Bubbles and fads in the stock market: another look at the experience of the US.

Piergiorgio Alessandri[*]

December 28, 2004

Abstract

This paper considers a standard present-value equity price formula with time-varying discount rates, and proposes a state-space formulation that allows a decomposition of price fluctuations into fundamental and non-fundamental components. By "fundamentals" we refer to dividends, interest rates and risk premia, both actual and expected; the "non-fundamental" price component is defined residually allowing for the possibility of a rational bubble.

The empirical application uses annual US data, postulating a simple discount factor driven by the real return on short-term public debt. The stochastic factor explains part of the volatility in prices, but it is not sufficient to exclude the occurrence of near-exponential bubbles analogous to those found in the constant discount literature. On the contrary, definition and measurement of the fundamentals, and particularly of the dividend payout, prove to be crucial in this respect.

1 Introduction.

Was there a bubble in the nineties? The behaviour of the US stock market in the last ten years has been exceptional in many respects. In the post-war period, the value of equity fluctuated between 40% and 100% of the gross domestic product. Between 1995 and 2000 the stock market capital-isation increased from 100% to 180%; by the end of 2002 it was back to roughly 120%. A cursory glance at the price-dividend and price-earnings ratios delivers the same, simple message: prices were "unusually high" for a few years, and then suddenly reverted to more "normal" levels. Some ana-lysts attempted an explanation of these facts on the basis of fundamentals only. It has been suggested that once the dividend is replaced by a more sophisticated measure of the net cashflow generated by the firms, the behav-iour of the market appears to be consistent with some version of the classic Gordon (1962) model (Robertson and Wright 2003, Wright 2004). Others admit that the decade was extraordinary, but explain it by pointing to ex-traordinary structural conditions. High prices may have been determined for instance by firms accumulating large stocks of intangible capital (e.g. Hall, 2001) or by the baby-boom cohort saving for retirement (Geanakoplos *et al.*, 2002). These models maintain that shares are priced according to a present-value formula, and that all low-frequency market movements are driven by observed or expected changes in fundamentals. A common criticism is that, even when they convincingly explain the price runup, these theories are silent on the causes of the subsequent slump.

The alternative is to consider non-fundamental explanations, namely de-partures from the present-value pricing principle. Not surprisingly, the nineties revived the interest for price bubbles on the media and in the academic litera-ture: the observed boom-and-bust cycle is in a way easier to rationalize after accepting the idea that price changes are not always related to changes in the present value of rationally forecasted future dividends. This line of investi-gation presents of course its own difficulties. Rational, self-fulfilling bubbles can be ruled out in many contexts by some backward induction argument, and speculative phenomena of a non-rational nature are difficult to model in a non-arbitrary way. Furthermore, the empirical investigation has not pro-duced clear answers on the existence and nature of price misalignments, and many factors suggest that the task is intrinsically very difficult.

This paper proposes a strategy to detect equity price misalignments, and

2

presents an empirical application based on US data. The next section surveys the literature on bubbles, focussing on the empirical research. In section 3 we discuss a general framework that allows to test for bubbles under different assumptions on the nature of the stochastic discount factor. The key component of the model is a linearised price formula obtained by a first order Taylor approximation around the unconditional mean of the discount factor (Shiller (1981), Poterba and Summers (1986), West (1987)). The approximation can be used to model prices, fundamentals and bubbles in a state-space framework, which allows maximum likelihood estimation of the bubble through the Kalman filter (Burmeister and Wall (1982), Wu (1995, 1997), Chen *et al.* (2001)). An original feature of the model is that, due to the stochastic discount factor, the rate of growth of the bubble changes over time.

Section 4 examines the implications of the model when (i) agents discount on the basis of a safe return R(t) augmented by a constant risk premium, (ii) R(t) follows a stationary autoregressive process of order one and (iii) dividends are expected to grow at a constant rate. The main prediction of the model is a negative linear relationship between the price-dividend ratio and the safe rate, the strength of which depends on the persistence of the shocks to R(t). If the shocks are persistent, a high R(t) is expected to be followed by a period of high rates, or low discount factors, which lowers the contemporaneous price-dividend ratio. The model is estimated on three different sets of annual observations on the US stock market. We use a Standard&Poors dataset, a non-financial industry dataset, and an adjusted non-financial industry dataset where the dividend is computed netting out new share issues and buybacks. The safe return R(t) is taken to be the real rate on short-term public debt. In section 5 we compare the explanatory power of the exponential bubble to that of the "intrinsic" bubble proposed by Froot and Obstfeld (1991). Section 6 concludes.

Our results can be summarised as follows. The assumption on the discount factor holds in all datasets; the interest rate has a significant negative impact on the price-dividend ratio, which also confirms that there is a monetary policy channel operating through the stock market. We estimate significant bubbles in two datasets out of three: the S&P price index and, to a smaller extent, the non-adjusted industry share price are inflated by speculation in the sixties and late ninenties. In both cases, exponential bubbles with time-varying rates of growth fit the data better than the intrinsic

bubble. There is no evidence of bubbles in the adjusted dataset.

The analysis suggests two conclusions. Firstly, stochastic discount factors and rational bubbles may well coexist. It is known that US equity prices appear to be "too volatile" even after accounting for time variation in interest rates (Campbell and Shiller (1988), West (1988)); our work shows that speculative bubbles are indeed a plausible explanation for the excess volatility. Secondly, measurement issues are crucial: a price-dividend ratio adjusted for new issues and buybacks is clearly incompatible with bubbles, be them extrinsic or intrinsic. This is a strong incentive to think more carefully about which fundamental the agents consider when evaluating shares.

2   Testing for bubbles.

The theoretical conditions under which rational, self-fulfilling bubbles can arise are known (Blanchard and Watson (1982), Tirole (1982, 1985), Stiglitz (1990), LeRoy (2004) among others). Bubbles cannot occur in a finite-horizon economy, or in an economy with a finite number of infinitely lived traders. In a deterministic overlapping generations model, the bubble can only arise if there are dynamic inefficiencies leading to over-accumulation of capital. If the economy is efficient, the bubble will eventually exceed the value of all available resources; as long as the agents are aware of this potential inconsistency, bubbles can again be ruled out by a backward induction argument[1]. Other departures from the present-value formula are possible. Price misalignments may occur due to the existence of "noise traders" who do not behave in a fully rational way (Black (1986), Abreu and Brunnermeier (2003)). The misalignment, or fad, is then the difference between the actual price and the price that would be observed if all traders were rational. Since this will be in general a mean-reverting process, in this case there are no long-run consistency issues. Though, a limit of this literature is that the behaviour of noise traders is often taken as given or modelled relying on *ad hoc* assumptions.

---

[1]This type of awareness is widely assumed but not completely uncontroversial. LeRoy (2004) suggests that agents who by definition never experienced a model failure may not appreciate the difference between a consistent and an inconsistent path - which would make the latter as likely as the former.

A large literature stimulated by Shiller (1981) and LeRoy and Porter (1981) has shown that equity prices are too volatile to be consistent with a constant discount present-value formula. Models with stochastic discount factors linked to interest rates, consumption growth or variance of market returns attenuate the problem but do not solve it completely: a substantial portion of the variance of the price-dividend ratio remains unexplained (Poterba and Summers (1986), Campbell and Shiller (1988), West (1988)). Bubbles and fads are a potential explanation for the excess volatility, but any attempt to test this hypothesis faces a basic conceptual difficulty: price movements can in principle be rationalised by misalignments or by omitted "fundamental" state variables (Hamilton and Whiteman (1985), Flood and Hodrick (1990)).

It is possible to test for price misalignments without relying on a specific parametrisation of the misalignment itself. West (1987) notes that the discount factor can be estimated from a one-period arbitrage equation or from its forward solution, under some assumption on the process used to forecast dividends. If there is no bubble, the estimates should coincide. With a rational bubble, the estimates should diverge because the first equation holds but the second omits a variable. Hence, a test on the two set of estimates being equal is a test of the null that there is no bubble. The author uses annual US data and rejects the null that there are no bubbles under several alternative specifications of the dividend process. Chirinko and Shaller (1996, 2001) apply an analogous idea to Tobin's Q investment model; in this case the Euler and equilibrium equations describe the optimal investment policy of a representative firm. They find evidence of bubbles in the US and in Japan[2].

Blanchard *et al.* (1993) and Bond and Cummins (2001) ask whether deviations of equity prices from fundamentals can be responsible for the typically poor empirical performance of Q investment equations. Both papers compare a stock market measure of Q with one computed on the basis of fundamentals. Blanchard *et al.* (1993) use annual aggregate US data and find that, if the equation contains a "fundamental Q" based on profits, the

---

[2]These tests have some limitations. Any misalignment other than a rational bubble causes the failure of both the Euler and the equilibrium equation, which in theory invalidates the procedure. An exponential rational bubble would on the other hand affect the distribution of the test statistic and possibly make the test inconsistent (West (1987), Chirinko and Shaller (2001)). Finally, the procedure may fail to detect bubbles if these are completely uncorrelated with fundamentals.

marginal explanatory power of the market-related variable is negligible. The conclusion is that misalignments are small and/or firms ignore them. Bond and Cummins (2001) use data on a panel of American firms, and measure Q relying on published earnings forecasts; this variable appears to be a sufficient statistic for investment and delivers plausible estimates of the adjustment cost parameters. The authors infer that the Q model is correct, but its parameters cannot be identified using stock market data because share prices depart persistently from their fundamental level.

Sarno and Taylor (1999) show that, if there is no rational bubble, the log dividend-price ratio and the log *ex post* return must be either stationary or cointegrated (that is, time-variation in returns may imply the failure of the first condition, but not of the second). They analyse monthly stock market data for eight East Asian countries over the nineties; non-stationarity and lack of cointegration cannot be rejected for any of them, suggesting that asset prices were bubbly at the outset of the 1997 crisis.

It is also possible to test a specific bubble parametrisation. Flood and Garber (1980) develop a test for a monetary model of the German hyperinflation considering a deterministic bubble growing at a constant rate. The idea is to test the significance of an exponential time trend in a reduced-form inflation equation. The no-bubble hypothesis is not rejected, but the procedure raises some methodological issues (Flood and Hodrick (1990)); in particular, because of the explosive regressor the authors' statistical inference cannot rely on the standard asymptotic distribution theory. Flood *et al.* (1984) replicate a similar analysis exploiting a cross-sectional dimension to by-pass this issue, and reject the null; the problem in this case lies with the sample size, as only three simultaneous hyperinflations are considered. In Froot and Obstfeld (1991) the bubble is a power of the contemporaneous dividend, which is assumed to follow a logaritmic random walk. The bubble is thus completely deterministic and it is "intrinsic" in that all of its variability stems from economic fundamentals. Using aggregate annual US data up to 1988, the authors find that the price-dividend ratio is indeed positively correlated to some power of the dividend. This paper is discussed in greater detail in section 5.

Finally, a few papers introduce a parametric bubble process in a state-space model, as we do here. The bubble, a non-observable state variable, can then be estimated by maximum likelihood through a Kalman filter pro-

cedure. Burmeister and Wall (1982) pioneered this technique on the German hyperinflation data, Wu (1995) and Elwood *et al.* (1999) apply it to exchange rates; Wu (1997) and Chen *et al.* (2001) use it to analyse the US equity market. Wu (1997) assumes a constant discount factor and models prices using Campbell and Shiller's (1988) log-linear approximation, finding significant evidence of bubbles. Chen *et al.* (2001) assume that the discount factor is driven by an autoregressive risk premium and linearize the price around the mean discount factor; they do not reject the no-bubble hypothesis. These two papers are clearly close in spirit to ours, and they are discussed more extensively below.

