Sovereign Debt Recontracting: The Role of Trade Credit and Reserves

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Abstract

This paper analyzes sovereign debt in an economy in which the availability of short-term trade credit reduces international trade transaction costs. The model highlights the distinction between gross and net international reserve positions. Borrowed reserves may provide net wealth and liquidity services during a negotiation, as long as they are not fully attachable by creditors. Moreover, reserves strengthen the bargaining position of a country by shielding it from a cut-off from short-term trade credits thereby diminishing its degree of impatience to conclude a negotiation. We show that competitive banks do lend for the accumulation of borrowed reserves, that provide partial insurance against consumption risk associated with uncertain output.

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1 Introduction

Access to short-term trade credits has often been pointed out as key for understanding why countries repay their debts if not for reputational considerations alone. In his 1999 survey Kenneth Rogoff noted that *The strongest weapon of disgruntled creditors, perhaps, is the ability to interfere with short-term trade credits that are the lifeblood of international trade* (Rogoff (1999, p. 31). Nevertheless, short-term trade credits have not been formally incorporated into the sovereign debt literature.[1] This paper tries to bridge this gap. Although we are not aware of a study that quantifies the effects of trade finance on sovereign lending, a few papers do suggest that the effects are of first order. One such study is Rose (2002), that has found empirical support for the hypothesis that the downside of a non repayment strategy comes through the trade channel: changes in international debt contracts are generally followed by substantial reductions in trade flows between the creditor and the borrowing country.[2] The study mentions the use of retaliatory trade measures and reductions in the trade credit availability as candidate explanations for the means by which the fall in trade flows might come about. However, as an increasing number of countries are becoming WTO members and there is no exemption clause to the non-discrimination principle related to debt issues in the GATT articles, the scope for retaliatory measures seems

[1] The seminal paper of Bulow and Rogoff (1989a) incorporated retaliatory trade measures into the literature. The paper does refer to the importance of trade credits in its introduction.

[2] Rose and Spiegel (2002) find that there is more lending between countries that trade more.

to be rather narrow. Moreover, with multiple creditors each of them individually might be tempted to free ride and let other creditors incur the costs of punishment (Wright (2002)). No such legal impediment applies to trade credit however, which occurs on a voluntary basis and often comes from governmental trade agencies and private banks - most of which are also creditors in other types of lending operations with the borrowing country.

By introducing an explicit role for trade credit, we obtain two basic insights for the role of international reserves. First, we highlight a rationale for borrowed reserve holdings that relies on the role of gross reserves in providing liquidity services in the event of a debt renegotiation that impairs the borrower's access to short-term trade credits. The model thus provides an explanation for why developing countries often hold substantial stocks of reserves in spite of the fact that their external liabilities carry a considerably higher interest rate. The imperfect substitutability between gross reserves and undrawn credit was mentioned for instance when Brazilian Central Bank Governor Arminio Fraga added \$30bn of IMF funds to the bank's international reserves explaining that *It is much easier to negotiate the lowering of the net reserves limit, so that the funds can be used to intervene in the foreign exchange market or fund commercial trade lines than to negotiate a new financial assistance package.*[3]

Second, we find a theoretical underpinning for anecdotal evidence suggesting that the terms of actual rescheduling agreements may be sensitive

[3] In *Brazil presidential candidates would be foolish to reject IMF deal*, AFX Press, Aug 16th, 2002. The possibility of using reserves for the extension of commercial credit, which did materialize, had been previously discussed with the IMF.

to the ability of creditors and borrowers to 'wait out' a bargaining process. From the borrower's side, time pressure comes from the fact that short-term trade credit may dry up during the period in which outstanding loans are in default. In our model, the country's liquidity demand during a renegotiation can only be met by pre-existing reserves. International reserves therefore directly affect the bargaining position of debtors during a debt renegotiation, by reducing the country's degree of impatience to reach an agreement. Similarly lenders may face time pressure, deriving for instance from accounting practices that impose costs on bank loans that remain in arrears for sufficiently long time periods or fund managers may be forced to off-load bonds that systematically fail to pay interest from their portfolio at heavy discounts. The ongoing restructuring process of Argentine bonds is particularly illustrative in this respect. The Argentine government seems to have adopted a reserve accumulation and delay strategy. As reported by the Financial Times, *Mr. Lavagna's [the finance minister] most effective weapon has been time. By spinning the restructuring process out for almost three years, he has worn a lot of investors down. Investment funds tire of seeing on monthly reports Argentine bonds that pay no interest.*[4]

Two considerations imply that higher reserves shift bargaining power towards the borrower. First, although borrowed international reserves are offset by external liabilities and therefore do not provide net wealth *ex ante*, they are only partially attachable (at worst) and therefore do provide net wealth in the event of a repudiation. Hence, a higher stock of reserves increases the credibility of the borrower's threat to walk away from the negotiation

[4] In *Argentina closes in on sweet debt relief deal.* Sept 20th, 2004.

table. Second, while reserves and trade credits are perfect substitutes when the latter are available, reserves may provide liquidity services in case of repayment problems, until an agreement that restores the borrower's access to short-term credit markets is reached. A borrower with reserves is therefore less impatient to conclude a rescheduling negotiation. The net effect is that borrowed reserves allow the borrower to shift some consumption from a high-consumption state, associated with debt repayment, to a low-consumption state, in which debt is rescheduled (van Wijnbergen (1990)). It follows, as we show below, that they constitute an additional channel through which risk may be shifted from (probably less risk averse) lenders to borrowers.

While our model does allow for the possibility that higher reserve holdings lead to higher debt repayments, it can also explain why in some instances countries with sizable foreign reserves may obtain concessions from creditors or show reluctance to spend reserves on debt buy-back operations. A recent negotiation between the Russian and German governments provides an example. In the beginning of 2001, after its reserves had tripled to $24bn, the Russian government tried to pursue a hard line with its creditors, of which Germany was the principal, by declaring a *technical delay of repayments.* The timing coincided with ongoing negotiations with the German government over some $6.4bn worth of 'transfer roubles' - an artificial currency used for trade in Soviet times.[5] Following discontinuation of debt service,

[5] The first creditor affected was the German export credit guarantee group Hermes, that did not receive repayments of Soviet-era borrowings. The sovereign analyst of the rating agency Standard and Poor's commented the Russian threat of default by noting: *They are not desperate for funds, that obviously strengthens their position.* In *Russia's Threat of Default*, FT, Jan 5th, 2001.

Germany responded by withholding new export credit guarantees to Russia. Although the overall success of the Russian strategy is an open issue, Germany settled one year later for \$440m, at the same time agreeing to raise the insurance cover of business relations with Russia.

Relation to the literature. The sovereign debt literature has evolved around a controversy about the form of punishment that disgruntled creditors may impose on defaulting borrowers. Our model assumes that the country's assets can be partially seized in the event of repudiation.[6] In this sense it is closer to Bulow and Rogoff (1989a), who allow a fraction of export proceeds to be attached by lenders, than to Eaton and Gersovitz (1981) or Bulow and Rogoff (1989b) where the rationale for repayment is based on reputational aspects alone and non-repayment is punished with permanent exclusion from international credit markets. With rational expectations and perfect information however, asset seizures do not occur in equilibrium and deadweight losses are avoided (Eaton and Engers (1999)). All that creditors effectively do in case of default is to withhold voluntary short-term trade credits, although they are prepared to attach a fraction of the borrower's assets if negotiations prove unsuccessful. The possibility of attachment however defines the threat points, conditioning the outcome of the bargaining process.[7]

In a related paper, Detragiache (1996) argues that international reserves

[6] Alternatively, one could assume that an attempt to attach assets in court is successful with probability $\nu > 0$. This would not alter the implications of the paper.

[7] In practice, attachments have occasionally occurred during debt renegotiations. Delaume (1994) discusses attachability in the context of sovereign debt defaults and Wright (2002, p.35-37) provides an account of the recent legal battle between the Swiss Compagnie Noga d'Importacion et d'Exportacion and the Russian government.

holdings increase repayments in a debt renegotiation framework. As in the reserve demand model of Aizenman and Marion (2004), Detragiache relies on a combination of convex Barro-type tax distortions and the non-existence of domestic credit markets. The argument here is that international reserves reduce the adjustment cost associated with having to repay debt from current tax revenue, thereby reducing the borrower's bargaining power. One limitation of this argument, however, is that as long as the borrower holds international reserves, it can choose the timing of a default. If reserves increased renegotiated repayments, the borrower would always choose to get rid of reserves just ahead of formally entering default.[8] While in our model reserves may increase repayment to lenders, they unambiguously reduce the lenders *share* of the surplus. As the borrower's welfare is monotonically increasing with the level of reserves, the model may explain why we do not see debt buyback operations with greater frequency during debt crises. It can also explain why most borrowers that do default do so with positive reserve holdings, a case that has been referred to as *strategic default* in the literature.

Outline. The paper is organized as follows. Section 2 outlines the model. Section 3 analyzes the bargaining game that begins at the moment output is realized and debt service is due. Following the approach of Rubinstein (1982), we find a unique Nash equilibrium by exploiting the relative impatience of bargainers and the requirement that all threats be credible (i.e., the equilibrium is perfect). Section 4 scrutinizes the borrower's choice between

[8] Consider for instance the strategy of paying government expenditures in advance using reserves. If reserves increase the renegotiated repayment, this strategy reduces taxation and the distortions associated with it, with no effect on the level of government expenditures.

repayment and rescheduling. The model implies that the borrower has an incentive to accumulate gross reserves even though such balances would be an inefficient source of liquidity if debts were always repaid. Section 5 studies the reserve accumulation process by endogeneizing long-term borrowing in advance of a potential rescheduling. We show that competitive lenders do provide long-term finance not only for investment projects, but also for accumulation of international reserves. We conclude by discussing some empirical implications of the model and directions for further research.

2 The Model

The model is a hybrid of a two-period model and an infinite horizon model. At time zero the borrower enters a competitive loan market in which a large number of risk-neutral lenders compete to provide funds. Banks are assumed to maximize expected profits discounting at rate r, that is taken to be less than the country leader's (henceforth country's) rate of time preference, δ. Competition drives expected profits to zero.

