Optimal monetary policy in the euro area in the presence of heterogeneity

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Abstract

This paper examines the optimal design of monetary policy in the European monetary union in the presence of structural asymmetries across union member countries. It derives analytically an optimal interest rate rule under commitment and studies the dependence of its coefficients on the parameters of the structural model of each economy, the central bank's preferences for inflation and output stabilisation as shown in its loss function, and the relative size of each country. Based on a two-country, forward-looking, general equilibrium model, which is estimated for two euro area countries (Germany and France), we show that there are gains to be achieved by taking into account the heterogeneity of economic structures. This finding appears to be robust under alternative weights given by the central bank to the stabilisation of the target variables. Successful implementation of this type of rule, however, would have to deal with difficulties in assessing empirically differences in monetary transmission across countries and measurement uncertainties that are compounded in a disaggregate context.

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1 Introduction

On 1st January 1999 the European monetary union was established, initially with 11 member countries, which Greece joined on 1st January 2001 and Slovenia on 1st January 2007. Participating countries are subject to a centralised monetary policy conducted by the European Central Bank (ECB) and to a common currency. According to the mandate of the ECB, as defined in the Maastricht Treaty (article 105 (1)), the primary objective of monetary policy is to maintain price stability over the medium term in the euro area, and without prejudice to this objective, the ECB shall support the general economic policies in the Community, which include a sustainable and non-inflationary growth. Despite the existence of national asymmetries, the present official view is that the ECB should focus on maintaining price stability in the euro area *as a whole* and should not seek to address questions of relative prices or inflation differentials (ECB (2005)).

Research on the monetary policy strategy of the ECB has increased in recent years. Most researchers focus on the specification of the appropriate monetary policy rule and the welfare improvement that can be achieved by using this rule. Little attention, however, has been given to the issue of data aggregation and the importance of national differences for the success of the common monetary policy. Although the dispersion of economic developments across member countries is considered a normal feature of any monetary union related to the convergence process, in the European monetary union it is also, at least to some extent, attributed to diverging national policies and long-lasting structural inefficiencies, such as nominal and real rigidities in product and factor markets. In view of the enlargement of the European monetary union, national differentials are expected to become even larger with potential costs in terms of the union's economic performance. Therefore, the choice of the ECB to react to aggregate data can be questioned. A thorough examination of how national information could be incorporated in monetary policy decisions would thus be warranted.

A common conclusion from the literature (as presented in Section 2) is that the effectiveness of monetary policy in the euro area may be enhanced by paying attention to national information, as opposed to reacting solely to aggregate union-wide variables. Even though the objectives of the ECB are expressed exclusively in union-wide terms, the fact that the economies of the euro area are characterised by structural differences and may be hit by asymmetric shocks can make neglecting national developments very costly. The results of this paper suggest that an optimal

monetary policy rule should take into consideration not only the relative size of the countries (relative output or population), but also the structural characteristics of the economies. In order to substantiate this claim, we extend the analysis of De Grauwe and Piskorski (2001), Angelini *et al.* (2002) and Monteforte and Siviero (2002), and analytically derive an optimal interest rate reaction function of the monetary union's central bank by minimising its loss function subject to a multi-country structural model. The paper contributes to the literature on the optimal design of monetary policy in the European monetary union in the presence of structural asymmetries across union member countries by studying the dependence of the coefficients of the interest rate rule on the parameters of the structural model of each economy, the central bank's preferences for inflation and output stabilisation as shown in its loss function, and the relative size of each country. Furthermore, recognising the advantages of New-Keynesian models in describing the economy, our analysis adopts a forward-looking perspective in the spirit of Clarida, Galí and Gertler (1999). Finally, we evaluate the optimal weights the central bank should attach to each country's economic variables using the parameters of the multi-country structural model, allowing for more than one type of asymmetry as an extension to Benigno (2004), and we assess the welfare improvement that would be achieved by the implementation of such a rule compared to a rule that focuses only on union-wide variables.

The baseline model used to derive the optimal monetary policy rule is described in Section 3 and is a dynamic, general equilibrium model, with the aggregate demand equation resulting from the consumer's utility maximisation problem and the New-Keynesian Phillips curve being based on Calvo's (1983) staggered price setting. Following Svensson (1999), Giannoni and Woodford (2003b) and Svensson (2003), in Sections 4 and 5 we derive the optimal interest rate reaction function subject to the assumption that the union economy can be satisfactorily described by an aggregate union-wide model, and alternatively by a disaggregate two-country model. In Section 6, we estimate the structural equations of both models, the first with data from a hypothetical union between Germany and France, and the second with individual country data for Germany and France. Using these estimates, we calculate in Section 7 the optimal coefficients of the interest rate reaction function, the volatility of the variables of interest, the value of the loss function for both models and the loss ratio in order to compare the relative performance of the two interest rate reaction functions. Also, our main findings, as well as some qualifications on the applicability of these results are offered. Section 8 summarises and presents the main conclusions. The Appendix contains technical and other details.

2 Related literature

The importance of considering aggregate information when the central bank of the European monetary union decides on monetary policy has been studied empirically by Wieland (1996) who compares an asymmetric regime, under which one country (i.e. Germany) conducts monetary policy for the whole union (which also includes France, Italy and the UK) by targeting domestic variables (nationalistic perspective), and a symmetric regime, under which the center country (Germany) targets union-wide variables (consensus perspective). The simulation results show that the asymmetric regime allows the reduction of inflation and output variability in the German economy, but the other countries bear the full burden of adjustment. In contrast, under the symmetric regime, the negative impact on the other countries of the union is alleviated, at the cost of larger inflation and output variability in Germany. Similar results are found by Taylor (1999), who examines the efficiency of the ECB's monetary policy by simulating two different policy rules, the first with country-specific data (i.e. for Germany) and the second with union (weighted average) data (i.e. for Germany, France and Italy). The author finds that the latter, more symmetric, rule increases slightly the impact of nominal shocks on inflation in Germany, but reduces the impact in France and Italy, compared to the country-specific rule.

Further research by Aksoy, De Grauwe and Dewachter (2001, 2002) emphasises the prominent role played by national central bank governors within the Governing Council of the ECB and examines the welfare implications of alternative decision-making procedures, namely whether the governors adopt a nationalistic or consensus (union-wide) perspective. Similar studies by De Grauwe (2000), De Grauwe and Piskorski (2001), Angelini *et al.* (2002) and Monteforte and Siviero (2002) investigate the usefulness of country-specific information for monetary policy in the European monetary union. In particular, the authors evaluate, using the framework proposed by Rudebusch and Svensson (1999), the performance (relative loss) of rules targeting national variables as opposed to union-wide variables for calibrated aggregate demand and supply equations and all but De Grauwe and Piskorski (2001) find that the first type of rule may deliver large welfare gains. Recent research by Angelini *et al.* (2004) shows that the ECB is able to align national economic cycles by taking into account the inflation dispersion across member states, but at the expense of a larger variance of union-wide inflation.

Finally, Benigno (2004) and Lombardo (2002, 2006), using two-country optimising models,

examine optimal monetary policy in a currency area, like the European monetary union, characterised by asymmetric shocks across countries. According to these authors, an optimal inflation-targeting policy should attach a higher weight to the inflation of the country with the higher degree of nominal rigidity (Benigno, 2004) or to the inflation of the country with the higher market competition (Lombardo 2002, 2006). The rationale behind these results is that less flexible prices or a higher degree of competition affect relative prices across members of a currency union, cause output dispersion and worsen the welfare of the currency area.

3 The baseline model

The New-Keynesian model used is a dynamic, stochastic, general equilibrium model, based on optimising behaviour combined with some form of nominal price rigidity. Early examples of such models include Goodfriend and King (1997), Rotemberg and Woodford (1997, 1999) and McCallum and Nelson (1999) The equations of the model are derived from well-specified optimisation problems, i.e. the representative agent's problem and the pricing decisions of individual firms. Traditional aggregate demand and supply equations are often criticised of being too *ad hoc*. However, this criticism does not apply to the New-Keynesian framework, since the coefficients in these equations are explicit functions of the underlying structural parameters of the consumer's utility function, the production function and the price-setting process. Furthermore, both equations contain forward-looking elements and assume rational expectations, the omission of which was a serious shortcoming of the traditional models.

