Measuring inflation persistence: a structural time series approach*

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Abstract

Time series estimates of inflation persistence incur a strong upward bias if shifts in the inflation target of the central bank remain unaccounted for. Using a structural time series approach we measure inflation persistence allowing for an unobserved time-varying inflation target. Unobserved components are identified using Kalman filtering and smoothing techniques. Posterior densities of the model parameters and the unobserved components are obtained in a Bayesian framework based on importance sampling. Inflation persistence, measured by the sum of the autoregressive coefficients, is estimated to range between 0.45 and 0.79.

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1 Introduction

A measure of inflation persistence that is unaffected by historical policy shifts is particularly important for building realistic macroeconomic models to conduct monetary policy. A central bank that has established a credible inflation target needs to know how persistent the inflation process is. When shocks to inflation are causing deviations from the central bank's inflation target, the speed at which inflation returns back to its pre-shock level will determine the extent to which the central bank needs to use its policy instrument to get inflation back to target in a timely manner. If the central bank's decisions are based on historical measures of inflation persistence that are severely biased due to past regime shifts, it will miss its objective. The estimates will be susceptible to the Lucas Critique.

Levin and Piger (2004) use the notion *intrinsic* inflation persistence, which is determined by the structural way prices and wages are set. Intrinsic inflation persistence can also be defined as the tendency of inflation to converge slowly towards its long-run value, which is the steady-state inflation rate that will prevail after all shocks have run their course.

In the medium to long run, inflation is primarily determined by monetary policy. Permanent changes in the central bank's monetary policy strategy that have a bearing on its medium-term inflation objective will therefore lead to persistent changes in inflation. This type of inflation persistence is not part of intrinsic persistence. Over shorter horizons, various shocks including supply, demand, cost and temporary policy shocks will influence inflation and move it away from the steady state value that is compatible with the central bank's monetary policy strategy.

In empirical research using post-WW II data, convergence is often estimated to be very slow[1], close to that of a random walk[2]. Levin and Piger (2004) pointed out that it is important to take into account shifts in the mean[3] of the inflation series in order to get a reliable estimate of intrinsic inflation persistence. We illustrate their point with the following data generating process for inflation:

$$\pi_t = \pi_t^T + \alpha_1(\pi_{t-1} - \pi_t^T) + \varepsilon_t \quad (1)$$

$$\pi_t^T = \pi_{t-1}^T + \varepsilon_t^T \quad (2)$$

[1] See Fuhrer and Moore (1995), Pivetta and Reis (2004) or O'Reilly and Whelan (2004).

[2] Without accounting for possible shifts, Levin and Piger (2004) report median unbiased estimates for the sum of autoregressive coefficients of the US GDP deflator of 0.92 (sample 1984Q1-2003Q4). O'Reilly and Whelan (2004) find a median unbiased estimate of 1.02 for the euro area GDP deflator (sample 1970Q1-2002Q4). Both papers use the Hansen grid bootstrap procedure.

[3] This argument goes back to Perron (1990) who showed that in the case of shifts in the mean value of a time series, testing for a unit root crucially depends on how these breaks in the mean are accounted for.

The inflation process π_t is characterised by a degree of intrinsic inflation persistence α_1 ($0 \leq \alpha_1 < 1$) in reaction to all kinds of temporary[4] shocks ε_t. Furthermore, the steady-state inflation rate, which is determined by the central bank's inflation target π_t^T, can change in any time period. All target changes are considered to be permanent, therefore the target is formalised as a random walk. Estimating equation (3) empirically

$$\pi_t = \alpha_1 \pi_{t-1} + u_t \tag{3}$$

corresponds to estimating equation (4).

$$\pi_t = \alpha_1 \pi_t^T + \alpha_1 (\pi_{t-1} - \pi_t^T) + u_t \tag{4}$$

Hence, if the data generating process is given by equation (1), but we estimate equation (3), then the estimate of α_1 will be biased towards 1, unless π_t^T is a constant. This is exactly the point of Levin and Piger (2004). In order to get a reliable estimate for intrinsic inflation persistence it is thus necessary to take shifts in the inflation target into account. Unfortunately, the inflation target is not observed and therefore needs to be estimated.

Recent research persued three different directions to account for shifts. One[5] way of dealing with shifts uses techniques that search for discrete shifts in the inflation series. Identifying a discrete shift might however be difficult if the regime change occurred gradually. Moreover, even in the case where a discrete break is correctly accounted for, this technique does not discriminate between persistence in reaction to temporary innovations and persistence in reaction to a regime shift, which could produce inaccuracy in the estimate of intrinsic persistence if they are different from each other. A second[6] line of research uses a sub-sample containing a smaller number of observations moving gradually through a bigger sample. By reducing the sub-sample size the potential number of breaks that occur is limited. However, this approach does not entirely rule out the possibility of shifts, and also has limits in terms of degrees of freedom. A third[7] way estimates time-varying autoregressive coefficients conditional on a time-varying mean. The latter is typically assumed to behave like a pure random walk. In this way, the approach does not allow for the case where persistence in reaction to temporary innovations and persistence in reaction to a regime shift differ.

Using both a univariate and a multivariate structural time series approach, this paper measures inflation persistence as the persistence in the deviations from a time-varying mean. The time-varying mean captures the long-run inflation expectations of economic agents, i.e. the monetary inflation target as

[4] Note that in the limiting case where $\alpha_1$is equal to 1, all shocks are permanent.

[5] See Levin and Piger (2004), Gadzinski and Orlandi (2004) and Bilke (2004).

[6] See O'Reilly and Whelan (2004), Pivetta and Reis (2004).

[7] See Cogley and Sargent (2001, 2003), Pivetta and Reis (2004), Benati (2004), Canova and Gambetti (2004).

perceived by economic agents. Due to, among other, imperfect credibility and asymmetric information, this perceived inflation target does not necessarily coincide with the inflation target pursued by the central bank. Both the univariate and the multivariate model take this difference explicitly into account. Consequently, both models allow for a different persistence of inflation in response to monetary target shocks and temporary shocks.

In the univariate approach, which relies on inflation data only, the possibly slow convergence of inflation expectations to the monetary target is incorporated by modelling the perceived inflation target as an autoregressive process with one root on and a second close to the unit circle. In order to disentangle the perceived and the monetary inflation target, the multivariate setting adds data on real output and the key nominal central bank interest rate to the information set. Basically, the multivariate model we propose extends the widely used macroeconomic model of Rudebusch and Svensson (1999) with a time-varying perceived and monetary inflation target. More specifically, the perceived inflation target is modelled as an autoregressive process around the monetary inflation target, which is assumed to be a random walk process. Innovations to the latter process are identified mainly from shifts in the key interest rate of the central bank. In addition, the multivariate model enables us to measure inflation persistence in response to different macroeconomic shocks, e.g. shocks to the business cycle. After all, the dynamic response of inflation does not only depend on its own persistence, but also on the interaction between other variables that are affected by the same shock.

As both the univariate and the multivariate model include unobserved components, they are cast in a linear Gaussian state space representation. This allows for identification of the unobserved components using Kalman filtering and smoothing techniques. The unknown parameters of the state space model are estimated in a Bayesian framework. Posterior densities of the model parameters and the unobserved components are obtained using importance sampling.

The results of both the univariate and the multivariate model indicate that inflation persistence is lower than the inflation persistence traditionally found using time series analysis, i.e. the sum of the autoregressive coefficients is estimated to range between 0.45 and 0.79. This confirms that previously measured high inflation persistence is partly an artefact of unaccounted time-varying monetary policy. Moreover, the impulse response functions obtained from our multivariate model suggest that the speed at which inflation returns back to its initial level highly depends on the kind of shock that hits the economy. Persistence measured by the half life of a shock to inflation is relatively low in the case of a cost-push shock, whereas for a shock to the output gap or long run inflation expectations it is markedly higher. In the case of a shift in the policy target the half life even goes to infinity.

In Section 2 we explain how we model the inflation process. Section 3 lays out how a structural time series approach can identify the unobservable inflation target. Section 4 presents the results. Section 5 concludes.

2 Theoretical specification

2.1 Univariate model

As we want to allow for possible shifts in the steady state inflation rate at any point in time, we model the inflation process as a long run inflation rate around which short run deviations occur. We decompose inflation into a temporary and a permanent component. A univariate process is specified by the following equation:

$$\pi_t = (1 - \sum_{i=1}^{4} \alpha_i)\pi_t^P + \sum_{i=1}^{4} \alpha_i L^i \pi_t + \varepsilon_t^{\pi} \tag{5}$$

where π_t is the quarterly inflation rate, L is the lag operator so that $L^i \pi_t = \pi_{t-i}$. As we work with quarterly data, we take inflation lagged by four periods throughout the entire paper. The long run or steady state inflation rate as *perceived*[8] by economic agents is called π_t^P. The notion of a perceived inflation target is equivalent to inflation expectations at business cycle frequencies or long run inflation expectations[9]. In the case of imperfect credibility, the perceived inflation target does not coincide with the inflation target of the central bank.

Data on long term inflation expectations in figure 1 seem to suggest that in the case of shifts of the equilibrium inflation rate, convergence towards a new equilibrium evolves smoothly over time. Even if the central bank clearly announces a new inflation target, evidence (Castelnuovo et al, 2003) suggests that it takes quite some time before the new policy target is incorporated into long term inflation expectations of economic agents. In the literature this is often attributed to asymmetric information and signal extraction, imperfect credibility or, more recently, sticky information. Therefore, we model the long run inflation target as a stochastic process that when hit by a - by definition permanent - shock, takes time before arriving at the new equilibrium. In equation (6) the perceived inflation target is specified as a stochastic process that can either be I(1) or I(2)[10].

$$\pi_t^P = (1 + \delta)\pi_{t-1}^P - \delta\pi_{t-2}^P + \varepsilon_t^{\pi^P} \tag{6}$$

If the parameter δ lies between 0 and just below 1 it will determine the speed at which a new long run equilibrium value will be attained after inflation is hit

[8] See also Kozicki and Tinsley (2003) for this definition of the long run steady state inflation rate.

[9] We will use the terms long run inflation expectations, steady state or equilibrium inflation rate and perceived inflation target interchangeably throughout the rest of the paper.

[10] In the empirical literature on the NAIRU, which uses unobserved components, the stochastic process is often assumed to be I(2) in order to ensure enough "smoothness" of the estimated NAIRU. See for instance Fabiani and Mestre (2001) or Durbin and Koopman (2001).

by a permanent shock. In the case where δ is zero, the process corresponds to a pure random walk. A permanent shock is immediately and completely passed through. The higher is δ, the slower the convergence will take place, or in other words, the smoother the time series will look. In the case where δ takes the value of 1 the permanent component behaves like an I(2) process, so that the trend growth of the series is permanently changed in response to a shock.

The long run inflation rate or perceived inflation target is defined as the rate of inflation that will prevail after all shocks have run their course. From equation (7) it can be seen that in the long run ($L = 1$) the inflation rate π_t will be equal to π_t^P if ε_t^π is equal to 0.

$$\pi_t = \frac{(1 - \sum_{i=1}^{4} \alpha_i)}{(1 - \sum_{i=1}^{4} \alpha_i L^i)} \pi_t^P + \frac{1}{(1 - \sum_{i=1}^{4} \alpha_i L^i)} \varepsilon_t^\pi \tag{7}$$

The degree of intrinsic inflation persistence is reflected in $\sum_{i=1}^{4} \alpha_i$, which is determined by structural economic factors, such as price and wage setting. It also includes the extent to which the inflation process is characterised by indexation. In this way, the perceived inflation target π_t^P can be considered as not containing any backward-looking behaviour in response to short run disturbances.

