Inflation Targets as Focal Points*

 ${\rm Maria~Demertzis^{\dagger}}$ De Nederlandsche Bank and University of Amsterdam ${\rm Nicola~Viegi^{\dagger}}$ University of KwaZulu-Natal

November 29, 2004

Abstract

The benefits of inflation targeting by comparison to alternative regimes are understood to be in terms of providing clearer objectives that help pin down private sector expectations in the long run. We argue that the mechanism for achieving this rests on the fact that monetary policy can be perceived as a matching game in which private agents aim to coordinate their expectations and thus benefit from a clearly given signal that acts as a focal point. We therefore, argue that first, the credibility of the signal achieves coordination and second that the clarity of the signal achieves welfare improvements. To demonstrate that we use Bacharach's, Variable Universe Game framework, which allows for individuals' understanding and interpretation of the signal to differ. As private agents benefit from coordination, they rely a lot on the public signal given. Following this, the Central Bank can then help increase the welfare of agents by providing clear and precise signal that increase the chance of coordination and hence increased welfare.

JEL codes: C71, C78, E52

Keywords: inflation targeting, high order expectations, matching games,

focal points

^{*}Views expressed are our own and do not necessarily reflect those of the DNB. Without wishing to implicate, we would like to thank Wilko Bolt, Lex Hoogduin, Andrew Hughes Hallett and seminar participants at DNB and the University of Amsterdam for comments and suggestions. The usual disclaimer applies.

[†]Corresponding author: m.demertzis@dnb.nl, Research Division, De Nederlandsche Bank, P.O. Box 98, 1000 AB, Amsterdam, the Netherlands, tel: +31 (20) 524 2016, fax: +31 (20) 524 2529.

[‡]viegin@nu.ac.za, Economics Department, University of KwaZulu-Natal, King George V Avenue, Durban 400, South Africa, tel: +27 31 260 3558, fax: +27 31 260 2811.

1 Introduction

Modern monetary policy theory emphasizes the central role of private sector expectations in determining policy outcomes. As argued by Woodford "...insofar as it is possible for the Central Bank to affect expectations, this should be an important tool of stabilisation policy..." (Woodford 2003). It is thus widely acknowledged, that the success of maintaining a stable monetary environment depends crucially on the ability of the policy regime to control inflation expectations (Blinder et al, 2001). Evidence of that is shown by Orphanides and Williams (2004) in their analysis of US monetary policy history, where they argue that monetary policy failures are connected with changes in public sentiment about the future state of the economy. In other words, policy mistakes alone are not enough to produce long term negative effects on monetary stability. The practice of monetary policy in the past ten to fifteen years has thus concentrated on providing institutional set-ups that provide an explicit platform of information for expectations to be formed. The main features of such institutional set-ups are

- credible institutions, mainly through independence and the pursuit of the sole objective of price stability;
- clear policy frameworks, captured by well defined intermediate policy objectives and procedures, and finally,
- transparent policy making, implemented through publication and distribution of the information set used in the decision making process (inflation forecasts, modelling strategies, well defined assumptions) and a clear demonstration of accountability (publication of minutes, regular appearance in front of parliamentary committees and regular press conferences).

Practically every monetary policy authority nowadays defines its policies according to these criteria, emphasising one or another aspect, depending on preferences. The set-up of the twelve-country Euro area, for example, has emphasised the importance of building and sustaining credibility and independence from governments, as an instrument towards low expected inflation. In the US experience instead, credibility, independence but also flexibility in following multiple objectives has helped achieve a stable monetary environment. Alternatively, inflation targeting as implemented first, by the Reserve Bank of New Zealand and then the Bank of England, and increasingly more and more banks around the world, is understood to provide clear and immediate objectives for monetary policy. Inflation targeting practitioners argue that the main advantage of an explicit numerical inflation target is its ability to provide a focal point for private sector expectations. As Mervyn King (2004, p.4) has claimed for the UK case, inflation expectations have indeed been anchored to the pre-announced target. The ability of explicit quantitative targets to tie down expectations, is also confirmed by the empirical analysis of Levin et al (2004), Mishkin and

Schmidt-Hebbel (2001) and more recently by Fatás et al $(2004)^1$.

However, in analysing various monetary policy regimes (Svensson, 1999, 2003), we observe no difference in the way we approach inflation targeting by comparison to other regimes². There is thus no explicit analysis of the way the provision of a specific numerical target constitutes a better anchor for long term expectations. Is there a difference between giving a numerical target as opposed to an upper bound to desired inflation? And if there is a difference, what is the mechanism that achieves it? In our view, to be able to answer that, we need a more complex mechanism for the way expectations are formed.

The recent model put forward by Morris and Shin (2002a, 2002b) (and used in Amato et al, 2003 and Amato and Shin, 2003), constitutes such an attempt to identify first, how private agents form expectations based on both private as well as public information available to them, and second how policy makers (not exclusively Central Banks) can affect those expectations by providing greater or lesser information. We thus assume in this set-up that, when forming expectations, private agents care not only about their own views but also of other people's expectations as a way of confirming their own beliefs. And as this element of 'beauty contest', (based on Keynes, 1936), plays a greater role in expectations forming, signals provided by public institutions become tantamount to coordination devices. In fact Phelps (1983) noted that "...in order to reduce the price level (in relation to the accustomed trend), it is not sufficient that the central bank persuade each agent to reduce his private expectation of the money supply (in relation to the past trend) by the warranted amount. The prevalence of this expectation must be public knowledge - an accepted fact" (p.35). This therefore, implies that public information acquires a dual role - "...of conveying fundamentals information as well as serving as a focal point for beliefs" (Morris and Shin, 2002a, henceforth, MS). The questions that arise following this argument is therefore, what monetary policy regimes provide better signals and in which way those signals constitute focal points. The aim of this paper is to formalise the widely believed but little analysed benefits of inflation targeting in coordinating private individuals' expectations. To this end, we employ the Variable Universe Games approach put forward by Bacharach (1993).

