

Productivity and the real euro-dollar exchange rate

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Abstract

This paper analyses empirically how changes in productivity affect the real euro-dollar exchange rate. We derive impulse responses from a two-sector new open economy macro (NOEM) model. These are used as sign restrictions to identify a structural vector autoregression. Our results show that productivity shocks, whether economy-wide or sector-specific, are much less important in explaining the variation in the euro-dollar exchange rate than are demand and nominal shocks. In particular, productivity can explain part of the appreciation of the dollar in the late 1990s only to the extent that it created a boost to aggregate demand in the US. We find an insignificant contribution of the Balassa-Samuelson effect.

Keywords: real exchange rate, productivity, vector autoregression

JEL classification: F41, F31

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1 Introduction

This paper looks at the real appreciation of the dollar against the euro in the late 1990s. In policy circles, this exchange rate movement has been linked to the more rapid productivity growth in the US relative to the euro area. For instance, Bailey, Millard and Wells (2001) have argued that the productivity growth in the tradables sector (particularly in the information technology industry) has led to a real appreciation through the Balassa-Samuelson effect. We assess this argument by estimating a structural vector autoregression to explain the observed fluctuations in the real exchange rate. As identifying restrictions we use short run impulse responses derived from a calibrated two-sector new open economy macroeconomic (NOEM) model.

In an open economy with two sectors, the effect of a productivity shock on the real exchange rate depends on its sectoral location within the home country. An economy-wide productivity boom at home raises the supply of all home-produced goods on world markets. As a consequence, home-produced tradables become cheaper relative to imports: the terms of trade deteriorate. Through this terms of trade effect, the real exchange rate depreciates.

A productivity shock concentrated in the tradable goods sector has a similar effect on the real exchange rate through the terms of trade. However, there is also an additional transmission channel through spillovers across sectors. As workers in the export sector become more productive, they earn higher wages. Due to intersectoral labour mobility, wages increase also in the nontraded goods sector, where productivity has not changed. Prices of nontraded goods rise along with marginal costs. The result is a rise in the relative price of nontradable goods, known as the Balassa-Samuelson effect, which leads to a real appreciation.

The net effect of a traded sector productivity shock is a combination of the terms of trade effect and the Balassa-Samuelson effect and can go in either direction. Our estimation method identifies these two channels and lets the data determine their relative importance. We also control for two other important classes of shocks, namely demand shocks and nominal disturbances. By the latter, we mean money market shocks or exogenous exchange rate changes. Demand shocks could result from fiscal policy or shifts in consumption preferences.

The main contribution of the paper is that in estimating the effect of productivity on the real exchange rate, we explicitly distinguish between economy-wide and sector-specific shocks. On the one hand, research on the Balassa-Samuelson effect has concentrated on small countries which take the terms of trade as given. See, for example, De Gregorio and Wolf (1994). On the other hand, the one-good two-country model in Clarida and Gali (1994) looks only at the terms of trade, neglecting the possibility of intersectoral spillovers.

Our approach takes into account that for large countries, prices of tradables (and hence the terms of trade) are endogenous, while allowing for real exchange rate movements through the relative price of nontradables.

The new open economy macroeconomics has become the standard framework for analysing the international transmission of exogenous disturbances. It is a general equilibrium model with an optimising representative agent, rational expectations, sticky prices and imperfect competition. We extend the two-sector NOEM-type model in Benigno and Thoenissen (2003) to include government spending. Using numerical solution methods, we derive impulse response functions to the four types of shocks described above: economy-wide and sector-specific shocks to productivity, demand shocks (modelled as government spending on non-tradables), and nominal shocks (modelled as disturbances to the interest-rate rule).

Following the approach in Peersman (2004), the signs of the short run impulse responses are then used as restrictions to identify a VAR with four variables: output, traded goods prices, the real exchange rate and the relative price of nontradables. The latter variable, which we call the internal real exchange rate, helps to identify whether a productivity shock is an economy-wide phenomenon or is instead confined to the tradables sector. Crucially, the effect of productivity shocks on the real exchange rate is left unrestricted, which enables us to assess the relative importance of the terms of trade channel versus the Balassa-Samuelson channel. In addition, we estimate the importance of each type of shock for the in-sample variation in the euro-dollar rate.

The paper is structured as follows. In Section 2, we outline the two-sector model based on Benigno and Thoenissen (2003). A decomposition of the real exchange rate illustrates how the terms of trade effect and the Balassa-Samuelson effect work. We report and discuss the impulse responses implied by the model. The identification scheme and estimation results are given in Section 3. Section 4 concludes.

