The Yield Spread as a Symmetric Predictor of Output and Inflation*

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Abstract

The predictive ability of the yield spread for future economic activity is related to a symmetric predictive ability for future inflation: An increase in the slope of the nominal term structure predicts an increase in output growth and a decrease in inflation of equal magnitude. A monetary asset pricing model with sticky goods prices and an intertemporal rate of substitution larger than unity can explain these relations. The model also predicts that the slope of the real yield curve is negatively associated with future output growth and positively associated with future inflation, a prediction also borne out of the U.S. data over the period 1960:Q1 – 2004:Q2.

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1 Introduction

Following the original independent findings of Chen (1991), Estrella and Hardouvelis (1991), Harvey (1988), and Stock and Watson (1989), a large body of empirical literature has documented that the slope of the yield curve – defined as the difference between nominal long-term and short-term interest rates of Treasury securities – is positively related to future real economic activity. An increase in the nominal long-term relative to the nominal short-term interest rate is associated with an increase in real economic activity next quarter and a number of quarters into the future, with the predictability peaking out in approximately four to six quarters.[1]

In this paper we provide new evidence that the ability of the nominal yield spread to forecast output is related to a simultaneous forecasting ability for inflation. Specifically, an increase in the nominal yield spread is associated with an increase in future output and a simultaneous drop in prices of approximately the same percentage as the percentage increase in real output. Figure 1 depicts this symmetry by graphing the sample correlations of those variables with the nominal yield spread at different forecasting horizons.

The evidence on the symmetric predictability of the yield spread is robust to a number of econometric specifications. The first specification is the traditional multi-period forecasting regression with bootstrap simulations that check for the statistical significance of the results. The second specification is the one proposed by Jegadeesh (1991) and Hodrick (1992), in which the dependent variable is the one-quarter-ahead growth in output or the one-quarter-ahead inflation and the independent variable is the cumulative average of the current and lagged nominal yield spread. The third specification calculates the implied coefficients of multiperiod regressions from the dynamics of a vector autoregressive model.[2] All three econometric formulations point to the same result: A symmetric predictability of the yield spread for output and inflation.

The symmetry in the predictability of output and inflation is further corroborated by the remarkable finding that during periods when the forecasting ability of the yield spread for output deteriorates (especially after the mid-1980s), its forecasting ability for inflation also deteriorates by an approximately similar amount. The rolling sample regression coefficients of Figure 2 point this symmetry quite clearly. The predictability of inflation and real output seem to be mirror reflections of the same economic phenomenon!

[1] Later examples are the studies of Harvey (1989), Plosser and Rouwenhorst (1994), Haubrich and Dombrosky (1996), Bernard and Gerlach (1996), Davis and Fagan (1997), Estrella and Mishkin (1997), Smets and Tsatsaronis (1997), Dueker (1997), Kozicki (1997), Dotsey (1998), Ivanova, Lahiri and Seitz (2000), Hamilton and Kim (2002), Moneta (2003). These papers have shown that the predictability of output is also present in a number of countries outside the United States.

[2] This approach has been used by Campbell and Shiller (1988), Kandel and Stambaugh (1989) and Hodrick (1992) in predicting stock returns at various horizons.

The symmetric predictability of output and inflation via the nominal yield spread is a stylized fact, which requires an economic explanation. We, therefore, proceed to build the simplest possible general equilibrium monetary model that can explain not only output predictability, but the symmetric price predictability as well. A monetary model is required because the empirical evidence is based on the nominal yield spread, not the real yield spread, and the predictions refer to both output and inflation. The model follows the work of Rotemberg (1982, 1996). It is essentially an one-factor general equilibrium model of a monetary economy with sticky prices, which is able to explain the stylized facts as a result of intertemporal smoothing of rational consumers. We derive explicit analytic solutions of the model, which relate the predictive power of the yield spread to two main "deep" structural economic parameters: the degree of price stickiness and the elasticity of intertemporal substitution of the representative consumer.

One key feature of the model is the simplicity of its dynamics. The dynamics are driven entirely by the nature of price stickiness, which are embedded in the general equilibrium framework.[3] Because prices are sticky, current economic shocks lead to predictable changes in future prices and output. These expectations, coupled with consumption smoothing and arbitrage, lead to contemporaneous changes in real and nominal interest rates. A second key feature of the model is that the velocity of money is constant and, thus, productivity and money supply shocks lead to symmetric effects on future output and inflation, a characteristic which is required in order to explain the new empirical evidence of the paper. A third key feature is the opposite influence of shocks on real and nominal interest rates. Positive productivity shocks increase real but decrease nominal interest rates. Positive money supply shocks decrease real but increase nominal interest rates. A fourth key feature is the fact that the influence of shocks on short rates, nominal and real, is stronger than their influence on the corresponding long rates. Thus, the nominal yield spread moves in the opposite direction from the term structure of nominal rates and the real yield spread moves in the opposite direction from the term structure of real rates.

The model predicts that the nominal spread is positively correlated with future output growth and negatively correlated with future inflation. The model also predicts that the real yield spread is negatively correlated with future output growth and positively correlated with future inflation. Finally, the model explains why previous authors such as Fama (1990) and Mishkin (1990a,b), who regress the difference between future long-term and short-term inflation on the current nominal yield spread, find stronger evidence of predictability for long horizon inflation than short horizon inflation.

The model's implications for the predictive power of the *real* yield spread is

[3] This feature distinguishes our model from the class of affine yield models, which are econometric in nature and their dynamics are exogenous (Ang et al. (2003)).

subsequently explored in greater empirical detail. We use the earlier vector autoregression to calculate the implied regression slope coefficients for multiperiod regressions of output growth and inflation on the current real yield spread. These implied coefficients do show a negative relation of the real yield spread with future output growth and a positive relation with future inflation.

The remainder of the paper is organized as follows: Section 2 presents the empirical evidence on the predictive ability of the nominal yield spread for output and inflation. Section 3 presents the general equilibrium monetary model - whose detailed description is contained in Appendix A - and derives analytic solutions of the covariance between the yield spread and future output growth and inflation. Section 4 explores the additional empirical implications of the model regarding the predictive ability of the spread of real interest rates. Section 5 concludes and discusses possible extensions.

2 Empirical Evidence on the Predictive Ability of the Nominal Yield Spread

2.1 Data

The empirical analysis is based on quarterly data for the United States from 1960:Q1 to 2004:Q2. Data are from the Federal Reserve Bank of St. Louis (FRED II) database. As a measure of economic activity, we use seasonally adjusted data on real, chain-weighted Gross Domestic Product (GDP), expressed in 2000 prices. Prices are measured by the seasonally adjusted Consumer Price Index (CPI), and represent the middle month of the quarter. Long-term interest rates are annualized yields to maturity of the 3-year, 5-year and 10-year Treasury Bonds. Each yield spread is computed as the difference between the long-term interest rate and the 3-month Treasury bill rate.[4] All interest rate data are monthly averages of the second month of the quarter. Choosing the middle month of the quarter for prices and interest rates instead of the quarterly average alleviates the aggregation bias of the later regressions, but the results are very similar when we use average quarterly data.

Table 1 reports summary statistics (Panel A) and correlations (Panel B) of the data. All three yield spreads are positively correlated with the one-year ahead GDP growth, with correlations ranging between 0.41 and 0.44, and negatively correlated with one-year ahead inflation, with correlations between -0.28 to -0.34. The 10-year spread shows the highest correlations, although the differences between the correlations are minor. Observe that output and inflation are contemporaneously negatively correlated. Also, the three yield spreads are highly correlated with each

[4]Series codes: GDPC96, CPIAUCSL, TB3MS, GS3, GS5, GS10.

other, with bivariate correlations ranging between 0.97 and 0.99. In our subsequent analysis, we follow the earlier literature and utilize the 10-year spread.

Panel C of Table 1 presents the estimates of a first-order autoregressive model of the three variables of interest: The growth in output, Δy_{t+1},the level of inflation, Δp_{t+1}, and the spread between the 10-year nominal yield and the 3-month yield, s_{t+1}. The first-order VAR is a parsimonious representation which describes the dynamics of the vector of the three series quite adequately, as corroborated by the Schwarz criterion. Observe that the nominal spread at t, s_t, retains a positive association with next quarter's growth in output, Δy_{t+1}, in the presence of the other two contemporaneous variables, Δy_t, and Δp_t. Similarly, it retains a negative association with next quarter's inflation, Δp_{t+1}, in the presence of the other two variables.

All three variables are stationary, with output growth being the less persistent of the three. Indeed, Panel D of Table 1 reports Johansen's (1988) Likelihood Ratio tests of cointegrating rank, which are based on the vector error correction representation of the three variables. These tests confirm that all three variables are stationary, implying that our VAR(1) representation of the data is satisfactory.

2.2 Multiperiod Regressions

Table 2 presents formal evidence of the predictive ability of the nominal yield spread for future GDP growth and inflation. The table reports estimates of the typical OLS regression used by most researchers to measure the predictive ability of the yield spread for future output:

$$100(\frac{4}{k})(y_{t+k} - y_t) = a_{0,k} + a_{1,k}s_t + u_{y,t+k} \tag{1}$$

where y_t is log real GDP, $100(\frac{4}{k})(y_{t+k} - y_t)$ measures the annualized growth rate of real GDP from quarter t to quarter $t+k$ in percentage terms, and s_t is the nominal yield spread, measured as the difference between the 10-year and the 3-month yields. The table also reports estimates of a similar OLS regression for future annualized inflation:

$$100(\frac{4}{k})(p_{t+k} - p_t) = b_{0,k} + b_{1,k}s_t + u_{p,t+k} \tag{2}$$

where p_t is the log of the Consumer Price Index in the middle of quarter t. The two equations are estimated simultaneously as a system of seemingly unrelated regressions because of the need to subsequently test cross-equation restrictions.

One important issue is how to construct confidence intervals of the slope coefficients of multiperiod predictive regressions (1), (2). Stambaugh (1986, 1999) and Mankiw and Shapiro (1986) have demonstrated that the distribution of the estimates of slope coefficients in predictive regressions is mislocated in small samples

when the regressor follows an AR(1) process and its innovations are contemporaneously correlated with the innovations of the dependend variable. In the context of multiperiod predictive regressions, Hodrick (1992) provides an alternative standard error estimator when the dependent variable is not autocorrelated. More recently, Valkanov (2003) proposed a rescaled t-statistic when the predictor variable is a near unit root process. These authors have not provided analytic results for the small sample properties of the slope coefficients when the dependent variables are themselves autocorrelated, as it is the case with our data.

Hodrick proposes Monte Carlo analysis to correct for the bias in estimated coefficients and to construct standard errors. This is a natural suggestion and we follow it. Thus, in addition to the usual asymptotic Newey-West (1987) t-statistics, which correct for conditional heteroskedasticity and autocorrelation of order $k-1$, we also report the 5% and 95% fractiles of the slope coefficients, which originate from 5,000 bootstrap simulations.

In the simulations, we impose the null hypothesis of no predictability of output growth and inflation based on the current yield spread, $a_{1,k} = b_{1,k} = 0$. Specifically, in each simulation run, we construct artificial time series for each variable $\Delta y_t, \Delta p_t$ and s_t as independent AR(1) processes. The AR(1) coefficients are set equal to the diagonal elements of the estimated VAR coefficient matrix, reported in Panel C of Table 1. The starting value of each series is set equal to its unconditional mean (i.e., zero). We then draw with replacement from the empirical distribution of the VAR residuals of each original series. Subsequently, we calculate the multiperiod changes $y_{t+k} - y_t$ and $p_{t+k} - p_t$ and perform the k regressions per equation. After 5,000 simulations, we calculate the 5% and 95% fractiles of the slope coefficients of the multiperiod regressions from their simulated distribution. We also calculate the means of the coefficients $a_{1,k}$ and $b_{1,k}$ of the bootstrap distributions and subsequently subtract those means from the OLS estimates, which are the ones that are finally tabulated. Nevertheless, it turns out the bias is very small, so the adjustment does not make much difference.[5]

The estimates of coefficient $a_{1,k}$ are qualitatively similar to those obtained by a number of previous researchers. Both asymptotic t-statistics and 95% confidence bounds from bootstrap simulations confirm that the nominal yield spread has predictive power for future real GDP growth for horizons up to the two-year horizon that we explore. The adjusted $R^{2\prime}s$ peak at k between five and seven quarters. Economically, an increase in the 10-year yield spread by 100 basis points predicts an increase in output growth by about 0.8 percentage points in one year's time.

The inflation equation also shows substantial predictability that lasts for approximately five to six quarters. An increase in the 10-year nominal yield spread by 100 basis points predicts a decrease in consumer price inflation by about 0.9 percent

[5]The bias in the one-quarter-ahead real growth regression is approximately -0.02 and in the one-quarter-ahead inflation regression -0.08. The bias declines at longer forecasting horizons.

one quarter ahead, and by about 0.7 per cent in one year. In contrast to the GDP growth predictions, the adjusted $R^{2\prime}s$ are highest in the one-quarter ahead horizon and decline monotonically after that.