A common and plausible assumption widely used in the literature is that the central bank aims at minimising a quadratic loss function specified in terms of inflation (π_t) and the output gap ($\widetilde{y}_t$), which is defined as the deviation of actual from potential output. Although this assumption may seem rather *ad hoc*, Woodford (1999) has provided a formal justification for the use of such a loss function, which is derived as a quadratic approximation to the expected utility of the representative household. A useful extension can be obtained if one includes real money balances as an additional argument in the household's utility function. Thus, considering the welfare consequences of the transactions frictions related to money demand adds an extra term to the loss function, namely the squared deviation of the interest rate from a constant rate[1] ($i_t - \hat{\imath}$). In this case, the intertemporal loss function to be minimised can be

[1] This is the interest rate consistent with the inflation being equal to equilibrium level (Woodford, 1999).

written as:

$$E_t \sum_{j=0}^{\infty} \delta^j V_{t+j} = \Omega E_t \sum_{j=0}^{\infty} \delta^j \left\{ (\pi_{t+j})^2 + \lambda (\widetilde{y}_{t+j})^2 + \kappa (i_{t+j} - \hat{\imath})^2 \right\} + t.i.p. \tag{1}$$

where Ω includes constant parameters of the welfare maximisation problem, δ denotes the discount rate ($0 < \delta < 1$), λ and κ are the relative weights on output and interest rate stabilisation respectively and $t.i.p.$ denotes terms independent of policy.

The aggregate demand equation is derived from the Euler condition of a representative agent's optimisation problem and relates the output gap to the expected future output gap and the real interest rate. Thus, it shows the sensitivity of output to the monetary policy interest rate. Changes in the latter affect the real interest rate and this alters the optimal time path of consumption. The forward-looking aggregate demand curve is given by:

$$\widetilde{y}_t = \widetilde{y}_{t+1|t} - \frac{1}{\sigma} \left[i_t - \pi_{t+1|t} - \overline{r} \right] + u_t \tag{2}$$

where $\pi_{t+1|t}$ and $\widetilde{y}_{t+1|t}$ represent the expected inflation and output gap respectively for period $t+1$ on the basis of information in period t, σ is the intertemporal elasticity of substitution of consumption, $\overline{r}$ represents the Wicksellian natural rate of interest, which is required to bring output to the flexible-prices level and u_t denotes a productivity shock.

The source of the real effects of monetary policy in this model is the assumption that prices are adjusted at exogenous random intervals (Calvo, 1983). Calvo's model assumes that a fraction $(1-\alpha)$ of producers charge a new price at the end of a period, whereas the rest (α) continue charging the old price. The parameter α is a measure of the degree of price rigidity. The New-Keynesian Phillips curve relates inflation to expected future inflation, and also to the deviation of output from potential output that could be attained under flexible prices, namely:

$$\pi_t = \delta \pi_{t+1|t} + \theta \gamma \widetilde{y}_t + \epsilon_t \tag{3}$$

where $\theta = \frac{(1-\alpha)(1-\alpha\delta)}{\alpha}$, $\gamma = \eta + \sigma$, with η being the elasticity of real marginal cost with respect to output, and ϵ_t represents either changes in tastes that affect leisure or stochastic shifts in the markup of wages over the marginal rate of substitution between leisure and consumption.

4 An optimal rule based on a union-wide model

Using the baseline model[2] described in the previous section and following Giannoni and Woodford (2003a), we can derive the optimal reaction function of the central bank based on a union-wide model. We start by assuming that the central bank decides on the interest rate for the simplest case of a two-country union, taking into account the aggregate (weighted) variables of both countries. The central bank minimises a modified loss function, with positive weights 1, λ and κ on the squared deviations of inflation from the inflation target ($\widehat{\pi}$), squared output gap and squared interest rate deviations from a constant rate ($\hat{\imath}$) consistent with the inflation target, as follows:

$$\min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j L_{t+j} = \min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j \frac{1}{2} \left[\left(\pi_{t+j}^U - \widehat{\pi}\right)^2 + \lambda \left(\widetilde{y}_{t+j}^U\right)^2 + \kappa \left(i_{t+j} - \hat{\imath}\right)^2 \right] \tag{4}$$

subject to the union's forward-looking Phillips and aggregate demand curves:

$$\pi_t^U = \delta \pi_{t+1|t}^U + \rho \widetilde{y}_t^U + \epsilon_t \tag{5}$$

$$\widetilde{y}_t^U = \widetilde{y}_{t+1|t}^U - \frac{1}{\sigma} \left(i_t - \pi_{t+1|t}^U - \overline{r} \right) + u_t \tag{6}$$

where[3] $\pi_t^U = w\pi_t + (1-w)\pi_t^*$, $\widetilde{y}_t^U = w\widetilde{y}_t + (1-w)\widetilde{y}_t^*$ and w is the weight given to each country in the union according to its relative size.

The Lagrangian is given by:

$$\min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j \left\{ \begin{array}{c} \frac{1}{2} \left[\left(\pi_{t+j}^U - \widehat{\pi}\right)^2 + \lambda \left(\widetilde{y}_{t+j}^U\right)^2 + \kappa \left(i_{t+j} - \hat{\imath}\right)^2 \right] \\ +\phi_{t+j} \left[\pi_{t+j}^U - \delta \pi_{t+1+j|t}^U - \rho \widetilde{y}_{t+j}^U \right] \\ +\psi_{t+j} \left[\widetilde{y}_{t+j}^U - \widetilde{y}_{t+1+j|t}^U + \frac{1}{\sigma} \left(i_{t+j} - \pi_{t+1+j|t}^U - \overline{r} \right) \right] \end{array} \right\}$$

where ϕ_{t+j} and ψ_{t+j} are the Lagrange multipliers associated with the constraints in period $t+j$.

[2] We recognise that this model treats the union like a closed economy and disregards features that are present in currency areas, like terms-of-trade effects, relative-price effects, etc. However, it enables us to get manageable and straightforward solutions and trace the monetary policy implications we want to focus on.

[3] Asterisks are used to distingusish the variables of the second country.

Under commitment it is sufficient to minimise the Lagrangian for only two periods:

$$\min_{i_t}\left\{\begin{array}{c}\frac{1}{2}\left[\left(\pi_t^U-\widehat{\pi}\right)^2+\lambda\left(\widetilde{y}_t^U\right)^2+\kappa\left(i_t-\hat{\imath}\right)^2\right]\\ +\phi_t\left[\pi_t^U-\delta\pi_{t+1|t}^U-\rho\widetilde{y}_t^U\right]+\psi_t\left[\widetilde{y}_t^U-\widetilde{y}_{t+1|t}^U+\frac{1}{\sigma}\left(i_t-\pi_{t+1|t}^U-\overline{r}\right)\right]\\ +\frac{\delta}{2}\left[\left(\pi_{t+1|t}^U-\widehat{\pi}\right)^2+\lambda\left(\widetilde{y}_{t+1|t}^U\right)^2+\kappa\left(i_{t+1|t}-\hat{\imath}\right)^2\right]\\ +\delta\phi_{t+1|t}\left[\pi_{t+1|t}^U-\delta\pi_{t+2|t}^U-\rho\widetilde{y}_{t+1|t}^U\right]+\delta\psi_{t+1|t}\left[\widetilde{y}_{t+1|t}^U-\widetilde{y}_{t+2|t}^U+\frac{1}{\sigma}\left(i_{t+1|t}-\pi_{t+2|t}^U-\overline{r}\right)\right]\end{array}\right\}$$

The first-order conditions with respect to $\pi_{t+1|t}^U$, $\widetilde{y}_{t+1|t}^U$ and i_t yield respectively:

$$\delta\left(\pi_{t+1|t}^U-\widehat{\pi}\right)-\phi_t\delta+\phi_{t+1|t}\delta-\psi_t\frac{1}{\sigma}=0 \tag{7}$$

$$\delta\lambda\left(\widetilde{y}_{t+1|t}^U\right)-\psi_t-\delta\phi_{t+1|t}\rho+\delta\psi_{t+1|t}=0 \tag{8}$$

$$\kappa\left(i_t-\hat{\imath}\right)+\psi_t\frac{1}{\sigma}=0\Rightarrow\psi_t=-\kappa\sigma\left(i_t-\hat{\imath}\right) \tag{9}$$