2 A two-sector new open macro model

First, we look at how the real exchange rate is determined in the context of a two-country optimising sticky price model. The discussion is based on a simplified version of Benigno and Thoenissen (2003), where the traded and nontraded goods sectors are modelled separately. We extend the model to include government spending. Secondly, we decompose the real exchange rate, showing three channels through which it can deviate from purchasing power parity: the market segmentation channel, the internal real exchange rate channel and the terms of trade channel. Finally, we use numerical techniques to solve the model and compute impulse responses to four types of shocks: traded sector and economy-wide productivity

shocks, government spending shocks and monetary shocks.

2.1 Model setup

We have a two-sector model with imperfectly competitive product and labour markets. Home and foreign agents consume three types of goods: home-produced tradables, foreign-produced tradables and nontradables. Each country produces a continuum of tradables and a continuum of nontradable goods. Home agents, as well as home-produced goods, are indexed by $i \in [0, 1]$, while foreign agents and foreign-produced goods are indexed by $i^* \in [0, 1]$.¹ In the following, all starred variables refer to the foreign country.

Preferences, the budget constraint and consumption demand

Consumers are infinitely lived and maximise the present discounted value of lifetime utility. The utility of the representative agent in the home country depends positively on consumption, C_t^i , and negatively on labour supply, L_t^i .

$$U_t^i = E_t \sum_{s=t}^{\infty} \beta^{(s-t)} \{U(C_s^i) - V(L_s^i)\} \quad (1)$$

where we specify the functional forms $U(C_t) = (1 - \rho)^{-1} C_t^{1-\rho}$ and $V(L_t) = (1 + \mu)^{-1} L_t^{1+\mu}$. The home agent consumes traded goods, $C_{T,t}$, and nontraded goods, $C_{N,t}$. His consumption basket is given by $C_t = [\gamma^\gamma (1 - \gamma)^{(1-\gamma)}]^{-1} C_{T,t}^\gamma C_{N,t}^{(1-\gamma)}$ where γ is the relative weight that home individual puts on traded goods. The consumption-based price index (the price of the consumption basket C_t) in the home country is derived as $P_t = P_{T,t}^\gamma P_{N,t}^{(1-\gamma)}$ where $P_{T,t}$ is the price of the basket of traded goods and $P_{N,t}$ is the price of the basket of nontraded goods. Consumption of tradables is divided into domestically produced tradables, indexed by H , and imports, indexed by F . Tradable goods consumption is given by the following subindex $C_{T,t} = [\nu^\nu (1 - \nu)^{(1-\nu)}]^{-1} C_{H,t}^\nu C_{F,t}^{(1-\nu)}$ where ν is the relative weight that the home individual puts on domestically produced traded goods. We derive the price of a basket of traded goods as $P_{T,t} = P_{H,t}^\nu P_{F,t}^{(1-\nu)}$, where $P_{H,t}$ is a price subindex for the home-produced tradables goods and $P_{F,t}$ is the price subindex for the foreign-produced traded goods, expressed in the domestic currency. The consumption subindices for home-produced goods are defined as

$$C_{j,t} = \left[\int_0^1 c_{j,t}(i)^{\frac{\sigma^j - 1}{\sigma^j}} di \right]^{\frac{\sigma^j}{\sigma^j - 1}}$$

where $j = N, H, H^*$.

¹Note that we depart from Benigno and Thoenissen (2003) in several ways. Firstly, we do not worry about relative country size, but assume instead that the two countries are equally large. Secondly, we assume a cashless economy where money by itself has no intrinsic value and therefore does not provide any utility. These simplifications do not alter the major conclusions of the model.

The household maximises lifetime utility (1), subject to the following budget constraint.

$$\begin{aligned}
& P_{T,t}C_{T,t}^i + P_{N,t}C_{N,t}^i + \frac{B_{H,t}^i}{1+i_t} + \frac{S_t B_{F,t}^i}{(1+i_t^*)\Theta\left(\frac{S_t B_{F,t}^i}{P_t}\right)} \\
\leq & B_{H,t-1}^i + S_t B_{F,t-1}^i + W_{H,t}^i L_{H,t}^i + W_{N,t}^i L_{N,t}^i + \int_0^1 \Pi_{H,t}^i di + \int_0^1 \Pi_{N,t}^i di \quad (2)
\end{aligned}$$

There are two assets, home bonds denominated in domestic currency, $B_{H,t}^i$, and internationally traded foreign-currency denominated bonds, $B_{F,t}^i$. In order to trade in the foreign bond market, the home individual has to pay an intermediation cost, denoted by $\Theta(\cdot)$, which is a function of the foreign asset position of the whole economy.² On the right hand side of the budget constraint, we have bond holdings carried over from the previous period, labour income and firm profits, $\Pi_{j,t}^i$, which are assumed to be shared equally among home residents. On the left hand side, we have consumption spending on tradables and nontradables, as well as the purchase of home and foreign bonds at price $(1+i_t)^{-1}$ and $(1+i_t^*)^{-1}$, respectively. Note that the budget constraint is written in home currency terms, which requires that foreign bonds are multiplied by the nominal exchange rate, S_t .