2.1 The Technology

There are three goods. Since trade is central to the story, we assume that the only consumption good is an importable good that is not produced locally and is the international numeraire. The borrowing country is thought to be a small open economy whose production is unable to affect world prices. The other two goods are exportables that accrue to the country in period 1. The borrower has three sources of the importable good for consumption:

i) international reserves; ii) a storable export good that is the output of the investment project and has price 1 in terms of the importable; iii) a perishable export good that accrues to the country as a constant endowment stream of y per period (hy over any interval of length h), starting in period 1. The perishable export good can be traded internationally at price p. Since exportable goods are only obtained in period 1, the country has to borrow B to be able to consume C_0, invest in a risky project or accumulate reserves.

The production technology of the storable exportable good requires one unit of the imported good as input at $t = 0$, giving a stochastic output $Q(s)$ at $t = 1$, where s is a discrete random variable with finite support whose probability distribution is common knowledge. To keep things simple, we will assume that the country has no further need for project finance upon completion of the investment project. Hence, the amount borrowed B will be given by

$$B = 1 + C_0 + (R_1 - R_0)$$

where R_t represents the level of reserves held at the beginning of period t.

Note that since one exportable good is perishable, it must be traded immediately, with the proceeds either consumed or added to reserves. Also, we assume that the country cannot convert the output from the investment project into reserves during a debt renegotiation. This condition is satisfied endogenously as long as any attempt to sell the output is interpreted as repudiation, triggering attachment of a share of reserves and output.[9]

[9] Alternatively, the exportable output may be freezable during a negotiation.

2.2 The Borrower

The country's preference at time 0 is given by

$$U_0 = u(c_0) + \beta E v(W_1) \tag{1}$$

where $u(.)$ and $v(.)$ are twice differentiable, concave functions and W_1 represents an index of future consumption. The expectation in (1) is taken over the probability distribution of output from the investment project. Concavity of the utility function implies that the country will wish to insure against variability of W_1 deriving from the stochastic production technology.

Although the two period structure in (1) is all we need to study the insurance role of reserves, we want actual debt service on the original loan to be determined by a time consuming bargaining process. We therefore treat W_1 not as a consumption in a single terminal period, but as a measure of consumption over the indefinite future. In order to get closed form solutions, we assume that the borrower is risk neutral from time 1 onwards. At $t \geq 1$, then, the borrower maximizes the present value of consumption, W_t, given by

$$W_t = \sum_{i=0}^{\infty} [\beta(h)]^i \, c_{t+hi} \quad , \quad t \geq 1 \tag{2}$$

where $\beta(h) = \frac{1}{1+\delta h}$ is the country's discount factor and h will coincide with the interval between alternate proposals during a debt renegotiation (the dependence of β on h will be suppressed when this can be done without confusion). The country therefore maximizes utility over an infinite horizon, although at time 0 all that is relevant is the expected discounted value of future consumption.[10]

[10] The preferences given by (1) and (2) are not stationary, but this does not introduce

2.3 Trade Finance and Sanctions

Assumption. *In the absence of external short-term trade finance,* $p(R) \epsilon \; [p(0), 1]$, $p'(R) \geq 0$, $p''(R) \leq 0$ *and* $p(0) > 0$ with $\lim_{R \to \infty} p(R) = 1$. *If external short-term finance is available,* $p = 1$.

This assumption is intended to capture the potential liquidity services of international reserves. We take a reduced form approach, letting a more careful analysis of the micro-economic foundations for future research. It implies that when the borrower is cutoff from external short-term trade finance, its terms of trade are an increasing, concave function of the stock of reserves R.[11]

The dependency of terms of trade on liquidity gives the lenders the ability to harass a recalcitrant borrower by interfering with its access to short-term trade credit during a debt rescheduling process. Lenders have an incentive to limit availability of short-term credits to the country as much as possible, since by doing so they increase the borrower's impatience to reach an agreement. They are able to cut off short-term finance completely until the relationship with current creditors is terminated, either through a negotiated agreement or through unilateral repudiation, but not further.[12] This would

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a time consistency problem. To see this, notice that the marginal rate of substitution in consumption between any two future periods is the same regardless of the period from which it is viewed.

[11] As the storable export good is only traded after the end of negotiations, terms of trade equals the price of the perishable export good.

[12] One could extend the analysis to allow the possibility of cutoff of trade finance in the event of repudiation. Bulow and Rogoff (1989b) showed that in the absence of cash-in-advance insurance contracts, this kind of cutoff could sustain lending even if lenders were not able to extract debt service unilaterally. In Bulow and Rogoff (1989a) the lender may

be the case if trade credit were provided by the same lenders that provide the long term project finance or if debt instruments contained cross-default clauses.[13] (Also long-term credit is unavailable during a renegotiation.)

The lack of short-term trade credit increases the actual cost of exporting, being equivalent to a tax on exports whose proceeds are wasted. With no reserves, the country is restricted to international barter at terms of trade $p(0) > 0$. Reserves improve the terms of trade and in the limit substitute completely for the liquidity provided by access to short-term credit markets. Thus, the cost of operating as a financial autarky, $c(R_t)$, can be expressed in terms of the loss in real income per unit of time due to the terms of trade deterioration brought about by the cutoff from short-term trade finance, i.e.:

$$c(R_t) = (1 - p(R_t))y \tag{3}$$

Note that lenders do not freeze the country's reserve assets as long as a renegotiation process goes on.[14] Since the borrower suffers an utility loss of $c(R_t)$ in each period of the negotiation, we can focus on the implications of the cutoff from credit during the negotiation process.

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harass the borrower's trade forever if the borrower repudiates, but there is no trade during the negotiation. Incorporating a permanent cutoff from trade credit upon repudiation in our model increases the deadweight loss of repudiation and makes it less likely that the country can credibly threaten to do so.

[13] In reality, this is not always the case. Kaletsky (1985, p.37) gives examples in which LDC borrowers tried to discriminate among creditors, maintaining debt service for short-term lenders, while rescheduling longer term debts. We do not treat the implications of this observation here.

[14] Alternatively, one could allow lenders to attach a fraction γ of reserves at the outset of the negotiation.

If the country repudiates its foreign obligations, lenders can forcibly attach a fraction of the borrower's exportable output (as in Bulow and Rogoff (1989a)) and/or a portion of international reserves. Let $\gamma < 1$ and α be the fractions of international reserves and output, respectively, that the borrower loses as a result of the lenders' attempts to confiscate debt service.[15]

We also assume that there is no deadweight loss associated with the confiscation of reserves, but that the lender can only collect a fraction $(1 - \mu) < 1$ of the output lost by the borrower. The deadweight loss $\mu\alpha Q$ is an essential feature of the model, since it gives the country and its creditors an incentive to bargain to avoid the deadweight losses associated with the confiscation of output. With $\mu = 0$, repudiation by either party is Pareto efficient, leaving them with nothing to bargain over. In summary, the shares of the pie accruing to the country and the bank in the event of a repudiation, are given by $\lambda(t) = \frac{(1-\gamma)R_t+(1-\alpha)Q}{R_t+Q}$ and $1 - \lambda(t) - D(t)$ respectively, where $D(t) = \frac{\mu\alpha Q}{R_t+Q}$.

[15] In fact, central bank assets held in the U.S. are given protection by the Foreign Sovereign Immunities Act and there have been few successful freezes of reserves in association with the buildup of arrears and debt reschedulings (see Delaume (1994) for some instances), suggesting that the appropriate assumption is that gross reserves may not be fully confiscated by lenders. Note that freezing reserves of a country in default, but engaged in a 'good faith' rescheduling negotiation, is a different action than confiscating reserves of a borrower who has repudiated. The distinction is important for the discussion of the liquidity role of reserves in Section 4.

3 The Bargaining Game

Once we have the basic outline of the model, we start by looking at the outcome by backward induction, i.e., we take gross reserves, R_1, and debt service on long-term debt, B, as given. We then analyze the game that takes place when borrower and lender(s) observe output and debt service is due. We will assume that while debt may be owed to a large number of banks, the banks' interests in the event of a repudiation or rescheduling are represented by a single *lead bank* that acts on behalf of all lenders. The amount B, that represents interest and principal on all outstanding debts, is due at the instant that output is realized.

At time 1, the country's total resources consist of gross assets $R_1 + Q \geq 0$, where Q is the output of the investment project (remember that the perishable export good starts accruing only in period 1). On paper, these assets are offset by the stock of debt service obligations $B > 0$. Since the country has the option to repudiate its debt, however, the actual liability only amounts to the minimum of B and what it can be bargained into repaying.[16] We now focus on the bargaining solution.

3.1 The Negotiation Framework

To model the bilateral bargaining game we follow the alternating offers framework, developed by Rubinstein (1982), as outlined in Fig. 1. The bank and the country take turns at making proposals over how to divide the country's

[16] B will grow due to arrears during the negotiation. This is irrelevant to the solution because it does not affect penalties the lender can impose (reserve growth, in contrast, does matter).

resources at time t, denoted by $\pi_t = R_t + Q$. We denote the share of the pie to be received by the country by $q^*(t)$ when the bank makes the proposal and by $q(t)$ when the proposal is made by the country. Throughout the paper, starred variables will refer to bankers. Supposing that the bank has the first offer, the bargaining game will be characterized by a potential sequence of alternating offers $q^*(t)$, $q(t+h)$, $q^*(t+2h)$, $q(t+3h)$, etc.

After each proposal, the responding player either accepts or turns down the offer. In case of agreement, π_t is split according to the proposed terms. The agreement restores the country's creditworthiness and its access to external short-term trade credit, so that the country can trade the perishable exportable good at value $p = 1$, irrespective of reserves. The demand for reserves at that point will be zero and the pressure of discounting makes the country consume its share of the pie plus the current value of its exportable output immediately.

If players disagree, the responder may terminate the negotiation unilaterally by walking away from negotiations or it may wait to make a counteroffer.[17] In case of unilateral termination by either player, the bank will extract whatever debt service it can obtain by attaching the maximum fraction of reserves and/or confiscating a fraction of the country's (exportable) output.[18] To keep things simple, we assumed that once lenders have im-

[17]Sutton (1986) analyzes a game in which the responder has access to an outside option with propability p. The game here assumes that $p = 1$ and the outside option is unilateral termination of the negotiation.