We will argue therefore, that monetary policy is a game between policy makers and private agents in which two issues are important: 1) private agents care about the beliefs of others, very much à la Morris and Shin (2002a), and 2) inflation expectations are important to the current level of inflation and therefore, to the direct interest of the Central Bank. The first point implies that the monetary policy game is of a 'matching nature', in the sense that private agents' welfare improves when their expectations lie closer to the average. The second point in turn, implies that the Central Bank has an incentive to be transparent

 $^{^1\}mathrm{See}$ also Leiderman and Svensson (1995) and Bernanke et al (1999) for early experiences with inflation targeting.

² Kuttner 2004, also alludes to this fact. The benefits of inflation targeting as a coordination device have been discussed by Hughes Hallett and Viegi, 2002, but then in the context of two policy authorities, the policies of which might have strong "spillovers".

about its intentions in the hope that, the signal provided will be a focal point for agents to anchor their expectations at.

Both points from above imply that monetary policy can be viewed as a coordination game between the Central Bank and the public but also between the public themselves. The theory on coordination games provides valuable insight into the way that such games are resolved. For example, it is often observed that in matching games players coordinate much more frequently than by randomising (Casajus, 2000). Indeed, according to Wilson and Rhodes (1997), it is to the benefit of all actors to avoid the conflict that escalates as solutions are delayed. To achieve that, players rely heavily on salient features when deciding on their actions. And salience in this context, can be a "...social custom or convention, namely, a mode of behaviour that finds automatic acceptance" (Dixit and Skeath, 1999) and a salient item is "...one that stands out from the rest by its uniqueness in some conspicuous respect. Salience thus defined has two dimensions: conspicuousness or noticeability of some feature, and unique instantiation of this feature." (Bacharach 1993). Furthermore, Wilson and Rhodes (1997) argue that all that is required for such salience to be achieved is a signal from somebody that can be recognised as the "leader" in the game, to send a signal. The existence of a leader in a clearly defined leader-follower(s) game can thus provide such a focal point (Wilson and Rhodes, 1997). In our set up we will thus assume that the Central Bank sends a signal (of different degrees of precision), and only then do agents form expectations. The timing of the game, thus allows for it to be considered of a leader-follower sort. The precise model in which inflation is implemented is of no great relevance here as we concentrate on the way the signal affects (helps coordinate) private sector expectations.

The paper is organised as follows. In section 2, we describe how monetary policy can be described as a matching game between private sector agents. Section 3 then, provides Bacharach's (1993) framework for solving such matching games and section 4, applies it explicitly to monetary policy. We examine two monetary policy regimes differentiated by the degree of precision in the signal they provide. Section 5 concludes.

2 A Matching game

We describe the monetary policy game between the Central Bank and private individuals in terms of Bacharach 's (1993) "Variable Universe game", multiperson decision problem. The novelty of such an approach is that this framework allows for the fact that the way different players perceive the game may actually vary. This is in contrast to normal or extensive form games, in that the way players perceive their strategies is always unique. However, we believe that varying perceptions are very relevant in monetary policy and the inflation outcome. This is because agents form perceptions about what their actions should be based on their own understanding of the monetary policy regime but also on their reading of everybody else's views. Implicit in the analysis that follows is

therefore, a utility function based on that by Morris and Shin (2002a) in which there is a continuum of agents indexed by the unit interval [0, 1], i.e.

$$u_i(\mathbf{x}^e, x) \equiv -(1 - r)(x_i^e - x)^2 - r(L_i - \bar{L}) \tag{1}$$

where x_i^e is individual i's expectation of what the state will be and x is the ex post inflation outcome. We use \mathbf{x}^e to refer to the expectations profile over all agents. The objective of each individual i is thus to form expectations x_i^e which he will use in wage negotiations. But while she is interested in forming views about the underlying state, (first term in 1), at the same time she is also interested in second-guessing the views of other individuals in the economy. From the point of view of the individual, this constitutes an externality and is captured by the second term in (1) where,

$$L_{i} \equiv \int_{0}^{1} (x_{j}^{e} - x_{i}^{e})^{2} dj$$

$$\bar{L} \equiv \int_{0}^{1} L dj$$

Parameter r represents how strong this externality weighs on the utility function and is assumed to be constant, 0 < r < 1. As MS show, optimising (1) produces individual agents' actions given by

$$x_i^e = (1 - r)E_i(x) + rE_i(\bar{x}^e)$$
 (2)

The interesting feature of (2) is that it allows for the underlying state x, to be interpreted by the individual agents. And in doing so, they form their own views about what the state is but also of what everybody else thinks the state is (higher order expectations). In that respect, the game can arguably be considered of a 'matching' nature, in the sense that identical actions are preferable. Naturally, if x were common knowledge, then (2) would collapse to $x_i^e = x$. Note that we will not model how private information weighs against public information, as our objective is to show how the provision of a signal provides a focal point. In that respect, we will show, in a game-theoretic set up, how the signal generates the notion of collectively rational outcomes, based on Sugden's (1995) definition³. Further to that, we will show that actual coordination, will depend crucially, on every individual's perception about the credibility of the signal provided⁴.