Finally, it should be stressed that equation (5) allows for the possibility that the inflation process is characterised as a pure random walk process, where all innovations are permanent, or equivalently that the sum of autoregressive coefficients $\sum_{i=1}^{4} \alpha_i$ will be equal to 1. The hypothesis of inflation being a pure random walk, where there is no distinction between shocks having temporary and shocks having permanent effects, is thus nested in the model.

2.2 Multivariate model

The following model extends the widely used macroeconomic model of Rudebusch and Svensson (1999) with a time-varying equilibrium inflation rate. Because we want to measure inflation persistence as the sum of the coefficients on the lagged inflation terms, this non-expectational autoregressive model suits very well. In the case the economy is characterised by forward looking rational expectations, it can be considered as its reduced form representation. However, Rudebusch (2003) shows that in this case the reduced form representation of a simple forward looking monetary policy model will be subject to the Lucas critique. This is not so for our extension, as we model the economy in a reduced form around a time varying steady state inflation rate. In this way the reduced form parameters are not affected by policy changes. Lansing and Trehan (2003) for instance show that the reduced form parameters in the so-called lagged expectational model depend on the policy parameters (ρ_1 and ρ_2 in the model below), which do not change in our setting. In our model it is the steady state path of the economy that changes.

As the measured intrinsic persistence depends highly on the right measure of the long run equilibrium inflation rate, we want to test the robustness of the results of the univariate time series approach, and check whether the dynamics in the permanent component of inflation can be linked to shifts in the monetary policy regime.

$$
\begin{aligned}
\pi_t &= (1-\sum_{i=1}^{4}\alpha_i)\pi_t^P + \sum_{i=1}^{4}\alpha_i L^i \pi_t + \beta_1 z_{t-1} + \varepsilon_t^{*\pi} && (8)\\
y_t &= y_t^P + z_t && (9)\\
y_t^P &= \lambda + y_{t-1}^P + \varepsilon_t^{y^P} && (10)\\
z_t &= \beta_2 z_{t-1} + \beta_3 z_{t-2} - \beta_4\left(i_{t-1} - \pi_{t-1}^P\right) + \varepsilon_t^z && (11)\\
i_t &= \bar{r} + \pi_t^P + \rho_1\left(\pi_{t-1} - \pi_t^T\right) + \rho_2\left(i_{t-1} - \pi_t^P - \bar{r}\right) + \varepsilon_t^i && (12)\\
\pi_t^T &= \pi_{t-1}^T + \varepsilon_t^{\pi^T} && (13)\\
\pi_t^P &= (1-\delta^*)\pi_{t-1}^P + \delta^*\pi_t^T + \varepsilon_t^{*\pi^P} && (14)
\end{aligned}
$$

The model enables us to extract information on shifts in the monetary policy regime contained in the key interest rate. As can be seen from equation (12), the interest rate is composed of the sum of the steady state real interest rate r and the steady state inflation rate π_t^P (Fisher equation), and a term that captures the reaction of the central bank to deviations of inflation from its target $\left(\pi_t - \pi_t^T\right)$. If the equilibrium inflation rate is stable and inflation moves together with output, this means that over an entire business cycle the key nominal interest rate must on average be equal to the sum of the steady state real interest rate and the steady state inflation rate.

Figures 2 and 3 present data for key interest rates and inflation since 1970. In a stable inflation environment, inflation and key interest rates should over an entire business cycle move around a fixed point on the 45 degree line. The 45 degree line corresponds to the sum of the equilibrium real interest rate (intercept[11]) and the equilibrium inflation rate. However, the seven year moving average[12] line of the data, which filters out business cycle fluctuations, suggests that from the 1970s until now inflation and interest rates did not move around a fixed point. There have been substantial shifts in the steady state inflation rate. As in the long run inflation is primarily determined by monetary policy, the shifts should be caused by changes in the central bank inflation target.

The same figures also reveal to what extent the steady state inflation rate differed from the central bank inflation target at a certain point in time. If the steady state inflation rate deviates from the central bank inflation target, the

[11] The intercept is the mean of the real interest rate in the sample 1970Q2-2003Q4.

[12] As the sample begins in 1970Q2, the moving average will only start to contain seven years of data from 1977Q2. Therefore, the average is a bit more volatile in the beginning of the sample.

interest rate rule suggests that the central bank will react to this by setting the key interest rate different from its steady state. The latter is equal to the sum of the steady state real interest and inflation rates. In the case of a completely credible inflation or disinflation, the shifts in the central bank inflation target would be accompanied by the same changes in the steady state inflation and key interest rates. Graphically, this would correspond to shifts along the 45 degree line. As this is neither the case for the US nor for the euro area in most of the sample, this shows that changes in the central bank target are usually only slowly reflected in the perceived inflation target. The only time this observation seems not to hold, is for the period between 1994 and today in the United States. It suggests that during the last decade, the Federal Reserve was able to disinflate in a credible way by about 2 percentage points, which seems also to be confirmed by narrative evidence[13].

All this shows that the interest rate contains exploitable information on shifts in the policy regime. If we estimate shifts in the central bank target that coincide with shifts in the permanent component of inflation in the univariate model, it confirms the interpretation that they are driven by policy changes.

The model further consists of a Phillips curve (8) and an IS curve (11). Equations (9) and (10) relate real output y_t to potential output y_t^P. The interest rate rule depends on the inflation target of the central bank π_t^T. The central bank's inflation target affects long run inflation expectations π_t^P through equation (14). The equation for long run inflation expectations is a weighted average of the long run inflation expectation in the previous period and this period's central bank inflation target. The parameter δ^* determines the speed at which changes in the central bank's inflation target will affect changes in the long run inflation expectations of economic agents.

The reason why the equilibrium inflation target is slowly updated with information on the policy target could be imperfect credibility, asymmetric information and signal extraction or sticky information. The model does not distinguish between the different theories. The parameter δ^* can simultaneously correspond to the Kalman gain parameter k_g in the signal extraction[14] problem of Erceg and Levin (2003) and to the information updating parameter λ in a variant of the model of Mankiw and Reis (2002). The model neither excludes that δ^* is a weighted average of both parameters, which could be the case if reality is a

[13] Goodfriend (2002) writes: "... in February 1994, the Fed started to announce its current intended federal funds rate target immediately after each FOMC meeting. This new practice made Fed policy more visible than ever. Every increase in the federal funds rate since then has attracted considerable attention."

[14] Erceg and Levin (2003) specify the following - slightly simplified - equation of how economic agents update their beliefs about the the central bank target π_{pt}. $E_t\pi_{pt} = E_{t-1}\pi_{pt-1} + k_g(\pi_t^* - E_{t-1}\pi_{t-1}^*)$. Economic agents can infer an inflation target π_t^* from the interest rate rule, but do not know in how far changes are permanent or temporary. Therefore, their belief $E_t\pi_{pt}$ about the central bank target is only gradually updated with new information.

mixture of both theories. We refer to the appendix for more details on how λ in Mankiw and Reis (2002) can be linked to δ^*. Eventually, it is not excluded that another theory can explain δ^*. But at least δ^* can be considered as a parameter measuring persistence in reaction to shifts in the policy target.

2.3 Relation between parameters δ^* and δ

There exists a clear link between the parameter δ in the univariate model, that determines the gradual change from one equilibrium inflation rate to the next equilibrium inflation rate, and the parameter δ^*, that gets a structural interpretation in models with asymmetric or sticky information.

From equations (13) and (14) we get:

$$\pi_t^P = (1-\delta^*)\pi_{t-1}^P + \delta^*\pi_{t-1}^T + \delta^*\varepsilon_t^{\pi^T} + \varepsilon_t^{*\pi^P} \tag{15}$$

After substituting out $\delta^*\pi_{t-1}^T$ this becomes:

$$\pi_t^P = (2-\delta^*)\pi_{t-1}^P - (1-\delta^*)\pi_{t-2}^P + \delta^*\varepsilon_t^{\pi^T} + \varepsilon_t^{*\pi^P} - \varepsilon_{t-1}^{*\pi^P} \tag{16}$$

Rescaling equation (16) with $\delta^* = 1-\delta$ and $\delta^*\varepsilon_t^{\pi^T} + \varepsilon_t^{*\pi^P} - \varepsilon_{t-1}^{*\pi^P} = \varepsilon_t^{\pi^P}$ we get equation (6):

$$\pi_t^P = (1+\delta)\pi_{t-1}^P - \delta\pi_{t-2}^P + \varepsilon_t^{\pi^P} \tag{17}$$

So the smoothing parameter δ from the time series specification corresponds to 1 minus the learning rate or sticky information update parameter δ^*. The higher the degree of learning, the less smooth will be the change of the long run inflation rate from the old equilibrium to the new one. If δ^* lies between 1 and just above 0, the time series specification has a structural interpretation. In the case δ^* takes a value of 0, monetary policy cannot affect the steady state inflation rate, which will then wander around as a pure random walk. In the time series model, the stochastic process for π_t^P will then correspond to an I(2) trend. Only in this special case will the link between both models be broken.

3 Estimation methodology

3.1 State space representation

The structural time series models outlined in section 2 both include a number of unobserved series (e.g. y_t^P, π_t^P, π_t^T). In order to estimate these models, it is necessary to write them into state space form. In a state space model, the development over time of the system under study is determined by an unobserved series of vectors $\alpha_1, \ldots, \alpha_n$, which are associated with a series of observed vectors $y_1, \ldots, y_n$.[15] A general linear Gaussian state space model can be written in the following form:

[15] See e.g. Durbin and Koopman (2001) for an extensive overview of state space methods.

$$y_t = Z\alpha_t + Ax_t + \varepsilon_t, \qquad \varepsilon_t \sim N(0, H), \tag{18}$$

$$\alpha_{t+1} = T\alpha_t + R\eta_t, \qquad \eta_t \sim N(0, Q), \qquad t = 1, \ldots, n, \tag{19}$$

where y_t is a $p \times 1$ vector of observed endogenous variables, modelled in the observation equation (18), x_t is a $k \times 1$ vector of observed exogenous variables and α_t is a $m \times 1$ vector of unobserved states, modelled in the state equation (19). The disturbances ε_t and η_t are assumed to be independent sequences of independent normal vectors. The matrices Z, A, T, H, and Q are parameter matrices. The matrix R is a selection matrix, i.e. a matrix whose columns are a subset of the columns of the identity matrix.[16]

3.2 Kalman filter and smoother

Assuming that Z, A, T, R, H, and Q are known, the purpose of state space analysis is to infer the relevant properties of the α_t's from the observations $y_1, \ldots, y_n$ and $x_1, \ldots, x_n$. This can be done through the subsequent use of two recursions, i.e. the Kalman filter and the Kalman smoother. The objective of filtering is to obtain the distribution of α_t, for $t = 1, \ldots, n$, conditional on Y_t and X_t, where $Y_t = \{y_1, \ldots, y_t\}$ and $X_t = \{x_1, \ldots, x_t\}$. In a linear Gaussian state space model, the distribution of α_t is entirely determined by the filtered state vector $a_t = E(\alpha_t \mid Y_t, X_t)$ and the filtered state variance matrix $P_t = Var(\alpha_t \mid Y_t, X_t)$. The (contemporaneous) Kalman filter algorithm (see e.g. Hamilton, 1994, or Durbin and Koopman, 2001) estimates a_t and P_t by updating, at time t, a_{t-1} and P_{t-1} using the new information contained in y_t and x_t. The Kalman filter recursion can be initialised by the assumption that $\alpha_1 \sim N(a_1, P_1)$. In practice, a_1 and P_1 are generally not known though. Therefore, we assume that the distribution of the initial state vector α_1 is diffuse, i.e. $\alpha_1 \sim N(0, \kappa I_m)$ where we let $\kappa \to \infty$. This assumption requires no prior knowledge about the initial state. The Kalman filter is modified to account for this diffuse initialisation by using the exact initial Kalman filter introduced by Ansley and Kohn (1985) and further developed by Koopman (1997) and Koopman and Durbin (2001).

