

Evaluating Currency Crisis: A Bayesian Markov Switching Approach

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Abstract

In this paper we examine the nature of currency crisis. In line with Jeanne (1997) and Jeanne (2000), we provide an empirical support of the view that both fundamentals and animal spirits play an important role in the genesis of currency crisis. We do so by employing an out-of-sample forecasting exercise to analyse the Mexican crisis in 1994. We also extend the empirical framework suggested by Jeanne and Masson (2000) to test for the hypothesis that currency crisis was driven by sunspots. To this end we contribute to the existing literature by comparing Markov regime switching model with time-varying transition probabilities with two alternative models. The first is a Markov regime switching model with constant transition probabilities. The second is a linear benchmark model. Empirical results show that Markov regime switching model with time varying transition probabilities outperforms both linear and nonlinear alternative models but it fails to predict the Mexican currency crisis in 1994. The implication of these results are that fundamentals first put an economy into a crisis zone but the timing of speculative attack is driven by a sunspot.

1 Introduction

The currency crises of the EMS in 1992-1993, of Mexico in 1994 and the Asian crises in 1997 have been accompanied by considerable controversy over their causes. There are two main theoretical models, that explain currency crises. The first generation of currency crises model was determined by monetary and fiscal policy that are inconsistent with maintaining the fixed currency peg. The failure of the first generation model to explain the EMS currency crises led to the second generation

model. More concretely, although expansionary monetary and fiscal policy may have been an issue in some countries such as Italy and Spain this was not the case in some others such as UK and France.

In the second generation model the central bank act as an optimizer where the decision to devalue or not seems to be motivated by the desire to avoid adverse macroeconomic consequences of maintaining the peg. The second generation model add two new elements to the first generation model. The first new element concerns with the notion of macroeconomic fundamentals which in the second generation models, include any variables that might affect the loss function of central bank. In addition, the second generation model emphasizes the role that market expectations can have on the monetary authority's decision to devalue or not. This leads to the second new element of the second generation model. Specifically, the model provides the theoretical framework of self-fulfilling speculation and multiple equilibria.

The logic of self-fulfilling crises is based on the idea that devaluation expectation increases the cost of retaining a peg and therefore the desire of the policy-maker to devalue. One way to defend a currency peg is by raising the ex-ante nominal interest rate, which affects economic growth negatively. Under such circumstances, the policy maker might prefers to devalue rather than to maintain high interest rates. Therefore, the decision to devalue or not is affected by market expectations regarding changes in monetary policy.

The disconnection of fundamentals from market expectation is the main property that differentiate the first generation model from the second generation model. Jeanne (1997 and 2000) in the so-called escape clause model provided the theoretical framework which reconciles both models. More concretely, the escape-clause model of currency crises views the fixed exchange rate regime as a conditional commitment.

Jeanne (2000) argues that "the main message of the escape-clause model is that currency crises should be analysed in the context of a conflict among contradicting policy objectives. In the limit any type of currency crises can be analysed in the escape-clause perspective." This can be shown by endogenizing monetary and fiscal policy in the first generation model. Although the first generation model shows that currency crises is the consequence of the monetary and fiscal policies followed by government, it has not address the question of why these policies were pursued. Jeanne (op.cit) shows that there is a level of interest rate that authorities in the first generation model could adopted and defend the fixed exchange rate. If raising interest rate were not costly then currency crises would not occur. Therefore, once the role of interest rate is introduced into the first generation model, the logic

of first generation model is the same as the logic of second generation model.¹

Jeanne (2000) emphasize that the escape-clause model provides a political compromise between fundamentalist and the proponent of the self-fulfilling view.² Jeanne (op. cit) also argues that “self-fulfilling and fundamentalist are not mutually exclusive. For a currency to be vulnerable to self-fulfilling speculation, the fundamentals must first put it in a state of fragility. The occurrence and the precise time of crises may be impossible to predict solely on the basis of fundamentals, but the latter play a crucial role.”

Empirical literature shows that expectation of devaluations are subject to abrupt shifts unrelated to fundamentals . The regime shifts on market expectations can be interpreted as jumps between multiple equilibria. More concretely, Jeanne and Masson (2000) show that strategic complementarities between market expectations, about the intended policy rule and the policy actually adopted, produces multiple equilibria.³ It is important to observe that the presence of multiple equilibria is due to speculators sharing a common knowledge of the information set.⁴ In Jeanne and Masson (op. cit.) and in Jeanne (1997), what coordinates the public’s expectations and leads the economy across the different equilibria is a sunspot (waves of optimism or pessimism).

Jovanovic (1989) shows that, if a sunspot is independent from fundamentals, then it is necessary to distinguish the dynamic of fundamentals process from the sunspot process. A Markov regime-switching (MRS) model provides a framework that satisfies the distinction between the two processes. In particular, the data generating process of a MRS model consists of two components: the first component gives rise to the autoregressive dynamic of fundamentals, and the second component describes the dynamics of an unobserved state variable which follows a Markov process. The second component represents the sunspot. According to

¹The third generation model used to explain Asian crises can also be reconciled with the second generation model once raising interest rate to defending the currency are taken into account. The cost of increasing the interest rate is that it further weakens the banking system.

²Political compromise regarding the view that currency crises is due to market failure and the view that currency crises are self-inflicted by governments pursuing monetary and fiscal policies inconsistent with the fixed exchange rate regime.

³Cooper and John (1988) show that spillover and strategic complementarities give rise to multiple equilibria, the dynamics of which can be approached by regime switching. Spillover refers to a situation where other’s strategies affect one’s payoff. Strategic complementarity refers to a situation where others’ strategy affect one’s optimal strategy.

⁴In Morris and Shin (1998), the absence of common knowledge, hence the presence of heterogeneous expectations, gives rise to a unique currency crisis equilibrium.

Jeanne and Masson (2000), what defines a sunspot in the context of MRS model is the assumption of a constant transition probability matrix.⁵ This implies that switches of the unobserved state variable from one equilibrium to the other is independent of fundamentals.

Jeanne and Mason (2000), Piard (1997) and Psaradakis et. al. (1997) shows that MRS models do a better job in describing the speculative attack on the French franc in 1993 than the simple linear models. The same methodology applied by Gonzalez-Garcia (1999) to the 1994 Mexican crises and Cerra and Saxena (1999) to the 1997 Indonesian currency crisis. However, empirical support on sunspot by these studies was based on an in-sample forecast comparison of MRS with linear model. Evidence of better in-sample fit of MRS than the in-sample fit of linear models led these studies to conclude that speculative activity jumps up and down driven by a sunspot. However, although nonlinear models outperform linear models in an in-sample forecasting exercise this was not the case in out-of sample forecasting exercise where linear models found to perform often better than nonlinear models (Clements and Smith 1999; Diebold and Nason 1990).