³p. 542, "I shall say that a recommendation A^* is collectively rational if there exists a pair of utilities, $u^* = (u_1^*, u_2^*)$ such that (i) if each player i chooses a decision rule from A_i^* , the outcome in expected utility terms is (u_1^*, u_2^*) , and (ii) if either player j chooses a decision rule that is not in A_j^* , then whatever the decision rule followed by the other player, player 1's expected utility is strictly less than u_1^* and player 2's expected utility is strictly less than u_2^* ."

⁴MS show that as individual agents have an incentive to coordinate, they rely on the public signal available, to provide a focal point. Note that MS argue that this incentive leads to individuals relying disproprtionally on the public information available than justified by its quality. This implies that there are parameterisations of the model for which public information is welfare reducing. However, Hellwig (2004) challenges that by arguing that

Based on Wilson and Rhodes (1997) we argue that the nature and timing of the game allows us to consider the Central Bank as a Stackelberg leader. The Central Bank sends thus a signal x_s to the followers, and they in turn, have two options: to either follow the signal (x_s) in whichever way they interpret it, and pin their expectations to it in the hope that all other agent do so too, or formulate their expectations based on their own evaluations (x_i) and hence deviate from the target as dictated by their own private information. The extensive form representation of such a set-up, in a two-player game, is portrayed in figure 1.

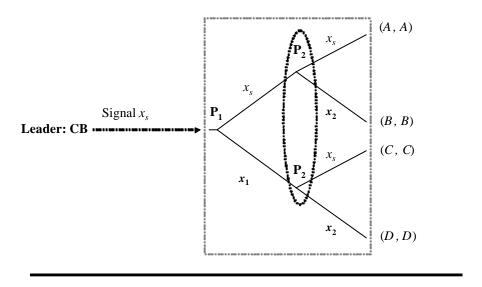


Figure 1: The Central Bank as a Leader

The dotted rectangular implies that the two followers are acting simultaneously. The game is therefore, sequential only vis-à-vis the leader. For simplicity we show the payoffs (expected utilities) for the two individuals only (represented by A, B, C and D). All four outcomes may result in players forming identical expectations. However, the probability of doing so, will be smaller than one when at least one of the two players chooses to follow his own private information. We will show how this is derived in detail, in section 4. By contrast, if both players follow the signal, then full benefits are accrued by both players. It is in this respect that, given perfect knowledge, the existence of a signal acts as a focal point.

agents place too little weight on the costs of aggregate volatility and too much weight on heterogeneity.

3 Variable Universe Games (Bacharach, 1993)

We describe next, Bacharach's Variable Universe Games framework, which helps describe how players evaluate their strategies to identify salient points when forming expectations in matching games⁵. The novelty of this approach is that it allows explicitly for differences in perceptions which then helps players choose rationally between alternative outcomes. The framework provided shows that in matching games, the players' incentive to coordinate induces them to look for salient points. However, as salience is subject to personal interpretation, the existence of such features is not necessarily uniquely defined.

3.1 An Expected Utility Approach

The game of blockmarking is played in the following way. Two players are shown a number of wooden blocks and each has to secretly pick a block. If both players pick the same one, they receive an identical pecuniary prize; otherwise they receive nothing. The author then describes three variants of the game, summarised in figure 2.

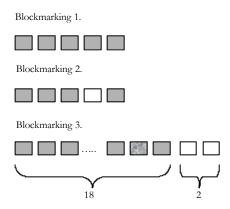


Figure 2: The Game of Blockmarking

In Blockmarking 1, (B1), the players are given five identical blocks (in size, colour, shape and material). In Blockmarking 2, (B2), the same game is repeated, except now one of the five blocs is of a different colour, (white). In Blockmarking 3, (B3), players are now given 20 blocks, eighteen of which are grey and two are white. Furthermore, closer inspection of the blocks, allows players to see that the grain of the wood in just one of the grey blocks is wavy. B3 can thus be described either in terms of colour, (C) or in terms of the grain of the wood, (G). As the game is of a matching nature, it is to the players' interest to look for means of achieving tacit coordination. In doing that, players

⁵Used and extended by Janssen (2001).

therefore look for salient features that help increase the chance of coordination. In example B1 above, there is no clear way of differentiating between the blocks, so one is inclined to simply pick at random. At example B2 however, the difference in colour allows players to distinguish between the blocks in such a way, that it is always wise to go for the one that is white. The unique instantiation of the white block thus provides the two players with a focal point. Similarly in B3, if colour is the distinguishing feature that occurs to the players, they are then inclined to pick one of the white blocks, even though such action does not automatically lead to coordination. However, if a player has managed to see that not only colour but also the grain of the wood differentiates the blocks, uniqueness is again guaranteed. The difficulty now however, is that the grain pattern of the wood is not necessarily identifiable (conspicuous) by (to) all players. In forming her choice therefore, having seen the difference in grain herself, player 1, needs to assess how likely her partner is to distinguish the blocks in terms of the grain as well. Bacharach's analysis shows, that if this likelihood is big, then it is to her interest to pick the grey block with the wavy pattern; otherwise she is better off picking one of the white blocks and face an, at most, 50% chance of matching the choice of her partner.