Substituting ψ_t from the last condition (9) into the first-order conditions (7) and (8), solving the second condition (8) for ϕ_t and substituting it in equation (7), we obtain:

$$i_t=\frac{\rho}{\kappa\sigma}\left(\pi_t^U-\widehat{\pi}\right)+\frac{\lambda}{\kappa\sigma}\Delta\widetilde{y}_t^U+\left[\frac{\rho}{\sigma\delta}+1\right]i_{t-1}+\left[\frac{1}{\delta}\right]\left(i_{t-1}-i_{t-2}\right)-\hat{\imath}\left[\frac{\rho}{\sigma\delta}\right] \tag{10}$$

This implicit instrument rule, which the central bank commits to follow, involves a positive contemporaneous response of the interest rate to deviations of union inflation from the target and to changes in union output gap. Furthermore, it involves history dependence as the interest rate responds positively to past interest rates. The coefficients of the rule satisfy the generalised Taylor principle of determinacy as proposed by Giannoni and Woodford (2003b).[4]

Concerning the response of the interest rate to the aggregate (weighted) variables, this varies directly with the size of ρ. Thus, the larger the slope of the Phillips curve, the stronger the interest rate reaction to inflation deviations from target. Note that a lower price rigidity (α), i.e. the fraction of firms not adjusting their prices in every period, implies a steeper Phillips

[4] The relevant condition is: $coef(\pi_t)+\frac{1-\delta}{\rho}coef\left(\widetilde{y}_t\right)>1-coef\left(i_{t-1}\right)$

curve and a stronger effect of output on inflation. Therefore, in case of disturbances, changes in inflation call for a more aggressive interest rate adjustment in order to stabilise inflation towards the target. Similarly, the interest rate can be seen to respond to the target variables in proportion to $\frac{1}{\sigma}$. Thus, when the slope of the aggregate demand curve is larger, the interest rate reaction to inflation deviations from target, as well as to output gap changes, should be stronger. It should be noted that a lower intertemporal elasticity of substitution of consumption (σ) makes the aggregate demand curve steeper and causes a larger real interest rate effect on the output gap. Therefore, if a demand shock occurs, there will be a large effect on the output gap and the central bank must adjust the interest rate sufficiently to bring the output gap close to zero.

Additionally, the interest rate response to past interest rates depends inverserly on the size of δ, so the more importance the consumers attach to the future level of the variables (which in turn implies lower inertia for these variables), the stronger the monetary policy leverage is. Therefore, the interest rate needs to adjust less to past interest rates changes.

Finally, the interest rate response to output gap changes is directly related to λ, i.e. the weight given by the central bank to output gap stabilisation. In contrast, the interest rate response to inflation deviations from target and output gap changes depends inverserly on κ; thus, in case the central bank is concerned about interest rate variability, it must adjust the interest rate to changes in target variables more smoothly.

5 Deriving the optimal rule based on a multi-country model

Recognising that asymmetries may exist across member countries of the European monetary union, it is important to derive the optimal reaction function of the central bank, taking into account national information explicitly. For the simplest case of a two-country union, one may assume that the central bank minimises the average of individual economies' loss functions, weighted according to the countries' relative size (w):

$$\min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j \left[w L_{t+j} + (1-w) L^*_{t+j} \right] \tag{11}$$

The loss functions of the two countries share the same features, namely the discount factor (δ), the inflation target ($\widehat{\pi}$) and the relative weights on the output gap (λ) and on interest rate

deviations from a constant rate (κ), while the monetary policy interest rate (i_t) is common for both countries - members of the union.

The loss functions of the two countries are given respectively by:

$$L_t = \frac{1}{2}\left[(\pi_t - \widehat{\pi})^2 + \lambda \widetilde{y}_t^2 + \kappa(i_t - \widehat{\imath})^2\right]$$

and

$$L_t^* = \frac{1}{2}\left[(\pi_t^* - \widehat{\pi})^2 + \lambda \widetilde{y}_t^{*2} + \kappa(i_t - \widehat{\imath})^2\right]$$

Next, we assume heterogeneous forward-looking Phillips and aggregate demand curves for both countries, i.e.:

$$\pi_t = \delta \pi_{t+1|t} + \beta \widetilde{y}_t + \varepsilon_t \tag{12}$$

$$\widetilde{y}_t = \widetilde{y}_{t+1|t} - \frac{1}{\tau}\left(i_t - \pi_{t+1|t} - \overline{r}\right) + \upsilon_t \tag{13}$$

$$\pi_t^* = \delta \pi_{t+1|t}^* + \beta^* \widetilde{y}_t^* + \varepsilon_t^* \tag{14}$$

$$\widetilde{y}_t^* = \widetilde{y}_{t+1|t}^* - \frac{1}{\tau^*}\left(i_t - \pi_{t+1|t}^* - \overline{r}\right) + \upsilon_t^* \tag{15}$$

The Lagrangian for two periods is given by:

$$\min_{i_t} \left\{ \begin{array}{c} \frac{1}{2}w\left[(\pi_t - \widehat{\pi})^2 + \lambda \widetilde{y}_t^2\right] + \frac{1}{2}(1-w)\left[(\pi_t^* - \widehat{\pi})^2 + \lambda \widetilde{y}_t^{*2}\right] + \frac{1}{2}\kappa(i_t - \widehat{\imath})^2 \\ +\phi_t\left[\pi_t - \delta\pi_{t+1|t} - \beta\widetilde{y}_t\right] + \chi_t\left[\widetilde{y}_t - \widetilde{y}_{t+1|t} + \frac{1}{\tau}\left(i_t - \pi_{t+1|t} - \overline{r}\right)\right] \\ +\psi_t\left[\pi_t^* - \delta\pi_{t+1|t}^* - \beta^*\widetilde{y}_t^*\right] + \omega_t\left[\widetilde{y}_t^* - \widetilde{y}_{t+1|t}^* + \frac{1}{\tau^*}\left(i_t - \pi_{t+1|t}^* - \overline{r}\right)\right] \\ +\delta\{\frac{1}{2}w\left[\left(\pi_{t+1|t} - \widehat{\pi}\right)^2 + \lambda\widetilde{y}_{t+1|t}^2\right] + \frac{1}{2}(1-w)\left[\left(\pi_{t+1|t}^* - \widehat{\pi}\right)^2 + \lambda\widetilde{y}_{t+1|t}^{*2}\right] + \frac{1}{2}\kappa(i_{t+1|t} - \widehat{\imath})^2 \\ +\phi_{t+1|t}\left[\pi_{t+1|t} - \delta\pi_{t+2|t+1} - \beta\widetilde{y}_{t+1|t}\right] + \chi_{t+1|t}\left[\widetilde{y}_{t+1|t} - \widetilde{y}_{t+2|t+1} + \frac{1}{\tau}\left(i_{t+1|t} - \pi_{t+2|t+1} - \overline{r}\right)\right] \\ +\psi_{t+1|t}\left[\pi_{t+1|t}^* - \delta\pi_{t+2|t+1}^* - \beta^*\widetilde{y}_{t+1|t}^*\right] + \omega_{t+1|t}\left[\widetilde{y}_{t+1|t}^* - \widetilde{y}_{t+2|t+1}^* + \frac{1}{\tau^*}\left(i_{t+1|t} - \pi_{t+2|t+1}^* - \overline{r}\right)\right]\} \end{array} \right\}$$

where ϕ_t, χ_t, ψ_t and ω_t are the multipliers associated with the constraints.

