

# Estimating the Natural Rates in the New Keynesian Framework \*

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December 22, 2006

## Abstract

The time-varying natural rate of interest and output and the implied medium-term inflation target for the US economy are estimated over the period 1983-2005. The estimation is conducted within the New-Keynesian framework using Bayesian and Kalman-filter estimation techniques. While there is considerable variation in the natural rate of interest, the inflation target estimate is close to 2% over the last decade. Employing the model-consistent estimate of the output gap, we get a small weight on the backward-looking component of the New-Keynesian Phillips curve – similar to studies which use labor share of income as a proxy for marginal costs (e.g., Galí et al., 2001, 2003). Model-consistency may therefore be a requirement if the output gap estimate shall represent marginal costs efficiently.

**JEL-codes:** C51, E32, E37, E52.

**Keywords:** Natural rate of interest, natural rate of output, New-Keynesian model, inflation target.

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\*We are grateful to Ida Wolden Bache, Leif Brubakk, Scott Schuh and seminar participants at the 2006 Dynare Conference in Paris for comments to an earlier draft of the paper. We also thank John Williams for providing information from the updated estimation of the model in Laubach and Williams (2003). The authors thank the *Norwegian Financial Market Fund* under the Norwegian Research Council for financial support. Views expressed are those of the authors and do not necessarily reflect the views of Norges Bank.

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# 1 Introduction

The macroeconomic theory as described in Clarida et al. (1999) and Woodford (2003), denoted as New Keynesian theory, has become the leading framework for the analysis of monetary and stabilization policies. It honors the proposition that monetary policy only affects nominal variables in the long run and that the steady-state inflation rate can be governed by monetary policy. The theory also assumes that the central bank implements its policy through the setting of the short-term interest rate. Monetary policy influences decisions about real magnitudes due to prices not being fully free to adjust to shocks (price rigidities). It is commonly assumed that the overriding objective of monetary policy is to alleviate the effects of these rigidities while keeping inflation expectations close to a target rate of inflation. An important point of reference for the policymaker is therefore how the economy would have developed had prices been fully flexible (no price rigidities). *The natural rates of interest rates and output* refer to the rate of interest and the output level in such an equilibrium (see Woodford, 2003). To appropriately alleviate the effects of price rigidities on the economy, the strategy of monetary policy is often formulated in terms of deviations from the natural rates. The well-known Taylor rule (Taylor, 1993) provides an illustration. Under the Taylor rule, the central bank raises the interest rate *relative* to the natural rate of interest if either inflation deviates from the inflation target and/or output deviates from the natural rate of output. In steady state, where there is no need for monetary policy in responding to shocks that brings the economy away from the flexible price equilibrium, the interest rate equals the natural rate. For these reasons, the natural rates are important indicators for the setting of the policy instrument and the characterization of a neutral monetary policy stance.

The natural rates and the inflation target are all non-observable variables that need to be estimated. In this paper, we provide estimates of the natural rates of output and interest as well as the a time-varying inflation target for the US economy using Bayesian and the Kalman filtering techniques. Using observations on GDP, CPI inflation and a short-term interest rate, we derive plausible time-varying estimates of the natural rates and a medium-term inflation target. Previous studies on the topic include the seminal paper by Laubach and Williams (2003) who use the Kalman filter to estimate the natural rates of interest and the output gap.<sup>1</sup> We extend their analysis by using an approach that is consistent with the New-Keynesian theory framework. Importantly, our framework allows expectations to be rational and forward-looking and equations to be structural in the sense of satisfying the first-order conditions of optimal consumption and pricing

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<sup>1</sup>The idea builds on the papers by Watson et al. (1997) and Gordon (1998), among others, that estimate the natural rate of unemployment (NAIRU) using the Kalman filter.

problems. Moreover, the natural rates are derived in a model-consistent manner. Our approach is similar to that of Ireland (2006) who also estimate the time-varying inflation target for the US economy using a New Keynesian model but assumes a constant natural rate of interest. We nevertheless reach similar conclusions as him regarding the inflation target.

We employ a hybrid New-Keynesian Phillips curve in the model (see, e.g., Galí et al., 2001, 2003) with both backward-looking and forward-looking components of inflation. The model-consistent estimate of deviations of output from the natural rate – the output gap – is used as a proxy for marginal costs and a driver of inflation. An interesting finding is that inflation is primarily a forward-looking process – a conclusions commonly found in studies who use labor’s share of income as a proxy for marginal costs. We find it interesting that once the output gap has been estimated in a model-consistent manner, it share some of the same properties as labor’s share of income in representing marginal costs.

Our approach share some similarities with Edge et al. (2005) and Juillard et al. (2005) who estimate the natural rates in a larger DSGE model. Note however that our approach does not require detrending of the data prior to analysis (using for instance the HP-filter) or making output stationary by deflating by a trending variable (for instance by assuming total factor productivity follows a trend stationary process) as has been common practice in most conventional DSGE analysis including those cited above.<sup>2</sup> Instead, we allow for persistent AR(1) and temporary shocks to the natural growth rates of output.

Section 2 presents the New Keynesian framework. Section 3 presents the estimation framework and results. Section 4 provides some concluding remarks.

## 2 The New Keynesian Framework

The New Keynesian framework<sup>3</sup> assumes that firms operate in monopolistic competitive markets and production is constrained by aggregate demand. Prices are assumed to be sticky and consequently do not move instantaneously to movements in marginal costs. Due to the price stickiness, the central bank affects aggregate demand through its influence on real interest rates. By lowering real interest rates, the central bank induces higher aggregate demand, marginal costs and prices than would otherwise materialize. As noted above, the natural rate of interest rate can be regarded as the neutral stance of monetary

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<sup>2</sup>A recent exception is Juillard et al. (2006). They allow for a more general stochastic process where there could be both temporary changes in the growth rate of total factor productivity as well as autocorrelated deviations from steady state.

<sup>3</sup>See Goodfriend and King (1997), Rotemberg and Woodford (1997), Rotemberg and Woodford (1999), McCallum and Nelson (1999) and Clarida et al. (1999).

policy - the real interest rate that produces zero output gap and stable inflation.

In estimating the natural rates, we build on the economic structure provided by the New Keynesian framework. The basic model is extended with external habit formation in consumption (Fuhrer, 2000) and a hybrid New-Keynesian Phillips curve that allows for both forward-looking and backward-looking elements. This set up is rationalized by the Calvo (1983) framework with some of the firms setting prices in accordance to an indexation scheme (Christiano et al., 2005) or in accordance with some rule-of-thumb (Galí and Gertler, 1999).

The approach remains nevertheless conservative regarding the extent of the economic structure imposed in estimation. In particular, we refrain from modeling the labor market. Neither do we use information about the stock of money when estimating the model. While the main reasons for doing this is simplicity and convenience, we are not imposing possible controversial structure regarding production structure and the structure of the sub-markets. By leaving labor market decisions as essentially exogenous processes, the paper sacrifices rigor at the gain of not being tied up to a specific description of the labor market, which may bias the results if proven wrong. In this regard, the paper draws on both the dynamic stochastic general equilibrium modeling approach and that of structural time-series estimation, see e.g., Harvey (1989).

## 2.1 Aggregate demand

We assume that the economy consists of a representative household that lives forever and maximizes expected utility given by

$$U = E_t \sum_{i=0}^{\infty} \left( \frac{1}{1+\delta} \right)^i \left[ \frac{1}{(1-\sigma)} \left( \frac{C_{t+i} V_{t+i}}{H_{t+i}} \right)^{(1-\sigma)} \right],$$

subject to the intertemporal budget constraint given by

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + I_{t-1} \frac{B_{t-1}}{P_t} - \frac{T_t}{P_t} + \Pi_t.$$

$\delta$  is the discount rate,  $\sigma$  is the intertemporal elasticity of substitution and  $C$  is an CES index of consumption goods.  $V$  is a consumption preference shock. The consumer is also assumed to have preferences over money and leisure, but for simplicity, these are not explicitly modeled.