Bacharach provides a thorough proof to B3 in the appendix to his paper, but the essence of the game faced by the two players individually can be summarised as follows. In solving B3, player 1 is effectively faced with two alternative strategies: $M\tilde{h}$, mark a white block at random, or Mw, mark the grey block with the wavy grain. Furthermore, as explained above, the crucial point in this analysis is the likelihood with which player 1 believes player 2 will have noticed the grain. She is thus left with the following two choices when forming her views about player 2. Either she believes that her opponent has seen the grain (and assigns probability v to that event), or she does not believe that he has seen the grain (and assigns probability 1-v to that event). It is reasonable to assume that if player 2 has indeed noticed the grain, then he will pick it with some non-zero probability. However, if he has not noticed the grain then he can never mark a block accordingly. From player 1's perspective therefore, her expected utility from choosing one of her two actions is the following.

Definition 1: Both players have an identical set of feasible strategies, $R^+ = \{C, G\}$ and possible actions, $A = \{M\widetilde{h}, Mw\}$. Define $U_1(x_{1,a(\bullet)}, x_{2,a(\bullet)})$, player 1's utility from following action $x_{1,a(\bullet)}$ and player 2 following action $x_{2,a(\bullet)}$, for $a \in A$ where $a(C) = M\widetilde{h}$ and a(G) = Mw.

We need to deal with two cases:

Case 1: Player 2 always marks a block according to colour, either because he has not seen the grain himself, or because he believes his partner has not. Then player 1' expected utility is

$$E_1U(M\widetilde{h}, M\widetilde{h}) = (1-v)U_1\left(M\widetilde{h}, M\widetilde{h}\right) + vU_1\left(M\widetilde{h}, M\widetilde{h}\right)$$

$$E_1U(Mw, M\widetilde{h}) = (1-v)U_1\left(Mw, M\widetilde{h}\right) + vU_1\left(Mw, M\widetilde{h}\right)$$

We normalise next $U(x_1 = x_2) = 1$, and calculate the expected utilities:

$$E_1 U(M\widetilde{h}, M\widetilde{h}) = (1 - v) \frac{1}{2} U(x_1 = x_2) + v \frac{1}{2} U(x_1 = x_2) = \frac{1}{2}$$

$$E_1 U(Mw, M\widetilde{h}) = (1 - v) * 0 + v * 0 = 0$$

This implies that $E_1U(M\widetilde{h}, M\widetilde{h}) > E_1U(Mw, M\widetilde{h})$ and therefore player 1 has an incentive to match her partner's action by also picking a white block at random.

Case 2: Player 2 now, marks a block based on the grain when he has noticed it. Otherwise, he marks a block according to colour. Then expected utility for player 1 is now

$$E_1 U \left[M\widetilde{h}, \left(M\widetilde{h} \text{ or } Mw \right) \right] = (1 - v)U_1 \left(M\widetilde{h}, M\widetilde{h} \right) + vU_1 \left(M\widetilde{h}, Mw \right)$$

$$E_1 U \left[Mw, \left(M\widetilde{h} \text{ or } Mw \right) \right] = (1 - v)U_1 \left(Mw, M\widetilde{h} \right) + vU_1 \left(Mw, Mw \right)$$

and therefore

$$E_{1}U\left[M\widetilde{h}, \left(M\widetilde{h} \text{ or } Mw\right)\right] = (1-v)\frac{1}{2}U\left(x_{1} = x_{2}\right) + v * 0 = \frac{1-v}{2}$$

$$E_{1}U\left[Mw, \left(M\widetilde{h} \text{ or } Mw\right)\right] = (1-v) * 0 + vU\left(x_{1} = x_{2}\right) = v$$

It follows that,

$$E_1U\left[Mw,\left(M\widetilde{h} \text{ or } Mw\right)\right] > E_1U\left[M\widetilde{h},\left(M\widetilde{h} \text{ or } Mw\right)\right], \text{ if } f \text{ } v > \frac{1}{3}.$$

But between the two cases, the necessary and sufficient condition for player 1 to decide to mark a block according to the grain, is

$$E_1U\left[Mw, \left(M\widetilde{h} \text{ or } Mw\right)\right] > E_1U(M\widetilde{h}, M\widetilde{h}) \iff v > \frac{1}{2}$$
 (3)

In other words, the balance of reasons favours marking the block with the wavy grain, only if v is a large enough number by comparison to $\frac{1}{m}$ where m is the number of white blocks. Bacharach argues therefore, that the relative rarity of the white blocks, captured here by $\frac{1}{2}$, is pulling against the conspicuousness v of the grain pattern, as the less rare the white blocks, the more likely the player is to pick the wavy grey block.