The household chooses the amount of good i that maximises utility (1) subject to the budget constraint (2). Denoting the price of the differentiated good i in sector j by $p_{j,t}^i$, the resulting home demand for good i in sector j is $c_{j,t}(i) = [p_{j,t}(i)/P_{j,t}]^{-\sigma_j} C_{j,t}$. Note that in this setup, $\sigma_j > 1$ is the elasticity of substitution between differentiated goods in consumption bundle j , as well as the price elasticity of home demand for good j .

Technology and labour demand

Labour is the only input and production is characterised by constant returns to scale. The production function has the form $Y_{j,t} = A_{j,t}L_{j,t}$, where $j = N, H$ and $A_{j,t}$ denotes the level of productivity³ in sector j . Labour supply is assumed to be immobile between countries and perfectly mobile between sectors, which implies that there is a common wage rate across sectors. The individual agent supplies a differentiated labour service to both sectors; working in the nontraded or the traded sector is equivalent. Household unions bundle individual labour types per sector, $L_{j,t}(i)$, such that total labour supply in sector j is written as

$$L_{j,t} = \left[\int_0^1 L_{j,t}(i)^{\frac{\phi-1}{\phi}} di \right]^{\frac{\phi}{\phi-1}} \quad (3)$$

The elasticity of substitution between labour inputs is denoted by $\phi > 1$ and is assumed to be the same across sectors. Let W_t denote the price of one unit of the bundle of labour inputs,

²For details on this financial market friction, see Benigno (2001).

³Note that Benigno and Thoenissen (2003) refer to their concept of productivity as total factor productivity (TFP). However, because labour is the only input in their model, TFP here corresponds to labour productivity.

$L_{j,t}$, and let $w_t(i)$ be the wage charged by labour type i in the home country. Firms choose the amount of labour type i to maximise profits, given the household union's labour supply (3). Total demand for agent i 's labour services is then given by $L_{j,t}(i) = [w_t(i)/W_t]^{-\phi} L_{j,t}$.

Price setting

Firms set prices to maximise profits, which are evenly distributed among the home country population and ultimately converted into utility terms. Thus, the representative agent chooses the price of good i in sector j , $p_{j,t}^f(i)$ to maximise utility (1). Due to imperfect competition in goods markets, prices are set as a markup $\sigma_j/(\sigma_j - 1)$ over marginal cost. Firms set prices in a forward-looking way, knowing that every period, they are able to change prices with a fixed probability $(1 - \alpha_p)$ (Calvo pricing). For a firm selling good i in sector H , for example, we have the following price setting equation.

$$E_t \left\{ \sum_{k=0}^{\infty} (\alpha_p^H \beta)^k \left[\frac{\sigma_H - 1}{\sigma_H} p_{H,t}^f(i) - \frac{W_{t+k}}{A_{H,t+k}} \right] \frac{U_c(C_{t+k})}{P_{t+k}} c_{H,t+k}(i) P_{H,t+k} \right\} = 0 \quad (4)$$

where $p_{H,t}^f(i)$ is the price set optimally in period t , $c_{H,t+k}(i)$ is the home demand for home-produced tradables.⁴

Wage setting

Household unions maximise utility (1) with respect to the individual wage rate $w_t(i)$, taking labour demand as given. As a result of imperfect competition in the labour market, wages are set as a markup $\phi/(\phi - 1)$ over the marginal rate of substitution between consumption and labour. As in product markets, we assume that wages are set according to Calvo-contracts i.e., each period, they are adjusted with a probability of $(1 - \alpha_w)$. This implies the following first order condition for wage setters in the home country.

$$E_t \left\{ \sum_{k=0}^{\infty} (\alpha_w \beta)^k \left[\frac{\phi - 1}{\phi} \frac{w_t^f(i)}{P_{t+k}} - \frac{V_l(L_{t+k}^i)}{U_c(C_{t,t+k})} \right] U_c(C_{t+k}) L_{t+k}(i) \right\} = 0 \quad (5)$$

where $w_t^f(i)$ is the wage rate set optimally at time t .

Monetary policy

Monetary policy takes the form of a Taylor rule with interest-rate smoothing, according to which the nominal interest rate i_t is set in response to current inflation π_t , the output gap y_t^{gap} (output less its flexible-price level) and the lagged interest rate i_{t-1} . We introduce nominal shocks to the interest rate rule as ε_t^M . The linearised policy reaction function at home (with an analogous equation for the foreign country) is given by $i_t = \Gamma_\pi \pi_t + \Gamma_y y_t^{gap} +$

⁴In Benigno and Thoenissen (2003), only a fraction $1 - \varepsilon$ of firms set price in this forward-looking way. The remaining firms are backward-looking and set prices equal to last period's price index corrected for lagged inflation. See Galí, Gertler and López-Salido (2001) for details.