3 A state-space formulation for the price of a share.

This section introduces a linear approximation for a standard present-value equity price formula with time-varying discount factor, and discusses the use of the approximation as a building block for a state-space formulation of the price process. The main references are Poterba and Summers (1986), that firstly proposed the approximation[3], Campbell and Shiller (1988), that introduced an alternative well-known logaritmic formula, and finally Chen *et al.* (2001) and Wu (1997), where these approximations are used to investigate equity price bubbles. It is well known after Lucas (1978) that, under risk neutrality, the current price of a share is the discounted value of the stream of expected dividends it will generate in the indefinite future:

$$P_t^f = E_t \sum_{i=0}^{\infty} \beta^{i+1} D_{t+i} \tag{1}$$

(the superscript f stands for "fundamental"). It has been argued that the assumption of a constant discount factor β is a straightjacket to be avoided. A more general alternative is given by the following formula:

$$P_t^f = E_t \sum_{i=0}^{\infty} \left(\prod_{j=0}^{i} (1 + r_{t+j} + \alpha_{t+j})^{-1} \right) D_{t+i} \tag{2}$$

---

[3] Analogous formulae appear in Shiller (1981), Abel and Blanchard (1986), West (1987).

The discounting is based here on a time-varying factor driven by the dynamics of a risk-free rate of return r_{t+j} and a risk premium α_{t+j}. Equation (2) is the no-bubble forward solution for the price of a share satisfying the condition $(E_t P_{t+1} - P_t)/P_t + D_t/P_t = r_t + \alpha_t$; if no assumption is made about the terms on the right-hand side, this is of course a mere accounting identity without a precise economic meaning. Poterba and Summers (1986) assume a constant interest rate r and develop a first order Taylor approximation of (2) around the mean risk premium $\bar{\alpha} \equiv E\alpha_t$. A more flexible model can be obtained by using a bivariate approximation. Assume that the safe rate and the premium have a finite unconditional mean $E(r_t, \alpha_t) \equiv (\bar{r}, \bar{\alpha})$. The existence of this moment is crucial: none of the equations below holds if, for instance, one of these variables is not covariance-stationary. If the mean exists, we can define an "average" discount factor[4] $\beta \equiv 1/(1 + \bar{r} + \bar{\alpha})$. A first-order Taylor approximation of $P_t^f(D_{t+i}, r_{t+i}, \alpha_{t+i})$ around $P_t^f(D_{t+i}, \bar{r}, \bar{\alpha})$ delivers the following equation:

$$P_t^f \simeq P_t^L + R_t + A_t \tag{3}$$

where:

$$P_t^L \equiv \sum_{i=0}^{\infty} \beta^{i+1} E_t D_{t+i} \tag{4}$$

$$A_t \equiv \sum_{i=0}^{\infty} \left\{ \left[-\beta^{i+1} \sum_{k=0}^{\infty} \beta^{k+1} E_t D_{t+i+k} \right] (E_t \alpha_{t+i} - \bar{\alpha}) \right\} \tag{5}$$

$$R_t \equiv \sum_{i=0}^{\infty} \left\{ \left[-\beta^{i+1} \sum_{k=0}^{\infty} \beta^{k+1} E_t D_{t+i+k} \right] (E_t r_{t+i} - \bar{r}) \right\} \tag{6}$$

Details on the derivation and the assumptions under which we can expect it to be accurate can be found in the appendix. The first term has been named P_t^L because it represents in a sense a "Lucas' price" - the dividend stream discounted on the basis of a constant factor β. The second and third terms show how expected deviations of risk premium and interest rates from their equilibrium values influence today's equity valuation. The fact that A_t and R_t contain products between dividends and deviations of α_t and r_t

---

[4]Because of the non-linearity of the relationship, this does not coincide with the un-conditional mean of the stochastic discount factor - hence the inverted commas.

from their means has a natural interpretation: the impact on today's price of an expected change in the discount factor at any future date depends on how big is the cashflow to be discounted from that date onwards. If for instance agents believe that dividends will be zero after $t + i^*$, for all $i \geqslant i^*$ the expression in square brackets is zero and the corresponding terms drop out of A_t and R_t: the price of the share today is thus independent of any expected movement in the discount factor after $t + i^*$. If the misalignment M_t is defined residually as the non-fundamental price component, we can write:

$$P_t \equiv P_t^f + M_t \simeq P_t^L + R_t + A_t + M_t \qquad (7)$$

This is a candidate for the measurement equation in the state-space model. The question is whether the right-hand side terms provide a set of suitable states.

P_t^L does not pose particular problem. As Hansen and Sargent (1980) firstly showed, if $D_t \sim AR(q)$ then P_t^L will be itself a linear function of q lags of the dividend. By vectorising the autoregressive D_t process into a VAR(1) of dimension q, the first block of states can be written as follows:

$$\begin{pmatrix} D_{t+1} \\ D_t \\ ... \\ D_{t-q+2} \end{pmatrix} = \begin{bmatrix} \phi_0 \\ 0 \\ ... \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1 & \phi_2 & ... & \phi_q \\ 1 & 0 & ... & 0 \\ & ... & ... & \\ 0 & ... & 1 & 0 \end{bmatrix} \begin{pmatrix} D_t \\ D_{t-1} \\ ... \\ D_{t-q+1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{t+1} \\ 0 \\ ... \\ 0 \end{pmatrix} \qquad (8)$$

Hansen and Sargent's result implies $P_t^L = c_0(\phi, \beta) + \sum_{i=1}^{q} c_i(\phi, \beta) D_{t-i+1}$, where each c_i is a known function of $\phi_i \ i = 0, ..., q$ and β.

A_t and R_t are less straightforward. Chen *et al.* (2001) show that A_t (N_t in their notation) follows an AR(1) process with a time-varying intercept and an innovation that is a complicated function of conditional moments of D_{t+i} and α_{t+i}. Since an analogous equation holds for R_t, one possibility is to model A_t and R_t as non-observable state variables. However, this procedure fails to impose some cross-equation restrictions and introduces two error terms that are well-behaved only under extra assumptions on the data-generating process. Again, a discussion of these issues is presented in the appendix. The next section shows that fortunately, despite the apparent cumbersomeness of

equations (5) and (6), R_t and A_t turn out to be reasonably simple functions of D_t, r_t and α_t in a range of economically interesting cases. Therefore, the system can be directly specified in terms of dividends, interest rates and risk premia without introducing any further variable.

As far as M_t is concerned, a completely agnostic approach would require not taking a stance on the nature of the misalignment. On the other hand, if M_t is simply considered part of the price equation error term, the occurrence of a serially correlated rational bubble could invalidate the model. The solution is to allow for both a rational bubble and a purely random deviation from fundamentals: $M_t = B_t + n_t$, where n denotes a "noise" satisfying $E(n_t) = 0$ for all t and $E(n_t n_s) = 0$ for all $t \neq s$. The bubble is modelled as a further non-observable forcing state:

$$B_{t+1} = (1 + r_t + \alpha_t)B_t + b_{t+1} \tag{9}$$

(where b_t is a serially uncorrelated error term). Equation (9) generalises the standard parametrisation of a rational bubble to the case where the discount factor, and consequently the growth rate of the bubble, changes over time. This non-linearity makes the model more interesting because it allows the examination of bubbles that roam about more irregularly than the usual, monotonically explosive variable examined by constant discount models. As we will show, there is no cost in terms of econometric complications as long as r_t and α_t can be observed. Note that the presence of n_t and b_t implies that both the measurement equation for P_t and the bubble are stochastic; the Kalman filter is run under the assumption that the contemporaneous covariance between the two error terms is zero. The n_t factor can be interpeted as the consequence of fads or measurement errors.

We conclude the section relating this work to the log-linear approximation of Campbell and Shiller (1988). Both approximations are based on the definition of return. Campbell and Shiller (1988) start from $(P_{t+1} + D_t)/P_t = 1 + R_t$, take the logaritm of the identity and then compute a first order Taylor approximation around the mean of the log dividend-price ratio $\delta_t \equiv d_{t-1} - p_t$. By contrast, we write the *ex ante* return as the sum of a safe rate and a risk premium, solve forward for the price level and then approximate the solution. The formulae in Campbell and Shiller (1988) have a clear advantage in terms of elegance and intuitive appeal; furthermore, they are linear in logaritms of prices and dividends, so they can be combined with log-linear models that

10

seem to fit the data better and explicitly take into account the non-negativity of these variables. Our defense on this issue is twofold. Firstly, equation (8) is meant to be an empirical approximation of the process investors use to forecast dividends rather than a description of the "true" process. Since dividends are positive and increase over time, the probability (8) attaches to the event $\{D_t < 0\}$ is small and approaches zero asymptotically; in this respect, the difference between modelling the dividend in levels or in logaritms is unlikely to be important. Secondly, the approximation and the implied state-space system can be derived also under the assumption that the log-dividend follows a random walk[5].

There are two more substantial differences. Campbell and Shiller (1988) approximate around the mean log dividend-price ratio $E(\delta_t)$. If there is a bubble, this moment does not exist: prices may indefinitely diverge from dividends, pushing the dividend-price ratio to zero and its logaritm to minus infinity. Since the approximation is only valid under the null that there are no bubbles, it should not be used to test for their existence. On the contrary, the mean discount factor is well-defined independently of the existence of bubbles. A related issue is the accuracy of the linearisations. Campbell and Shiller's (1988) formula holds exactly if the dividend yield is constant ($\delta_t = \delta$), whereas ours holds exactly if the discount factor is constant ($r_t + \alpha_t = r + \alpha$); the approximation errors respectively depend on the unconditional volatility of these variables. Poterba and Summers (1986) do not discuss the accuracy of their formula; numerical analysis is probably the only way to gain insight on the relative quality of the two approximations. However, the real interest rate is typically less volatile than the log dividend-price ratio, and the nineties have seriously weakened the evidence on the stationarity of the latter. Bubbles may or may not be responsible for this, but as a matter of fact the yield appears to depart persistently from its mean level. Hence, provided the risk premium does not vary much, it may be in principle a good idea to approximate around the discount factor.

The second difference has to do with the degree of generality of the models. Campbell and Shiller (1988) can only allow for one source of randomness in discounting. Their paper analyses four possibile specifications for the dis-

---

[5]In this case the state equation would be specified with $\log(D_t)$ and the measurement equation would contain an exponential term $D_t = e^{\log(D_t)}$; again, the non-linearity can be handled easily because the dividend (or its logaritm) is an observable state.

count rate, respectively based on a constant, the real return on short-term debt, the growth rate of real aggregate consumption and the variance of stock returns. In all cases, either the safe return or the risk premium (or both) are constant; this is unavoidable, because the equation only contains the overall discount factor. With the approximation used in this paper interest rates and risk premia can be modelled separately. Since safe return and risk premium are fundamentals that move under the influence of different forces, the development of econometric frameworks where they can be modelled independently is an important enterprise. The hope is that this work will give a contribution in that direction, despite the fact that in the simple application presented in the next section, the premium is again assumed to be constant[6].

