Central Bank Balance Sheet Concerns, Monetary and Fiscal Rules, and Macroeconomic Stability*

Feng Zhu†
Bank for International Settlements and Yale University
Centralbahnplatz 2
CH-4002, Basel, Switzerland

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†E-mail: feng.zhu@bis.org; Telephone: 44-(0)61-2809158; Webpage: http://pantheon.yale.edu/~fz9

Abstract

We provide a fully articulated theoretical foundation for the view that a central bank's (CB) balance sheet concerns may hinder monetary policy "activism" needed to achieve macroeconomic stability. We model the CB and the fiscal authority (FA) as two independent entities, each with its own separate budget constraint. To reflect the CB's balance sheet concerns, we augment the conventional Taylor rule with net worth targeting (NWT).

We show that in a monetary system with no fiscal backing for monetary policy, emphasis on NWT leads to local indeterminacy of the steady state equilibrium targeted by the CB, thereby reversing the well-known result that an active Taylor rule combined with a passive fiscal policy ensures local uniqueness of the target equilibrium. Moreover, a Hopf bifurcation emerges around the target equilibrium, suggesting extreme sensitivity of the local stability properties of the target equilibrium to a very small variation in the NWT parameter. CB balance sheet concerns build into the economic system an inherent tendency towards structural instability, making it more likely for an economy to drift away from the targeted steady state even under an active interest rate rule.

To avoid the undesirable consequences of CB balance sheet concerns, it may be necessary for a CB to engage in institutional reforms that ensure automatic and immediate fiscal backing for monetary policy, well before the zero lower bound on nominal interest rates becomes binding.

Keywords: central bank balance sheet concerns, Hopf bifurcation, local indeterminacy, liquidity trap, net worth targeting, Taylor rule.

JEL Classification: E31, E42, E52, E58

1 Introduction

Successful disinflation programs set in motion in the 1980s made price stability, defined as low and stable inflation, an economic reality in many industrial and developing economies. Achieving and maintaining price stability has become the main, if not the sole policy goal of most central banks[1]. Central bank (CB) independence has now been firmly incorporated in the legal framework of most industrial economies, and the credibility of central bankers engaged in rule-based policy-making has been greatly enhanced. The demise of the "Great Inflation" of the 1960s and 1970s seems to harbinger the advent of a new era of price stability, in which central banks face a new set of challenges to monetary policy-making. These include the risk of deflationary recessions, and the possibility of short-term nominal interest rates hitting the zero lower bound (ZLB)[2] and of an economy falling into a "liquidity trap". The decade-long "Great Recession" in Japan and recent economic slowdowns in the US and in some major European economies, combined with very low and occasionally negative inflation rates, raised the specter of a period characterized by prolonged deflationary recessions and by a loss of monetary policy effectiveness under binding ZLB.[3]

Starting with the seminal paper of Sargent and Wallace (1975), it has become an ongoing tradition studying macroeconomic stability focusing on local determinacy properties of a rational expectations equilibrium (REE) when a CB follows an interest rate rule. They illustrated how price level indeterminacy arises under a pure interest rate peg in a rational-expectations

[1]In a recent speech, Bernanke (2003b) emphasized the importance of price stability: "Achieving and maintaining price stability is the bedrock principle of a sound monetary policy. Price stability promotes economic growth and welfare by increasing the efficiency of the market mechanism, facilitating long-term planning, and minimizing distortions created by the interaction of inflation and the tax code, accounting rules, financial contracts, and the like. Price stability also increases economic welfare by promoting stability in output and employment."

[2]The US federal funds rate has been sliding downwards and is now targeted to the lowest level in more than 40 years, currently standing at 1%. The Japanese economy has experienced sustained deflation and close-to-zero overnight call money rate for the last few years. In fact, its call rate has been at or below 50 basis points since October 1995 and practically zero since February 1999. The European Central Bank is maintaining a rate of 2%, also a historical low.

[3]For a relatively early warning against the possibility of a loss of monetary policy effectiveness when the ZLB on nominal interest rate becomes binding in an environment of price stability, see Summers (1991).

model. McCallum (1981) argued that committing the interest rate rule to feedback from endogenous state variables renders the REE determinate. In that spirit, Taylor (1993) proposed an interest rate rule that reacts to both inflation rate and output gap. It has been shown that under the Taylor Principle, which dictates that the CB raises interest rate by more than one-to-one in response to an increase in inflation, such a rule, when coupled with a passive fiscal policy that ensures fiscal solvency, guarantees local uniqueness of the REE and promotes macroeconomic stability. In fact, this result has made the active Taylor rule a major policy prescription for central banks across the world.

In a recent paper, Benhabib, Schmitt-Grohé and Uribe (2001b) questioned the robustness of the local determinacy result for an active Taylor rule. They demonstrated that such a result may be sensitive to specifications of preferences and technology, for instance, when money enters the production function, and when money and consumption are Edgeworth substitutes. Furthermore, when a Taylor rule is constrained by the ZLB on the nominal interest rate, Benhabib, Schmitt-Grohé and Uribe (2001a, 2002) uncovered a second, low-inflation equilibrium, besides the desirable target equilibrium. The low-inflation equilibrium has the characteristics of a liquidity trap. In this case, more complex dynamics emerge and the possibility of aggregate instability is greater. The focus on local determinacy is misleading when multiple steady state equilibria arise naturally, a global analysis may be imperative.

Extending the line of argument followed by Sargent and Wallace (1981), Leeper (1991), Sims (1994) and Woodford (1994, 1995, 1996, 2003) showed that fiscal policy is no less important than monetary policy in the determination of price level. In fact, local uniqueness under an interest rate rule may be restored by a reconsideration of fiscal policy and the government budget constraint. According to the fiscal theory of price level, no matter how independent a CB is, price stability cannot be achieved without appropriate fiscal policies that ensure fiscal solvency. In a series of papers, Sims (1997, 1999, 2001, 2003) has been a tireless advocate of closer monetary-fiscal coordination as the institutional setup better suited to weather situations of severe economic distress.

We study the impact of a CB's balance sheet concerns[4] on aggregate sta-

[4] The asset side of a central bank's balance sheet usually consists of gold, foreign currencies and debt instruments, and domestic government bonds, while the liabilities are

bility in a situation where an economy edges towards a deflationary recession and the ZLB is almost binding. It has long been argued, albeit informally, that in the face of major stock and housing market crashes in the late 1980s, the Bank of Japan (BoJ) acted too late and did too little to prevent the Japanese economy from sliding into a deflationary recession, and the BoJ's concern over its own balance sheet position has been repeatedly cited as the reason for its policy conservatism and inertia. In a recent policy speech delivered to the Japan Society of Monetary Economics, Bernanke (2003a) called the BoJ's balance sheet concern a "barrier to more aggressive policies". Because of it, "not all the possible methods for easing monetary policy in Japan have been fully exploited". For Bernanke (2003a), "one possible approach to ending deflation in Japan would be greater cooperation, for a limited time, between the monetary and the fiscal authorities".[5]

In this paper, we provide a fully articulated theoretical foundation for the view that a central bank's balance sheet concerns may hinder monetary policy "activism" needed for achieving local uniqueness of the intended steady state equilibrium, thereby opening the door for greater macroeconomic instability. At a time of low inflation, almost binding ZLB on nominal interest rates and persistent economic difficulties, conventional open market operations are no longer effective as money and short-term government bonds become perfect substitutes. To prevent an economy from slipping into a deflationary spiral, the CB may have to take unconventional policy measures which have a fiscal dimension, such as the purchases of long-term government bonds, foreign currencies and debt instruments, and private bonds and equities.[6] Asset prices fluctuate according to market conditions, which may cause large capital loss and unfavorable revaluations of the CB's asset portfolio. Without a promise of fiscal support by the Treasury to replenish the CB's balance sheet whenever the need arises, a CB that targets inflation or price level to

primarily composed of domestic currency. The CB's net worth is defined as the difference between total assets and liabilities. The CB is concerned with its balance sheet when it runs into negative net worth.

[5]Sims (2001) expressed similar views earlier: "a bank with such (balance sheet) concerns could also hoard interest earnings and refrain from bold, risky open market purchases to sustain fiscal institutions or end the deflation." He also observes that "to the extent that the central bank has the power to make risky open market purchases to end the deflation, it requires an understanding that it will if necessary have fiscal backing."

[6]For instance, the purchase of private debt or private equities by the CB may be seen as government bailout or outright acquisition of control over private firms, respectively. These are fiscal actions that usually pertain to the domain of the Treasury.

achieve price stability will be reluctant to take these measures in order to avoid a negative net worth.

There are both economic and institutional reasons for a CB to be concerned with its balance sheet. From the institutional perspective, a CB may cherish its independence so much that it will do everything in its power to fend off possible interference from the Treasury, and balance sheet imbalance may be an open invitation for troublesome meddling by the fiscal authority. In a democratic society, the management of public funds at a CB's disposal is closely monitored and the central bankers want to be perceived as managing its own financial situation in a responsible way. Large capital losses and CB balance sheet deficit may draw unwanted attention and criticisms from the public, when the CB has to defend its credentials as the prudent caretaker of a country's monetary and financial systems.

There is also economic rationale for CB balance sheet concerns. In so far as seignorage is transferred to the Treasury, or when it becomes insignificant when inflation is low and nominal interest rates draw close to zero, negative net worth may develop. For a CB that endeavors to achieve price stability through inflation or price level targeting, efforts to control the value of money will not be enough to guarantee a stable price level. Without fiscal support, the prospect of suffering large capital losses and running into negative net worth implies a possible loss of control over inflation when the economy eventually pulls out of deflation. The CB's asset holdings become insufficient to cover its money liabilities and rational economic agents become reluctant to hold money.

When the CB's assets diminish in value faster than its money liabilities because of, say, asset price fluctuations, the CB may run out of reserves before it can retire enough money out of circulation in an attempt to tighten monetary policy and control the ensuing inflation. Unless the Treasury issues new debt and make a free transfer of bonds to the CB, ultimately the CB may lose control of inflation although it manages to escape deflation. Fearing this outcome, a CB may be reluctant to adopt unconventional policy measures that might end the recession, or it may not carry them out to the full extent, because these measures may aggravate the CB's balance sheet and lead to negative net worth. If the fiscal authority is willing to help and is capable of supporting the CB's policy, it can use the power of taxation to replenish the CB's balance sheet and the CB will have its hands free to pursue price stability.

We model central bank and fiscal authority as two distinct entities, each with its own budget constraint and can go bankrupt separately. Since the ZLB is almost binding, the CB is assumed to conduct monetary policy entirely through the purchase and sale of a real asset. The central bank follows an interest rate rule that reacts to deviations of its net worth and inflation from the corresponding target levels. When a CB is sufficiently worried about its net worth, monetary activism embodied in the Taylor Principle cannot be applied to its full extent, the well-known result that an active interest rate rule combined with a passive fiscal policy ensures local uniqueness of the desired steady state equilibrium is reversed. Even a small dose of the CB's balance sheet concerns may lead to local indeterminacy of the targeted steady state equilibrium, which implies the existence of an infinite number of stable equilibrium solutions that are compatible with the same monetary policy rule, and there is no reason to expect the economy to converge to the targeted equilibrium. In fact, the very feasibility of using an active interest rate rule to achieve aggregate stability is put in doubt.

When a CB follows a passive interest rule and the Treasury takes the lead and carries out an active policy, we show that the liquidity trap equilibrium described in Benhabib, Schmitt-Grohé and Uribe (2001a, 2002) becomes locally determinate. Therefore liquidity trap becomes a focal point for economic agents' expectations, making it much more difficult for an economy to pull out of a deflationary equilibrium. The lengthy and ongoing Japanese recession can be a reflection of this scenario: for a good part of 1990s, the Japanese Ministry of Finance expanded fiscal spending through several significant fiscal stimulus packages, but the Bank of Japan was slow to react. Fiscal expansions did little to contain deflation, as expectations became deeply mired in the liquidity trap equilibrium.

With net worth targeting, an active Taylor rule also leads to a local Hopf bifurcation and the emergence of a deterministic cycle around the targeted equilibrium. There is a dramatic change in the local stability properties of the equilibrium due to very small changes in the value of the NW targeting parameter. CB balance sheet concerns build into the economic system an inherent tendency towards structural instability, making it more likely for an economy to drift away from the targeted equilibrium even under an active interest rate rule. Practicing monetary policy without adequate fiscal backup is fundamentally flawed and may lead to greater macroeconomic instability.

Completely severing links between the CB and the Treasury as a way to shore up credibility in the campaign to bring under control high and persistent inflation may hurt, rather than benefit an economy in a deflationary environment. Better monetary-fiscal coordination in unusual economic circumstances, by eliminating CB balance sheet concerns, enhances monetary policy effectiveness and helps to stabilize the economy in a timely manner and to prevent it from falling into a liquidity trap. In fact, solid and immediate fiscal backing for the CB's actions is essential for the success of monetary policy making in a new era of price stability. The conclusions of this paper has immediate import for the institutional setup of the BoJ and other central banks with similar institutional structure, such as the US Federal Reserve System.

In section II we provide a discussion of the nature of monetary policy making in an environment of low and stable inflation. In section III, we lay out our simple flexible-price general equilibrium model, with separate budget constraints for the monetary and fiscal authorities and an interest rate rule augmented with net worth targeting by the CB. In section IV we present and discuss the main results. Section V concludes. Derivation of the linearized version of the dynamical system and proofs of our results are provided in the Appendices.

2 Monetary Policy under Price Stability

Our analysis focuses on central banks which target inflation or price level in order to achieve price stability. In an environment of low and stable inflation, adverse exogenous shocks or policy mistakes may easily push an economy into a deflationary situation, and a CB's ability to pursue stabilization policies. When the zero lower bound (ZLB) on short-term nominal interest rates becomes binding, usual open market operations exchanging money for short-term government bonds are no longer effective. Around the ZLB, there may be a structural change in the transmission mechanism of monetary policy, potentially unorthodox policy measures have to be experimented. Under these unusual circumstances, balance sheet imbalance may become a concern and potentially a policy constraint for a CB that has no fiscal support.