An out of sample forecast comparison of MRS model with the linear model is required in order to provide empirical support to currency crises model driven by sunspot. Furthermore, we argue that the definition of the sunspot given by Jeanne and Mason (2000) in the context of MRS model is rather limitative. We allow the switch in market expectations to be function of fundamentals. We do so by using MRS model with time-varying transition probabilities. A statistically significant effects of fundamentals on transition probabilities would lead to the conclusion that speculative attacks are driven not only by an external uncertainty but also by fundamentals. This is consistent with the argument of Jeanne (2000) that the first and second generation models are not mutually exclusive.

The main contribution of the papers is that it provides an empirical support of the view that first bad fundamentals put a currency in a state of fragility and then a sunspot trigger a speculative attack. We do so by employing an out-of sample forecasting exercise to analyse the Mexican crisis in 1994. An out-of-sample forecasting comparison of duration independent MRS⁶ model with MRS model with time-varying transition probabilities indicates whether market expectation are subject to

⁵Jeanne and Masson (op. cit) show that the case of simple selffulfilling currency crises can be viewed as a special case of sunspot. This occurs when the transition probability matrix in equation is equal to an identity matrix.

⁶Duration independent MRS model are MRS models with constant transition probability matrix.

abrupt shifts unrelated to fundamentals. Evidence that the time-varying transition probability models outperform model with fixed transition probabilities indicates that market expectations changes gradually up to certain point on the basis of fundamentals and jumps suddenly due to an external uncertainty independent of fundamentals. Furthermore, evidence that the later MRS model outperforms a linear benchmark model provides empirical support of the currency crisis model with multiple equilibria.

This paper proceeds by introducing the empirical methodology utilised in this paper. Section 3 presents the econometric methodology adopted to bring the model to the data. Section 4 explain data and empirical results from an application to Mexican crises in 1994.

2 Empirical methodology

The first part of this section describes briefly a debt crisis model used by Brasiotis et. al. (2004). The second part presents the model of multiple equilibria developed by Jeanne (1997) and extended by Jeanne and Masson (2000). We also explain, in line with Jeanne and Masson (op.cit), how currency crises models with multiple equilibria can be estimated using MRS model. We further explain how MRS with time-varying transition probabilities model provides an empirical framework that reconciles the first generation model of currency crises with the second generation model.

2.1 Debt Crises Model

We adopt a debt crisis model used by Brasiotis et. al. (2004) to evaluate the Mexican currency crises in 1994. Brasiotis (op. cit.) modified the economic fundamentals in Jeanne's (1997) escape clause model. Jeanne (op. cit.) shows that currency crises arises as a result of deterioration of fundamentals and of animal spirits. More concretely, policy makers have a benefit function which depends both on economic fundamentals and of expectation of devaluation. The decision to devalue or not depends on whether the benefit function takes on negatives values. This can be described as follows:

$$B_t = V_t - \gamma q_{t-1} \tag{1}$$

where V_t denotes a function of fundamentals and q_{t-1} is the probability of devaluation formed at period $t - 1$ that policy makers will devalue at period t . The net benefit of maintaining the peg is given by the welfare loss difference between a devaluation and that of maintaining the peg

$$B_t = L_t^d - L_t^f \tag{2}$$

where L_t^d and L_t^f denote the loss function of devaluation and of maintaining the peg respectively. Brasiotis et. al. (2004) analysed the Mexican currency crises assuming that the policy makers loss function is positively related to unemployment and debt growth

$$L_t = u_t^2 + \psi \Delta d_t + \delta C_t \quad (3)$$

where u_t is the unemployment rate, Δd_t is the growth of government debt proportional to nominal GDP, ψ is a preference parameter, C_t denotes the credibility cost that policy makers received when devalue and δ is a dummy variable that takes on the value 1 when policy makers devalue and zero when they maintain the peg. Brasiotis et. al. (2004) shows that⁷

$$\begin{aligned} \Delta d_t &= (g_t - \tau_t - i^* d_t) - (\Delta s_t - E_{t-1} \Delta s_t) d_t - (\Delta s_t - E_{t-1} \Delta s_t) s_t (f_t^* - b_t^*) \\ \Delta d_t &= \widehat{g}_t - (\Delta s_t - E_{t-1} \Delta s_t) d_t - (\Delta s_t - E_{t-1} \Delta s_t) z_t^* \end{aligned} \quad (4)$$

$$u_t = \bar{u} + \rho u_{t-1} - \beta (\Delta s_t - E_{t-1} \Delta s_t) \quad \beta > 0 \quad (5)$$

where s_t is the nominal exchange rate with respect the US dollar, g denotes government expenditures, τ is tax revenues; s is nominal exchange rate; f^* and b^* denote foreign exchange reserves and US\$-denominated debt respectively. Therefore, $\widehat{g} = (g_t - \tau_t - i^* d_t)$ is the primary deficit plus the debt service estimated at the world interest rate; $z_t^* = s_t (f_t^* - b_t^*)$ denotes the net foreign position of central bank in terms of domestic currency. Introducing the probability of devaluation implied by (1) we can write (4) and (5) as a function of probability of devaluation q_{t-1} .⁸ Then substituting the resulting equations into (2) we obtain (1) in terms of fundamentals and expectation of devaluation:

$$B_t = C_t - 2\beta(\bar{u} + \rho u_{t-1}) \Delta s_t - \psi(d_t + z_t^*) \Delta s_t + (\beta \Delta s_t)^2 - 2(\beta \Delta s_t)^2 q_{t-1} \quad (6)$$

Using (2) we can write

$$\begin{aligned} V_t &= C_t - 2\beta(\bar{u} + \rho u_{t-1}) \Delta s_t - \psi(d_t + z_t^*) \Delta s_t + (\beta \Delta s_t)^2 \\ \gamma &= 2(\beta \Delta s_t)^2 \end{aligned} \quad (7)$$

2.2 Multiple equilibria, Sunspots and MRS

The probability of devaluation at time t is given by the probability that the government is in a soft mood and the net benefit function at time

⁷Note that all variables are in logs.

⁸For more details see equations 8-11 in Brasiotis et. al (2004).

$t + 1$ is negative, i.e.:⁹

$$\begin{aligned} q_t &= \mu \Pr ob(B_{t+1} < 0) \\ q_t &= \mu \Pr ob(V_{t+1} - \gamma q_t < 0) \end{aligned} \tag{8}$$

Agents expect that the benefit of maintaining the peg next period is equal to the value of current fundamentals. Denoting fundamentals as ϕ_t and assuming rationality we can write $V_{t+1} = E_t V_{t+1} + \varepsilon_{t+1}$ or $V_{t+1} = \phi_t + \varepsilon_{t+1}$, where $\phi_t = E_t V_{t+1}$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. Therefore (8) can be written as

$$q_t = \mu \Pr ob(\varepsilon_{t+1} < \gamma q_t - \phi_t) \tag{9}$$

Since $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, (9) can be written as

$$q_t = \mu F(\gamma q_t - \phi_t) \tag{10}$$

where $F()$ denotes the cumulative normal distribution function of $f()$. Given that both sides of (10) are an increasing function of devaluation probability q_t , multiple equilibria can arise.