The first-order conditions with respect to $\pi_{t+1|t}$, $\pi_{t+1|t}^*$, $\widetilde{y}_{t+1|t}$, $\widetilde{y}_{t+1|t}^*$ and i_t are respectively:

$$\delta w\left(\pi_{t+1|t}-\widehat{\pi}\right)-\delta\phi_t-\frac{1}{\tau}\chi_t+\delta\phi_{t+1|t}=0 \tag{16}$$

$$\delta\left(1-w\right)\left(\pi^*_{t+1|t}-\widehat{\pi}\right)-\delta\psi_t-\frac{1}{\tau^*}\omega_t+\delta\psi_{t+1|t}=0 \tag{17}$$

$$\delta w\lambda\widetilde{y}_{t+1|t}-\chi_t-\delta\beta\phi_{t+1|t}+\delta\chi_{t+1|t}=0 \tag{18}$$

$$\delta\left(1-w\right)\lambda\widetilde{y}^*_{t+1|t}-\omega_t-\delta\beta^*\psi_{t+1|t}+\delta\omega_{t+1|t}=0 \tag{19}$$

$$\kappa(i_t-\hat{\imath})+\frac{1}{\tau}\chi_t+\frac{1}{\tau^*}\omega_t=0 \tag{20}$$

Solving equations (16 and 18) to eliminate ϕ_t and equations (17 and 19) to eliminate ψ_t, and substituting χ_t and ω_t into equation (20) we get the following rule:

$$\begin{aligned} i_t &= \frac{\beta w}{\kappa\tau}\left(\pi_t-\widehat{\pi}\right)+\frac{\lambda w}{\kappa\tau}\Delta\widetilde{y}_t+\frac{\beta^*\left(1-w\right)}{\kappa\tau^*}\left(\pi^*_t-\widehat{\pi}\right)+\frac{\lambda\left(1-w\right)}{\kappa\tau^*}\Delta\widetilde{y}^*_t \\ &\quad +[1+\frac{1}{\delta}]i_{t-1}-\frac{1}{\delta}i_{t-2}+\frac{1}{\delta}\left[\frac{\beta}{\tau}+\frac{\beta^*}{\tau^*}\right]\left(i_{t-1}-\hat{\imath}\right) \end{aligned} \tag{21}$$

In contrast to the rule based on the union-wide model, in the above rule there is differentiated adjustment of the interest rate to individual economies' variables beyond that justified by the relative size of the countries. In particular, the interest rate responds differently to each country's target variables, the response being related to that country's structural parameters, i.e. the slopes of the Phillips and the aggregate demand curves. The central bank is thus seen to react more aggressively to macroeconomic developments in the country with the lower price rigidity (steeper Phillips curve) and the lower intertemporal elasticity of substitution (steeper aggregate demand curve) in order to avoid large fluctuations of inflation and output gap and minimise welfare losses.[5] The rationale behind this result is that the effect of exogenous demand and supply shocks on inflation and the output gap increases with the slope of the Phillips curve

[5] Note that the interest rate response to inflation deviations from target relates to both these structural parameters, while the response to output gap changes depends only on the intertemporal elasticity of substitution.

or the aggregate demand curve. Similar results are found in De Grauwe and Piskorski (2001) and Angelini *et al.* (2002) who derive numerically the optimal coefficients of the interest rate rule by minimising the central bank's loss function subject to the estimated structural models for the countries forming the union.

Finally, given our assumptions about a common discount factor (δ) and central bank relative preferences for output and interest rate stabilisation (parameters κ and λ), the dependence of the interest rate on these parameters does not vary across countries. Thus, the relevant conclusions of the previous section hold also here.

6 Model estimation

In order to evaluate the optimal interest rate reaction function of the previous sections, we need to have estimates of the structural parameters of the Phillips and the aggregate demand equations.[6] For the first model (eqs. 5 and 6), we assumed the existence of a hypothetical monetary union between Germany and France and estimated it using aggregate data for the two countries,[7] weighted according to the OECD weighting scheme,[8] for the period 1965:1-1998:4. The second model, which consists of the individual country equations (eqs. 12, 13, 14 and 15), was estimated using data for Germany and France over the same period. Of course, the calculation of the coefficients of the interest rate reaction function depends on the choice of the countries, on the empirical estimates of the model parameters and on the assumptions about the relative preferences of the central bank as shown in the loss function. A more complete model for the euro area should include all member countries and would presumably result in more pronounced asymmetries. However, the results of our exercise can give us a lower bound to the welfare improvement that can be achieved if the central bank of a monetary union focuses on the structural characteristics of each member economy, and not only on union-wide variables.

All data series used for the estimation of the Phillips and aggregate demand curves in both

[6] Previous research on empirical estimation of New-Keynesian Phillips and aggregate demand curves include Fuhrer and Rudebusch (2003), Galí, Gertler and López-Salido (2001) and Jondeau and Le Bihan (2001).

[7] Most researchers justify the use of synthetic data for the euro area constructed by aggregating weighted national data before EMU by the fact that the gradual process of monetary convergence and the participation in the Exchange Rate Mechanism (ERM) since the beginning of the '90s. Moreover, statistical evidence by Mihov (2001) rejects the presence of a structural break in January 1999 and suggests that the use of pooled data from euro area member countries can be a good approximation to the euro area data.

[8] OECD's weights are based on 2000 GDP and purchasing power parities. The weights were rescaled to sum to unity and are 0.42 for France and 0.58 for Germany.

models have a quarterly frequency and are drawn from the OECD Economic Outlook database. Inflation is measured in terms of the Harmonised Index of Consumer Prices. The output gap is given by the deviation of real GDP from its potential level, the calculation of which is based on a production function approach.[9] The nominal interest rate is the 3-month interbank rate. The parameters were estimated using the Generalised Method of Moments (GMM), which is widely used in models with forward-looking variables. The instruments chosen are lagged values of the explanatory variables (inflation, output gap and interest rate), so that they are predetermined at the time the central bank decides on the level of the interest rate. Furthermore, they are uncorrelated with the residuals, but strongly correlated with the forward-looking variables.

[Insert Table 1]

Our estimates show almost flat aggregate demand curve in all cases, similarly to Fuhrer and Rudebusch's (2003) findings for the US.[10] Also, the Phillips curve estimates show high values of the discount factor (δ), as reported in most empirical studies. Furthermore, we obtain very low estimates for the slope of the Phillips curve for Germany and the Union (β and ρ), which are significant only at the 10 percent level, as in Jondeau and Le Bihan (2001).[11] In contrast, the Phillips curve for France (with slope β^*) is found comparatively steeper, revealing a stronger effect of the output gap on inflation for that country. The difference between the slopes of the Phillips curve in the two countries can be attributed to the higher price level stickiness in Germany relative to France. This is also supported by Leith and Malley (2006), who estimate the New Keynesian Phillips curve for the G7 countries and provide evidence that, compared with the other countries, Germany is characterised by the longest time (close to two years) required for price adjustment by all firms in the economy. Similar evidence is given in Rumler (2005) and Peersman (2000) who find higher price persistence in Germany than in France. The values of the *J-statistic* verify the validity of the instruments used and the values of $adjR^2$ are reasonably high.

[9] For explicit details, see Giorno *et al.* (1995).

[10] Fuhrer and Rudebusch (2003) estimate a modified output Euler equation which includes both past and future values of the output gap.

[11] Jondeau and Le Bihan (2001) estimate a hybrid Phillips curve, which includes a lag and a lead of inflation in addition to the output gap.

7 Monetary policy performance under the alternative rules and sensitivity analysis

In order to evaluate the relative performance of the optimal interest rate reaction functions derived in Sections 4 and 5, we need to compare the value of the loss generated under each model. Both models are written in state space form (presented in Appendices A1 and A2) and are solved numerically under commitment following Söderlind (1999). We can then derive the dynamics of the system and in particular the interest rate reaction function. We can also calculate the variance of the target variables, namely the inflation deviations from the target, the output gap and the interest rate deviations from the constant rate, in order to estimate the expected value of the loss function, which (as shown in Appendix A3) is given by:

$$E\left(L_t\right) = \frac{\delta}{1-\delta} trace\left(K\Sigma_{YY}\right)$$

where K is a diagonal matrix, the diagonal elements of which are the parameters that represent the relative preferences of the central bank for the stabilisation of inflation, the output gap and the interest rate in its loss function (1, λ and κ respectively), and Σ_{YY} denotes the covariance matrix of the target variables.[12] Based on the literature, the discount factor (δ) is set equal to 0.99.

As a robustness check, we calculated the loss generated under the alternative interest rate rules assuming that the parameters in the loss function (λ and κ) take values ranging from 0.1 (in which case the central bank focuses almost exclusively on inflation) to 1 (case where the central bank attaches a high cost to deviations of actual output from potential and to interest rate deviations from a constant rate). The resulting loss ratio, i.e. the loss associated with the interest rate rule based on the union-wide model (10) relative to that from the rule based on the multi-country model (21) is shown in Figure 1. Table 2 presents the coefficients of the interest rate reaction function, the variance of the target variables, the value of the loss function and the loss ratio under the alternative interest rate rules for specific combinations of λ and κ.