Under normal economic circumstances, central banks conduct open market operations primarily on short-term homogeneous domestic nominal bonds as the counterpart of high-powered money. The return distributions for the CB's assets and liabilities, which are seen as close substitutes and quoted in the same unit of account, are quite uniform and almost perfectly hedged. Both government bonds and money are simply claims to a fraction of the stream of current and future government tax revenues net of spending. Seignorage in the form of unpaid interest on the CB's money liabilities (*i.e.*, the private sector's opportunity cost of holding money) accumulates and a positive net worth is almost assured, taking into account transfers to the Treasury. The issue of fiscal backing for monetary policy does not arise, and one consolidated government budget constraint suffices. In such an institutional framework, ultimately it is the fiscal power of taxation which backs up the value of money.

However, at a time of economic distress, facing the threat of deflation and an almost binding ZLB, money and short-term government bonds become almost perfect substitutes and conventional open market operations are ineffective. A CB which cares about price stability and growth will be obliged to implement unorthodox policy measures, such as the purchases of long-term government bonds, foreign currencies or debt instruments, or private bonds and securities. The return distributions of the CB's assets and liabilities become mismatched. Vigorously pursuing such measures will leave the CB with a large pool of diverse assets, which may have different maturity structures, and may be denominated in foreign currencies. This makes the CB's balance sheet susceptible to market risk, exchange rate and asset price fluctuations, and exposed to unexpected policy contingencies. For instance, large-scale purchases by the CB of long-term government bonds, if successful in pulling the economy out of a deflationary recession, will eventually raise both long and short nominal interest rates, depressing bond prices. This entails a potentially large capital loss for the CB. The CB may shy away from taking these measures, which may be necessary for economic recovery, if these foreshadow balance sheet problems.

Seignorage is small around the ZLB. It is often seen as part of a country's fiscal revenue and by law much of it has to be transferred to the government. In a situation of economic distress, automatic and immediate fiscal support for monetary policy is necessary to make sure that the CB is recapitalized whenever it runs into negative net worth. Without fiscal backing, a CB that pursues price stability through inflation targeting is obliged to monitor its net worth more closely. The CB, in an attempt to avoid any tangible risk to its balance sheet position, may not carry out certain policy actions to the extent that is required by economic rationale. In this situation, a CB is effectively disconnected from the FA, and it becomes natural to model CB and FA as two mutually independent entities, each having a separate budget constraint.

The decade-long deflationary recession best exemplifies the difficulties of monetary policy-making in a new era of price stability. Most CB's are legally independent from the fiscal authority (FA). The BoJ formally earned its independence from the Japanese Ministry of Finance (MoF) only recently, after the Bank of Japan Law took effect in April 1998. Cautious in its approach to monetary policy, the BoJ has made financial soundness of the Bank, seen as a safeguard of its independence from the MoF, its top priority.[7] The BoJ has long been criticized for letting its financial concerns meddle with proper policy making when drastic measures might be required in the face of an unprecedented deflationary recession. Until recently, the BoJ has resisted pressure to purchase long-term government bonds in order to bring down the yield curve, in fear of accumulating a large amount of such bonds that may entail an "unacceptable" level of capital losses once the economy recovers. Outright purchase of asset-based securities (ABS) by the BoJ, which has recently gained its cautious approval, presents even greater risk to the BoJ's balance sheet.[8]

[7]For instance, excerpts from the Minutes of the (BoJ) Monetary Policy Meeting on April 7 and 8, 2003 (italics added by the author) make the BoJ's "obsession" with its balance sheet transparent:

"Members agreed that, when examining the new scheme, the Bank should give due consideration to how it would secure the *soundness of its financial condition.* One member warned that if the Bank were not able to prove the soundness of its financial condition clearly to the public, its *credibility* and the *effectiveness of policy* could decline";

"A different member said that ... if the Bank were to take bold action to purchase risk assets, this might create market concern that the Bank would not be able to allocate sufficient capital to purchase other risk assets."

"Many members agreed that the Bank's proposed participation in fostering markets by taking on credit risk would inevitably cause the Bank to enter the *domain of fiscal policy.* One member said that it would therefore be important to cooperate with relevant government institutions in addition to market participants in fostering markets."

[8]In an article published on the Wall Street Journal (March 6, 2000), Kazuo Ueda, a member of the BoJ's policy board, expressed his view that (italics added by the author) "to hold down long-term interest rates, the Bank of Japan would be forced to purchase a huge amount government bonds. At some point, these purchases would need to be reversed at a higher interest rate, generating serious capital losses for the central bank. In such a case, *the government would need to agree in advance that it would recapitalize the central bank.* These costs would be even greater if the law governing the central bank was amended and the Bank of Japan was allowed to buy up equities or real estate."

As of September 30, 2003, long-term and short-term Japanese government bonds make up about 47% and 22% of the BoJ's pool of assets, respectively. Foreign currencies and asset-backed securities and stocks held as trust property make up about 3.2% and 1.4%, Although one's best guess is that the MoF will be happy to provide support for the BoJ's balance sheet once the latter runs into financial trouble, it is not clear at what price, in terms of fiscal intervention, this support will come. The Bank therefore does everything in its power to make sure that its finances are sound and it acts as if a reserve is maintained so as to guarantee a small but positive net worth.[9] CB independence, narrowly interpreted as the institutional separation of the CB from the FA, is one means to achieve credibility and price stability, but when the means becomes itself a rigorously pursued policy goal that directly impinges on the making of monetary policy, the consequences may be far-reaching.

3 A Flexible-Price Model

In a simple flexible-price representative agent model, we investigate possible macroeconomic consequences of the balance sheet concerns of a central bank in pursuit of price stability, taking into account a variety of different fiscal policy rules. We augment Benhabib, Schmitt-Grohé and Uribe's (2001a, 2002) model with separate budget constraints and an interest rate rule which reacts to changes in the CB's net worth besides inflation rate. We study how CB balance sheet concerns induce an element of conservatism in monetary policy-making and lead to greater macroeconomic instability.

To analyze the effects of net worth targeting in a monetary system where fiscal support for monetary policy is absent, we assume that the economy has access to a Lucas tree type real asset (F) that is of fixed supply ($F = \bar{F}$). The asset can be held by both the representative household (F^H) and the central bank (F^{CB})[10]

$$F(t) = F^H(t) + F^{CB}(t)$$

Assume that the asset yields a fixed amount of dividend per unit of F. For simplicity, dividend per unit of F is fixed at one in terms of the consumption good. We define the rate of capital gain for the asset as $\pi_F = \dot{Q}/Q$, where

respectively.

[9]As of September 30, 2003, the Bank of Japan's net worth, consisting of reserves for possible loan losses, for possible losses on securities and foreign exchange transactions, for retirement allowances and for net accumulated profits totals 5,32 trillion yen, about 4% of its total assets.

[10]To simplify notation, we will drop the time argument t whenever the context is clear.

$Q(t)$ is the price of asset F in terms of the consumption good. The real rate of return on F is then $r(t) = 1/Q(t) + \pi_F(t)$, which depends on both the asset price and the size of capital gain (or loss) by holding one unit of asset F. Holders of F units of the asset receive a dividend at the amount of F, which is paid in kind and can be consumed.

3.1 Household

We postulate that money facilitates transactions and enters the model as an argument in the instantaneous utility function.[11] In the familiar Brock-Sidrauski setup, the representative household maximizes lifetime utility,

$$U_0 = \int_0^{\infty} e^{-\beta t} u(c(t), m(t)) dt \tag{1}$$

subject to the budget constraint

$$c(t) + Q(t)\dot{F}^H(t) + \frac{\dot{M}(t) + \dot{B}(t)}{P(t)} = y + F^H(t) + \frac{iB(t)}{P(t)} - \tau(t)$$

where c is consumption, τ is the household's tax payment, P is the price level, $M \geq 0$ and B are the household's holdings of nominal money balances and government bonds, respectively, F^H stands for both the household's holdings of the Lucas tree type asset F, which pays a real rate of return r, and the payment of dividends to the household, and i is the nominal rate of interest on government bonds. The same asset F also serves the central bank, either as its reserve asset, or as the object of choice for its monetary policy operations. The endowment income y is exogenous and fixed.

Define $m = M/P$, $b = B/P$, and the economy's rate of inflation as $\pi = \dot{P}/P$. Moreover, define the household's financial asset holding as $A = PQF^H + M + B$ and the real financial asset portfolio as $a = A/P$, then

[11] Money can be introduced in other ways, notably as an argument in the production function, or through a standard cash-in-advance constraint for consumption and/or investment goods, or through a shopping time constraint or a transaction cost function in the household's budget constraint. For a detailed account, see Sargent (1987), Walsh (2003) and Woodford (2003).

These modeling devices are shortcut mechanisms to introduce monetary frictions in a simple manner, and many of these yield results that are similar to those derived from our money-in-utility specification.

$\dot{a} = \dot{Q}F^H + Q\dot{F}^H + \dot{m} + \dot{b}$. In real terms, the household's budget constraint is

$$\dot{a} = (i - \pi)\, a - (i - \pi - r)\, QF^H - im + y - c - \tau \tag{2}$$

We make the following assumption about the utility function.

Assumption A *Assume that (1) the utility function $u(\cdot,\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable in both arguments; (2) $u_c u_{mm} - u_m u_{cm} < 0$ and $u_c u_{cm} - u_m u_{cc} > 0$.*

Assuming an interior solution, the necessary first-order conditions for the household's maximization problem include

$$u_c(c, m) = \lambda \tag{3a}$$
$$u_m(c, m) = \lambda i \tag{3b}$$
$$\dot{\lambda} = (\pi + \beta - i)\, \lambda \tag{3c}$$
$$i = \pi + r \tag{3d}$$

where λ is the marginal utility of wealth and hence consumption. Notice that equation (3c) is the usual Euler equation, and equation (3d) is the equilibrium Fisher relation.

The necessary transversality at infinity condition (TVC) is

$$\lim_{t\to\infty} e^{-\beta t}\lambda\left[m(t) + b(t) + Q(t)\, F^H(t)\right] = 0$$

Since $\lambda(t) = \lambda(0)\exp\left(-\int_0^t [i(s) - \pi(s) - \beta]\, ds\right)$, the TVC in equilibrium can be simplified as

$$\lim_{t\to\infty} \exp\left(-\int_0^t [i(s) - \pi(s)]\, ds\right) a(t) = 0 \tag{4}$$

To simplify the analysis, we assume that the instantaneous felicity function takes the Cobb-Douglas form[12]

$$u(c, m) = c^{\eta} m^{1-\eta} \tag{5}$$

[12]The Cobb-Douglas specification of instantaneous utility function is not suitable for global analysis, since it implies that utility may be driven to infinity if consumption is held constant. We focus, at this stage, on the local analysis of our model.

where $\eta \in (0,1)$ is the utility share of consumption goods. Under this specification, consumption goods and money are Edgeworth complements ($u_{cm} > 0$). From conditions (3a) and (3b), we obtain the usual liquidity preference function

$$m = m(i,c) = \frac{1-\eta}{\eta}\frac{c}{i} \tag{6}$$

Since $u_{cc}, u_{mm} < 0$, the fact that consumption and money are Edgeworth complements ($u_{cm} > 0$) for Cobb-Douglas utility is a sufficient condition for us to have $m_i < 0$ and $m_c > 0$.

3.2 Government Sector

Conventional economic models postulate one single consolidated budget constraint for the government sector, through which the Treasury implicitly provides fiscal backing for monetary policy. In this case, money and government bonds are alternative paper claims to the flow of future fiscal surpluses and they resemble equity claims to private firms. When the CB's net worth turns negative, its money liabilities in excess of existing assets can be quickly converted into government debt and hence claims to future tax revenues.

We analyze a monetary system where the Treasury does not provide such automatic fiscal support for monetary policy. As a result, the central bank (CB) faces its own budget constraint and CB balance sheet concerns may arise in a natural way in unfavorable economic circumstances. Departing from the existing literature, we model the government sector as consisting of two separate and mutually independent entities, the Central Bank and the Fiscal Authority (FA). Each entity has its own budget constraint and policy rules. Without fiscal backing for monetary policy, the value of money becomes intimately related to the CB's own asset portfolio. Assuming that a CB's quantitative easing works, the economy becomes flooded with liquidity and inflation finally picks up. In order to control inflation, the CB needs to tighten monetary policy by raising nominal interest rates. Once the value of a CB's reserve of unconventional assets drops to a level that is insufficient to cover the expanded money liabilities, its net worth becomes negative, and the CB may not be able to further tighten monetary policy and retire money from circulation. If a CB is determined to maintain price stability through inflation or price level targeting,[13] negative net worth implies significant restrictions

[13]For a CB that is committed to a constant level of money supply ($M = \bar{M}$), there will

on a CB's capability of controlling inflation and pegging the value of money[14]. The CB's policy goal of long-term price stability may be compromised.

The fear of uninvited outside interference in its affairs and of possible loss of control over inflation propels the CB to target the level of its net worth. We incorporate the CB's concerns over its own balance sheet explicitly in its policy rule in the form of net worth targeting. The fact that the CB's financial soundness may take priority over more fundamental policy objectives, such as price stability and economic growth, imposes constraints on the making of monetary policy and has important implications for local determinacy of equilibria and macroeconomic stability.