The sixth column in the table (Column "W") presents Wald tests of the null hypothesis of symmetry, i.e. that the coefficients $a_{1,k}$ and $b_{1,k}$ are of opposite sign and equal magnitude, $b_{1,k} = -a_{1,k}$. The hypothesis cannot be rejected in any of the horizons. The coefficient magnitudes are also economically very close to each other. The last column in the table reports the sum of the coefficients along with the 5% and 95% fractiles of its bootstrap distribution. Again, we cannot reject the hypothesis of symmetry in any of the horizons.

2.3 An Alternative Specification of the Forecasting Equation

Researchers have criticized the use of long horizon regressions with overlapping forecasting horizons in the presence of highly persistent regressors. For example, Valkanov (2003) shows that the t-statistics in very long-horizon regressions do not converge to well-defined distributions if the regressor is close to a unit root process. Similar results are provided by Campbell and Yogo (2004) and Rossi (2005). Our earlier multiperiod regressions of Table 2 do not fall in this category, as our regressor variable is safely away from a unit root process (the AR(1) coefficient of the yield spread is 0.85, see Table 1, Panel C) and the forecasting horizon is short relative to the sample size. Moreover, we did present simulation results on the statistical significance of the estimated coefficients. Nevertheless, it is worthwhile exploring alternative specifications, which were utilized by previous researchers in order to partially circumvent the overlapping horizons problem. One such specification was proposed by Jegadeesh (1991) and was later also utilized by Hodrick (1992) and Plosser and Rouwenhorst (1994). It avoids the overlapping horizons problem, by estimating the predictive equation only for one quarter ahead and, instead of cumulating the dependent variable, it cumulates the independent variable, as follows:

$$100(4)\Delta y_{t+1} = c_{0,k} + c_{1,k} s_{t,k} + e_{y,t+1} \tag{3}$$

$$100(4)\Delta p_{t+1} = d_{0,k} + d_{1,k} s_{t,k} + e_{p,t+1} \tag{4}$$

where $s_{t,k} = \frac{1}{k}\sum_{i=0}^{k-1} s_{t-i}$ is the average nominal yield spread between time t and time $t-k-1$. For $k=1$, the regression coefficients $c_{1,k}$ and $d_{1,k}$ are identical to the corresponding regression coefficients $a_{1,k}$ and $b_{1,k}$ of earlier equations (1) and (2). For $k>1$, these coefficients differ but they still capture the same covariance between future output growth or inflation and the current nominal yield spread that the earlier ones did.

Table 3 presents the estimates of $c_{1,k}$ and $d_{1,k}$ for the different forecasting horizons k. The Newey-West (1987) $t-$statistics, which are in parentheses below the

coefficient estimates, correct for conditional heteroskedasticity and autocorrelation of order four. As in the previous table, the slope estimates are adjusted for small-sample bias by subtracting the mean of their distribution from the same earlier 5,000 bootstrap simulations. The 5% and 95% fractiles of the simulated distribution of coefficients are reported in curly brackets. Recall that in each simulation run, we generate independent time series Δy_t, Δp_t and s_t. In the present table, we have also calculated the average nominal yield spread $s_{t,k}$ from the artificial data and have subsequently performed the k regressions per equation.

The results in Table 3 are similar to those in Table 2. There is output and inflation predictability in all horizons, although the significance of price predictability decreases after seven quarters. The hypothesis of symmetry is not rejected. In fact, the magnitudes of the coefficients are economically very close to each other, confirming our previous results.

2.4 Implied Slope Coefficients from a Vector Autoregression

A third way to examine the predictive power of the nominal yield spread is to construct the implied multiperiod regression slope coefficients from the short-run dynamics of the VAR estimates of Table 1, Panel C. This vector autoregressive approach was previously utilized by a number of authors to conduct inference about the ability of dividend yields to predict stock returns at various horizons (Campbell and Shiller (1988), Kandel and Stambaugh (1989) and Hodrick (1992), among others). The slope coefficients of multiperiod regressions can be backed out from the parameter estimates of the VAR. These slope coefficients reflect the predictive power of the nominal spread when the information set of economic agents includes only the current and past history of the nominal spread, the rate of growth of real output and the level of inflation.

Let $z_{t+1} = [\Delta y_{t+1}, \Delta p_{t+1}, s_{t+1}]$ represent the vector of de-meaned variables and assume that z_{t+1} can be modeled as a first order autoregressive model: $z_{t+1} = A\ z_t + u_t$, with the error process satisfying the standard properties $E(u_{t+1}) = 0$, $E(u_{t+1}u'_{t+1}) = V$.

Since $z_{t+1} = (I - AL)^{-1}u_{t+1}$, the variance of the z_t process is: $C(0) = \sum_{j=0}^{\infty} A^j V A^{j\prime}$.[6] Also, the covariance between z_t and z_{t+j} is $C(0)A^{j\prime}$ and the covariance between z_t and $\frac{1}{k}\sum_{j=1}^{k} E_t z_{t+j}$ is $\frac{1}{k}C(0)[A + A^2 + ... + A^k]'$.

The slope coefficient $a_{1,k}$ in the output regression (1) is the covariance of the yield spread with the k-periods ahead cumulative growth, divided by the variance of the yield spread. Thus, the estimate of this coefficient, as implied by the VAR, is:

$$a_{1,k} = \frac{(1/k)i'_1 C(0)[A + A^2 + ... + A^k]'i_3}{i'_3 C(0) i_3} \tag{5}$$

[6] In computing $C(0)$, we truncate the infinite sum at j =200.

where i_m, $m = 1, 2, 3$ is the $m-$th column of the (3×3) identity matrix. Similarly, the slope coefficient $b_{1,k}$ in the inflation regression (2) can be calculated from the VAR as:

$$b_{1,k} = \frac{(1/k)i_2'C(0)[A + A^2 + ... + A^k]'i_3}{i_3'C(0)i_3} \tag{6}$$

The distribution of the implied slope coefficients is computed from 5,000 bootstrap simulations of the VAR under the null hypothesis that each of the series Δy_{t+1}, Δp_{t+1}, s_{t+1} follows a univariate AR(1) process. In particular, we generate artificial data by drawing with replacement from the vector of estimated VAR residuals as $\widetilde{z}_{t+1} = diag(A)\widetilde{z}_t + \widetilde{u}_{t+1}$, where $diag(A)$ is the main diagonal of the estimated VAR coefficient matrix A, $\widetilde{u}_{t+1}$ are the bootstrap residuals and the initial values are set equal to the unconditional mean of the variables, $\widetilde{z}_0 = 0$. Subsequently, we estimate the VAR with the artificial data and calculate the implied slope coefficients $a_{1,k}$, $b_{1,k}$ and the sum $a_{1,k} + b_{1,k}$ for horizons of 1 to k quarters ahead. In order to correct for bias, we subtract the mean of the bootstrap distribution of the slope coefficients from their VAR estimates, given by equations (5), (6).

Table 4 presents estimates of bias-adjusted slope coefficients from the VAR. This bias is very small. We report in curly brackets below the coefficient estimates the 5% and 95% fractiles of their bootstrap distribution. The implied output coefficients $a_{1,k}$ are all positive and statistically significant in all horizons. The implied inflation coefficients $b_{1,k}$ are negative and statistically significant as well. The hypothesis of symmetry is not rejected in any of the horizons. Observe that the magnitude of the implied coefficients $a_{1,k}$ and $b_{1,k}$ is very close to the magnitude of the corresponding coefficients in Tables 2 and 3. This is an indication that our parsimonious VAR model is an adequate forecasting tool for economic agents and that the short-term dynamics of the three variables of the VAR are consistent with the long-run forecasting behavior of the nominal spread. [7]

Summing up, all three econometric specifications arrive at the same result: The nominal yield spread is a symmetric predictor of output and inflation.

2.5 Exploring the Symmetric Predictability in Greater Detail

We now explore the symmetry in predictability in more detail. We ask two questions: First, is the predictive ability present throughout the sample period? Second, does the yield spread have independent information about the future evolution of output and prices over and above the information contained in the past performance

[7] Expanding the VAR to include a fourth variable, the nominal short-term rate, as in Ang, Piazzesi and Wei (2003), does not alter our conclusions. We prefer the parimonious VAR representation.

of each series or in other predictive variables?

Table 5 presents the four-quarter-ahead forecasting regressions of the earlier Table 2 over four separate subperiods. Each subperiod spans a decade, with the exception of the last one which is longer, including the last years of the sample period up to year 2004. The table reveals that the ability of the yield spread to predict one-year-ahead GDP growth broke down during the 1990s, confirming the earlier results of Haubrich and Dombrovsky (1996) and Dotsey (1998). However, this predictive ability may be coming back after the end of the prolonged expansion of the 1990's.[8]

The interesting new information in Table 5 is the behavior of the price equation. We observe that, remarkably, the predictability of inflation followed the decline in the predictability of output in the latter part of the sample period. This close relation between output and price predictability is even more striking when we run rolling regressions and tabulate the time-varying regression coefficients.

Figure 2 presents rolling regression estimates for the regression equations of Table 5. The rolling sample window is 40 quarters long. Observe that the rolling estimates $a_{1,4}$ and $b_{1,4}$ are almost a mirror reflection of each other. This evidence suggests that when looking for an explanation for output predictability one has to tie that explanation to a simultaneous price predictability in the opposite direction.

Table 6 presents evidence on the marginal predictive ability of nominal yield spread, when the regression includes additional regressors, the lagged dependent variable plus a measure of the stance of monetary policy. Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994) and others have provided evidence that the nominal yield spread retains its predictive power for output in the presence of additional regressors. Here we choose the level of the short-term nominal interest rate, $r(1)_t$ as a measure of the stance of monetary policy. The estimates in Table 6 are bias-adjusted, as was the case in the earlier Table 4.[9]

The estimates in Table 6 suggest that the yield spread does have extra predictive power for both output and inflation in the presence of those regressors. In particular, the slope coefficients of the yield spread in the output forecasting regressions are positive and significant at the 5% level up to six quarters ahead. Similarly, the yield spread coefficients of the price forecasting regressions are negative and significant at the 10% level up to five quarters ahead. Finally, the hypothesis of symmetry cannot be rejected at any horizon.

[8] The yield spread did in fact a good job in predicting the 2001 recession. The estimated slope coefficient in (1) over the period 2000:Q1-2004:Q2 is 0.70 with a standard error of 0.15 and an $\overline{R}^2$ of 0.57. Of course, the number of observations is still too small to make any reliable inference.

[9] In order to compute the distribution of the OLS slope coefficients, we estimate a VAR(1) in z_{t+1} = $[\Delta y_{t+1}, \Delta p_{t+1}, s_{t+1}, r(1)_{t+1}]$ and estimate the distributions from 5,000 bootstrap simulations of the VAR under the null hypothesis that each of the series follows a univariate AR(1) process.

3 A Monetary Asset Pricing Model with Price Rigidities

In this section we present a general equilibrium asset pricing model of a monetary economy in order to provide an explanation for the joint behavior of output, prices and the term structure of interest rates that we documented in Section 2. Our model is relatively simple and is, indeed, able to describe the qualitative features of the observed correlations in terms of very few deep structural economic parameters. We begin by providing a brief overview of earlier theoretical attempts to explain the predictability of output.

3.1 A Brief Review of the Theoretical Literature on the Predictive Ability of the Nominal Yield Spread

The previous literature has focused on explaining the predictability of output, that is, half of the empirical evidence that was presented in Section 2. Early attempts to explain the correlation of the yield spread and subsequent output or consumption growth essentially provided heuristic stories of the correlation. Estrella and Hardouvelis (1991), for example, interpret the positive association between the yield spread and future output growth as arising from market expectations of future shifts in investment opportunities and/or consumption (an expected future shift in the IS curve that would affect future output and future short rates, hence the current long rate). They claim the association is not due to the current behavior of the central bank (a current shift in the LM curve, which affects short-term rates and future economic activity), as they control for the central bank's behavior in their regression analysis. Later on, Estrella (2005) built an IS – Phillips Curve - Policy reaction function model, in which the behavior of the central bank is important. In the context of this model, Estrella shows that the predictive power of the yield spread depends on the preferences of the central bank and, in particular, on the importance of inflation targeting relative to the importance of output stabilization in the monetary policy rule.[10]

Others have concentrated on models of the real economy and the Consumption-based CAPM (Harvey (1988), Hu (1993), Den Haan (1995), Rendu de Lint and Stolin (2003) and Estrella, Rodrigues and Schich (2003)). These authors have ignored the difference between nominal and real yield spreads and attempted to explain the empirical positive association of the nominal yield spread and output growth as a reflection of a possible positive association between the real yield spread and output (consumption) growth within the C-CAPM. Recall that, according to

[10] Frankel and Lown (1994) view a steep term structure as an indication of loose monetary policy, but do not explicitly relate it to output growth.

the C-CAPM, there is a positive relation between the real yield to maturity of a $\tau-$period bond, $rr(\tau)_t$, and the average expected growth rate of consumption between period t and period $t+\tau$, $\frac{1}{\tau}E_t(c_{t+\tau}-c_t)$:

$$rr(\tau)_t = \alpha_\tau + \frac{1}{\sigma}[\frac{1}{\tau}E_t(c_{t+\tau}-c_t)] \tag{7}$$

where α_τ is a constant and σ is the elasticity of intertemporal substitution between present and future consumption with respect to the real rate of interest and is equal to the inverse of the coefficient of relative risk aversion, γ.