$\Gamma_{i-1}i_{t-1} - \varepsilon_t^M$. In this model, ε_t^M is interpreted as a monetary policy shock. More generally, we could think of ε_t^M as an exogenous disturbance equivalent to a loosening of monetary conditions, for example, an exogenous nominal depreciation of the home currency.

The current account

The current account equation is derived by aggregating the individual budget constraints over all home agents.

$$\frac{S_t B_{F,t}}{P_t (1 + i_t^*) \Theta(\cdot)} = \frac{S_t B_{F,t-1}}{P_t} + \frac{P_{H,t} C_{H,t}}{P_t} + \frac{S_t P_{H,t}^* C_{H,t}^*}{P_t} - \frac{P_{T,t} C_{T,t}}{P_t}$$

where $C_{H,t}$ and $C_{H,t}^*$ denote the aggregate home and foreign demand for domestic tradables.

2.2 Deviations from PPP

The real exchange rate, S_t^R , is defined as the cost of a basket of goods in the home country relative to the foreign country, $S_t^R = S_t P_t^* / P_t$. S_t denotes the nominal exchange rate, the price of one unit of the foreign currency in terms of domestic currency. When S_t increases (decreases), this is called a depreciation (appreciation).

Purchasing power parity (PPP) holds when the typical consumption basket costs the same in the two countries. Absolute PPP implies that $S_t^R = 1$. The relative price of a basket of goods between countries may vary due to the presence of nontraded goods, home bias in tradables consumption and market segmentation. Below, we show how the real exchange rate can be rewritten to show these three channels of deviations from PPP. In our empirical analysis we will however focus on the first two only.

2.2.1 Nontraded goods

We use the expressions for the price indices P_t and P_t^* derived in Section 2.1 to express the real exchange rate as the product of two components.

$$S_t^R = \frac{S_t P_t^*}{P_t} = \frac{S_t P_{T,t}^*}{P_{T,t}} \frac{(P_{N,t}^* / P_{T,t}^*)^{(1-\gamma^*)}}{(P_{N,t} / P_{T,t})^{(1-\gamma)}} \quad (6)$$

The first part is the real rate of exchange for traded goods. The second part, which we call the internal real exchange rate ($IRER_t$), is the relative prices of nontraded goods in the two countries. In the presence of nontradables in consumption ($\gamma, \gamma^* \neq 1$), this term is different from one.

2.2.2 Market segmentation and deviations from the law of one price

Using the expressions for $P_{T,t}$ and $P_{T,t}^*$ from Section 2.1, the real rate of exchange for traded goods can be further split into deviations from the law of one price for traded goods on the

one hand and an expression in the terms of trade on the other hand.

$$\frac{S_t P_{T,t}^*}{P_{T,t}} = \left[\left(\frac{S_t P_{H,t}^*}{P_{H,t}} \right)^{\nu^*} \left(\frac{S_t P_{F,t}^*}{P_{F,t}} \right)^{(1-\nu^*)} \right] \left(\frac{P_{F,t}}{P_{H,t}} \right)^{(\nu-\nu^*)} \quad (7)$$

Consider the term in brackets. Under local currency pricing, each firm producing tradable goods sets two prices for the same good, one for the home market in home currency, $P_{H,t}$, and one for the foreign market in foreign currency, $P_{H,t}^*$. If home and foreign agents have different demand elasticities for the same good, $\sigma_H \neq \sigma_H^*$, and there is no goods market arbitrage, producers will price discriminate between the two countries. Then the law of one price fails, $S_t P_{H,t}^*/P_{H,t} \neq 1$.

2.2.3 Terms of trade and home bias

The relative price of home imports in terms of home exports, $P_{F,t}/S_t P_{H,t}^*$, is called the inverse terms of trade. Here, we use a different definition of the inverse terms of trade, namely the home currency price of imports divided by the home currency price of home-produced tradables, $T_t = P_{F,t}/P_{H,t}$.⁵ When T_t increases, we speak of a terms of trade deterioration as imports become dearer relative to exports (conventional definition), or as imports become more expensive relative to home-produced tradables (our definition). Relative consumption of home- versus foreign-produced tradables is a function of the terms of trade.

$$\frac{C_{H,t}}{C_{F,t}} = \frac{\nu}{1-\nu} T_t \text{ and } \frac{C_{H,t}^*}{C_{F,t}^*} = \frac{\nu^*}{1-\nu^*} T_t^*$$

Home bias arises when for a given terms of trade, home residents consume more home-produced tradables (relative to foreign-produced tradables) than do foreign consumers. I.e., $C_{H,t}/C_{F,t} > C_{H,t}^*/C_{F,t}^*$ at any given relative price, which requires that $\nu > \nu^*$. Therefore, with home bias in consumption, the second component in (7) is different from one.