Jeanne (1997) shows that when the slope of the left hand-side of (10) is higher than 1 (i.e. $\mu\gamma f(0) > 1$) then there are three equilibria. This implies that the left-hand side of equation 10 cross the 45° line representing the right-hand side of (10) at three points. Alternatively, when $\mu\gamma f(0) < 1$ then the slope of left-hand-side cross the 45° line at one point indicating the existence of unique equilibrium. Jeanne (op.cit.) also shows that multiple equilibria occurs only when fundamentals enter a zone of vulnerability (i.e. $\phi_t \in [\underline{\phi}, \bar{\phi}]$). More concretely, if $\phi_t < \underline{\phi}$ then the probability of devaluation is determined uniquely and is close to 1. If $\phi_t > \bar{\phi}$ then the devaluation probability is uniquely defined and is close to 0.

Jeanne and Masson (2000) show that the number of equilibria can become arbitrary large by assuming that the net benefit function presented by (1) depend on devaluation expectation formed at current period¹⁰

$$B_t = V_t - \gamma q_t \tag{11}$$

This implies that (10) is equal to

$$q_t = \mu F(\gamma q_{t+1} - \phi_t) \tag{12}$$

⁹Policy makers can be in a soft mood with probability μ and in a tough mood with probability $1 - \mu$. When central bank is in a tough mood, it maintains the peg regardless the circumstances. Alternatively, when it is in a soft mood, it maintains the beg only when the net benefit function is positive.

¹⁰Each equilibrium characterised by the level of fundamentals that a speculative attack is imminent.

Jeanne and Masson (op.cit) went a step further and they argue that shifts across equilibria are driven by an extrinsic uncertainty-a sunspot. This implies that devaluation probability is the sum of devaluation probability next period weighted by the transition probability that the economy switch across different equilibria. Under such circumstances (10) can be written as

$$q_t = \mu\{S_t\}F(\gamma q_t - \phi_{t+1}) \quad (13)$$

where $\{S_t\}$ is the unobserved state variable that takes values in the finite set $\Omega = \{1, \dots, n\}$. $\{S_t\}$ follows a Markov process with transition probability $P = (p_{ij})'_{i,j \in \Omega}$ where $p_{ij} = P(S_{t+1}=j|S_t = i)$. Equation 13 consists of two components. The first components describes the dynamic of the unobserved state that drives market expectations. The second component describes the dynamic of observed fundamentals given market expectations.¹¹

According to (13) a sunspot arises as a result that the transition probability matrix P is time invariant. Under such circumstances market's expectation are independent of fundamentals and are driven by Keynesian "animal spirits". However, evidence that the transition probability matrix is time varying indicates that extrinsic uncertainty is not disconnected from fundamentals. More concretely, if the macroeconomic variables that affect the loss function of policy makers provides information about the transition of the unobserved state variables across different regimes then both fundamentals and "animal spirits" complement each other in the genesis of speculative attack. This is consistent with Jeanne (1997) and Jeanne (2000) where in the words of Jeanne (1997) "both fundamentals and animal spirits have a role to play in the ignition of the crisis: while the deterioration of fundamentals prepares the grounds for speculation, the occurrence and precise timing of the crisis is determine by animal spirits".

We employed an econometric methodology that provides the empirical framework that (13) can be brought to the data. More concretely, in line with Jeanne and Masson (2000), we use Markov regime switching model to distinguish the impact of animal spirits from the dynamic of fundamentals. We further extent (13) by allowing the transition probability of switching across different equilibria to be function of fundamentals.¹² A statistical significant effects of fundamentals on the transition probabilities would implies that speculative attack was not independent of fundamentals. Alternatively, a failure of MRS model with

¹¹Equation 12 is a special case of (13) when the transition probability matrix of $\{S_t\}$ across different states is an identity matrix.

¹²See the studies of Filardo (1994), Diebold et. al. (1994) and Filardo and Gordon (1998) on the use of time varying transition probability models.

time-varying transition probabilities to forecast a speculative attack ex ante indicates that animal spirits play an important role of determining the timing of crisis.

3 Econometric Methodology

This section describes the credibility indicator used in this study to measure the probability of devaluation presented in (13). We also described the econometric methodology which employed to test for interaction between sunspots and fundamentals.

3.1 Measure of Credibility Indicator

The obvious difficulty is how to measure the probability of devaluation. Early work focus on the interest rate differential as a proxy of devaluation expectation based on the assumption that interest rate differential follows a Markov process. In the context of target zones models the drift adjustment method also used as a measure of the probability of devaluation. Although the interest rate differential and drift-adjustment method can be sensibly applied in target zones models their reliance on uncovered interest rate parity (UIP) and on past fundamentals makes them inappropriate proxies of expectation for emerging markets.

We therefore, in line with Eichengreen et. al. (1996), employ the actual exchange market pressure as a credibility indicator.¹³ This measure is the weighted average of the changes of exchange rate Δs_t , of the interest rate differentials between domestic country's and the US short-term interest rate, $\Delta(i_t - i_t^{US})$ and of the changes of foreign exchange reserves, ΔR_t

$$EMP_t = w_1 \Delta s_t + w_2 \Delta(i_t - i_t^{US}) - w_3 \Delta R_t \quad (14)$$

where w_1 , w_2 , and w_3 are the weights of changes of exchange rate, of interest rate differential and of foreign exchange reserves. Each of the weights is calculated as the inverse of the series variance in the past. The implication of this measure is that when policy makers facing pressure on their currency they can either devalue or increase domestic interest rate and/or running down foreign exchange reserve.

¹³The same indicator also used in various studies such as Sacks et al. (1996) and Fratzcher (2001). Jeanne (1997), Jeanne and Masson (2000), Gomez-Puig and Montalvo (1997) and Cipollini et. all (2005) used interest rate differential as a proxy of market expectation to study the speculative attack on the French franc in 1993. Brasiotis et al. (2004) used interest rate differential (IRD) as a proxy of devaluation expectation to study the Mexican currency crises. Gomez-Puig and Montalvo (1997) argue that IRD might be a reasonable proxy of devaluation probability for EMS member countries because the risk premium component of UIP is small due to the fact that risk was diversified in the EMS.