4 A tentative model.

Equations (4)-(6) and (9) may be regarded as the basis for the specification of simpler pricing models. The structure of each specific model will depend on the assumptions we are willing to make on dividends, interest rates and risk premia, which in turn have to be consistent with the dataset at hand. The application proposed here is designed for a set of low-frequency (possibly annual) aggregate data. For the time being, ignore the bubble process and consider the case where:

i. $D_t = \phi D_{t-1} + \varepsilon_t$
ii. $r_t = \rho_0 + \rho_1 r_{t-1} + \eta_t$
iii. $\alpha_t = \alpha$

Dividends are expected to grow at a constant rate, the interest rate follows an autoregressive process of order one and the risk premium is constant; ε_t and η_t are zero-mean, serially uncorrelated disturbances. These assumptions are discussed in the appendix; here we just note that the analysis has the same basic implications if (i) is replaced by the commonly used assumption that $log(D_t)$ follows a random walk, and that (iii) could also be augmented by an error term without consequences, because all we need is a process for which the unconditional mean is also the best conditional predictor at all horizons ($E_t \alpha_{t+i} = E \alpha_t = \alpha$). With $\phi > 1$, two sufficient conditions in order

---

[6]If a simplification has to be introduced, a constant risk premium is in our view more appropriate than a constant safe rate of return. In the appendix we discuss this choice together with the alternative proposed by Chen *et al.* (2001).

for the series of conditional expectations in (4), (5) and (6) to be defined are $\beta\phi < 1$ and $|\rho_1| < 1$. Under these assumptions we have $E_t D_{t+i} = \phi^i D_t$ and $E_t(r_{t+i} - \bar{r}) = \rho_1^i(r_t - \bar{r})$, with $\bar{r} = \rho_0/(1 - \rho_1)$, which substantially simplifies the three equations:

$$P_t^L = \frac{\beta}{1 - \beta\phi} D_t \tag{10}$$

$$R_t = \left(\frac{\rho_0}{1 - \rho_1} \frac{\beta^2}{1 - \beta\phi} \frac{1}{1 - \beta\phi\rho_1} \right) D_t - \left(\frac{\beta^2}{1 - \beta\phi} \frac{1}{1 - \beta\phi\rho_1} \right) D_t r_t \tag{11}$$

$$A_t = 0 \tag{12}$$

It is clear now that the dynamics of R_t are dictated by those of the underlying forcing states D_t and r_t, and that there is no need to specify a transition equation for this variable. The transition equation (8) is replaced by (i) and, using the expressions above and collecting the two D_t terms, the price equation (3) becomes:

$$
\begin{aligned}
P_t^f & \simeq \frac{\beta}{1 - \beta\phi} \left(1 + \frac{\rho_0}{1 - \rho_1} \frac{\beta}{1 - \beta\phi\rho_1} \right) D_t - \left(\frac{\beta^2}{(1 - \beta\phi)(1 - \beta\phi\rho_1)} \right) D_t r_t \\
& \equiv c_0 D_t + c_1 D_t r_t
\end{aligned} \tag{13}
$$

This equation has two simple implications. The first one is that the mean price-dividend ratio also depends on the parameters of the process driving the discount factor: $E(P_t^f/D_t) = c_0 + c_1 \bar{r}$. A simple algebraic manipulation shows that, for positive values of ρ_0 and ρ_1, this mean value is the product of the coefficient implied by a constant discount model $(\beta(1 - \beta\phi)^{-1})$ by a number larger than one. The difference between the two is small as long as the mean safe rate $\bar{r}$ is small. More importantly, dividend and interest rate interact. The interaction effect has a negative sign, meaning that *coeteris paribus* an increase in r_t depresses prices, and its magnitude depends on ρ_1, the persistence of the innovations in the interest rate process. If ρ_1 is large, a positive shock to r_t has to be followed by a prolonged period of high returns and low discount rates, and this is reflected in a strong negative correlation between price and interest rate at time t. It is an interesting result, because basically the equation contains an "extra" term that depends

on the contemporaneous dividend and, contrary to the intrinsic bubble of Froot and Obstfeld (1991), does not have a speculative nature. The equation can be re-cast for the price-dividend ratio:

$$\frac{P_t}{D_t} = c_0 + c_1 r_t + n_t^*$$ (14)

where $n_t^* = n_t/D_t$. Note that the system given by (i),(ii) and (13) imposes two non-linear restrictions on β. We may now reintroduce the non-observable bubble by embedding the price equation into a fully specified state-space system:

$$P_t = \begin{bmatrix} c_0 & c_1 & 1 \end{bmatrix} \begin{pmatrix} D_t & D_t r_t & B_t \end{pmatrix}' + n_t$$ (15)

$$\begin{pmatrix} D_t \\ r_t \\ B_t \end{pmatrix} = \begin{bmatrix} 0 \\ \rho_0 \\ 0 \end{bmatrix} + \begin{bmatrix} \phi & 0 & 0 \\ 0 & \rho_1 & 0 \\ 0 & 0 & (1 + r_{t-1} + \alpha) \end{bmatrix} \begin{pmatrix} D_{t-1} \\ r_{t-1} \\ B_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \eta_t \\ b_t \end{pmatrix}$$ (16)

The system has one measurement variable (P_t) and three state variables, two of which (D_t, r_t) are observed. The measurement equation can alternatively be specified in terms of the price-dividend ratio by simply diving through by D_t. In either case, the system contains two non-linear elements: one in the measurement equation (either a product or a ratio of variables) and one in the transition equation for the bubble (because of the time-varying rate of growth). These do not pose particular problems because D_t and r_t are observable, i.e. they are known with certainty at time t, and the transition matrix is diagonal. Another way of looking at it is that, since the only hidden state is B_t, we may simply formulate a system of a price equation and a bubble equation, considering D_t and r_t "exogenous"; such a state-space is *conditionally linear* and it allows consistent maximum likelihood estimation of the parameters and minimum mean-squared error filtering of the bubble (Harvey (1989)). An advantage of estimating a two-equations system for P_t/D_t and B_t is that no assumption has to be made on the distribution of the error term in the dividend equation; the cost is clearly that it is not possible to impose the restrictions on the c_i coefficients.

4.1 Standard&Poor companies.

This section analyses a dataset containing annual observations on Cowles-Standard&Poors quoted companies starting from 1871. This data has been used in many empirical studies of the US stock market, so the results discussed here can be directly related to a large present-value pricing literature[7].

The estimation of (14) relies on the exogeneity (at least in a weak sense) of r_t, which is taken to be the one-year return on government debt deflated by the consumer price index[8]. Unless otherwise specified, the estimates are obtained using data on the 1900-1995 period; this choice is due to the fact that the composition of the portfolio of shares becomes more restrictive as one goes further back in time, and the last years of the XX century deserve special attention. The price equation tends to have serially correlated residuals (e.g. Froot and Obstfeld (1991), Driffill and Solá (1998)); this is accounted for by modelling an AR(1) error term[9]. For the basic form of (14), the least squares estimates are the following:

$$
\text{a.} \quad \frac{P_t}{D_t} \quad = \quad \underset{23.7^*}{c_0} \quad \underset{-25.2^*}{+c_1 r_t} \quad +n_t, \quad n_t \sim AR(1)
$$

Asterisks denote significance at the 5% level. The prediction that r_t has a negative impact on P_t/D_t is not rejected; with $c_1 = -25$, a four percentage points increase in the real rate implies a one unit decrease in the price-dividend ratio. As far as the intercept is concerned, in Froot and Obstfeld (1991) estimates range more or less from 14 to 16.5, while Driffill and Solá (1998) obtain figures of 15.01 for the "low-mean" state and 17.97 for

---

[7]The data can be downloaded from Robert Shiller's web site at Yale (http://www.econ.yale.edu/~shiller/data.htm), and is analysed for instance in Shiller (2000).

[8]We test the weak exogeneity of r_t in the system (ii)-(14) using the LM test proposed by Engle (1984); the test has the advantage that no instruments for r_t are needed. The null hypothesis is $E(\eta_t n_t^*) = 0$, and the test is run by augmenting (14) with the residual $\hat{\eta}_t$ obtained from (ii). The residual is insignificant at the 10% level, so the null cannot be rejected. Strong exogeneity is more elusive, in the sense that the results of Granger-causality tests depend to some extent on the sample. Further tests suggest that $\log(P_t)$ and $\log(D_t)$ cointegrate and the latter is at least weakly exogenous. Froot and Obstfeld (1991) obtain analogous results. Details are available on request.

[9]The autoregressive error term may capture both noise and bubbles, especially because the last, apparently explosive observations are excluded from the sample. At this stage we are simply testing the implications of the linearized model without discriminating between different types of misalignment.

the "high-mean" state. The estimate above is larger, probably because of sample differences. Do the restrictions make sense in terms of the underlying discount factor? With $\phi = 1.02$, $\rho_0 = .02$, $\rho_1 = .33$ (obtained from univariate models for D_t and r_t), the estimated c_i coefficients imply a β between .93 and 1.04 - an acceptable result once sample variability is taken into account. If we make the further assumptions that the innovations $(n_t, \varepsilon_t, \eta_t)$ are approximately normally distributed, we can estimate the trivariate system by Full-Information Maximum Likelihood and test the restrictions more formally. Table 1 reports FIML estimates for the restricted and unrestricted model. The restricted system gives $\hat{\beta} = .94^{10}$; the likelihood ratio statistic is about .10, well below any conventional significance level, so the restrictions on c_0 and c_1 are not rejected. Normality is of course a strong assumption, and in the case of ε_t and n_t it can only be interpreted as an approximation to the true distribution because of the non-negativity of P_t/D_t and D_t/D_{t-1}. Nevertheless, the estimate of β and the extremely low value of the likelihood ratio statistic indicate that model and data are broadly consistent.

Equation (a) performs well in all the standard specification tests, displaying normal i.i.d. residuals, and it is qualitatively robust to subsampling in the 1870-1995 period; in fact, c_1 is negative and significant even if the 1995-2002 years are included, but in this case the model fails most specification tests. As a further check on the robustness of the estimates, we also consider two expanded versions of the price-dividend ratio equation:

$$
\text{b.} \quad \frac{P_t}{D_t} = \underset{23.7^*}{c_0} \quad \underset{-25.9^*}{+c_1 r_t} \quad \underset{.0}{+c_2 \Delta P_{t-1}} \quad \underset{-.0}{+c_3 \Delta D_{t-1}} \quad \underset{.87^*}{+c_4 \frac{P_{t-1}}{D_{t-1}}} \quad +n_t
$$

$$
\text{c.} \quad \frac{P_t}{D_t} = \underset{17.5^*}{c_0} \quad \underset{-22.3^*}{+c_1 r_t} \quad \underset{.01^*}{+c_1 D_t^{2.61}} \quad +n_t, \quad n_t \sim AR(1)
$$

Equation (b) simply includes a lag of the price, the dividend and the ratio of the two; (c) is an attempt to incorporate an "intrinsic bubble" *à la* Froot and Obstfeld (1991)[11]. Asterisks denote again significance at the 5% level;

---

[10]Note that the mean one-period discount factor is $E(1 + r_t + \alpha)^{-1} \equiv E f(r_t)$, whereas $\beta \equiv (1 + E(r_t) + \alpha)^{-1} = f(E(r_t))$. Since f is convex, by Jensen's inequality $\beta \leq E f(r_t)$; in other words, the actual one-period expected discount rate is above β.

[11]The exponent 2.61 is taken from table 3 on page 1201 of Froot and Obstfeld (1991); results are not sensitive to this choice. We further elaborate on the intrinsic bubble model in section 5.

neither the extra lags nor a power of D_t swamp the interest rate effect: c_1 remains negative and significant.

The estimates are encouraging, in the sense that the assumption on the structure of the discount factor (and the approximation) prove to have something to say on the behaviour of P_t independently of the stance one takes on the existence of speculative bubbles. In particular, the model rationalises the negative relationship between interest rate and price-dividend ratio found in the data, and it does it in a way which is consistent with plausible values of the underlying average discount factor.

The remainder of this section examines the contribution of the bubble to the explanation of the price pattern. Some estimates for the unrestricted "reduced" model, where P_t/D_t is the measurement variable and B_t the hidden state, are presented in table 2 and figures 1 to 4. We report results obtained with two samples, 1900-1990 and 1900-2002. Again, the inclusion of the 1871-1900 period implies only marginal changes, the most interesting of which is an increase in the significance of the interest rate term. This is clearly not the case for the last decade, that stretches the model to its limits. Each observation after 1995 pushes the estimated average risk premium up, and the estimate is as large as 11% when the full sample is used; for this reason we also report the estimates obtained imposing $\alpha = .0\dot{6}$[12]. Furthermore, the residuals corresponding to the last observations are very large: the behaviour of the price index after 1998 is somewhat exceptional even after accounting for a near-exponential bubble. Nevertheless, the analysis delivers some clear conclusions. The parameters of the P_t/D_t equation, including the interest rate term, are significant, rightly signed and of a reasonable magnitude. The pattern of the bubble is realistic: it remains latent during the first 50 years (in fact, it is also insignificantly different from zero throughout the XIX century), and it drives prices up in the 60's and in the second half of the 90's[13]. For

---

[12]As LeRoy (2004) stresses, the existence of rational bubbles cannot explain the equity premium puzzle of Mehra and Prescott (1985). The excess return on equity has been very high in the past decades, and this is puzzling independenly of whether the price level contained a bubble or not. Note also that as long as $\alpha > 0$ the bubble is explosive and $\hat{\alpha}$ has a non-standard distribution; the significance levels reported in table 2 are valid insofar as this is approximately normal.