The consumer can either hold money ( $M$ ) or bonds ( $B$ ) as a store of wealth. Money yields utility (not modeled) whereas bonds yield a gross risk-free return of  $I_t$  in every period. Consumption preferences are subject to a shock  $V_t \equiv (1 - v_t)$  where

$$v_t = \rho_v v_{t-1} + \tilde{v}_t \quad (1)$$

and  $\tilde{v}_t$  is a white-noise shock.  $H_t$  represents external habit persistence. We allow habit persistence to be of order 2 and specify as follows

$$H_t = C_{t-1}^{\gamma_1} C_{t-2}^{\gamma_2}$$

where  $\gamma_1$  and  $\gamma_2$  are habit parameters. This more general setup allows agents to form habits with respect to the changes in as well as the level of consumption.

The first-order condition for the solution to the problem implies the consumption Euler equation

$$\left( \frac{C_t V_t}{C_{t-1}^{\gamma_1} C_{t-2}^{\gamma_2}} \right)^{1-\sigma} \frac{1}{C_t} = \left( \frac{1}{1+\rho} \right) I_t E_t \left( \frac{C_{t+1} V_{t+1}}{C_t^{\gamma_1} C_{t-1}^{\gamma_2}} \right)^{1-\sigma} \frac{1}{C_{t+1}} \frac{P_t}{P_{t+1}}. \quad (2)$$

Taking the logarithm of the Euler equation and using the resource constraint, we have

$$\begin{aligned} y_t = & \frac{\sigma}{A} E_t y_{t+1} + \frac{(\gamma_1 - \gamma_2)(\sigma - 1)}{A} y_{t-1} \\ & + \frac{\gamma_2(\sigma - 1)}{A} y_{t-2} - \frac{1}{A} (i_t - E_t \pi_{t+1} - \rho) + \frac{(\sigma - 1)}{A} (v_t - E_t v_{t+1}), \end{aligned} \quad (3)$$

where  $A \equiv \sigma + \gamma_1(\sigma - 1)$  and  $\pi_t$  is quarterly inflation at an annual rate. A small letter denotes the log of the corresponding capital letter variable.<sup>4</sup>

Note that due to dynamic homogeneity, we can write the aggregate demand schedule

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<sup>4</sup>Note that we have for simplicity ignored Jensen's inequality and used the approximation for small number approximation, both implying  $\ln E(1+x) = E \ln(1+x) = Ex$ .

(3) as

$$\begin{aligned}\Delta y_t = & \frac{\sigma}{\gamma_1(\sigma-1)} E_t \Delta y_{t+1} - \frac{\gamma_2}{\gamma_1} \Delta y_{t-1} \\ & - \frac{1}{\gamma_1(\sigma-1)} (i_t - E_t \pi_{t+1} - \rho) + \frac{1}{\gamma_1} (v_t - E_t v_{t+1}).\end{aligned}\tag{4}$$

## 2.2 Aggregate supply

Aggregate supply is represented with the hybrid Phillips curve

$$\pi_t = \mu E_t \pi_{t+1} + (1 - \mu) \sum_{j=1}^4 \alpha_j \pi_{t-j} + \zeta m c_t + \varepsilon_t$$

where  $(1 - \mu)$  is the weight on the backward-looking component and  $m c_t$  is marginal costs as of time  $t$ . We will assume that log marginal costs are linear in log deviation of output from the natural rate of output, i.e.,

$$m c_t = \tilde{\gamma} (y_t - y_t^n),$$

where  $y_t^n$  is the natural rate of output. By defining the deviation of output from the natural rate as the output gap,  $x_t \equiv y_t - y_t^n$ , we can write the Phillips curve as

$$\pi_t = \mu E_t \pi_{t+1} + (1 - \mu) \sum_{j=1}^4 \alpha_j \pi_{t-j} + \kappa x_t + \varepsilon_t,\tag{5}$$

where  $\kappa \equiv \zeta \tilde{\gamma}$ . The natural rate of output is given exogenously by the process

$$\Delta y_t^n = v + \omega_t\tag{6}$$

where  $\nu$  is the unconditional expected growth rate of output and  $\omega_t$  is an AR(1) shock to the growth rate

$$\omega_t = \phi\omega_{t-1} + \varrho_t. \quad (7)$$

The output gap follows the process

$$x_t = x_{t-1} + \Delta y_t - \Delta y_t^n. \quad (8)$$

### 2.3 Monetary policy

The monetary authority is setting the interest rate in accordance with a dynamic Taylor rule as

$$i_t = \psi i_{t-1} + (1 - \psi) \left( i_t^n + \theta_\pi (\bar{\pi}_t - \pi_t^T) + \theta_x x_t \right) + u_t, \quad (9)$$

where  $\psi$  measures the smoothing in the interest rate setting.  $i_t^n$  is the nominal natural interest rate (defined below) and

$$\bar{\pi}_t \equiv \frac{1}{4} \sum_{j=0}^3 \pi_{t-j}$$

is the four-quarter inflation at an annual rate. We assume that the intermediate-term inflation target evolves according to

$$\pi_t^T = (1 - \rho_\pi) \pi^* + \rho_\pi \pi_{t-1}^T + \xi_t, \quad (10)$$

where  $\pi^*$  is the equilibrium inflation rate and  $\xi_t$  is an AR(1) shock to the inflation target, following

$$\xi_t = \rho_{\pi}\xi_{t-1} + \varkappa_t. \quad (11)$$

## 2.4 The natural interest rate

The process for the natural nominal rate of interest can be found by replacing output and the interest rate in equation (4) with the natural rates and then solving for the interest rate, i.e.,

$$\begin{aligned} y_t^n = & \frac{\sigma}{A} E_t y_{t+1}^n + \frac{(\gamma_1 - \gamma_2)(\sigma - 1)}{A} y_{t-1}^n \\ & + \frac{\gamma_2(\sigma - 1)}{A} y_{t-2}^n - \frac{1}{A} (i_t^n - E_t \pi_t - \rho) + \frac{(\sigma - 1)}{A} (v_t - E_t v_{t+1}), \end{aligned} \quad (12)$$

or

$$\begin{aligned} \Delta y_t^n = & \frac{\sigma}{\gamma_1(\sigma - 1)} E_t \Delta y_{t+1}^n - \frac{\gamma_2}{\gamma_1} \Delta y_{t-1}^n \\ & - \frac{1}{\gamma_1(\sigma - 1)} (i_t^n - E_t \pi_t - \rho) + \frac{1}{\gamma_1} (v_t - E_t v_{t+1}). \end{aligned} \quad (13)$$

and isolating for the natural interest rate

$$\begin{aligned} i_t^n = & \rho + E_t \pi_{t+1} + \sigma E_t \Delta y_{t+1}^n - \gamma_1(\sigma - 1) \Delta y_t^n - \gamma_2(\sigma - 1) \Delta y_{t-1}^n \\ & + (\sigma - 1)(v_t - E_t v_{t+1}). \end{aligned} \quad (14)$$

The natural real interest rate is then found from the Fisher equation as

$$r_t^n \equiv i_t^n - E_t \pi_{t+1}. \quad (15)$$



The output gap process can be expressed as a function of the natural interest rate by subtracting equation (12) from equation (3) which gives

$$x_t = \frac{\sigma}{A} E_t x_{t+1} + \frac{(\gamma_1 - \gamma_2)(\sigma - 1)}{A} x_{t-1} + \frac{\gamma_2(\sigma - 1)}{A} x_{t-2} - \frac{1}{A} (i_t - i_t^n) \quad (16)$$

where the natural rate of interest is given in equation (14) above.