3.2 MRS model

In what follows we model currency crises in Mexico by assuming that the market pressure indicator computed by (14) follows Markov process with time varying transition probability. Let denote $EMPI_t = y_t$ and assuming that y_t follows an autoregressive process then we can write

$$y_t = \alpha_0 + \alpha_1 S_t + \Psi(L)(y_{t-1} - \alpha_0 - \alpha_1 S_{t-1}) + \varepsilon_t \quad (15)$$

where $S_t \in \{0, 1\}$, $\Psi(L) = \psi_1 L + \psi_2 L^2 + \dots + \psi_p L^p$ is a lag polynomial in the lag operator L and $\varepsilon_t \sim iidN(0, \sigma_\varepsilon^2)$. The value $S_t = 0$ indicates a period of low pressure for devaluation. Alternatively, $S_t = 1$ denotes a period of high devaluation pressure. The evolution of unobservable state variable depends on $(k \times 1)$ vector of time series X_t . This implies that

$$\begin{aligned} P(S_t = 1) &= P(S_t^* \geq 0) \\ P(S_t = 0) &= P(S_t^* < 0) \end{aligned} \quad (16)$$

where S_t^* is a latent variable defined as

$$S_t^* = \beta_0 + \beta'_X X_t + \beta_s S_{t-1} + u_t \quad (17)$$

Thus we have

$$\begin{aligned} p_t &= P(S_t = 1 | S_{t-1} = 1) = P(u_t \geq -(\beta_0 + \beta'_X X_t + \beta_s)) \\ &= 1 - F(-(\beta_0 + \beta'_X X_t + \beta_s)) \\ q_t &= P(S_t = 0 | S_{t-1} = 0) = P(u_t < -(\beta_0 + \beta'_X X_t)) = F(-(\beta_0 + \beta'_X X_t)) \end{aligned} \quad (18)$$

Setting β_X equal to zero we obtain a fixed transition probability model.

We adopt a Bayesian Gibbs sampling approach to estimate both MRS models presented by equations (15)-(18) and linear models.¹⁴ Unlike the classical approach, in the Bayesian approach inference about S_t and the vector of unknown parameters $\theta = (p, q, \alpha_0, \alpha_1, \beta_0, \beta_X, \beta_s, \Psi(L))$ drawn from their joint distribution conditional on the data $p(S_t, \theta | Y_t)$ rather than the conditional distribution $p(S_t = j | \hat{\theta}, Y_t)$. This implies that estimates of the states takes into account the parameter uncertainty $\hat{\theta}$ inherited in the classical approach.¹⁵ This is so because the Bayesian

¹⁴We also estimate a model that allows the variance of ε_t to be regime dependent. This implies that $\varepsilon_t \sim iidN(0, \sigma_{\varepsilon, st}^2)$. It can be shown that $\sigma_{\varepsilon, st}^2 = \sigma_{\varepsilon, 0t}^2(1 + H_1 S_t)$ and $\sigma_{\varepsilon, 1t}^2 = \sigma_{\varepsilon, 0t}^2(1 + H_1)$ for $H_1 > 0$. For a detailed description of the estimation see Filardo and Gordon (1998) and Appendix.

¹⁵A drawback of estimating Markov switching models via maximum likelihood approach is that inference about the state variable based on the conditional probability $p(S_t = j | \hat{\theta}, Y_t)$. This is a two-step process. First estimates $\hat{\theta}$ of θ are obtained and then $p(S_t = j | \hat{\theta}, Y_t)$ is computed. Therefore, estimates of the states do not reflect parameter uncertainty inherent in the estimates of $\hat{\theta}$.

approach treats both the unobserved state S_t and the parameters of the model as random variables.

In this study we use only real exchange rate as a fundamental indicator that affects the unobserved state variable. We do so for two reasons. First, introducing more variables into the transition probability, it will not affect our main argument that fundamentals matter in the genesis of currency crises. More concretely, including extra variables into the transition probability beside that of real exchange rate it will only reinforce our conclusion that fundamentals affects market expectation before the economy enters into a crisis zone. The second reason is that extra variables in the transition probability will increase computational time.

4 Recursive an out-of-sample estimation

We use monthly average money rate to construct the interest rate differential component of EMP indicator and CPI to construct real exchange rate. Out of sample forecast are computed recursively from January 1991 to December 1996. The sample starts in April 1981, the first month of the IFS data-base obtained from datastream.

The forecast performance of linear and nonlinear models employed in this study are first evaluated based on the root mean square forecast error (RMSFE) criterion. We also used density forecast criterion, in line with the recent literature in monetary economics that policy makers have asymmetric loss functions. Dolado et. al. (2002a,b) and Kim et. al. (2005) show that central bank have asymmetric loss function. Under such circumstances, RMSFE is not optimal and policy makers need to focus on density rather just on point forecast; see Granger and Pesaran (2000a,b), Pesaran and Skouras (2002) and Clements (2005).

Diebold et. al. (1998) show how the density forecast criterion is optimal regardless the loss function of decision maker. The density forecast criterion, based on the probability integral transform of standardised forecast errors, test whether the model employed by the policy maker is the true model. Given that we used an estimator that takes into account parameter uncertainty the use of density forecast criterion will reflect the impact of any source of uncertainty. Diebold et. al. (1998) show that if a sequence of density forecast is correctly conditionally calibrated then the sequence of the probability integral transform of standardised forecast errors are *i.i.d* and $U(0, 1)$. Berkowitz (2001) suggests an alternatively goodness of fit test where instead of testing for uniformity of probability integral transform it might be more fruitful to test for normality of the inverse cumulative distribution function (*CDF*) of standardised forecast

errors.¹⁶

Although the out-of-sample period of this exercise goes beyond the crises in 1994, we emphasise more the period before the crises than the period after the crisis. This is so because evidence that the MRS model with time-varying transition probabilities outperforms both the linear and the constant transition probability model indicates that the currency crisis in Mexico was consistent with the view of the escape-clause model. More concretely, fundamentals put economy into the crises zone but the precise time of crisis is not possible to be predicted. However, if the time-varying model is outperformed by a linear AR(1) benchmark model, this implies that the Mexican currency crisis was consistent with the view of first generation model.

4.1 Empirical results

Table 1 presents the posterior means and standard deviations of four MRS models.¹⁷ Results presented in Tabled 1 are obtained based on full-sample estimation. We estimate MRS models both allowing the variance to switch and to remain constant across regimes. We select a model with two autoregressive lags based on an in-sample correlation with the outturn. Exception to this is the time-invariant transition probability model with the variance allowed to switch across regimes where only one autoregressive lag is included.