2.2.4 Decomposing the real exchange rate

Combining equations (6) and (7), we have the final real exchange rate decomposition

$$S_t^R = \left[\left(\frac{S_t P_{H,t}^*}{P_{H,t}} \right)^{\nu^*} \left(\frac{S_t P_{F,t}^*}{P_{F,t}} \right)^{(1-\nu^*)} \right] \left(\frac{P_{F,t}}{P_{H,t}} \right)^{(\nu-\nu^*)} \frac{(P_{N,t}^*/P_{T,t}^*)^{(1-\gamma^*)}}{(P_{N,t}/P_{T,t})^{(1-\gamma)}} \quad (8)$$

The first part is the market segmentation component, the second part is the home bias component and the third part is the internal real exchange rate component. PPP holds if all components are equal to one.

⁵In a world where the law of one price holds, the two are the same.

2.3 Impulse responses

We solve the model using the same calibration as Benigno and Thoenissen (2003).⁶ Below, we analyse the dynamic adjustment of the economy to four types of shocks: traded sector productivity shocks, economy-wide productivity shocks, demand shocks and nominal shocks. In Figure 1 of the appendix, we graph the impulse responses of selected variables to these shocks. In Section 3, it will become apparent why we have chosen those particular variables. There, we show how the sign of these impulse responses can be used to identify a vector autoregression.

Traded sector productivity shocks: $A_{H,t} \uparrow \Rightarrow Y_t \uparrow, P_{T,t} \downarrow, S_t^R \uparrow$ (here \uparrow), $IRER_t \downarrow$

Productivity improvements benefit consumers (in terms of lower prices) and workers (in terms of higher wages). From the production function, we see that a positive shock to productivity in the traded goods sector, $A_{H,t}$, raises the amount of goods that can be produced with a given labour input. Due to the monopolistic distortion in the product markets, firms set prices as a markup over marginal cost. The increase in productivity directly reduces their marginal cost, and therefore prices in the traded goods sector, $P_{H,t}$, must fall as we move down the demand curve. Output rises to accommodate the extra demand, but by less than productivity. This implies that labour effort decreases in sector H . The labour market is also imperfectly competitive; household unions set the wage rate as a markup over the marginal rate of substitution between labour and consumption, $mrs = V_l/U_c$. We have established that consumption rises and labour effort falls. Given that the utility function $U(C)$ is concave in consumption, we know that the marginal utility of consumption, U_c , must fall. As the disutility of labour function, $V(L)$, is convex in labour, marginal labour disutility, V_l , falls. In our calibration, V_l falls less than U_c and as a result, wages rise along with the marginal rate of substitution.

Because labour is mobile between the two sectors, nominal wages increase in the whole economy. In the nontradables sector, where productivity has not changed, these higher wages imply higher marginal costs. Prices of nontradable goods, $P_{N,t}$, are set as a markup over marginal costs and therefore have to rise as we move up the demand curve. As $P_{N,t}$ increases relative to $P_{T,t}$, the internal real exchange rate appreciates (see equation (6)). Since the internal real exchange rate is a component of the (overall) real exchange rate, S_t^R , as we can see from equation (8), this effect in isolation would lead to a real appreciation. However, an increase in the supply of home tradables leads to a deterioration in the terms of trade. In terms of equation (8), $T_t = P_{F,t}/P_{H,t}$ increases. Therefore, the real exchange rate

⁶The model in Benigno and Thoenissen (2003) is calibrated to fit the UK and the Euro area, while in our empirical model we will look at the US and the Euro area. Since we are only interested in the signs of the short run impulse responses, we suppose that these will not change if we adjust the parameters to match US data. A full robustness analysis is beyond the scope of this paper.

depreciates via the home bias channel. The net effect of tradable sector productivity shocks on the real exchange rate is ambiguous. The parameter values used in Benigno-Thoenissen (2003) imply a real depreciation.

Economy-wide productivity shocks: $A_{H,t}, A_{N,t} \uparrow \Rightarrow Y_t \uparrow, P_{T,t} \downarrow, S_t^R \uparrow, IRR_t \uparrow$

Similar arguments apply to an increase in productivity in the nontradable goods sector, $A_{N,t}$. Due to the assumption of a linear production function, an economy-wide increase in productivity can be derived from summing the responses to shocks to traded sector and nontraded sector productivity shocks. In this scenario, there are no spillover effects between sectors and the Balassa-Samuelson effect does not arise. We expect an equal reduction in the prices of home-produced goods ($P_{H,t}$ and $P_{N,t}$), an increase in total output and a real depreciation. In addition, a productivity shock across both sectors leads to a depreciation in the internal real exchange rate, as nontraded goods become cheaper relative to the basket of imported and home-produced tradables. This is because although the ratio of home-produced tradables to non-tradables prices, $P_{H,t}/P_{N,t}$, remains unchanged, both prices fall relative to the price of foreign-produced tradables, $P_{F,t}$. So $P_{N,t}$ falls more than $P_{T,t}$.