[Insert Figure 1]

[Insert Table 2]

[12] For comparison reasons, we solved both models using the parameters that were empirically estimated for each model, but by imposing the same covariance matrix, which was derived by averaging the variance of the residuals from the multi-country model.

The estimated coefficients of the optimal interest rate reaction functions presented in Table 2 exhibit positive responses to inflation and output gap changes and pronounced interest rate smoothing, which can be attributed to the inclusion of the interest rate changes in the loss function. A further and rather obvious result is that increasing the weight (λ) given to output gap stabilisation in the loss function leads to a higher interest rate response to output gap changes, while increasing the weight (κ) given to deviations of the interest rate from the rate consistent with the inflation target reduces the interest rate reaction both to inflation and to output gap changes. Furthermore, the welfare loss increases in proportion to the relative importance the central bank attaches to the goals of output gap and interest rate stabilisation compared to inflation stabilisation. This can be explained by the fact that increasing the weights attached to the stabilisation of the output gap and the interest rate deviations from a constant rate, weakens inflation targeting and increases welfare losses.

A result evident in all cases is the higher volatility of the common interest rate under the rule based on the union-wide model compared to the rule based on the multi-country model. The intuition behind this result is that the rule which responds to national variables attaches relatively higher importance to the country which is characterised by more responsive inflation and output gap to interest rate movements and is better off with smaller shifts of the common interest rate. As a result, the interest rate from the rule based on the multi-country model, adjusts less. Similar conclusions are reached by De Grauwe and Piskorski (2001).

The comparison of the alternative rules supports our proposition that the central bank should take into account the national structural characteristics, as presented in Section 5. In particular, the rule based on the union-wide model (10) responds to each country's variables according to their weight in the aggregate variable. In contrast, the rule based on the multi-country model (21) suggests an adjustment to each country's variables depending on the structural parameters of the economy. This is why the interest rate in the case of the multi-country model adjusts more to inflation in France and to the output gap in Germany. The fact that the Phillips curve is steeper in France justifies a stronger (almost double) response of the single interest rate to French inflation, taking also into account that the aggregate demand curve is steeper in Germany and that Germany's weight is higher. On the other hand, the steeper aggregate demand curve in Germany, combined with the higher weight given to Germany's variables, calls for a much stronger (three times as high) response to German output gap changes. For any

combination of λ and κ reported in Table 2, but also in Figure 1, the loss ratio generated from the monetary policy decisions based on the union-wide interest rate rule relative to the multi-country interest rate rule remains above unity. This provides evidence that it can be welfare improving if the central bank commits to follow an interest rate rule that focuses on the individual countries' variables taking into account their structural characteristics, especially in case there are sizeable differences. This finding is at odds with the conclusion reached by Benigno (2004) that an inflation targeting policy in a two-country currency union, which attaches higher weight to the inflation of the country with the higher degree of nominal rigidity, is optimal. One possible explanation is that in this paper we allow for more than one type of asymmetry among countries, namely in the degree of price rigidity and in the intertemporal elasticity of substitution. As also indicated by Lombardo (2006), the result in Benigno (2004) could be altered if, additionally, the degree of competition differs.

Indeed, the level of dispersion in the European monetary union is higher than that observed in other currency areas like the United States, even though the process of nominal convergence across union members has advanced notably in recent years. This dispersion, which can be attributed to various temporary and permanent factors, can be significantly destabilising for the national economies and may be aggravated as more countries join the European monetary union. Therefore, the ECB's commitment to set the interest rate in response to individual countries' variables would give rise to substantial welfare gains.

There may, however, be difficulties in the implementation of such a rule, as identified in ECB (2005). First, there are problems to precisely estimate the differential impact of the common monetary policy on the individual countries of the union. The relatively persistent inflation and output differentials observed in the euro area (ECB (2003), Benalal *et al.* (2006)) are mainly resulting from the convergence process in EMU. At the same time, they can be attributed, to some extent, to structural differences across countries, for example in price and wage-setting mechanisms and in agents' preferences regarding consumption as reflected in the coefficients of the Phillips and the aggregate demand curve respectively (see Mojon and Peersman (2003), Angeloni *et al.* (2003)). Second, possible measurement constraints, especially those regarding the estimation of potential output and the output gap, are likely to be compounded when disaggregate data are considered. Both these factors could introduce uncertainty in the conduct of monetary policy and affect negatively the transparency and accountability of the central bank.

Third, a differentiated response by the central bank taking into account national structural characteristics would imply that monetary policy is accommodating the existing structural divergences, thus creating disincentives for real convergence across euro area countries. The first two of the above difficulties could be overcome by better measurement and estimation methods, while the last one should be addressed by national economic policies, given that such divergences cannot be expected to be influenced by the ECB's single monetary policy.

8 Conclusions

This paper studied the optimal design of monetary policy in a monetary union in the presence of structural asymmetries among countries by deriving analytically an optimal interest rate rule that a central bank can commit to follow and examining the dependence of its coefficients on the parameters of the structural model of each country, the central bank's preferences in the loss function and the relative size of each country. We provided empirical evidence on the gains to be achieved by taking into account the heterogeneity in the structure of the economies using data from Germany and France. In particular, according to our results, the ratio of the loss generated by an interest rate reaction function based on the union-wide model over that based on the multi-country model, remains above unity. Thus, our evidence suggests that the interest rate must be adjusted so that it stabilises more the variables of the country with the lower nominal rigidity and lower intertemporal elasticity of substitution. Otherwise, monetary policy decisions may cause large fluctuations of the target variables in this country and generate welfare losses. This finding appears to be robust under alternative values of central bank preferences for the stabilisation of the target variables. Although the implementation of the proposed rule involves difficulties relating to data and estimation constraints as well as risks of accommodating structural divergences, it is important that the ECB should take into consideration national characteristics in formulating its monetary policy, especially in view of more countries joining the European monetary union.

References

Aksoy, Yunus, Paul De Grauwe and Hans Dewachter. 2001. "From EMS to EMU: Are We Better Off?" In *The Political Economy of European Monetary Unification*, second edition, ed. Barry Eichengreen and Jeffry A. Frieden. Boulder: Westview Press.

———. 2002. "Do Asymmetries Matter for European Monetary Policy?" *European Economic Review* 46 (3): 443-69.

Angelini, Paolo, Paolo Del Giovane, Stefano Siviero and Daniele Terlizzese. 2002. "Monetary Policy Rules for the Euro Area: What Role for National Information?" *Banca d' Italia Temi di discussione* No. 457.

Angelini, Paolo, Stefano Siviero and Daniele Terlizzese. 2004. "Monetary Policy as a Sheep Dog?" Mimeo, Banca d' Italia.

Angeloni, Ignazio, Anil K. Kashyap, Benoît Mojon and Daniele Terlizzese. 2003. "Monetary Policy Transmission in the Euro Area: Where Do We Stand?" In *Monetary Policy Transmission in the Euro Area,* ed. Ignazio Angeloni, Anil K. Kashyap and Benoît Mojon. Cambridge: Cambridge University Press.

Benalal, Nicholai, Juan Luis Diaz del Hoyo, Beatrice Pierluigi and Nick Vidalis. 2006. "Output Growth Differentials across Euro Area Countries: Some Stylised Facts", *ECB Occasional Paper* No. 45.

Benigno, Pierpaolo. 2004. "Optimal Monetary Policy in a Currency Area." *Journal of International Economics* 63 (2): 293-320.

Calvo, Guillermo. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12 (3): 383-98.

Chow, Gregory. 1970. *Analysis and Control of Dynamic Economic Systems.* New York: John Wiley & Sons.

Clarida, Richard, Jordi Galí and Mark Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37 (2): 1661-707.

De Grauwe, Paul. 2000. "Monetary Policies in the Presence of Asymmetries." *Journal of Common Market Studies* 38 (4): 593-612.

De Grauwe, Paul and Tomasz Piskorski. 2001. "Union-wide Aggregates versus National Data Based Monetary Policies: Does it Matter for the Eurosystem?" *CEPR Discussion Paper* No. 3036.