### 3 Estimation

We estimate the parameters of the model comprising of equations (1, 4, 5, 6, 7, 8, 9, 10 and 11) using Bayesian methods and the Kalman filter. The focus of the analysis will be on the estimation of the natural real rate of interest and the output gap. The use of Bayesian methods to estimate DSGE models has increased over recent years, in a variety of contexts, see An and Schorfheide (2006) for a recent evaluation. The focus is on methods that are built around a likelihood function, typically derived from a DSGE model (see for instance Adolfson et al. (2005)). While the econometric analysis of these models initially had to cope with several challenges including potential model misspecification and identification problems, the Bayesian framework has helped overcome these challenges. For instance, Fernandez-Villaverde and Rubio-Ramirez (2004) show that parameter estimation is consistent in the Bayesian framework even under model misspecification. With sensible priors, Bayesian techniques offer a major advantage over other system estimators such as maximum likelihood, which in small samples can often allow key parameters to wander off in nonsensical directions.

#### 3.1 Data

We estimate the model laid out in the previous section using U.S. quarterly time series for three variables: real output, inflation and interest rates. The sample period is 1983q1 to 2005q4. The period covers the last part of the Volcker period and the major part of the Greenspan period. The choice of periods follows from the assumption that these two Chairmen shares approximately the same dislike for inflation. The monetary policy regime is therefore roughly constant over the sample period. We use the quarterly average daily readings of US 3-month deposit rates as the relevant nominal interest rate. For real

output and inflation we use real GDP and the CPI, all items, for total USA. GDP and CPI are seasonally adjusted by their original source (OECD). We treat inflation, output growth, and the nominal interest rate as stationary, and express them in deviations from their sample mean. Note that all changes are measured at an annual rate. Inflation, output and 3-month deposit rates are plotted in Figure 1.1 in the appendix.

### 3.2 Parameter estimation

As is well known from Bayes's rule, the posterior distribution of the parameters is proportional to the product of the prior distribution of the parameters and the likelihood function of the data. This prior distribution describes the available information prior to observing the data used in the estimation. The observed data is then used to update the prior, via Bayes theorem, to the posterior distribution of the model's parameters.

To implement the Bayesian estimation method, we need to be able to evaluate numerically the prior and the likelihood function. Then we use the Metropolis-Hastings algorithm to obtain random draws from the posterior distribution, from which we obtain the relevant moments of the posterior distribution of the parameters.

More specifically, the model is first log-linearized and then estimated in two steps in Dynare-Matlab. In the first step we compute the posterior mode using 'csmnwel', an optimization routine developed by Christopher Sims. We use the first three years of the full sample 1983q1 to 2005q4 to obtain a prior on the unobserved state, and use the subsample 1986q1 to 2005q4 for inference. To calculate the likelihood function of the observed variables we apply the Kalman filter. In the second step, we use the mode as a starting point to compute the posterior distribution of the parameters and the marginal likelihood by simulations of the Metropolis-Hasting (MH) algorithm (see Schorfheide, 2000, for details). The debugging features of Dynare are used to determine if the optimization routines have found the optimum and if enough draws have been executed for the posterior distributions to be accurate. Having estimated the parameters, they can then be used to construct the natural rates of interest rates and output.

### 3.3 Prior and posterior distributions of the estimated parameters

The Bayesian estimation technique allows us to use prior information from previous micro and macro based studies in a formal way. Table 1 below summarizes the assumptions for the prior distribution of the estimated parameters and structural shocks. In the first three columns, the list of structural coefficients with their associated prior mean, standard deviations and distribution are shown. Following standard conventions we use

Beta distributions for parameters that fall between zero and one, (inverted) gamma (inv $\gamma$ ) distributions for parameters that need to be constrained to be greater than zero and normal (norm) distributions in other cases. For some of the parameters, the distribution is constrained further, as indicated in column four ('support').

The next three columns indicate the posterior mean and the associated 95 percent confidence interval. Starting with the Phillips curve, we provided a prior for  $\mu = 0.50$  that put equal weight on the forward-looking and backward-looking components with a large standard deviation providing a rather diffuse prior. This choice is rationalized by the fact that the literature has suggested estimates in the whole zero-unity interval. We wanted data to determine this coefficient without pushing it in either direction. In the estimation,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  and  $\alpha_4$  were restricted to sum to one, (with  $\alpha_4$  determined by this identity). However, since we do not have a strong prior on their magnitudes, we give them the same weight with the standard deviation set to 0.1.

We find that the Phillips curve is primarily forward looking, it has nevertheless a non-negligible weight on the backward-looking component, with  $(1-\mu)$  just below 0.4. This is consistent with the estimates of the New Keynesian Phillips curve found when using labor's share of income as the proxy for marginal costs<sup>5</sup> as opposed to using detrended output. Our results are consistent with the estimation results in Galí et al. (2003, 2005) using a full information, system estimation. We find this result interesting because it suggests that the output gap may proxy marginal costs on par with labor's share of income once we use a model-consistent estimate of the output gap. Further, our results also support that monetary policy can be studied within the simple two-equation model framework which explains the development of inflation and the output gap conditional on the policy instrument (as suggested by Clarida et al. (1999) and Woodford (2003)).

Regarding the expectational IS curve, we find that our prior on the intertemporal elasticity of substitution  $\sigma = 2$  is well within the range of the estimates in the literature. The posterior has increased somewhat from the prior, although not significantly so (posterior mean equals 2.05). Moreover, the preference shocks display a high degree of persistence, with a coefficient of  $\rho_v = 0.95$ . In addition, the habit parameters  $\gamma_1$  and  $\gamma_2$  are restricted to lie between zero and one, with the prior for  $\gamma_1$  being the largest, assuming more habit from the immediate past. However, we choose a large standard deviation of 0.2 that provides us with a fairly diffuse prior. The second-order habit persistence is well accounted for in data, as both  $\gamma_1$  and  $\gamma_2$  turn out to be above, yet close to 0.5. Finally, the prior for the annual discount rate  $\delta$  is set to 0.04, reflecting a quarterly discount factor of 0.99. Rather surprisingly, we find that data push the annual discount rate from the prior of four percent to 1.6 percent.

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<sup>5</sup>See, e.g., Galí et al. (2001, 2003, 2005) and Sbordone (2002, 2005)

Table 1: Estimation results for the US economy

Coefficients	Prior mean	Prior s.d.	Distr.	Support	post. mean	5%	95%
<i>Phillips curve</i>							
$\mu$	0.50	0.20	beta	$[0, 1]$	0.626	0.314	0.908
$\alpha_1$	0.25	0.10	norm	none	0.353	0.200	0.506
$\alpha_2$	0.25	0.10	norm	none	0.240	0.100	0.366
$\alpha_3$	0.25	0.10	norm	none	0.227	0.094	0.369
$\alpha_4$	0.25	n/a	n/a	n/a	0.180	n/a	n/a
$\kappa$	0.20	0.15	gamm	$[0, \infty]$	0.089	0.005	0.163
<i>IS curve</i>							
$\delta$	0.04	0.02	gamm	$[0, \infty]$	0.016	0.006	0.027
$\sigma$	2.00	0.50	beta	$[1.05, 5]$	2.047	1.625	2.448
$\gamma_1$	0.50	0.20	beta	$[0, 1]$	0.537	0.332	0.727
$\gamma_2$	0.40	0.20	beta	$[0, 1]$	0.599	0.396	0.870
$\rho_v$	0.85	0.10	beta	$[0, 1]$	0.945	0.916	0.980
<i>Natural rate process</i>							
$\phi$	0.850	0.10	beta	$[0, 1]$	0.788	0.678	0.909
$v$	0.030	0.005	gamm	$[0, \infty]$	0.029	0.024	0.035
<i>Monetary policy</i>							
$\rho_{pi}$	0.800	0.10	beta	$[0, 1]$	0.853	0.751	0.950
$\rho_\chi$	0.800	0.10	beta	$[0, 1]$	0.795	0.662	0.939
$\theta_\pi$	0.500	0.10	beta	$[0.1, 1.5]$	0.578	0.420	0.720
$\theta_x$	0.500	0.10	beta	$[0.1, 1.5]$	0.449	0.284	0.570
$\psi$	0.700	0.10	beta	$[0, 1]$	0.828	0.793	0.872
<i>Standard deviations of shocks</i>							
$\sigma_\varkappa$	0.002	Inf	invg	$[0, \infty]$	0.0024	0.0008	0.0045
$\sigma_\varepsilon$	0.001	Inf	invg	$[0, \infty]$	0.0110	0.0091	0.0127
$\sigma_u$	0.001	Inf	invg	$[0, \infty]$	0.0024	0.0020	0.0028
$\sigma_{\tilde{v}_t}$	0.001	Inf	invg	$[0, \infty]$	0.1983	0.1181	0.3045
$\sigma_\varrho$	0.001	Inf	invg	$[0, \infty]$	0.0096	0.0074	0.0119