[18]If repudiation penalties do not transfer resources to the bank, the bank will never find it optimal to repudiate. In contrast to Eaton and Gersovitz (1981) and Bulow and Rogoff (1989b), the lender does collect part of the penalty and therefore may prefer repudiation

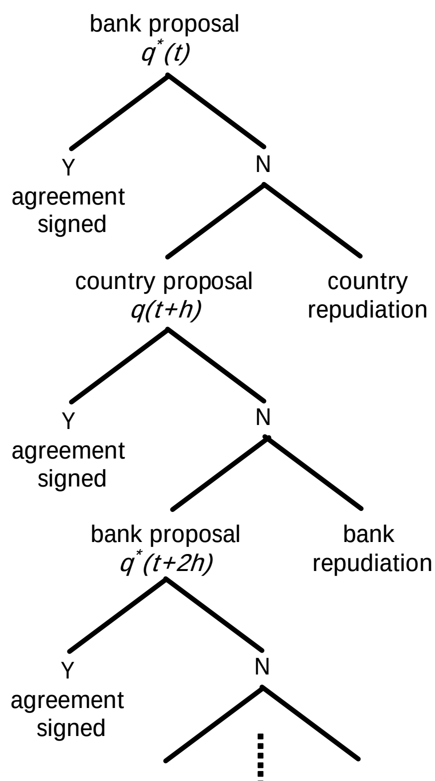

Figure 1: The bargaining game

posed this penalties, their claim on the country is regarded as settled. In other words, the original lenders cannot preclude new lenders from lending to the country after termination of a negotiation. If, on the other hand, a proposal by one of the players is rejected and a counter-proposal is made, the obligation remains on the table and the borrower remains in formal default.

There are three possible ways in which a negotiation can end: by agreement to the bank's proposal, by agreement to the country's proposal or by unilateral repudiation of one of the players. If $C_t \leq \pi_t$ denotes the country's consumption of reserves and output at time t given that negotiation ended at that time, the country's post-negotiation utility will be given by

$$W_t = C_t + \sum_{i=0}^{\infty} \frac{hy}{(1+\delta h)^i} = C_t + \frac{y}{\beta\delta} \tag{4}$$

where

$$C_t = \begin{cases} q(t)\pi_t & \text{if agreeing to country's proposal} \\ q^*(t)\pi_t & \text{if agreeing to bank's proposal} \\ (1-\gamma)R_t + (1-\alpha)Q & \text{if unilateral repudiation} \end{cases}$$

and the last term represents the present value of trade from the perspective of the country when the borrower has access to trade credit.[19]

3.2 The Bargaining Solution

To solve the model, we exploit the recursive nature of the game. Consider first the case in which the bank places the offer at time t. The best strategy

to bargaining.

[19]If $\delta > r$, the country could consider selling its output stream to the lenders, who attribute a higher value to it. We consider such contract to be suboptimal because of the adverse incentives it would generate on the production of a good (see Lucas (1979)) and/or enforceability problems in the delivery of goods.

for the bank will be to offer the minimum acceptable share to the borrower. If the country is to accept the offer, however, the utility deriving from its implementation must be at least equivalent to what the country would get by turning it down and either making the minimum acceptable counter-offer to the bank at $t+h$ or repudiating. Hence,

$$q^*(t)\pi_t + \frac{y}{\beta\delta} = \max\left[\lambda(t)\pi_t + \frac{y}{\beta\delta}; \beta\left(q(t+h)\pi_{t+h} + \frac{y}{\beta\delta}\right) + p(R_t)hy\right] \quad (5)$$

The second term in the brackets measures the country's utility if it waits to make the minimum acceptable offer in the next round. Note that we are assuming that the borrower consumes the proceeds from the sale of the perishable good immediately. As we show in Section 3.3, this results from optimal reserve policy during a renegotiation. Using the fact that $\frac{y}{\beta\delta} = \frac{y}{\delta} + hy$, and substituting with expressions (3) and (4) in (5), gives us the following expression for the bank's minimum acceptable offer to the country:

$$q^*(t) = \max\left[\lambda(t); \beta\frac{\pi_{t+h}}{\pi_t}q(t+h) - \frac{hc(R_t)}{\pi_t}\right] \quad (6)$$

Note that the ability of the bank to cutoff credit during the negotiation affects the minimum offer, even though banks cannot impose any penalty beyond the period of repudiation. The country looses the amount $c(R_t)$ each period in which it remains in default, by virtue of having to finance its trade using its reserves rather than trade credit. In the particular case in which the stock of reserves is constant, this term is equivalent to the fixed bargaining cost introduced by Rubinstein (1982) in his original article.

Equation (6) provides one relationship between the offers $q^*(t)$ and $q(t+h)$. A second relationship can be obtained by considering the country's counter-offer at time $t+h$. As with the bank, the optimal minimum acceptable offer leaves the bank indifferent between accepting and refusing. If the bank were to wait to make the minimum acceptable counter-offer, it would receive the discounted value of its share of the pie, $\beta^* \frac{\pi_{t+2h}}{\pi_{t+h}} (1 - q^*(t+2h))$. The payoff obtained from unilateral termination of the negotiation is

$(1 - \lambda(t+h) - D(t+h)) \pi_{t+h}$.[20] This gives us

$$1 - q(t+h) = \max \begin{bmatrix} 1 - \lambda(t+h) - D(t+h); \\ \beta^* \frac{\pi_{t+2h}}{\pi_{t+h}} (1 - q^*(t+2h)) \end{bmatrix} \tag{7}$$

Taken together, equations (6) and (7) yield the following recursion for the country's share $q(t)$:

$$q^*(t) = \max \begin{bmatrix} \lambda(t); \\ \min \begin{bmatrix} \beta \frac{\pi_{t+h}}{\pi_t} \left(1 - \beta^* \frac{\pi_{t+2h}}{\pi_{t+h}} (1 - q^*(t+2h))\right); \\ \beta \frac{\pi_{t+h}}{\pi_t} (\lambda(t+h)) + D(t+h)) \end{bmatrix} - \frac{hc(R_t)}{\pi_t} \end{bmatrix} \tag{8}$$

Let ρ represent the growth rate of reserves. As long as $1+\rho h < \sqrt{(1+rh)(1+\delta h)}$, the unique convergent solution to the second-order difference equation in the minimum subgame perfect equilibrium bank offer $q^*(t)$ is given by

$$q_N^*(t) = 1 - \sum_{i=0}^{\infty} (\beta\beta^*)^i \left(\frac{\pi_{t+2ih}}{\pi_t} - \beta \frac{\pi_{t+(2i+1)h}}{\pi_t} + \frac{hc(R_{t+2ih})}{\pi_t} \right) \tag{9}$$

[20] Recall that we assumed that it is not costly for the bank to interfere with the country's access to trade credits. This is the natural assumption if the banks are the providers of trade credit. As noted by Bulow and Rogoff (1989a), the action may affect the utility of the country's trading partners and thereby bring them into negotiation. We do not model this possibility here.

(see Appendix A). The overall solution to expression (8) therefore takes the form

$$q^*(t) = \max\left[\lambda(t); \min\left[q_N^*(t); \beta\frac{\pi_{t+h}}{\pi_t}\left(\lambda(t+h) + D(t+h)\right) - \frac{hc(R_t)}{\pi_t}\right]\right] \tag{10}$$

Although we have been referring to q as the minimum share the country receives in a perfect equilibrium, it is also the maximum perfect equilibrium share (Appendix B).[21] The equilibrium strategy for the bank is to propose $q^*(t)$ given by (10) when it is its turn to make an offer and refuse any offer below $1 - q(t)$, given by equation (7), after substituting from (10) for $q^*(t+2h)$. Conversely, for the country, the optimum strategy is to offer the amount given by equation (7) and refuse any offer below the quantity $q^*(t)$ as defined by equation (10). The solution is immediate, i.e., the first offer will be implemented, so that deadweight losses due to delay or repudiation are avoided.

So far, we have arbitrarily assumed that the bank had the advantage of making the first proposal. One way to eliminate this arbitrary advantage is to reduce the time between offers to an arbitrarily small period of time.[22] If

[21] Rubinstein (1982) studied the cases of discounting and (constant) bargaining costs separately. In the constant bargaining costs case, the solution is discountinuous in the bargaining costs and possibly non-unique, with the player with the lower cost receiving either the entire 'pie' (if he moves first) or anything greater than or equal to the pie less his bargaining cost (the solution is not unique if the high cost player moves first). We get uniqueness and continuity in the bargaining cost due to the simultaneous presence of discounting in our setup.

[22] The *first mover advantage* shows up in the two last terms in equation (10): the first of these is reflected in the fact that the bank receives more than half of the pie even if $c(R_t) = 0$ and $\delta = r$; the second has the country receiving less than $\lambda + D$.

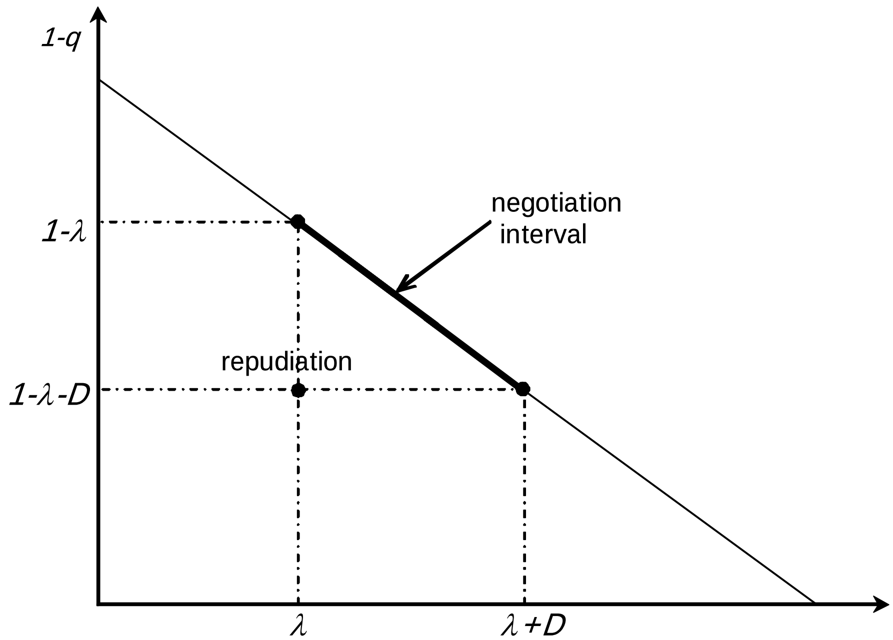

Figure 2: The contract curve

h is negligible, the non-proposing part can refuse the offer at negligible cost and place a new proposal on the table (i.e. continuous negotiation). The bargaining solution then reduces to

$$q^* = \max\left[\lambda; \min\left[q_N^*; \lambda + D\right]\right] \quad (11)$$

where $\lambda = \lambda(1)$ and $q_N^* = \lim_{h \to 0} q^*(1)$.