3.2.1 Monetary Authority

Assume that there has been a one-time negative shock[15] to the economy that is sufficiently large so that the economy enters the initial period $t = 0$ already edging close to the zero lower bound on nominal interest rates, conventional open market operations lose effectiveness. Since bonds and money are almost perfect substitutes, the CB cannot make much use of its bond holdings. For simplicity, we assume that the CB has exhausted all its holdings of short-term government bonds in contractionary operations prior to the adverse shock so that $B^{CB}(t) = 0$, $\forall t \geq 0$.[16] Furthermore, the CB has initially purchased and maintained a reserve of real asset F^{CB}, probably as a result of some experiments with unconventional policy interventions. Then the only asset the CB holds is the real asset ($F^{CB}(t) \geq 0$, $\forall t$).

The CB has its own separate budget constraint

$$Q(t)\,\dot{F}^{CB}(t) = \frac{\dot{M}(t)}{P(t)} + F^{CB}(t) - \gamma i(t)\,\frac{M(t)}{P(t)} \tag{7}$$

not be persistent inflationary pressure.

[14]Only if the Treasury agrees to issue and transfer new bonds to retire money from circulation will the value of money remain stable at the previous target level. This is in fact covert fiscal support.

[15]Consider the stock and housing market crashes in Japan in late 1990s. Asset prices collapsed and monetary policy had to shift quickly from a contractionary to an expansionary stance. Changes in the US target federal funds rate were equally dramatic during the most recent recession in the US, following a series of bad shocks which include the stock market collapse and the 9/11 terrorist attack.

[16]With $B^{CB} = 0$, the CB may conduct open market operations buying or selling the real asset.

where the real net worth ϕ is defined as the difference between the CB's holding of the Lucas tree type real asset (QF^{CB}) and outstanding real balances (m): $\phi = QF^{CB} - m$. The CB uses proceeds from real asset holdings (F^{CB}) and new issues of money ($\dot{M}/P$), net of the transfer (γim) to the Treasury, to finance further purchases of assets ($Q\dot{F}^{CB}$).[17] The coefficient of transfer γ is assumed to be very close to but different from one.[18] Because the asset price Q may fluctuate over time, it is possible for the values of assets and liabilities of the CB to become mismatched and negative net worth may develop.

In equilibrium, using the Fisher equation (3d), we obtain from (7)[19]

$$\dot{\phi} = (i - \pi)\,\phi + (1 - \gamma)\,im\,(i, c) \tag{8}$$

Clearly, the evolution of ϕ depends on the CB's interest rate policy, which itself reacts to fluctuations in ϕ. With a separate budget constraint and by not holding Treasury bonds, the CB is effectively disconnected from the FA. CB independence from fiscal authority can be defined as below.

Definition 1 *A central bank is said to be independent from the fiscal authority if (1) the CB is not required to support the prices of government bonds; (2) the CB alone determines how much the CB's seignorage will be transferred to the FA and how much financial support it may demand from the FA in case of negative net worth.*

This definition of CB independence is probably more stringent than what has often been used in the "Rules *versus* Discretion" literature. In our model, the CB has its own budget constraint so that the FA does not consider itself responsible for any outstanding liabilities and balance sheet deficits of the

[17]This formulation of seignorage transfer is not suitable for global analysis but is sufficient for our present purpose.

[18]For $\gamma = 1$, the stationary level of the net worth is $\phi^* = 0$. With an appropriate interest rate rule where the CB targets $\phi^* = 0$, CB balance sheet concerns can be shown to have no effects on aggregate stability.

[19]We can also write equation (8) as

$$\dot{\phi} = (i - \pi)\,QF^{CB} + \pi m - \gamma im$$

Changes in net worth are therefore determined by real returns on the CB's holding of real asset (rQF^{CB}) and on inflation tax revenues earned upon its money liabilities (πm), net of transfers (γim) to the FA. If an economy is in deflation ($\pi < 0$), the πm term is negative and may lead to a reduction in net worth ($\dot{\phi} < 0$).

CB. The CB is therefore "disconnected" from the Treasury and insulated from fiscal interference, but tax revenues no longer support the value of money and automated fiscal backing for monetary policy actions cannot be expected. For this reason, we can postulate separate budget constraints for the CB and FA. For the CB in our model to become truly independent, it must have fiscal backing, be it implicit or explicit.

There is little doubt that central bankers are often concerned with their balance sheet, particularly in a situation of economic distress. In a deflationary environment, when the ZLB on nominal interest rate is attained or nearly so, unorthodox measures need to be taken but they often imply increased market risk and the possibility of large capital losses for the CB. By taking these measures, monetary operations clearly acquire a fiscal dimension and require a firm commitment from the FA that guarantees the financial viability of such monetary policy actions. When this commitment is not forthcoming, the CB, worried about its balance sheet, will refrain from taking these actions in a timely and quantitatively significant manner, or even works in the wrong direction by tightening monetary policy and reducing real asset holdings, in an attempt to shore up its net worth and to avoid any unpleasant implications of such a policy on its balance sheet.

The balance sheet concern is transformed, often implicitly, into a more conservative monetary policy stance, the fact being captured in our model by assuming, as a "reduced form", an interest rate rule that also reacts to fluctuations in the net worth of the CB besides inflation rate. Suppose that the monetary policy takes the general form of an interest rate rule that reacts to current values of inflation and CB net worth

$$i\,(t) = \rho\,(\pi\,(t)\,,\,\phi\,(t)) \tag{9}$$

The CB adjusts the nominal interest rate to minimize deviations from both a target rate of inflation π^* and a target level of net worth ϕ^*. By inducing changes in the portfolio composition of the private sector, the CB varies the nominal interest rate, and its policy rule implicitly determines money supply $M\,(t)$. Equation (8) now becomes

$$\dot{\phi} = [\rho\,(\pi, \phi) - \pi]\,\phi + (1-\gamma)\,\rho\,(\pi, \phi)\,m\,(\rho\,(\pi, \phi)\,, c) \tag{10}$$

For analytical tractability, we specify the following log-linear rule which imposes a ZLB on the nominal interest rate[20]:

$$i = i^H \exp\left[\frac{\alpha_\pi}{i^H}\left(\pi - \pi^H\right) + \frac{\alpha_\phi}{i^H}\left(\frac{\phi - \phi^*}{\phi^*}\right)\right] \tag{11}$$

where $i, i^H, \alpha_\pi > 0$, $\alpha_\phi \leq 0$. Also $\pi^H > 0$ and ϕ^* are the target levels of inflation rate and CB's net worth, respectively. We have implicitly assumed that the CB targets the steady state equilibrium values of inflation and net worth, so that at the desired steady state, $\pi = \pi^H$, $\phi = \phi^*$ and $i = i^H$. For a given policy rule of this type, it can always be rewritten equivalently in terms of the lower steady state equilibrium values. Taking $\alpha_\pi^L/i^L = \alpha_\pi^H/i^H$ and $\alpha_\phi^L/i^L = \alpha_\phi^H/i^H$, then

$$i^H \exp\left[\frac{\alpha_\pi^H}{i^H}\left(\pi - \pi^H\right) + \frac{\alpha_\phi^H}{i^H}\left(\frac{\phi - \phi^*}{\phi^*}\right)\right] = i^L \exp\left[\frac{\alpha_\pi^L}{i^L}\left(\pi - \pi^L\right) + \frac{\alpha_\phi^L}{i^L}\left(\frac{\phi - \phi^*}{\phi^*}\right)\right]$$

Notice from (11) that the ZLB is never attained but the nominal rate i can be arbitrarily close to zero. Moreover, from the equilibrium Fisher relation (3d), we obtain

$$\dot{Q} = \left[\rho\left(\pi, \phi\right) - \pi\right] Q - 1 \tag{12}$$

The evolution of the price of the real asset is determined by the rate of inflation, and more importantly, by the monetary policy rule. At the steady state equilibrium, $\dot{Q} = 0$ and $Q^* = \beta^{-1}$.

The CB is assumed to adjust the interest rate symmetrically[21] around a target reserve-to-money coverage ratio ϕ^*, which is the steady state equilib-

[20]Given this interest rate rule, we have

$$\frac{\partial i}{\partial \pi} = \alpha_\pi \frac{i}{i^H} \qquad \frac{\partial i}{\partial \phi} = \frac{\alpha_\phi}{\phi^*}\frac{i}{i^H}$$

In the higher-inflation steady state equilibrium, these become

$$\left.\frac{\partial i}{\partial \pi}\right|_{i=i^H} = \alpha_\pi \qquad \left.\frac{\partial i}{\partial \phi}\right|_{i=i^H} = \frac{\alpha_\phi}{\phi^*}$$

In the low-inflation liquidity trap steady state where $i = i^L$, since $\alpha_\pi^H/i^H = \alpha_\pi^L/i^L$ and $\alpha_\phi^H/i^H = \alpha_\phi^L/i^L$, the same relations hold true.

[21]In reality, the CB might be willing to maintain or even accumulate a positive level of net worth and loath to allow negative deviations to unfold. There may be some asymmetry and nonlinearity in the reaction function. This kind of asymmetry makes the CB balance sheet concerns all the more important in an unfavorable economic environment.

rium value of the CB's net worth ϕ. The assumption that the CB intends to maintain a relatively high reserve-to-money coverage ratio is in line with the BoJ's practice of maintaining an extra reserve fund for unexpected consequences of riskier policy actions.

When nominal interest rates draw close to the ZLB, conventional open market operations lose leverage. When unconventional policy measures such as the purchase of private securities and bonds are taken, the CB no longer expands money supply through the interest rate channel. Rather the CB is acquiring control over private firms or bailing out private companies, loosening money through the credit channel. To induce the private sector to hold extra money, the nominal interest rate has to be reduced.

Assumption B *Assume that (1) $i, i^* > 0, \forall \pi$; (2) The parameter values are such that there exists an inflation rate $\pi^H > -\beta$ at which $i^H = \pi^H + \beta$ and $\alpha_\pi^H > 1$;*[22] *(3) $\alpha_\pi > 0$ and $\alpha_\phi \leq 0$.*

Remark 1 *The monetary system under study is characterized by separate budget constraints for the CB and FA, resulting from a lack of fiscal support for monetary policy. This gives rise to the need for the CB to closely monitor its net worth. By our definition, a CB without fiscal backing is not fully independent from fiscal authority.*

Remark 2 *Condition (2) in Assumption B guarantees that the interest rate rule (11) and the steady state Fisher equation $i = \pi + \beta$ intersect twice, as a result of the fundamental nonlinearity in (11). This implies the existence of a second steady state equilibria, where $\pi = \pi^L < \pi^H$ and $\alpha_\pi^L < 1$. We assume that parameter values are such that the higher inflation steady state equilibrium is the one that the model economy intends to achieve through a combination of monetary and fiscal policies.*

As in Leeper (1991), monetary policy is termed *active* if $\alpha_\pi > 1$ and *passive* if $\alpha_\pi < 1$. To a certain extent, the assumption that $\alpha_\phi < 0$ in a simple deterministic setup like ours is debatable and needs careful elaboration. It describes the behavior of the CB in anticipation of an exogenous shock that

[22] Essentially, this assumption postulates that for any given $\phi = \bar{\phi}$, the nonlinear interest rate rule $i = \rho\left(\pi, \bar{\phi}\right)$ intersects the steady state Fisher equation $i = \pi + \beta$ (where $\dot{\lambda} = 0$) twice. This rules out cases of no intersection (non-existence of steady state equilibrium) and one single tangency point (unique stationary equilibrium). Indeed, as we later solve the model, ϕ^*, the steady state equilibrium value of ϕ, is shown to be unique.

causes large and unfavorable movement in the asset price Q. This assumption is made as a first step towards a more realistic model setup that accommodates stochastic shocks and in which both short-term nominal government bonds and the Lucas tree type asset are included in the CB's operational balance sheet. If the referred exogenous shock is not expected to occur, indeed we would have assumed $\alpha_\phi > 0$.

The present interest rate rule implies that, when its net worth is below the target level, the CB raises the nominal interest rate, selling off instead of piling up the Lucas type asset F. In normal circumstances, a CB with a below-target net worth level should lower the nominal interest rate by purchasing the real asset, which amounts to a policy of intervening in the market to support asset prices. This in general will increase the asset price Q and therefore improve the CB's balance sheet in the course of monetary intervention. However, suppose that the CB anticipates a large one-time reduction in Q in the near future, then the more real asset it accumulates, the worse its balance sheet problem will become once the collapse in asset price materializes. The fear of such a "sudden death" propels the CB to adopt a perverse rule that dictates $\alpha_\phi < 0$. Such a fear has been a key factor in preventing the BoJ from taking bold expansionary measures for years. In fact, even when the BoJ adopted the so-called "Quantitative Easing" policy in March 2001, which implies policy actions that will provide the economy liquidity beyond the level that keeps the nominal interest rate at zero, it only grudgingly sanctioned the outright purchase of private "asset-based" securities (ABS) as a temporary measure in June 2003. On September 30th, 2003, ABS made up only 1.4% of the BoJ's total assets. In a deflationary environment, reluctance to loosen monetary policy in anticipation of unfavorable asset price movement is counterproductive as it may push the economy into deeper recession.

We differentiate the degree of monetary conservatism according to the magnitude of the net worth targeting policy parameter α_ϕ.