Table 2: Error variance decomposition

Variables & shocks	Inf.-tar. ( $\varkappa$ )	Cost-push ( $\varepsilon$ )	Mon.pol. ( $u$ )	Preference ( $\tilde{v}$ )	Nat. rate ( $\varrho$ )
$r^n$	0.00	0.00	0.00	91.52	8.48
$x$	25.14	21.75	6.51	44.04	2.56
$\pi$	31.58	48.25	1.46	17.56	1.14
$i$	10.79	3.04	0.85	83.91	1.41
$i^n$	10.81	2.89	0.44	78.08	7.78
$\pi^T$	100.00	0.00	0.00	0.00	0.00

The prior for the equilibrium natural output growth rate is set equal to the (annual) growth rate in the model (3 percent), with the posterior mean estimated to  $v = 0.029$ .

The data seems to support a dynamic Taylor rule specification of monetary policy reasonably well. The monetary policy shock has standard deviation of 0.024. Moreover, the weight on inflation and output gap is deviating only marginally from the priors and what Taylor (1993) suggested as likely coefficients (0.5). There is a pronounced gradual adjustment of the interest rate with  $\psi = 0.83$ . Finally, we calibrate the equilibrium inflation rate  $\pi^*$  to be equal to steady state inflation. The results seems to indicate fairly persistent movements in the medium-run inflation target ( $\rho_\pi = 0.85$ ), with also rather persistent shocks to this process ( $\rho_\chi = .80$ ). The latter suggest that movements in the medium-run inflation target is done gradually over time.

### 3.4 Error variance decomposition and impulse responses

Table 2 shows the decomposition of the unconditional variance. Some interesting observations can be made from the table. We first note that the main drivers of inflation variations are the cost-push and inflation-target shocks. These shocks account for about 80% of the variation in inflation. If the central bank adheres to an inflation-targeting loss specification with the loss function having inflation and output gap variations as the two arguments (see Svensson (1997) and Clarida et al. (1999)), efficiency in policymaking requires that inflation should be driven *only* by cost-push and inflation-target shocks. The ratio is high and can be taken as an indication of efficiency in policymaking. However, by the same logic, the central bank should neutralize the impact of preference shocks on both the output gap and inflation. This does not seem to be the case. Although the Taylor rule has allowed strong responses to the preference shocks as they can explain more than 80% of the variation in the interest rate, preference shocks have still influenced inflation and, in particular, the output gap to a large extent. Hence, the estimated Taylor rule does less well in insulating the economy from this type of shock.

The natural real interest rate is driven mainly by preference shocks. Shocks to the natural rate of output play only a minor role in explaining the variation observed. This partly reflects the fact that the standard deviation preference shocks is large relative to the standard deviation of shocks to the natural rate of output.

The impulse response functions are shown in the appendix. None of these responses deviate from what we understand as conventional thinking, although the responses to some of the shocks seem to be rather fast (preference shocks in particular). The impulses from the monetary policy shock correspond well with results generated from VARs: For a positive shock to the interest rate, the output gap falls on impact and inflation reacts with a lag. The short-term interest rate falls relatively quickly and enters a period in which the policymaker corrects for the shock. Finally, a shock to the medium-run inflation target raises inflation expectations and the current inflation rate on impact due to the expectations channel. The nominal interest rate increases, but the real interest rate falls and creates a temporary increase in the output gap which again increases inflation. Inflation peaks after 5 quarters and is then brought slowly back to the equilibrium rate of inflation over a 5 – 7 years period. Hence, the medium-term is relatively long, approximately equal to the average business cycle. This gives some indication of the medium-term inflation target being used as an instrument to smooth output as a result of pursuing a constant inflation target over the business cycle.

### 3.5 The estimated variables

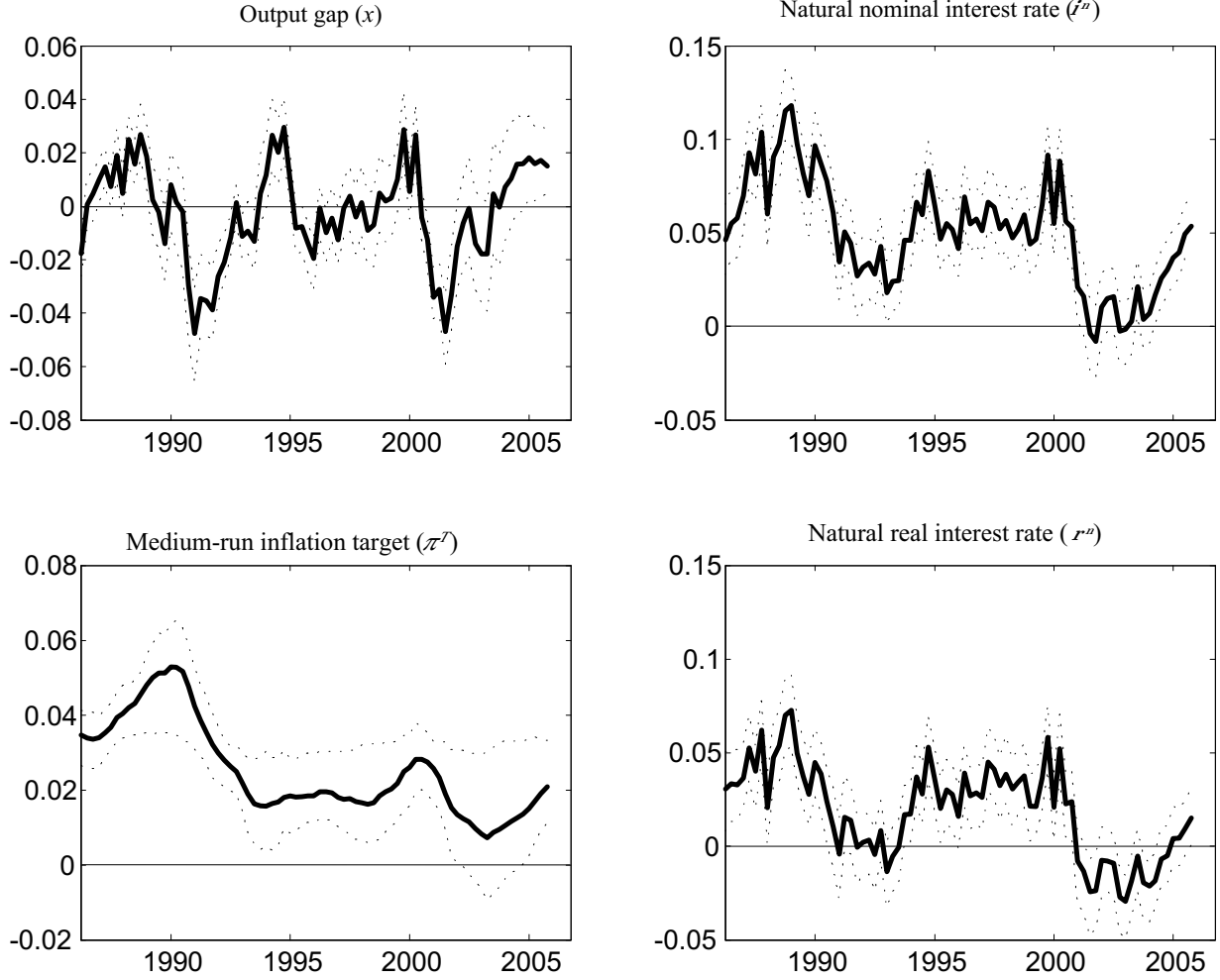
The smoothed output gap, the medium-run inflation target and the nominal and real natural interest rates are shown with 95% confidence intervals in Figure 1. Furthermore, Figure 2 shows the smoothed natural rate of output and the natural real interest rate plotted with actual output and the real interest rates respectively, as well as the real interest rate gap ( $r - r^n$ ) and the estimated inflation gap ( $\bar{\pi}_t - \pi^T$ ).

