest rate, if the inflation rate reaction is strong enough. Turning to details, a more open economy is associated with a larger determinacy region, and a smaller ζ_π is necessary for a unique REE. Moreover, inertia in policy making decreases the determinacy region, at least when there is no output gap reaction, but it is always the case that a larger ζ_π is necessary for a unique REE. Finally, reacting to the output gap when setting the interest rate increases the determinacy region, and a smaller ζ_π is necessary for a unique REE.

Because of lack of space in the letter, we only demonstrate the latter result graphically. See Figures 1 a-b.

[Figures 1 a-b about here.]

According to the figures, the Taylor principle holds in monetary policy when $\zeta_e=0$, and this is still true for a very large value of ζ_x.

[5] MATLAB routines for this purpose are available on request from the author.

3.1.2 Contemporaneous data in the Taylor rule

The Taylor rule that is substituted into the baseline model in (3)-(4) is (6), and it turns out that the complete model is (8), where the relevant coefficient matrices are

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \gamma(\delta-1) & 1 & -\delta & 0 \\ 0 & 0 & 0 & 0 \\ -\zeta_x & -\zeta_\pi & -\zeta_e & -\zeta_r \end{bmatrix}, \tag{13}$$

and

$$\mathbf{\Theta} = \begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & \frac{\alpha\delta}{\delta-1} & -\alpha \\ 0 & \beta & -\beta\delta & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \tag{14}$$

and where the state of the economy is

$$\mathbf{y}_t = [x_t, \pi_t, \Delta e_t, r_{t-1}]'. \tag{15}$$

As in the case with lagged data in the rule, exactly one eigenvalue of the coefficient matrix $\mathbf{\Theta}^{-1} \cdot \mathbf{\Gamma}$ must be inside the unit circle to have a unique and stable REE.

Most importantly, $\zeta_e = 0$ does not belong to the determinacy region for any parameter setting in the model. Instead, the monetary authority should "lean with the wind" to have a unique REE. Turning to details, a more open economy is associated with a smaller determinacy region, which also are the cases when there is inertia in policy making, and when there is a reaction to the output gap when setting the interest rate.

In Figures 2 a-b, it is demonstrated that inertia in monetary policy calls for a stronger "lean with the wind" than when the interest rate in the previous time period does not matter in policy making.

[Figures 2 a-b about here.]

3.1.3 Forward expectations in the Taylor rule

The complete model is (8), after substituting the Taylor rule in (7) into the baseline model in (3)-(4), where the state of the economy is (15), and the relevant coefficient matrices are

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \gamma(\delta-1) & 1 & -\delta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\zeta_r \end{bmatrix}, \tag{16}$$

and

$$\mathbf{\Theta} = \begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & \frac{\alpha\delta}{\delta-1} & -\alpha \\ 0 & \beta & -\beta\delta & 0 \\ 0 & 0 & 1 & -1 \\ \zeta_x & \zeta_\pi & \zeta_e & -1 \end{bmatrix}. \tag{17}$$

As in previous cases, exactly one eigenvalue of the coefficient matrix $\mathbf{\Theta}^{-1} \cdot \mathbf{\Gamma}$ must be inside the unit circle to have a unique and stable REE.

When a forward expectations specification of the rule is used in monetary policy, we do not find any determinacy region for the parameter settings investigated. Consequently, there is no reason to react to the exchange rate when setting the interest rate since neither a "lean with the wind" nor a "lean against the wind" policy will resolve the indeterminacy problem.

3.2 E-stability

Since the dating of the variables in (15) are not the same, it is not appropriate to investigate under what conditions the model in (8) is characterized by least squares learnability when the Taylor rule is (6) or (7), even if it simplifies the analysis for determinacy. Instead, the state of the economy should be (9), which implies that the complete model is

$$\mathbf{\Gamma} \cdot \mathbf{y}_t = \mathbf{\Theta} \cdot \mathbf{y}_{t+1}^e + \mathbf{\Lambda} \cdot \mathbf{y}_{t-1} + \mathbf{\Xi} + \mathbf{\Pi} \cdot \overline{rr}_t, \tag{18}$$

where, of course, the coefficient matrices differ for different specifications of the rule. Note that when lagged data are used in the rule as in (5), $\mathbf{\Lambda}$ is the null matrix.