Many authors inadvertedly transplant the positive association of the level of real rates with consumption growth in equation (7) to a similar positive association of the *spread* in real interest rates with future consumption growth. This, however, is misleading. Rendu de Lint and Stolin (2003) explain that equation (7) results in a *negative* relation between the real yield *spread* and future consumption growth. To see this, rewrite equation (7) for the case of $\tau = 1$, and subtract the result from (7):

$$rr(\tau)_t - rr(1)_t = \alpha + \frac{1}{\sigma}\left[\frac{1}{\tau}E_t(c_{t+\tau}-c_t) - E_t(c_{t+1}-c_t)\right] \tag{8}$$

Observe that the left-hand-side of equation (8) is, indeed, the real yield spread or the slope of the real term structure. However, the right-hand-side of equation (8) is no longer the expected growth in consumption but the expected *difference* between average growth in consumption over τ periods and the one-period growth. To translate this difference in growth rates into a level of growth rates, suppose that consumption growth follows an autoregressive process of order one, with an autoregressive parameter ϕ, $0 < \phi < 1$. Then, equation (8) becomes:

$$rr(\tau)_t - rr(1)_t = \alpha - \frac{1}{\sigma}\left[1 - \frac{1}{\tau}(1+\phi+\ldots+\phi^{\tau-1})\right]E_t(c_{t+1}-c_t) \tag{9}$$

The slope coefficient in the above relation is always negative. This is because the growth of consumption is a stationary process and thus shocks to consumption affect the short-run growth rates a lot more than they affect the long-run ones.[11] Indeed, later in Section 4, we show that the empirical relationship between the real yield spread and future output growth is negative.[12]

It is clear that the theoretical C-CAPM alone cannot accommodate the positive association between consumption growth and the nominal yield spread. To account

[11] Rendu de Lint and Stolin (2003) show that this result holds even when the level of log consumption is an autoregressive process as opposed to the growth in consumption.

[12] Estrella, Rodrigues and Schich (2003) suggest that the C-CAPM could accomodate a positive association between the real yield spread and expected future consumption growth, provided the model includes habit formation that behaves in a specific manner. Their story is heuristic and not fully modeled. Moreover, as mentioned above, the empirical evidence points to a negative association, as is predicted by the simple C-CAPM.

for the positive association without abandoning the C-CAPM, the model has to be extended to include the temporal behavior of prices and, in particular, it has to have features which transform the negative association of the real yield spread with future output growth into a positive association of the nominal yield spread with future output growth.

3.2 The Elements of the Proposed Model

Our model is a monetary general equilibrium model, in which prices and output are determined endogenously. The model includes a mechanism for generating the evolution of prices over time and retains the fundamental Euler equation of the Consumption CAPM, expressed in nominal terms. Thus, the model is able to generate correlations between the nominal yield spread and real output growth and inflation. These correlations are consistent with the empirical evidence, provided that the intertemporal elasticity of substitution is larger than unity and prices in the economy are not perfectly flexible.

There is a long theoretical literature on general equilibrium models of inflation and the term structure (Danthine and Donaldson (1986), Constantinides (1992), Sun (1992) and others). Benningha and Protopapadakis (1983) were the first to emphasize the breakdown of the Fisher Theorem. Stulz (1986) points out the presence of a negative relation between expected inflation and real asset returns. Marshall (1992) provides empirical evidence consistent with Stulz. Donaldson, Jonsen and Mehra (1990) build a model of the real term structure, not the nominal one, but are among the first to examine its properties across the business cycle. Labadie (1994) builds a model of the nominal term structure, by introducing a cash-in-advance constraint and explores the behavior of both the nominal and the real spread across the business cycle. Den Haan (1995) introduces money via a shopping-time technology. Bakshi and Chen (1996) introduce money in the utility function.

Our model is a modification of Rotemberg (1982, 1996) and differs from earlier ones mainly in the way it introduces dynamics. The dynamics are endogenous and are driven by price stickiness, which arises from the existence of costs of price adjustment. Price rigidities imply that shocks to output and money supply lead to forecastable changes in future price and output growth. These forecastable changes lead consumers to adjust their savings in order to smooth their consumption over time, generating a correlation between the current yield spread and future economic activity and inflation.

In the model, the economy is populated by identical, infinitely-lived households. Each household produces a type of intermediate good which is an imperfect substitute for the other goods and sells it under conditions of monopolistic competition. Prices of intermediate goods adjust with a lag to changes in demand and costs of production due to the existence of a cost of adjusting prices. Firms purchase intermediate goods from households and use them to produce a single consumption good

with a constant returns to scale technology. We modify Rotemberg's model by using a power utility function and by adding a bond market, in which households can borrow or lend their proceeds for $1, ..., N$ periods. Indeed, households can buy or sell nominally risk-free $\tau-$ period discount bonds which promise to pay $R_{\tau,t}$ dollars in all states of the world at time $t+\tau$, $\tau = 1, ..., N$. Consumption goods must be paid for with money, i.e. households are subject to a Cash-In-Advance constraint. Money is a non-interest bearing security. Each period, the central bank makes a lump-sum money transfer to households.

The full exposition of the model is contained in Appendix A. Here, we begin the analysis by moving directly to the solution of the model for the nominal interest rate and for the prices of output:

$$r_t(\tau) = -\log(\beta) + \frac{1}{\tau}\left[\frac{1}{\sigma}E_t(c_{t+\tau} - c_t) + E_t(p_{t+\tau} - p_t)\right] + \theta(\tau) \tag{10}$$

$$p_t = \alpha p_{t-1} + (1-\alpha)(1-\delta)E_t\sum_{k=0}^{\infty}\delta^k(m_{t+k} - x_{t+k}) \tag{11}$$

where $r_t(\tau)$ is the continuously compounded, annualized nominal yield to maturity at time t of a zero coupon bond with maturity τ periods, c_t, p_t, m_t, x_t are the natural logarithms of consumption, prices, money supply and productivity, respectively, $\theta(\tau)$ is a constant term premium, $\alpha \in (0,1)$ is the degree of price stickiness, $\delta \in (0,1)$ is a constant, and σ is the elasticity of intertemporal substitution of consumption with respect to the real rate of interest, which with power utility equals the inverse of the coefficient of relative risk aversion, $\sigma = 1/\gamma$.

Equation (10) is the well-known optimality condition of the Consumption-CAPM, but is now expressed in a monetary environment with inflation. It says that the nominal yield to maturity of a τ-period zero coupon bond at time t is determined by the sum of the expected average consumption growth and the expected average inflation between time t and time $t+\tau$ plus a term premium.

Equation (11) says that prices are a linear combination of lagged prices and long-run equilibrium prices. The latter are given as the discounted value of expected excess money supply over productivity. An expected increase in money supply increases current prices because it increases the demand for the final product. An expected increase in productivity decreases current prices because it decreases production costs per unit of output. Due to the existence of costs of price adjustment, there is a lagged adjustment of prices towards their long-run equilibrium. The speed of this adjustment depends negatively on the degree of price stickiness, α.

In order to derive a simple price equation in terms of observables, we specify the stochastic processes driving money supply and productivity:

$$m_t = \mu_m + m_{t-1} + \varepsilon_{m,t} \tag{12}$$

$$x_t = \mu_x + x_{t-1} + \varepsilon_{x,t} \tag{13}$$

Money supply and productivity follow random walks with drift factors μ_m, μ_x and independent innovation processes $\varepsilon_{m,t}$ and $\varepsilon_{x,t}$, respectively. Taking expectations of equations (12) and (13), conditional on information up to time t, gives: $E_t(m_{t+k}) = m_t + k\mu_m$, $E_t(x_{t+k}) = x_t + k\mu_x$ for all $k = 0, \ldots, \infty$. Substituting in equation (11), we obtain:

$$p_t = \alpha p_{t-1} + (1-\alpha)(m_t - x_t) + \frac{\delta(1-a)\mu}{1-\delta} \tag{14}$$

where $\mu \equiv \mu_m - \mu_x$. Taking the first difference of equation (14) gives the rate of inflation as a function of contemporaneous and past innovations to money supply and productivity:

$$\Delta p_t = \frac{(1-\alpha)}{(1-\alpha L)}(\Delta m_t - \Delta x_t) = \mu + \psi(L)(\varepsilon_{m,t} - \varepsilon_{x,t}) \tag{15}$$

where $\psi(L) = (1-\alpha)/(1-\alpha L)$ is an infinite-order polynomial in the lag operator L. L is defined as: $L^i z_t \equiv z_{t-i}$, thus $\psi(L)z_t = (1-\alpha)(z_t + \alpha z_{t-1} + \alpha^2 z_{t-2} + \cdots + \alpha^p z_{t-p} + \cdots)$. Note that $\psi(1) = 1$, meaning that an one-off monetary shock leads to a proportional long-run increase in the price level, whereas an one-off productivity shock leads to a proportional decrease in the price level. The conditional expectation of the long-run rate of inflation is given by $E_t \Delta p_{t+\infty} = \mu + \varepsilon_{m,t} - \varepsilon_{x,t}$ (long-run quantity theory).

The relationship between output, money and prices is given by the cash-in-advance constraint (A6) - described in Appendix A, together with the condition that in equilibrium consumption is equal to output, i.e., in logs: $y_t = m_t - p_t$. Substituting (14) in this equation for p_t and taking first differences, we obtain:

$$\Delta y_t = \mu_x + (1 - \psi(L))\varepsilon_{m,t} + \psi(L)\varepsilon_{x,t} \tag{16}$$

According to equation (16), real output growth is a function of current and past monetary and productivity shocks. Since $\psi(1) = 1$, the monetary shock, $\varepsilon_{m,t}$, represents the transitory component, whereas the productivity shock, $\varepsilon_{x,t}$, represents the permanent component of output growth.

3.3 Why does the Yield Spread Predict Future Economic Activity and Inflation?

In order to derive the term structure of interest rates as a function of unexpected changes in money supply and productivity, we first compute the conditional expectation of the continuously compounded output growth and inflation. From equations (16) and (15) we obtain for the conditional expectation of the growth rate of output (consumption) and prices from period $t+k-1$ to period $t+k$ for $k \geq 1$:

$$E_t(\Delta y_{t+k}) = -E_t(\Delta p_{t+k}) = \alpha^k \psi(L)\varepsilon_t \tag{17}$$

where, for convenience, we have excluded the constants and re-defined the innovation process as the productivity minus the money supply shock: $\varepsilon_t \equiv \varepsilon_{x,t} - \varepsilon_{m,t}$. It follows that the continuously compounded, annualized rate of output growth between time t and time $t+k$, given information up to time t, is:

$$\frac{1}{k}E_t(y_{t+k} - y_t) = -\frac{1}{k}E_t(p_{t+k} - p_t) = \alpha\kappa(k)\psi(L)\varepsilon_t \tag{18}$$

where $\kappa(k) = \frac{(1-\alpha^k)}{k(1-\alpha)}$.

Next, setting $k = \tau$ in (18) and substituting the resulting equation in (10), we obtain for the time t yield to maturity of a $\tau-$period nominal discount bond as:

$$r(\tau)_t = -\log(\beta) - (1 - \frac{1}{\sigma})\alpha\kappa(\tau)\psi(L)\varepsilon_t + \theta(\tau) \tag{19}$$

where $\kappa(\tau) = \frac{(1-\alpha^\tau)}{\tau(1-\alpha)}$.

Using equation (19) and noting that $\kappa(1) = 1$, the τ-period nominal yield spread, defined as $s_{\tau,t} = r(\tau)_t - r(1)_t$, can be written as:

$$s_{\tau,t} = (1 - \frac{1}{\sigma})\alpha(1 - \kappa(\tau))\psi(L)\varepsilon_t \tag{20}$$

and the conditional mean of the τ-period real yield spread, defined as $E_t(rs_{\tau,t}) = s_{\tau,t} - (\frac{1}{\tau}E_t(p_{t+\tau} - p_t) - E_t(p_{t+1} - p_t))$, can be written as:

$$E_t(rs_{\tau,t}) = -\frac{1}{\sigma}\alpha(1 - \kappa(\tau))\psi(L)\varepsilon_t \tag{21}$$

Equations (20) and (21) demonstrate that the effects of productivity and monetary shocks on the nominal and the real yield spread depend on the degree of price stickiness, α, the elasticity of intertemporal substitution, σ, and the term to maturity, τ. Note that $1-\kappa(\tau) > 0$ for $\tau > 1$ and $0 < \alpha < 1$, implying that long-term nominal and real interest rates react less strongly than one-period nominal and real interest rates to a productivity or monetary shock. This occurs because most of the change in expected inflation and output takes place in the first periods following the shock, implying that the average expected one-period interest rate over a horizon of τ periods changes less than the current one-period interest rate. Observe also that the nominal yield spread reacts in the opposite direction from the direction of the real yield spread, provided that the elasticity of intertemporal substitution, σ, is larger than unity.

To understand the mechanics of the model, let us trace the effects of a positive productivity shock, $\varepsilon_{x,t}$. In the model, the shock is permanent, hence, once it occurs, it is expected to influence the level of output forever. Also, in our experiment, the increase in productivity occurs with the money supply process unaltered. Thus, given price stickiness and the fact that the cash-in-advance constraint has to be

satisfied, the positive productivity shock drives contemporaneous output and consumption up by $\alpha\varepsilon_{x,t}$ and contemporaneous prices down by $(1-\alpha)\varepsilon_{x,t}$, creating the base of comparisons with expected future levels of consumption, output and prices.