[13]There is an issue with the sign of the price bubble. Negative bubbles are usually ruled out because they imply a non-zero probability of prices becoming negative in the future. However, temporary undervaluation due to a short-lived negative bubble appears to be a realistic possibility (Wu (1997)). Our estimation procedure does not impose a

these two periods, and notably for the second one, the estimated bubble is significant well above the 5% level independently of whether the risk premium is restricted or not[14]. It is interesting to compare the results to Wu (1997), where the bubble is estimated by Kalman-filtering under the assumption of a constant discount factor. The bubble estimated by Wu follows a remarkably similar path, and between 1960 and 1970 it accounts on average for 40-50% of the price level; our estimates suggest a figure of 30-40% (more details are given in table 5). The type of time-variation in the discount factor assumed in this model reduces the estimated size of the bubble by roughly 10%.

As a further robustness test, the model was also estimated expanding the measurement equation to include alternatively a lagged price-dividend ratio and an "intrinsic bubble" (again with a constant exponent). The extra terms are generally significant and the lagged price-dividend ratio reduces the size of the bubble; however, B_t remains highly significant in both cases and follows a very similar path. Besides, the lagged price-dividend ratio is completely *ad hoc* and its presence in the equation is even more difficult to justify than that of the bubble itself. The residuals do not display any clear pattern. Though, the diagnostics in table 2 show that the last observations weaken the evidence that the "noise" is an i.i.d. normal variable; this is partially due to the model's inability to anticipate the burst of the bubble, which generates inaccurate forecasts for 2001 and 2002.

What to make of this result? We already noted that there is a long and articulated debate on the theoretical and empirical plausibility of rational bubbles. A few recent empirical works provided ground for scepticism by showing that fundamentals can explain a lot depending on how they are measured and modelled. Wright (2004) adjusts the dividend series by netting out new issues and buybacks and shows that the adjusted dividend yield behaves more regularly, displaying no negative drift in the last decade of the century. Using ordinary S&P data and assuming constant discounting, Driffill and Solá (1998) show that a dividend process switching between a "bad" state (low mean-high variance) and a "good" state (high mean-low variance) also goes a long way towards explaining movements in the price

---

non-negativity constraint on the bubble process.

[14]The error bands shown in the figures 2 and 4 quantify the "filter uncertainty" ignoring the "parameter uncertainty" (Hamilton (1994)); since the system is very parsimonious and the parameters are estimated with good accuracy, this is unlikely to be a serious shortcoming.

index.

More generally, it is still possible (and probably it will always be) to object that any finding of speculative price components in the data can be potentially eroded or annihilated by modelling a more complex stochastic discount factor and/or dividend process. However, two points are worth stressing. Firstly, the nineties are unlikely to fit comfortably in the framework proposed by Driffill and Solá (1998), who use data up to 1987. Since the behaviour of the dividend series does not change dramatically after that year, on the basis of a cursory look at the data one is tempted to conclude that a bubble is indeed a plausible explanation for what happened after 1995. The compatibility of the bubble with the history of the market in the previous decades obviously reinforces this view. Secondly, previous attempts to estimate share price bubbles have been criticised because of their reliance on the assumption of a constant discount factor. This paper shows that rational bubbles are in this sense quite resistant, and they survive the introduction of a simple but realistic stochastic factor. Time variation in the latter explains an extra portion of the volatility of the price (in our case through the interest rate term), but it also implies that a potential bubble will follow more complex dynamics. Interestingly, the net outcome of these two factors on the Cowles/S&P data is such that the occurrence of a near-exponential price bubble cannot be ruled out.

4.2   The non-financial corporate sector.

A dataset containing annual observations for the whole non-financial US corporate sector from 1900 to 2000 is available from Steven Wright's web page (*www.econ.bbk.ac.uk/faculty/wright*). All data for the post-World War II period come from the Flow of Funds for the United States. For the pre-war period, the aggregate dividend series is obtained by assembling data from three different sources and the price index is derived assuming the existence of a stable relationship between the dividend yield or the return on the sector and those on the more restricted Cowles/S&P index. Sources and adjustments are described in detail in Wright (2004). Wright also computes an "adjusted" aggregate dividend series by subtracting to the ordinary dividend all net new issues, namely new issues minus buybacks; the adjusted variables (P_t^a, D_t^a) basically describe an hypothetical market where the number

of shares remains constant at the 1900 level[15].

Figure 5 plots the price-dividend ratio for S&P companies together with the ordinary and adjusted ratios for the whole non-financial sector. Clearly, both the expansion of the set of companies and the adjustment for net new issues have a significant impact on the variable. The pattern of the non-adjusted sector ratio is qualitatively similar to the S&P benchmark; the main difference is that the peak in the late nineties is much smaller, leaving less room for rational bubbles. As Robertson and Wright (2003) and Wright (2004) document, the consequences of adjusting for issues and buybacks are dramatic. The adjusted ratio has no clear upward trend in the second half of the sample, and it displays "fat tailed volatility"; in particular, two large spikes in the late twenties and early seventies (in coincidence with large net equity issues) complicate the statistical modelling of the variable. The adjusted ratio is the only one for which the null hypothesis of a unit root is confidently rejected by the standard augmented Dickey-Fuller tests.

Preliminary analyses involve checking the adequacy of (i) and the weak exogeneity of r_t in the price-dividend ratio equation[16]. Figure 6 shows the growth rates of ordinary and adjusted dividends (D_t/D_{t-1}, D_t^a/D_{t-1}^a). A regression of D_t/D_{t-1} on a constant delivers serially uncorrelated residuals and an estimated dividend growth rate of about 3%. The growth rate of the adjusted dividend is more volatile and it peaks above 100% three times over the sample. In this case, a simple regression of the growth rate on a constant generates serially correlated and heteroscedastic residuals. The correlation is significantly mitigated if the extreme observations are ignored or the innovation is modelled as conditionally heteroscedastic (via an ARCH(1)).

---

[15]Robertson and Wright (2003) and Wright (2004) argue that this "broad dividend" is theoretically more appropriate than the usual, "narrow" dividend. Others (e.g. Cole *et al.* (1996)) express doubts on the equivalence between dividends and repurchases. According to LeRoy (2004), the two variables simply describe different porfolio strategies, either of which may have a bubble or not. But arguably a representative agent holding a representative share can only implement one such strategy, and this is linked to the "broad" dividend measure.

[16]In what follows, the safe rate r_t is again the nominal return on one-year public debt deflated by the Consumer Price Index, available from Shiller. As long as the dependent variable is the price-dividend ratio, any deflator can potentially be used to obtain the real interest rate.

Furthermore, a BDSL[17] test shows that the null of the observations being i.i.d. cannot be rejected at any reasonable significance level. Hence, we proceed on the assumption that agents expected both the ordinary dividend and the broad dividend, or net cash flow, to grow at a constant rate. It should be noted that the accuracy of this assumption is only relevant to the simplification of the approximation formulae (which incidentally would remain unchanged if the process were conditionally heteroscedastic or subject to unpredictable jumps), and it has no implications for the statistical validity of the estimates of the price equations. The weak exogeneity of r_t for the parameters of the price-dividend equation is again tested following the procedure of Engle (1984) (see footnote 10). Exogeneity holds in the case of P_t^a/D_t^a but not in the case of the non-adjusted ratio P_t/D_t; for the latter, the estimates are thus obtained by a *Two-Stages Least Square* procedure using as instruments one lag of r_t (consistently with the variable following an AR(1) process) and three lags of P_t/D_t; alternative choices of the instruments deliver analogous results.

Tables 3 and 4 report estimates of equation (14) for the adjusted and non-adjusted datasets. The equation is estimated on the whole sample 1900-2000, the period 1900-1990, that excludes the last anomalous years, and the period 1900-1970, that excludes the oil shocks and their aftermath. The c_1 coefficient is always negative and of the same order of magnitude of the intercept, and it is significant at the 10% level in all cases but one. The error term appears to have a richer autoregressive structure than in the S&P case, so the equation is specified including lags 2 and/or 3 whenever significant. However, c_1 is negative and significant even when the innovation is modelled as an AR(1). In all cases, the Newey-West (1987) correction is used to compute standard errors. The estimation of the P_t^a/D_t^a equation is complicated by two sharp jumps in the dependent variable in 1929 and 1971; adopting a simple (and admittedly not entirely satisfactory) approach, we introduce two dummy variables for these observations[18]. The 1900-1970 subsample provides a robustness check allowing to ignore at least one of the two spikes. Given that the adjusted ratio does not trend upwards in the nineties, an-

---

[17]Brock W., Dechert D., Sheinkman J. and LeBaron B. (1996): "A Test for Independence Based on the Correlation Dimension", *Econometric Reviews*, August, 15(3), 197-235.

[18]The financial crisis is a natural explanation for the 1929 outlier. A possible cause for the 1971 spike is the collapse of the Bretton Woods Agreements, even though its connection with the firms' retention/issuance policy is less clear.

other check can be made by estimating the equation on the 1975-2000 period - which again delivers a negative and significant c_1 (results are not reported for brevity). All models are structurally stable and generate well-behaved residuals.

Independently of whether the relevant fundamental is the dividend or a more sophisticated cash flow measure, the data is consistent with the view that a safe one-year return augmented by a constant premium provides a reasonable description for the discount factor. However, the measurement of D_t is crucial when it comes to investigating the existence of speculative bubbles. Figure 5 clearly suggests that the pattern of P_t^a/D_t^a is incompatible with the presence of an explosive price component. This conclusion is strengthened by the fact that, when the equation for the price-dividend ratio is estimated on the full sample, the real root of the autoregressive innovation is only .83 and the residuals do not persistently depart from zero for the last observations.

In the case of P_t/D_t the picture is more ambiguous: the estimated root for the innovation is .92 on the 1900-1990 sample and .95 if the last decade is included, and the residuals trend upwards in the last 5-6 years. Unfortunately the usual state-space formulation cannot be used to measure the bubble because of the endogeneity of r_t. However, it is still possible to gain some insight on the likelihood and magnitude of the bubble by means of a small simulation; the Kalman filter can be run assuming that c_0 and c_1 are known, so that the only free parameters are the risk premium and the variances of the two error terms. Figure 7 shows the filtered bubble $B_{t|t}$ when c_0 and c_1 are fixed at the values given in table 3. The estimated risk premium is between 6% and 11%, and the standard error bands (not reported) show that the bubble departs significantly from zero in the 60's and in the 90's; between 1995 and 2000 the estimated bubble is significant at the 5% level in all three experiments[19]. The similarities with the Standard&Poor data are apparent. There is however an interesting difference as far as the magnitude of the bubble is concerned. Table 5 shows the percentage of P_t that can be attributed to the bubble during the two decades when it is significant

---

[19]Note that $B_{t|t}$ is on average larger when the c_i are set equal to the estimates obtained with the 1900-1990 subsample. A possible explanation is that the last ten observations push up the estimate of the intercept c_0, which leaves less room for the bubble.

at the 5% level in both the S&P and the non-financial sector data[20]. Both prices inflate by an average 30-40% during the first surge of the bubble. The situation is different in the nineties: the price of a non-financial share increases by about 20% with a peak of 50% in 2000, whereas for an S&P share the average price increase is around 40% and the 2000 maximum is +73%. These figures are obviously subject to some margin of error. However, this is likely to be roughly of the same size in the two datasets, and it is presumably unrelated to the sample period. Hence, such a large discrepancy in the estimates is a reliable indicator of a substantial divergence between the true underlying bubble processes.