Subsequently, the Kalman smoother algorithm is used to estimate the distribution of α_t, for $t = 1, \ldots, n$, conditional on Y_n and X_n, where $Y_n = \{y_1, \ldots, y_n\}$ and $X_n = \{x_1, \ldots, x_n\}$. Thus, the smoothed state vector $\widehat{a}_t = E(\alpha_t \mid Y_n, X_n)$ and the smoothed state variance matrix $\widehat{P}_t = Var(\alpha_t \mid Y_n, X_n)$ are estimated using all the observations for $t = 1, \ldots, n$. In order to account for the diffuse initialisation of α_1, we use the exact initial state smoothing algorithm suggested by Koopman and Durbin (2001).

[16] The exact elements of the vectors y_t, x_t and α_t and the matrices Z, A, T, R, H, and Q for both the univariate and the multivariate model are specified in appendix 3.

Given the complexity of the multivariate model, we do not use the entire observational vector y_t in the filtering and smoothing algorithm. Following Koopman and Durbin (2000), the elements of y_t are introduced into the filtering and smoothing algorithms one at a time, i.e. the multivariate analysis is converted into a univariate analysis. As the data can then be analysed in univariate form, this approach offers significant computational gains, particulary for the treatment of initialisation by diffuse priors.

3.3 Bayesian analysis

The filtering and smoothing algorithms both require that Z, A, T, R, H, and Q are known. In practice, these matrices generally depend on elements of an unknown parameter vector ψ. One possible approach is to derive, from the exact Kalman filter, the diffuse loglikelihood function for the model under study (see de Jong, 1991, Koopman and Durbin, 2000 and 2001) and replace the unknown parameter vector ψ by its maximum likelihood estimate. This is not the approach persued in this paper. Given the fairly large number of parameters and unobserved series that need to be identified, especially in the multivariate model, the numerical optimisation of the sample loglikelihood function becomes quite cumbersome. On the other hand, the models we use have been estimated in the past for different countries and samples. Therefore, we analyse the state space models from a Bayesian point of view, i.e. we treat ψ as a random parameter vector with a known prior density $p(\psi)$ and estimate posterior densities $p(\psi \mid y, x)$ and $p(\alpha_t \mid y, x)$ for the parameter vector ψ and the state vector α_t by combining information contained in $p(\psi)$ and the sample data. Essentially, this boils down to calculating the posterior mean $\overline{g}$:

$$\overline{g} = E\left[g(\psi) \mid y, x\right] = \int g(\psi)\, p(\psi \mid y, x)\, d\psi \tag{20}$$

where y and x denote the stacked vectors $(y_1', \ldots, y_n')'$ and $(x_1', \ldots, x_n')'$ respectively and g is a function which expresses the moments of the posterior densities $p(\psi \mid y, x)$ and $p(\alpha_t \mid y, x)$ in terms of the parameter vector ψ. The vector α_t equals the stacked vector $(\alpha_1', \ldots, \alpha_n')'$.

Equation (20) is evaluated using a simulation approach known as importance sampling. The idea is to obtain a sequence $\psi^{(1)}, \ldots, \psi^{(n)}$ of n random vectors from a density $g(\psi \mid y, x)$ which is as close to $p(\psi \mid y, x)$ as possible. Such a density is known as an importance density for $p(\psi \mid y, x)$. By Bayes' theorem and after some manipulations, equation (20) can be rewritten as

$$\overline{g} = \frac{\int g(\psi)\, z^g(\psi, y, x)\, g(\psi \mid y)\, d\psi}{\int z^g(\psi, y, x)\, g(\psi \mid y)\, d\psi} \tag{21}$$

with

$$z^g(\psi, y, x) = \frac{p(\psi)\, p(y \mid \psi)}{g(\psi \mid y)} \tag{22}$$

and where $p(y \mid \psi)$ is given by the likelihood function derived from the exact Kalman filter. Using a sample of n independent draws of ψ, denoted by $\psi^{(i)}$, from $g(\psi \mid y, x)$, an estimate of $\overline{g}$ can be obtained as

$$\overline{g}_n = \left(\sum_{i=1}^{n} g\left(\psi^{(i)}\right) z^g\left(\psi^{(i)}, y, x\right) \right) / \left(\sum_{i=1}^{n} z^g\left(\psi^{(i)}, y, x\right) \right) \quad (23)$$

Geweke (1989) shows that if $g(\psi \mid y, x)$ is proportional to $p(\psi \mid y, x)$, and under a number of weak regularity conditions, $\overline{g}_n$ will be a consistent estimate of $\overline{g}$ for $n \to \infty$.

As an importance density $g(\psi \mid y, x)$, we take a large sample normal approximation to $p(\psi \mid y, x)$, i.e.

$$g(\psi \mid y, x) = N\left(\widehat{\psi}, \widehat{\Omega}\right) \quad (24)$$

where $\widehat{\psi}$ is the mode of $p(\psi \mid y, x)$ obtained from maximising

$$\log p(\psi \mid y, x) = \log p(y \mid \psi) + \log p(\psi) - \log p(y) \quad (25)$$

with respect to $\widehat{\psi}$ and where $\widehat{\Omega}$ denotes the varaince-covariance matrix of $\widehat{\psi}$. Note that we do not need to calculate $p(y)$ as it does not depend on ψ.

4 Results

We use quarterly data for the US and euro area over the period 1970Q1-2003Q4. Due to lags and differencing of the price series, the actual sample runs from 1971Q2 to 2003Q4. Data sources are reported in appendix. The inflation series is the annualised first difference of the log of the seasonally adjusted GDP deflator. For the interest rate we use the annualised central bank key interest rate, as this interest rate should be most appropriate to infer changes in the central bank's behaviour. Output is measured as seasonally adjusted GDP at constant prices.

4.1 Parameter estimates

4.1.1 Prior distribution[17] of the parameters

Univariate model The priors for the autoregressive coefficients in the univariate model are chosen from studies allowing a break in the mean of the inflation rate. Levin and Piger (2004) for instance find a value of 0.36 for the sum of autoregressive coefficients of the US GDP deflator. Gadzinski and Orlandi (2004) found a somewhat higher figure of 0.6 for the euro area. Finally

[17] For all coefficients we assume a normal distribution around the prior mean. For the variances we assume a gamma distribution.

we chose a prior for the sum of the autoregressive coefficients of respectively 0.3 and 0.4 for the United States and the euro area. Our prior for δ lies around 0.8, which is close to what one can infer from the parameter values determining signal extraction in Erceg and Levin (2003) and Kozicki and Tinsley (2003), or sticky information in Mankiw and Reis (2002). As we want to stay quite agnostic about the time series characteristics of inflation, we leave the uncertainty around the priors relatively high.

Multivariate model Our multivariate priors[18] are consistent with the results of previous studies, estimating the model of Rudebusch and Svensson (1999) for different countries and samples[19]. As a prior for the sum of the autoregressive coefficients we chose a value of 0.45. The prior for the parameter δ^* is 1 minus the posterior mean of δ in the univariate model. For the impact of the lagged output gap on inflation we chose a value of 0.2. The autoregressive coefficients of the output gap are chosen in order to generate a hump-shaped response of output in reaction to a shock. This feature is often found in the previous empirical studies. For the equilibrium real interest rate we took a value just above 2 p.c., comparable to the sample means. The parameter value for ρ_2 assumes considerable interest rate smoothing, which is consistent with numerous studies on interest rate rules. The parameter value for ρ_1 is chosen so that the Taylor principle ($\frac{\rho_1}{1-\rho_2} > 1$) holds. The variances are also consistent with other empirical results. The variance of the inflation objective shock is close to what Smets and Wouters (2004) find. We argued that this variance in the United States could be higher relative to the euro area, as for the bigger part of the sample it is not an aggregate of different national key interest rates. As in our univariate setting we found that the permanent shocks occurred mainly in periods where monetary policy was changing its objective, we assumed low variance for the perceived inflation target shock.

4.1.2 Posterior distribution of the parameters

Univariate model Table 1 and 2 present posterior estimates for the sum of the autoregressive coefficients in the univariate model. The sum of coefficients respectively amounts to 0.4 and 0.58 for the euro area and the US. On the other hand, our estimate of δ, which reflects persistence following a permanent shift in the steady state inflation rate, is much higher than intrinsic inflation persistence, namely higher than 0.8. This measure of persistence is however significantly different from 1, so that the steady state inflation rate is not estimated to be an I(2) process.

Figures 4 and 10 show the dynamics of the inflation rate together with the

[18] In the case of rational expectations the priors we use could be susceptible to the Lucas critique, therefore we leave considerable uncertainty around the prior means.

[19] The studies we reviewed are: Rudebusch and Svensson (1999), Gerlach and Smets (1997), Peersman and Smets (1999), Rudebusch (2003), Domenech and Gomez (2003), Laubach and Williams (2001), Gerlach and Smets (1999).

steady state inflation rate for both the euro area and the US, estimated with the univariate model. The steady state inflation rate has significantly shifted since the 1970s. The shifts seem indeed to take place at times when monetary policy in both economies considerably changed. For the US, Paul Volcker became president of the Federal Reserve in 1979 and started a process of disinflation. In the euro area, it is more difficult to match this with narrative evidence as no unified monetary policy existed yet. Still, most euro area countries where disinflating in the beginning of the eighties. A second disinflation period is also clear in the case of the euro area in the beginning of the nineties. Future euro area countries were then disinflating in order to comply with the Maastricht criteria. In the United States there seems to have been a somewhat weaker change during that period.

The standardised permanent shocks are presented in Figures 5 and 11. For the interpretation one has to keep in mind that the smoothed shocks are the result of a weighted average of the filtered shocks in the time period before and after a specific point in time. That is why the smoothed estimates are sometimes also called two-sided estimates. We find three significant negative smoothed shocks that for the euro area culminate in 1974:Q1, 1982:Q1 and 1991:Q2. In the case of the United States the shocks occur in 1974:Q2 and 1980:Q3. These observations seem to be in line with common knowledge about historical monetary policy. In order to judge whether our results are significantly better than approaches that look for discrete breaks in the inflation series, one could calculate the likelihood of our model in the case of discrete breaks previously identified by other authors. This we leave for future research.