Definition 2 *Let*

$$\bar{\alpha}_\pi = \frac{\pi^* + \beta}{\pi^* + (2-\eta)\,\beta} \in \left(\frac{\pi^* + \beta}{\pi^* + 2\beta}, 1 \right) \tag{13}$$

For any fixed $\alpha_\pi > \bar{\alpha}_\pi$*, define*[23]

$$\bar{\alpha}_\phi = \frac{\pi^* + \beta}{1-\eta} - \frac{\pi^* + (2-\eta)\,\beta}{1-\eta}\alpha_\pi \tag{14}$$

Monetary policy is conservative if $\alpha_\phi \leq \bar{\alpha}_\phi$*, it is moderate if* $\alpha_\phi \in (\bar{\alpha}_\phi, 0)$*, and it is liberal if* $\alpha_\phi = 0$*.*

We require that $\alpha_\pi > \bar{\alpha}_\pi$ so as to guarantee that $\bar{\alpha}_\phi < 0$. Notice that the value of $\bar{\alpha}_\phi$ varies with the specific value of policy parameter α_π selected by the monetary authority. The more active the CB is (larger α_π), the larger $\bar{\alpha}_\phi$ will be in terms of absolute value (larger $|\bar{\alpha}_\phi|$). When the monetary policy is passive (active), a relatively smaller (larger) $|\bar{\alpha}_\phi|$ is needed to classify the CB as conservative. Intuitively, monetary activism, interpreted as adherence to the Taylor Principle ($\alpha_\pi > 1$), contains elements that work against excessive net worth targeting and it necessarily entails a larger $|\bar{\alpha}_\phi|$ for a CB to be classified as conservative. In short, net worth targeting injects a degree of passivism into monetary policy-making.

Taking an annual target inflation rate $\pi^* = 1\%$ and an annual discount rate $\beta = 0.03$ as our benchmark values, since $\bar{\alpha}_\pi$ varies inversely with $\eta \in (0, 1)$, the value of $\bar{\alpha}_\pi$ falls into the empirical range $(0.57, 1)$. Empirical estimates of η range from $\eta = 0.5$ to $\eta = 0.95$.[24] If $\eta = 0.5$, then $\bar{\alpha}_\pi = 0.73$; if $\eta = 0.95$, then $\bar{\alpha}_\pi = 0.96$. For the same values of π^* and β, and taking $\alpha_\pi = 1.5$ as in Taylor (1993), then for $\eta = 0.5$, $\bar{\alpha}_\phi = -0.085$, and for $\eta = 0.95$, $\bar{\alpha}_\phi = -0.45$. If the monetary policy is passive, say, $\alpha_\pi = 0.97$, then for $\eta = 0.5$, $\bar{\alpha}_\phi = -0.027$, and for $\eta = 0.95$, $\bar{\alpha}_\phi = -0.005$. The cutoff values for the net worth targeting parameter are fairly small for empirically plausible parameter values, suggesting even a very little dose of balance sheet concern may have large practical consequences for aggregate stability. Changes in the nominal interest rate i caused by a unit change in ϕ/ϕ^* is just $\bar{\alpha}_\phi$ at $\alpha_\phi = \bar{\alpha}_\phi$.

We have assumed that $B^{CB} = 0$, so the CB cannot issue money against short-term government debt to directly finance the FA's fiscal spending. Instead of conducting open market operations by purchasing and selling short-term government bonds, the CB may provide or withdraw liquidity through open market transactions on the real asset F. From the policy rule (11) and the assumption that $\alpha_\phi < 0$, it is clear that net worth targeting potentially

[23]The values for $\bar{\alpha}_\pi$ and $\bar{\alpha}_\phi$ are derived from our equilibrium analysis.

[24]See, for example, Holman (1998).

undermines monetary policy activism. The mechanism through which net worth targeting may jeopardize monetary policy objectives can be exemplified as follows.

Suppose that, at a time of undesirably low inflation or even deflation ($\pi < \pi^*$), the CB wishes to loosen monetary policy by keeping the nominal interest rate as close to zero as possible.[25] This implies that the CB should purchase the real asset F in order to inject liquidity into the economy. However, the CB anticipates a large exogenous shock that will cause steep reduction in the asset price Q. The smaller the CB's stock of real asset (F^{CB}), the smaller the expected future capital loss ($\dot{Q}F^{CB}$) will be. In anticipation of this scenario, the CB will be reluctant to expand its holding of real asset, instead it will follow the "perverse" policy rule (11), selling off the asset F in an attempt to reduce exposure to balance sheet risk due to the anticipated collapse in the asset price Q. These actions will increase the nominal interest rate and tighten money supply, undoing much of the original expansionary efforts. The mechanism[26] works through the interest rate rule (11), when the CB is moderate or conservative ($\alpha_\phi < 0$).

The effect of monetary policy is immediately attenuated by actions entailed by the CB's desire to fend out balance sheet risks and to maintain a target level of net worth ϕ^*. The CB's balance sheet concerns as reflected in net worth targeting become a major impediment to implementing the policy rule to the full extent in the desired direction. Even under the Taylor Principle ($\alpha_\pi > 1$), local indeterminacy becomes a distinct possibility when the CB is conservative and unduly concerned with its balance sheet position ($\alpha_\phi < \bar{\alpha}_\phi$). In fact, net worth targeting builds an "automatic destabilizer" into the monetary transmission mechanism, and monetary conservatism may jeopardize the CB's capability of achieving its policy goals.

[25] This has indeed been the official "zero interest rate policy" followed by the Bank of Japan. Since March 2001, the BoJ went even further by implementing the so-called "quantitative easing policy", supplying extra liquidity to the Japanese banking system beyond what is necessary to maintain a zero nominal rate.

[26] The described interest rate rule and the related mechanism of adjustment are more reasonable in a realistic setup where the CB operates on both the real asset F^{CB} and short-term government bonds B^{CB}. Our simplifying assumption of $B^{CB} = 0$ renders the analysis less powerful. We intend to explore the model with two operational assets in a future paper, and the new model may lead to much more interesting dynamics and more realistic account of Japan's Great Recession in the 1990s and the US Great Depression in the 1930s.

3.2.2 Fiscal Authority

The fiscal authority (FA) has its own separate budget constraint

$$i(t)\frac{B(t)}{P(t)} = \tau(t) + \frac{\dot{B}(t)}{P(t)} + \gamma i(t)\frac{M(t)}{P(t)} \tag{15}$$

The government deficit, which consists of interest payments on its outstanding debt, must be financed by tax revenues, issues of new debt and seignorage transfer from the CB. The assumption of separate budget constraints and the lack of any transfer payments from the FA to the CB imply that taxes cannot be used to back up the value of money. In fact, from the representative household's perspective, the value of government liabilities issued by the CB can only be supported by assets held by itself. The CB does not expect any form of fiscal backing. The value of money, liabilities of the CB, is backed up by the real asset reserve, while bonds issued by the Treasury are supported by the stream of future tax revenues plus seignorage transfers from the CB.[27] Separate budget constraints imply that the CB and Treasury are insulated from each other and can go bankrupt independent of the balance sheet position of the other entity. Monetary-fiscal separation has important implications for macroeconomic stability.

In real terms, the fiscal budget constraint is

$$\dot{b}(t) = [i(t) - \pi(t)]\, b(t) - \tau(t) - \gamma i(t)\, m(t)$$

From the representative household's view, only the consolidated government budget constraint matters for its optimization problem. This takes the following form

$$\frac{\dot{M} + \dot{B}}{P} + F^{CB} + \tau = Q\dot{F}^{CB} + i\frac{B}{P}$$

The left-hand side is the sum of new issues of government liabilities plus the FA's tax and the CB's net seignorage revenues in terms of interest earnings, whereas the right-hand side consists of changes in the CB's reserve position and interest payment on Treasury bonds. In equilibrium, the consolidated government budget constraint is equivalent to

$$\dot{a} = (i - \pi)\, a - \bar{F} - im - \tau \tag{16}$$

[27] In a low-inflation environment, when the nominal interest rate is close to zero, seignorage revenues are insignificant.

Combined with the household's budget constraint, and given that $F = \bar{F}$ and $\dot{F} = 0$, we obtain the social resource constraint

$$c = \bar{F} + y \tag{17}$$

Therefore, consumption is constant at $\bar{c} \equiv \bar{F} + y$ because y is exogenously fixed.

We simplify the government's fiscal policy-making by assuming that the Treasury follows certain tax rules. The simplest rule has tax per household taking the form of a constant lump-sum

$$im + \tau^C = \bar{\tau} \tag{18}$$

The FA can also follow variable fiscal rules, letting the tax change, respectively, with the level of real debt, real total government liabilities, or real asset holdings of the representative household

$$\begin{aligned} im + \tau_1^V &= \delta_0 + \delta_1 b \\ &= \delta_0 - \delta_1 Q\bar{F} + \delta_1 (a + \phi) \end{aligned} \tag{19a}$$

$$\begin{aligned} im + \tau_2^V &= \delta_0 + \delta_2 (m + b) \\ &= \delta_0 - \delta_2 Q\bar{F} + \delta_2 (a + \phi + m) \end{aligned} \tag{19b}$$

$$\begin{aligned} im + \tau_3^V &= \delta_0 + \delta_3 \left(m + b + QF^H\right) \\ &= \delta_0 + \delta_3 a \end{aligned} \tag{19c}$$

where the subsidy $\delta_0 < 0$ and tax rates $\delta_1, \delta_2, \delta_3 \in (0, 1)$ are constant and exogenously set by the FA. With the constant tax rule (18),

$$\dot{a} = (i - \pi) a - \bar{F} - \bar{\tau} \tag{20}$$

Under the proposed variable tax rules (19a)-(19c), we have, respectively,

$$\dot{a} = (i - \pi - \delta_1) a + \delta_1 Q\bar{F} - \delta_1 \phi - \delta_0 - \bar{F} \tag{21a}$$

$$\dot{a} = (i - \pi - \delta_2) a + \delta_2 Q\bar{F} - \delta_2 (\phi + m) - \delta_0 - \bar{F} \tag{21b}$$

$$\dot{a} = (i - \pi - \delta_3) a - \bar{F} - \delta_0 \tag{21c}$$

We classify the constant tax rule (18) and the variable tax rules (19a)-(19c) as follows.

Definition 3 *For $\delta_j > 0$, $j = 1, 2, 3$, a variable fiscal rule (19) is said to be active or non-Ricardian if $i(t) - \pi(t) - \delta_j > 0$, and it is passive or Ricardian otherwise. The constant tax rule (18) is always active.*

With an active fiscal rule, the Treasury commits to a tax policy that is consistent with intertemporal budget constraint only under a subset of all possible time paths for prices. Since the intertemporal budget constraint always holds in equilibrium, the Treasury thereby plays a role in the determination of the equilibrium path of prices. Under a passive fiscal rule, the intertemporal budget constraint always holds under any time path of prices, so long as the Fisher equation holds, therefore the FA has no role in determining the equilibrium path of prices.

4 Solving the Model: Local Analysis

From equations (3a), (3c), (10) and (17), we obtain

$$\dot{\pi} = \beta \frac{\lambda}{\rho_\pi \lambda_i} - [\rho(\pi, \phi) - \pi] \left(\frac{\lambda}{\rho_\pi \lambda_i} + \frac{\rho_\phi}{\rho_\pi} \phi \right) - (1 - \gamma) \frac{\rho_\phi}{\rho_\pi} \rho(\pi, \phi) m \quad (22)$$

where $\lambda = \lambda(i) = \lambda(\rho(\pi, \phi)) > 0$, and $\lambda_i = u_{cm}(c, m) m_i(i, \bar{c})$. With our Cobb-Douglas utility specification, consumption and money are Edgeworth complements ($u_{cm} > 0$), hence $\lambda_i < 0$.

Define the perfect foresight equilibrium for the flexible price model as follows:

Definition 4 *A flexible-price perfect foresight equilibrium (PFE) is defined as functions of time* $\{c, a, m, \pi, \phi, i, Q\}_{t=0}^{\infty}$ *and* $P(0)$ *such that: (1) they solve the household's constrained utility maximization problem, taking as given initial conditions* $A(0) = M(0) + B(0) + Q(0) F^H(0)$*, price* P *and government policies* $\{\tau,\ i\}$*; (2) all markets clear*[28]*, i.e.,* $c = \bar{F} + y$ *and* $F^H + F^{CB} = \bar{F}$*.*

Alternatively, a more operational definition of PFE for the flexible-price model is

Definition 5 *A flexible-price perfect foresight equilibrium (PFE) is defined as functions of time* $\{\pi, \phi, Q, a\}_{t=0}^{\infty}$ *and* $P(0)$ *that satisfy the dynamical system (10), (12), (20) or (21), and (22) and the Transversality Condition (4).*

[28]In equilibrium, households' demand for nominal financial assets must equal the government's net supply of liabilities. Money and bond markets clear as a consequence of our notation.

In this second definition of PFE, to arrive at the equilibrium dynamical system, we have used the equilibrium Fisher relation (3d) and the money demand function (6), which are derived from the household's optimality conditions. We have also used the stipulated monetary rule (11), the fiscal rule (18) or (19), and the social resource constraint (17).

4.1 Equilibria Converging to the Steady State

Solving the model for the steady state equilibrium, we obtain, respectively, the following unique stationary values for ϕ and Q

$$\begin{aligned} \phi^* &= -\frac{(1-\gamma)(1-\eta)}{\beta\eta}\left(\bar{F}+y\right) \\ Q^* &= \beta^{-1} \end{aligned}$$

The unique steady state value for a, under the constant tax rule (18), is

$$a^* = \beta^{-1}\left(\bar{F}+y\right)$$

and under the proposed variable tax rules (19a)-(19c), we have, respectively,

$$\begin{aligned} a^* &= \beta^{-1}\bar{F} + \frac{\delta_1(1-\eta)}{(\beta-\delta_1)\beta\eta}\left(\bar{F}+y\right) + \frac{\delta_0}{\beta-\delta_1} \\ a^* &= \beta^{-1}\bar{F} + \frac{\delta_2(1-\eta)}{(\beta-\delta_2)\beta\eta}\frac{\pi^*+2\beta}{\pi^*+\beta}\left(\bar{F}+y\right) + \frac{\delta_0}{\beta-\delta_2} \\ a^* &= (\beta-\delta_3)\left(\bar{F}+\delta_0\right) \end{aligned}$$

With our log-linear interest rate rule (11), the ZLB on nominal interest rate i never binds ($i > 0$). Because of nonlinearity in the interest rate rule and the assumption that $\alpha_\pi^H > 1$, corresponding to ϕ^*, there are two different steady state equilibria[29] as described in detail in Benhabib, Schmitt-Grohé and Uribe (2001a, 2002): the higher-inflation steady state equilibrium

[29]For the unique steady state value ϕ^* such that $\dot{\phi} = 0$, the interest rate rule becomes

$$i = i^* \exp\left[\frac{\alpha_\pi}{i^*}(\pi - \pi^*)\right]$$

The interest rate rule is nonlinear and convex. Combined with the steady state Fisher equation $i = \pi + \beta$, it results in two steady states, with distinct steady state inflation rates π^H and π^L.