As long as we stick with the traditional definition of dividends, it seems hard to exclude that an exponential speculative bubble may have pushed share prices up at least twice in the last century, and most notably in the 90's. At the same time, though, the dimensions of the presumed bubble depend quite substantially on how broad is the set of firms considered. In a way, even a "broad market index" like S&P500 is not entirely representative of the whole non-financial US sector: overvaluation is quantitatively less important when looking at industry-level data.

If one accepts the idea that the Miller-Modigliani theorem has to be taken more seriously, and that net share repurchases are equivalent to dividends, the picture changes substantially. In this case, a more technical analysis confirms what a qualitative assessment of the data already suggests - namely that the bubble does not exist. Since there is again evidence of time-varying discounting of the type discussed in the previous sections, the approximation still gives some guidance in investigating price dynamics; in particular, it provides a more articulated definition of "fundamental price" than that implied by constant discount models. The difference is that, on adjusted data, the deviations of actual prices from this benchmark fit better the concept of short-lived fads or generic noise than that of a proper speculative bubble.

---

[20]The bubble for the non-financial sector is filtered fixing c_0 and c_1 at 26.1 and -39.3, namely their estimates on the 1900-1990 sample. These values are preferable because they exclude the obviously bubbly years 1990-2000 (so that, for instance, c_0 is very close to the sample mean of P_t/D_t) without any further, unnecessary shortening of the sample. As noted above, this is likely to give an upper bound for the actual bubble. Again, the significance level is computed disregarding any uncertainty stemming from the estimated parameters.

5 Intrinsic or extrinsic?

Among the bubble-based explanations of the behaviour of the American stock market in post-war years, one that has received significant attention is the already mentioned *intrinsic bubble* studied by Froot and Obstfeld (1991). They define bubbles as "intrinsic" when "they derive all of their variability from exogenous economic fundamentals and not from extraneous factors" (page 1). Note that according to this definition the bubble examined in the previous section does *not* qualify as intrinsic: it depends on fundamentals because the rate of growth is a function of r_t, but, due to the presence of the b_t shocks, it also introduces an extraneous source of variability in the price process. Froot and Obstfeld derive a simple parametrisation by which the bubble is a power of the dividend itself. They examine the 1900-1988 period using Standard&Poors dividend and price series, and show that the non-linearity of the price-dividend relation is not rejected by the data. The objective of this section is to replicate the analysis of Froot and Obstfeld (1991) on a more comprehensive set of data: how does the intrinsic bubble fare in the 90's? And what happens when the whole non-financial industry is considered, or the dividend series is adjusted by netting out issues and buybacks? The purpose of this exercise is not only to compare the performance of two different formulations of the bubble process, but also - and more importantly- to find out whether the intrinsic bubble can be considered a candidate to explain the failure of the Q model of investment.

Froot and Obstfeld (1991) consider the case where the instantaneous rate of interest is constant (r) and the log dividend follows a random walk: $d_{t+1} = \mu + d_t + \xi_{t+1}$, with $\xi_{t+1|t} \sim N(0, \sigma^2)$. The focus is thus on the following period-by-period equation: $P_t = e^{-r} E_t(P_{t+1} + D_t)$. The authors show that, if no transversality condition is imposed, the price equation has a forward solution of the type $P_t = kD_t + cD_t^{\lambda}$, where $k \equiv (e^r - e^{\mu+\sigma^2/2})^{-1}$, c is an arbitrary constant and λ is the positive root of $\lambda^2\sigma^2/2 + \lambda\mu - r = 0$. An attractive feature of this approach, not shared by the exponential bubble, is that it can explain the high sensitivity of prices to firms' dividend announcements. The empirical analysis focusses on equation (13), page 1198:

$$\frac{P_t}{D_t} = c_0 + cD^{\lambda-1} + \eta_t,$$

where the error term η_t is by assumption independent of dividends at all leads and lags. Note that, since this equation is derived under the assumption that

24

the discount rate is constant, a direct comparison between the two bubble specifications (intrinsic and exponential) is not straightforward. In other words, an equation like (b) on page 12, that contains both r_t and $D_t^{\lambda-1}$, cannot be given a rigorous theoretical interpretation.

As Froot and Obstfeld point out, a random walk can be considered at best an approximation to the true process agents use to forecast dividends. In particular, stock prices, which reflect a broad (possibly the broadest) information set available at each point in time, may well contain information on future dividends beyond that given by the current dividend. Obviously this also applies to the discussion in the previous sections - the BDSL test simply shows that an agent expecting the dividend to grow at a constant rate did not commit gross, systematic mistakes over the sample period. When it comes to estimating the equation above, though, the issue becomes critical in two respects. Firstly, consistent estimation of c by OLS requires $E_t(\eta_t|D_t) = 0$. Secondly, Froot and Obstfeld conduct inference on c relying on the assumption that ξ_t is independently distributed of η_t at all leads and lags: appendix B of their paper shows that in this case the t statistic for the null hypothesis $\hat{c}_{OLS} = 0$ is asymptotically normal despite the explosive regressor. In the non-financial industry data (both adjusted and non-adjusted) there is evidence of Granger-causality running from prices to dividends; independence between ξ_t and η_s for all t and s is admittedly a strong assumption, and it does not fare too well on these series. Fortunately, it is possible to extend Froot and Obstfeld's discussion to show that a *2SLS* procedure where D_t is instrumented by its own lag preserves both consistency of $\hat{c}$ and asymptotic normality of the t statistic under the less stringent assumption that η_t is independent of past dividend innovations ξ_s $(s = 1, ..., t-1)$.

Table 6 expands the available evidence on the american intrinsic bubble. For each of the three datasets (S&P, non-financial industry, adjusted non-financial industry) the price-dividend ratio equation is estimated with and without the 1990-2000 observations[21]. For the S&P data we report *OLS*

---

[21] All equations allow for an AR(1) error term, as in rows 2 and 4 of table 3 of the original paper, and are estimated imposing the value of λ implied by the estimates of σ^2, μ and r. Depending on the dataset and the sample period, λ goes from a minimum of 1.33 to a maximum of 2.81; Froot and Obstfeld compute $\lambda = 2.74$. Sensitivity analysis confirms that the results are qualitatively robust to the value of λ. If this is estimated concurrently with the other parameters by a non-linear least squares procedure, the estimates are of a similar magnitude (as in Froot and Obstfeld), but the standard errors tend to be large.

estimates: *2SLS* estimates are very similar, which is consistent with both Froot and Obstfeld's assessment of the exogeneity of the dividend and our own; for the other two datasets we report *2SLS*.

The first row of table 6 is basically a replica of Froot and Obstfeld's result: with a positive and significant c, prices become increasingly overvalued as dividends rise, consistently with the model's prediction[22]. The fourth row shows that the model fails to explain the behaviour of the S&P price index in the 90's: when the 1990-2000 observations are considered, the intrinsic bubble looses its significance and the estimated root of the autoregressive error jumps well above unity. If the 90's are to be explained by assuming the existence of an explosive price component, an exponential process appears to be a better choice than a power of the dividend.

Rows two and five test the intrinsic bubble hypothesis using non-adjusted data for the whole non-financial sector. On pre-1990 data, the c coefficient is correctly (i.e. positively) signed and significantly different from zero at the 10% level, but it is estimated with a relatively large standard error. Interestingly, the significance of the bubble increases when the full sample is used. Furthermore, the unexplained serial correlation appears in this case to be stationary. The estimate $c = 4.79$ is not entirely convincing, in the sense that it implies a surprisingly high degree of "sensitivity" of the price to the contemporaneous dividend, but the theory places no restriction on the magnitude of this coefficient. Finally, results for the adjusted series (rows four and six) leave little room for ambiguity: c is far from any acceptable significance level in both samples, and wrongly signed in one of them.

The adjustment for new issues and buybacks is as lethal for the intrinsic bubble as it is for the exponential bubble; again, this is hardly surprising given the strong mean-reversion displayed by the adjusted price-dividend ratio. As far as the non-adjusted data is concerned, it seems fair to say that on the whole the intrinsic bubble performs worse than the exponential bubble studied in the previous sections. The two formulations are both broadly compatible with price patterns between 1900 and 1990, but the following ten years provide a clear case for the exponential bubble. The growth of the

---

[22]Froot and Obstfeld note that the equation "does a better job" on post-war data (p.1204). Indeed, a formal Chow test rejects the null hypothesis of structural stability before and after 1945 at the 1% confidence level. However, magnitude and significance of c are more or less unchanged when the pre-war period is ignored.

non-financial industry share price can be explained within Froot and Obstfeld's (1991) framework, even though this comes at the cost of a somewhat implausibly strong link between price and contemporaneous dividend. But the rise in the S&P index is simply too large and sudden to be justified by this type of mechanism.

These findings cast some doubts on the overall plausibility of the intrinsic bubble story. One would expect that, if there is a bubble of any type in the industry-level data, this will also appear in market indices based on a large set of firms, such as Standard&Poor's. This is the case for the exponential bubble, but not for the intrinsic one. Furthermore, the bubble modelled by Froot and Obstfeld is a deterministic function of the dividend and, as such, it cannot pop and re-start at different points in time: the lack of significance of the intrinsic bubble in a period of widely aknowledged price misalignment is in this sense particularly difficult to justify. Of course one may argue that prices contain both a permanent intrinsic bubble and occasional, stochastic exponential bubbles, and that the latter swamp the former during their expansion phases. Albeit theoretically admissible, this possibility strikes us as unrealistic.

6   Conclusion

Economists have been puzzled by stock markets for a long time, and recent experiences in the US and elsewhere have revived debates dating back at least to the eighties: how much can fundamentals explain? Are prices really "too volatile"? Price misalignments, and rational bubbles in particular, are a fascinating hypothesis, but one on which there is no consensus on either theoretical or empirical grounds. In this paper we discuss a state-space model that allows maximum likelihood estimation of share price bubbles conditional on different assumptions on the stochastic discount factor. The bubbles are also stochastic, and they grow at a time-varying rate given by the inverse of the one-period discount factor. The model is used to analyse three long series of annual observations on the US, namely a Standard&Poors dataset, a non-financial industry dataset, and an adjusted non-financial industry dataset where the dividend is computed netting out new share issues and buybacks. We assume that investors discount expected dividends on the basis of a safe one-year return augmented by a constant risk premium.

This assumption fits the data reasonably well: the price-dividend ratio is negatively correlated to the contemporaneous real interest rate, and the magnitude of this effect is consistent with the interpretation we propose. As far as misalignments are concerned, our estimates sketch the following picture. The S&P price index was inflated by rational bubbles twice in the last century, in the sixties and in the nineties. This is also true for the price of a non-financial company share, though the bubble of the nineties was proportionally much smaller. Finally, there are no bubbles in the adjusted share price; variation in the interest rate does not completely explain price dynamics in this case either, but what is left resembles short-lived fads rather than a bubble. Our work suggests two conclusions that should be of some interest independently of the position one takes on the existence of rational bubbles. The first one is that self-fulfilling bubbles and stochastic discounting may coexist; in particular, bubbles are a possible explanation for the excess volatility of prices over dividends and interest rates already documented in the literature. The second one is that investigations of this type are very sensitive to the way variables are defined; for a shareholder looking not just at dividends but also at share issues and repurchases, the nineties were a perfectly ordinary decade.