Multivariate model Table 3 and 4 summarise the parameter estimates for the multivariate model. Intrinsic inflation persistence, measured by the sum of the autoregressive coefficients, is estimated to amount to respectively 0.45 and 0.79 in the euro area and the United States. In the case of the euro area, the estimate for intrinsic persistence seems to be quite in line with what was estimated in the univariate specification. In the case of the United States, intrinsic inflation persistence is however somewhat higher. The estimate of δ^*, which determines the speed at which changes in the central bank target feed into long run inflation expectations, is 0.27 for the euro area and 0.15 for the United States. This comes close to 1 minus the estimate of the smoothing parameter δ of the univariate specification.

Figures 6 to 9 and 12 to 15 present the estimated dynamics of the different components of inflation in the euro area and the United states. In the case of the euro area, the steady state inflation rate again seems to have shifted considerably during the last three decades. From figure 7 we can infer that the shifts in the perceived inflation target were mainly induced by shifts in the central bank inflation target. We find that significant changes in the key interest rate coincide with changes in the central bank inflation target that slowly feed into the perceived inflation target of economic agents.

For the euro area the standardised smoothed shocks to the central bank inflation target are significantly negative during the periods 1976:Q2 and 1983:Q3. The identified shocks to the central bank inflation target seem to lie close to the permanent shocks we identified with our univariate model. As can be seen in equations (16) and (17), the difference can be explained by the fact that the permanent shock to inflation also comprises exogenous shifts to the steady state that are not caused by the central bank. Moreover, Figure 9 shows that our multivariate estimate of the perceived inflation target does not seem to differ significantly from our univariate estimate. Therefore, the estimates of intrinsic inflation persistence that depend on the correct identification of the steady state inflation rate should not be affected too much. However, our estimates could be sensitive to our identification of potential output and the equilibrium real interest rate. One could argue that the assumption of a constant natural or equilibrium real interest rate and trend potential output growth are too simplistic. Laubach and Williams (2003) estimate potential output as an I(2) process, where the time-varying trend growth rate is positively correlated with the equilibrium real interest rate. These authors conclude that in the United States trend growth and natural real interest rate vary considerably over the post-WWII period. This significantly affects the assessment of past monetary policy. We leave this extension to our model for future research.

Our results for the United States are generally close to those of the euro area. However, the link between the univariate permanent shocks and the multivariate central bank target shocks seems to be less close than in the case of the euro area. This is possibly due to a still too simplistic modelling of potential output. However, as in the case of the euro area the multivariate estimate of the perceived inflation target does not significantly differ from our univariate estimate.

4.2 Additional measures of persistence

4.2.1 Impulse response analysis

Assessing the persistence of the inflation process can also be done using impulse response functions of the different variables. Figures 16 to 19 present impulse response functions[20] in reaction to a temporary inflationary shock $\varepsilon_t^{*\pi}$, a shock to the perceived inflation target $\varepsilon_t^{*\pi^P}$, a shock to the central bank inflation target $\varepsilon_t^{\pi^T}$ and a shock to the output gap ε_t^z.

In the case the central bank wants to increase its inflation target by one p.c., it decreases its key interest rate in order to adjust its policy instrument to the new target. As the perception of economic agents about the inflation target

[20] The impulse response functions concern the euro area multivariate estimation results. The figures for the United States are very similar and therefore not included. They are available on request from the authors.

slowly reacts to this change, the decrease in the interest rate stimulates economic activity. Observed inflation closely follows the perceived inflation target. As the perceived inflation target starts to converge to the new central bank inflation target, the central bank can start to increase key interest rates. Moreover, the positive output gap creates inflation higher than the perceived inflation target, so that as soon as the perceived inflation target is close to the new central bank inflation target, the central bank will keep its interest rates up for some more time in order to close the output gap. In the new equilibrium, inflation, the perceived inflation target and the central bank inflation target will coincide at a level that lies one p.c. higher than before.

If the perception of economic agents about the inflation target is hit by a shock of 1 p.c., but the central bank inflation target remains at the same level, then economic agents will slowly learn from the behaviour of the central bank about their mistake. The initial uprise of economic agents' long run inflation expectations will stimulate economic activity as they believe the real interest rate has decreased. All this will cause inflation to be higher than the central bank's inflation target. The central bank will then react with a significant increase in interest rates. She will show to the economic agents that its inflation target has not changed, and will have to create a recession due to the slowly changing beliefs of economic agents.

In the case of a cost-push or temporary shock to inflation, the central bank will react to this with an increase in interest rates. However, economic agents understand that this is a temporary shock instead of a change in the central bank inflation target, and therefore will not adjust their perception of the inflation target. As intrinsic inflation persistence is not very high, inflation quickly returns back to its initial level.

When the economy is hit by a shock to the output gap, inflation will rise due to higher economic activity. The central bank will react to this with an increase in interest rates. The perceived and central bank inflation target will remain unchanged. Although intrinsic inflation persistence is low, the reaction of inflation to this shock will be rather persistent. This is due to the high persistence in the output gap.

Comparing the four shocks, it becomes clear that the reaction of inflation and the speed at which inflation evolves to a new equilibrium is highly dependent on the kind of shock. When the central bank changes its target, inflation never comes back to its inital level. In the case of a shock to the perceived inflation target, it takes several years before it returns to its initial level. However, in the case of a temporary shock it only takes a few quarters before it is again at its starting point.

Another interesting observation is that the impact of the shock on output is inversely related to the inflation persistence in reaction to the shock that is hitting the economy. In the case of an increase in the central bank's inflation target, the output gap increases to approximately 0.25 p.c. If one would simulate a decrease in the central bank inflation target of -1 p.c., the output gap would decrease to -0.25 p.c. On the other hand, the output gap decreases to -0.1 p.c. in the case of an upward shock to the perceived inflation target, and only -0.05 p.c. in the case of a temporary upward shock to inflation. Disinflations are thus very costly in terms of output compared to temporary inflation shocks due to the high persistence they are accompanied by.

4.2.2 Half life analysis

Another persistence parameter is the so-called half life (see table 5). It is defined as the number of periods for which the effect of a shock to a variable remains above half its inital impact. Before inflation is again below half of the initial impact of a tempoary shock it takes only one quarter, while for a shock to the perceived inflation target, it takes respectively 6 and 14 quarters in the euro area and the United States. For a shock to the output gap it takes even 19 and 17 quarters in the euro area and the United States. Finally, after the economy is hit by a shock to the inflation target, which by definition is permanent, it never returns below the initial impact of the shock. In other words, the half life is equal to infinity. These results show that making a distinction between the kind of shocks that are hitting inflation is particularly important for measuring inflation persistence.

5 Conclusions

Contrary to previous estimations, recent research concluded that without accounting for shifts in the steady state inflation rate, historical measures of inflation persistence incur a strong upward bias. In this paper, we have measured inflation persistence while accounting for potential shifts in the steady state inflation rate that can occur in every time period. We model the inflation process in a univariate and multivariate model, that is consistent with both views. Moreover, instead of assuming a pure random walk we propose an alternative stochastic process for the steady state inflation rate with a clear link to structural models of imperfect or sticky information.

We estimate the univariate and multivariate models using quarterly data for the United States and the euro area for the sample 1971Q2-2003Q4. We find evidence that post war inflation is indeed characterised by breaks in the steady state inflation rate. Therefore, intrinsic inflation persistence, which is determined by the structural way prices and wages are set, is not close to that of a random walk. Moreover, we also estimate a parameter that measures the persistence in reaction to changes in the policy target of the central bank. This parameter is linked to theories of imperfect credibility, asymmetric information and signal extraction, or sticky information. We find that the dissemination of changes in the policy target is typically very slow compared to temporary shocks, which next to the breaks in the mean inflation rate explains the high degree of post war inflation persistence.

The impulse response functions obtained from our multivariate model suggest that the speed at which inflation returns back to its initial level highly depends on the kind of shock that hits the economy. Persistence measured by the half life of a shock to inflation is relatively low in the case of a cost-push shock, whereas for a shock to the output gap or long run inflation expectations it is markedly higher. In the case of a shift in the policy target the half life even goes to infinity.

The implications for monetary policy are important. First, our evidence suggests that a measure of historical intrinsic inflation persistence, that is unaffected by previous policy shifts is relatively low compared to what was previously found. This suggests that a central bank with a credible inflation target needs to react less vigourously to shocks hitting inflation than one would infer from estimates that find inflation persistence close to that of a random walk. Second, the results also imply that in the case monetary policy makers would again get tempted to exploit the trade-off between inflation and economic activity, it would afterwards be very costly to disinflate due to the high persistence in response to changes in the inflation target.

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Appendix

Appendix 1: Sticky information and the pass-through of central bank target changes to the perceived inflation target

We here reformulate the sticky information model into a model that captures the gradual dissemination of information about the central bank's target π_t^T into long run inflation expectations π_t^P. As in Mankiw and Reis (2002), we assume that every period firms reset their prices. They infrequently gather information about the central bank inflation target π_t^T which is readily available in every period.

A firm's optimal price is:

$$p_t^* = p_{t-1}^P + \pi_t^T \tag{26}$$

Firms that last updated their beliefs about the inflation target j periods ago set their price:

$$\begin{aligned} x_t^j &= E_{t-j} p_t^* & (27) \\ &= p_{t-1-j}^P + (j+1)\pi_{t-j}^T & (28) \end{aligned}$$

The aggregate price level consistent with the perceived inflation target is:

$$p_t^P = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j x_t^j \tag{29}$$

$$p_t^P = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j (p_{t-1-j}^P + (j+1)\pi_{t-j}^T) \tag{30}$$

$$p_t^P = \lambda(p_{t-1}^P + \pi_t^T) + \lambda \sum_{j=0}^{\infty} (1-\lambda)^{j+1} (p_{t-2-j}^P + (j+2)\pi_{t-j-1}^T) \tag{31}$$

$$p_{t-1}^P = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j (p_{t-2-j}^P + (j+1)\pi_{t-j-1}^T) \tag{32}$$

subtracting (32) from (31) we get:

$$\pi_t^P = \lambda(p_{t-1}^P + \pi_t^T) + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \pi_{t-j-1}^T - \lambda^2 \sum_{j=0}^{\infty} (1-\lambda)^j (p_{t-2-j}^P + (j+2)\pi_{t-j-1}^T) \tag{33}$$

Rearranging (31):

$$\frac{1}{1-\lambda} p_t^P = \frac{\lambda}{1-\lambda}(p_{t-1}^P + \pi_t^T) + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j (p_{t-2-j}^P + (j+2)\pi_{t-j-1}^T) \tag{34}$$

Rearranging:

$$\frac{-\lambda}{1-\lambda}(p_t^P - \lambda p_{t-1}^P - \lambda \pi_t^T) = -\lambda^2 \sum_{j=0}^{\infty}(1-\lambda)^j (p_{t-2-j}^P + (j+2)\pi_{t-j-1}^T) \quad (35)$$

Substituting the third term in (33) gives:

$$\pi_t^P = \lambda \pi_t^T + \lambda \sum_{j=0}^{\infty}(1-\lambda)^{j+1}\pi_{t-j-1}^T) \quad (36)$$

This is equivalent to:

$$\pi_t^P = (1-\lambda)\pi_{t-1}^P + \lambda \pi_t^T \quad (37)$$

Setting λ equal to δ we get equation (14).