>From the next quarter on, prices are expected to slowly decline towards their long-run equilibrium, since they have a negative gap to close. Prices will adjust downward by $\alpha[(1-a)\varepsilon_{x,t}]$ in period $t+1$, by $\alpha^2[(1-\alpha)\varepsilon_{x,t}]$ in period $t+2$, by $\alpha^3[(1-\alpha)\varepsilon_{x,t}]$ in period $t+3$, and so on. Given an unchanged money supply process and the need to satisfy the cash-in-advance constraint at the end of each period, output is thus expected to rise *symmetrically* by $\alpha[(1-\alpha)\varepsilon_{x,t}]$ at time $t+1$, $\alpha^2[(1-\alpha)\varepsilon_{x,t}]$ at time $t+2$, $\alpha^3[(1-\alpha)\varepsilon_{x,t}]$ at time $t+3$, etc. Observe that as the horizon increases, the successive percentage drops in prices and percentage increases in output decline in absolute magnitude. The absolute value of the expected average cumulative percentage drop in prices and increase in output from period t to period $t+\tau$, which equals $\alpha\kappa(\tau)[(1-\alpha)\varepsilon_{x,t}]$, also declines as the horizon τ increases.

Real annualized interest rates of maturity τ increase proportionately to the corresponding increase in real output over the next τ periods: $(1/\sigma)\alpha\kappa(\tau)[(1-\alpha)\varepsilon_{x,t}]$, while the spread between the τ-period real rate and the one-period real rate declines by $-(1/\sigma)\alpha(1-\kappa(\tau))[(1-\alpha)\varepsilon_{x,t}]$. Nominal interest rates are influenced by the increase in real rates and by the simultaneous decrease in expected inflation. The τ-period nominal rate will change by $-(1-\frac{1}{\sigma})\alpha\kappa(\tau)[(1-\alpha)\varepsilon_{x,t}]$, which is negative as long as the elasticity of intertemporal substitution, σ, is larger than unity. The spread between the τ-period nominal rate and the one-period nominal rate increases by $(1-\frac{1}{\sigma})\alpha(1-\kappa(\tau))[(1-\alpha)\varepsilon_{x,t}]$.

A positive monetary shock has exactly the opposite influence on interest rates. It increases expected inflation as prices fail to adjust immediately upward, but are instead expected to increase gradually over time. The gradual increase in expected future prices drives expected future output down by a symmetric amount. Real interest rates decline and nominal rates increase, provided that $\sigma > 1$. The real spread widens and the nominal spread shrinks.[13]

More formally, one can compute the conditional covariance of the nominal and real term structure spread with the $k-$period ahead continuously compounded annualized output growth, $\frac{1}{k}E_t\sum_{i=1}^{k}\Delta y_{t+i}$, and the $k-$period ahead inflation, $\frac{1}{k}E_t\sum_{i=1}^{k}\Delta p_{t+i}$. From equation (20) of the nominal spread and equation (18), and noting that the innovations are i.i.d. with constant variance σ_ε^2, the conditional covariance between the time t nominal yield spread and the $k-$period ahead continuously compounded annualized output growth and inflation is:

[13]It should be noted that, as emphasized by Rotemberg (1996), a positive money supply shock generates a *positive* contemporaneous correlation between changes in prices and output and if monetary shocks dominate productivity shocks, then the model allows for a contemporaneous positive correlation between output and price growth. On the other hand, in the model, the revisions in expected future changes in output and prices always move in opposite directions.

$$
\begin{aligned}
&Cov_t(s_{\tau,t},\ \frac{1}{k}(y_{t+k}-y_t)) = \\
&-Cov_t(s_{\tau,t},\ \frac{1}{k}(p_{t+k}-p_t) = \\
&(1-\frac{1}{\sigma})\kappa(k)(1-\kappa(\tau))\alpha^2(1-\alpha)^2\sigma_\varepsilon^2
\end{aligned} \tag{22}
$$

Similarly, the conditional covariance between the time t *real* yield spread and the $k-$period ahead continuously compounded annualized output growth and inflation is:

$$
\begin{aligned}
&Cov_t(rs_{\tau,t},\ \frac{1}{k}(y_{t+k}-y_t)) = \\
&-Cov_t(rs_{\tau,t},\ \frac{1}{k}(p_{t+k}-p_t)) = \\
&-\frac{1}{\sigma}\kappa(k)(1-\kappa(\tau))\alpha^2(1-\alpha)^2\sigma_\varepsilon^2
\end{aligned} \tag{23}
$$

Figure 3 displays the above two conditional covariances between the nominal and real yield spread on the one hand and future output growth on the other, for various values of α, ranging from zero to one. In the figure we set $\tau = 40$ and $k = 4$, to match the covariance of the 10-year yield spread and 4 quarter ahead GDP growth. Furthermore, we set $\sigma_\varepsilon^2 = 3.6$, the sample variance of the difference in innovations of quarterly changes in GDP and M3 money supply.[14] Finally, we set $\sigma = 1.5$, in line with Vissing-Jorgensen (2002) and Bansal and Yaron (2004).

There are several results worth noticing from equations (22) and (23), and Figure 3. First, the conditional covariance of the real yield spread with future output (price) growth is always negative (positive). This relationship is consistent with Rendu de Lint and Stolin (2003) and our earlier heuristic discussion for the growth in consumption. Subsequently, in Section 4, we examine whether or not this prediction is supported by the empirical evidence.

Second, for the conditional covariance of the nominal yield spread with future output (price) growth to be positive (negative) in the model, as is the case empirically, the elasticity of intertemporal substitution has to be larger than unity. There is considerable variance among earlier studies that used aggregate consumption data

[14] Innovations were estimated using an AR(1) model for both output and money supply. Seasonally adjusted M3 money supply is taken from the IMF database, code: USI59MCCB. We use quarter averages from monthly data in order to ensure comparability with GDP, which is a flow variable.

in order to estimate the size of parameter σ.[15] However, the latest work on household income and asset allocation data by Vissing-Jorgensen (2002) and Vissing-Jorgensen and Attanasio (2004) shows that the condition $\sigma > 1$ is a good characterization of bondholders. Namely, for bondholders the elasticity of intertemporal substitution is larger than that of stockholders and is larger than unity, perhaps closer to 2.6. The higher the elasticity of intertemporal substitution, the smaller the required response of real interest rates to exogenous shocks in order to restore equilibrium in the economy. Thus for $\sigma > 1$, exogenous shocks do not affect real interest rates as much as they affect expected inflation. The influence of real interest rates on the level of nominal interest rates is, therefore, overwhelmed by the opposite influence of expected inflation on nominal rates.

Third, the size of the conditional covariance between the (real or nominal) yield spread and output growth is highest for intermediate values of the degree of price stickiness α. When α is very close to zero, prices are very flexible and, hence, the serial correlation in price and output growth is small, preventing the shocks of the model from generating large revisions in the expectations of future changes in prices or output and in the yield spreads. At the opposite extreme, when α is very close to unity, prices are very sticky and, although there is very high serial correlation in output and prices, the size of the revisions themselves are very small relative to the size of the shocks.

Finally, the model generates a positive conditional covariance between the current nominal term structure spread, $s_{\tau,t}$, and the future *change* in the rate of inflation, $\frac{1}{k}(p_{t+k} - p_t) - (p_{t+1} - p_t)$, provided, as before, that $\sigma > 1$:

$$\begin{aligned} &Cov_t(s_{\tau,t},\ \frac{1}{k}(p_{t+k} - p_t) - (p_{t+1} - p_t) \\ &= (1 - \frac{1}{\sigma})(1 - \kappa(k))(1 - \kappa(\tau))\alpha^2(1-\alpha)^2\sigma_\varepsilon^2 \end{aligned} \tag{24}$$

The above covariance is zero at horizon $k = 1$, since at that horizon the change in inflation is by construction zero. Then, at horizon $k = 2$, the covariance becomes positive but small and, subsequently, as the forecasting horizon k increases, it keeps rising, but at a declining rate. For very large forecasting horizons, i.e. as $k \to \infty$, the above covariance approaches the value of $(1-\frac{1}{\sigma})(1-\kappa(\tau))\alpha^2(1-\alpha)^2\sigma_\varepsilon^2$, which equals minus the covariance of the nominal spread with the one-period ahead inflation. Thus, equation (24) provides an explanation of the previous findings of Fama (1990),

[15] The earlier time-series studies concentrate on estimating the parameter $\gamma = 1/\sigma$, using aggregate consumption data. Brown and Gibbons (1985) estimate a range of γ between 0.09 and 7, implying a value of σ from 0.15 to 11. Mankiw, Rotemberg and Summers (1985) estimate γ between 0.09 and 0.51, implying a value of σ between 2 and 11. Harvey (1988) estimates a range of γ between 0.33 and 0.96, implying a value of σ between 1 and 3. Hall (1988) finds a very small σ.

Mishkin (1990a,b, 1991), Jorion and Mishkin (1991) and others, that the nominal spread can predict the change in inflation at long horizons a lot better than at short horizons.

4 Examining the Predictive Ability of the Real Yield Spread

Our model is able to explain the symmetric predictive power of the nominal yield spread for output and inflation based on a stripped down parsimonious framework of optimizing agents. While both parsimony and optimizing behavior are desirable features of an economic model, the question always remains how realistic such models are. Our model would gain credibility as a more realistic descriptor of the behavior of the variables of interest, should it make additional predictions on the behavior of those economic variables, which would not be rejected by the evidence.

Indeed, as was discussed earlier, the model does make sharp predictions about the behavior of the spread of real interest rates. Recall that equations (18) and (21) completely characterize the dynamics of output growth, inflation and the real yield spread, while equation (23) describes the symmetric predictive power of the real yield spread. In this section, we explore those additional empirical implications in greater detail. We examine whether or not the real term structure spread is negatively related to future output growth and positively related to future inflation. We also examine if those output and inflation predictions are symmetric, as predicted by the model.

The chosen econometric framework resembles the earlier one. We begin by writing down the equilibrium relationships of the real yield spread with expected future output growth and inflation in the familiar form of predictive equations, as follows:

$$\frac{1}{k}E_t(y_{t+k} - y_t) = \alpha_0 + \alpha_{1,k}E_t(rs_{\tau,t}) \tag{25}$$

$$\frac{1}{k}E_t(p_{t+k} - p_t) = \beta_0 + \beta_{1,k}E_t(rs_{\tau,t}) \tag{26}$$

where $E_t(rs_{\tau,t})$ is the $\tau-$period ex-ante real yield spread, defined as: $E_t(rs_{\tau,t}) = s_{\tau,t} - \left[\frac{1}{\tau}E_t\left(\sum_{j=1}^{\tau}\Delta p_{t+j}\right) - E_t\Delta p_{t+1}\right]$, with $s_{\tau,t}$ denoting the nominal yield spread, which is measured as the difference between the $\tau-$period and the $1-$period nominal yields.

According to our model, the slope coefficients in the above equations are governed by the relationship: $-\alpha_{1,k} = \beta_{1,k} = \sigma\frac{\kappa(k)}{1-\kappa(\tau)}$ and α_0, β_0 are two constants.[16] This

[16] Adding the constant terms α_0 and β_0 in equations (25) and (26) is justified by our assumption that the money supply and productivity follow random walks with drift – see equations (12), (13). Earlier, we omitted these constants from equation (18) in order to simplify the notation.

is easily seen by substituting equation (18) for $\psi(L)\varepsilon_t$ into equation (21). Put differently, the real yield spread is negatively related to expected output growth and, in a symmetric way, positively related to expected inflation.

The empirical assessment of the predictive properties of the real yield spread is not as straightforward as the earlier one for the nominal yield spread. Real rates and, hence, the real yield spread, are unobservable and have to be somehow approximated. The approximation creates measurement error, which causes inconsistency in the estimated parameters. One solution would ordinarily have been to use the ex-post real yield spreads and a set of instrumental variables that proxy for the ex-ante real yield spread. However, instrumental variables techniques such as GMM are inappropriate in our setup, as the instruments that are correlated with the regressand (the real yield spread) are, by construction, also correlated with the regressor (inflation), destroying the required orthogonality condition between instruments and error terms.[17] We, therefore, follow an alternative approach, which utilizes the earlier estimates of the vector autoregression of Table 1, Panel C, in a manner similar to the implied VAR coefficients of Table 4. Specifically, we construct an estimate of the real term structure spread by generating inflation forecasts within the earlier vector autoregressive model and subsequently compute the implied slope coefficients $\alpha_{1,k}$, $\beta_{1,k}$ of equations (25) and (26) for a hypothetical regression of the multiperiod output or price growth on the current real term structure spread, assuming agents 'information set consists of the variables in the VAR.

The VAR methodology has three main advantages, compared to instrumental variables techniques such as GMM. First, the VAR can be based on the nominal yield spread, thus avoiding issues of exogenous measurement of the real spread. Equations (25) and (26) can be estimated directly from the VAR of Table 1, Panel C, as the VAR can generate simultaneous forecasts of output growth, inflation and the real spread. Second, when the VAR model of Table 1, Panel C, is utilized, the results for the predictive power of the real spread are internally consistent with the results of the predictive power of the nominal spread, shown earlier in Table 4. Finally, confidence intervals of slope coefficients can be easily constructed using the standard simulation techniques that we employed earlier.