"Demand" shocks: $G_{N,t} \uparrow \Rightarrow Y_t \uparrow, P_{T,t} \uparrow, S_t^R \downarrow, IRR_t \downarrow$

We extend the Benigno-Thoenissen model to incorporate government spending shocks, where we assume that the government consumes only nontradables. Consider an exogenous rise in demand for nontradables raising $P_{N,t}$. As home output increases to accommodate this extra demand, workers need to raise their labour effort. They are compensated for working harder in the form of higher wages. These higher labour costs drive up prices in sector H . We see from equation (8) that the real exchange rate appreciates through the home bias channel. The internal real exchange rate also appreciates, because nontradables prices rise by more than tradables prices ($P_{N,t}$ rises more than $P_{T,t}$). We conclude that in the case of a government spending shock, the real exchange rate appreciates unambiguously.

"Nominal" shocks: $\varepsilon_t^M \uparrow \Rightarrow Y_t \uparrow, P_{T,t} \uparrow, S_t^R \uparrow, IRR_t \uparrow$

Monetary shocks, which are incorporated as an exogenous variable, ε_t^M , in the Taylor rule, lower the nominal interest rate. This has the effect of increasing current consumption at the expense of future consumption. Current output at home increases to meet demand. Long run money neutrality implies that the home price index and the nominal exchange rate should ultimately rise one-for-one with ε_t^M . However, because of price rigidities, the nominal exchange rate adjusts faster to the shock than prices. Consequently, there is a real depreciation in the short run. The internal real exchange rate depreciates due to the higher weight given to nontradables in consumption. As argued above, ε_t^M can alternatively be

interpreted as a shock to the foreign exchange market equivalent to a loosening of monetary policy. For this reason, we prefer to call this a nominal instead of a monetary shock.

3 Empirical analysis

In this Section, we estimate a structural vector autoregression to examine the effect of productivity shocks (in the whole economy as well as in the traded sector) on the real euro-dollar exchange rate, while controlling for demand and nominal shocks. To identify the structural shocks to the system, we use sign restrictions⁷ on short run impulse responses, building on the model outlined in Section 2. This technique was pioneered by Uhlig (1999), Canova (2002) and Canova and De Nicoló (2002). Using sign restrictions to identify a VAR avoids some problems that arise in the context of short run or long run zero restrictions. See Canova and De Nicoló (2002) for details.

Peersman and Farrant (2004) use short run sign restrictions⁸ to identify supply, demand and nominal shocks in a three-variable VAR model of output, prices and the real exchange rate. All three shocks are labelled positive if they give a short run boost to output. Supply shocks (equivalent to productivity shocks in our setup) reduce inflation, while demand and nominal shocks are both inflationary. Finally, demand shocks lead to an appreciation of the real exchange rate, while nominal shocks cause a real depreciation.

Our model distinguishes between the traded and the nontraded sectors, opening up the possibility of sectoral productivity shocks and a real appreciation through the Balassa-Samuelson effect. Therefore, in addition to identifying productivity shocks to the whole economy, we want to control for the effect of sector-specific productivity shocks. We do this by adding the internal real exchange rate, $IRER_t$, as defined in equation (6). This variable allows us to distinguish between productivity shocks that affect the whole economy and those that are limited to the tradable goods sector.

3.1 Data

Our sample runs from 1981Q1 to 2003Q1. Let ∇ denote a cross-country ratio, e.g. $\nabla x = x_{US}/x_{Euro}$. Our four data series⁹, shown in Figure (2) in the appendix, are the ratio of US to euro area real GDP, ∇Y_t , the ratio of US to euro area traded goods prices, $\nabla P_{T,t}$, the real euro-dollar exchange rate, S_t^R , and the euro-dollar internal real exchange rate, $IRER_t$, as defined in Section (2). The data sources are given in the appendix. The US is regarded as the home country. The data series for the euro area before 1999Q1 are from Fagan et al. (2001).

⁷I thank Gert Peersman for sharing his programming code.

⁸The sign restrictions are based on the model predictions in Clarida and Gali (1994).

⁹We take logs of all variables and rescale them by a factor 100, which gives us the percentage difference between the US and the euro area.