In a system with globally linear interest rate rule, three equations in three unknowns lead to a unique solution. In a system with a fundamentally nonlinear interest rate rule (11), corresponding to ϕ^*, there emerge two stationary equilibria.

$\left(\pi^H, \phi^*, Q^*, a^*\right)$ where the monetary policy is locally active ($\alpha_\pi > 1$); and the low-inflation steady state equilibrium $\left(\pi^L, \phi^*, Q^*, a^*\right)$ where the monetary policy is locally passive ($\alpha_\pi < 1$). We assume that the values of the fundamental and monetary policy parameters are such that the target inflation rate π^H is at a satisfactorily high level. As i approaches the zero lower bound, the interest rate rule becomes increasingly passive and the low-inflation steady state equilibrium involves a value of inflation π^L well below the intended target level. In fact, π^L can be negative, and the nominal interest rate i^L is low. The low-inflation steady state equilibrium therefore has the characteristics of a "liquidity trap". Henceforth we refer to the higher-inflation steady state equilibrium $\left(\pi^H, \phi^*, Q^*, a^*\right)$ as the "target equilibrium" and the low-inflation steady state equilibrium $\left(\pi^L, \phi^*, Q^*, a^*\right)$ as the "liquidity trap" equilibrium.

To illustrate the emergence of the liquidity trap equilibrium because of the fundamental nonlinearity in the interest rate rule, in Figure **??**, we borrow a diagram from Benhabib, Schmitt-Grohé and Uribe (2001a, 2002).

ftbpFU336.9375pt221.8125pt0ptTarget and Liquidity Trap Steady State Equilibria under a Nonlinear Interest Rate RuleBSU03BSU03.wmf

We use $(\pi^*, \phi^*, Q^*, a^*)$ to denote generically both the higher and low-inflation steady state equilibria, where $\dot{\pi} = \dot{\phi} = \dot{Q} = \dot{a} = 0$ and $\rho(\pi^*, \phi^*) = \pi^* + \beta = \pi^* + r$. Price stability, the main policy goal of the central bank, entails a targeting of the desirable stationary equilibrium with $\pi^* = \pi^H$, where $\dot{\pi} = 0$.

We focus on equilibria in which the sequences $\{\pi, \phi, Q\}$ converge to steady state equilibria (π^*, ϕ^*, Q^*). Under a variable fiscal rule of the type (19c), all sequences $\{\pi, \phi, Q\}$ converging to (π^*, ϕ^*, Q^*) satisfy the transversality condition (4), so the latter places no restriction on the equilibrium solutions and any sequence $\{\pi, \phi, Q\}$ that satisfies equations (10) and (22) can be supported as a perfect foresight equilibrium. In general, under active or non-Ricardian fiscal rules, the $\{\pi, \phi, Q\}$-sequences that converge to (π^*, ϕ^*, Q^*) also involve those with the household's financial portfolio a growing at such a rate that the transversality condition (4) is violated. Therefore equations (10) and (18) or (19) impose restrictions on the set of sequences $\{\pi, \phi, Q\}$ that are consistent with our definition of perfect foresight equilibrium. In particular, only those $\{\pi, \phi, Q\}$ converging to (π^*, ϕ^*, Q^*) that also imply a sequence $\{a\}$ that converges to a constant a^* ($\dot{a} = 0$) will be considered a PFE. We can therefore study the dynamic properties of the model by focusing on a linear approximation of the equilibrium dynamical system (10), (12),

(20) or (21), and (22).

A first-order Taylor expansion around the steady state $(\pi^*, \phi^*, Q^*, a^*)$ delivers the following linear system

$$\begin{bmatrix} \dot{\pi} \\ \dot{\phi} \\ \dot{Q} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & 0 & 0 \\ J_{21} & J_{22} & 0 & 0 \\ J_{31} & J_{32} & \beta & 0 \\ J_{41} & J_{42} & J_{43} & J_{44} \end{bmatrix} \begin{bmatrix} \pi - \pi^* \\ \phi - \phi^* \\ Q - Q^* \\ a - a^* \end{bmatrix} \tag{23a}$$

where the non-zero entries of the Jacobian matrix J are evaluated at the steady state. Denote as J^S the 2×2 submatrix consisting of $\{J_{11}, J_{12}, J_{21}, J_{22}\}$. Notice that J^S is independent of the fiscal policy which enters the dynamical system through a. It is easy to see that two of the eigenvalues of J are $J_{33} = \beta$ and J_{44}, and the remaining two eigenvalues of J are those of J^S. Under our Cobb-Douglas preference specification (5) and a log-linear interest rate rule (11), the elements of J^S can be simplified as follows (see Appendix A for derivation)

$$J_{11} = \left(1 - \frac{1}{\alpha_\pi}\right)\left(\frac{\pi^* + \beta}{1-\eta} - \alpha_\phi\right) \tag{23b}$$

$$J_{12} = \frac{\alpha_\phi}{\alpha_\pi \phi^*}\left(\frac{\pi^* + \eta\beta}{1-\eta} - \alpha_\phi\right) \tag{23c}$$

$$J_{21} = (\alpha_\pi - 1)\,\phi^* \tag{23d}$$

$$J_{22} = \alpha_\phi + \beta \tag{23e}$$

Observe that $J_{12} < 0$ and $sgn\,(J_{11}) = sgn\,(\alpha_\pi - 1)$. In particular, if the monetary policy is active ($\alpha_\pi > 1$), then $J_{11}, J_{21} > 0$; if the monetary policy is passive ($\alpha_\pi < 1$), $J_{11}, J_{21} < 0$. In the case that the CB is not concerned with its balance sheet ($\alpha_\phi = 0$), $J_{12} = 0$ and $J_{22} = \beta$.

Corresponding to the constant tax rule (20), we have

$$J_{44}^C = \beta$$

For alternative variable fiscal rules, linearizing around the steady state yields, respectively

$$\begin{aligned} J_{44}^{V1} &= \beta - \delta_1 \\ J_{44}^{V2} &= \beta - \delta_2 \\ J_{44}^{V3} &= \beta - \delta_3 \end{aligned}$$

For variable fiscal rules, determinacy and stability results will differ from those in the case of a constant tax rule ($\tau = \bar{\tau}$), because for $j = 1, 2, 3$, the fourth eigenvalue J_{44}^{Vj} ($= \beta - \delta_j$) of the Jacobian matrix J can now take non-positive values, *i.e.*, $\beta - \delta_j \leq 0$. We only consider the cases of either active or passive fiscal rules, where $\beta - \delta_j \neq 0$, since it is unlikely that the Treasury happens to follow a knife-edge "neutral" rule with $\beta - \delta_j = 0$. In the case of $\beta - \delta_j < 0$, the fiscal authority passively adjusts taxes to balance the consolidated government budget.

4.1.1 Without Net Worth Targeting ($\alpha_\phi = 0$)

We first investigate the issue of local determinacy in a monetary system where the CB is sufficiently independent that it can decide the size and timing of fiscal backing by the FA for monetary policy. The CB is therefore unconcerned with its balance sheet in its conduct of policy, and it does not need to monitor its net worth position ($\alpha_\phi = 0$).

Studying the signs the eigenvalues for the Jacobian matrix J of the linearized dynamical system allows us to determine whether steady state equilibria are locally unique. Our local determinacy results for a CB without balance sheet concerns (*i.e.*, no net worth targeting) conform to those obtained in Leeper (1991), except that now we have two steady state equilibria arising from the fundamental nonlinearity in the interest rate rule.

In particular, under the non-Ricardian fiscal rules, which include the constant tax rule (18) and variable tax rules (19a)-(19c) with $r - \delta > 0$, no stable equilibrium solution exists under an active monetary policy rule. The low-inflation liquidity trap equilibrium, on the other hand, is locally unique under the mix of an active fiscal and a passive monetary policy. Under this set of policy rules, the liquidity trap equilibrium becomes a focal point for economic agents' expectations. Once an economy falls into such a trap, it is likely that it stays there for a long time, unless policies shift and expectations change. This may have been the reason for the extraordinarily long deflationary recession in Japan, where the government expanded fiscal spending and lavished funds on public projects in the mid-1990s, but the Bank of Japan was slow to react to the ongoing recession, and these fiscal stimulus packages turned out to be largely ineffective and had little effect on public expectations. We summarize these results in the following Proposition.

Proposition 1 (Non-Ricardian Fiscal Rules) *Assume that* $\alpha_\phi = 0$*, then under the non-Ricardian fiscal rules: (1) If the monetary policy is active*

($\alpha_\pi > 1$), no stable equilibrium solution exists; (2) If the monetary policy is passive ($\alpha_\pi < 1$), the liquidity trap equilibrium where $\pi = \pi^L$ is locally determinate.

Proof. See Appendix B. ■

Under Ricardian fiscal rules, *i.e.*, variable tax rules with $r - \delta < 0$, fiscal solvency is assured. When combined with a Ricardian tax rule, it is well-known that an interest rate rule that observes the Taylor Principle ($\alpha_\pi > 1$) ensures that the target steady state equilibrium ($\pi = \pi^H$) is locally determinate. Local uniqueness of the target equilibrium under such a fiscal-monetary policy mix has been shown by Leeper (1991), Sims (1994), Woodford (1994, 1996, 2003) and Clarida, Galí and Gertler (2000). In fact, this result has become a main justification for any CB wishing to follow an active Taylor rule, since it implies that monetary activism will be sufficient to stabilize the economy at the desired equilibrium once the Treasury takes care of fiscal solvency of the government. On the contrary, if the monetary policy is passive ($\alpha_\pi < 1$), the liquidity trap equilibrium with $\pi = \pi^L$ is locally indeterminate and a passive monetary policy stance may destabilize the economy by inducing expectations-driven fluctuations around the liquidity trap.

In our local analysis, the possibility that a passive monetary policy combined with an active fiscal rule leads to a locally unique liquidity trap is disturbing, and monetary activism in the form of Taylor Principle is justified as one way to prevent the economy from drifting away from the target equilibrium towards a deflationary trap.[30] Furthermore, monetary activism has the power to stabilize the real economy by ensuring local uniqueness of the desired equilibrium once the right kind of fiscal rules are followed. These results, proved in our model setup, are summarized in Proposition 2.

Proposition 2 (Ricardian Fiscal Rules) *Assume that $\alpha_\phi = 0$. Under Ricardian fiscal rules ($r - \delta < 0$), (1) If the monetary policy is active ($\alpha_\pi > 1$), the target equilibrium where $\pi = \pi^H$ is locally determinate; (2) If the monetary policy is passive ($\alpha_\pi < 1$), the liquidity trap equilibrium where $\pi = \pi^L$ is locally indeterminate.*

[30]In the global analyses of Benhabib, Schmitt-Grohé and Uribe (2001a, 2002), liquidity trap equilibrium is possible even when the CB follows an interest rate rule that satisfies the Taylor Principle. This is a result of the global dynamics beyond the immediate neighborhood of the steady states that they uncovered. However, the interest rate rule becomes locally passive as the economy approaches the liquidity trap steady state equilibrium.

Proof. See Appendix B. ■

Clearly, without net worth targeting, artificially setting up separate budget constraints will have no effect on the flexible-price model's equilibrium solutions. In fact, separate budget constraints are equivalent to one single consolidated government budget constraint. The above results are standard and are provided in our own model setup for comparability with results in the following subsection, where CB balance sheet concerns and net worth targeting are introduced in the model.

4.1.2 With Net Worth Targeting ($\alpha_\phi < 0$)

We now introduce CB balance sheet concerns and net worth targeting ($\alpha_\phi < 0$) into the model. According to our earlier definition, in this case the CB is insufficiently independent from the FA, and fiscal support for monetary policy will not be forthcoming once the CB runs into negative net worth. We show, in the following two propositions, that CB balance sheet concerns and net worth targeting have a large impact on macroeconomic stability and may indeed be responsible for much of the BoJ's policy conservatism and prolonged deflationary recession.

Proposition 3 (Non-Ricardian Fiscal Rules) *Assume that $\alpha_\phi < 0$ and the FA follows a non-Ricardian tax rule. Then, (1) If the monetary policy is passive ($\alpha_\pi \in (\bar{\alpha}_\pi, 1)$), then independent of the degree of CB conservatism (α_ϕ), the low-inflation equilibrium is locally determinate; (2) If the monetary policy is active ($\alpha_\pi > 1$) and if the CB is conservative ($\alpha_\phi < \bar{\alpha}_\phi$), then the target equilibrium is locally indeterminate; (3) If the monetary policy is active ($\alpha_\pi > 1$) and if the CB is moderate ($\alpha_\phi > \bar{\alpha}_\phi$), then no stable equilibrium solution exists.*

Proof. See Appendix B. ■

First, notice that from Propositions 3 and 4 (see below), a passive monetary policy renders the liquidity trap equilibrium locally unique when the fiscal rule is active, and the liquidity trap equilibrium is locally indeterminate when the fiscal rule is passive. This result holds true regardless of the degree of the CB's balance sheet concerns (α_ϕ). Indeed, it is true whether the CB targets net worth or not. Intuitively, CB balance sheet concerns and net worth targeting inject a further dose of passivism in monetary policy making, therefore they preserve any local determinacy results that already obtain under a passive monetary rule.