The slope coefficient $\alpha_{1,k}$ in the output regression (25) on the, say 10-year, real yield spread ($\tau = 40$) is the covariance of the real yield spread with the k-quarters-ahead cumulative annualized growth, divided by the variance of the real yield spread:

[17] We have actually estimated the system of equations (25) and (26) using GMM and the following set of instruments: a constant, 4 lags of quarterly real GDP growth, 4 lags of quarterly inflation, and 4 lags of the nominal yield spread. The estimated slope coefficients are consistent with the model's predictions and are qualitatively similar to those of the VAR analysis that follows.

$$\alpha_{1,k}=\frac{Cov\left(s_t-\left[\frac{1}{40}E_t\left(\sum_{j=1}^{40}\Delta p_{t+j}\right)-E_t\Delta p_{t+1}\right],\frac{1}{k}\sum_{j=1}^{k}\Delta y_{t+j}\right)}{Var\left(s_t-\left[\frac{1}{40}E_t\left(\sum_{j=1}^{40}\Delta p_{t+j}\right)-E_t\Delta p_{t+1}\right]\right)} \tag{27}$$

Using the VAR(1) model $z_{t+1} = A\ z_t + u_t$, where $z_{t+1} = [\Delta y_{t+1},\ \Delta p_{t+1},\ s_{t+1}]$ is the vector of de-meaned variables, the estimate of the above slope coefficient can be represented as a nonlinear function of the VAR coefficients (see Appendix B):

$$\alpha_{1,k}=\frac{\frac{1}{k}i_1'C(0)A(k)'i_3-\frac{1}{40}\frac{1}{k}\sum_{j=1}^{40}\left(i_2'A^jC(0)A(k)'i_1\right)+\frac{1}{k}\left(i_2'AC(0)A(k)'i_1\right)}{\left[i_3'I_{3\times3}+i_2'\left(A-\frac{1}{40}\sum_{j=1}^{40}A^j\right)\right]C(0)\left[i_3'I_{3\times3}+i_2'\left(A-\frac{1}{40}\sum_{j=1}^{40}A^j\right)\right]'} \tag{28}$$

where i_m is the $m-$th column of the (3×3) identity matrix $I_{3\times3}$ and $A(k) = [A + A^2 + ... + A^k]$. The first term of the numerator is the covariance between nominal yield spread and k-quarters-ahead expected annualized growth. The second term of the numerator is minus the covariance between k-quarters-ahead expected annualized growth and the 40-quarters-ahead expected annualized inflation. The third term of the numerator is the covariance between the k-quarters-ahead expected annualized growth and the one-quarter-ahead expected annualized inflation.

Similarly, the slope coefficient $\beta_{1,k}$ in the inflation regression (26) can be calculated from the VAR as:

$$\beta_{1,k}=\frac{\frac{1}{k}i_2'C(0)A(k)'i_3-\frac{1}{40}\frac{1}{k}\sum_{j=1}^{40}\left(i_2'A^jC(0)A(k)'i_2\right)+\frac{1}{k}\left(i_2'AC(0)A(k)'i_2\right)}{\left[i_3'I_{3\times3}+i_2'\left(A-\frac{1}{40}\sum_{j=1}^{40}A^j\right)\right]C(0)\left[i_3'I_{3\times3}+i_2'\left(A-\frac{1}{40}\sum_{j=1}^{40}A^j\right)\right]'} \tag{29}$$

Table 7 presents the results of this exercize. As in the earlier implied VAR coefficients of Table 4, we present bias-adjusted estimates of the slope coefficients along with the 5% and 95% fractiles of their distribution from 5,000 bootstrap simulations. As predicted by theory, the output coefficients are negative and statistically significant for all horizons. Also, the inflation coefficients are positive, but are statistically significant only up to four quarters ahead. [18] The last column of Table 7 reports the sum of the (bias-adjusted) slope coefficients along with the 5% and 95% fractiles of its bootstrap distribution. Once again, the hypothesis of symmetry cannot be rejected.

[18] It is worth mentioning that the bias in the estimated slope coefficient $\beta_{1,k}$ of the inflation equation is very large and positive, especially at short forecasting horizons. This is apparently due to the fact that the estimate of expected inflation appears on both sides of equation (26). At the one-quarter horizon, the estimated bias in $\beta_{1,k}$ is 0.79, that is, the unadjusted original estimate of the coefficient is 0.96, instead of the reported adjusted estimate of 0.17. This bias declines monotonically as the horizon increases to reach the value of 0.35 at the two-year horizon. By contrast, no substantial bias is observed in the output equation. The bias in $\alpha_{1,k}$ is less than 0.02.

5 Concluding Remarks

We examined the predictive ability of the nominal yield spread for future output and inflation in the United States and discovered a symmetric predictability. Over the period 1960-2004, an increase in the yield spread is associated with a future increase in real output and a decline in inflation of approximately the same magnitude. This symmetry is present in the various supberiods as well.

In order to explain this new stylized fact, we developed a parsimonious monetary consumption based asset pricing model, whose main innovation is the introduction of nominal rigidities in the economy in form of sticky prices of the consumption good. Due to price stickiness, shocks to the economy generate predictable changes in future output and prices, hence, allowing for intertemporal consumption smoothing effects on interest rates. This generates a correlation between the current yield spread and future expected output growth and inflation. We derived analytic solutions of the model, which relate output growth, inflation and the term structure to unanticipated changes in productivity and money supply.

In the model, productivity shocks in excess of shocks to the money supply generate a positive correlation between the nominal yield spread and future output growth and a negative correlation between the nominal yield spread and future inflation. The theoretical model can explain the observed stylized facts, provided that there exists some price stickiness and that the elasticity of intertemporal substitution is larger than unity.

The model makes a sharp distinction between the behavior of nominal and real interest rates. In the model, real interest rates move in the opposite direction from the direction of nominal interest rates, and the real yield spread moves in the opposite direction from the direction of the nominal yield spread. This distinction was passed over by the earlier literature, which attempted to explain the output predictability of the nominal yield spread as a direct implication of the Consumption-CAPM Euler equation. As Rendu deLint and Stolin (2003) show, this Euler equation predicts a negative relation between the real yield spread and future real output growth, not a positive one, as was typically assumed. Our model incorporates the Consumption-CAPM Euler equation and predicts a negative association between the real yield spread and future real output growth and, at the same time, is able to also predict the observed positive association between the nominal yield spread and future real output growth.

We explored the empirical implications of the model for the real yield spread and found them to be consistent with the data. Namely, we found that the real yield spread is negatively associated with future output growth and positively associated with future inflation.

Overall, a simple one-factor model goes a long way in explaining the sign of the correlation of both nominal and real yield spreads with future output growth and

inflation. Nevertheless, it is clear that such a model is too restrictive to account for the observed magnitude of the correlations between the variables. For example, the model predicts that the magnitude of the association between the yield spreads and future output and price growth declines with the horizon, whereas the evidence does not necessarily show a significant decrease in this magnitude as the horizon increases. Also, in the model, the term premium is constant and the yield spreads are perfectly correlated with future output growth and inflation. This is because all variables are driven by the same stochastic disturbance, namely the innovation of productivity in excess of money supply.

In order to account for less than perfect correlation between the variables, one has to include more than one stochastic disturbance driving the variables. One possibility would be to relax the assumption of a constant velocity of money in order to allow for asymmetric effects of productivity and monetary shocks on expected output growth and inflation and, perhaps, allow for a monetary policy rule as well. Another possibility would be to allow for a time-varying term premium, which affects the slope of the yield curve independently of productivity and money supply shocks, leading to changes in the yield spread which are uncorrelated with predictable changes in future output and prices.[19] A further modification of the model would be to introduce richer dynamics in the driving processes of money supply and productivity. For example, assuming an AR(1) process for money growth allows for predictable changes in output and inflation due to mean-reversion in money supply in addition to predictable changes related to price stickiness.[20] Allowing for richer dynamics of the stochastic disturbances driving the economy will lead to more flexibility in the dynamic adjustment of the term structure to economic shocks.

[19] A decline in the term premium due to lower output volatility might be able to explain the breakdown of the predictive power of the yield spread during the 1990s.

[20] See Rotemberg (1996), equations (12)-(16), for a modification of the model in this direction.

Appendix A: Description and Solution of the Model

Let Y_t be the output of the final good. It is produced using a continuum of intermediate goods as inputs indexed by $i \in [0,1]$. The production function of the final good is given by:

$$Y_t = \left[\int_0^1 Y_t(i)^q di \right]^{\frac{1}{q}}, \qquad 0 < q \leq 1 \tag{A1}$$

where $Y_t(i)$ is the input of intermediate good i and $1/(1-q)$ is the elasticity of substitution between goods. Final goods are produced under conditions of perfect competition. Firms take prices as given and choose $Y_t(i)$ in order to maximize profits, given by: $P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$, where P_t is the price of the final good and $P_t(i)$ is the price of intermediate good i. The resulting demand functions for intermediate goods have the form:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\frac{1}{1-q}} \tag{A2}$$

Households produce intermediate goods using labor, $L_{i,t}$, as the only input, according to the production function

$$Y_{i,t} = L_{i,t} \cdot X_t \tag{A3}$$

where X_t is a productivity shock. There is monopolistic competition in the market for intermediate goods. Households face the demand curve given by equation (A2) and set the price $P_{i,t}$ in order to maximize their utility function.

The utility function of the representative household depends on consumption of the final good, leisure (which we model directly as disutility of work) and negatively on the cost of adjusting prices.

Utility of household i is given by:

$$\begin{aligned} U_{i,t} = E_t \sum_{k=0}^{\infty} \beta^k \{ & \frac{1}{1-\gamma} C_{i,t+k}^{1-\gamma} - \psi X_{t+k}^{1-\gamma} \frac{1}{1-n} L_{i,t+k}^{1-n} \\ & - \frac{c}{2} X_{t+k}^{1-\gamma} [\ln P_{i,t+k} - \ln P_{i,t+k-1}]^2 \} \end{aligned} \tag{A4}$$

where E_t is the conditional expectations operator given information up to time t, $\beta \in (0,1)$ is a discount factor, γ is the coefficient of relative risk aversion (which, with power utility is equal to the reciprocal of the elasticity of intertemporal substitution, $\gamma = 1/\sigma$), n is the elasticity of labor supply w.r.t. real wages and ψ and c are positive constants with c depending positively on the cost of price adjustments.

As in Rotemberg (1996), we add a multiplicative productivity shock in the two disutility terms in order to ensure that technological progress does not lead to a secular decrease in labor input.[21]

At the beginning of period t and prior to any trading, the money supply and productivity shocks are observed. A currency transfer takes place of size $T_{i,t}$. Later, in deriving explicit solutions to the model, we assume that the cash transfer $T_{i,t}$ is a function of last period's money stock, $M_{i,t-1}$ and a current shock $\epsilon_{m,t}$. Besides the currency transfer, the household also carries wealth from earlier periods in the form of currency, which originates from two sources. The first source is its income in period $t-1$, $P_{i,t-1}Y_{i,t-1}$. The second source is the gross interest received from zero coupon bonds it had bought during the earlier periods, $t-1, t-2, ..., t-N$. At the time they were acquired, these bonds, $B_{i,1,t-1}$, $B_{i,2,t-2}$, ..., $B_{i,N,t-N}$, had an original maturity of $1, 2, ..., N$ periods respectively. The gross nominal interest received at time t is $\sum_{\tau=1}^{N} R_{\tau,t-\tau}B_{i,\tau,t-\tau}$, where $R_{\tau,t-\tau}$ is the gross nominal interest rate (not annualized) of bond $B_{i,\tau,t-\tau}$.