One can see the logic of equation (11) in Fig. 2, where for a given value of π_1, we measure the country's share on the horizontal axis and the bank's on the vertical axis. Since the bank's share is $1 - q^*$, potential bargaining solutions lie on the efficient sharing locus $\overline{ab}$. As confiscation of output involves a deadweight loss, the repudiation payoffs $[\lambda, 1-\lambda-D]$ lie strictly inside the $\overline{ab}$ locus.

The bargaining outcome depends on the position of q_N relative to the negotiation interval $[\lambda, \lambda+D]$, the endpoints of which are determined by the value of the outside option represented by repudiation to the two players. If q_N falls within this interval, the bargaining is resolved as if there were no outside option. In this region, players know that repudiation threats will ultimately not be carried out. Such non-credible threats are excluded by the requirement of subgame perfection.

If q_N falls outside of the negotiation interval, the equilibrium offer lies at the nearest endpoint, with the relevant party's repudiation threat determining the split of the pie. If $q_N \leq \lambda$, for example, the country has no incentive to continue bargaining and can therefore credibly threaten to walk away. In this case, the bank 'buys off' the country and consumes what would otherwise be a deadweight loss.

3.3 Optimal Reserve Policy During Renegotiation

At time $t = 0$, the borrower might want to accumulate reserves to smooth consumption between states in which debt is repaid in full and states in which debt is renegotiated. During a renegotiation, reserves also improve the borrower's terms of trade. Since the borrower is risk neutral from $t = 1$ onwards, only the latter rationale applies during a renegotiation. The optimal reserve policy during a renegotiation might involve consuming out of the stock of reserves or using some portion of export proceeds to add to reserves.

The basic feature of the optimal reserve policy can be understood by considering the autonomous reserve policy the country would run following a repudiation, if repudiation were accompanied by a permanent cutoff of trade credits. In this case it is straightforward that, since the country is risk neutral, the optimal reserve policy involves attaining the target level of reserves, $\widetilde{R}$, immediately, where $\widetilde{R}$ is the level of reserves for which the marginal increase in the discounted value of liquidity services equals the marginal return to immediate consumption (1).[23] This reserve policy is reminiscent of the target-adjustment models in the reserve demand literature (e.g., Frenkel (1983)). Hence, the optimal consumption policy is:

$$C_{t+kh} = \begin{array}{ll} R_{t+kh} - \widetilde{R} & \text{if } R_{t+kh} \geq \widetilde{R} \\ 0 & \text{if } R_{t+kh} < \widetilde{R} \end{array} \tag{12}$$

Expression (12) implies that once the target level of reserves has been reached, the optimal consumption plan involves consuming whatever income that may accrue in each subsequent period. The rationale underlying this policy extends to the case of optimal reserve management during a debt renegotiation. Consider that the country is able to allocate reserves and export proceeds optimally between reserves and consumption in between offers. Ignoring the repudiation option, the optimal policy would again be characterized by an interior reserve target, $\widehat{R}$, with the property that in each period, the marginal return to consuming an additional unit of reserves would equal the marginal deterioration in the value of the bargaining game due to the fall in reserves. The country would follow a policy similar to (12), so as to approach $\widehat{R}$ as rapidly as possible. The bargaining cost would adjust

[23] Appendix C shows that $\widetilde{R}$ is implicitly defined by $-c'(\widetilde{R}) = \delta$. Since $c''(R_t) > 0$, the optimal policy is to approach $\widetilde{R}$ monotonically.

endogenously over time, reaching a constant level as soon as $R_t = \widehat{R}$.[24]

For simplicity, we focus on the case where the level of reserves acquired in period 0 is such that the country may attain the target level immediately at $t = 1$.[25] Reserves are then kept at that level and proceeds obtained from selling the perishable export good are immediately consumed.

As the stock of reserves is held constant during negotiations, the perfect equilibrium offer $q_N^*(t)$ is given by

$$q_N^*(t) = \frac{r - \frac{c(R_t)}{\pi_t}}{r + \delta} \tag{13}$$

The share is constant since both, the size of the pie π_t and the bargaining cost are fixed. The benchmark bargaining solution of a half-and-half split would emerge if trade credit were irrelevant ($c(R_t) = 0$) and the two players had identical discount rates ($r = \delta$). In this case, there is nothing to differentiate the bargaining strength of the two players, and the Rubinstein game yields the familiar symmetric Nash bargaining solution for a static bargaining problem. However, since the country is more impatient than the bank ($\delta > r$), its share will be less than $\frac{1}{2}$, decreasing further as the cost of cutoff from trade credit grows. Note also that, since the borrower's part of the pie is decreasing in R, a buyback operation using the stock of reserves unambiguously decreases the welfare of the country in the negotiation region.

The optimal level of reserve holdings is given by the condition $\frac{dV_N}{dR} = 1$.

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[24] The outside option complicates matters. Since the marginal return on reserves conditional on $q = \lambda$ or $q = \lambda + D$ is less than one, the optimal reserve policy may be to consume all reserves in the first period of bargaining.

[25] I.e., we do not consider the case of gradual convergence to the target by conversion of the perishable export proceeds into reserves.

In order to get a closed form solution, we must specify the functional form of the bargaining cost. For example, with $c(R_t) = \frac{m}{R_t}$ we get $R = \sqrt{\frac{m}{\delta}}$: the level of reserves held during a negotiation increases with the responsiveness of the cost function to reserves and is inversely proportional to the square root of the borrower's impatience.

3.4 Extension: Fixed Costs to Lenders

The framework allows us to analyze the outcome in the presence of banking regulations that may act to increase the bank's impatience and thereby reduce their bargaining power. Suppose that the lender has to pay a fixed cost K if the negotiation if the bargaining is still unresolved at time $T + 1 > 1$. The deadline at $T + 1$ can be thought of as coming from regulations stating that a loan in arrears for T periods has to be declared as non-performing. Such action calls for provisions which can lower bank equity values. With the help of one additional technical assumption, one can derive the following bargaining solution for the case of constant reserves (see Appendix D):[26]

$$q^*(t) = \max\left[\lambda(t); \min\left[\frac{r - \frac{c(R_1)}{\pi_1}}{r + \delta} + \frac{K}{2\pi_1} e^{-\frac{r+\delta}{2}(T-t)}; \lambda(t) + D(t)\right]\right] \tag{14}$$

It is clear from the expression above that the cost faced by the bank shifts bargaining power towards the country, raising its share $q^*(t)$. As before, the

[26]The one-time cost K renders the problem nonstationary up to time T. After T, however, the stationary solution of equation (9) holds. Note that the solution at $t \leq T$ hinges on who has the last proposal before time T. To avoid the problems associated with taking the limit as $h \to 0$, we follow the approach of Binmore (1980) to remove the first mover advantage, assuming that the proposer is decided by the flip of a coin in each period.

share of the country in the negotiation region is capped by its share deriving from the situation in which the bank chooses to abandon negotiations. Moreover, the country's share is non-decreasing in the proximity of the deadline T, i.e., the bank would increase its offer to the country if the deadline were anticipated.

4 The Repayment Decision

In this section we examine the effect of the country's assets on its choice to repay debts in full or reschedule and, in case the latter option is chosen, on the terms of the rescheduling agreement.

Since the country may always settle the claims by repaying outstanding debts at face value, its payoff in period 1 will be given by

$$W_1 = \max\left[V^p, V^r\right] + \frac{y}{\beta\delta}$$

where V^p and V^r are the values of repaying in full and rescheduling, respectively, net of future trade proceeds which will accrue either way.

4.1 The Value of Rescheduling

The value of rescheduling can be expressed as

$$V^r = \max\left[V; \min\left[V_N; V^*\right]\right] \ ,$$

where $V = \lambda\pi_1$, $V_N = q_N\pi_1$, and $V^* = (\lambda + D)\pi_1$. To streamline terminology, we will define the *bank region*, *country region* and *negotiation region*, as the range of reserve levels for which bank's threat to repudiate is credible (i.e.

$V^r = V^*$), the country's threat is credible ($V^r = V$), and neither is credible ($V^r = V_N$), respectively. While the exact configuration of V^r will depend on all the parameters, one can see from (11) that V^r is a differentiable function of R_1 except at a finite number of switch points where the equilibrium moves from one region to another. Since $\lambda\pi_1$, $q_N\pi_1$, and $(\lambda + D)\,\pi_1$ are all nondecreasing in R_1, a rise in the level of reserves cannot decrease the value of rescheduling. Put alternatively,

Proposition 1: The return to gross reserves is strictly positive conditional on debt renegotiation, as long as reserves are not fully attachable ($\gamma < 1$).