4 Inflation Expectations Coordination

We consider next a one-shot monetary policy game, in which we assume two players forming the public, P_1, P_2 , who are faced with private information, (X), and public information in the form of a signal given by the Central Bank, (S). Based on these strategies, we will compare the expected values of following a particular action

In understanding the game, the two players are faced thus with two concepts: $\mathbb{F} = \{X, S\}$ and adopt strategies accordingly, $\mathbb{R}^+ = \{X, S\}$. The actions that correspond to this set of strategies are $a_i = x_i$ for $i = P_1, P_2$ if they rely on their own private information r(X), and $a_i = x_s$ when they follow the signal r(S). The set of distinct actions is therefore, $\mathbb{A} = \{x_i, x_s\}$. In summary, the game of monetary policy is simpler than that of blockmarking in the sense that there is no uncertainty about noticing the signal. The information required is summarised as follows:

	Sets	Elements
Concepts	$\mathbb{F} = \{X, S\}$ $\mathbb{R}^+ = \{X, S\}$	F(X), F(S)
Feasible Strategies	$\mathbb{R}^+ = \{X, S\}$	r(X), r(S)
Distinct Actions	$\mathbb{A} = \{x_i, x_s\}$	$a(X) = x_i, \ a(S) = x_s$

Player P_1 now has to form a view on whether to follow the signal, either because she thinks the signal is credible or because she thinks her partner will follow it. Similarly her partner will be thinking along the same lines. In forming her views therefore, P_1 is left with two choices. Either she believes that her partner will follow the signal (and assigns probability v to that event), or she believes that he will not follow it (and assigns probability 1-v to that event). Each individual forms expectations within a range of discrete values: i.e. $\mathbb{X}_i = \{x_{i,1}...x_{i,n}\}$. We assume that all values in \mathbb{X}_i occur with the same likelihood. This is a simplifying assumption that can be extended to capture different distributions. Furthermore, we also assume that $\mathbb{X}_1 = \mathbb{X}_2$.

We examine two monetary policy regimes that differ in the degree of precision of the signal they provide. The regime that has one precise value as numerical target will be identified with Inflation Targeting. We will compare that to non-inflation targeting, where the monetary objective is not identified with one value but with a range of values. Note, that we do recognise that inflation targeting is not always associated with a unique numerical target in the literature; we apply this terminology however, in order to differentiate the two regimes in terms of their level of target precision. Furthermore, we will differentiate according to the degree by which the signal given by the Central Bank differs from public perceptions.

4.1 A need to coordinate: $x_s \in X_i$

In first instance, we assume that the signal given by the Central Bank falls inside the range of discrete private sector expectations, i.e. $x_s \in X_i$. As the elements

in X_i are equally likely, then P_1 is less worried about believing the signal and more concerned with achieving coordination with her partner. We show next that the very existence of a signal will induce coordination while the precision of the signal is going to increase the likelihood of achieving it.

4.1.1 Inflation Targeting: $x = x_s$

We examine first a Central Bank that is an inflation targeter, in the sense that it announces a precise numerical target x_s . Following the notation presented in Figure 1, the normal representation of payoffs is given in Table 1.

Table 1			
P_1	$\setminus P_2$	x_s	x_2
x_s		A, A	B, B
x_1		C, C	D, D

By construction of the game, the following taxonomy of expected utilities holds.

$$A : = U(x_1 = x_2)$$

$$B : = \Pr(x_1 = x_2|_{x_1 = x_s, x_2 \in \mathbb{X}_i}) * U(x_1 = x_2) = \zeta A \quad 0 < \zeta < 1$$

$$C : = \Pr(x_1 = x_2|_{x_2 = x_s, x_1 \in \mathbb{X}_i}) * U(x_1 = x_2) = B$$

$$D : = \Pr(x_1 = x_2|_{x_1, x_2 \in \mathbb{X}_i}) * U(x_1 = x_2) = B$$

Individuals are indifferent which level of inflation they coordinate at, provided they do coordinate. However, as A>B, for a discrete choice of inflation expectations, Table 1 shows that coordination of the two players at a level other than x_s will happen with a probability less than one. By contrast, coordination at the unique signal provided guarantees payoffs equal to the utility achieved through matching expectations, $U(x_1=x_2)$. We show next how the solution of the matching game implies coordination of the two players at the target. For simplification we normalise $A:=U(x_1=x_2)=1$.

Proposition 1: A precise signal $x_s \in X_i$ provided by the central Bank, induces players to always apply action $a_i = \{x_s\}$.

Proof 1:

The expected utility for P_1 , of applying either of the two actions, is:

$$E_{1}U\left[x_{1},(x_{2} \text{ or } x_{s})\right] = (1-v)U\left(x_{1},x_{2}\right) + vU\left(x_{1},x_{s}\right) = (1-v)D + vC = \zeta$$

$$E_{1}U\left[x_{s},(x_{2} \text{ or } x_{s})\right] = (1-v)U\left(x_{s},x_{2}\right) + vU\left(x_{s},x_{s}\right) = (1-v)B + vA = (1-v)\zeta + v$$
As $B = C = D = \zeta$, it is straight forward to demonstrate that

$$E_1U[x_s, (x_2 \text{ or } x_s)] > E_1U[x_1, (x_2 \text{ or } x_s)]$$

and therefore P_1 does better by following the signal irrespective of P_2 's action. By analogy player 2 follows the same action and monetary policy achieves perfect coordination at the signal provided.

4.1.2 Non-Inflation Targeting: $x_s \in [\tilde{x}, \hat{x}]$

We examine next the case when the Central Bank has a less precise target⁶. This time therefore, the monetary policy objective takes the form of a range of discrete values, for example $x_s \in [\tilde{x}, \hat{x}]$, (where \tilde{x} and \hat{x} , two constant values), rather than a single numerical value. The signal provided is still assumed to be a subset of \mathbb{X}_i , or in other words $[\tilde{x}, \hat{x}] \subset \mathbb{X}_i$. The payoffs can now be summarised in Table 2, again in accordance with notation in Figure 1.