The difference between the imperfect information and sticky information models is how information arrives. In the first the exact information is not available which leads to a signal extraction problem, in the second the exact information is available, but is not updated every period due to for instance information gathering costs.

Appendix 2: Data

- **Inflation**: quarterly inflation rate, defined as $400(\ln P_t - \ln P_{t-1})$, with P_t the seasonally adjusted quarterly GDP deflator, the seasonally adjusted quarterly CPI and seasonally adjusted underlying CPI (United States only). Sources: AWM (Fagan et al, 2001) and BIS;
- **Real output**: quarterly $\ln(GDP_t)$, with GDP_t the seasonally adjusted quarterly GDP in constant prices. Sources: AWM (Fagan et al, 2001) and BIS;
- **Key interest rate**: quarterly central bank key interest rate. Sources: NCB and ECB calculations and BIS.
- **Long term inflation expectations**: Six to ten years ahead inflation expectations. Sources: Consensus Forecasts and ECB calculations.

Appendix 3: State space representation

$$y_t = Z\alpha_t + Ax_t + \varepsilon_t \qquad \varepsilon_t \sim N(0,H) \tag{38}$$

$$\alpha_{t+1} = T\alpha_t + R\eta_t \qquad \eta_t \sim N(0,Q) \tag{39}$$

Univariate model: state space representation

$$y_t = \begin{bmatrix} \pi_t \end{bmatrix};\ Z = \begin{bmatrix} (1-\sum_{i=1}^{4}\alpha_i) & 0 \end{bmatrix};\ \alpha_t = \begin{bmatrix} \pi_t^P \\ \pi_{t-1}^P \end{bmatrix};$$

$$A = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{bmatrix};\ x_t = \begin{bmatrix} \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ \pi_{t-4} \end{bmatrix};\ H_t = [\varepsilon_t^{\pi}];\ T = \begin{bmatrix} 1+\delta & -\delta \\ 1 & 0 \end{bmatrix};$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix};\ Q_t = \left[\varepsilon_t^{\pi^P}\right]$$

Multivariate model: state space representation

$$y_t = \begin{bmatrix} \pi_t \\ y_t \\ i_t \end{bmatrix};\ Z = \begin{bmatrix} 0 & -\beta_1 & 0 & 0 & 0 & \left(1-\sum_{i=1}^{4}\alpha_i\right) & 0 \\ 1 & -\beta_2 & -\beta_3 & 0 & 0 & 0 & \beta_4 \\ 0 & 0 & 0 & 0 & -\rho_1 & (1-\rho_2) & 0 \end{bmatrix};$$

$$\alpha_t = \begin{bmatrix} y_t^P \\ y_{t-1}^P \\ y_{t-2}^P \\ \lambda \\ \pi_t^T \\ \pi_t^P \\ \pi_{t-1}^P \end{bmatrix};\ A = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \beta_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_2 & \beta_3 & -\beta_4 & 0 \\ \rho_1 & 0 & 0 & 0 & 0 & 0 & \rho_2 & (1-\rho_2)r \end{bmatrix};$$

$$x_t = \begin{bmatrix} \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ \pi_{t-4} \\ y_{t-1} \\ y_{t-2} \\ i_{t-1} \\ 1 \end{bmatrix};\ H_t = \begin{bmatrix} \sigma^2_{\varepsilon^{*\pi}} & 0 & 0 \\ 0 & \sigma^2_{\varepsilon^z} & 0 \\ 0 & 0 & \sigma^2_{\varepsilon^i} \end{bmatrix};\ T = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta & (1-\delta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix};$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}; Q_t = \begin{bmatrix} \sigma^2_{\varepsilon^{y^P}} & 0 & 0 \\ 0 & \sigma^2_{\varepsilon^{\pi^T}} & 0 \\ 0 & 0 & \sigma^2_{\varepsilon^{*\pi^P}} \end{bmatrix}$$

Tables and Figures

Table 1: Univariate model Euro Area (1971Q2-2003Q4)

		Prior distribution			Posterior distribution		
Parameter	Distribution	5 p.c.	Mean	95 p.c.	5 p.c.	Mean	95 p.c.
α_1	normal	0.04	0.2	0.37	0.14	0.26	0.37
α_2	normal	-0.07	0.1	0.27	-0.00	0.11	0.21
α_3	normal	-0.12	0.05	0.22	-0.18	-0.08	0.03
α_4	normal	-0.12	0.05	0.22	0.01	0.11	0.21
$\sum_{i=1}^{4} \boldsymbol{\alpha}_i$	-	-	**0.4**	-	-	**0.4**	**-**
$\boldsymbol{\delta}$	**normal**	**0.59**	**0.75**	**0.92**	**0.73**	**0.82**	**0.90**
$\sigma^2\ \varepsilon_t^{\pi}$	gamma	0.33	1.22	2.60	1.35	1.65	2.03
$\sigma^2\ \varepsilon_t^{\pi^P}$	gamma	+0.00	+0.00	0.01	+0.00	+0.00	0.01

Table 2: Univariate model United States (1971Q2-2003Q4)

		Prior distribution			Posterior distribution		
Parameter	Distribution	5 p.c.	Mean	95 p.c.	5 p.c.	Mean	95 p.c.
α_1	normal	-0.05	0.2	0.45	0.26	0.4	0.55
α_2	normal	-0.02	0.1	0.22	0.06	0.16	0.25
α_3	normal	-0.03	0.0	0.03	-0.02	0.01	0.04
α_4	normal	-0.03	0.0	0.03	-0.02	0.01	0.04
$\sum_{i=1}^{4} \boldsymbol{\alpha}_i$	**-**	**-**	**0.3**	-	**-**	**0.58**	**-**
$\boldsymbol{\delta}$	**normal**	**0.83**	**0.85**	**0.87**	**0.83**	**0.85**	**0.87**
$\sigma^2\ \varepsilon_t^{\pi}$	gamma	0.27	1.22	2.12	1.07	1.32	1.64
$\sigma^2\ \varepsilon_t^{\pi^P}$	gamma	+0.00	+0.00	0.01	+0.00	+0.00	0.01

Table 3: Multivariate model Euro Area (1971Q2-2003Q4)

		Prior distribution			Posterior distribution		
Parameter	Distribution	5 p.c.	Mean	95 p.c.	5 p.c.	Mean	95 p.c.
α_1	normal	0.14	0.30	0.47	0.20	0.29	0.4
α_2	normal	-0.02	0.15	0.31	0.02	0.13	0.24
α_3	normal	-0.20	-0.03	0.13	-0.21	-0.11	-0.00
α_4	normal	-0.13	0.04	0.20	0.04	0.14	0.25
$\sum_{i=1}^{4} \boldsymbol{\alpha}_i$	**-**	**-**	**0.46**	**-**	**-**	**0.45**	**-**
$\boldsymbol{\delta}^*$	**normal**	**0.02**	**0.18**	**0.35**	**0.17**	**0.27**	**0.38**
β_1	normal	0.18	0.20	0.22	0.19	0.20	0.21
β_2	normal	1.37	1.40	1.43	1.39	1.42	1.44
β_3	normal	-0.53	-0.5	-0.47	-0.50	-0.47	-0.45
β_4	normal	-0.00	0.15	0.03	-0.06	0.02	0.1
r	normal	1.89	2.05	2.21	1.89	2.03	2.20
ρ_1	normal	0.17	0.20	0.23	0.14	0.17	0.19
ρ_2	normal	0.87	0.90	0.93	0.88	0.91	0.93
$\sigma^2_{\varepsilon^{*\pi}}$	gamma	0.43	1.50	3.16	1.29	1.60	1.99
$\sigma^2_{\varepsilon^z}$	gamma	0.08	0.30	0.6	0.24	0.3	0.36
$\sigma^2_{\varepsilon^i}$	gamma	0.08	0.30	0.62	0.28	0.35	0.42
$\sigma^2_{\varepsilon^{y^P}}$	gamma	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma^2_{\varepsilon^{\pi^T}}$	gamma	0.02	0.09	0.18	0.06	0.11	0.20
$\sigma^2_{\varepsilon^{*\pi^P}}$	gamma	+0.00	+0.00	+0.00	+0.00	+0.00	0.01

Table 4: Multivariate model United States (1971Q2-2003Q4)

		Prior distribution			Posterior distribution		
Parameter	Distribution	5 p.c.	Mean	95 p.c.	5 p.c.	Mean	95 p.c.
α_1	normal	0.06	0.20	0.36	0.24	0.35	0.42
α_2	normal	-0.01	0.15	0.31	0.09	0.17	0.28
α_3	normal	-0.11	0.05	0.21	0.05	0.15	0.23
α_4	normal	-0.11	0.05	0.21	0.06	0.14	0.23
$\sum_{i=1}^{4} \boldsymbol{\alpha}_i$	**-**	**-**	**0.45**	**-**	**-**	**0.79**	**-**
$\boldsymbol{\delta}^*$	**normal**	**0.13**	**0.15**	**0.17**	**0.14**	**0.15**	**0.16**
β_1	normal	0.18	0.2	0.21	0.18	0.20	0.21
β_2	normal	1.41	1.45	1.48	1.42	1.45	1.47
β_3	normal	-0.58	-0.55	-0.51	-0.56	-0.54	-0.51
β_4	normal	-0.01	0.16	0.3	0.03	0.06	0.09
r	normal	1.89	2.05	2.21	1.89	2.05	2.22
ρ_1	normal	0.18	0.20	0.21	0.18	0.19	0.21
ρ_2	normal	0.88	0.90	0.91	0.88	0.89	0.90
$\sigma^2_{\varepsilon^{*\pi}}$	gamma	0.40	1.50	3.14	1.06	1.29	1.48
$\sigma^2_{\varepsilon^z}$	gamma	0.08	0.30	0.6	0.48	0.57	0.66
$\sigma^2_{\varepsilon^i}$	gamma	0.08	0.30	0.6	0.87	1.06	1.22
$\sigma^2_{\varepsilon^{y^P}}$	gamma	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma^2_{\varepsilon^{\pi^T}}$	gamma	0.10	0.40	0.84	0.07	0.17	0.53
$\sigma^2_{\varepsilon^{*\pi^P}}$	gamma	0.00	0.00	0.00	0.00	0.00	0.00

Table 5: Half lives of inflation (quarters)

	Euro area	United States
Temporary inflation shock	1	1
Perceived inflation target shock	6	14
Output gap shock	19	17
Central bank target shock	∞	∞

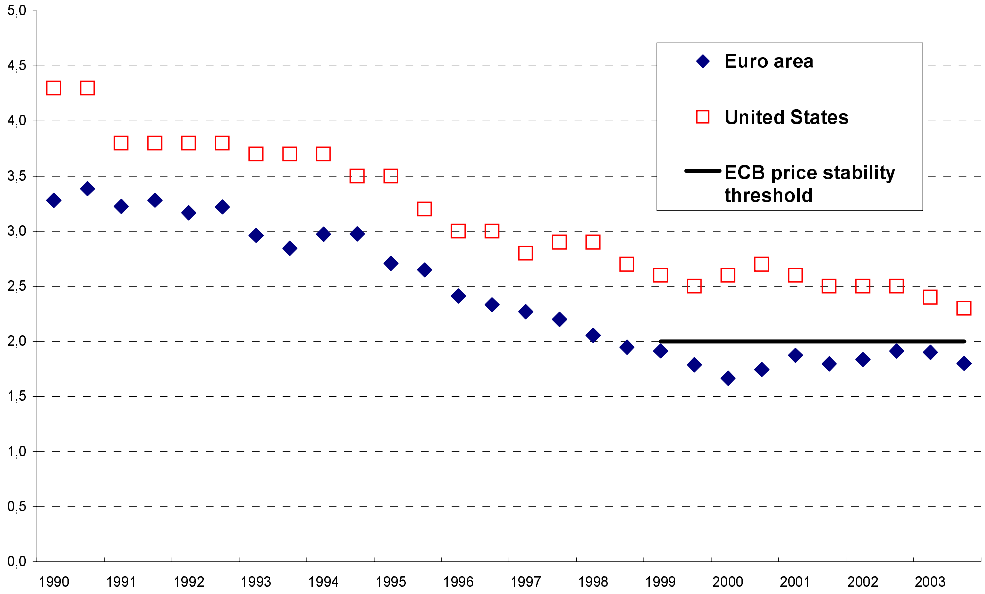

Figure 1: Long run inflation expectations. Source: Consensus Economics.