Proof. By equations (11) and (13), the value of rescheduling is

$$\begin{aligned} V^r\left(R_1, Q\right) \;&=\; \max[(1-\gamma)\,R_1 + (1-\alpha)\,Q; \min[\frac{r - \frac{c(R_1)}{R_1+Q}}{r+\delta}\left(R_1+Q\right); \\ &\quad (1-\gamma)\,R_1 + (1-\alpha+\mu\alpha)\,Q] \end{aligned}$$

In the bank and country regions, $\frac{\partial V_r}{\partial R_1} = 1-\gamma > 0$. In the negotiation region, the return to reserves is[27]

$$\frac{\partial V_N}{\partial R_1} = \frac{r - c'(R_1)}{r+\delta} > 0 \tag{15}$$

which is strictly positive since $c'(R_1) \geq 0$. QED. ■

The return to gross reserves has two distinct components in a world with debt renegotiation. Under default, a portion $1-\gamma$ of gross reserves constitutes *net wealth*; this is nonnegative as long as lenders cannot fully attach reserves. In the renegotiation region, reserves also have a *liquidity role.* They substitute for trade credit, making the borrower appear more patient; this puts the

[27] Note that if $c''(R_1) > 0$ the V_N schedule is strictly concave.

borrower in a position to demand a greater share of the surplus. Figs. 3 and 4 illustrate these two roles of gross reserves. In Fig. 3 [28] we set $c(R) \equiv 0$, so that the value of reserves stems entirely from their imperfect appropriability by lenders.[29] In Fig. 3 we set $\gamma = 0$ so that reserves cannot be attached in default; but we allow $c'(R) < 0$ so that the borrower gains from the liquidity services provided during a negotiation. Note that the relationship between the level of reserves and the operative bargaining region depends on the parameters: in Fig. 3, for example, higher reserves eventually shift the surplus in favor of the bank, because the marginal return to reserves in the negotiation region is below that in the country and bank regions. The opposite would be true if the ranking were reversed, as it is in Fig. 4.[30]

[28] To ensure that all three regions are non-empty we imposed the restriction that $1-\alpha+D < \frac{r}{r+\delta} < 1-\gamma$, so that in the absence of reserves the country would prefer negotiation to repudiation, whereas the bank prefers repudiation.

The V and V^* schedules differ by the amount of the deadweight loss, DQ, having a common slope of $1-\gamma$, that is equal to the fraction of non-attachable reserves. The shape of the V_N schedule hinges on whether creditors are able to interfere with trade finance during the negotiation. Fig. 3 is drawn assuming that creditors cannot affect terms of trade (that is the case if $R \to \infty$). In this case, the slope of V^N is determined by the relative impatience rates $\frac{r}{r+\delta}$.

[29] Note that if the country could not touch its reserves during a renegotiation and they earned the risk-free rate, they would effectively be fully attachable (i.e. V_N would be flat). This is so because the remuneration of reserves would make the bank infinitely patient with respect to that portion of the pie.

[30] More complicated configurations are clearly possible when reserves offer liquidity services as in Fig. 4. With $\gamma > 0$, for example, the V and V^* schedules are upward-sloping. In the case of strictly concave V^N, a negotiation region of the type shown in Fig. 4 would be followed, at higher reserve levels, by another negotiation region with a transition back to the country region.

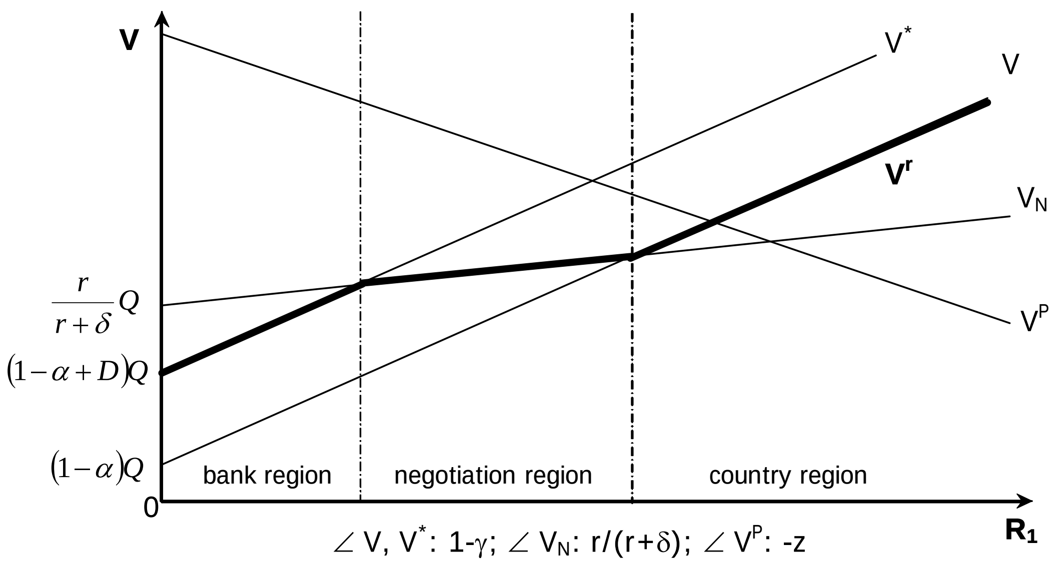

Figure 3: Reserves as net wealth

Note that there is no case in which the value of rescheduling depends on the stock of debt. In the country and the bank regions, this is because the default penalty incides only on existing assets. In the negotiation region it is because repayment is limited to what the country can be bargained into repaying. In either case, it follows that *net* reserves, $R_1 - D$, are irrelevant for the value of rescheduling given the level of gross reserves.

Fig. 4 depicts the case in which reserves are fully confiscated in the event of a repudiation - implying that V and V^* are flat - and creditors can impose a terms of trade loss on the country by interfering with trade finance during negotiation. We assumed that liquidity services are substantial enough to ensure that $V^N(R_1 = 0) < V(R_1 = 0)$.

One can see from the diagrams that the ex post marginal gross return

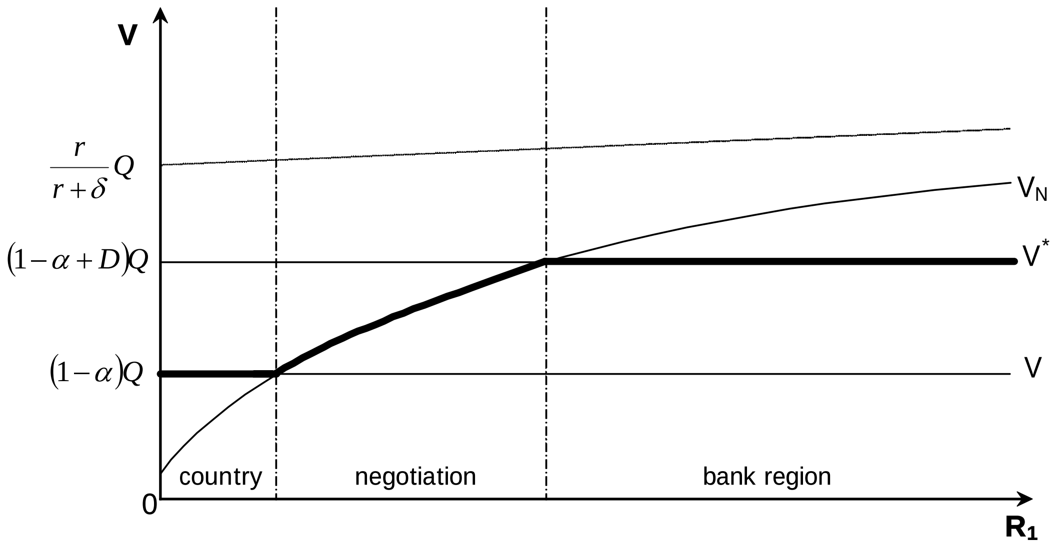

Figure 4: Reserves as liquidity

of reserves conditional on rescheduling can only exceed 1 in case of a trade credit cutoff with the agreement falling in the negotiation region. There is a strong sense, therefore, in which the liquidity role is more central than the net wealth role in explaining the demand for reserves. In the model presented here, liquidity services are a *necessary* condition for reserves to be held past the first negotiation period if the country is following an optimal reserve policy. If trade credit were always readily available, demand for reserves would be zero.

4.2 The Value of Repayment vs. Rescheduling

Proposition 2: V^p **is a strictly decreasing function of** R_1 **and a strictly increasing function of** R_o**.**

Proof. Recall that the country borrowed for consumption, accumulation of reserves and one unit for the investment project. If z is the promised interest rate on debt incurred in period 0, the borrowers repayment value is

$$V^p = Q + R_1 - (1 + C_0 + R_1 - R_0)(1 + z) = Q - zR_1 - (1 - R_0)(1 + z) \quad (16)$$

QED. ■

The above proposition makes two important points. First, gross reserves will be dominated in rate of return - and will therefore not be held at all at $t = 1$ - unless the borrower reschedules its debt in some states of the world. This is because reserves carry a strictly positive opportunity cost of $z > 0$ in states of the world in which the borrower repays. Second, while we have just noted that *net* reserves do not affect the payoff to rescheduling, they do affect the value of repaying, and in the opposite direction to gross reserves. Given the level of gross reserves, an increase in net reserves implies a reduction in debt and therefore an *increase* in the probability of repayment.

The country repays if $V^P \geq V^r$ and reschedules otherwise. Since the value of rescheduling is non-decreasing in reserves and the value of repayment is strictly decreasing in reserves, the impact of reserves on the rescheduling decision is straightforward:

Proposition 3: For given values of Q, R_o and z, either the country reschedules for all values of R_1, or there is a unique level of reserves, $R^*(Q, R_o, z)$, above which the country reschedules and below which the country repays. This cutoff level of reserves is continuous and piecewise differentiable in its arguments, with $\frac{\partial R^*(.)}{\partial Q} > 0$, $\frac{\partial R^*(.)}{\partial R_o} \geq 0$ and $\frac{\partial R^*(.)}{\partial z} < 0$.

Proof. Follows from Propositions 1 and 2, (15) and (16). ■

In Fig. 5 we plot the cutoff level of reserves for selected values of Q, holding R_0 and z constant. We assume that reserves are not fully attachable ($\gamma < 1$) and that reserves deliver liquidity services ($c'(R) < 0$). Kinks in the schedule may occur where the bargaining solution switches between regions. For R_1 sufficiently large, the outcome will fall in the country or bank regions as Figures 3 and 4 make clear, and the R^* schedule will approach a horizontal asymptote. Given z, the cutoff value rises with output because the bank is not a residual claimant of the storable export good under repayment; this means that for the country, the value of repaying rises by more than that of rescheduling as output rises.