Dixit, Avinash K. and Joseph E. Stiglitz. 1977. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review* 67 (3): 297-308.

ECB. 2003. *Inflation Differentials in the Euro Area: Potential Causes and Policy Implications.*

ECB. 2005. "Monetary Policy and Inflation Differentials in a Heterogeneous Currency Area." *Monthly Bulletin* (May): 61-77.

Friedman, Milton. 1969. "The Optimum Quantity of Money." In *The Optimum Quantity of Money and Other Essays.* Chicago: Aldine.

Fuhrer, Jeffrey C. and Glenn D. Rudebusch. 2004. "Estimating the Euler Equation for Output." *Journal of Monetary Economics* 51(6): 1133-53.

Galí, Jordi, Mark Gertler and J. David López-Salido. 2001. "European Inflation Dynamics." *European Economic Review* 45 (7): 1237-70.

Giannoni, Mark P. and Michael Woodford. 2003a. "Optimal Interest-Rate Rules: I. General Theory." *NBER Working Paper* No. 9419.

———. 2003b. "Optimal Interest-Rate Rules: II. Applications." *NBER Working Paper* No. 9420.

Giorno, Claude, Pete Richardson, Deborah Roseveare and Paul Van den Noord. 1995. "Estimating Potential Output, Output Gaps and Structural Budget Balances." *OECD Economics Department Working Paper* No. 152.

Goodfriend, Marvin and Robert G. King. 1997. "The New Neoclassical Synthesis and the Role of Monetary Policy." In *NBER Macroeconomics Annual* 12, ed. Ben Bernanke and Julio Rotemberg. Cambridge MA: MIT Press.

Jondeau, Eric and Hervé Le Bihan. 2001. "Testing for a Forward-Looking Phillips Curve: Additional Evidence from European and US Data." *Banque de France Notes d' Études et de Recherche* No. 86.

Leitemo, Kai and Ulf Söderström. 2005. "Simple Monetary Policy Rules and Exchange Rate Uncertainty." *Journal of International Money and Finance* 24 (3): 481-507.

Leith, Campbell and Jim Malley. 2006. "Estimated Open Economy New Keynesian Phillips Curves for the G7." *Open Economies Review,* forthcoming.

Lombardo, Giovanni. 2002. "Imperfect Competition, Monetary Policy and Welfare in a Currency Area." *Deutsche Bundesbank Discussion Paper* No. 21.

———. 2006. "Inflation Targeting Rules and Welfare in an Asymmetric Currency Area." *Journal of International Economics* 68 (2): 424-42.

McCallum, Bennett T. and Edward Nelson. 1999. "An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis." *Journal of Money, Credit, and Banking* 31 (3): 296-316.

Mihov, Ilian. 2001. "Monetary Policy Implementation and Transmission in the European Monetary Union." *Economic Policy* 33: 369-406.

Mojon, Benoît and Gert Peersman. 2003. "The Monetary Transmission Mechanism in the Euro Area: Evidence from VAR analysis." In *Monetary Policy Transmission in the Euro Area,* ed. Ignazio Angeloni, Anil K. Kashyap and Benoît Mojon. Cambridge: Cambridge University Press.

Monteforte, Libero and Stefano Siviero. 2002. "The Economic Consequences of Euro Area Modelling Shortcuts." *Banca d' Italia Temi di discussione* No. 458.

Peersman, Gert. 2004. "The Transmission of Monetary Policy in the Euro Area: Are the Effects Different across Countries?" *Oxford Bulletin of Economics and Statistics* 66 (3): 285-308.

Rotemberg, Julio J. and Michael Woodford. 1997. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy." In *NBER Macroeconomics Annual* 12: 297-346.

———. 1999. "Interest Rate Rules in an Estimated Sticky Price Model." In *Monetary Policy Rules*, ed. John B. Taylor. Chicago: University of Chicago Press.

Rudebusch, Glenn D. and Lars E.O. Svensson. 1999. "Policy rules for inflation targeting." In *Monetary Policy Rules*, ed. John B. Taylor. Chicago: University of Chicago Press.

Rumler, Fabio. 2005. "Estimates of the Open Economy New Keynesian Phillips Curve for Euro Area Countries." *ECB Working Paper* No. 496.

Sargent, Thomas J. 1987. *Dynamic Macroeconomic Theory.* Cambridge MA: Harvard University Press.

Söderlind, Paul. 1999. "Solution and Estimation of RE Macromodels with Optimal Policy." *European Economic Review* 43 (4-6): 813-23.

Svensson, Lars E.O. 1997."Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets." *European Economic Review* 41 (6): 1111-46.

———. 1999. "Inflation Targeting as a Monetary Policy Rule." *Journal of Monetary Economics* 43 (3): 607-54.

———. 2003. "What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules." *Journal of Economic Literature* 41 (2): 426-77.

Taylor, John B. 1999. "The Robustness and Efficiency of Monetary Policy Rules as Guidelines for Interest Rate Setting by the European Central Bank." *Journal of Monetary Economics* 43 (3): 655-79.

Wieland, Volker. 1996. "Monetary Policy Targets and the Stabilization Objective: A Source of Tension in the EMS." *Journal of International Money and Finance* 15 (1): 95-116.

Woodford, Michael. 1999. "Optimal Monetary Policy Inertia." *NBER Working Paper* No. 7621.

———. 2003. "Optimal Interest-Rate Smoothing." *Review of Economic Studies* 70 (4): 861-86.

A Appendix

A.1 State space representation of the union-wide model

The structural equations of the model (eqs. 5 and 6) can be written as:

$$\pi^U_{t+1|t} = \frac{1}{\delta}\pi^U_t - \frac{\rho}{\delta}\widetilde{y}^U_t - \frac{1}{\delta}\epsilon_t$$

$$\widetilde{y}^U_{t+1|t} = \widetilde{y}^U_t + \frac{\rho}{\sigma\delta}\widetilde{y}^U_t + \frac{1}{\sigma}i_t - \frac{1}{\sigma\delta}\pi^U_t - \frac{\overline{r}}{\sigma} + \frac{1}{\sigma\delta}\epsilon_t - u_t$$

and their state-space representation is:

$X^U_{t+1} = A^U X^U_t + B^U i_t + V^U_t$, i.e.

$$\begin{bmatrix} \epsilon_{t+1} \\ u_{t+1} \\ \widetilde{y}^U_t \\ i_t \\ i_{t-1} \\ 1 \\ E_t\pi^U_{t+1} \\ E_t\widetilde{y}^U_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{\delta} & 0 & 0 & 0 & 0 & 0 & \frac{1}{\delta} & -\frac{\rho}{\delta} \\ \frac{1}{\sigma\delta} & -1 & 0 & 0 & 0 & -\frac{\overline{r}}{\sigma} & -\frac{1}{\sigma\delta} & 1+\frac{\rho}{\sigma\delta} \end{bmatrix} * \begin{bmatrix} \epsilon_t \\ u_t \\ \widetilde{y}^U_{t-1} \\ i_{t-1} \\ i_{t-2} \\ 1 \\ \pi^U_t \\ \widetilde{y}^U_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sigma} \end{bmatrix} * i_t + \begin{bmatrix} \epsilon_{t+1} \\ u_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The target variables are given by:

$Y^U_t = C^U_X X_t + C^U_I i_t$, i.e.

$$\begin{bmatrix} \pi^U_t - \widehat{\pi} \\ \widetilde{y}^U_t \\ i_t - \widehat{i} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\widehat{\pi} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -\widehat{i} & 0 & 0 \end{bmatrix} * \begin{bmatrix} \epsilon_t \\ u_t \\ \widetilde{y}^U_{t-1} \\ i_{t-1} \\ i_{t-2} \\ 1 \\ \pi^U_t \\ \widetilde{y}^U_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} * i_t$$

The loss function can be written in state-space form as:

$$L_t = Y_t^{U\prime} K Y_t^U = \begin{bmatrix} \pi_t^U - \widehat{\pi} \\ \widetilde{y}_t^U \\ i_t - \widehat{i} \end{bmatrix}' * \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \kappa \end{bmatrix} * \begin{bmatrix} \pi_t^U - \widehat{\pi} \\ \widetilde{y}_t^U \\ i_t - \widehat{i} \end{bmatrix}$$