Following Lucas (1978, 1982), the exchange of money, bonds and goods takes place in two phases. In the first phase of trading, the household divides its post-transfer wealth among bonds, $B_{i,\tau,t}$, maturing at $\tau = t+1, t+2, ..., t+N$, and cash, $M_{i,t}$. Thus,

$$M_{i,t} + \sum_{\tau=1}^{N} B_{i,\tau,t} = P_{i,t-1}Y_{i,t-1} + T_{i,t} + \sum_{\tau=1}^{N} R_{\tau,t-\tau}B_{i,\tau,t-\tau} \tag{A5}$$

Goods trading takes place in the second phase. In this phase, the household must finance its consumption purchases, $C_{i,t}$,with the currency it accumulated previously, so that

$$C_{i,t} \leq M_{i,t}/P_t$$

As is typical in this literature, we assume that since money has no other use than facilitating transactions of goods, the cash-in-advance constraint is binding, so that real money balances acquired during the previous periods determine consumption. Also, there is no money left over in the form of wealth to be carried into period t+1, an assumption which was already incorporated in earlier equation (A.5):

$$C_{i,t} = M_{i,t}/P_t \tag{A6}$$

Next, using equations (A5) and (A6), we can write the equation for consumption as:

[21] See Rotemberg (1996) p. 509 for a discussion.

$$C_{i,t} = \frac{P_{i,t-1}}{P_t} Y_{t-1} \left(\frac{P_{t-1}}{P_{i,t-1}} \right)^{\frac{1}{1-q}} + \frac{T_{i,t}}{P_t} + \sum_{\tau=1}^{N} R_{\tau,t-\tau} \frac{B_{i,\tau,t-\tau}}{P_t} - \sum_{\tau=1}^{N} \frac{B_{i,\tau,t}}{P_t} \tag{A7}$$

Substituting equations (A2), (A3) and (A7) into (A4), we obtain:

$$\begin{aligned} U_{i,t} = E_t \sum_{k=0}^{\infty} \beta^k \frac{1}{1-\gamma} [& \frac{P_{i,t+k-1}}{P_{t+k}} Y_{t+k-1} \left(\frac{P_{t+k-1}}{P_{i,t+k-1}} \right)^{\frac{1}{1-q}} \\ & + \frac{T_{i,t+k}}{P_{t+k}} + \sum_{\tau=1}^{N} R_{\tau,t+k-\tau} \frac{B_{i,\tau,t+k-\tau}}{P_{t+k}} - \sum_{\tau=1}^{N} \frac{B_{i,\tau,t+k}}{P_{t+k}}]^{1-\gamma} \\ & - E_t \sum_{k=0}^{\infty} \beta^k \psi X_{t+k}^{1-\gamma} \frac{1}{1-n} \left(\frac{Y_{t+k}}{X_{t+k}} \right)^{1-n} \left(\frac{P_{t+k}}{P_{i,t+k}} \right)^{\frac{1-n}{1-q}} \\ & - E_t \sum_{k=0}^{\infty} \beta^k \frac{c}{2} X_{t+k}^{1-\gamma} [\ln P_{i,t+k} - \ln P_{i,t+k-1}]^2 \end{aligned} \tag{A8}$$

Maximizing (A.8) w.r.t. $B_{i,\tau,t}$ and $P_{i,t}$ leads to the following optimality conditions, evaluated at the symmetric equilibrium where $P_{i,t} = P_t$ and $Y_{i,t} = Y_t$:

$$E_t \left[\beta^\tau \left(\frac{C_{t+\tau}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+\tau}} R_{\tau,t} \right] = 1, \qquad \tau = 1, ..., N \tag{A9}$$

$$\begin{aligned} 0 = E_t [& - \beta \frac{q}{1-q} \frac{P_t}{P_{t+1}} \left(\frac{M_t}{P_t X_t} \right)^{1-\gamma} + \psi \frac{1}{1-q} \left(\frac{M_t}{P_t X_t} \right)^{1-n} \\ & + c \left(\ln P_t - \ln P_{t-1} \right) - \beta c \left(\frac{X_{t+1}}{X_t} \right)^{1-\gamma} \left(\ln P_{t+1} - \ln P_t \right)] \end{aligned} \tag{A10}$$

Equation (A9) is the well-known asset pricing formula of Consumption-CAPM. We assume that the joint conditional distribution of consumption and consumption prices is i.i.d. lognormal. Thus, defining the continuously compounded, annualized yield to maturity at time t on a nominal discount bond with term τ as $r_t(\tau) = \log(R_{\tau,t})/\tau$ and taking logs of equation (A.9) leads to equation (10) in the text. Note that, in general, the parameter $\theta(\tau)$ is time-varying,

$\theta(\tau)_t \equiv -\frac{1}{2\tau} \left[\gamma^2 var_t(c_{t+\tau} - c_t) \ + var_t(p_{t+\tau} - p_t) + 2\gamma cov_t(c_{t+\tau} - c_t, \ p_{t+\tau} - p_t) \right]$. However, when we loglinearize the optimality condition, we make the assumption that the joint conditional distribution of consumption and prices is i.i.d. lognormal. Hence, the conditional variance and covariance terms are constant and, as a result,

the term premium is equal to a constant $\theta(\tau) = -\frac{1}{2}\left[\gamma^2\sigma^2_{c,\tau} + \sigma^2_{p,\tau} + 2\gamma\sigma_{cp,\tau}\right]$, where $\sigma_{c,\tau}$ and $\sigma_{p,\tau}$ are the variance of $c_{t+\tau} - c_t$ and $p_{t+\tau} - p_t$ and $\sigma_{cp,\tau}$ is their covariance.

Next, we log-linearize equation (A10) around the sample means of P_{t+1}/P_t, M_t/P_tX_t, X_{t+1}/X_t. Denoting these sample means $1+\pi$, M/PX, $1+g$ and ignoring constants, the loglinearized version of equation (A10) reads:

$$\begin{aligned} E_t[\beta &\left(-c(1+g)^{1-\gamma} + \tfrac{q}{1-q}\tfrac{(M/PX)^{1-\gamma}}{(1+\pi)}\right)(p_{t+1} - p_t) + c(p_t - p_{t-1}) \\ &+ \left(\psi\tfrac{1-n}{1-q}(M/PX)^{1-n} - \beta(1-\gamma)\tfrac{q}{1-q}\tfrac{(M/PX)^{1-\gamma}}{(1+\pi)}\right)(m_t - p_t - x_t) \\ &-\beta c\pi(1-\gamma)(1+g)^{1-\gamma}(x_{t+1} - x_t)] = 0 \end{aligned} \tag{A11}$$

where lowercase letters, p_t, m_t, x_t denote logs of the upper case variables.

Equation (A11) is a second-order difference equation in p_t. As in Rotemberg (1982, 1986), this equation has a unique, nonexplosive solution if one of the two roots of the characteristic equation is smaller than one while the other is larger than one. Thus, the solution to equation (A11) is equation (11) in the text, where α is the root smaller than one and $1/\delta$ is the other root of the characteristic equation.

Appendix B: Implied slope coefficients of multi-period regressions of GDP growth and inflation on the real yield spread from VAR(1)

The OLS slope coefficient of regression (25) is:

$$a_{1,k} = \frac{Cov\left(s_t - \left[\frac{1}{40}\sum_{j=1}^{40} E_t\Delta p_{t+j} - E_t\Delta p_{t+1}\right], \frac{1}{k}\sum_{j=1}^{k} E_t\Delta y_{t+j}\right)}{Var\left(s_t - \left[\frac{1}{40}\sum_{j=1}^{40} E_t\Delta p_{t+j} - E_t\Delta p_{t+1}\right]\right)} \tag{B1}$$

The numerator can be expanded as:

$$\begin{aligned} &Cov\left(s_t - \left[\frac{1}{40}\sum_{j=1}^{40} E_t\Delta p_{t+j} - E_t\Delta p_{t+1}\right], \frac{1}{k}\sum_{j=1}^{k} \Delta y_{t+j}\right) = \\ &Cov\left(s_t, \frac{1}{k}\sum_{j=1}^{k} E_t\Delta y_{t+j}\right) \\ &-Cov\left(\left[\frac{1}{40}\sum_{j=1}^{40} E_t\Delta p_{t+j} - E_t\Delta p_{t+1}\right], \frac{1}{k}\sum_{j=1}^{k} E_t\Delta y_{t+j}\right) \end{aligned} \tag{B2}$$

Using the VAR model $z_{t+1} = A\ z_t + u_t$, where $z_{t+1} = [\Delta y_{t+1},\ \Delta p_{t+1},\ s_{t+1}]$, we can compute the unconditional variance of the z_t process as: $C(0) = \sum_{j=0}^{\infty} A^j V A^{j\prime}$. Also, the covariance between z_t and z_{t+j} is $C(0)A^{j\prime}$ and the covariance between z_t and $\frac{1}{k}\sum_{j=1}^{k} E_t z_{t+j}$ is $\frac{1}{k}C(0)[A + A^2 + ... + A^k]'$

Hence, the first term in the RHS of (B2) is given by $\frac{1}{k} i_1' C(0)[A + A^2 + ... + A^k]' i_3$.

The second term in the RHS of (B2) is:

$$
\begin{aligned}
&-Cov\left(\frac{1}{40}\sum_{j=1}^{40}E_t\Delta p_{t+j},\frac{1}{k}\sum_{j=1}^{k}E_t\Delta y_{t+j}\right)+Cov\left(E_t\Delta p_{t+1},\frac{1}{k}\sum_{j=1}^{k}E_t\Delta y_{t+j}\right)\\
=&-\frac{1}{40}\frac{1}{k}\sum_{j=1}^{40}Cov\left(E_t\Delta p_{t+j},\ \sum_{j=1}^{k}E_t\Delta y_{t+j}\right)\\
&+\frac{1}{k}Cov\left(i_2'Az_t,\ i_1'[A+A^2+...+A^k]z_t\right)\\
=&-\frac{1}{40}\frac{1}{k}\sum_{j=1}^{40}Cov\left(i_2'A^jz_t,\ i_1'[A+A^2+...+A^k]z_t\right)\\
&+\frac{1}{k}\left(i_2'AC(0)[A+A^2+...+A^k]'\ i_1\right)\\
=&-\frac{1}{40}\frac{1}{k}\sum_{j=1}^{40}\left(i_2'A^jC(0)[A+A^2+...+A^k]'i_1\right)\\
&+\frac{1}{k}\left(i_2'AC(0)[A+A^2+...+A^k]'\ i_1\right)
\end{aligned}
$$

Next, the denominator of (B1) is:

$$
\begin{aligned}
&Var\left(s_t-\frac{1}{40}\sum_{j=1}^{40}E_t\Delta p_{t+j}+E_t\Delta p_{t+1}\right)\\
=&Var\left(i_3'z_t-\frac{1}{40}\sum_{j=1}^{40}i_2'A^jz_t+i_2'Az_t\right)\\
=&Var\left(i_3'z_t+i_2'\left[A-\frac{1}{40}\sum_{j=1}^{40}A^j\right]z_t\right)\\
=&\left[i_3'I_{3\times3}+i_2'\left[A-\frac{1}{40}\sum_{j=1}^{40}A^j\right]\right]C(0)\left[i_3'I_{3\times3}+i_2'\left[A-\frac{1}{40}\sum_{j=1}^{40}A^j\right]\right]'
\end{aligned}
$$

The OLS slope coefficient of regression (26):

$$
\beta_{1,k}=\frac{Cov\left(s_t-\left[\frac{1}{40}\sum_{j=1}^{40}E_t\Delta p_{t+j}-E_t\Delta p_{t+1}\right],\frac{1}{k}\sum_{j=1}^{k}E_t\Delta p_{t+j}\right)}{Var\left(s_t-\left[\frac{1}{40}\sum_{j=1}^{40}E_t\Delta p_{t+j}-E_t\Delta p_{t+1}\right]\right)}
$$

can be derived in a similar way.

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Table 1: Descriptive Statistics

Panel A:
Summary Statistics, 1960:Q1 - 2004:Q2

variable	mean	variance	skewness	kurtosis
GDP growth	3.33	5.01	-0.44	0.17
Inflation	4.21	7.83	1.36	1.41
3-m T-bill	5.66	7.72	1.12	2.12
3-y yield	6.69	7.44	0.95	1.05
5-y yield	6.91	6.94	0.97	0.84
10-y yield	7.12	6.46	0.95	0.59
3y-3m spread	1.03	0.63	0.02	0.56
5y-3m spread	1.25	1.01	-0.12	-0.07
10y-3m spread	1.46	1.57	-0.16	-0.45

Panel B:
Correlations, 1960:Q1 - 2004:Q2

	GDP growth	Inflation	3y-3m spread	5y-3m spread	10y-3m spread
GDP growth	1.00	−0.40	0.41	0.42	0.44
Inflation		1.00	−0.28	−0.32	−0.34
3y-3m spread			1.00	0.99	0.97
5y-3m spread				1.00	0.99
10y-3m spread					1.00

Panel C:
Vector Autoregression Estimates, 1960:Q1 - 2004:Q2

	Δy_t	Δp_t	s_t	$\overline{R^2}$
Δy_{t+1}	0.20^{*} (2.87)	-0.26^{*} (−3.08)	$0.36^{\dagger}$ (1.85)	0.15
Δp_{t+1}	0.06 (1.49)	0.76^{*} (15.37)	-0.31^{*} (−2.57)	0.63
s_{t+1}	-0.03^{*} (−2.17)	−0.01 (−0.50)	0.85^{*} (19.68)	0.71

Panel D:
Tests of cointegrating rank, 1960:Q1 - 2004:Q2

λ	H_0	$\lambda-$max	99% cv	Trace	99% cv
0.32	$r = 0$	66.25	25.52	95.25	35.65
0.11	$r \leq 1$	19.63	18.63	29.00	20.04
0.05	$r \leq 2$	9.36	6.65	9.36	6.65

Notes: In Panels A and B, GDP growth is the four-quarter ahead difference of quarterly log GDP. Inflation is the four-quarter ahead difference of the log of the mid-quarter CPI. 3-m T-Bill is the 3-month Treasury Bill rate. Bond yields are yields to maturity of 3-year, 5-year, and 10-year Treasury bonds. All yields are annualized and represent average values for the second month of the quarter. Yield spreads are calculated over the 3-month T-Bill rate.

In Panel C, a first-order VAR is estimated for (i) real quarterly GDP growth, Δy_{t+1}, (ii) quarterly consumer price inflation, Δp_{t+1}, and (iii) the 10-year minus the 3-month nominal yield spread, s_{t+1}. All variables in the VAR are annualized and de-meaned.

In Panel D, we report Johansen's test for cointegration between the three variables of the VAR: $(\Delta y_{t+1},\ \Delta p_{t+1}, s_{t+1})$. Column "$\lambda$" reports the eigenvalues of the long-run Π matrix of the Error Correction representation, columns "$\lambda-$max" and "Trace" report Johansen's (1988) Likelihood Ratio tests of the null hypothesis stated in column H_0, where r is the rank of the Π matrix. Column "99% cv" reports 99% critical values.