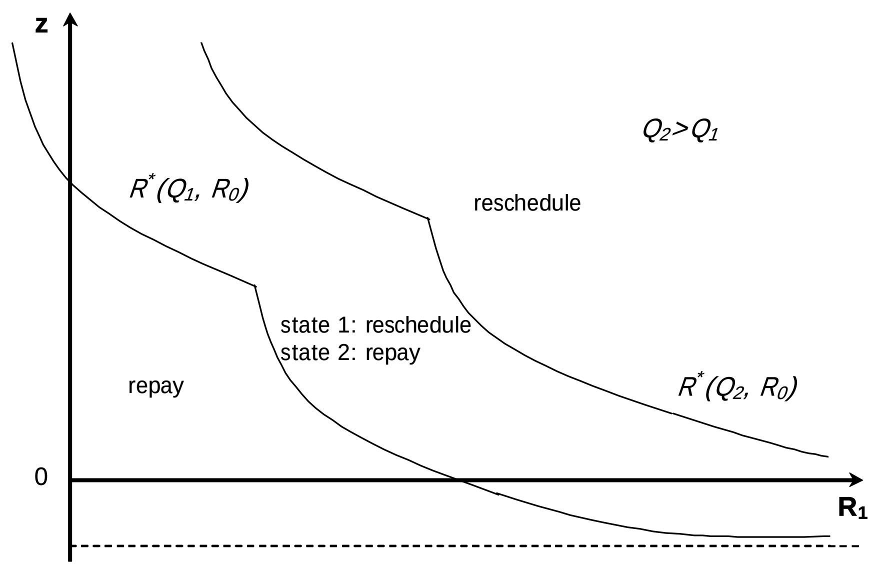

Figure 5: The rescheduling decision

The R^* schedule partitions the (z, R) plane into areas in which the pattern of rescheduling and repayment is clearly defined. If a country chooses to repay (reschedule) for a given level of output, it will always choose to repay for any higher (lower) output level. More generally, the comparative statics of the repayment decision satisfy:

Corollary: The country repays (reschedules) when output is above (below) a critical level $Q^*(R_0, R_1, z)$, where $\frac{\partial Q^*}{\partial R_0} < 0$, $\frac{\partial Q^*}{\partial R_1} < 0$ and $\frac{\partial Q^*}{\partial z} < 0$. Given z, the probability of repayment is a non-decreasing function of R_1 and a non-decreasing function of R_0.

5 The Supply and Demand of Borrowed Reserves

In the previous section we concluded that gross reserves may increase the value of rescheduling, and at the same time reduce the value of repayment. In this section we show that rational banks will lend reserves to the country - in spite of the fact that they increase the bargaining power of the country - as long as penalties on output are large enough. As we assume that banks are perfectly competitive ex ante, this amounts to showing that reserve lending in the first period satisfies the zero-profit condition.

We shall assume that there are two possible states in the economy, s_1 and s_2, that are associated with the output realizations Q_1 and Q_2 respectively, where $Q_2 > Q_1$. The arbitrage condition requires that $E(z(s_i)) = r$, where the expectation is taken given all information available at $t = 0$, which includes the specification of the bargaining problem that players will face in period $t = 1$. Below the $R^*(Q_1, R_0)$ schedule in Fig. 6, repayment occurs in both states so that lending is risk-free (i.e. $z(s_1) = z(s_2) = z$). Competition among banks drives the promised rate z down to r. Notice that the existence of the horizontal segment $\overline{ab}$ in the zero-profit locus on Fig. 6 requires that the condition $R^*(Q_1, r) > -R_o$ is met. The range of borrowed reserves in which lending is risk-free increases with Q_1, α and $c(.)$.

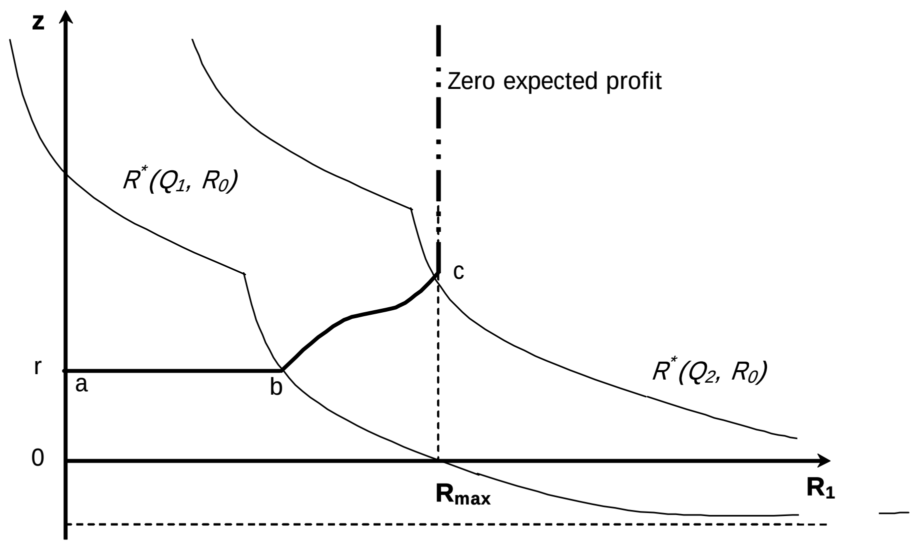

Figure 6: The supply of borrowed reserves

Between the $R^*(Q_1, r)$ and the $R^*(Q_2, r)$ schedules, the country repays only in the high-output state. Notice that the return in the low output state falls with R_1, so that the promised return (which is paid only in the high output state) must rise with R_1 in this interval. This gives the segment $\overline{bc}$ in the zero-profit locus, that must be above r. There is no discontinuity at

b because the rescheduling process is efficient and involves no deadweight loss.[31]

At point c the country reschedules in the low output state and is indifferent between rescheduling and repaying in the high output state. Hence, any further rise in the promised interest rate z is irrelevant, as both players anticipate that it will never be honored. Since the return conditioned on rescheduling can never exceed r, the zero profit locus becomes vertical at c. We denote the maximum amount of borrowed reserves by $R_{\max}$, so that the country's overall long-term credit ceiling is $1 + R_{\max}$. The supply schedule is given by $\overline{abc}$. [32]

Credit ceilings are a well known characteristic of the sovereign debt literature (e.g., Eaton and Gersovitz (1981)). Defining $q_i(s)$ as the share of reserves or output $(i = R, Q)$ received by the borrower in a rescheduling agreement in state s and assuming that reserves earn the risk-free rate from $t = 0$ to 1, $R_{\max}$ satisfies

$$R_{\max} = \frac{E\left[\left(1 - q_Q(s)\right) Q(s)\right] - (1 + r)\left(1 - R_0\right)}{E\left[q_R(s)\right]}$$

If $R_{\max} \leq -1$, the country is excluded from long-term credit markets, and its investment can only be self-financed, i.e. via accumulation of current account surpluses. If $R_{\max}$ is positive but less than R_0, the investment project can

[31] If the rescheduling process involves a deadweight loss, there would be a discontinuity at b and the possibility of two equilibrium promised interest rates over some interval of reserves.

[32] We are implicitly assuming that the reserve generating debt instruments are issued sequentially and contain a seniority clause, so that rational competitive lenders will never be willing to hold such instruments beyond the credit ceiling.

be financed, but only if the country uses part of its reserve endowment.

The credit ceiling $1+R_{\max}$ is a non-decreasing function of the penalties the lender can impose in case of repudiation, with comparative statics depending on the bargaining region that is operative in each output state at the credit limit.

Proposition 4: i) $\frac{\partial R_{\max}}{\partial (EQ)} \geq 0$;**ii)** $\frac{\partial R_{\max}}{\partial \alpha} \geq 0$**, with strict inequality if the bargaining equilibrium is in the country or bank region in either state at** $R_{\max}$;**iii)** $\frac{\partial R_{\max}}{\partial (1-\mu)} \leq 0$**, with strict inequality if the bargaining equilibrium is in the country region in either state at** $R_{\max}$;**iv)** $\frac{\partial R_{\max}}{\partial c} \geq 0$ **and** $\frac{\partial R_{\max}}{\partial \delta} \leq 0$**, with strict inequalities if the bargaining equilibrium is in the negotiation region in either state at** $R_{\max}$;**v)**$\frac{\partial R_{\max}}{\partial \gamma} = 0$ **if** $R_{\max} = 0$**. Otherwise** $sign\left(\frac{\partial R_{\max}}{\partial \gamma}\right) = sign\left(R_{\max}\right)$;**vi)**$\frac{\partial R_{\max}}{\partial R_0} \geq 1$ **and** $\frac{\partial R_{\max}}{\partial r} \leq 0$**.**

The results are intuitive. Part v) implies that borrowers do not have an incentive to increase the attachability of reserves (i.e., raise γ) so as to make long-term investments possible. This contrasts with the output penalty and the terms of trade loss. A rise in α, for example, increases the borrower's credit ceiling if either the country or the bank can credibly threaten to walk away in at least one of the states; similarly, a rise in c (for all R) increases the borrower's impatience and raises $R_{\max}$ as long as the outcome lies in the negotiation region in one of the states. In either case, an appropriate alteration in the penalty structure is capable of increasing the credit ceiling. An increase in the attachability of reserves, on the other hand, will increase the credit ceiling if and only if it already is positive. It does not help the borrower to turn the borrowing limit positive however. The reason is simple:

if the bank is not willing to lend enough so as to allow the borrower to retain positive reserves while financing its investment project, an increase in γ has no effect on the bank's expected rate of return.[33]

Since lenders are competitive *ex ante*, the country obtains the entire surplus from the relationship with lenders. It can choose the equilibrium level of reserves taking the bank's zero expected profit locus as given. Hence, equilibrium occurs at the point on the zero expected profit locus that maximizes the country's utility. When reserves are remunerated at the risk-free rate until $t = 1$, the country augments its consumption by $S = E(Q) - (1 + r)$, regardless of the level of reserves it holds. In this case, reserves serve a pure insurance role, redirecting consumption from high output states to low output states without changing its expected value.[34] The two state case when reserves are remunerated at the risk-free rate r is summarized in the proposition below:

Proposition 5: If the country is risk-neutral ($u'' = 0$), it is indifferent to the amount of borrowed reserves held, including zero. If the country is risk-averse ($u'' < 0$), the country borrows up to its credit ceiling and holds the maximum amount of borrowed reserves. Borrowed reserves provide partial insurance.