The output gap estimates suggest two recessions over the sample period: the first one with a trough in 1991 and the other with a trough in 2001/2002. The recessions are of approximately the same order of magnitude, suggesting a deviation of output from the natural rate output of approximately 5%. Furthermore, the recessions show similar patterns of evolution and degree of persistence. The recessions correspond to periods with large positive interest-rate gaps. The dates for the turning points and the length of the business cycles seem consistent with NBER and CBO estimates. The output gap estimate is also broadly consistent with that found in the updated version (2006) of Laubach and Williams (2003).<sup>6</sup> Interestingly, both indicate that the output gap became

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<sup>6</sup>We thank John Williams for providing us with the updated simulation results.

Figure 1: Inflation target, output gap and natural interest rates.

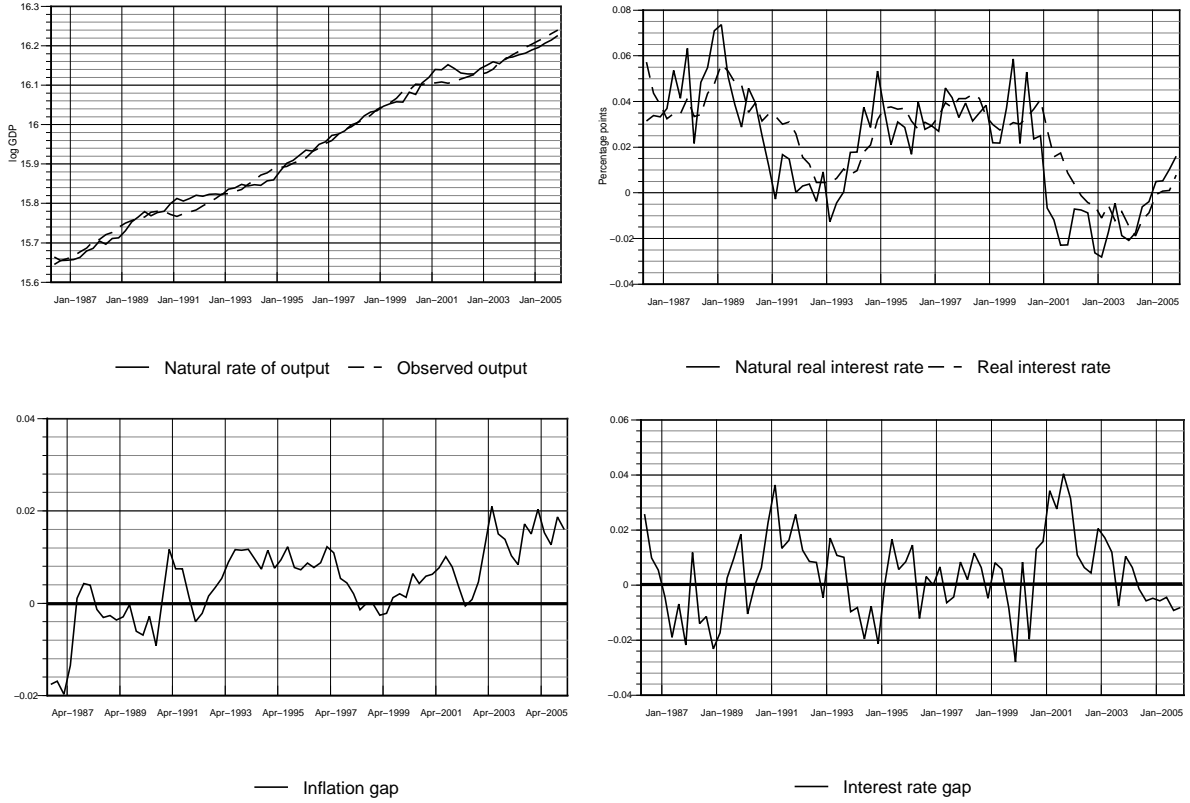


The figures show the estimated smoothed variables over the sample period.

positive from 2004 onwards.

The sample average CPI inflation over the period is 3.3%. The medium-run inflation target estimates suggest that the mild run-up of inflation in the late 1980s, due to a positive output gap, was partly accommodated by an increase in the inflation target over the period. The reduction in the rate of inflation of the first part of the 1990s, accompanied by the recession in the same period, can partly be explained by a reduction in the inflation target. From 1994 to the end of the sample, the medium-run inflation target is estimated to be around 2% with a confidence band of about  $\pm 1$  p.p. For most of the period, the inflation target is significantly above zero. The inflation gap suggests that for the major part of the 1990s and the period after 2002, inflation has in general been above the medium-term inflation target, and therefore has exerted an upward pressure on interest rates.

Figure 2: The natural rate of output, interest-rate and inflation gaps.



From the top-left panel and moving clockwise: The panels show the natural rate of output and the real rate with their observed variables, the inflation gap and the interest rate gap

The estimate of the natural real interest rate shows considerable variation over the period – varying between  $-3\%$  and  $6\%$ . The variation in the natural real interest rate is in periods greater than the equivalent real interest rate. This is also found in the DSGE study of Edge et al. (2005), but not by Laubach and Williams (2003) where the natural interest rates appear as smoothed interest rates. Here, the natural rate follows instead from the stochastic processes governing the preference shocks and shocks to the natural rate of output (see equations (14) and (15)). These processes which determine the interest rate under the assumption of flexible prices are unaffected by the potential smoothing of interest rates done by the central bank in the sticky-price equilibrium.<sup>7</sup> Moreover, the mode of the natural rate of interest is in the range  $3 - 4\%$  which does not seem unreasonable for the average real interest rate. The average natural interest

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<sup>7</sup>By the same logic, there is nothing that ensures that the evolvement of the natural rate of output is smoother than output itself. Woodford (2001, p.234) notes “In theory, a wide variety of real shocks should affect the growth rate of potential output[...] [T]here is no reason to assume that all of these factors follow smooth trends. As a result, the output-gap measure that is relevant for welfare may be quite different from simple detrended output.”



rate is remarkably stable over the period 1994-2000 where the variation is in the region  $\pm 1$ p.p. This is a result also found by Edge et al. (2005). The recession of the first half of 2000s imply negative real interest rates for this period, suggesting a rather expansionary monetary policy that would have been needed in order to keep aggregate demand equal to the natural rate of output.

It has been relatively common to estimate monetary policy reaction functions conditional on the natural rate of interest being equal to a constant plus the inflation rate. The relatively large variation in the normal interest rate suggests that the resulting estimates could be severely biased if the central bank is taking account of the fact that the natural rate of interest is time-varying. In particular, the high degree of persistence in the natural rates might show up in a larger coefficient on past interest rates in a regression analysis. The estimate is then likely to be biased upwards. Moreover, failing to take account of the interdependence between the output gap and natural interest rates (estimates) may also distort the estimates.