It is worth to note two points. First the coefficient of high market pressure α_1 is negative instead of positive. This is so because α_1 is expected to have positive coefficient the period before the crisis in 1994. After devaluation takes place market expectation for devaluation will fall. More concretely, the coefficient of high market pressure is expected to have positive value before the crisis and negative after the crisis. Alternative, the coefficient of low market pressure α_0 is expected to be positive and high but with negative slope. This implies that the credi-

¹⁶The density forecast is constructed as follows. We assume that disturbances are i.i.d. Gaussian. Then if \hat{y}_{t+1} is the one-step-ahead forecast of y_{t+1} made at time t and $\hat{\sigma}_{t+1}$ is the standard deviation of \hat{y}_{t+1} then the Gaussian density forecast is $F(y_{t+1}) = N(\hat{y}_{t+1}, \hat{\sigma}_{t+1})$. Then the probability integral transform values are given by $\{z_{t+1}\} = \{\Phi(\frac{y_{t+1} - \hat{y}_{t+1}}{\hat{\sigma}_{t+1}})\}$ where Φ is the Normal *CDF*. $\{z_{t+1}^*\} = \{(\frac{y_{t+1} - \hat{y}_{t+1}}{\hat{\sigma}_{t+1}})\}$ are the standardised forecast errors that are distributed $N(0, 1)$ under the null. Here to test for normality we employ the Dornik-Hansen (1994) test. To test for independence of z_{t+1}^* we use the Ljung-Box for autocorrelation see Harvey et. al. (1989) p. 259- we consider up to the third moment.

¹⁷The estimates presented in this section are based on 4000 draws of the Gibbs sampler. We disregard the first 1000 to ensure the convergence of the Gibbs sampler. Regarding the issue of convergence of Gibbs-sampling see McCullon and Rocci (1994) and Gelman and Rubin (1992). Visual inspection of draws suggest that convergence achieved after 400 of iterations.

bility of fixed exchange rate regime is high a long before the crises but it declines gradually. Figure 1 indicates that credibility of fixed exchange rate regime declines before the crisis while expectation of devaluation built up gradually. Figures 2 indicates that after the 14% devaluation in December 1994, the coefficient of high market pressure α_1 falls sharply taking on negatives values. Furthermore, the high credibility coefficient α_0 become stable just before crisis but does not returning to the high before devaluation values. Second, Table 1 indicates that models with switching variance have insignificant autoregressive coefficient. This implies that introducing exogenous shock into the model the impact of model uncertainty becomes insignificant.¹⁸

In the out-of-sample forecasting exercise we select the models with constant variance. This is so because models with constant variance have higher in-sample correlation with the outturn than the model with switching variance. In addition to low in-sample correlation with the outturn, models with switching variance have some coefficients that are implausible. More concretely, the model with two autoregressive lag has no significant variance plus the model with one autoregressive lag has a negative and no-significant low market pressure coefficient α_0 .

Results from an out of sample experiments shows that the time varying transition probability model outperforms both the fixed transition probability model and the linear AR(1) model. The fixed transition probability model performs poorly because the coefficient of low market pressure got implausible and not significant coefficient. Figure 3 presents the out-of-sample t-statistics of α_0 and α_1 . Figure 4 presents the RMSFE of both the MRS with time varying transition probability model and of the benchmark AR(1) model. More concretely, the RMSFE was computed recursively for the period before the devaluation in December 1994.¹⁹ We also test whether these differences in RMSFE are statistically significant at 95% using the corrected-Diebold-Mariano test of Harvey et. al. (1997).²⁰ The DM statistic rejects the null that

¹⁸This is consistent with Sims (2001) and Sims and Zhang (2002) who study parameters shifts of the US economy and find evidence of changes in the variance rather than changes in the model structure. In contrast to Sims (2001), Cogley and Sargent (2002) show through a Bayesian VAR with time varying coefficient that there are evidence of drifts both on the mean and variance.

¹⁹The RMSFE of AR(1) is in general lower than the RMSFE of the MRS TVP model for the period after the devaluation in December 1994.

²⁰The large-sample statistic that Diebold and Mariano (1995) proposed to test for the null that two forecast are equal is given by:

$$\frac{\bar{d}}{\sqrt{\widehat{V}(\bar{d})}}$$

the two forecast are equal for the period before the crisis in 1994 and accept the null for the period after the crisis.²¹ The DM accepts the null of equal forecast for a period which includes the observation of the large devaluation in December 1994. This implies that both models perform badly in forecasting extreme events. The DM test also fails to reject the null for the period after devaluation where on average the corresponding p-values is 0.545.

Forecast encompassing test rejects the null hypothesis that MRS model forecast encompass the AR(1) model and vice-versa. This implies that an optimal forecast combination will result in a forecast with lower RMSFE than the best performing model.²² Furthermore, a model that includes lagged dependent variables in the transition probabilities, it will be an interesting alternative beside the optimally combined based on a linear regression model. This is so because an improved forecasting performance of the model with lagged dependent variables in the transition probabilities, it will provide further support to our conclusion about the nature of currency crisis. Tests concerning the density forecast criterion rejects the null hypothesis that standardised forecast errors both for MRS model with time-varying transition probabilities and for the AR(1) model are *iid* and $N(0, 1)$.²³ Thus an optimal forecast depend on the loss function of policy makers.

where $\bar{d} = \frac{1}{T} \sum_{i=1}^T \hat{d}_i$, $\hat{d}_i = \hat{\varepsilon}_{1i}^2 - \hat{\varepsilon}_{2i}^2$, $\hat{\varepsilon}_{1i}^2$ are the respective forecast errors and $\hat{V}(\bar{d})$ is the estimated variance of \bar{d} . Harvey et. al. (1997) propose a small sample correction which result in the revised statistic of DM test:

$$S_{DM} = T^{-1}[T + 1 - 2h + T^{-1}h(h - 1)]\hat{V}(\hat{d})^{-1/2}\bar{d}$$

where now $\hat{V}(\hat{d}) = T^{-1}(\hat{\gamma}_0 + \sum_{i=1}^{h-1} \hat{\gamma}_i)$, $\hat{\gamma}_i$ $i = 1, 2, ..h - 1$ are the estimated autocovariances of the series of the square prediction errors differences $\{\hat{d}_i\}_{i=1}^T$. Critical values are taken from a t -distribution with $T - 1$ degrees of freedom.

²¹The p-values of the DM test for the period up to November 1994 is 0.036. It is important to note that the average p-values for a recursive estimation of DM test statistic is 0.013.

²²Forecast combination in line with Newbold and Granger (1973) and Granger and Ramanathan (1984) is optimal under the assumption of quadratic loss function. Optimal forecast combination in terms of density forecast has been developed by Hall and Mitchell (2004), Mitchell and Hall (2005) and Sarno and Valente (2004). Optimal density combination extend the concept of optimal forecast for more general than the quadratic loss function.

²³More concretely, although the Dornik-Hansen accept the null of normality, test concerning the assumptions of mean equal to zero and of variance equal to one fails to accept the null. Test for autocorrelation of standardised forecast errors up to the third moment accept the null of uncorrelated errors.