All data thereafter are from IMF International Financial Statistics. Nontraded goods prices are proxied by the consumer price index, traded goods prices are proxied by the producer price index. The nominal euro-dollar exchange rate is constructed as described in Schnatz et al. (2003). It is computed as the geometric weighted average of the dollar exchange rates of the euro legacy currencies. The real exchange rate is the nominal euro-dollar exchange rate multiplied by the ratio of consumer prices in Euro area and the US.

3.2 Preliminary analysis and unrestricted VAR

We conduct augmented Dickey-Fuller tests with four lags, a constant and a trend, on all series in levels and in first differences. The null hypothesis of nonstationarity cannot be rejected at the 5% significance level for all of the level series. The first differences of relative output, relative traded goods prices and the internal real exchange rate are stationary at the 5% level, the first difference of the real exchange rate is stationary at the 10% level. We proceed assuming that all four variables are integrated of order 1, which allows us to estimate a VAR in first differences. The lag length of the VAR was set equal to three as selected by the likelihood ratio test. We therefore estimate the following model

$$\Delta \mathbf{x}_t = c + \beta(L) \Delta \mathbf{x}_{t-1} + e_t$$

where $\mathbf{x}_t = (\log \nabla Y_t, \log \nabla P_{T,t}, \log S_t^R, \log IRE R_t)$, and $\beta(L) = \beta_0 + \beta_1 L + \beta_2 L^2$ and c is a vector of constants and linear trends.

3.3 Identification of the structural shocks

Having estimated the unrestricted VAR, we obtain reduced form residuals, e_t , which are a linear combination of the four underlying structural shocks, ε_t . The unknown matrix A links the two types of shocks: $e_t = A\varepsilon_t$. Imposing the normalisation that all structural shocks have unit variance and are uncorrelated, we have the following relation.

$$\Sigma_e = AA' \tag{9}$$

where Σ_e is the covariance matrix of the residuals. We want to obtain estimates of the orthogonal structural shocks, ε_t . Therefore, our aim is to find a matrix A for which equation (9) holds. Since the number of possible matrices A is infinitely large, we impose further restrictions. The impulse responses arising from the structural shocks should have the signs given in Table (1). Using the method in Peersman (2003), explained in detail in the appendix, we search over the space of orthogonalisation and check the signs of the impulse responses each time. If they match our priors, we save them. We order the resulting impulse response functions and variance decompositions and report the median, as well as the 16th

and 84th percentile (one standard deviation) error bands. For each type of shock, the signs of the short run responses are summarised in the Table below.

	Variable			
	Y_t	$P_{T,t}$	S_t^R	$IRER_t$
Type of shock				
overall productivity shock	≥ 0	≤ 0	?	≥ 0
demand shock	≥ 0	≥ 0	≤ 0	?
nominal shock	≥ 0	≥ 0	≥ 0	?
traded sector productivity	≥ 0	≤ 0	?	≤ 0

Table 1: **Signs of responses used for identification**

We identify the four shocks as follows. In the short run, both economy-wide productivity shocks and traded sector productivity shocks lead to a rise in output and a fall in traded sector goods prices, but can be discriminated by their effect on the internal real exchange rate. Nominal shocks and demand shocks raise prices and also increase total output in the short run. We distinguish between the two shocks through the restriction that demand shocks reduce the real exchange rate, while nominal shocks increase it.

The horizon over which the sign restriction is binding, is set equal to 4 quarters for output, traded goods prices and the internal real exchange rate and to 1 quarter for the real exchange rate. The idea behind this is that the real exchange rate is a more flexible variable than output or prices. Setting a higher value reduces the number of plausible decompositions.

The sign of the response of the real exchange rate to productivity shocks (economy-wide and in the traded sector) is not restricted, but is instead determined by the data. Similarly, the response of the internal real exchange rate to demand and nominal shocks is unrestricted.

3.4 Estimation results

3.4.1 Impulse response functions

Graphs of the impulse response functions are given in the appendix, Figure (3). The median response to a positive shock is given by the continuous black line, while the dotted lines represent the 16th and 84th percentile error bands. Impulse responses are significant in the cases where the upper and lower error bands have the same sign. On the x-axis, the forecast horizon is given in quarters.

First of all, we note that the impulse responses of output and prices make sense economically. The impulse responses for output show that productivity and demand shocks lead to significant increases in output in the long run, which is consistent with many macroeconomic models. Nominal shocks, by construction, lead to temporary booms, but are insignificant

at longer horizons. Prices of tradable goods rise permanently following a nominal shock and fall permanently in response to productivity shocks. Demand shocks raise traded goods prices only at short horizons.

In response to sectoral productivity shocks, the real exchange rate appreciates significantly in the first quarter. The effect at longer horizon is, however, uncertain. A permanent appreciation of the *internal* real exchange rate indicates that the Balassa-Samuelson effect plays a role in the determination of relative prices, but this does not necessarily translate into a permanent appreciation of the (overall) real exchange rate.