Table 2		
$P_1 \setminus P_2$	$x_s \in [\tilde{x}, \hat{x}]$	x_2
$x_s \in [\tilde{x}, \hat{x}]$	A', A'	B', B'
x_1	C', C'	D', D'

where

$$\begin{array}{lll} A' & : & = \Pr \left({{x_1} = {x_2}|_{{x_1},{x_s} \in \left[{\tilde x,\hat x} \right]}} \right)U({x_1} = {x_2}) = \xi A & 0 < \xi < 1 \\ B' & : & = \left\{ {\phi \Pr \left({{x_1} = {x_2}|_{{x_1} \in \left[{\tilde x,\hat x} \right],{x_2} \in \left[{\tilde x,\hat x} \right]}} \right) + \left({1 - \phi } \right){Z_1}} \right\} *U({x_1} = {x_2}) = \phi A' & 0 < \phi < 1 \\ C' & : & = \left\{ {\phi \Pr \left({{x_1} = {x_2}|_{{x_2} \in \left[{\tilde x,\hat x} \right],{x_1} \in \left[{\tilde x,\hat x} \right]}} \right) + \left({1 - \phi } \right){Z_2}} \right\} *U({x_1} = {x_2}) = \phi A' \\ D' & : & = \Pr \left({x_1} = {x_2}|_{{x_1},{x_2} \in {\mathbb{X}_i}}} \right) *U({x_1} = {x_2}) = D = \phi A' \\ \end{array}$$

and

$$Z_1 : = \Pr(x_1 = x_2|_{x_1 \in [\tilde{x}, \hat{x}], x_2 \notin [\tilde{x}, \hat{x}]}) = 0$$

$$Z_2 : = \Pr(x_1 = x_2|_{x_2 \in [\tilde{x}, \hat{x}], x_1 \notin [\tilde{x}, \hat{x}]}) = 0$$

But since D'=D, it follows that $\phi A'=\phi \xi A=\zeta A$ and $\phi \xi=\zeta$. Similarly to inflation targeting from above, A'>B'=C'=D'.

Proposition 2: A signal x_s provided by the Central Bank, although imprecise, $(x_s \in [\tilde{x}, \hat{x}])$, when still part of X_i will induce players to always apply action $a_i = \{x_s\}$.

Proof 2:

Again, the expected utility for P_1 , of applying either of the two actions, is:

$$E_1U[x_1, (x_2 \text{ or } x_s)] = (1-v)U(x_1, x_2) + vU(x_1, x_s) = (1-v)D' + vC' = \xi\phi$$

$$E_1U[x_s, (x_2 \text{ or } x_s)] = (1-v)U(x_s, x_2) + vU(x_s, x_s) = (1-v)B' + vA' = (1-v)\xi\phi + v\phi$$

It is straight forward to demonstrate that

⁶ Note that we are only interested here in how the precision of the signal alters the way people coordinate their actions. This approach therefore, does not allow for potential benefits (if any) to be had from having a more flexible (albeit less precise) signal.

$$E_1U[x_s, (x_2 \text{ or } x_s)] > E_1U[x_1, (x_2 \text{ or } x_s)].$$

This implies that the provision of the signal is sufficient in providing a coordination mechanism.

Corollary1: The benefits of Inflation Targeting accrue in the form of a higher probability of coordination amongst the agents and therefore, higher expected utility.

Although the signal is followed in both regimes, the probability of coordination is higher for inflation targeting, as A > A'. Furthermore, we compare the expected utility of following the signal, for the two regimes.

$$E_1^{IT} U[x_s, (x_2 \text{ or } x_s)] = (1-v)\zeta + v$$

 $E_1^{NIT} U[x_s, (x_2 \text{ or } x_s)] = (1-v)\xi\phi + v\phi$

As $\zeta = \xi \phi$ from Table 2, and $0 < \phi < 1$, it follows that inflation targeting produces higher expected utility. Precision in the target announced therefore, is beneficial to the public.

4.2 Credibility before coordination: $x_s \notin X_i$

We examine next the case when $x_s \notin \mathbb{X}_i$. In other words, the signal provided by the Central Bank lies outwith the range of expectations formed by the agents which implies greater divergence in views between the public and the Central Bank, by comparison to the previous case. From the point of view of player 1, the problem now is not only one of coordination, but also of assessing how player 2 perceives the signal, in the form of the v value. This is because two of the four possible pure strategies do not lead to matched expectations. If one of the two players chooses to follow the signal (x_s) when the other one does not, then expectations are never matched and therefore, the payoff is zero for both players. We differentiate again according to the precision of the signal provided.

4.2.1 Inflation Targeting: $x = x_s$

The normal representation is now:

Tab	le 3		
P_1	$\setminus P_2$	x_s	x_2
$\overline{x_s}$		A, A	0,0
x_1		0,0	D, D

As indicated above, the game in its pure representation shows that unless both players adopt the same action, a_i , there is no probability of coordination. Furthermore, while A > D, it is not immediately obvious that the expected value

from following the signal is unequivocally to the advantage of P_1 . This will depend crucially on the value of v.

Proposition 3: The necessary and sufficient condition for coordination to occur at the precise signal when $x_s \notin X_i$, is $v > \frac{\zeta}{1+\zeta}$.