7 Appendix

7.1 Derivation of equation (3)

This appendix illustrates the derivation of the Taylor expansion. We stress again that this is merely a bivariate extension of an approximation originally used in Poterba and Summers (1986). Define the sequences $\{r_t\} \equiv r_t, r_{t+1}, ...$ and $\{\alpha_t\} \equiv \alpha_t, \alpha_{t+1}, ...$ and consider the following function[23]:

$$f(\{r_t\}, \{\alpha_t\}) \equiv \sum_{i=0}^{\infty} \left(\prod_{j=0}^{i} (1 + r_{t+j} + \alpha_{t+j})^{-1} \right) D_{t+i} \qquad \text{(A1)}$$

A first-order Taylor approximation of f in a neighborhood of $(\bar{r}, \bar{\alpha}) \equiv E(r_t, \alpha_t)$ has the following form:

---

[23]We follow Poterba and Summers (1986) in letting the summation start in $i = 0$; the implicit timing convention is that the contemporaneous dividend accrues to the time-t buyer of the share.

$$f(\{r\}, \{\alpha\}) \simeq f(\bar{r}, \bar{\alpha}) +$$

$$+ \left[\left. \frac{\partial f}{\partial \{r_t\}} \right|_{(\bar{r}, \bar{\alpha})} \right] (\{r_t\} - \bar{r})$$

$$+ \left[\left. \frac{\partial f}{\partial \{\alpha_t\}} \right|_{(\bar{r}, \bar{\alpha})} \right] (\{\alpha_t\} - \bar{\alpha}). \qquad \text{(A2)}$$

The computation of the first term is straightforward:

$$\tilde{P}_t^L \equiv f(\bar{r}, \bar{\alpha}) = \sum_{i=0}^{\infty} \left(\frac{1}{1 + \bar{r} + \bar{\alpha}} \right)^{i+1} D_{t+i} \equiv \sum_{i=0}^{\infty} \beta^{i+1} D_{t+i} \qquad \text{(A3)}$$

where $\beta \equiv (1 + \bar{r} + \bar{\alpha})^{-1}$. Because r_{t+j} and α_{t+j} enter f in the same way, the partial derivatives of f with respect to these variables have the same functional form. To derive it, note first that:

$$\frac{\partial f}{\partial r_{t+i}} = -(1 + r_{t+i} + \alpha_{t+i})^{-1} \sum_{k=0}^{\infty} \left(\prod_{j=0}^{i+k} (1 + r_{t+j} + \alpha_{t+j})^{-1} \right) D_{t+i+k}$$

Consequently:

$$\left. \frac{\partial f}{\partial r_{t+i}} \right|_{(\bar{r}, \bar{\alpha})} = -\beta^{i+1} \sum_{k=0}^{\infty} \beta^{k+1} D_{t+i+k}$$

$$\left[\left. \frac{\partial f}{\partial r_{t+i}} \right|_{(\bar{r}, \bar{\alpha})} \right] (r_{t+i} - \bar{r}) = \left[-\beta^{i+1} \sum_{k=0}^{\infty} \beta^{k+1} D_{t+i+k} \right] (r_{t+i} - \bar{r})$$

The first-order Taylor term for $\{r_t\}$ is given by the summation over i of the equations above:

$$\tilde{R}_t \equiv \left[\left. \frac{\partial f}{\partial \{r_t\}} \right|_{(\bar{r}, \bar{\alpha})} \right] (\{r_t\} - \bar{r})$$

$$= \sum_{i=0}^{\infty} \left\{ \left[-\beta^{i+1} \sum_{k=0}^{\infty} \beta^{k+1} D_{t+i+k} \right] (r_{t+i} - \bar{r}) \right\} \qquad \text{(A4)}$$

An analogous formula can be derived for $\tilde{A}_t$, the term involving the risk premium. By the linearity of the expectation operator we can write:

$$P_t^f = E_t\left[f(\{r_t\},\{\alpha_t\})\right] \simeq E_t\left[\tilde{P}_t^L + \tilde{R}_t + \tilde{A}_t\right] = E_t\tilde{P}_t^L + E_t\tilde{R}_t + E_t\tilde{A}_t \quad \text{(A5)}$$

Hence, the only condition needed to derive (A5) is the existence of a finite unconditional mean for r_t and α_t. If $E_t\tilde{P}_t^L$, $E_t\tilde{R}_t$ and $E_t\tilde{A}_t$ are equal to P_t^L, R_t and A_t as defined in (4), (5), and (6), we then obtain a bivariate version of equation (4) of Poterba and Summers (1986). This requires further assumptions on the processes generating dividends, interest rates and risk premia.

A sufficient condition for $E_t\tilde{P}_t^L = P_t^L$ is the absolute summability of the series: $\sum_{i=1}^{\infty} E_t\left|\beta^i D_{t+i}\right| < \infty$. This condition holds (or is assumed to hold) for most common specifications of the dividend process. In the case of the other two variables on the right hand side of (A5) the derivation is more cumbersome, even though it does not require sophisticated mathematics. A general discussion is beyond the scope of the paper; we briefly comment on the relationship between $E_t\tilde{R}_t$ and R_t in the following case:

$D_t = \phi D_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0,\sigma_\varepsilon^2)$;
$r_t = \rho_0 + \rho_1 r_{t-1} + \eta_t$, $\eta_t \sim N(0,\sigma_\eta^2)$, $|\rho_1| < 1$;
$\forall t : E(\varepsilon_t\eta_t) = \sigma_{\epsilon\eta}$;
$\forall t \neq s : E(\varepsilon_t\varepsilon_s) = E(\eta_t\eta_s) = E(\varepsilon_t\eta_s) = 0$.

If $\sigma_{\varepsilon\eta} = 0$, we obtain $E_t\tilde{R}_t = R_t$. If $\sigma_{\varepsilon\eta} \neq 0$, $E_t\tilde{R}_t$ also contains a covariance term; it is possible to show that this is bounded by a multiple of $\sigma_{\varepsilon\eta}$ itself, so it is finite (despite the infinite summation over i and k) and it is "small" for "small" values of the contemporaneous covariance between the innovations in D_t and r_t. On the three datasets used in the paper, bivariate systems for the growth rate of the dividend and the real interest rate deliver $\hat{\sigma}_{\varepsilon\eta} \simeq .003$. Finally, as long as $\sigma_{\varepsilon\eta}$ is constant, ignoring the covariance term in R_t can at worst complicate the interpretation of the intercept of the price equation but it has no impact on the dynamics. Since there is no evidence of a time-varying conditional covariance, the result is quite reassuring. Of course this point may become more delicate when modelling higher frequency data. The equality $E_t\tilde{A}_t = A_t$ can be derived along the same lines insofar as the underlying process for the risk premium α_t has suitable properties. Details on the derivation are available upon request.

7.2 Caveats on the state-space formulation

This section highlights some issues that should be taken into account when considering further specifications and applications of the general state-space model used in the paper. Chen *et al.* (2001) provides a good starting point to clarify the risks connected to "assuming away" variation in one of the components of the discount factor. Let us define $x_t \equiv \alpha_t + r_t$ and collect the terms with the same index i in (3):

$$P_t^f \simeq \sum_{i=1}^{\infty} \frac{E_t D_{t+i}}{(1+\bar{x})^i} + \sum_{i=1}^{\infty} \frac{\partial P_t^f}{\partial x_{t+i}} (E_t x_{t+i} - \bar{x}).$$

The authors assume $r_t = r$ and $(\alpha_{t+1} - \bar{\alpha}) = \xi(\alpha_t - \bar{\alpha}) + \varepsilon_{t+1}$, where $\xi \equiv E(\alpha_t)$. What happens with a time-varying interest rate? Consider the simple case where (i) $(r_{t+1} - \bar{r}) = \rho_1(r_t - \bar{r}) + \eta_{t+1}$, (ii) $\rho_1 = \xi$ and (iii) $E(\eta_t \varepsilon_s) = 0 \ \forall t, s$. Then x_t is itself an AR(1) process with root equal to ξ and the state-space formulation of Chen *et al.* (2001) is statistically correct, even though interpreting the estimation results is difficult because the filtered state is x_t and not α_t as the authors claim. But these assumptions are arbitrary: r_t and α_t need not be AR(1) processes; even if they are, there is clearly no reason why they should have identical roots. Risk premium and riskless return have to be modelled separately, and ideally the state-space system should include an equation for each of them.

If a simplifying assumption is to be introduced, a constant premium seems to be better than a constant interest rate. One obvious reason is that when the empirical analysis is based on long low-frequency data a constant safe return is clearly counterfactual: we do not know if and how α_t changed during the last century, but we know for sure that r_t moved continuously. Furthermore, r_t typically displays a strong autoregressive pattern (this need not be the case for the risk premium), and ignoring it amounts to passing it to the residuals. Finally, devicing an equation for α_t is not a trivial task. Two routes are available in the context of the state-space system. The first one, followed by Chen *et al.* (2001), is to treat the premium as a further non-observable state variable and simply postulate its process. This allows maximum likelihood estimation of the pattern of α_t. However, since it is difficult to justify the choice of any particular process for α_t on theoretical grounds, we end up appending to the model an arbitrary component. Furthermore, the estimation becomes technically more problematic: in this scenario the B_t equation

contains a product between non-observables, so the model can only be estimated using an approximate filtering procedure that relies on a first-order Taylor expansion of the transition equation (Harvey (1989)). Since the measurement equation is itself the outcome of a linearization, this is clearly not desirable. In Chen *et al.* (2001) this problem is avoided by assuming that the bubble grows at a constant rate $(1 + r + \alpha)$, but this is inconsistent: a price bubble cannot grow at a constant rate in a world were the discount factor changes over time.

The alternative is to link the premium to some observable variable and identify its generating process in the data. For instance, Poterba and Summers (1986) use the linear relationship between equity premium and variance of equity returns derived by Merton (1973); they model α_t as an AR(1) because in their daily dataset the variance is well described by this type of process. On annual data, the variance of market returns does not display clear autoregressive dynamics; if Merton's (1973) equation is to be used, it is necessary to think of more sophisticated processes. An interesting possibility is to model the premium as a regime-switching variable. If α_t has a low-mean and a high-mean state, A_t has to be computed taking into account the possibility of α_t moving between the two in the future. The occurrence of isolated, short periods of high volatility in the stock market may then help explain high equity prices over long horizons. The implementation of this idea is left for future research.

The dynamics of A_t and R_t also deserve a comment. Chen *et al.* (2001) show that A_t follows by construction a process of this type:

$$A_{t+1} = \beta^{-1} A_t + P_t^L (E_t \alpha_{t+1} - \bar{\alpha}) + \nu_{t+1},$$

$$
\begin{aligned}
\nu_{t+1} \quad &\equiv \quad -\sum_{i=1}^{\infty} \beta^i \sum_{k=0}^{\infty} \beta^{k+1} E_{t+1} D_{t+1+i+k} (E_{t+1} \alpha_{t+1+i} - \bar{\alpha}) \\
&+ \sum_{i=2}^{\infty} \beta^{i-1} \sum_{k=0}^{\infty} \beta^{k+1} E_t D_{t+i+k} (E_t \alpha_{t+i} - \bar{\alpha}).
\end{aligned}
$$

This can be derived by simply substituting the definition of A_t and A_{t+1} in $(A_{t+1} - \beta^{-1} A_t)$ and rearranging the terms in an appropriate fashion. With

32

the bivariate approximation, an analogous equation also holds for R_{t+1} once α_t is replaced with r_t. The estimation procedure the authors follow relies on the claim that $E_t \nu_{t+1} = 0$ by the law of iterated expectations, and implicitly assumes that ν_t is serially uncorrelated. For the case where D_t grows at a constant expected rate and $\alpha_t \sim AR(1)$, ν_t can be derived analytically and it turns out that $E(\nu_t \nu_{t-1}) \neq 0$ unless further assumptions are made on the innovations of these two processes. It is thus preferable to model the conditional expectation of D_{t+i+k}, α_{t+i} and r_{t+i} explicitly and use (5) and (6) to place restrictions on the price-dividend ratio equation. This strategy may obviously be unattractive (or unfeasible) if D_t, α_t and r_t follow particularly complicated processes.

References

[1] Abel A.B., Blanchard O.J. (1986) - The present value of profits and cyclical movements in investment, Econometrica 54(2), 249-273.