Another issue related to the relatively high degree of variation in the natural rate of interest, is that even a “neutral” monetary policy stance requires considerable changes in the interest rate. If the policymaker regards the natural rate of interest as a constant, policy is likely to induce inefficient movements in inflation and output. Some may object to the arguments by claiming that interest rates should be smoothed over time and for this reason the variability in the natural rate should largely be ignored. Although we disagree with this argument, we recognize that the policymaker may still desire smoothed changes in the interest rate. Such preferences may even be welfare-enhancing in terms of producing smaller variation in inflation and output (see Woodford (1999)), but for other reasons than ignoring the variability in the natural interest rate.

## 4 Concluding remarks

This paper provides estimates of the natural real interest rate, the output gap and the implicit inflation target for the US economy. The inflation target since 1994 has been remarkably stable around 2 percent. The natural real interest rate has, however, been varying a lot. The assumption often made in the monetary policy literature that the natural real interest rate is exogenous or even constant, might be very misleading and biasing the results. For the conduct of monetary policy, acknowledging the variation in the real interest rate and conducting policy in accordance with it, seems to be important.

By estimating the hybrid New-Keynesian Phillips curve with a model-consistent estimate of the output gap, we find that the structure of the curve is very similar to that found by estimating the Phillips curve with the labor share of income as a proxy for marginal

costs. The debate whether it is the output gap or the labor share of income that provide the best proxy for marginal costs, may therefore be misleading. The paper suggests that the output gap might represent marginal costs on par with labor's share of income once a model-consistent estimate of the output gap is used instead of ad-hoc estimates, e.g. detrended output. If the output gap can be taken as a good proxy for marginal costs, our results strengthen the idea that a simple two-variable system in inflation and the output gap (see, Clarida et al. (1999) and Woodford (2003)) can capture the effects of monetary policy on the private sector.

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## Appendix

### 1 Extra figures

#### 1.1 Data

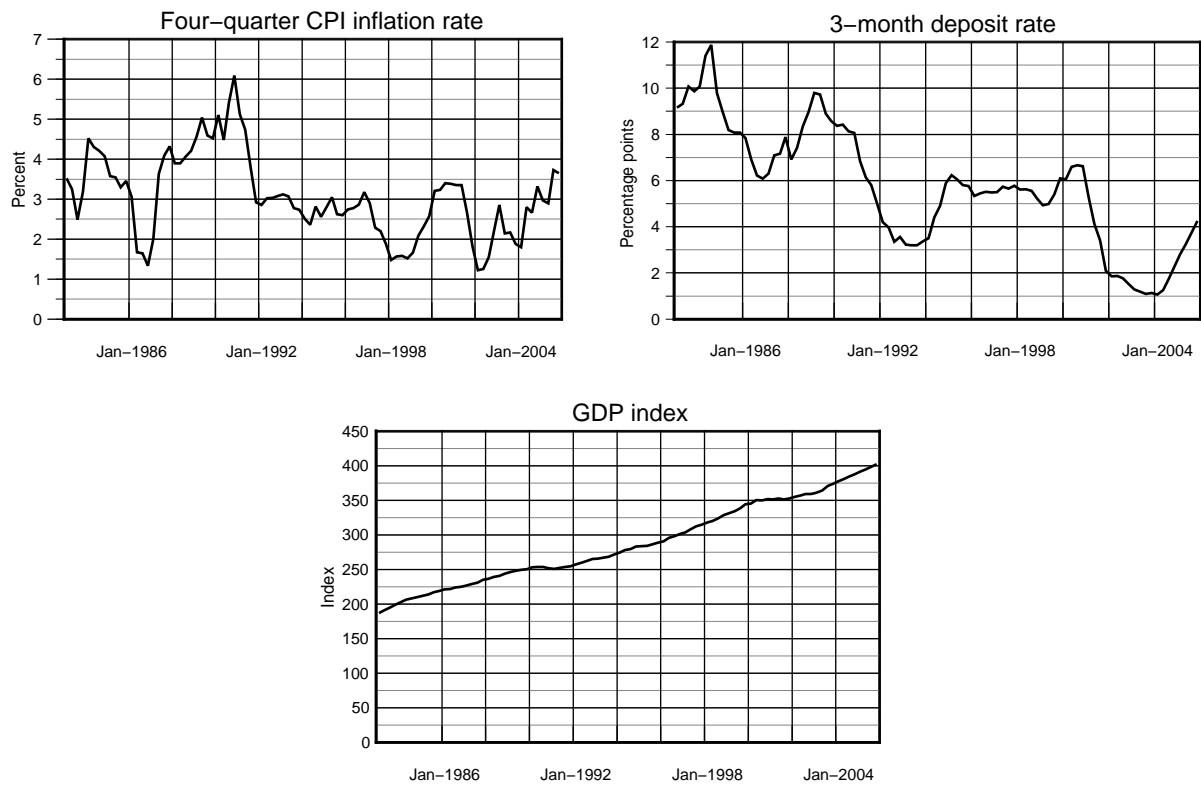
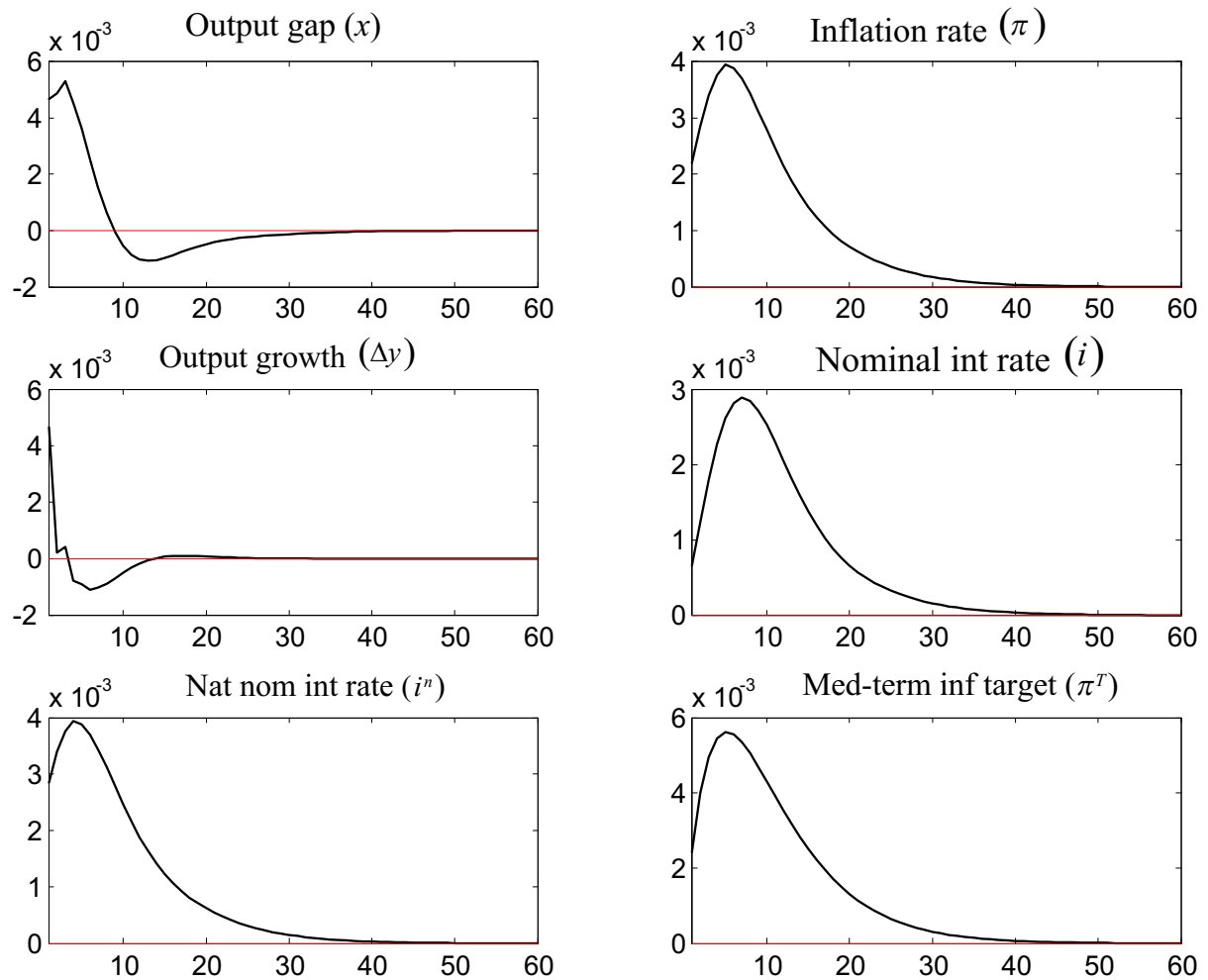