Whether a CB is concerned with its balance sheet or not, the policy mix of a non-Ricardian fiscal rule and a passive interest rate rule always produces a locally unique liquidity trap equilibrium, providing an undesirable focal point for the formation and coordination of people's expectations. Once an economy falls into a liquidity trap, it is likely that it stays in the trap for a long time. As we discussed earlier, fiscal spending in Japan increased greatly in the mid-1990s but the monetary policy remained quite cautious. Various fiscal stimulus packages failed to have any impact on aggregate demand and on shifting the public expectations, let alone jump-start the economy. The result is a prolonged period of deflationary recession that has continued unabated until this day.

When a non-Ricardian fiscal rule is coupled with an active monetary policy, either no stable equilibrium solutions exist when the CB is moderate or when it shows no concern at all with its balance sheet, or the target equilibrium becomes locally indeterminate because the CB is paying too much attention to its net worth.

We now consider local dynamic properties of our model when the CB has balance sheet concerns and the FA follows passive or Ricardian fiscal rules. Here CB balance sheet concerns have a major impact on the existence and local uniqueness of stable equilibrium solutions to the model.

Proposition 4 (Ricardian Fiscal Rules) *Assume that $\alpha_\phi < 0$ and the FA follows a Ricardian tax rule ($r - \delta < 0$). Then, (1) If the monetary policy is passive ($\alpha_\pi \in (\bar{\alpha}_\pi, 1)$), then independent of the degree of CB conservatism (α_ϕ), the liquidity trap equilibrium is locally indeterminate; (2) If the monetary policy is active ($\alpha_\pi > 1$) and if the CB is conservative ($\alpha_\phi < \bar{\alpha}_\phi$), then the target equilibrium is locally indeterminate; (3) If the monetary policy is active ($\alpha_\pi > 1$) and if the CB is moderate ($\alpha_\phi > \bar{\alpha}_\phi$), then the target equilibrium is locally determinate.*

Proof. See Appendix B. ■

As our results indicate, a passive monetary policy stance ($\alpha_\pi \in (\bar{\alpha}_\pi, 1)$) leads to local indeterminacy of the low-inflation steady state equilibrium. In general, monetary passivism should be avoided. Indeed, local determinacy of the target equilibrium under monetary activism has been the main justification of the use of active Taylor rule in the central banks. However, results in the Proposition 4 also suggest that adherence to active monetary policy is not a panacea. It is a necessary but not a sufficient condition for a CB to stabilize the economy at the target higher-inflation steady state equilibrium. It is here that CB balance sheet concerns enter into consideration and become an element that may potentially destabilize the economy. When the Taylor Principle is adhered to but the CB is sufficiently concerned with its balance sheet position ($\alpha_\phi < \bar{\alpha}_\phi$), the desired steady state equilibrium becomes locally indeterminate. If the CB is conservative, even though it follows an active interest rate rule, there emerge an infinite number of stable equilibrium solutions around the target equilibrium, which are also compatible with the same active monetary rule. The well-known result that an active Taylor rule, when combined with a passive fiscal rule, leads to local uniqueness of the target equilibrium and helps stabilize the economy, is completely reversed.

If the CB is moderate ($\alpha_\phi \in (\bar{\alpha}_\phi, 0)$), the result that Taylor Principle leads to local determinacy of the target equilibrium is preserved, so macroeconomic stability can still be achieved if the CB is only slightly concerned with it balance sheet. However, monetary conservatism ($\alpha_\phi < 0$), be it weak or strong, ties the hands of central bankers and may destabilize the economy. The "success formula" of monetary activism and fiscal passivism no longer suffices to guarantee stabilization of the economy when the CB is sufficiently constrained by concerns over its own financial soundness. The desirable target equilibrium becomes locally indeterminate and the economy is more likely to be driven into a deflationary recession of the type observed in Japan, even if the fiscal-monetary policy mix follows the recommended rules. For this policy formula to work, institutional reform that ensures fiscal backing for monetary policy is necessary.

4.2 Equilibria Converging to a Deterministic Cycle

In this section, we study conditions under which there exist perfect foresight equilibria in which $\{\pi, \phi, Q, a\}$, instead of converging to a steady state equilibrium $(\pi^*, \phi^*, Q^*, a^*)$, converge to a deterministic periodic cycle. Here we entertain local cyclical equilibrium dynamics that are bounded in a small neighborhood around the steady state equilibrium, but they do not converge asymptotically to the steady state equilibrium.

Fundamental to the dynamics around a deterministic cycle is the existence of a local Hopf bifurcation at some critical value $\bar{\alpha}_\phi$ of the bifurcation parameter α_ϕ, which in our model represents the degree of a CB's balance sheet concerns. At the ascertained bifurcation value, a unique limit cycle emerges when the steady state equilibria change their stability.[31] In its turn, the existence of a Hopf bifurcation implies the existence of a family of cycles for values of α_ϕ in a small neighborhood to the left or right of $\bar{\alpha}_\phi$. The existence of a stable limit cycle implies cyclical fluctuations in (π, ϕ, Q, a) which still satisfy all equilibrium conditions, but the equilibrium dynamics do not converge to the steady state equilibria.

4.2.1 With Net Worth Targeting

Taking the net worth targeting parameter α_ϕ as the bifurcation parameter, we first prove the existence of a Hopf bifurcation when the CB is concerned with its balance sheet and follows an interest rate rule that targets both inflation and the CB net worth. In the case that the FA follows a constant lump sum fiscal rule (18), we have the following result.

Proposition 5 (Constant Fiscal Rule) *Assume that $\alpha_\phi < 0$, then under active monetary policy ($\alpha_\pi > 1$) and constant fiscal rule (18), a Hopf Bifurcation exists at the target steady state equilibrium $\left(\pi^H, \phi^*, Q^*, a^*\right)$ with $\alpha_\phi = \bar{\alpha}_\phi$ as the bifurcation value.*

Proof. See Appendix B. ■

When the FA follows variable fiscal rules (19), as long as $r - \delta \neq 0$, we have essentially the same result.

Proposition 6 (Variable Fiscal Rules) *Assume that $\alpha_\phi < 0$, then under active monetary policy ($\alpha_\pi > 1$) and variable fiscal rules (19) where $r - \delta \neq 0$, a Hopf Bifurcation exists at the steady state equilibrium $\left(\pi^H, \phi^*, Q^*, a^*\right)$ with $\alpha_\phi = \bar{\alpha}_\phi$ as the bifurcation value.*

Proof. See Appendix B. ■

Propositions 5 and 6 state that under an active interest rate rule, for both constant and variable fiscal rules, a local Hopf bifurcation emerges at the intended target equilibrium $\left(\pi^H, \phi^*, Q^*, a^*\right)$, when the net worth targeting parameter α_ϕ passes through the cutoff value $\bar{\alpha}_\phi$. Therefore, when a CB is concerned with its balance sheet, adherence to the Taylor Principle

[31] Whether the Hopf bifurcation is supercritical or subcritical, *i.e.*, whether the steady state equilibrium generates a stable or unstable limit cycle as α_ϕ passes through the bifurcation value $\bar{\alpha}_\phi$, depends on the sign of the Lyapunov coefficient of our dynamical system.

leads to the emergence of a unique deterministic cycle at the desirable target steady state equilibrium. This result obtains irrespective of the magnitude of monetary activism ($\alpha_\pi > 1$) or the degree of CB balance sheet concerns ($\alpha_\phi < 0$).

If the Lyapunov coefficient of the dynamical system is negative, the uncovered Hopf bifurcation will be supercritical, the target equilibrium will then generate a stable, attracting limit cycle as α_ϕ passes through the bifurcation value $\bar{\alpha}_\phi$. There exist values of α_ϕ below $\bar{\alpha}_\phi$, for which any equilibrium trajectory $\{\pi, \phi, Q, a\}$ starting out in a small neighborhood close to the target steady state $\left(\pi^H, \phi^*, Q^*, a^*\right)$ converges to the limit cycle. The PFE will be indeterminate, and π eventually fluctuates in the cycle, dampening any hope of achieving price stability.

At the bifurcation value $\alpha_\phi = \bar{\alpha}_\phi$ and the non-hyperbolic steady state equilibrium $\left(\pi^H, \phi^*, Q^*, a^*\right)$, the Jacobian matrix J has a simple pair of pure imaginary eigenvalues and the other eigenvalues has no zero real part. For each α_ϕ near the bifurcation value $\bar{\alpha}_\phi$, there corresponds a unique equilibrium $(\pi^*_\alpha, \phi^*_\alpha, Q^*_\alpha, a^*_\alpha)$ near the steady state equilibrium $\left(\pi^H, \phi^*, Q^*, a^*\right)$. When the Jacobian matrix J crosses the imaginary axis at $\bar{\alpha}_\phi$, the dimensions of the stable and unstable manifolds of this unique equilibrium point change, and a periodic orbit or limit cycle is created as the stability properties of this equilibrium point $(\pi^*_\alpha, \phi^*_\alpha, Q^*_\alpha, a^*_\alpha)$ change. The extreme sensitivity of the local stability properties of the steady state equilibrium $\left(\pi^H, \phi^*, Q^*, a^*\right)$ to a small change in the degree of the CB conservatism around the bifurcation value $\bar{\alpha}_\phi$ is disturbing. An institutional framework that eliminates CB balance sheet concerns and the very cause of monetary conservatism by promoting closer fiscal-monetary cooperation and more CB independence is important for ensuring macroeconomic stability.

4.2.2 Without Net Worth Targeting

When the CB is unconcerned with it balance sheet, instead of α_ϕ, we take the CB's other policy parameter α_π as the bifurcation parameter. In this case, by inspection, the eigenvalues of the submatrix J^S are purely imaginary only if the trace of J^S is zero, or equivalently, $\alpha_\pi = \bar{\alpha}_\pi < 1$. But this implies that the determinant of J^S becomes $-\beta^2$, therefore no simple pair of purely imaginary eigenvalues can exist at the trial bifurcation value $\alpha_\pi = \bar{\alpha}_\pi$. Therefore the Hopf bifurcation cannot occur in this case. We first present the result for the case in which a Treasury follows the constant fiscal rule (18).

Proposition 7 (Constant Fiscal Rule) *Assume that* $\alpha_\phi = 0$*, and take* α_π *as the free parameter, then under the constant fiscal rule (18), the Hopf Bifurcation does not occur at the steady state equilibrium* $\left(\pi^L, \phi^*, Q^*, a^*\right)$*.*

Proof. See Appendix B. ■

This result differs from that contained in Proposition 7 in Benhabib, Schmitt-Grohé and Uribe (2001b). However, their result, obtained in a sticky-price model, relies on how money enters into utility and production functions, and on the specific functional form assumed by the interest rate rule at the target steady state equilibrium. The result for the cases of variable fiscal rules is similar and we summarize it as follows.

Proposition 8 (Variable Fiscal Rules) *Assume that* $\alpha_\phi = 0$ *and* $r - \delta \neq 0$*. Taking* α_π *as the free parameter, then under variable fiscal rules (19), the Hopf Bifurcation does not occur at the steady state equilibrium* $\left(\pi^L, \phi^*, Q^*, a^*\right)$*.*

Proof. See Appendix B. ■

The non-existence result for the Hopf bifurcation when the CB does not target net worth is significant, and it holds regardless of the type of fiscal rules in place, be it constant or variable, active or passive. It is valid as long as the CB is liberal and does not target its own net worth. This generally holds true when monetary policy has full fiscal backing. Therefore, to avoid the type of bifurcation results derived in the last sub-section, and to rid the economy of possibly dramatic changes in the local stability properties of the target equilibrium due to vary small variations in the degree of CB balance sheet concerns, it is best to proceed with reforms that would allow the CB to decide on the size and timing of fiscal backing for monetary policy. Indeed, enhanced CB independence helps to eliminate its balance sheet concerns and stabilize the economy.

5 Conclusion

In a new era of price stability, monetary policy making faces new challenges, such as the risk of deflationary recessions, and the possibility of short-term nominal interest rates hitting the zero lower bound (ZLB) and of an economy falling into a "liquidity trap". It has long been argued, albeit informally, that in the face of major stock and housing market crashes in the late 1980s, the

Bank of Japan (BoJ) has done too little and acted too late to prevent the Japanese economy from sliding into a deflationary recession, and the BoJ's concern over its own balance sheet position has been repeatedly cited as the reason for its policy conservatism and inertia.

In this paper, we provide a fully articulated theoretical foundation for the view that a CB's balance sheet concerns may hinder monetary policy "activism" needed for achieving and sustaining macroeconomic stability in a low-inflation environment, thereby opening the door for local indeterminacy and bifurcation. As in Benhabib, Schmitt-Grohé and Uribe's (2001a, 2002), we study the situation where an interest rate rule is constrained by the ZLB. Nonlinearity in the interest rate rule implies the emergence of a second steady state equilibrium that has the properties of a liquidity trap.

To analyze a monetary system where the CB is not fully independent and there is no automated fiscal support for monetary policy, we model the CB and the FA as two distinct entities, each with its own budget constraint and can potentially go bankrupt separately. To incorporate a CB's balance sheet concerns into our model, we augment the conventional interest rate rule with net worth targeting, so that the CB adjusts the interest rate in response to deviations of both inflation rate and net worth from the target levels.

It has now become conventional wisdom that the combination of a passive fiscal rule that guarantees fiscal solvency and an active Taylor rule ensures local determinacy of the desired target steady state equilibrium and therefore promotes macroeconomic stability. Based on this result, active interest rate rules have been advocated as the right policy prescription for any CB that strives for price stability. Our analysis demonstrates that the introduction of CB's balance sheet concerns into a simple flexible-price model reverses this well-known result of Taylor stability. With net worth targeting and insufficient CB independence, *i.e.*, no fiscal backing for monetary policy, monetary activism embodied in the Taylor Principle cannot be applied to its full extent. Monetary conservatism, resulting from the CB's balance sheet concerns, may lead to local indeterminacy of the desired target equilibrium: even though the CB follows an active interest rate rule, there emerge an infinite number of stable equilibrium solutions around the target equilibrium, which are also compatible with the same active monetary rule. It is no longer guaranteed that an active monetary policy combined with a passive fiscal rule will achieve aggregate stability.