Results from RMSFE and DM test indicate that a model with multiple equilibria is more suitable to explain the Mexican currency crises than the first generation model.²⁴ Although a model with multiple equilibria describes better the Mexican currency crisis, switches between equilibria is not independent of fundamentals. This is so because the coefficient of real exchange rate on the transition probability is significant in all out-of-sample observations.²⁵ Therefore, market expectation are driven not only by a sunspot but also by fundamentals. However, it is worth to emphasise that the RMSFE starts increasing only two months before the crisis and jumps to implausible values at exactly the time of crisis. Evidence of this study are consistent with the argument of Jeanne (1997), Jeanne(2000) and Brasiotis et.al. (2004) that fundamentals drive market expectation up to a time that economy enters into a crisis zone and then speculative attack is driven by a sunspot.

5 Conclusion

The aim of this paper to test whether currency crisis can be explained by the escape-clause model suggested by Jeanne (1997). The key element of the escape-clause element is that both fundamentals and external uncertainty play a significant role in the genesis of currency crisis. To bring the model to the data we extend the Markov regime switching model suggested by Jeanne and Masson (2000) by allowing transition probabilities to be time-varying. We employ an out-of-sample forecasting exercise to compare MRS model with time-varying transition probability both with an alternative MRS model where transition probabilities are constant and with a linear benchmark model. Empirical results of this studies from the Mexican crisis in 1994 show that the MRS model with time-varying transition probability outperforms both linear and non-linear alternatives. This implies that fundamentals and animal spirits are not mutually exclusive in explaining currency crisis but they complement each other.

²⁴We also focus on the bias component of RMSFE, and considered whether forecasts are systematically over or underpredict. More concretely, we test for bias by testing whether the mean of the forecast errors is significantly different from zero using a New-West estimator of standard error. We also test for rationality based on a linear regression of forecast error on an intercept and the forecast. Both tests show that the mean of forecast error is not statistically different from zero and are not correlated with the information set used to forecast the outturn. Thus, forecast were both unbiased and efficient.

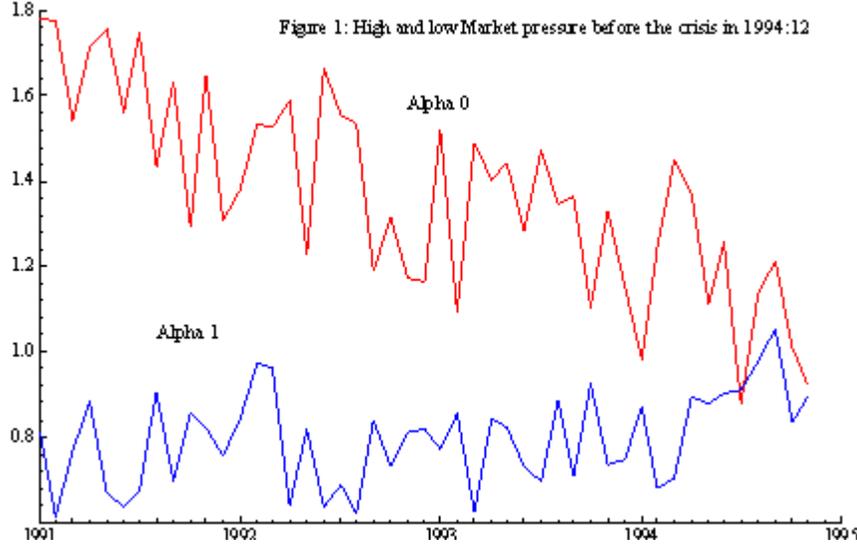
²⁵see Figure 5.

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6 Appendix 1: Results

Table1
Markov Regime Switching Models.

| Parameters | MRS TVP | MRS TVP SV | MRS FTP | MRS FTP SV |
|------------|-------------------------|------------------------|-----------------------|-----------------------|
| α_0 | 2.1666 [0.35625] | 2.4826 [27.605] | 1.7729 [0.21925] | -2.4495 [22.445] |
| α_1 | -0.14424 [0.39332] | -0.33313 [0.40644] | 6.2503 [0.39212] | 6.2964 [0.26692] |
| ψ_1 | 0.28647 [0.092249] | 0.32357 [0.62688] | 0.40277 [0.094221] | 0.94963 [0.052972] |
| ψ_2 | 0.43506 [0.091991] | 0.44953 [0.57814] | 0.26386 [0.090229] | |
| β_0 | -1.718 [0.11275] | -1.7548 [0.57814] | -1.8402 [] | -1.2721 [0.048128] |
| β_x | 0.534762 [0.0018626] | 0.50653 [0.0029316] | | |
| β_s | 2.9938 [0.13116] | 3.1710 [0.11862] | 2.6957 [] | 3.1526 [0.045436] |
| σ_0 | 2.2552 [0.23537] | 0.13982 [0.13796] | 0.88536 [0.084003] | 0.21510 [0.081282] |
| σ_1 | | 107.86 [389.48] | | 2.6875 [0.60327] |

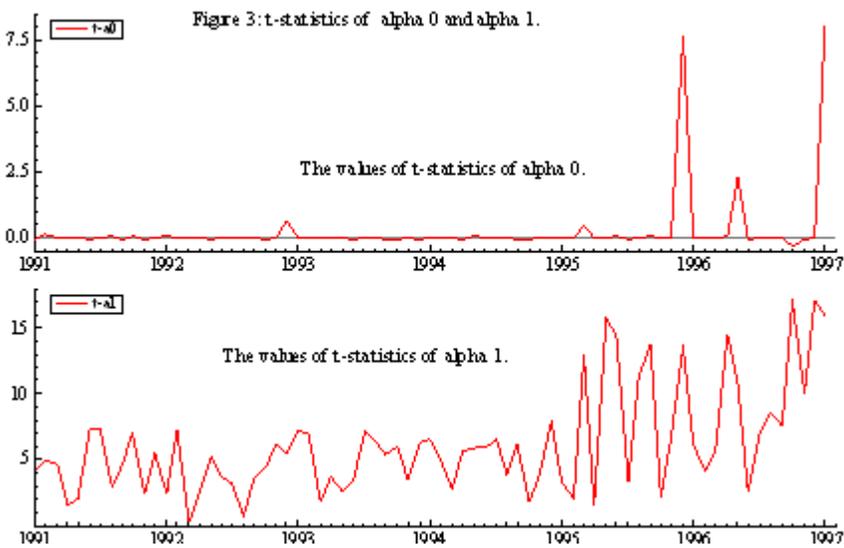
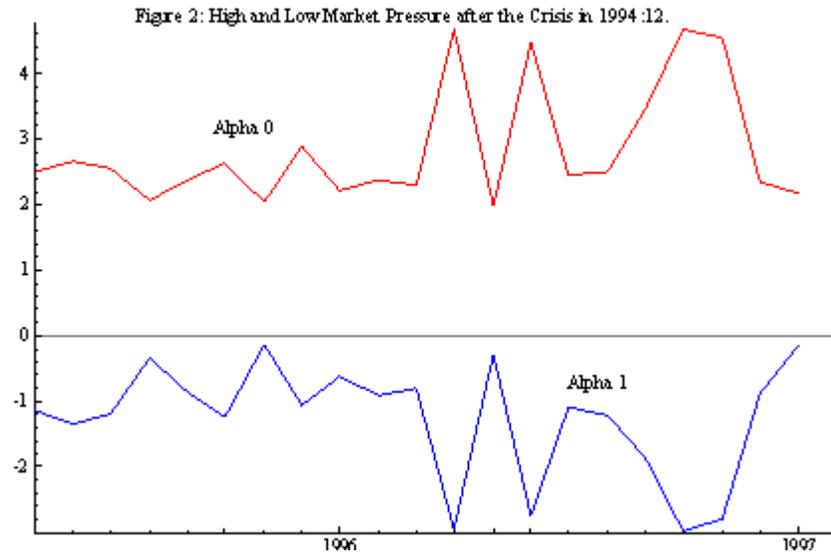
Notes:1) MRS TVP denotes the MRS model with time-varying transition probabilities.