A.2 State space representation of the multi-country model

The structural equations (eqs. 12, 13, 14 and 15) can be written as:

$$\pi_{t+1|t} = \frac{1}{\delta}\pi_t - \frac{\beta}{\delta}\widetilde{y}_t - \frac{1}{\delta}\varepsilon_t$$

$$\widetilde{y}_{t+1|t} = \widetilde{y}_t + \frac{\beta}{\delta\tau}\widetilde{y}_t + \frac{1}{\tau}i_t - \frac{1}{\delta\tau}\pi_t - \frac{\overline{r}}{\tau} + \frac{1}{\delta\tau}\varepsilon_t - \upsilon_t$$

$$\pi^*_{t+1|t} = \frac{1}{\delta}\pi^*_t - \frac{\beta^*}{\delta}\widetilde{y}^*_t - \frac{1}{\delta}\varepsilon^*_t$$

$$\widetilde{y}^*_{t+1|t} = \widetilde{y}^*_t + \frac{\beta^*}{\delta\tau^*}\widetilde{y}^*_t + \frac{1}{\tau^*}i_t - \frac{1}{\delta\tau^*}\pi^*_t - \frac{\overline{r}}{\tau^*} + \frac{1}{\delta\tau^*}\varepsilon^*_t - \upsilon^*_t$$

and represented in state-space form as:

$X_{t+1}^T = A^T X_t^T + B^T i_t + V_t^T$, i.e.

$$
\begin{bmatrix} \varepsilon_{t+1} \\ \upsilon_{t+1} \\ \varepsilon^*_{t+1} \\ \upsilon^*_{t+1} \\ \widetilde{y}_t \\ \widetilde{y}^*_t \\ i_t \\ i_{t-1} \\ 1 \\ E_t\pi_{t+1} \\ E_t\widetilde{y}_{t+1} \\ E_t\pi^*_{t+1} \\ E_t\widetilde{y}^*_{t+1} \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-\frac{1}{\delta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\delta} & -\frac{\beta}{\delta} & 0 & 0 \\
\frac{1}{\delta\tau} & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\overline{r}}{\tau} & -\frac{1}{\delta\tau} & 1+\frac{\beta}{\delta\tau} & 0 & 0 \\
0 & 0 & -\frac{1}{\delta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\delta} & -\frac{\beta^*}{\delta} \\
0 & 0 & \frac{1}{\delta\tau^*} & -1 & 0 & 0 & 0 & 0 & -\frac{\overline{r}}{\tau^*} & 0 & 0 & -\frac{1}{\delta\tau^*} & 1+\frac{\beta^*}{\delta\tau^*}
\end{bmatrix}
*
$$

$$
\begin{bmatrix} \varepsilon_t \\ \upsilon_t \\ \varepsilon^*_t \\ \upsilon^*_t \\ \widetilde{y}_{t-1} \\ \widetilde{y}^*_{t-1} \\ i_{t-1} \\ i_{t-2} \\ 1 \\ \pi_t \\ \widetilde{y}_t \\ \pi^*_t \\ \widetilde{y}^*_t \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\tau} \\ 0 \\ \frac{1}{\tau^*} \end{bmatrix}
* i_t +
\begin{bmatrix} \varepsilon_{t+1} \\ \upsilon_{t+1} \\ \varepsilon^*_{t+1} \\ \upsilon^*_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

The target variables are given by:

$Y_t^T = C_X^T X_t^T + C_I^T i_t$, i.e.

$$\begin{bmatrix} \pi_t - \widehat{\pi} \\ \widetilde{y}_t \\ \pi_t^* - \widehat{\pi} \\ \widetilde{y}_t^* \\ i_t - \widehat{i} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\widehat{\pi} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\widehat{\pi} & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\widehat{i} & 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} \varepsilon_t \\ \upsilon_t \\ \varepsilon_t^* \\ \upsilon_t^* \\ \widetilde{y}_{t-1} \\ \widetilde{y}_{t-1}^* \\ i_{t-1} \\ i_{t-2} \\ 1 \\ \pi_t \\ \widetilde{y}_t \\ \pi_t^* \\ \widetilde{y}_t^* \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} * i_t$$

The loss function is then given by:

$$L_t = Y_t^{T\prime} K^T Y_t^T = \begin{bmatrix} \pi_t - \widehat{\pi} \\ \widetilde{y}_t \\ \pi_t^* - \widehat{\pi} \\ \widetilde{y}_t^* \\ i_t - \widehat{i} \end{bmatrix}' * \begin{bmatrix} w & 0 & 0 & 0 & 0 \\ 0 & w\lambda & 0 & 0 & 0 \\ 0 & 0 & (1-w) & 0 & 0 \\ 0 & 0 & 0 & (1-w)\,\lambda & 0 \\ 0 & 0 & 0 & 0 & \kappa \end{bmatrix} * \begin{bmatrix} \pi_t - \widehat{\pi} \\ \widetilde{y}_t \\ \pi_t^* - \widehat{\pi} \\ \widetilde{y}_t^* \\ i_t - \widehat{i} \end{bmatrix}$$

A.3 Estimation of the loss

The evaluation of the loss function relies on the solution of rational expectations models proposed by Söderlind (1999). Under commitment, the problem of the central bank is to minimise the loss function, by choosing an optimal sequence of the policy instrument i_t:

$$\min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j L_t$$

where the period loss function is:

$$L_t = Y_t' K Y_t = \begin{bmatrix} X_t' & i_t' \end{bmatrix} \begin{bmatrix} C_X' \\ C_I' \end{bmatrix} K \begin{bmatrix} C_X & C_I \end{bmatrix} \begin{bmatrix} X_t \\ i_t \end{bmatrix} =$$

$$X_t^{'}C_X^{'}KC_XX_t + X_t^{'}C_X^{'}KC_Ii_t + i_t^{'}C_I^{'}KC_XX_t + i_t^{'}C_I^{'}KC_Ii_t$$

subject to the structural constraints:

$$X_{t+1} = AX_t + Bi_t + V_t$$

The structural variables can be distinguished into predetermined or backward-looking (X_{1t}) and non-predetermined or forward-looking (X_{2t}) variables. Therefore, the structural equations are written as:

$$\begin{bmatrix} X_{1t+1} \\ E_tX_{2t+1} \end{bmatrix} = A \begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} + Bi_t + \begin{bmatrix} e_{1t} \\ 0 \end{bmatrix}$$

The Lagrangian is given by:

$$J0 = E_t \sum_{j=0}^{\infty} \delta^j \begin{bmatrix} X_t^{'}C_X^{'}KC_XX_t + X_t^{'}C_X^{'}KC_Ii_t + i_t^{'}C_I^{'}KC_XX_t + i_t^{'}C_I^{'}KC_Ii_t \\ +2\xi_{t+1}^{'}\left[AX_t + Bi_t + V_t - X_{t+1}\right] \end{bmatrix}$$

where ξ denotes the Lagrange multipliers.

When the central bank minimises the Lagrangian under commitment, the solution is given by:

$$k_{1t+1} \equiv \begin{bmatrix} X_{1t+1} \\ \xi_{2t+1} \end{bmatrix} = M_c \begin{bmatrix} X_{1t} \\ \xi_{2t} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ 0 \end{bmatrix}$$

and

$$k_{2t+1} \equiv \begin{bmatrix} X_{2t+1} \\ i_t \\ \xi_{1t+1} \end{bmatrix} = N_c \begin{bmatrix} X_{1t} \\ \xi_{2t} \end{bmatrix}$$

where M_c and N_c are functions of the submatrices resulting from the generalised Schur decomposition of the first-order conditions.[13]

[13] See Söderlind (1999) for an analytical derivation.

As in Leitemo and Söderström (2001), the covariance matrix of k_{1t+1} is given:

$$vec\left(\Sigma_{k_1}\right) = \left[I - M_c \otimes M_c\right]^{-1} vec\left(\Sigma_{VV}\right)$$

where Σ_{VV} is the variance-covariance matrix of disturbances.