Proof 3:

The expected utility for P_1 , of applying either of the two actions, is now:

$$E_1 U [x_1, (x_2 \text{ or } x_s)] = (1-v)U(x_1, x_2) + vU(x_1, x_s) = (1-v)\zeta$$

$$E_1 U [x_s, (x_2 \text{ or } x_s)] = (1-v)U(x_s, x_2) + vU(x_s, x_s) = v$$
(4)

It follows that in this case, P_1 will only follow the signal if and only if

$$v > (1-v)\zeta \Leftrightarrow v > \frac{\zeta}{1+\zeta}.$$
 (5)

Compared to the case when $x_s \in \mathbb{X}_i$, the fact that now $x_s \notin \mathbb{X}_i$ has given rise to a critical threshold for v before agents choose to coordinate at the signal. In other words, they need to have a minimum level of confidence that their partner will also follow the signal, before it is beneficial for them to defy their own private information and follow the signal as well.

4.2.2 Non-Inflation Targeting: $x_s \in [\tilde{x}, \hat{x}]$

We examine again the case when the Central Bank has a less precise target, in the sense that the signal takes values within a range. Contrary to above however, this time $[\tilde{x},\hat{x}] \nsubseteq \mathbb{X}_i$, in other words, the signal is not a subset of individuals expectations set. The payoffs can now be summarised in Table 4, again in accordance with notation in Figure 1.

	Table 4		
	$P_1 \setminus P_2$	$x_s \in [\tilde{x}, \hat{x}]$	x_2
•	$x_s \in [\tilde{x}, \hat{x}]$	A', A'	0,0
	x_1	0, 0	D', D'
		$ \begin{array}{c c} A', A' \\ 0, 0 \end{array} $	$0,0 \\ D',D'$

Proposition 4: The necessary and sufficient condition for coordination to occur in the range of the signal when $x_s \in [\tilde{x}, \hat{x}] \nsubseteq X_i$, is $v > \frac{\varphi}{1+\phi}$.

Proof 4:

 P_1 's expected utility from applying either of the two actions, is

$$E_1 U [x_1, (x_2 \text{ or } x_s)] = (1 - v) U (x_1, x_2) + v U (x_1, x_s) = (1 - v) \xi \phi$$

$$E_1 U [x_s, (x_2 \text{ or } x_s)] = (1 - v) U (x_s, x_2) + v U (x_s, x_s) = v \xi$$
(6)

It follows that P_1 will only follow the signal if

$$v\xi > (1-v)\xi\phi \iff v > \frac{\varphi}{1+\phi}$$
 (7)

Similar to the case of inflation targeting, agents need a minimum level of confidence before they coordinate within the range signalled. Following this, we can then compare how the choice of monetary policy regime affects this degree of confidence. It follows from above that:

Corollary 2: The degree of confidence required for agents before they coordinate at the signal, is higher under a regime of non-inflation targeting by comparison to when the signal is precise (inflation targeting).

To see this we compare the critical level for parameter v for the two regimes, namely conditions (5) and (7).

$$\frac{\varphi}{1+\phi} = \frac{\xi\varphi}{\xi(1+\phi)} = \frac{\zeta}{\xi+\zeta}$$

As $\xi < 1$, it follows that

$$\frac{\zeta}{\xi + \zeta} > \frac{\zeta}{1 + \zeta} \tag{8}$$

and therefore, the critical threshold before which agents choose to coordinate, increases as the signal becomes more imprecise. As mentioned above, now that $[\tilde{x}, \hat{x}] \nsubseteq \mathbb{X}_i$, individuals are not only worried about coordination, but first and foremost, about how the signal is perceived by other individuals. When this is the case, condition (8) indicates, that when a CB worries about the credibility of its target, then concentrating in providing a clear signal maybe an easier way of helping agents coordinate at the desired outcome.

5 Conclusions

We model the benefits of inflation targeting in terms of providing a clear and precise inflation objective. The motivation behind our efforts stemmed from the fact that while most agree that a clear signal is beneficial in terms of tying down private sector expectations, theoretical attempts to examine monetary policy do not in fact differentiate in terms of how well this is achieved. This paper makes an attempt to show how inflation targeting achieves better inflation expectations coordination and provides a platform for assessing different monetary policy regimes. Our approach encompasses both Morris and Shin's framework, based on which we argue that monetary policy is a coordination game of a matching nature between private agents, as well as Bacharach's Variable Universe game,

which provides for a rational way of achieving such coordination. Our analysis leads to the following two conclusions: 1) it is the provision of the signal that achieves coordination. This justifies the recent attempts by Central Banks to communicate with the public. 2) It is the clarity of the signal that achieves welfare improvements. In other words, investing in clear and more effective ways of operating helps provide a "unique instantiation" of the signal and hence coordination at it.

Our conclusions are best understood in the context of a given level of credibility that a Central Bank is faced with. Comparing then monetary policy regimes that differ in terms of the precision of their objectives, our analysis shows that the incentive to coordinate is better served by a unique and precise signal rather than a range of values. In other words, inflation targeting sets clearer objectives and therefore, provides a better framework for monetary policy, by means of lowering the threshold above which high order expectations are matched. However, what our analysis does not show is the (admittedly very important) issue of which features monetary policy needs to have in order to build the necessary credibility to achieve the inflation objective. Here one could envisage that on the road to credibility, the precision of the signal could conceivably pull against the degree of flexibility maintained by aiming at a range of values. This issue remains unresolved and in order for it to be properly addressed, we would have to link explicitly every regime's inflation record to its ability to build and maintain its credibility.