$^{*}(^{\dagger})$ denotes significance at the 5% (10%) level.

Table 2: Regressions of k−quarter-ahead cumulative real GDP growth and inflation on the current nominal yield spread

$$100(\tfrac{4}{k})(y_{t+k} - y_t) = a_{0,k} + a_{1,k}s_t + u_{y,t+k}$$
$$100(\tfrac{4}{k})(p_{t+k} - p_t) = b_{0,k} + b_{1,k}s_t + u_{p,t+k}$$

SUR system estimates, 1960:Q1 - 2004:Q2

k	$a_{1,k}$	$b_{1,k}$	$\overline{R_y^2}$	$\overline{R_p^2}$	W	$a_{1,k} + b_{1,k}$
1	0.65^* (3.21) $\{-0.40,\ 0.40\}$	-0.88^* (-5.12) $\{-0.62,\ 0.65\}$	0.06	0.16	1.68 [0.19]	-0.23 $\{-0.76,0.79\}$
2	0.80^* (3.73) $\{-0.39,\ 0.39\}$	-0.83^* (-3.18) $\{-0.61,\ 0.64\}$	0.13	0.15	0.16 [0.69]	-0.03 $\{-0.74,0.77\}$
3	0.82^* (3.48) $\{-0.37,\ 0.38\}$	-0.76^* (-2.55) $\{-0.60,\ 0.63\}$	0.18	0.14	0.00 [0.97]	0.06 $\{-0.73,0.75\}$
4	0.81^* (3.33) $\{-0.36,\ 0.37\}$	-0.73^* (-2.23) $\{-0.59,\ 0.63\}$	0.20	0.12	0.01 [0.93]	0.08 $\{-0.72,0.73\}$
5	0.78^* (3.24) $\{-0.36,\ 0.36\}$	-0.68^* (-2.00) $\{-0.59,\ 0.62\}$	0.22	0.11	0.02 [0.87]	0.10 $\{-0.72,0.72\}$
6	0.70^* (3.13) $\{-0.35,\ 0.35\}$	-0.60^* (-1.82) $\{-0.59,\ 0.61\}$	0.21	0.09	0.04 [0.84]	0.10 $\{-0.71,0.70\}$
7	0.64^* (2.97) $\{-0.35,\ 0.33\}$	-0.51 (-1.69) $\{-0.58,\ 0.60\}$	0.20	0.07	0.06 [0.80]	0.13 $\{-0.70,0.70\}$
8	0.55^* (2.80) $\{-0.37,\ 0.38\}$	-0.44 (-1.60) $\{-0.57,\ 0.58\}$	0.17	0.05	0.08 [0.78]	0.11 $\{-0.68,0.68\}$

Notes: y_t : log real GDP. p_t : log CPI of the middle month of the quarter. s_t: 10-year minus 3-month average nominal annualized yield spread of the middle month of the quarter. Columns $a_{1,k}$ and $b_{1,k}$ report bias-adjusted estimates of the slope coefficients. The bias is computed as the mean of the distribution of the coefficients using 5,000 bootstrap simulations of the VAR under the null hypothesis that inflation and real output follow their historical autoregressive pattern, but cannot be

predicted by the nominal yield spread. Inside the parentheses below the coefficient estimates are Newey and West t-statistics, which take into account the conditional heteroskedasticity and autocorrelation in the residuals up to $k-1$ lags. Inside the curly brackets { } below the t-statistics are the 5% and 95% fractiles of the distribution of coefficients from the bootstrap simulations. Columns $\overline{R_y^2}$ ($\overline{R_p^2}$) report the adjusted R^2. Column W reports Wald statistics, which are distributed as $\chi^2(1)$, of the null hypothesis of symmetry, $H_0 : b_{1,k} = -a_{1,k}$, i.e. that the nominal yield spread predicts opposite cumulative changes in real log GDP and log prices. The numbers in square brackets below the Wald statistics are asymptotic p-values. The last column reports the sum of the estimated coefficients. Inside the curly brackets below the estimates are the 5% and 95% fractiles of the distribution of $a_{1,k} + b_{1,k}$ from the bootstrap simulations. $^*(^\dagger)$ denotes significance at the 5% (10%) level based on the simulated distributions of coefficients.

Table 3: Regressions of one-quarter-ahead real GDP growth and inflation on the average nominal yield spread of the current and k − 1 previous quarters

$100(4)\Delta y_{t+1} = c_0 + c_1 s_{t,k} + e_{y,t+1}$

$100(4)\Delta p_{t+1} = d_0 + d_1 s_{t,k} + e_{p,t+1}$

SUR System estimates, 1960:1-2004:2

k	$c_{1,k}$	$d_{1,k}$	$\overline{R_y^2}$	$\overline{R_p^2}$	W	$c_{1,k} + d_{1,k}$
1	0.65^* (2.46) $\{-0.40,\ 0.40\}$	-0.88^* (-2.38) $\{-0.62,\ 0.65\}$	0.06	0.16	0.82 [0.36]	-0.23 $\{-0.76,0.79\}$
2	0.88^* (2.92) $\{-0.42,\ 0.43\}$	-0.89^* (-2.27) $\{-0.67,\ 0.70\}$	0.10	0.14	0.08 [0.78]	-0.01 $\{-0.82,0.84\}$
3	0.98^* (2.98) $\{-0.44,\ 0.45\}$	-0.88^* (-2.13) $\{-0.70,\ 0.74\}$	0.11	0.13	0.01 [0.93]	0.10 $\{-0.86,0.88\}$
4	1.02^* (2.99) $\{-0.46,\ 0.46\}$	-1.02^* (-2.06) $\{-0.75,\ 0.78\}$	0.12	0.12	0.00 [0.99]	0.00 $\{-0.91,0.93\}$
5	1.04^* (2.90) $\{-0.48,\ 0.48\}$	-1.00^* (-1.98) $\{-0.78,\ 0.82\}$	0.11	0.11	0.04 [0.84]	0.03 $\{-0.99,0.98\}$
6	1.01^* (2.79) $\{-0.49,\ 0.49\}$	-0.95^* (-1.85) $\{-0.82,\ 0.86\}$	0.10	0.09	0.04 [0.84]	0.06 $\{-1.00,1.02\}$
7	0.98^* (2.71) $\{-0.50,\ 0.50\}$	-0.88^* (-1.72) $\{-0.85,\ 0.89\}$	0.08	0.07	0.06 [0.80]	0.10 $\{-1.04,1.05\}$
8	0.90^* (2.60) $\{-0.52,\ 0.52\}$	$-0.80^\dagger$ (-1.59) $\{-0.89,\ 0.91\}$	0.07	0.06	0.06 [0.80]	0.10 $\{-1.08,1.08\}$

Notes: See the notes of Table 2. Δy_{t+1} : annualized one-quarter-ahead real GDP growth, Δp_{t+1} : annualized one-quarter-ahead CPI inflation, $s_{t,k}$: average yield spread between time $t+1-k$ and t ($s_{t,k} = \frac{1}{k}\sum_{i=0}^{k-1} s_{t-i}$). Here the Newey-West t-statistics correct for autocorrelation in the residuals up to 4 lags. All coefficient estimates are bias-adjusted.

Table 4: Implied slope coefficients for hypothetical multiperiod regressions of cumulative annualized future output growth and inflation on the current nominal yield spread calculated from the estimates of a VAR(1)

Sample: 1960:Q1 - 2004:Q2

k	$a_{1,k}$	$b_{1,k}$	$a_{1,k} + b_{1,k}$
1	0.65^* {−0.38, 0.38}	-0.89^* {−0.63, 0.67}	-0.24 {−0.78, 0.77}
2	0.66^* {−0.36, 0.36}	-0.90^* {−0.61, 0.65}	-0.24 {−0.76, 0.74}
3	0.65^* {−0.34, 0.33}	-0.89^* {−0.60, 0.63}	-0.24 {−0.73, 0.71}
4	0.62^* {−0.31, 0.31}	-0.88^* {−0.59, 0.60}	-0.26 {−0.70, 0.69}
5	0.60^* {−0.29, 0.29}	-0.85^* {−0.57, 0.59}	-0.25 {−0.67, 0.66}
6	0.57^* {−0.27, 0.27}	-0.82^* {−0.55, 0.57}	-0.25 {−0.64, 0.63}
7	0.54^* {−0.26, 0.26}	-0.79^* {−0.53, 0.54}	-0.25 {−0.62, 0.60}
8	0.51^* {−0.24, 0.24}	-0.75^* {−0.51, 0.53}	-0.24 {−0.60, 0.58}

Notes: See Table 1, Panel C, where the VAR(1) is estimated for the de-meaned Δy_t, Δp_t, s_t, with s_t representing the spread between the 10-year and the 3-month nominal yield. The implied estimates of multiperiod regression slope coefficients are computed as in equations (5), (6) of the text. The table reports bias-adjusted estimates. The bias has been computed as the mean of the distribution of the coefficients using 5,000 bootstrap simulations of the VAR under the null hypothesis that inflation and real output follow their historical autoregressive pattern, but cannot be predicted by the nominal yield spread. The numbers inside the curly brackets are the 5%- and 95%-fractiles of the distribution of the slope coefficients, which are based on the same 5,000 bootstrap simulations of the VAR. Column $a_{1,k} + b_{1,k}$ reports the sum of the bias-adjusted slope coefficients, with its 5% and 95% fractiles in curly brackets. $^*(^\dagger)$ denotes significance at the 5% (10%) level based on the simulated distributions of coefficients.

Table 5: The forecasting ability of the nominal yield spread during the subperiods

Sub-sample SUR results for horizon $k = 4$

$$100(y_{t+4} - y_t) = a_{0,4} + a_{1,4}s_t + u_{y,t+k}$$
$$100(p_{t+4} - p_t) = b_{0,4} + b_{1,4}s_t + u_{p,t+k}$$

	1960:1969	1970:1979	1980:1989	1990:2004
$a_{1,4}$	1.96^* (2.38)	1.74^* (9.88)	1.12^* (6.90)	0.21 (0.60)
$b_{1,4}$	-2.06^* (-2.90)	-1.53^* (-4.14)	-0.90^* (-4.20)	0.01 (0.01)
$\overline{R_y^2}$	0.22	0.70	0.49	0.04
$\overline{R_p^2}$	0.41	0.44	0.39	0.02
$W : b_{1,4} = -a_{1,4}$	0.02 [0.87]	0.32 [0.57]	0.46 [0.49]	0.40 [0.52]

Notes: See the notes of Table 2 for the definitions of variables. Inside the parentheses below the coefficient estimates are Newey and West t-statistics, which take into account the conditional heteroskedasticity and autocorrelation in the residuals up to 3 lags. Rows $\overline{R_y^2}$ ($\overline{R_p^2}$) report the adjusted R^2. Row W reports Wald statistics, which are distributed as $\chi^2(1)$, of the null hypothesis of symmetry, $H_0 : b_{1,4} = -a_{1,4}$, i.e. that the nominal yield spread predicts opposite cumulative changes in real log GDP and log prices. The numbers in square brackets below the Wald statistics are asymptotic p-values. $^*(^\dagger)$ denotes significance at the 5% (10%) level.