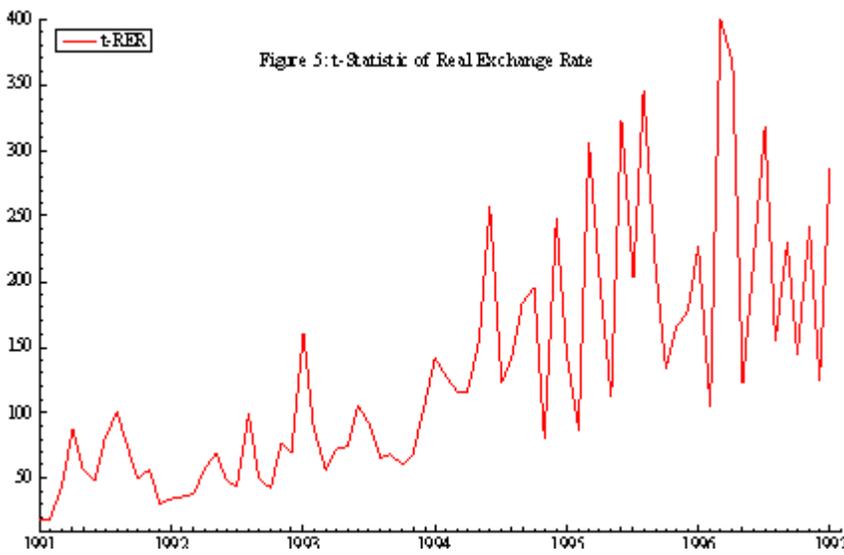
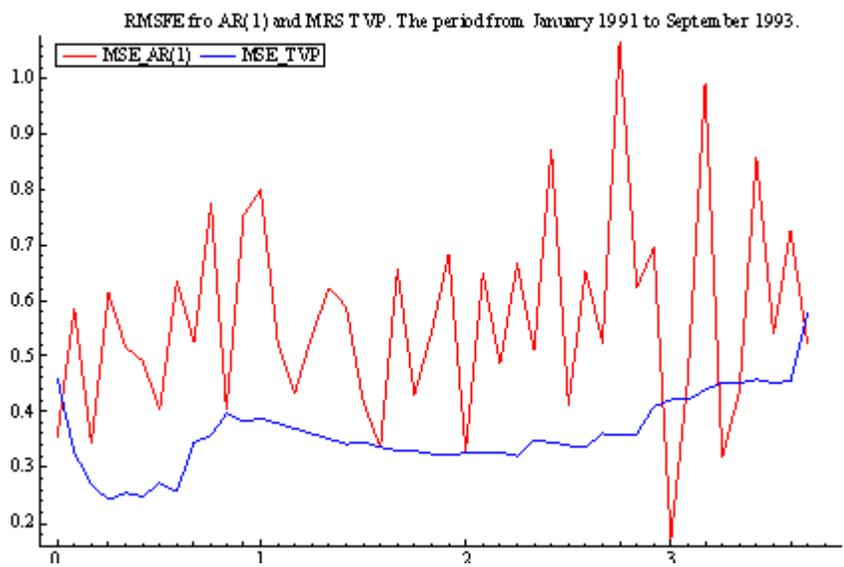
2)MRS TVP SV is the MRS model with time-varying transition probabilities and state dependent variance

3)MRS FTP is the MRS model with constant transition probabilities .

4)MRS FTP SV is the MRS model with constant transition probabilities and state dependent variance

5) P-values are reported in squared brackets





7 Estimation of MRS TVP

7.1 Step 1: Generate ψ

Define $\tilde{y}_t = y_t - \alpha_0 - \alpha_1 S_t$. Given initial values for σ_y^2 , S_t , α_0 and α_1 (15) can be written as

$$\tilde{y}_t = \psi_1 \tilde{y}_{t-1} + \dots + \psi_1 \tilde{y}_{t-r} + \varepsilon_t \quad t = 1 + r \dots T \quad (19)$$

Define \tilde{Y} the $(T \times k)$ matrix of regressors and \tilde{y} the vector of dependent variable. Using non-informative priors the posterior distribution of ψ is given by the normal distribution

$$\psi \sim N(\bar{\psi}, \bar{\Sigma}_\psi) \quad (20)$$

where

$$\bar{\Sigma}_\psi = [\Sigma_{0,\psi}^{-1} + \sigma_y^{-2}(\tilde{Y}'\tilde{Y})]^{-1} = \sigma_y^2(\tilde{Y}'\tilde{Y})^{-1} \quad (21)$$

$$\bar{\psi} = \bar{\Sigma}_\psi[\Sigma_{0,\psi}^{-1}\psi_0 + \sigma_y^{-2}(\tilde{Y}'\tilde{y})] = (\tilde{Y}'\tilde{Y})^{-1}(\tilde{Y}'\tilde{y}) \quad (22)$$

where $\Sigma_{0,\psi}$ and ψ_0 are the non-informative priors for ψ .