[2] Abreu D., Brunnermeier M.K. (2003) - Bubbles and crashes, Econometrica, 71, 173-204.

[3] Black F. (1986) - Noise, Journal of Finance, 41, 529-543.

[4] Blanchard O., Rhee C., Summers L. (1993) - The stock market, profit and investment, Quarterly Journal of Economics, 108(1), 115-136.

[5] Blanchard O., Watson M. (1982) - Bubbles, rational expectations and financial markets, in P. Wachtel editor "Crisis in the economic and financial structure", Lexington Books.

[6] Bond S.R., Cummins J.G. (2001) - Noisy share prices and the Q model of investment, Institute for Fiscal Studies Working Paper 01/22.

[7] Burmeister E., Wall K. (1982) - Kalman filtering estimation of unobserved rational expectations with an application to the German hyperinflation, Journal of Econometrics, 20, 255-284.

[8] Campbell J.Y., Lo A.W., MacKinlay A.C. (1997) - The econometrics of financial markets, Princeton University Press.

[9] Campbell J.Y., Shiller R.J. (1988) - The dividend-price ratio and the expectations of future dividends and discount factors, Review of Financial Studies, 1(3), 195-228.

[10] Chen L.T., Hueng C.J., Lin C.J. (2001) - Do bubbles and time-varying risk premiums affect stock prices? A Kalman filter approach, University of Alabama, working paper n. 00-04-02.

[11] Cole K., Helwege J., Laster D. (1996) - Stock market valuation indicators: is this time different?, Federal Reserve Bank of New York Research Paper n. 9520.

[12] Driffill J., Solá M. (1998) - Intrinsic bubbles and regime-switching, Journal of Monetary Economics, 42, 357-373.

[13] Elwood K., Ahmed E., Rosser J.B. (1999) - State-space estimation of rational bubbles in the Yen/Deutschemark exchange rate, Weltwirtschaftliches Archiv, 135(2), 317-331.

[14] Flood R.P., Garber P.M. (1980) - Market fundamentals versus price level bubbles: the first tests, Journal of Political Economy, 88, 745-770.

[15] Flood R.P., Garber P.M., Scott L. (1984) - Multi-country tests for price level bubbles, Journal of Economic Dynamics and Control, 8, 329-340.

[16] Flood R.P., Hodrick R.J. (1990) - On testing for speculative bubbles, Journal of Economic Perspectives, 4(2), 85-101.

[17] Froot K.A., Obstfeld M. (1991) - Intrinsic bubbles: the case of stock prices, American Economic Review, 81(5), 1189-1214.

[18] Geanakoplos J., Magill M., Quinzii M. (2002) - Demography and the long-run predictability of the stock market, mimeo, Yale University.

[19] Gordon M.J. (1962) - Variability of price-earnings ratio, American Economic Review, 52, 203-208.

[20] Hall R.E. (2001) - The stock market and capital accumulation, American Economic Review, 91, 1185-1202.

[21] Hamilton J.D. (1994) - Time series analysis, Princeton University Press.

[22] Hamilton J.D., Whiteman C.H. (1985) - The observable implications of self-fulfilling expectations, Journal of Monetary Economics, 16, 353-374.

[23] Hansen L.P., Sargent T.J. (1980) - Formulating and estimating dynamic linear rational expectations models, Journal of Economic Dynamics and Control, 2, 7-46.

[24] Harvey A.C. (1989) - Forecasting, structural time series and the Kalman filter, Cambridge University Press.

[25] LeRoy S.F. (2004) - Rational exuberance, mimeo.

[26] LeRoy S.F., Porter R. (1981) - The present value relation: tests based on implied variance bounds, Econometrica, 49, 555-674.

[27] Lucas R. (1978) - Asset prices in an exchange economy, Econometrica, 46, 1429-1445.

[28] Mehra R., Prescott E.C. (1985) - The equity premium: a puzzle, Journal of Monetary Economics, 15, 145-161.

[29] Merton R.C. (1973) - An intertemporal capital asset pricing model, Econometrica, 41, 867-887.

[30] Newey W.K., West K.D. (1987) - A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, Econometrica, 55, 703-708.

[31] Poterba J.M., Summers L.H. (1986) - The persistence of volatility and stock market fluctuations, American Economic Review, 76(5), 1142-1151.

[32] Robertson D., Wright S. (2003) - Dividends, total cashflow to shareholders and predictive return regressions, mimeo.

[33] Sarno L., Taylor M.P. (1999) - Moral hazard, asset price bubbles, capital flows, and the East Asian crisis: the first tests, Journal of International Money and Finance, 18, 637-657.

[34] Shiller R. (1981) - Do stock prices move too much to be justified by subsequent changes in dividends?, American Economic Review, 71(3), 4121-436.

[35] Shiller R. (2000) - Irrational Exuberance, Princeton University Press, Princeton.

[36] Stiglitz J.E. (1990) - Symposium on bubbles, Journal of Economic Perspectives, 4, 13-17.

[37] Tirole J. (1982) - On the possibility of speculation under rational expectations, Econometrica, 50, 1163-1181.

[38] Tirole J. (1985) - Asset bubbles and overlapping generations, Econometrica, 53, 1499-1528.

[39] West K.D. (1987) - A specification test for speculative bubbles, Quarterly Journal of Economics, 102(3), 553-580.

[40] West K.D. (1988) - Bubbles, fads and stock price volatility tests: a partial evaluation, Journal of Finance, 43(3), 639-656.

[41] Wright S. (2004) - Measures of stock market value and returns for the US nonfinancial corporate sector, 1900-2002, (forthcoming) Review of Income and Wealth.

[42] Wu Y. (1995) - Are there rational bubbles in foreign exchange markets? Evidence from an alternative test, Journal of International Money and Finance, 14 (1), 27-46.

[43] Wu Y. (1997) - Rational bubbles in the stock market: accounting for the US stock price volatility, Economic Inquiry, 35, 309-319.

Table 1 - A consistency test for the fundamental model.

(1) $\quad \frac{P_t}{D_t} = c_0 + c_1 r_t + n_t, \qquad n_t \sim AR(1)$

(2) $\quad \frac{D_t}{D_{t-1}} = \phi + \varepsilon_t$

(3) $\quad r_t = \rho_0 + \rho_1 r_{t-1} + \eta_t$

Unrestricted estimates:

c_0	c_1	ϕ	ρ_0	ρ_1	$R^2_{(1)}$	$R^2_{(2)}$	$R^2_{(3)}$	LLik.:
23.94**	-33.74**	1.02**	.01	.44**	.72	.00	.21	-465.65
(2.77)	(14.58)	(.01)	(.00)	(.10)				

Restricted estimates:

β	ϕ	ρ_0	ρ_1	$R^2_{(1)}$	$R^2_{(2)}$	$R^2_{(3)}$	LLik.:
.94**	1.02**	.01*	.44**	.71	.00	.21	-465.70
(.01)	(.02)	(.00)	(.08)				

L-ratio statistic:	.104
10% critical value for $\chi^2(1)$:	2.71

Notes: The system is estimated by Full-Information Maximum Likelihood under the assumption of joint normality of the innovations. The data is S&P, 1900-1995.

Table 2 - State-space estimates for the S&P data.

$$\frac{P_t}{D_t} = c_0 + c_1 r_t + \frac{B_t}{D_t} + n_t$$
$$B_t = (1 + r_{t-1} + \alpha)B_{t-1} + b_t$$

	c_0	c_1	α	Mll	CVM	BDSL
[i] 1900-1990	22.06**	-16.6**	-.07	-247	.096	.015
	(2.64)	(8.43)	(.06)		.60	.09
[ii] 1900-1990†	21.98**	-17.20*	[.06]	-253	.064	.014
	(2.84)	(9.47)			.78	.11
[iii] 1900-2002	21.11**	-18.97*	.11**	-315	.277	.037
	(3.92)	(10.03)	(.03)		.15	.00
[iv] 1900-2002†	17.66**	-18.40	[.06]	-317	.418	.046
	(3.84)	(18.97)			.06	.00

Notes: Standard errors in brackets; (*) and (**) denote significance at the 10% and 5% level. (†) means that α is restricted. Mll is the maximised log-likelihood. CVM and BDSL are diagnostic tests on the standardised prediction errors of the P_t/D_t equation, namely $\hat{n}_{t|t-1}$. CVM is the Cramer-Von Mises test of the null hypothesis $n_t \sim N(0,1)$; the table reports the test statistic and, below, the p-value. BDSL is the Brock *et al.* test of the null hypothesis $n_t \sim i.i.d.$ (see footnote 18, p.17); the table reports the statistic and the bootstrapped p-value (the "embedding dimension" is set equal to two; p-values are consistently larger for higher dimensions).

Table 3 - Price-dividend ratio equation, non-adjusted industry data.

$P_t/D_t = c_0 + c_1 r_t + n_t$				
	c_0	c_1	R^2	DW
1900-2000	31.77**	-35.44*	.71	2.01
	(10.31)	(19.79)		
1900-1990	26.10**	-39.35*	.67	2.02
	(4.51)	(21.50)		
1900-1970	28.22**	-31.53*	.67	1.91
	(9.31)	(16.04)		

Notes: All equations estimated by 2sls; $n_t \sim AR(i)$, where the choice of $i = 1, 2, 3$ is data-driven. Newey-West standard errors in brackets (truncation at lag 3).

Table 4 - Price-dividend ratio equation, adjusted industry data.

$P_t^a/D_t^a = c_0 + c_1 r_t + n_t$				
	c_0	c_1	R^2	DW
1900-2000	18.92**	-18.06*	.80	2.02
	(2.29)	(10.83)		
1900-1990	17.73**	-16.35	.82	2.03
	(2.73)	(10.62)		
1900-1970	19.67**	-18.58*	.77	1.90
	(3.58)	(10.67)		

Notes: All equations estimated by ordinary least squares; $n_t \sim AR(i)$, where the choice of $i = 1, 2, 3$ is data-driven. Newey-West standard errors in brackets (truncation at lag 3). Two dummy variables are introduced for 1929 and 1971; they are both positive and significant at the 1% level.

Table 5 - Magnitude of the bubble.

	S&P companies:			Non-financial sector:		
	$\frac{P_t}{D_t}$	$\frac{B_{t|t}}{D_t}$	%	$\frac{P_t}{D_t}$	$\frac{B_{t|t}}{D_t}$	%
1960	30.2	9.3	*30.9*	34.6	9.9	*28.8*
1961	29.7	9.5	*32.0*	41.5	14.5	*35.0*
1962	32.8	11.5	*35.1*	37.4	13.5	*36.2*
1963	29.0	9.8	*33.8*	37.2	12.8	*34.5*
1964	30.8	10.3	*33.4*	39.9	14.4	*36.0*
1965	32.2	11.3	*35.2*	40.0	14.7	*36.8*
1966	33.6	12.8	*38.2*	32.6	11.7	*35.9*
1967	29.9	11.5	*38.5*	40.7	14.3	*35.2*
1968	32.3	12.1	*37.6*	44.2	17.1	*38.8*
1969	34.2	13.8	*40.3*	37.4	16.1	*43.0*
1970	30.2	12.3	*40.9*	38.5	16.6	*43.1*
[...]						
1990	29.6	8.6	*29.2*	25.0	4.1	*16.4*
1991	27.3	8.3	*30.6*	32.1	5.6	*17.7*
1992	34.7	12.1	*34.9*	32.6	6.4	*19.8*
1993	35.4	14.4	*40.6*	32.6	6.9	*21.3*
1994	36.8	16.0	*43.3*	30.5	6.7	*22.1*
1995	34.6	15.7	*45.3*	36.0	8.0	*22.4*
1996	42.4	19.7	*46.4*	37.9	9.4	*24.8*
1997	50.2	26.5	*52.8*	44.7	12.6	*28.3*
1998	60.4	35.3	*58.5*	48.4	15.5	*32.1*
1999	76.8	49.4	*64.3*	60.5	21.8	*36.1*
2000	90.8	66.5	*73.2*	46.1	23.4	*50.8*

Notes: The bubble is significant at the 5% level for all t; however, in the case of the non-financial sector the Kalman filter is run assuming that the parameters of the price-dividend ratio equation are known (see text).