The MSV solution of the complete model in (18), which is the solution of a linear difference equation that depends linearly on a set of variables such that there does not exist a solution that depends linearly on a smaller set of variables (see McCallum [5]), is

$$\mathbf{y}_t = \mathbf{\Phi} \cdot \mathbf{y}_{t-1} + \mathbf{\Psi} + \mathbf{\Omega} \cdot \overline{rr}_t, \tag{19}$$

where $\mathbf{\Phi}$, $\mathbf{\Psi}$ and $\mathbf{\Omega}$ are parameter vectors to be determined with, for example, the method of undetermined coefficients.

To have a REE that is least squares learnable, the parameter values in the PLM of the economy[6] have to converge to the economy's ALM, and it was recently shown in McCallum [6] that for a broad class of linear rational expectations models, which includes the model in (18), a determinate solution is also E-stable when the dating of expectations is time period t. Consequently, since E-stability is linked to least squares learnability, all determinacy regions that we found in Section 3.1 are also regions for learnability of the REE.

4 Short discussion

The findings in this letter are mixed. If lagged data are used in the Taylor rule, it is not necessary to care about the exchange rate when setting the interest rate, if the inflation rate reaction is strong enough. If, however, contemporaneous data are used in the rule, the monetary authority must "lean with the wind" to have a unique REE.

An exercise for the future is to derive the equilibrium outcome when it is unique, and evaluate this outcome using a welfare function to find the most desirable parameter setting in a rule. Such an analysis would complement the derivation of the optimal policy with and without commitment.

[6] The suggested MSV solution in (19) is also the PLM of the economy.

References

[1] Blanchard, O.J. and Kahn, C.M. (1980). The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, **48**, 1305-1311.

[2] Bullard, J. and Mitra, K. (2002). Learning About Monetary Policy Rules. *Journal of Monetary Economics*, **49**, 1105-1129.

[3] Evans, G.W. and Honkapohja, S. (2001). *Learning and Expectations in Macroeconomics*. Princeton, New Jersey: Princeton University Press.

[4] Galí, J. and Monacelli, T. (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies*, **72**, 707-734.

[5] McCallum, B.T. (1983). On Non-Uniqueness in Rational Expectations Models: An Attempt at Perspective. *Journal of Monetary Economics*, **11**, 139-168.

[6] McCallum, B.T. (2006). E-Stability vis-a-vis Determinacy Results for a Broad Class of Linear Rational Expectations Models. Forthcoming in *Journal of Economic Dynamics and Control*.

[7] Taylor, J.B. (2001). The Role of the Exchange Rate in Monetary-Policy Rules. *American Economic Review (Papers and Proceedings)*, **91**, 263-267.

Appendix

Firstly, shift the first equation in (2) one time period forward in time:

$$\pi^e_{H,t+1} = \pi^e_{t+1} - \delta \Delta s^e_{t+1}. \tag{A.1}$$

Secondly, shift the second equation in (2) one time period forward in time, and take differences:

$$\Delta s^e_{t+1} = \Delta e^{e,m}_{t+1} + \Delta p^{e,*}_{t+1} - \Delta p^e_{H,t+1} = \Delta e^{e,m}_{t+1} + \pi^{e,*}_{t+1} - \pi^e_{H,t+1}. \tag{A.2}$$

Thirdly, substitute (A.2) into (A.1):

$$\pi^e_{H,t+1} = \frac{1}{1-\delta} \cdot \left(\pi^e_{t+1} - \delta \left(\Delta e^{e,m}_{t+1} + \pi^{e,*}_{t+1}\right)\right). \tag{A.3}$$

Fourthly, shift (A.3) one time period backward in time:

$$\pi_{H,t} = \frac{1}{1-\delta} \cdot \left(\pi_t - \delta \left(\Delta e_t + \pi^*_t\right)\right). \tag{A.4}$$

Finally, substitute (A.3) into the first equation in (1), substitute (A.3)-(A.4) into the second equation in (1), and (3) is derived.