Table 6: Controlling for other sources of predictable variation in the k−quarter-ahead cumulative real GDP growth and inflation

$$100(\tfrac{4}{k})(y_{t+k} - y_t) = a_{0,k} + a_{1,k}s_t + a_{2,k}(100(\tfrac{4}{k})(y_t - y_{t-k})) + a_{3,k}r(1)_t + u_{y,t+k}$$
$$100(\tfrac{4}{k})(p_{t+k} - p_t) = b_{0,k} + b_{1,k}s_t + b_{2,k}(100(\tfrac{4}{k})(p_t - p_{t-k})) + b_{3,k}r(1)_t + u_{p,t+k}$$

SUR system estimates, 1960:Q1 - 2004:Q2

k	$a_{1,k}$	$b_{1,k}$	$\overline{R_y^2}$	$\overline{R_p^2}$	W	$a_{1,k} + b_{1,k}$
1	$0.33^{\dagger}$ (1.85) $\{-0.35,\ 0.35\}$	-0.23^{*} (-1.80) $\{-0.21,\ 0.22\}$	0.16	0.64	0.17 [0.68]	0.10 $\{-0.42,0.42\}$
2	0.49^{*} (2.51) $\{-0.38,\ 0.38\}$	$-0.26^{\dagger}$ (-2.01) $\{-0.32,\ 0.32\}$	0.28	0.70	1.41 [0.23]	0.23 $\{-0.51,0.51\}$
3	0.55^{*} (2.40) $\{-0.39,\ 0.38\}$	$-0.31^{\dagger}$ (-2.10) $\{-0.40,\ 0.41\}$	0.31	0.71	2.55 [0.11]	0.24 $\{-0.59,0.57\}$
4	0.57^{*} (2.27) $\{-0.39,\ 0.39\}$	$-0.33^{\dagger}$ (-2.01) $\{-0.45,\ 0.46\}$	0.33	0.64	2.29 [0.13]	0.24 $\{-0.63,0.61\}$
5	0.56^{*} (2.08) $\{-0.39,\ 0.37\}$	$-0.38^{\dagger}$ (-2.15) $\{-0.49,\ 0.50\}$	0.33	0.57	1.34 [0.25]	0.18 $\{-0.65,0.64\}$
6	0.54^{*} (2.00) $\{-0.38,\ 0.36\}$	-0.39 (-2.40) $\{-0.53,\ 0.53\}$	0.30	0.51	1.13 [0.29]	0.15 $\{-0.68,0.67\}$
7	$0.50^{\dagger}$ (1.80) $\{-0.38,\ 0.35\}$	-0.40 (-2.75) $\{-0.56,\ 0.56\}$	0.27	0.45	0.55 [0.46]	0.10 $\{-0.70,0.68\}$
8	0.43 (1.42) $\{-0.37,\ 0.35\}$	-0.42 (-3.06) $\{-0.56,\ 0.57\}$	0.17	0.05	0.08 [0.92]	0.11 $\{-0.70,0.68\}$

Notes: y_t : log real GDP. p_t : log CPI of the middle month of the quarter. s_t: 10-year minus 3-month average nominal annualized yield spread of the middle month of the quarter. $r(1)_t$: 3-month nominal Treasury bill rate. Columns $a_{1,k}$ and $b_{1,k}$ report bias-adjusted estimates of the slope coefficients. The bias is computed as the mean of the distribution of the coefficients using 5,000 bootstrap simulations of the VAR

under the null hypothesis that inflation and real output follow their historical autoregressive pattern, but cannot be predicted by the nominal yield spread. Inside the parentheses below the coefficient estimates are Newey and West t-statistics, which take into account the conditional heteroskedasticity and autocorrelation in the residuals up to $k-1$ lags. Inside the curly brackets { } below the t-statistics are the 5% and 95% fractiles of the distribution of coefficients from the bootstrap simulations. Columns $\overline{R_y^2}$ ($\overline{R_p^2}$) report the adjusted R^2. Column W reports Wald statistics, which are distributed as $\chi^2(1)$, of the null hypothesis of symmetry, $H_0 : b_{1,k} = -a_{1,k}$, i.e. that the nominal yield spread predicts opposite cumulative changes in real log GDP and log prices. The numbers in square brackets below the Wald statistics are asymptotic p-values. The last column reports the sum of the estimated coefficients. Inside the curly brackets below the estimates are the 5% and 95% fractiles of the distribution of $a_{1,k} + b_{1,k}$ from the bootstrap simulations. $^*(^\dagger)$ denotes significance at the 5% (10%) level based on the simulated distributions of coefficients.

Table 7: Implied slope coefficients for hypothetical multiperiod regressions of cumulative annualized future output growth and inflation on the current *real* yield spread calculated from the estimates of the earlier VAR(1)

Sample: 1960:Q1 - 2004:Q2

k	$\alpha_{1,k}$	$\beta_{1,k}$	$\alpha_{1,k} + \beta_{1,k}$
1	-0.31^{*}	$0.17^{\dagger}$	-0.14
	{−0.21, 0.22}	{−0.21, 0.19}	{−0.30, 0.29}
2	-0.32^{*}	0.19^{*}	-0.13
	{−0.18, 0.18}	{−0.17, 0.16}	{−0.25, 0.25}
3	-0.30^{*}	0.16^{*}	-0.14
	{−0.16, 0.17}	{−0.16, 0.15}	{−0.24, 0.23}
4	-0.27^{*}	0.18^{*}	-0.09
	{−0.14, 0.15}	{−0.16, 0.15}	{−0.22, 0.22}
5	-0.24^{*}	0.11	-0.13
	{−0.13, 0.14}	{−0.16, 0.15}	{−0.22, 0.21}
6	-0.21^{*}	0.08	-0.13
	{−0.12, 0.13}	{−0.16, 0.15}	{−0.21, 0.20}
7	-0.19^{*}	0.06	-0.13
	{−0.11, 0.12}	{−0.15, 0.14}	{−0.20, 0.19}
8	-0.18^{*}	0.05	-0.13
	{−0.10, 0.11}	{−0.15, 0.14}	{−0.19, 0.18}

Notes: See Table 1, Panel C, where the VAR(1) is estimated for the de-meaned Δy_t, Δp_t, s_t, with s_t representing the spread between the 10-year and the 3-month nominal annualized yield. The implied estimates of multiperiod regression slope coefficients are computed as in equations (28), (29) of the text. The table reports bias-adjusted estimates. The bias has been computed as the mean of the distribution of the coefficients using 5,000 bootstrap simulations of the VAR under the null hypothesis that inflation and real output follow their historical autoregressive pattern, but cannot be predicted by the nominal yield spread. Inside the curly brackets, we report the 5% and 95% fractiles of the distribution of the slope coefficients, which are based on 5,000 bootstrap simulations of the VAR of Table 4, under the null hypothesis that inflation and real output follow their historical autoregressive pattern, but cannot be predioncted by the nominal yield spread. Column $\alpha_{1,k} + \beta_{1,k}$ reports the sum of the slope coefficients. Inside the curly brackets, we report the 5% and 95% fractiles of the distribution of $\alpha_{1,k} + \beta_{1,k}$ from bootstrab simulations of 5,000 runs. $^{*}(^{\dagger})$ denotes significance at the 5% (10%) level based on the simulated distributions of coefficients.

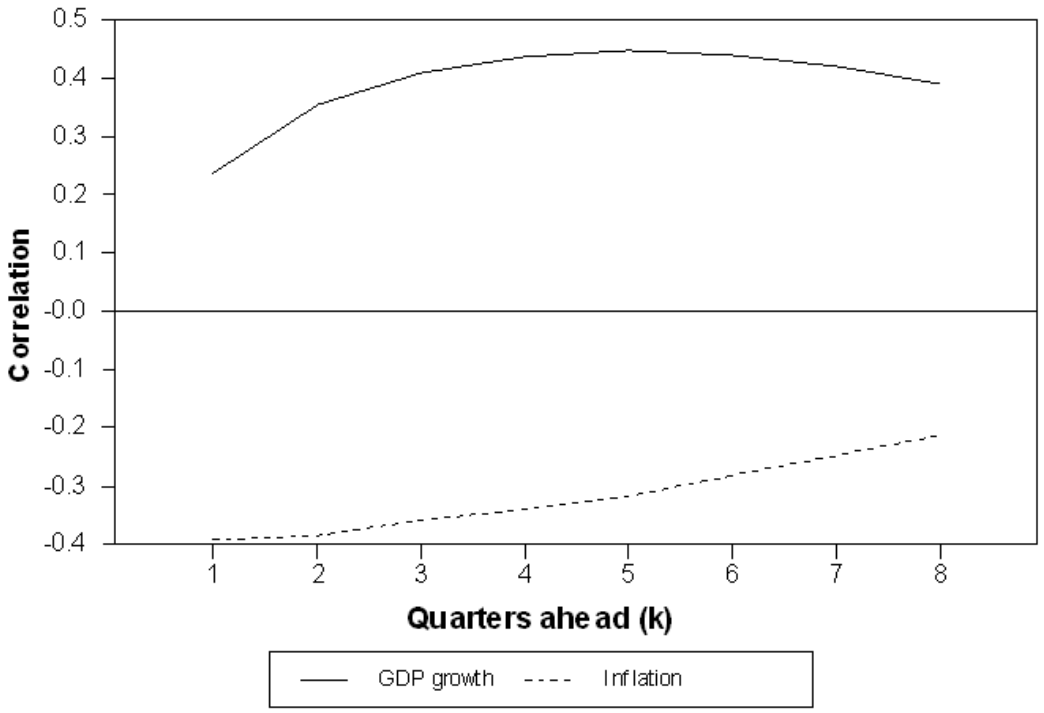

Figure 1: Sample correlations with the current 10-year minus 3-month interest rate spread: 'GDP growth' denotes the correlation of the $k-$quarters ahead annualized real GDP growth. 'Inflation' denotes the correlation of the $k-$quarters ahead annualized change in the Consumer Price Index.

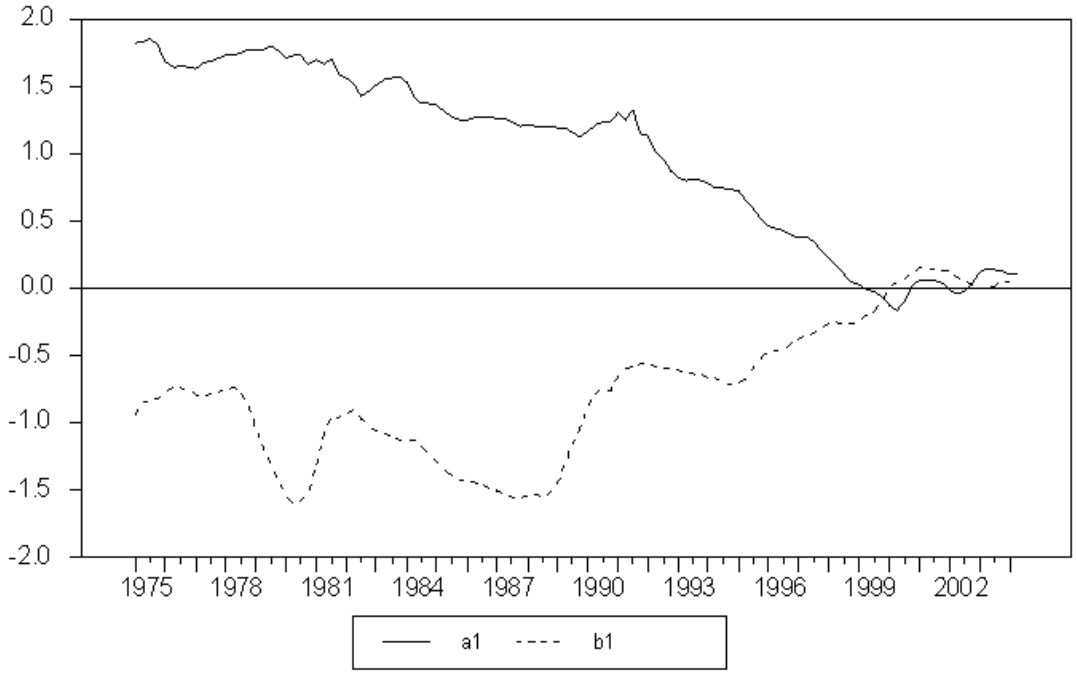

Figure 2: Recursive slope estimates of predictive regressions for one-year ahead real GDP growth ("`a1`") and inflation ("`b1`") on the spread between the 10-year and the 3-month yield, using a moving window of width = 40 quarters.

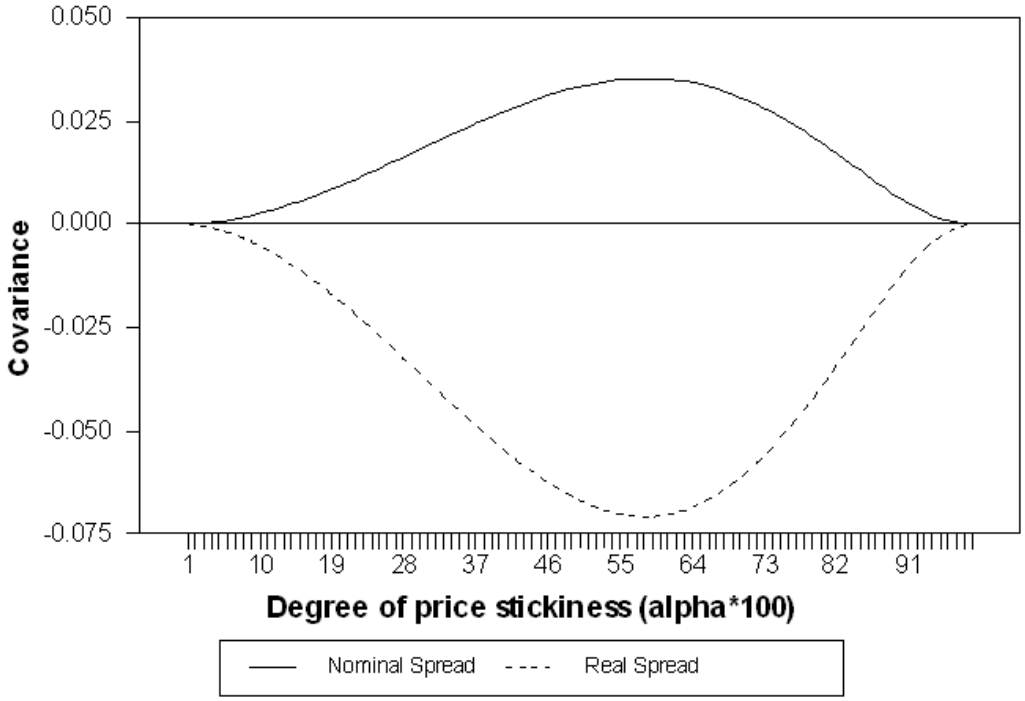

Figure 3: Theoretical covariance between one-year-ahead GDP growth and the 10-year minus 3-month yield spread for $\sigma = 1.5$, $\sigma_\varepsilon^2 = 3.6$. The straight line is the covariance between GDP growth and the nominal spread. The dotted line is the covariance between GDP growth and the real spread. The horizontal axis measures the degree of price stickiness α ($\times 100$).