Furthermore, CB balance sheet concerns also lead to the emergence of a local Hopf bifurcation around the target equilibrium, suggesting extreme sensitivity of the local stability properties of the target equilibrium to a small change in the net worth targeting parameter α_ϕ. CB balance sheet concerns build into the economic system an inherent tendency towards structural instability, making it more likely for an economy to drift towards the liquidity trap equilibrium even under an active interest rate rule.

These unpleasant local indeterminacy results and the extreme sensitivity of the structural stability of the model to a small variation in the policy parameter α_ϕ are unsettling. To achieve macroeconomic stability, institutional reforms are necessary to eliminate the CB's balance sheet concerns and to enhance CB independence, thereby providing full fiscal backing for monetary policy that is automatic, immediate and readily understood by the public. Better monetary-fiscal cooperation may be imperative for an economy striving to achieve macroeconomic stability in the face of deflationary pressure and possibly binding ZLB. Our results have important import for the institutional design for the Bank of Japan and other central banks with similar balance sheet structure and monetary-fiscal relationship.

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Appendices

A Derivation of Equation (22)

Linearizing around the steady state equilibria $(\pi^*, \phi^*, Q^*, a^*)$, we obtain

$$\begin{aligned}
J_{11} &= \frac{\beta\left(\rho_\pi\lambda_i\right)^2 - \beta\lambda\left(\rho_{\pi\pi}\lambda_i + \rho_\pi^2\lambda_{ii}\right)}{\left(\rho_\pi\lambda_i\right)^2} - \left(\frac{\lambda}{\rho_\pi\lambda_i} + \frac{\rho_\phi}{\rho_\pi}\phi\right)\left(\rho_\pi - 1\right) \\
&\quad -\beta\left[\frac{\left(\rho_\pi\lambda_i\right)^2 - \lambda\left(\rho_{\pi\pi}\lambda_i + \rho_\pi^2\lambda_{ii}\right)}{\left(\rho_\pi\lambda_i\right)^2} + \frac{\rho_{\phi\pi}\rho_\pi\phi - \rho_\phi\rho_{\pi\pi}\phi}{\rho_\pi^2}\right] \\
&\quad -\left(1-\gamma\right)\frac{\rho_\pi\left(\rho_{\phi\pi} im + \rho_\phi\rho_\pi m + \rho_\phi\rho_\pi i m_i\right) - \rho_\phi i m \rho_{\pi\pi}}{\rho_\pi^2} \\
&= -\frac{\rho_\pi\rho_{\phi\pi} - \rho_\phi\rho_{\pi\pi}}{\rho_\pi^2}\left[\beta\phi + \left(1-\gamma\right) im\right] \\
&\quad -\left(\frac{\lambda}{\lambda_i} + \rho_\phi\phi\right)\left(1 - \frac{1}{\rho_\pi}\right) - \left(1-\gamma\right)\rho_\phi\left(m + i m_i\right) \qquad (23f)
\end{aligned}$$

$$\begin{aligned}
J_{12} &= \frac{\beta\left(\lambda_i\right)^2\rho_\phi\rho_\pi - \beta\lambda\left(\rho_{\pi_\phi}\lambda_i + \rho_\phi\rho_\pi\lambda_{ii}\right)}{\left(\rho_\pi\lambda_i\right)^2} - \left(\frac{\lambda}{\rho_\pi\lambda_i} + \frac{\rho_\phi}{\rho_\pi}\phi\right)\rho_\phi \\
&\quad -\beta\left[\frac{\left(\lambda_i\right)^2\rho_\phi\rho_\pi - \lambda\left(\rho_{\pi_\phi}\lambda_i + \rho_\phi\rho_\pi\lambda_{ii}\right)}{\left(\rho_\pi\lambda_i\right)^2} + \frac{\left(\rho_{\phi\phi}\phi + \rho_\phi\right)\rho_\pi - \rho_\phi\rho_{\pi_\phi}\phi}{\rho_\pi^2}\right] \\
&\quad -\left(1-\gamma\right)\frac{\rho_\pi\left(\rho_{\phi\phi} im + \rho_\phi^2 m + \rho_\phi^2 i m_i\right) - \rho_\phi i m \rho_{\pi\phi}}{\rho_\pi^2} \\
&= -\frac{\rho_\pi\rho_{\phi\phi} - \rho_\phi\rho_{\pi_\phi}}{\rho_\pi^2}\left[\beta\phi + \left(1-\gamma\right) im\right] \\
&\quad -\frac{\rho_\phi}{\rho_\pi}\left(\frac{\lambda}{\lambda_i} + \rho_\phi\phi + \beta\right) - \left(1-\gamma\right)\frac{\rho_\phi^2}{\rho_\pi}\left(m + i m_i\right) \qquad (23g)
\end{aligned}$$

and

$$\begin{aligned}
J_{21} &= \left(\rho_\pi - 1\right)\phi + \left(1-\gamma\right)\rho_\pi\left(m + i^* m_i\right) \qquad (23h) \\
J_{22} &= \beta + \rho_\phi\phi + \left(1-\gamma\right)\rho_\phi\left(m + i^* m_i\right) \qquad (23i)
\end{aligned}$$

where J_{11}, J_{12}, J_{21} and J_{22} are evaluated at the steady state equilibria.

A.1 Alternative Interest Rate Rules

Under the log-linear Taylor rule (11), we have

$$
\begin{aligned}
\rho_\pi &= \alpha_\pi \frac{i}{i^*} \qquad \rho_\phi = \frac{\alpha_\phi}{\phi^*}\frac{i}{i^*} \\
\rho_{\pi\pi} &= \alpha_\pi^2 \frac{i}{i^{*2}} \qquad \rho_{\phi\phi} = \frac{\alpha_\phi^2}{\phi^{*2}}\frac{i}{i^{*2}} \\
\rho_{\pi\phi} &= \rho_{\phi\pi} = \frac{\alpha_\pi \alpha_\phi}{\phi^*}\frac{i}{i^{*2}}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\rho_\pi \rho_{\phi\pi} - \rho_\phi \rho_{\pi\pi} &= 0 \\
\rho_\pi \rho_{\phi\phi} - \rho_\phi \rho_{\pi_\phi} &= 0 \\
\frac{\rho_\phi}{\rho_\pi} &= \frac{\alpha_\phi}{\alpha_\pi \phi^*}
\end{aligned}
$$

At the steady state equilibrium $(\pi^*, \phi^*, Q^*, b^*)$, $i = i^* = \pi^* + \beta$. All elements J_{ij}'s of the submatrix J^S are evaluated at the SS and can be simplified as[32]

$$J_{11} = -(1-\gamma)\frac{\alpha_\phi}{\phi^*}(m + i^* m_i) - \left(\frac{\lambda}{\lambda_i} + \alpha_\phi\right)\left(1 - \frac{1}{\alpha_\pi}\right) \tag{23j}$$

$$J_{12} = -\frac{\alpha_\phi}{\alpha_\pi \phi^*}\left(\frac{\lambda}{\lambda_i} + \alpha_\phi + \beta\right) - (1-\gamma)\frac{\alpha_\phi^2}{\alpha_\pi \phi^{*2}}(m + i^* m_i) \tag{23k}$$

$$J_{21} = (\alpha_\pi - 1)\phi^* + (1-\gamma)\alpha_\pi (m + i^* m_i) \tag{23l}$$

$$J_{22} = \alpha_\phi + \beta + (1-\gamma)\frac{\alpha_\phi}{\phi^*}(m + i^* m_i) \tag{23m}$$

[32]If we assume a globally linear Taylor rule instead of the nonlinear rule

$$i = i^H + \alpha_\pi\left(\pi - \pi^H\right) + \alpha_\phi\left(\frac{\phi - \phi^*}{\phi^*}\right)$$

then

$$
\begin{aligned}
\rho_\pi &= \alpha_\pi \qquad \rho_\phi = \frac{\alpha_\phi}{\phi^*} \\
\frac{\rho_\phi}{\rho_\pi} &= \frac{\alpha_\phi}{\alpha_\pi \phi^*} \\
\rho_{\pi\pi} &= \rho_{\pi\phi} = \rho_{\phi\phi} = 0
\end{aligned}
$$

Therefore, the elements of the submatrix J^S take the same form as in (23j)-(23m). However, in this case, there is only one steady state equilibrium.

A.2 Non-Separable Preferences

To further simplify the expressions for J_{11}, J_{12}, J_{21} and J_{22}, we assume the instantaneous utility function takes the non-separable Cobb-Douglas form

$$u(c,m) = c^{\eta} m^{1-\eta} \tag{23n}$$

We have

$$m = m(i) = \frac{1-\eta}{\eta}\frac{\bar{c}}{i}$$

Substitute $m(i)$ into $u_c(c,m)$,

$$\begin{aligned} \frac{\lambda}{\lambda_i} &= -\frac{i}{1-\eta} \\ \frac{\lambda\lambda_{ii}}{(\lambda_i)^2} &= \frac{2-\eta}{1-\eta} \end{aligned}$$

Also

$$m + i m_i = 0 \tag{23o}$$

Therefore,

$$J_{11} = \left(1 - \frac{1}{\alpha_\pi}\right)\left(\frac{\pi^* + \beta}{1-\eta} - \alpha_\phi\right) \tag{23p}$$

$$J_{12} = \frac{\alpha_\phi}{\alpha_\pi \phi^*}\left(\frac{\pi^* + \eta\beta}{1-\alpha} - \alpha_\phi\right) \tag{23q}$$

$$J_{21} = (\alpha_\pi - 1)\,\phi^* \tag{23r}$$

$$J_{22} = \alpha_\phi + \beta \tag{23s}$$

Notice that the interest elasticity for money demand $m = m(i,c)$ is

$$\varepsilon_i = -\frac{\partial m/m}{\partial i/i} = -\frac{\partial m}{\partial i}\frac{i}{m} = -\frac{i m_i}{m} > 0$$

and the consumption elasticity for money demand $m = m(i,c)$ is

$$\varepsilon_c = \frac{\partial m/m}{\partial c/c} = \frac{\partial m}{\partial c}\frac{c}{m} = \frac{c m_c}{m} > 0$$

Therefore

$$m + im_i = m(1 - \varepsilon_i)$$

For coefficients J_{11} and J_{12}, assuming a constant ε_i and a simple money demand function would be sufficient to simplify their expressions.

B Proofs of Main Results for Local Analysis

The dynamical system (10), (12), (20) or (21), and (22) has four jump variables (π, ϕ, Q, a), which are forward-looking and can move discontinuously and respond instantaneously when new information arrives, but only three of them are linearly independent.[33] This is because B is constrained to move smoothly due to the separate fiscal budget constraint, and $b = B/P$ can only jump if P jumps. The household's holding of financial asset $a = A/P$ is forward-looking and it jumps whenever the price level P jumps. The CB's real net worth $\phi = QF^{CB} - M/P$ and the household's money demand $m = M/P$ are also forward-looking and can jump according to the CB's monetary policy or the household's portfolio adjustment decisions. In fact, both M and PQF^H can jump at any given time, making the household's portfolio choice forward-looking. The economy's rate of inflation π is forward-looking and it depends on the monetary policy rule $\rho(\pi, \phi)$, which reacts to current inflation and net worth.

The trace and determinant of the Jacobian transformation matrix J are

$$\begin{aligned} tr(J) &= J_{11} + J_{22} + \beta + J_{44} \\ \det(J) &= (J_{11}J_{22} - J_{12}J_{21})\beta J_{44} \end{aligned}$$

[33]Notice that A and B cannot jump. In fact, A and $\Phi = PQF^{CB} - M$ are correlated as follows:

$$\begin{aligned} A &= PQF^H + M + B \\ &= PQ\bar{F} + B - \Phi \end{aligned}$$

It is clear that the linear combination $A + \Phi$ can only jump when P or Q jumps, *i.e.*, discrete changes in $PQ\bar{F}$ must be completely offset by changes in Φ. For our dynamical system, only $(\pi, Q, a + \phi)$ are linearly independent jump variables.

And the trace and determinant of the submatrix J^S of J,

$$\begin{aligned} tr\left(J^S\right) &= J_{11} + J_{22} \\ &= \left(1 - \frac{1}{\alpha_\pi}\right)\left(\frac{\pi^* + \beta}{1-\eta}\right) + \frac{\alpha_\phi}{\alpha_\pi} + \beta \\ \det\left(J^S\right) &= J_{11}J_{22} - J_{12}J_{21} \\ &= \frac{\beta\left(\pi^* + \beta\right)}{1-\eta}\left(1 - \frac{1}{\alpha_\pi}\right) \end{aligned}$$

Assuming $\pi^* > -\beta$, then the sign of $\det\left(J^S\right)$ is the same as that of $1 - \alpha_\pi^{-1}$. That is, $\det\left(J^S\right) > 0$ whenever the monetary policy is active ($\alpha_\pi > 1$), and $\det\left(J^S\right) < 0$ if $\alpha_\pi < 1$. On the other hand, the sign of $tr\left(J^S\right)$ depends on both α_ϕ and α_π. Specifically, $tr\left(J^S\right) > 0$ ($tr\left(J^S\right) < 0$) if $\alpha_\phi > \bar{\alpha}_\phi$ ($\alpha_\phi < \bar{\alpha}_\phi$), respectively.