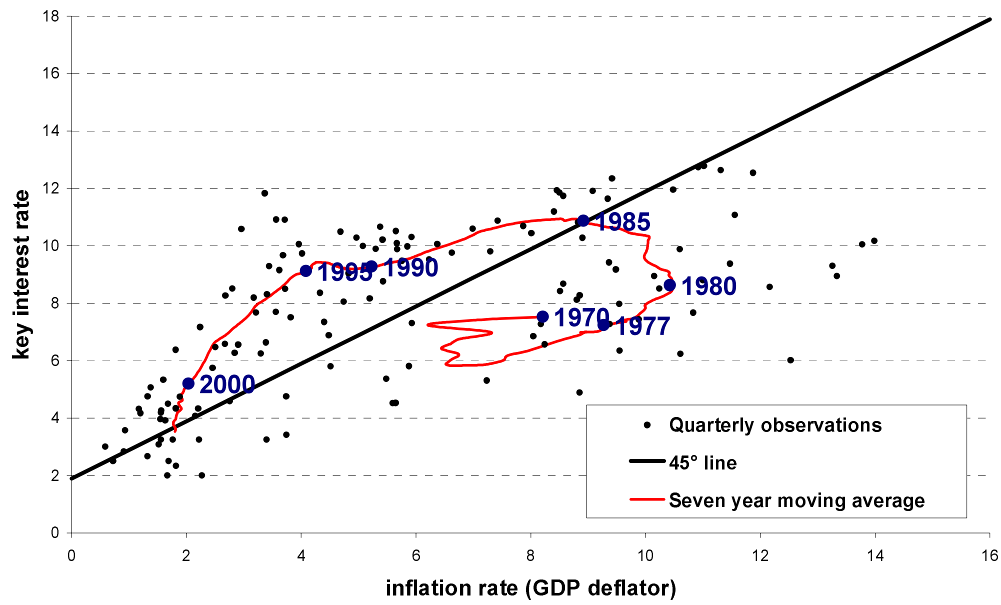

Figure 2: Shifts in the euro area inflation target

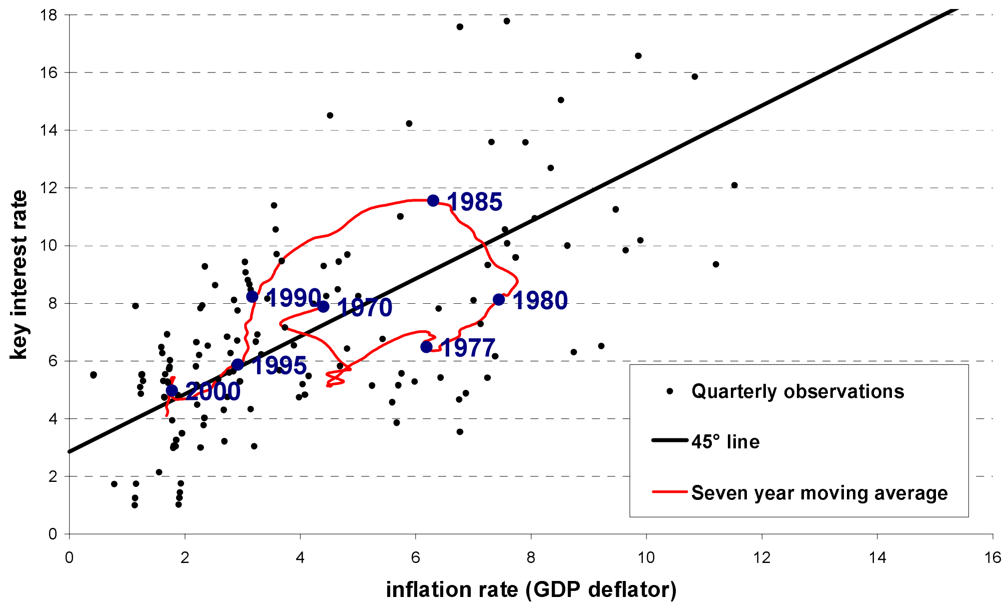

Figure 3: Shifts in the United States inflation target

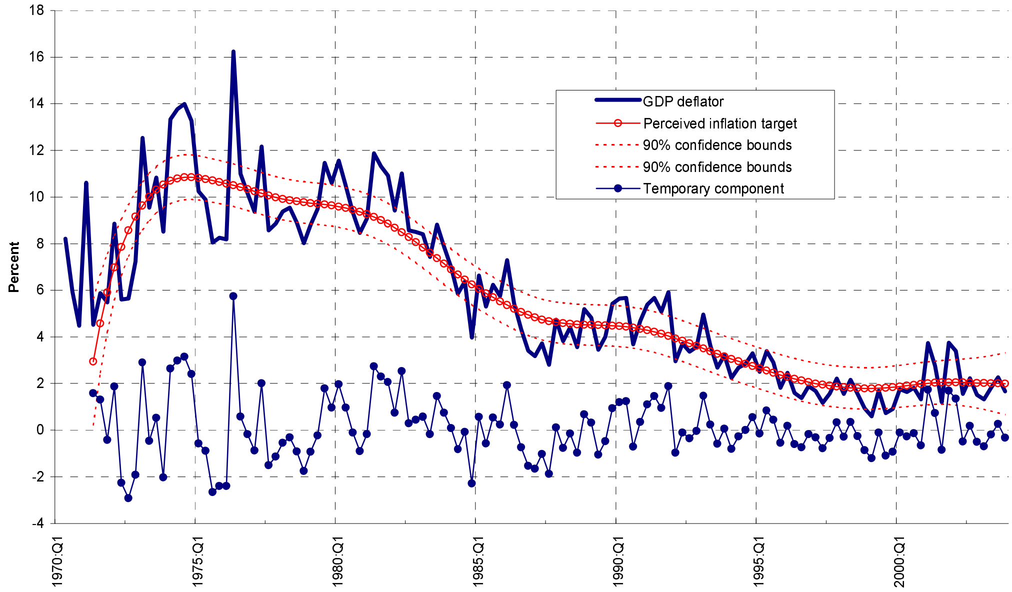

Figure 4: Euro area smoothed univariate states

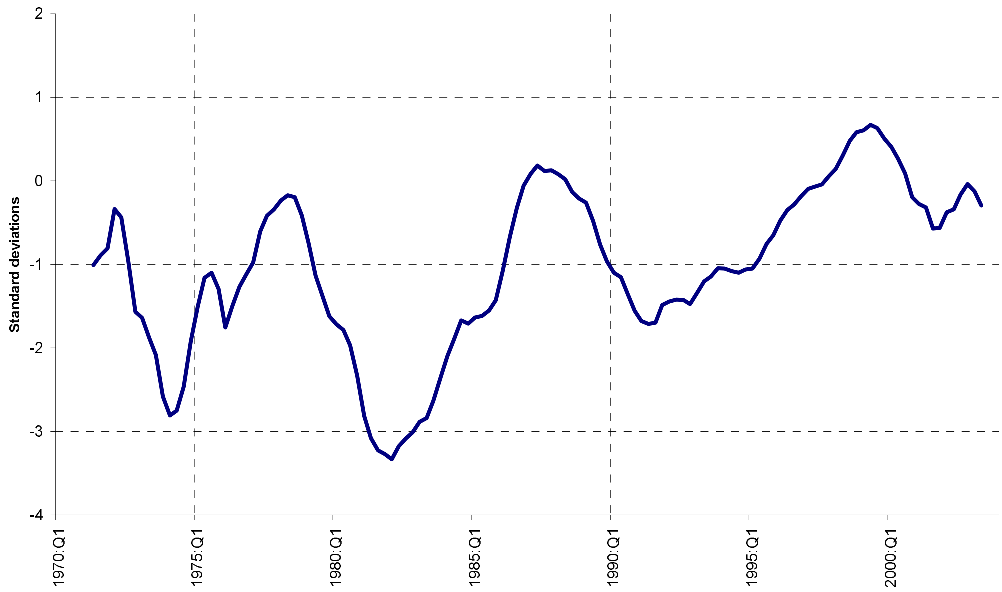

Figure 5: Euro area standardised and smoothed permanent shocks (univ. model)