7.2 Step 2: Generate α_0, α_1

Define $\tilde{\tilde{y}}_t = y_t - \psi_1 y_{t-1} + \dots + \psi_1 y_{t-r}$, $\tilde{\tilde{S}}_t = S_t - \psi_1 S_{t-1} - \dots - \psi_r S_{t-r}$ and $d = 1 - \psi_1 - \dots - \psi_r$. Using values of ψ generated in step 1 we can write (15) as

$$\tilde{\tilde{y}}_t = \alpha_0 d + \alpha_1 \tilde{\tilde{S}}_t + \varepsilon_t \quad (23)$$

Define $\tilde{\tilde{y}}$ and $\tilde{\tilde{Y}}$ as the vector of dependent variable and the matrix of regressors. Given non-informative priors, initial values for σ_y^2 , S_t and generated values of ψ from step1, the posterior distribution of $\alpha = (\alpha_0, \alpha_1)$ is a truncate normal

$$\alpha \sim I_{\alpha_1 > 0} N(\bar{\alpha}, \bar{\Sigma}_\alpha) \quad (24)$$

where

$$\bar{\Sigma}_\alpha = [\Sigma_{0,\alpha}^{-1} + \sigma_y^{-2}(\tilde{\tilde{Y}}'\tilde{\tilde{Y}})]^{-1} = \sigma_y^2(\tilde{\tilde{Y}}'\tilde{\tilde{Y}})^{-1} \quad (25)$$

$$\bar{\alpha} = \bar{\Sigma}_\alpha[\Sigma_{0,\alpha}^{-1}\alpha_0 + \sigma_y^{-2}(\tilde{\tilde{Y}}'\tilde{\tilde{y}})] = (\tilde{\tilde{Y}}'\tilde{\tilde{Y}})^{-1}(\tilde{\tilde{Y}}'\tilde{\tilde{y}}) \quad (26)$$

where $\Sigma_{0,\alpha}$ and α_0 are the non-informative priors for α .

7.3 Step 3: Generate σ_0^2, σ_1^2 and σ_y^2

Define $\sigma_{st}^2 = \sigma_{0t}^2(1 + H_1 S_t)$ where $H_1 > 0$. Given initial values of S_t and values of α_s and ψ generated above, we generate σ_{0t}^2 and σ_{1t}^2 in two steps. First, given H_1 we generate σ_{0t}^2 from an inverse Gama distribution. Second, given the generated σ_{0t}^2 we generate H_1 from an inverse gama distribution. We divide (19) by $\sqrt{1 + H_1 S_t}$ to obtain

$$y_t^* = \bar{\psi}_1 y_{t-1}^* + \dots + \bar{\psi}_r y_{t-r}^* + \varepsilon_t^* \quad (27)$$

where

$$y_t^* = \frac{\tilde{y}_t}{\sqrt{1 + H_1 S_t}}$$

$$\varepsilon_t^* = \frac{\varepsilon_t}{\sqrt{1 + H_1 S_t}}$$

Given no informative prior for σ_0^2 the posterior distribution for σ_0^2 is obtained as follows

$$\sigma_0^2 | \alpha, \psi, H_1 \sim IG\left(\frac{\nu_1}{2}, \frac{\delta_1}{2}\right) \quad (28)$$

²⁶where

$$\nu_1 = \nu_0 + T = T$$

$$\delta_1 = \delta_0 + \sum_{t=1}^T (y_t^* - \bar{\psi}_1 y_{t-1}^* - \dots - \bar{\psi}_r y_{t-r}^*)^2$$

$\nu_0 = \delta_0 = 0$ are the non-informative priors of an inverse gama distribution and T is the sample size. To generate H_1 conditional on σ_0^2 we divide (19) by σ_0

$$y_t^{**} = \bar{\psi}_1 y_{t-1}^{**} + \dots + \bar{\psi}_r y_{t-r}^{**} + \varepsilon_t^{**} \quad (29)$$

where

$$y_t^{**} = \frac{\tilde{y}_t}{\sigma_0}$$

$$\varepsilon_t^{**} = \frac{\varepsilon_t}{\sigma_0}$$

The likelihood function of H_1 depends only on observations of regime $S_t = 1$. Keeping this in mind and using no-informative priors the posterior distribution for H_1 is given as

$$H_1 | \alpha, \psi, H_1 \sim IG\left(\frac{\nu_4}{2}, \frac{\delta_4}{2}\right) \quad (30)$$

²⁶IG denotes the inverse gama distribution.

where

$$\begin{aligned}\nu_4 &= \nu_3 + T_1 = T_1 \\ \delta_4 &= \delta_3 + \sum_{t=1}^{N_1} (y_t^{**} - \bar{\psi}_1 y_{t-1}^{**} - \dots - \bar{\psi}_r y_{t-r}^{**})^2\end{aligned}$$

where $\nu_3 = \delta_3 = 0$ are the non-informative priors for H_1 ; $N_1 = \{t : S_t = 1\}$; T_1 is the sample of regime $S_t = 1$. Once H_1 is generated, σ_1^2 is calculated by $\sigma_0^2(1 + H_1)$ and $\sigma_{yt}^2 = \sigma_{st}^2 = \sigma_0^2(1 + H_1 S_t)$.

7.4 Step 4: Generate S_t

In line with Filardo and Gordon (1998) we generate S_t using the single-move Gibbs-sampling procedure suggested by Albert and Chib (1993). More concretely, Filardo and Gordon (op.cit) modified the formula of Albert and Chib (1993) to incorporate time-varying transition probabilities. Define $\widehat{Y}_T = \{y_1, y_2, \dots, y_T\}$, $\widehat{S}_T = \{S_0, S_1, \dots, S_T\}$, $\widehat{S}_{\neq t} = \{S_0, \dots, S_{t-1}, S_{t+1}, \dots, S_T\}$ and $\widehat{X} = \{X_1, X_2, \dots, X_T\}$ where X_t is the vector of information variables in (17). In the time-varying transition probability model, the full conditional distribution is

$$P(S_t | \widehat{Y}_T, \widehat{S}_{\neq t}, \widehat{X}) \propto P(S_t | S_{t-1}) P(S_{t+1} | S_t) \prod f(y_k | \widehat{Y}_{k-1}, \widehat{S}_k) \quad (31)$$

Using (31), $P(S_t = j | \widehat{Y}_T, \widehat{S}_{\neq t}, \widehat{X})$ with $j = 0, 1$ can be calculated by

$$P(S_t = j | \widehat{Y}_T, \widehat{S}_{\neq t}, \widehat{X}) = \frac{P(S_t = j | \widehat{Y}_T, \widehat{S}_{\neq t}, \widehat{X})}{\sum_{j=0}^1 P(S_t = j | \widehat{Y}_T, \widehat{S}_{\neq t}, \widehat{X})} \quad (32)$$

Once $P(S_t = 1 | \widehat{Y}_T, \widehat{S}_{\neq t}, \widehat{X})$ is calculated we compute S_t by generate random numbers from uniform distribution between 0 and 1. If the generated number is less than $P(S_t = 1 | \widehat{Y}_T, \widehat{S}_{\neq t}, \widehat{X})$, we set $S_t = 1$. Alternatively, we set $S_t = 0$.

7.5 Step 5: Generate S_t^*, β, p_t, q_t for $t = 1, \dots, T$

Given S_t generated in step 4 equations (16) and (17) can be estimated by a probit model. However, S_t^* is not observed and needs to be calculated by the data augmentation method suggested by Tarner and Wong (1987). The values of S_t^* are drawn from a truncated standard normal distribution. Given S_t^* equation 17 becomes a simple linear regression model where the vector of parameters β can be generated similarly to ψ in step 1. The transition probabilities p_t and q_t are obtained from the CDF of (17) for $t = \{1, \dots, T\}$.