Stacking k_{1t+1} and k_{2t+1}, the covariance matrix of all variables ($k \equiv k_1 + k_2$) is given by:

$$\Sigma_{kk} = \begin{bmatrix} I \\ N_c \end{bmatrix} \Sigma_{k_1} \begin{bmatrix} I' & N_c' \end{bmatrix}$$

Picking out the covariance matrix of the structural variables and the instrument $\left(\Sigma_{Xi}\right)$,the covariance matrix of the target variables (Y_t) can be written as:

$$\Sigma_{YY} = \begin{bmatrix} C_X & C_I \end{bmatrix} \Sigma_{Xi}' \begin{bmatrix} C_X' \\ C_I' \end{bmatrix}$$

Therefore, the expected value of the loss function can be estimated as:

$$E\left(L_t\right) = \frac{\delta}{1-\delta} trace\left(K\Sigma_{YY}\right)$$

A.4 Tables and Figures

Table 1. GMM estimates of the Phillips curve and of the aggregate demand curve at individual country level and union level

Germany		France		Monetary union of France and Germany	
δ	0.99 (0.016)	δ	0.99 (0.014)	δ	0.99 (0.014)
β	0.04 (0.026)	β^*	0.25 (0.073)	ρ	0.05 (0.031)
$\frac{1}{\tau}$	0.04 (0.019)	$\frac{1}{\tau^*}$	0.02 (0.011)	$\frac{1}{\sigma}$	0.03 (0.012)
$adjR^2(PC)$	0.85	$adjR^2(PC)$	0.97	$adjR^2(PC)$	0.91
$adjR^2(AD)$	0.87	$adjR^2(AD)$	0.94	$adjR^2(AD)$	0.89
$SE\ (PC)$	0.70	$SE\ (PC)$	0.70	$SE\ (PC)$	0.65
$SE\ (AD)$	0.48	$SE\ (AD)$	0.57	$SE\ (AD)$	0.87
J-stat	0.11	*J-stat*	0.12	*J-stat*	0.14
J-stat p-value	0.52	*J-stat p-value*	0.64	*J-stat p-value*	0.62

Note: Numbers in parentheses are standard errors.

Figure 1: Loss ratio under the union-wide model relative to the multi-country model

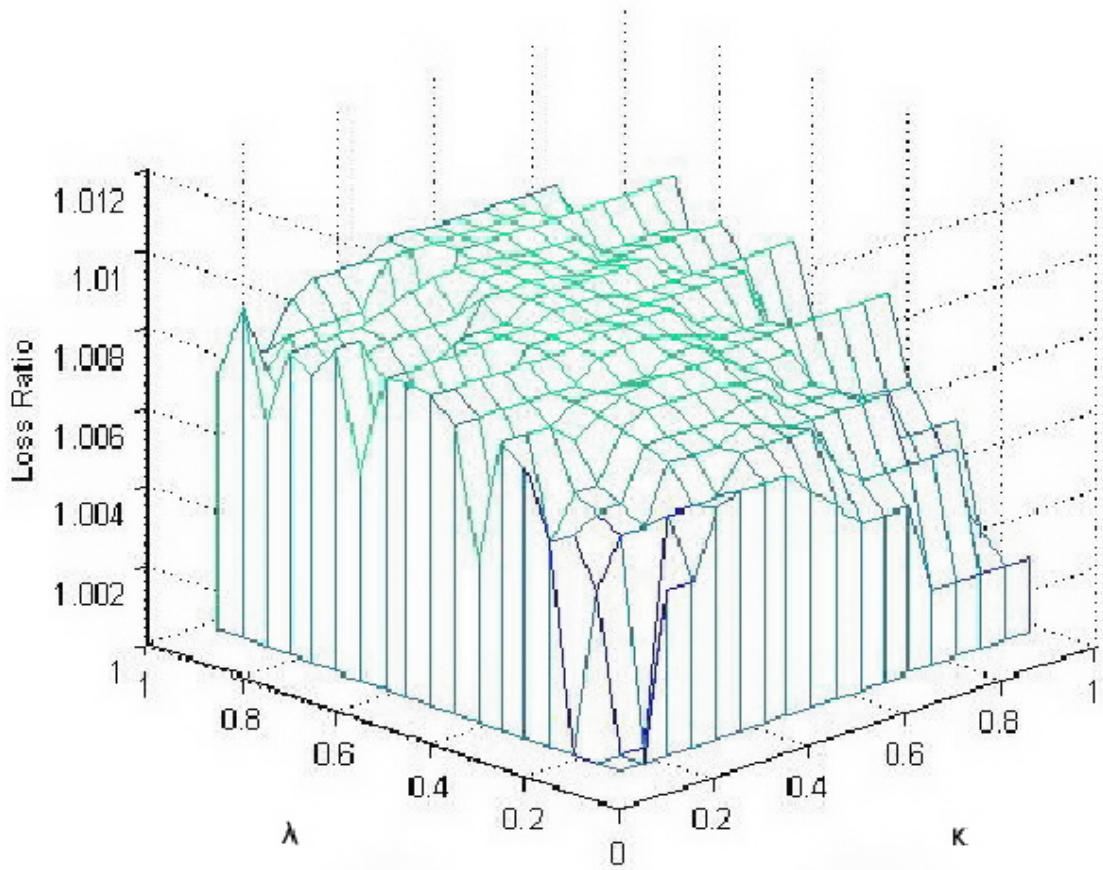

Table 2: Coefficients and variances of the target variables and loss value under alternative models

			Coefficient of						Variance of								
λ	κ	Model	π_t^U	$\widetilde{y}_t^U$	π_t	$\widetilde{y}_t$	π_t^*	$\widetilde{y}_t^*$	$\pi_t^U - \widehat{\pi}$	$\widetilde{y}_t^U$	$\pi_t - \widehat{\pi}$	$\widetilde{y}_t$	$\pi_t^* - \widehat{\pi}$	$\widetilde{y}_t^*$	$i_t - \hat{\imath}$	Loss	Loss Ratio
0.1	0.1	Union-wide	0.0168	0.0336					0.4335	0.3324					0.0647	46.85	
		Multi-country			0.0090	0.0210	0.0188	0.0076			0.4478	0.3184	0.4246	0.2701	0.0473	46.84	1.0001
	0.5	Union-wide	0.0034	0.0067					0.4460	0.3061					0.0082	47.62	
		Multi-country			0.0018	0.0042	0.0038	0.0015			0.4528	0.3048	0.4376	0.2681	0.0053	47.39	1.0048
	0.9	Union-wide	0.0019	0.0037					0.4492	0.3012					0.0034	47.79	
		Multi-country			0.0010	0.0023	0.0021	0.0008			0.4544	0.3021	0.4404	0.2678	0.0022	47.51	1.0059
0.5	0.1	Union-wide	0.0168	0.1680					0.4497	0.2626					0.0559	58.07	
		Multi-country			0.0090	0.1050	0.0188	0.0380			0.4582	0.2671	0.4278	0.2644	0.0432	57.75	1.0055
	0.5	Union-wide	0.0034	0.0336					0.4544	0.2715					0.0057	58.67	
		Multi-country			0.0018	0.0210	0.0038	0.0076			0.4598	0.2724	0.4385	0.2650	0.0045	58.21	1.0080
	0.9	Union-wide	0.0019	0.0187					0.4559	0.2726					0.0023	58.81	
		Multi-country			0.0010	0.0117	0.0021	0.0042			0.4604	0.2731	0.4411	0.2651	0.0018	58.31	1.0085
0.9	0.1	Union-wide	0.0168	0.3024					0.4590	0.2428					0.0849	68.33	
		Multi-country			0.0090	0.1890	0.0188	0.0684			0.4654	0.2499	0.4331	0.2607	0.0514	68.31	1.0003
	0.5	Union-wide	0.0034	0.0605					0.4613	0.2572					0.0094	69.06	
		Multi-country			0.0018	0.0378	0.0038	0.0137			0.4662	0.2594	0.4404	0.2624	0.0061	68.66	1.0059
	0.9	Union-wide	0.0019	0.0336					0.4623	0.2602					0.0036	69.25	
		Multi-country			0.0010	0.0210	0.0021	0.0076			0.4666	0.2613	0.4427	0.2628	0.0024	68.77	1.0070

Note: The coefficient of i_{t-1} is equal to 2.0118 under union-wide model and 2.0129 under multi-country model and that on i_{t-2} is -1.0101 under both models.