Figure 1: Data

## 1.2 Impulse response functions

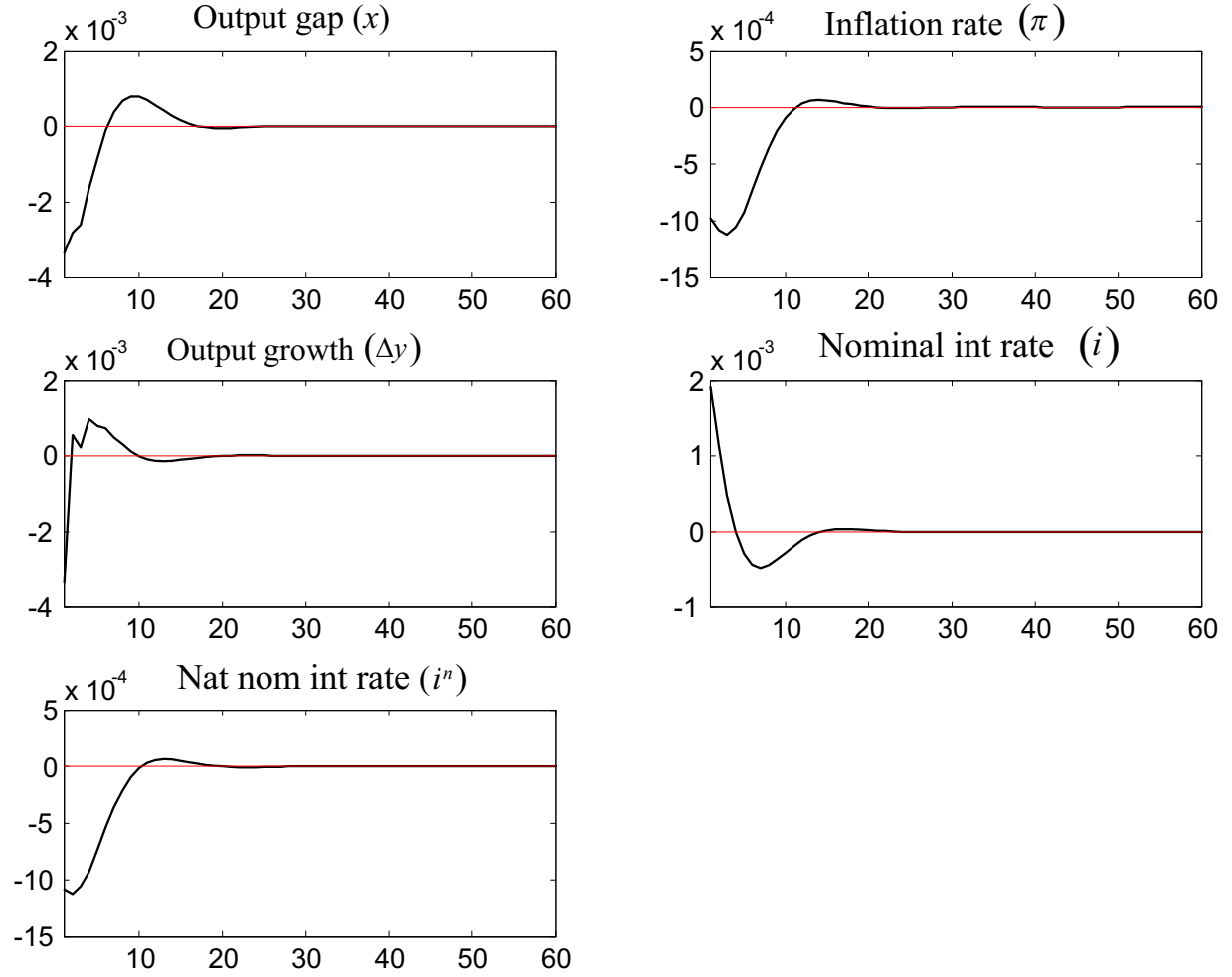
Figure 2: Monetary policy shock to the medium-term inflation target



The impulse response function due to a shock to the medium-term inflation target.