References

- [1] Amato, Jeffery D, Stephen Morris and Hyun Song Shin, 2003, Communication and Monetary Policy, BIS Working Papers, No. 123, January.
- [2] Amato, Jeffery and Hyun Song Shin, 2003, Public and Private Information in Monetary Policy Models, BIS Working Papers, No. 138, September.
- [3] Bacharach, Michael, 1993, Variable Universe Games, in Frontiers of Game Theory, by Ken Binmore, Alan Kirman and Piero Tani, MIT Press, 255-275.
- [4] Bernanke, Ben S., Thomas Laubach, Frederic S. Mishkin and Adam Posen, 1999, Inflation Targeting: Lessons from International Experience, Princeton: Princeton University Press.
- [5] Blinder, Alan, Charles Goodhart, Philipp Hildebrand, David Lipton and Charles Wyplosz, 2001, *How do Central Banks Talk?*, Geneva Report on the World Economy, No. 3, London, Centre for Economic Policy, Research/International Centre for Monetary and Banking Studies.
- [6] Casajus, André, 2000, Focal Points in Framed Strategic Forms, Games and Economic Behaviour, 32, 263-291.
- [7] Demertzis, Maria and Nicola Viegi, 2003, Aiming for the Bull's Eye: Inflation Targeting Under Uncertainty, DNB Mimeo, August.
- [8] Dixit, Avinash and Susan Skeath, 1999, Games of strategy, W.W.Norton and Company.
- [9] Fatás, Antonio, Ilian Mihov and Andrew Rose, 2004, Quantitative Goals for Monetary Policy, Centre for Economic Policy Research, London, Discussion Paper, No. 4445, June.
- [10] Hellwig, Christian, 2004, The Social Cost of Heterogeneous Information, UCLA, Mimeo, June.
- [11] Hughes Hallett, Andrew and Nicola Viegi, 2002, Inflation Targeting as a Coordination Device, Open Economies Review, 13, 341-362.
- [12] Janssen, Maarten C.W., 2001, Rationalising Focal Points, Theory and Decision, 50, 119-148.
- [13] Keynes, John Maynard, 1936, The general theory of employment interest and money, London: Macmillan.
- [14] King, Mervyn, 2002, The Inflation Target 10 Years On, Speech delivered to the London School of Economics, 19 November.
- [15] Knittel, Christopher R., and Victor Stango, 2003, Price Ceilings as Focal Points for Tacit Collusion: Evidence from Credit Cards, The American Economic Review, Vol. 93, No. 5, December, 1703-1729.

- [16] Kuttner, Kenneth, N., 2004, The Role of Policy Rules in Inflation Targeting, Federal Reserve Bank of St. Louis Review, Vol. 86, 4, pp.89-111.
- [17] Leiderman, Leonardo and Lars E.O. Svensson, 1995, *Inflation Targets*, (eds), London: Centre for Economic Policy Research, 1995.
- [18] Levin, Andrew, Fabio M. Natalucci and Jeremy M. Piger, 2004 The Macroeconomic Effects of Inflation Targeting, Federal Reserve Bank of St. Louis Review, Vol. 86, 4, 51-80.
- [19] Mishkin, Frederic S., and Klaus Schmidt-Hebbel, 2001, One Decade of Inflation Targeting in the World: What Do We Know and What Do We Need to Know? in Norman Loayza and Raimundo Soto, (eds), Inflation Targeting: Design, Performance, Challenges, Central Bank of Chile, Santiago, 117-219.
- [20] Morris, Stephen and Hyun Song Shin, 2002a, Social Value of Public Information, The American Economic Review, Vol. 92, No. 5, December, 1521-1534.
- [21] Morris, Stephen and Hyun Song Shin, 2002b, Global Games: Theory and Applications, in Advances in Economics and Econometrics, the Eighth World Congress, M. Dewatripont, L. Hansen and S. Turnovsky, (eds), Cambridge University Press
- [22] Orphanides, Athanasios and John C. Williams, 2004, The Decline of Activist Stabilisation Policy: Natural Rate Misperceptions, Learning and Expectations, Board of Governors of the Federal Reserve, April.
- [23] Phelps, Edmund S. 1983, The trouble with rational expectations and the problem of inflation stabilisation, in R. Frydman and E.S. Phelps, (eds), Individual forecasting and aggregate outcomes. New York, Cambridge University Press, 31-40.
- [24] Sugden, Robert, 1999, A theory of focal points, The Economic Journal, May, 105, 533-550.
- [25] Svensson, Lars 1999, Inflation Targeting as a Monetary Policy Rule, Journal of Monetary Economics, 43, 607-654.
- [26] Svensson, Lars 2003, What is Wrong with Taylor Rules? Using Judgement in Monetary Policy through Targeting Rules, Journal of Economic Literature, Vol. XLI, June, 426-477.
- [27] Wilson, Rick K. and Carl M. Rhodes, 1997, Leadership and Credibility in N-Person Coordination Games, Journal of Conflict Resolution, Vol. 41, No. 6, December, 767-791.

- [28] Woodford, Michael, 2002, Imperfect Common Knowledge and the Effects of Monetary Policy, in P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, eds., Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps, Princeton Univ. Press, 2002.
- [29] Woodford, Michael, 2003, Interest and Prices, Princeton University Press.