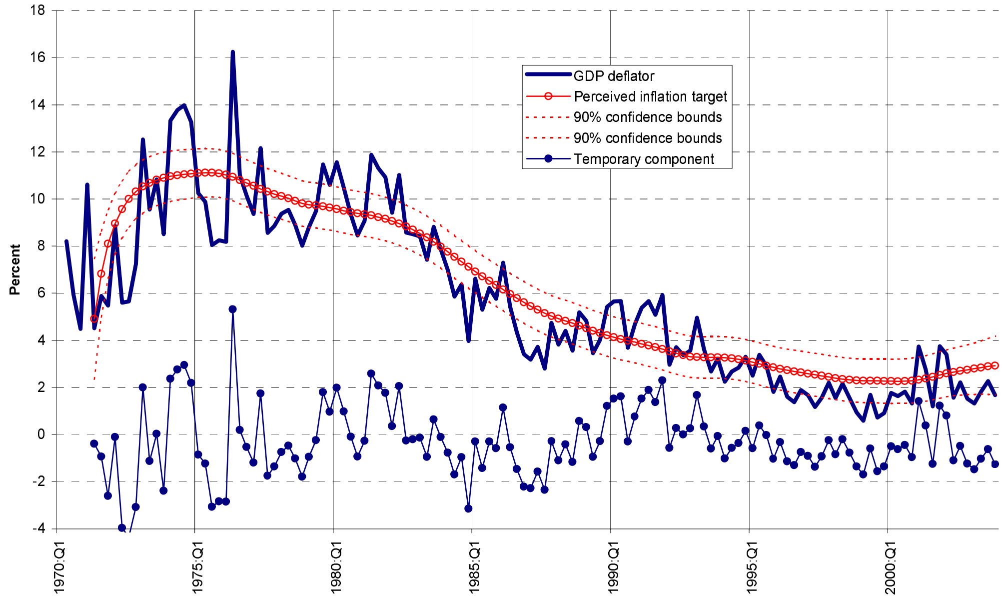

Figure 6: Euro area smoothed multivariate states

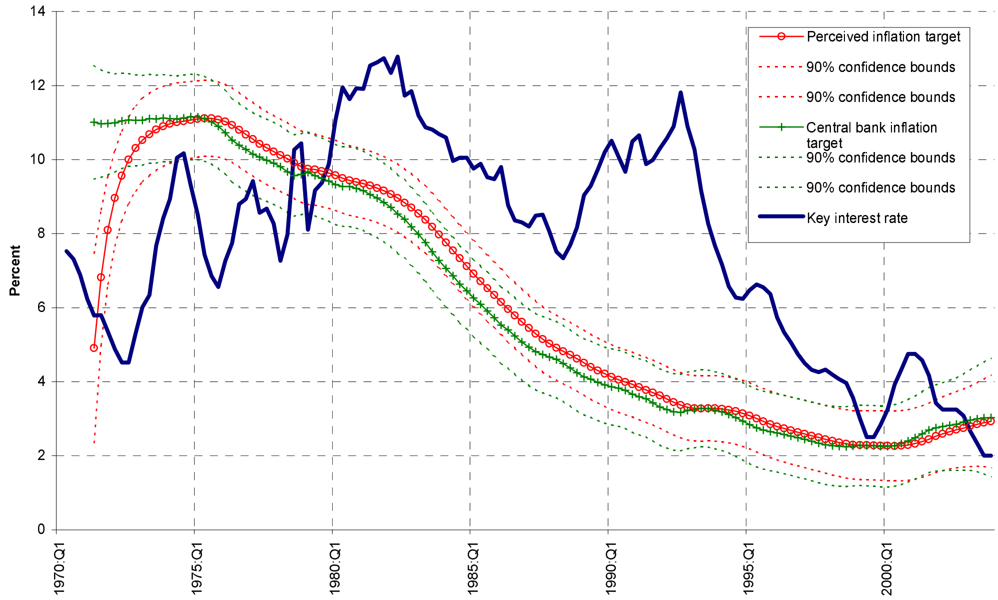

Figure 7: Euro area smoothed multivariate states (continued)

Figure 8: Euro area smoothed and standardised central bank target shocks

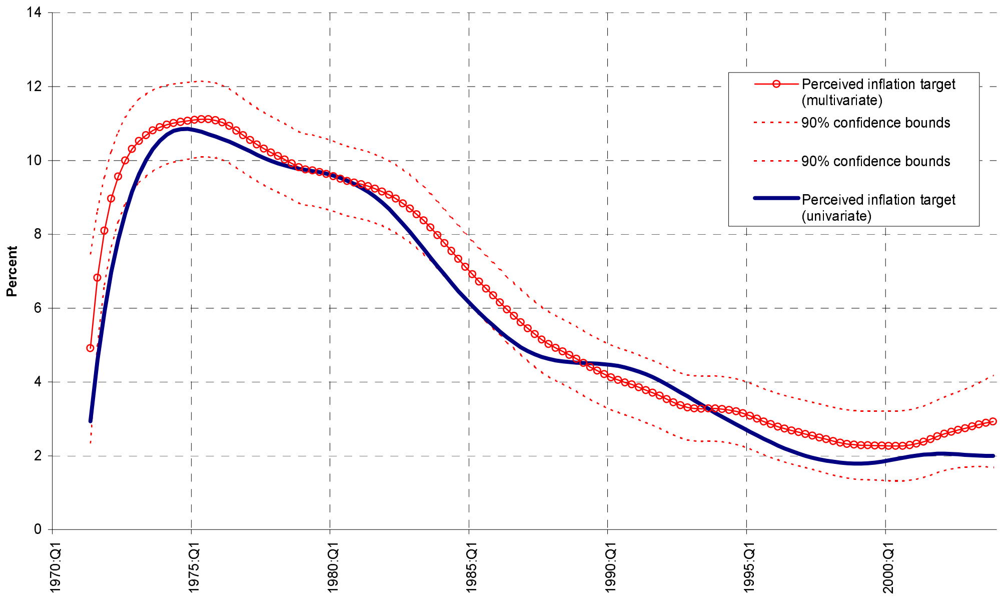

Figure 9: Euro area smoothed univariate and multivariate states

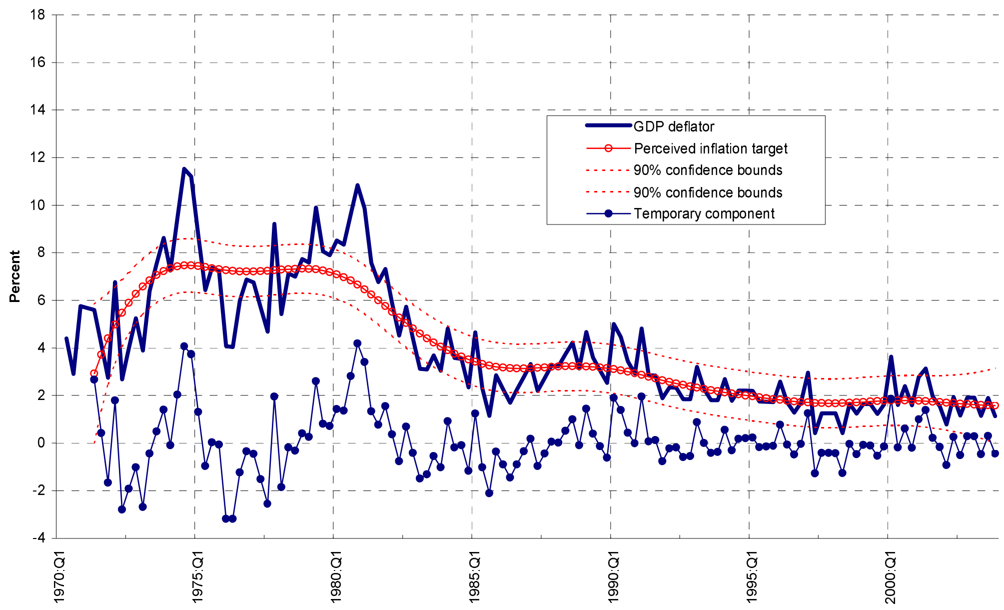

Figure 10: United States smoothed univariate states

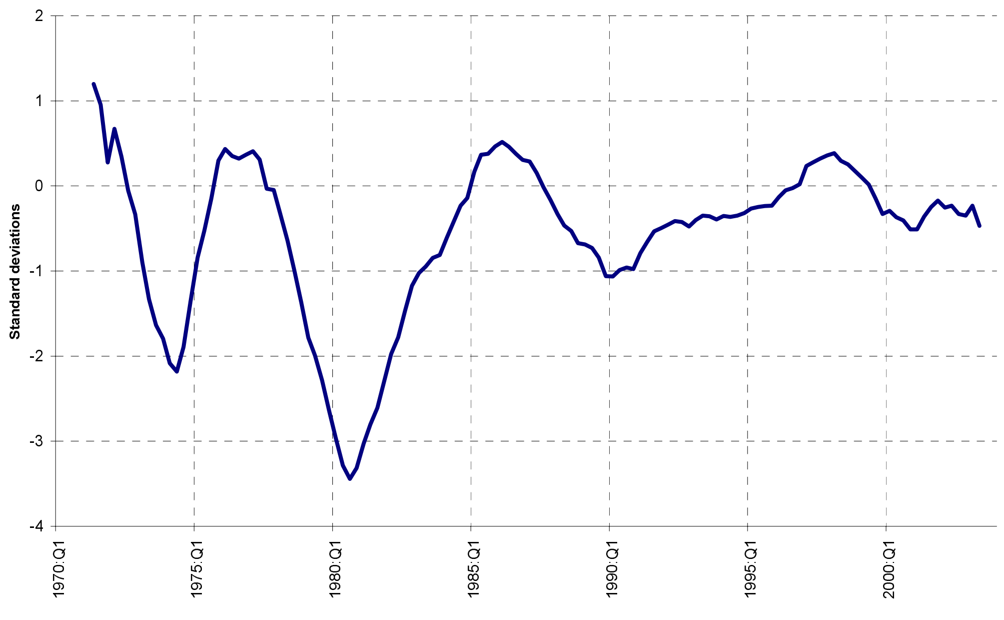

Figure 11: United States smoothed and standardised permanent shocks (univ.model)

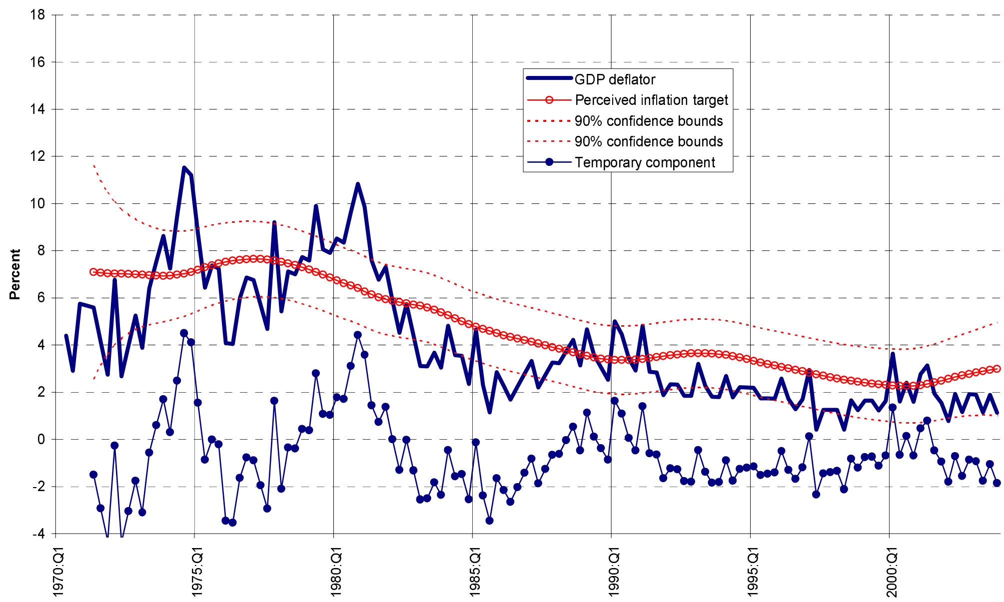

Figure 12: United States smoothed multivariate states

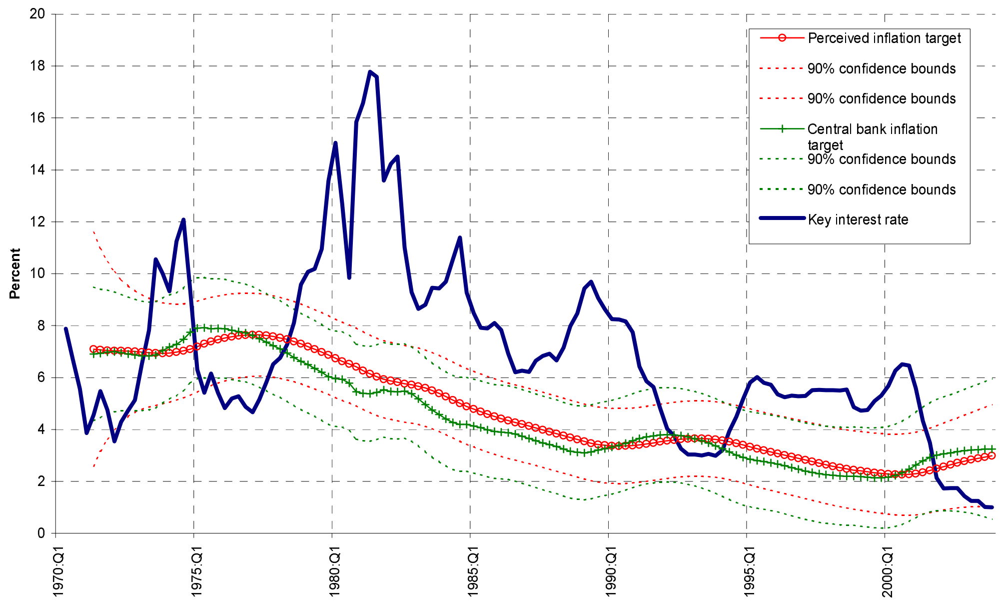

Figure 13: United States smoothed multivariate states (continued)