Table 6 - A reassessment of the "intrinsic bubble" hypothesis.

$$P_t/D_t = c_0 + cD_t^{\lambda-1} + \eta_t, \quad \eta_t = \theta\eta_{t-1} + u_t$$

		c_0	c	θ	R^2	DW
1900-1990:	S&P[1]	14.74**	.72**	.58**	.66	1.71
		(.00)	(.00)	(.00)		
	Sector, na[2]	21.02**	11.03	.78**	.63	1.95
		(0.00)	(0.08)	(.00)		
	Sector, a[2]	22.04*	-5.46	.33**	.15	1.84
		(0.02)	(.61)	(.00)		
1900-2000:	S&P[1]	21.90**	-.04	1.12**	.86	1.83
		(.00)	(.43)	(.00)		
	Sector, na[2]	23.65**	4.79**	.76**	.71	2.03
		(.00)	(.00)	(.00)		
	Sector, a[2]	17.45**	.26	.32**	.24	1.87
		(.00)	(.96)	(.00)		

Notes: Asterisks denote significance at the 5% or 1% level (the p-value is in brackets). [1] Standard&Poors data and OLS estimation. [2] industry data, adjusted (a) and non-adjusted (na), 2SLS estimation with D_t instrumented by D_{t-1}. The equation for adjusted data also contains two dummy variables for 1929 and 1971.
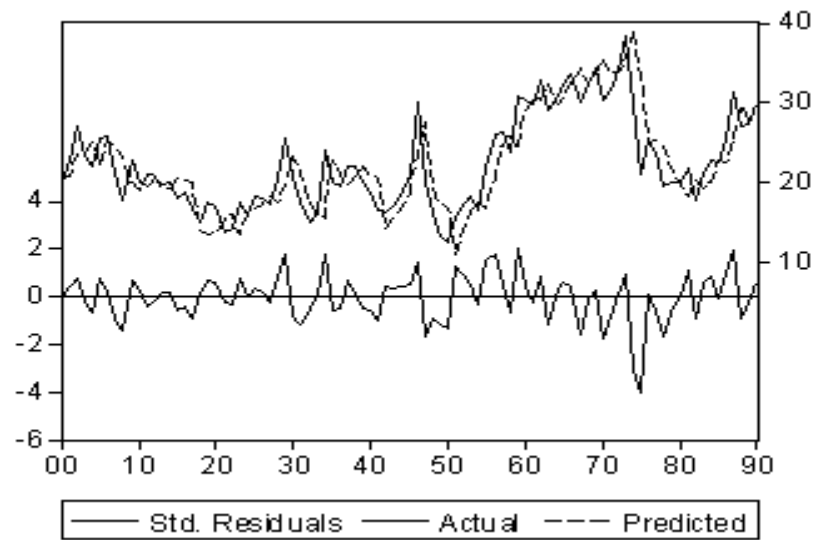
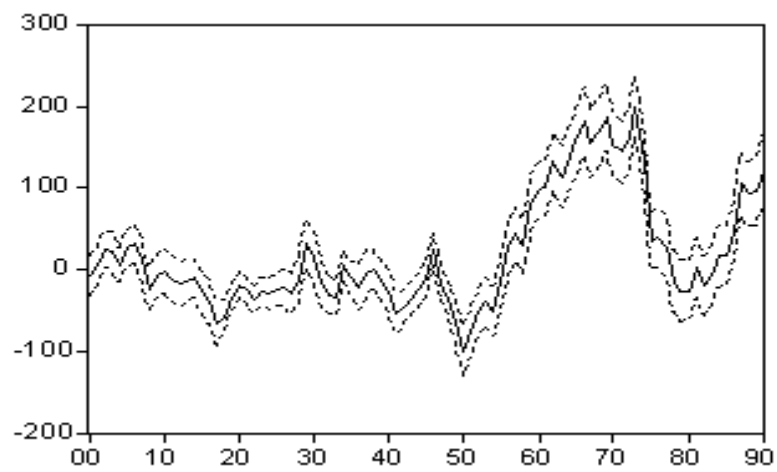Figure 1 - Price-dividend ratio ($\alpha = .06$; sample 1900-1990).

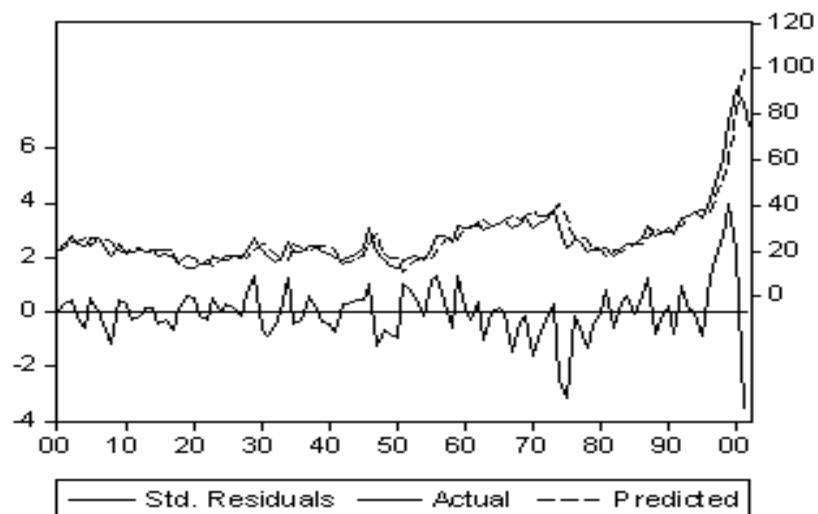Figure 2 - $B_{t|t} \pm 2$ standard error ($\alpha = .06$; sample 1900-1990).

42

Figure 3 - Price-dividend ratio ($\hat{\alpha} = .11$; sample 1900-2002).
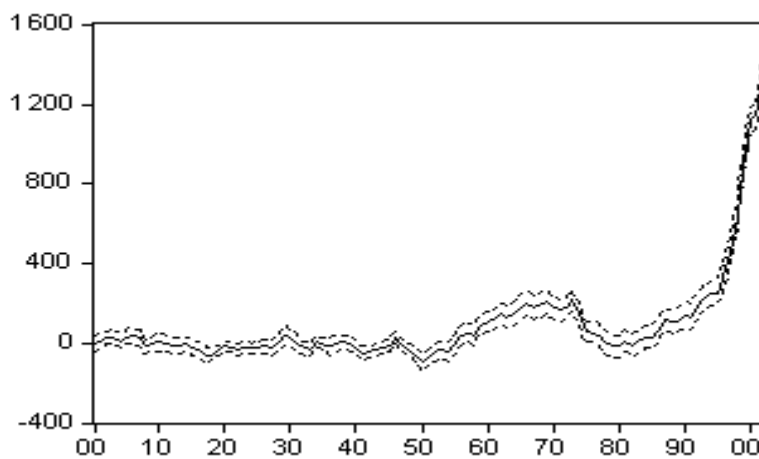

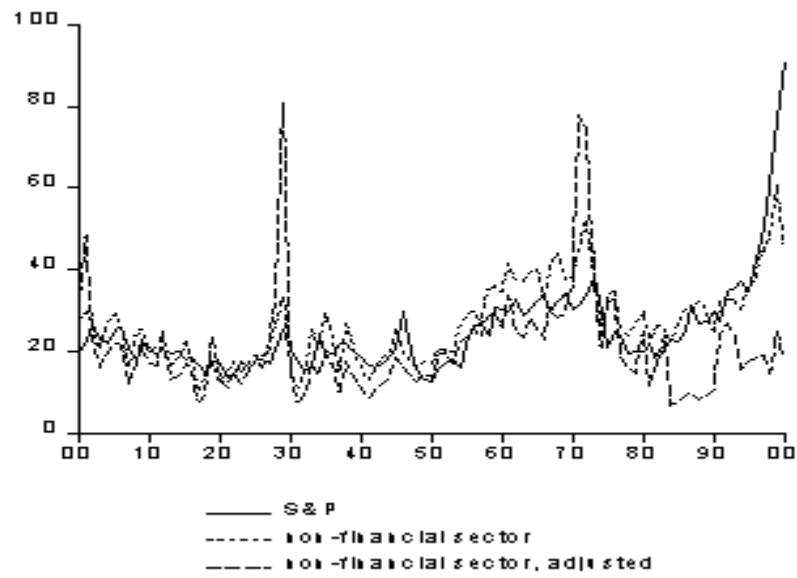Figure 4 - $B_{t|t} \pm 2$ standard error ($\hat{\alpha} = .11$; sample 1900-2002).

Figure 5 - Price-dividend ratio for S&P and non-financial companies.
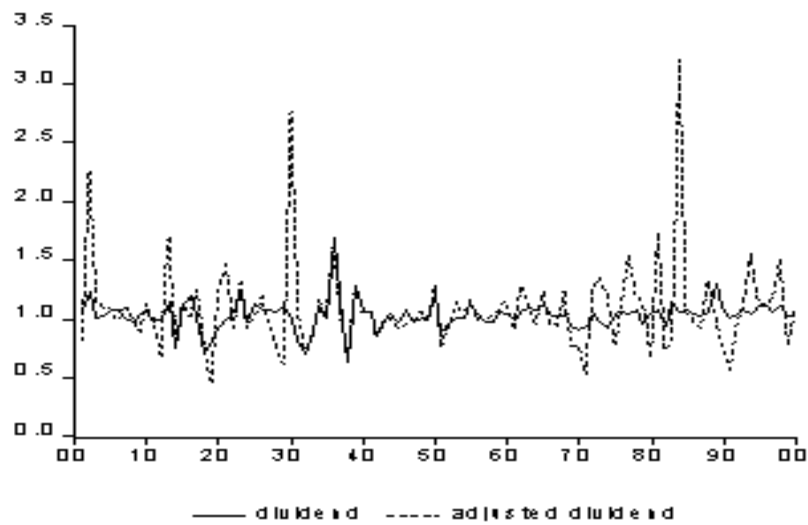

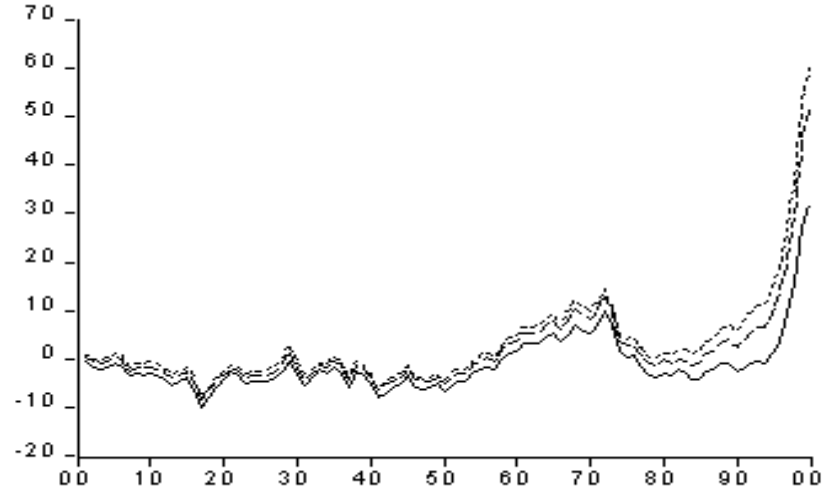Figure 6 - Growth rates, adjusted and non-adjusted dividends.

44

Figure 7 - $B_{t|t}$ under alternative values of c_0, c_1.

Notes: The bubble is filtered under the alternative assumptions that $[c_0, c_1]$ is equal to [31.77,-35.44] (continuous line), [28.22,-31.53] (long dash) and [26.10,-39.35] (short dash).