Fig. 7 shows the consumption allocation across the two states of nature

[33]The role of precommitments to high penalties as a way of facilitating long-term borrowing has been emphasized in the sovereign debt literature. See for example Cohen and Sachs (1986).

[34]In the case where reserves earn less than the risk-free rate, it is Pareto inefficient for the country to hold reserves if its debt is positive. The country still gets the entire surplus of the relationship at $t = 0$, but the surplus is a declining function of borrowed reserves.

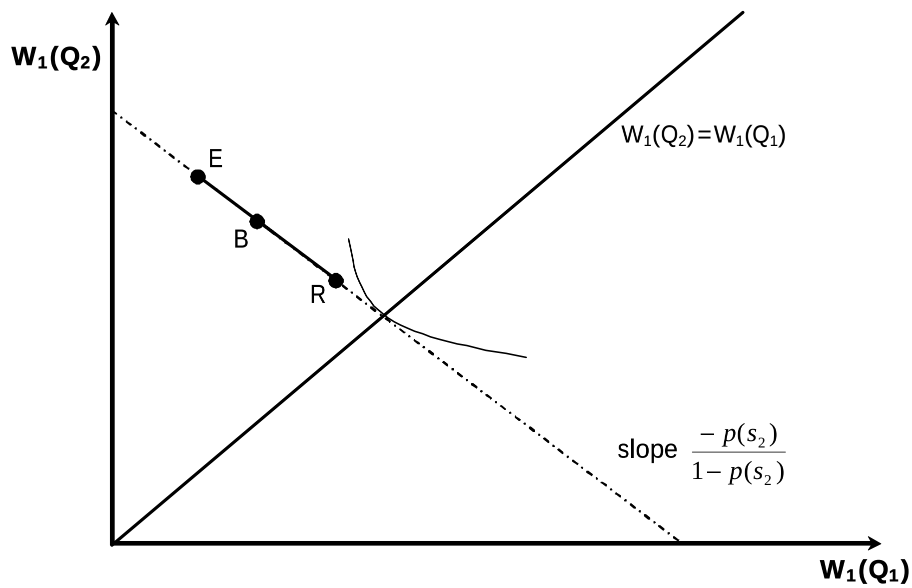

Figure 7: The insurance role of borrowed reserves

that can be achieved by various contracts. Taken at face value, a debt contract has the borrower bearing all the risk, with consumption on a point like E. As Hellwig (1986) and others have pointed out, this makes the use of standard international debt instruments somewhat puzzling, given that lenders are probably less risk-averse than borrowers. It would seem efficient to have payments contingent on output, thus shifting some of the risk to the lender.[35]

Point E however represents only *enforceable* debt contracts, and in equilibrium, the promised rate on debt contains a premium above the risk-free rate to compensate the lender for losses in case of a rescheduling (e.g., Gross-

[35] Atkeson (1991) argues that the optimal contract does not provide full insurance because of moral hazard.

man and van Huyck (1988)). The actual, ex post return paid by the borrower is below the risk-free return in the states in which debt is rescheduled. Sovereign lending, in the absence of borrowed reserves, moves the equilibrium to B, i.e., the possibility of rescheduling provides some of the missing insurance.

Borrowed reserves expand the range of achievable consumption allocations further. As borrowed reserves move from 0 to $R_{\max}$, the consumption allocation moves from B to R. A risk averse country will clearly choose the maximal amount of insurance given that lenders are competitive. This involves borrowing up to the credit ceiling and holding the excess over investment needs as reserves. It is easy to see that the insurance that is made available through the resort to borrowed reserves is only partial: full insurance would require the transfer from the borrower to the lender to rise one-for-one with output. Since at its credit limit the country reschedules in the low output state and is indifferent between rescheduling and repayment in the high output state, the difference in payments in the two states is just the difference between the rescheduling payments. As long as $\alpha < 1$, these payments differ by less than output.

6 Discussion

Since the landmark paper of Eaton and Gersovitz (1981) many studies of the sovereign debt market have been presented and many more debt reschedulings have taken place. Yet some key aspects within the sovereign debt literature remain puzzling. This study tried to shed light on some of the aspects involved, leaving others for future research. In our judgement the model brings the analysis closer to the empirical evidence that points to the importance of diminished trade flows[36] and adds to the realism of punishment strategies to which unlucky creditors may resort. We assumed that a debt renegotiation does not imply a halt to international trade. Nevertheless, export seizing 'gun-boats' are not deployed. All that creditors effectively do is to stop rolling over short-term trade finance during the negotiation process. The cut-off from trade finance has the effect of increasing the impatience of the borrower to seek an agreement in order to maximize the proceeds that accrue from its exports. In this sense, creditors are less active than in Bulow and Rogoff (1989a) and are likely to incur less costs, attenuating the free-rider problem.

The side effect of the assumed punishment strategy is to highlight a new rationale for reserve holdings: borrowing countries may accumulate reserves to guarantee its liquidity in anticipation of a bargaining game. This is certainly not always the main reason for reserve accumulation and many borrowers go considerable lengths in reducing their reserve holdings to avoid falling into arrears. Conditional on renegotiation, however, greater liquidity plays into the hands of the borrower. The relative degree of impatience of players - that ultimately defines the outcome - is the endogenous result of the liquidity position of the borrower. For this reason, the distinction between gross and net international reserves is predicted to be central to the outcome of the bargaining process. Borrowers with higher gross reserves find themselves in a position to reach a better deal during a debt renegotiation.

[36] Recent debt restructurings in Argentina and Uruguay have been associated with a drastic reduction in the availability of export credits.

Hence, the model may explain why some borrowers may not risk to exhaust their reserves to meet repayments, defaulting with positive reserve holdings. It may also explain the notable reluctance of borrowers in arrears to engage in immediate debt buyback operations.

Further extensions of the model could incorporate the IFIs as a third player into the bargaining game. The recent involvement of multilateral organizations, as the IFC and the IADB, in trade financing and the policy of export credit agencies of lending or not into arrears is likely to affect the degree of impatience of creditors and borrowers and consequently shift bargaining power. Also, the set up of contingent credit lines as the one arranged between Mexico and private financial institutions in 1997 or more recently by a group of South-East Asian economies may provide a cheaper alternative to borrowed reserves. In the case of Mexico however, some financial institutions objected to the exercise of the line in 1998, although contingencies were reasonably broad. Future research might focus on the conditions under which such arrangements may be substitutes for borrowed reserves.

References

Aizenman, J., Marion, N. (2004) International reserve holdings with sovereign risk and costly tax collection. Forthcoming in the Economic Journal.

Atkeson, A. (1991) International lending with moral hazard and risk of repudiation. Econometrica 59, 4, 1069-1089.

Binmore, K.G. (1980) Nash bargaining theory I-III, in Binmore, K.G. and Dasgupta, P. (eds.), Essays in Bargaining Theory. Blackwell.

Bulow, J., Rogoff, K. (1989a) A constant recontracting model of sovereign debt. Journal of Political Economy 97, 1, 155-178.

Bulow, J., Rogoff, K. (1989b) Sovereign debt: is to forgive to forget ? American Economic Review 79.

Cohen, D., Sachs, J. (1986) Growth and external debt under risk of debt repudiation. European Economic Review 30, 529-560.

Delaume, G. (1994) The foreign sovereign immunities act and public debt litigation: some fifteen years later. American Journal of International Law 88, 2, 257-279.

Detragiache, E. (1996) Fiscal adjustment and official reserves in sovereign debt negotiations. Economica 63, 81-95.

Dornbusch, R. (1984) Argentina since Martinez de Hoz. Mimeo. MIT.

Eaton, J., Engers, M. (1999) Sanctions: some simple analytics. American Economic Review 89, 2, 409-414.

Eaton, J., Gersovitz, M. (1981) Debt with potential repudiation: theoretical and empirical analysis. Review of Economic Studies 48, 289-309.

Fernandez, R., Rosenthal, R. (1990) Strategic models of sovereign-debt renegotiations. Review of Economic Studies 57, 331-349.

Frenkel, J., Boyanovic, J. (1981) Optimal international reserves: a stochastic framework. Economic Journal 91, 507-514.

Grossman, H., van Huyck, J. (1988) Sovereign debt as a contingent claim: Excusable default, repudiation, and reputation. American Economic Review 78, 5, 1088-1097.

Hellwig, M. (1986) Comments on 'The pure theory of country risk' by J. Eaton, M. Gersovitz and J. Stiglitz. European Economic Review 30, 3, 521-527.

Kaletsky, A. (1985) The costs of default. Priority Press.

Lucas, R. (1979) Sharing, monitoring, and incentives: Marshallian misallocation reasessed. Journal of Political Economy 79, 3, 501-521.

Osborne, M.J., Rubinstein, A. (1990) Bargaining and markets. Academic Press, Inc. ISBN 0-12-528632-5.

Rogoff, K. (1999) International institutions for reducing global financial instability. Journal of Economic Perspectives 13, 4, 21-42.

Rose, A. (2002) One reason countries pay their debts: renegotiation and international trade. Forthcoming in the Journal of Development Economics.

Rose, A., Spiegel, M. (2002) A gravity model of sovereign lending: trade, default and credit. Forthcoming IMF Staff Papers.

Rubinstein, A. (1982) Perfect equilibrium in a bargaining model. Econometrica 50, 97-109.

Sutton, J. (1986) Non-cooperative bargaining theory: An introduction. Review of Economic Studies LIII, 709-724.

van Wijnbergen (1990) Cash/debt buy-backs and the insurance value of reserves. Journal of International Economics 29, 123-131.

Wright, M.L.J. (2002) Reputations and sovereign debt. Mimeo. Stanford University.