Define

$$U = tr\left(J^S\right) = \left(1 - \frac{1}{\alpha_\pi}\right)\left(\frac{\pi^* + \beta}{1-\eta}\right) + \frac{\alpha_\phi}{\alpha_\pi} + \beta \tag{23t}$$

$$V = 4\det\left(J^S\right) = \frac{4\beta\left(\pi^* + \beta\right)}{1-\eta}\left(1 - \frac{1}{\alpha_\pi}\right) \lessgtr 0 \tag{23u}$$

Hence, $sgn\left(U\right) = sgn\left(\alpha_\phi - \bar{\alpha}_\phi\right)$ and $sgn\left(V\right) = sgn\left(\alpha_\pi - 1\right)$. The eigenvalues E_1 and E_2 of matrix J^S are,

$$\begin{aligned} E_{1,2} &= \frac{tr\left(J^S\right) \pm \sqrt{tr\left(J^S\right)^2 - 4\det\left(J^S\right)}}{2} \\ &= \frac{1}{2}U \pm \frac{1}{2}\sqrt{U^2 - V} \end{aligned}$$

B.1 Equilibria Converging to the Steady States

In this section, we provide proofs for the local determinacy results, for both the cases with and without CB balance sheet concerns.

B.1.1 Without Net Worth Targeting

Under the constant lump sum tax rule, linearizing (20), we obtain

$$J_{44}^C = \beta$$

Under alternative Ricardian fiscal rules, linearizing around the steady state,

$$\begin{aligned} J_{44}^{V1} &= \beta - \delta_1 \\ J_{44}^{V2} &= \beta - \delta_2 \\ J_{44}^{V3} &= \beta - \delta_3 \end{aligned}$$

The system has three linearly independent jump variables. Local determinacy results differ with respect to non-Ricardian and Ricardian fiscal policy rules, because for $j = 1, 2, 3$, the fourth eigenvalue J_{44} of the Jacobian matrix J can now take non-positive values, *i.e.*, $\beta - \delta_j \leq 0$.

Proof of Proposition 1. First, notice that the dynamical system has three linearly independent jump variables. Since $\alpha_\phi = 0$, $J_{12} = 0$, the submatrix J^S has two eigenvalues J_{11} and $J_{22} = \beta$. Now, $sgn\,(J_{11}) = sgn\,(\alpha_\pi - 1)$. In particular, $J_{11} > 0$ if the monetary policy is active $(\alpha_\pi > 1)$ and $J_{11} < 0$ if the monetary policy is passive $(\alpha_\pi < 1)$.

In the first case of constant non-Ricardian tax rule, the fourth eigenvalue of J is just $E_4 = J_{44} = \beta$. Therefore of the four eigenvalues of the Jacobian matrix J, three take real positive value $E_2 = E_3 = E_4 = \beta$. The first eigenvalue $E_1 = J_{11}$ is positive if $\alpha_\pi > 1$ and it is negative otherwise. Since the dynamical system has three linearly independent jump variables, there is no stable equilibrium solutions under the active Taylor rule. Under a passive monetary rule, the liquidity trap equilibrium is locally determinate.

Under a variable non-Ricardian fiscal rule, $\beta - \delta > 0$, the fourth eigenvalue of the Jacobian matrix is again positive and the submatrix J^S remains unchanged. Hence all determinacy results remain the same as those under the non-Ricardian constant rule $\tau = \bar{\tau}$. ■

Proof of Proposition 2. Under a passive or Ricardian fiscal rule, the fourth eigenvalue $E_4 = \beta - \delta$ is negative. Since $\alpha_\phi = 0$, $J_{12} = 0$. In this case, the submatrix J^S has two eigenvalues J_{11} and $J_{22} = \beta$. Now, $J_{11} > 0$ if the monetary policy is active $(\alpha_\pi > 1)$ and $J_{11} < 0$ if the monetary policy is passive $(\alpha_\pi < 1)$.

Since the dynamical system has three linearly independent jump variables, with active monetary policy, the target equilibrium is locally determinate. Under passive monetary policy, the liquidity trap equilibrium is locally indeterminate, *i.e.*, there is an infinite number of stable equilibrium solutions for the model that are consistent with the passive monetary rule. ■

B.1.2 With Net Worth Targeting

To introduce CB balance sheet concerns, we assume that $\alpha_\phi < 0$ so that the CB adjusts interest rate to target its net worth besides the inflation rate π. Again, $sgn\,(U) = sgn\,(\alpha_\phi - \bar{\alpha}_\phi)$ and $sgn\,(V) = sgn\,(\alpha_\pi - 1)$, *i.e.*, for $\bar{\alpha}_\phi$ defined as in (13), $U > 0$ if $\alpha_\phi > \bar{\alpha}_\phi$, and $U < 0$ otherwise. When the monetary policy is active ($\alpha_\pi > 1$), $V > 0$. A passive monetary policy ($\alpha_\pi < 1$) implies $V < 0$.

Proof of Proposition 3. Notice first that under the constant non-Ricardian tax rule, $J_{44} = \beta$. Therefore $E_3 = E_4 = \beta$. Consider the case of $\alpha_\phi = \bar{\alpha}_\phi$, which implies $U = 0$. Then if $\alpha_\pi > 1$, $U^2 - V = -V < 0$ and the submatrix J^S has one simple pair of purely imaginary eigenvalues. If $\alpha_\pi \in (\bar{\alpha}_\pi, 1)$, $U^2 - V > 0$ and the submatrix J^S has one real positive and one real negative eigenvalues. The low-inflation equilibrium is locally determinate.

When $\alpha_\phi \neq \bar{\alpha}_\phi$, $U \neq 0$. Since $V \neq 0$, the real part of the eigenvalues of the submatrix J^S is non-zero. If $\alpha_\pi < 1$, $V < 0$ and $U^2 - V > 0$ and $\sqrt{U^2 - V} > |U|$. Then if $\alpha_\phi > \bar{\alpha}_\phi$, $U > 0$, the submatrix J^S has one positive and one negative eigenvalues. The same is true for the case of $\alpha_\phi < \bar{\alpha}_\phi$ and $U < 0$. So the liquidity trap equilibrium is locally unique independent of the degree of monetary conservatism.

If $\alpha_\pi > 1$, $V > 0$ and $\sqrt{U^2 - V} < |U|$. Then if $\alpha_\phi > \bar{\alpha}_\phi$, $U > 0$, the submatrix J^S has either two positive eigenvalues (if $U^2 - V \geq 0$), or two complex eigenvalues with equal real positive parts (if $U^2 - V < 0$). In this case no stable equilibrium solution exists. If $\alpha_\phi < \bar{\alpha}_\phi$ and $U < 0$, the submatrix J^S has either two negative eigenvalues (if $U^2 - V \geq 0$), or two complex eigenvalues with equal real negative parts (if $U^2 - V < 0$). Then the target equilibrium is locally indeterminate.

Under a variable non-Ricardian fiscal rule, $\beta - \delta > 0$, the fourth eigenvalue of the Jacobian matrix is again positive. Since the submatrix J^S remains unchanged, all determinacy results remain the same as those under the constant non-Ricardian fiscal rule $\tau = \bar{\tau}$. ■

Proof of Proposition 4. Under a passive or Ricardian fiscal rule, $\beta - \delta < 0$, the fourth eigenvalue J_{44} is negative. The third eigenvalue is $E_3 = \beta$. In the case of $\alpha_\phi = \bar{\alpha}_\phi$, if $\alpha_\pi > 1$, the submatrix J^S has one simple pair of purely imaginary eigenvalues[34]. If $\alpha_\pi \in (\bar{\alpha}_\pi, 1)$, $U^2 - V > 0$ and the submatrix J^S has one real positive and one real negative eigenvalues. Therefore the liquidity

[34] In the case of $\alpha_\phi = \bar{\alpha}_\phi$ and $\alpha_\pi > 1$, if $\beta - \delta = 0$, there are one simple pair of imaginary eigenvalues and one third eigenvalue that is zero. In this case, we will have a Fold-Hopf or Garvrilov-Guckenheimer Bifurcation of codimension two. For details, see Kuznetsov (1995), p257.

trap equilibrium is locally indeterminate and there is an infinite number of stable equilibrium solutions.

When $\alpha_\phi \neq \bar{\alpha}_\phi$, since $U \neq 0$ and $V \neq 0$, the real part of the eigenvalues of the submatrix J^S is non-zero. If $\alpha_\pi < 1$, then $V < 0$. When $\alpha_\phi > \bar{\alpha}_\phi$, $U > 0$, the submatrix J^S has one positive and one negative eigenvalues. The same is true for the case of $\alpha_\phi < \bar{\alpha}_\phi$ and $U < 0$. So the liquidity trap equilibrium is locally indeterminate independent of the degree of monetary conservatism.

If $\alpha_\pi > 1$, then $V > 0$. When $\alpha_\phi > \bar{\alpha}_\phi$, the submatrix J^S has either two positive eigenvalues (if $U^2 - V \geq 0$), or two complex eigenvalues with equal real positive parts (if $U^2 - V < 0$). In this case the target equilibrium is locally determinate. If $\alpha_\phi < \bar{\alpha}_\phi$, the submatrix J^S has either two negative eigenvalues (if $U^2 - V \geq 0$), or two complex eigenvalues with equal real negative parts (if $U^2 - V < 0$). Then the target equilibrium is locally indeterminate. ■

B.2 Equilibria Converging to a Deterministic Cycle: Local Bifurcation

In this section, we prove the (non-)existence results of local bifurcation, taking α_ϕ as the bifurcation parameter. The critical bifurcation value $\bar{\alpha}_\phi$ is defined as in (14).

B.2.1 With Net Worth Targeting

When the CB has concerns over its own balance sheet and targets its own net worth ($\alpha_\phi < 0$), a Hopf bifurcation is proved to emerge under the Taylor Principle ($\alpha_\pi > 1$), independently of the type of fiscal rules under consideration.

Proof of Proposition 5. The dynamical system is continuous and differentiable. Under the constant tax rule, $E_4 = \beta$. Assume $U^2 \leq V$ so the submatrix J^S has a pair of complex conjugate eigenvalues. Notice that at $\alpha_\phi = \bar{\alpha}_\phi$, the real part of the complex eigenvalues of J^S vanishes and the dynamical system has a simple pair of pure imaginary eigenvalues and no other eigenvalues with zero real part. So there is a smooth curve of equilibrium points $(\pi(\alpha_\phi), \phi(\alpha_\phi), Q(\alpha_\phi), a(\alpha_\phi))$, with $(\pi(\bar{\alpha}_\phi), \phi(\bar{\alpha}_\phi), Q(\bar{\alpha}_\phi), a(\bar{\alpha}_\phi)) = (\pi^H, \phi^*, Q^*, a^*)$. The eigenvalues of the submatrix J^S, $(E_1(\alpha_\phi), E_2(\alpha_\phi))$, which are pure imaginary at $\bar{\alpha}_\phi$, vary smoothly with α_ϕ. Taking derivative of the real part of this pair of complex conjugate eigenvalues with respect to α_ϕ and evaluate it at $\bar{\alpha}_\phi$, we obtain

$$\frac{d}{d\alpha_\phi}\left[ReE(\alpha_\phi)\right]_{\alpha_\phi=\bar{\alpha}_\phi} = \frac{1}{2\alpha_\pi} > 0$$

This is known as the Transversality Condition in the Bifurcation Theory. It says that the pair of complex conjugate eigenvalues of the submatrix J^S, $E_{1,2}(\alpha_\phi)$, crosses the imaginary axis with non-zero speed.

Using the Poincaré-Andronov-Hopf Theorem (see Guckenheimer and Holmes (1983), p. 151-152, or Perko (2000), p. 353-354), there is a unique two-dimensional center manifold passing through the steady state equilibrium $(\pi^H, \phi^*, Q^*, a^*)$ at the bifurcation value $\alpha_\phi = \bar{\alpha}_\phi$. A Hopf bifurcation exists at this point. Specifically, the steady state equilibrium is asymptotically stable for $\alpha_\phi < \bar{\alpha}_\phi$ and it is unstable for $\alpha_\phi > \bar{\alpha}_\phi$ (see Wiggins 1990, p. 275). ■

Proof of Proposition 6. First, notice that at $\alpha_\phi = \bar{\alpha}_\phi$, since $\beta - \delta \neq 0$ under a variable fiscal rule, the dynamical system has a simple pair of pure imaginary eigenvalues and no other eigenvalues with zero real part. The rest of the proof is similar to the case of the constant fiscal rule. ■

B.2.2 Without Net Worth Targeting

When CB balance sheet concerns are eliminated ($\alpha_\phi = 0$) but the CB continues to hold the real asset, no Hopf bifurcation exists when we take α_π as the bifurcation parameter.

Proof of Proposition 7. The proof is simple. Under the constant tax rule $\tau = \bar{\tau}$, $E_4 = \beta$. In the case of $\alpha_\phi = 0$,

$$U = \left(1 - \frac{1}{\alpha_\pi}\right)\left(\frac{\pi^* + \beta}{1 - \eta}\right) + \beta$$

If $\alpha_\pi > 1$, we have both $U > 0$ and $V > 0$. If $\alpha_\pi \in (\bar{\alpha}_\pi, 1)$, then $U > 0$ and $V < 0$. If $\alpha_\pi < \bar{\alpha}_\pi$, $U < 0$ and $V < 0$. There are no simple pair of complex eigenvalues in these cases. In particular, with $\alpha_\phi = 0$, $U = 0$ if and only if $\alpha_\pi = \bar{\alpha}_\pi$, in which case

$$-V = 4\beta^2 > 0$$

so the Jacobian matrix J has eigenvalues $E_1 = E_3 = E_4 = \beta$ and $E_2 = -\beta$, there are no simple pair of complex eigenvalues in this case. ■

Proof of Proposition 8. Since $\beta - \delta \neq 0$ under a variable fiscal rule, the fourth eigenvalue of the Jacobian matrix J has a nonzero real part. The rest of the proof is similar to the case of a non-Ricardian fiscal rule. ■