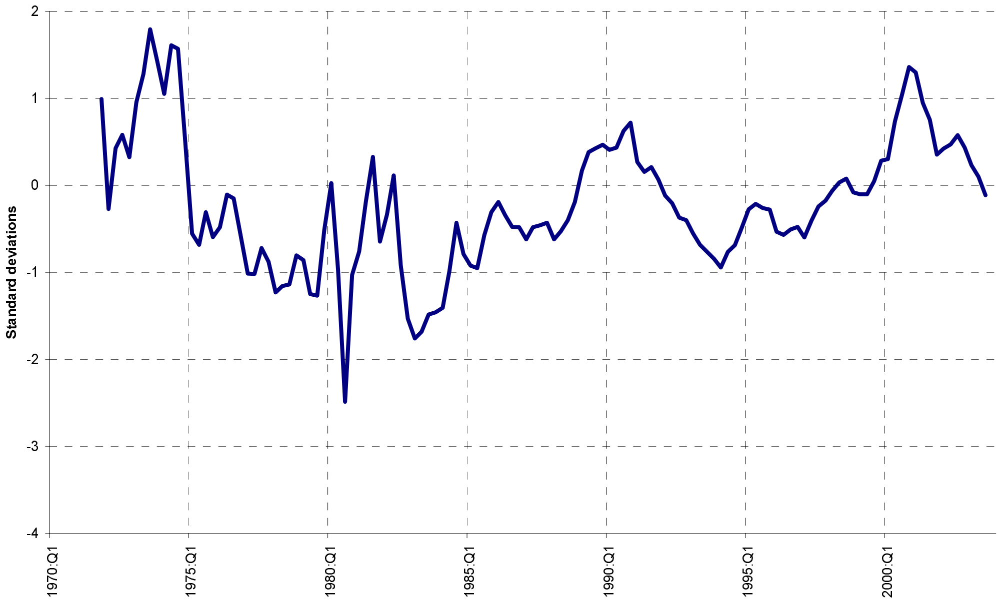

Figure 14: United States smoothed and standardised central bank target shocks

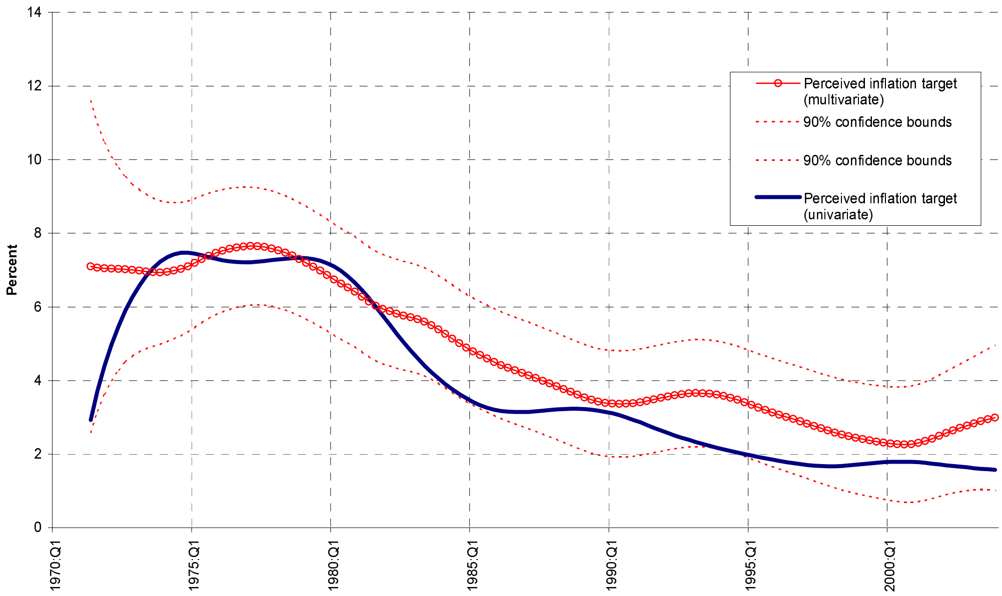

Figure 15: United States smoothed univariate and multivariate states

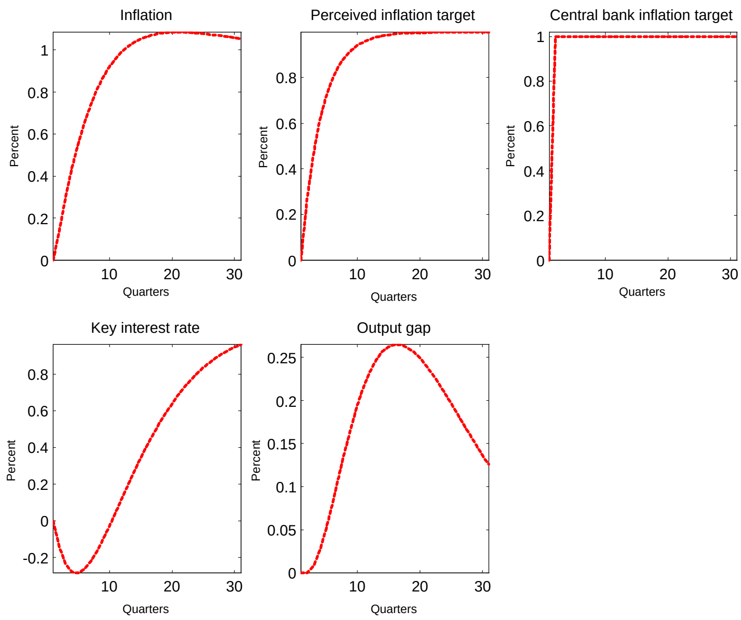

Figure 16: Impulse responses to a central bank target shock (euro area)

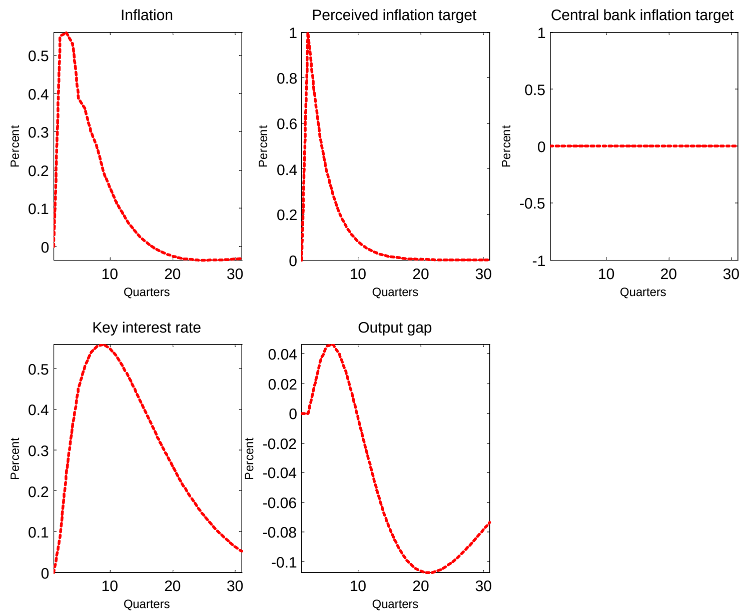

Figure 17: Impulse responses to a perceived inflation target shock (euro area)

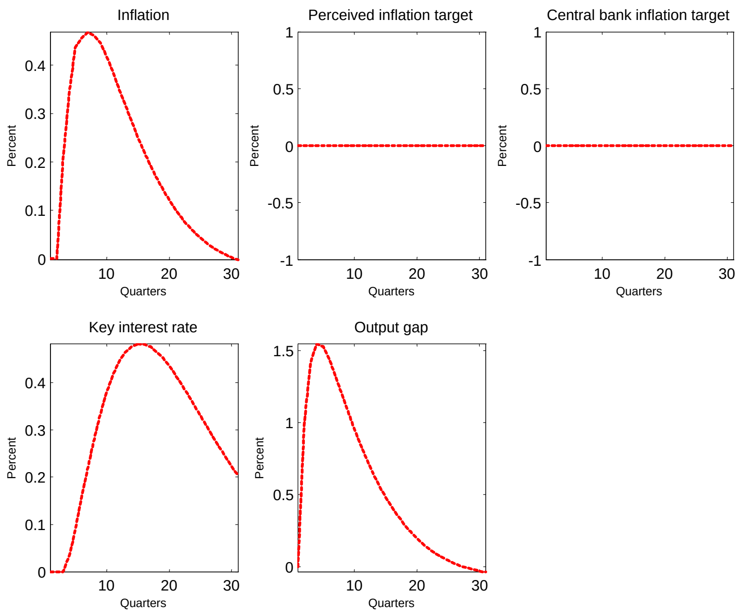

Figure 18: Impulse responses to an output gap shock (euro area)

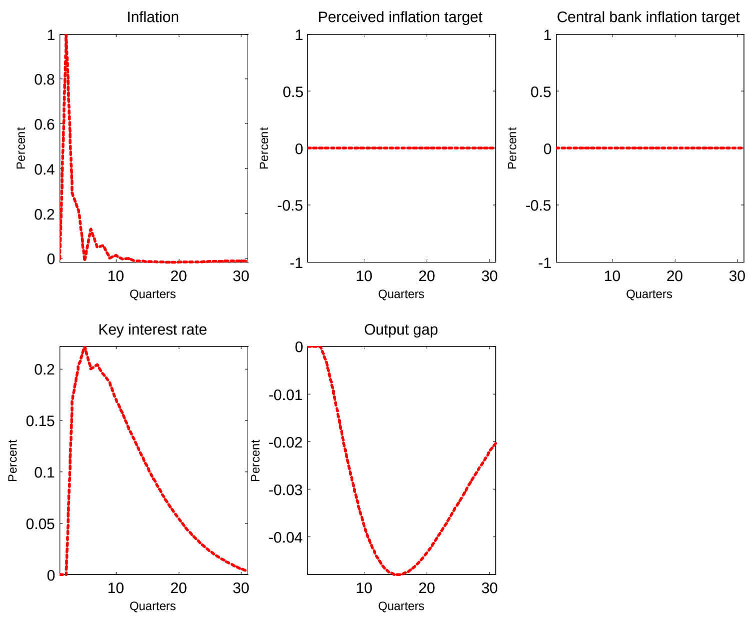

Figure 19: Impulse responses to a temporary inflation shock (euro area)