Appendix A - Derivation of the Bargaining Solution

If parameters are such that repudiation is not chosen in equilibrium, equation (8) reduces to the second-order difference equation

$$q^*(t) = \beta\frac{\pi_{t+h}}{\pi_t} - \beta\beta^*\frac{\pi_{t+2h}}{\pi_t} - \frac{hc(R_t)}{\pi_t} + \beta\beta^*\frac{\pi_{t+2h}}{\pi_t}q^*(t+2h) \qquad (17)$$

Define the variable

$$\Gamma_{t+kh} = (\beta\beta^*)^k \frac{\pi_{t+2kh}}{\pi_t} \qquad (18)$$

where we assume that parameters are such that $\Gamma_{t+kh} < 1 \quad \forall\ k \ \epsilon\ \mathbb{Z}_+$.

After iterating expression (17) and using (18), we can rewrite (17) as

$$q^*(t) = \sum_{i=0}^{T-1}\Gamma_{t+ih}\left(\beta\frac{\pi_{t+(2i+1)h}}{\pi_{t+2ih}} - \frac{hc(R_{t+2ih})}{\pi_{t+2ih}} - 1\right) + \Gamma_t - \Gamma_{t+Th}\left(1 - q^*(t+2Th)\right) \qquad (19)$$

Since $q^*(t)$ is bounded between 0 and 1, and $\lim_{k\to\infty}\Gamma_{t+kh} = 0$, the last term in this equation vanishes as $T \to \infty$. It follows that the general solution to (19) is given by

$$q^*(t) = 1 - \sum_{i=0}^{\infty}\Gamma_{t+ih}\left(1 - \beta\frac{\pi_{t+(2i+1)h}}{\pi_{t+2ih}} + \frac{hc(R_{t+2ih})}{\pi_{t+2ih}}\right)$$

Appendix B - Unicity

Let the proposer in period t be determined by the flip of a coin and $hc_i(t)$ represent the cost of delay of h in reaching an agreement for player i. Also, let $\pi_i(t)$ represent i's expected continuation value in a perfect game before the proposer is determined and $v_i(t)$ and $v_i'(t)$ represent the continuation value conditioned on being the proposer at time t or not respectively. Further,

$M_i(t) = \sup_\Omega \pi_i(t)$ and $m_i(t) = \inf_\Omega \pi_i(t)$ where Ω represents the set of subgame perfect equilibria and $\overline{\beta} = \max[\beta; \beta^*]$.

Lemma: If there exists $D(t) < \infty$ such that $M_i(t) - m_i(t) \leq D(t)$, then $M_i(t-h) - m_i(t-h) \leq \overline{\beta} D(t)$.

Proof: Suppose the country proposes the split (x, y) at $t - h$. The bank will surely reject if

$$y < \beta^* m^*(t) - c^*(t-h)$$

and accept if

$$y > \beta^* M^*(t) - c^*(t-h) \tag{20}$$

In case the bank rejects, the country will have to wait a period and will receive at least $m(t)$ in period t. The country will offer at most the value on the RHS of expression (20), since at this value the bank would already accept the offer for sure. Since the country has the offer, it will do no worse than receiving the better of this two payoffs:

$$v(t-h) \geq \max[\beta m(t) - c(t-h); x + y - \beta^* M^*(t) + c^*(t-h)] \tag{21}$$

$v(t-h)$ is also limited from above by the highest equilibrium payoff offered by the bank after a rejection by the country, $M(t)$, and the value given by least offer that is accepted by the bank. Hence, we also have

$$v(t-h) \leq \max[\beta M(t) - c(t-h); x + y - \beta^* m^*(t) + c^*(t-h)] \tag{22}$$

Similarly, if the bank makes the offer at $t - h$, the country rejects if

$$x < \beta m(t) - c(t-h)$$

and accepts if

$$x > \beta M(t) - c(t-h)$$

$$\beta m(t) - c(t-h) < v'(t-h) < \beta M(t) - c(t-h) \tag{23}$$

Substituting $M_i(t) \leq D(t) + m_i(t)$ in expressions (21), (22) and (23) we get

$$\begin{aligned} & \max\left[\beta m(t) - c(t-h); x + y - \beta^* m^*(t) - \beta^* D(t) + c^*(t-h)\right] \\ \leq \quad & v(t-h) \leq \max\left[\beta m(t) + \beta D(t) - c(t-h); x + y - \beta^* m^*(t) + c^*(t-h)\right] \end{aligned}$$

and

$$\beta m(t) - c(t-h) < v'(t-h) < \beta m(t) - c(t-h) + \beta D(t)$$

Since $\pi_i(t) = E\left[v_i(t)\right]$, it follows that the bounds on $\pi(t-h)$ will be

$$\begin{aligned} & \max\left[\beta m(t) - c(t-h); \frac{1}{2}\left[x + y - \beta^* m^*(t) - \beta^* D(t) + c^*(t-h) + \beta m(t) - c(t-h)\right]\right] \\ \leq \quad & \pi(t-h) \leq \max\left[\begin{array}{c} \beta m(t) + \beta D(t) - c(t-h); \\ \frac{1}{2}\left[x + y - \beta^* m^*(t) + c^*(t-h) + \beta m(t) + \beta D(t) - c(t-h)\right] \end{array}\right] \end{aligned} \tag{24}$$

A similar expression holds for $\pi^*(t-h)$.

Since $M(t-h)$ and $m(t-h)$ are defined as bounds to the equilibrium payoff, the difference $M(t-h) - m(t-h)$ must be bounded by the outer quantities in equation (24). Hence, the inequalities above imply

$$M(t-h) - m(t-h) \leq \beta D(t) + \frac{1}{2}\left(\max\left[0; \omega - \beta D(t)\right] - \max\left[0; \omega - \beta^* D(t)\right]\right) \tag{25}$$

, where $\omega = x + y - (\beta m(t) - c(t-h)) - (\beta^* m^*(t) - c^*(t-h))$. It is easy to see that for all values ω this implies

$$M(t-h) - m(t-h) \leq \max\left[\beta D(t); \frac{\beta^* + \beta}{2} D(t)\right] \leq \overline{\beta} D(t) \tag{26}$$

Similarly, one can also show that

$$M^*(t-h) - m^*(t-h) \leq \max\left[\beta^* D(t); \frac{\beta^* + \beta}{2} D(t)\right] \leq \overline{\beta} D(t) \tag{27}$$

QED.

Let $D(t) = R_t + Q$. From (26) and (27), as $t \to \infty$, $M_i(\tau) - m_i(\tau) = 0$ $\forall\ \tau$, i.e., each player has a unique equilibrium expected payoff for any finite time period.

Appendix C - Optimal Reserve Policy

Under financial autarky, the optimal reserve policy is given by the solution to

$$\max_{R_{t+(i+1)h}} \sum_{i=0}^{\infty} \frac{C_{t+ih}}{(1+\delta h)^i}$$

s.t.

$$\begin{aligned} C_{t+ih} + R_{t+(i+1)h} &= R_{t+ih} + p(R_{t+ih})hy \\ C_{t+ih} &\geq 0 \quad \text{and} \quad R_{t+ih} \geq 0 \end{aligned}$$

The Euler equation that characterizes the optimal policy is

$$(1 + \theta_i (1 + \delta h))(1 + p'(R_{t+h})hy) + \lambda_i = (1 + \theta_{i-1})(1 + \delta h)$$

where λ_i and θ_i are the shadow prices on the last two constraints, respectively. An interior solution is obtained when $\lambda_i = \theta_i = \theta_{i-1} = 0$. Letting $h \to 0$, we obtain the condition for the interior optimum:

$$\frac{p'(R_{t+h})y}{\delta} = 1$$

Appendix D - The Solution with a Fixed Cost to Lenders

Assume that in each period players put their proposal in an envelope and the relevant offer is decided by the flip of a coin. Moreover, let V_b and V_c

denote the country's payoff if the bank or the country gets to make the offer in a period t, respectively. We have

$$V(t) = \frac{E\left[V_c(t) + V_b(t)\right]}{2}$$

The optimal strategy for each player will be to make the minimum acceptable offer, i.e., to offer the amount that leaves the responder indifferent between accepting and turning the offer down. Hence, we get

$$V(t) = \frac{\left[1 - \left(\beta^* V^*(t+h) - hc^*(t)\right)\right] + \left[\beta V(t+h) - hc(t)\right]}{2} \tag{28}$$

where $c(t)$ and $c^*(t)$ represent the cost of delay in reaching an agreement for the country and the bank respectively. But perfect information implies $V^*(t) = 1 - V(t)$ for all t, so that we can rewrite (28) as

$$V(t) = \frac{1 - \beta^* + h\left(c^*(t) - c(t)\right) + \left(\beta^* + \beta\right) V(t+h)}{2} \tag{29}$$

Starting at $T+h$, bargaining costs are constant at $c(t) = c$ and $c^*(t) = 0$. The subgames starting at T and $T+h$ (before the coin toss) are identical, rendering the solution

$$V(T+kh) = \frac{1 - \beta^* - hc}{2 - \beta^* - \beta} \qquad \forall\, k \geq 1 \tag{30}$$

Now consider that the bank incurs a one time cost of k if the offer at time T is refused. We can obtain $V(T)$ by substituting equation (30) in (29) at time T:

$$V(T) = \min\left[\frac{1 - \beta^* - hc}{2 - \beta^* - \beta} + \frac{k}{2}; 1\right]$$

where we ensured that the country share does not exceed 1.

Consider that the time between offers is given by $h = \frac{T}{n}$ with $n \epsilon \mathbb{N}$. Iterating (29) and defining ϕ as the arithmetic average of β and β^* leads us to

$$V(t) = \frac{1 - \beta^* - hc}{2 - \beta^* - \beta} + \frac{1}{2}\sum_{i=0}^{n-1} \phi^i hc^*(t + ih) + \phi^n V(t + nh)$$

If the interval h goes to zero (i.e. $n \to \infty$), the last term vanishes and we obtain

$$V(t) = \begin{array}{ll} \min\left[\frac{r-c}{r+\delta} + \frac{k}{2}e^{-\frac{r+\delta}{2}(T-t)}; 1\right] & \text{if } t \leq T \\ \frac{r-c}{r+\delta} & \text{if } t > T \end{array}$$