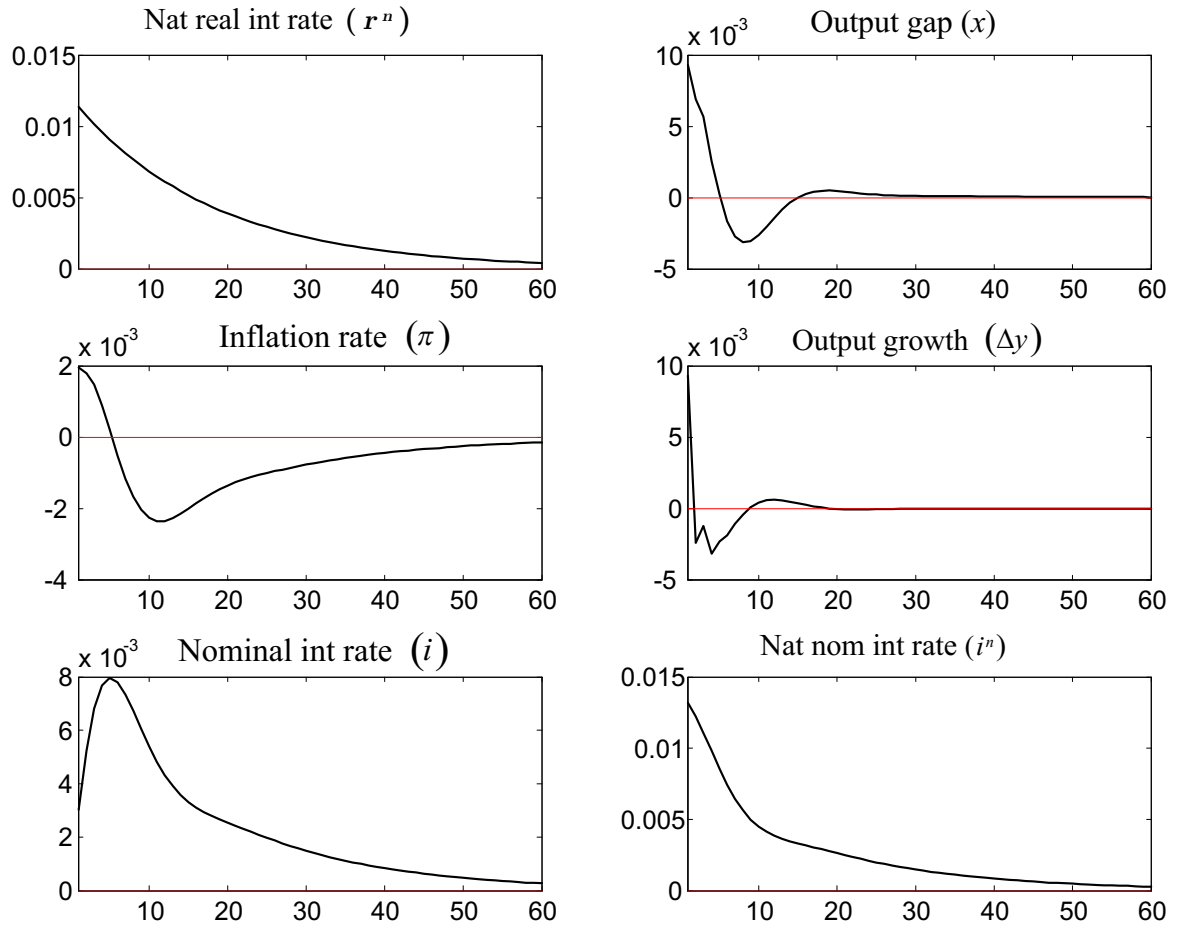


Figure 3: Monetary policy shock to short-term interest rate



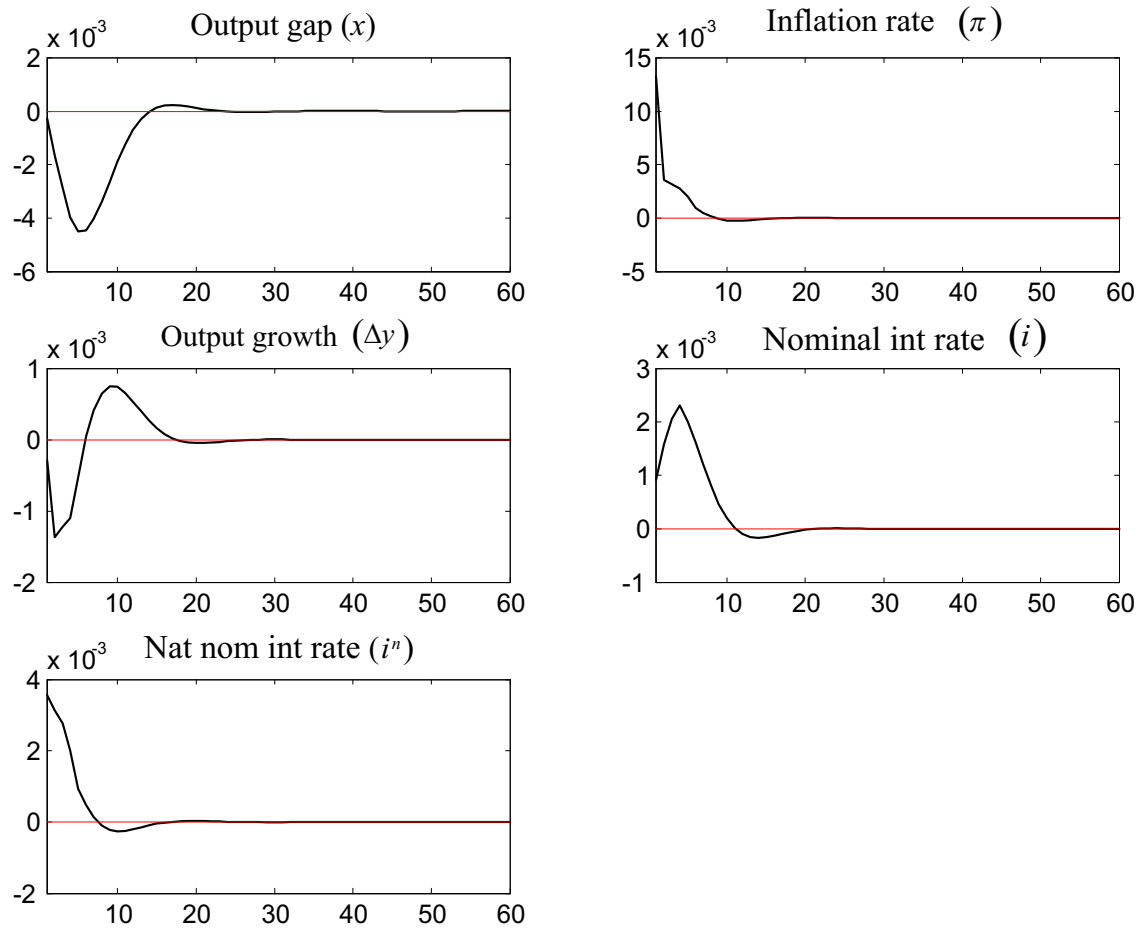
The impulse response functions due to a shock to the short-term interest rate.

Figure 4: Preference shock



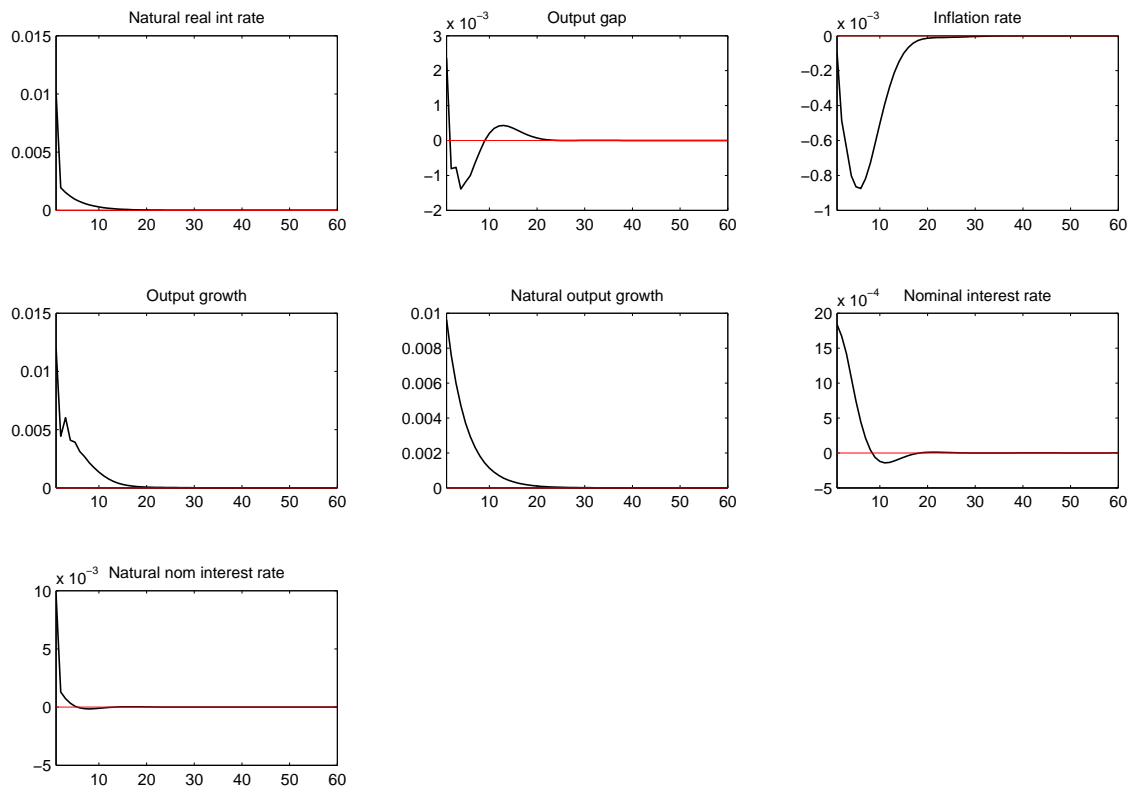
The impulse response functions due to a preference shock.

Figure 5: Cost-push shock



The impulse response functions due to a cost-push shock.

Figure 6: Natural rate shock



The impulse response functions due to the natural rate of output.