In the case of economy-wide productivity shocks, we expect the real exchange rate to depreciate¹⁰. The corresponding impulse response function suggests that an appreciation is more likely, although the result is not significant. This is a puzzling result. Based on our macroeconomic model, we have filtered out the only channel through which productivity shocks may cause an appreciation, which is the Balassa-Samuelson effect. Yet, even controlling for this effect, we still find evidence that an overall productivity improvement makes a currency stronger. We conclude that the standard macroeconomic model outlined above cannot capture the link between productivity and the real exchange rate very well. In other words, productivity shocks affect exchange rates in ways that are missing in the standard macro model. Of course, this result holds only for the euro-dollar exchange rate and for the period under study. Further research on other exchange rates and sample periods should make out if this result holds more widely.

Another striking finding, which confirms Peersman and Farrant (2004) is that nominal shocks have significant effects on the real exchange rate at long horizons. Many empirical studies using VARs impose the restriction that nominal shocks have no permanent effect on real variables, reflecting long run money neutrality. Our result might just demonstrate that this restriction is rather stringent in small samples, especially if the long run turns out to be very long. Nevertheless, the result that nominal shocks have permanent effects is rather worrying.

We label any shock that increases output, prices and the real exchange rate as a nominal shock. It is conceivable that instead of monetary policy shocks, we have identified positive non-fundamental shocks to the real exchange rate, which have the same effects: they boost output through increased exports and raise import prices, which enter the general price level. Peersman and Farrant (2004) find that these "pure exchange rate shocks" explain a substantial amount of the real exchange rate variability in the very short run. Finally, demand shocks result in significant long run appreciations in the real exchange rate.

¹⁰Even though this is an unambiguous result of a number of macroeconomic models, many empirical papers find a perverse supply effect of productivity on the real exchange rate, i.e., an appreciation. See, for example, Clarida and Gali (1994) and other papers that have estimated their model.

3.4.2 Variance decompositions

Decompositions of the forecast error variances are given in Table (2). Again, we report the median value and the 16th and 84th percentile error bands. Notice that the variance decompositions do not sum to one, as they would in the case of a single decomposition.

As we would expect, productivity shocks explain most of the output variation in the long run. Traded goods prices are affected mostly by sectoral productivity shocks and, at longer horizons, by nominal shocks.

Demand and nominal shocks explain most of the variation in the real euro-dollar exchange rate. Nominal shocks dominate at short horizons. The importance of nominal shocks contrasts with earlier findings of researchers who use long run neutrality restrictions to identify shocks. Empirical papers estimating the Clarida-Gali (1994) model tend to find only a small role for nominal disturbances. The results of estimating of structural VARs are very sensitive to the type of restrictions imposed.

Productivity shocks (economy-wide or sector-specific) explain little of the observed variation in the real exchange rate. For instance, productivity shocks in the traded sector account for 9.8% (33.4% and 1.3%) of the variation in the long run real exchange rate (upper and lower error bands given in brackets).

One problem with our approach is that we identify only uncorrelated shocks. However, an increase in productivity might itself boost demand, as people expect to earn more in the future. Then the positive demand effect on prices may overwhelm the negative price effect of the productivity increase. In that case, we only identify this as a "demand shock", while the underlying cause is an anticipated rise in productivity.

3.4.3 Historical contributions

Having identified the structural shocks to our system, we can divide each data series into a base projection (the path that the variable would have followed had there been no exogenous shocks) and the various shock component series, reflecting the deviations from the base projection. This allows us to compute the median contribution of each shock to the path of, say, the real exchange rate in a particular period.

A graph showing the decomposition for the period 1999Q1-2002Q4 is given in the appendix, Figure (4). On this graph, the real exchange rate is shown in deviations from the baseline projection. Consider the sub-period 1999Q1-2000Q4, which was characterised by a continuous fall in the real euro-dollar rate by 26%. Note that the sum of the shock components and the baseline projection is not equal to the real exchange rate series, as it would be in the case of a single orthogonalisation. We note that nominal shocks alone accounted for a 15% appreciation of the dollar, while demand shocks were responsible for a 11% appreci-

ation. Economy-wide productivity shocks had hardly any effect on the real exchange rate, while sectoral productivity shocks accounted for a 2% *depreciation* of the dollar. Similar pictures emerge from the analysis of other subperiods (not shown). The relative importance of each shock is about the same throughout our sample.

Our findings show that higher productivity growth in the US relative to the euro area cannot be directly responsible for the appreciation of the dollar against the euro at the beginning of Economic and Monetary Union. It may have played a role to the extent that it triggered an aggregate demand shock. Nominal disturbances weigh even stronger than a US demand shock and swamp any productivity effects.

4 Conclusion

The motivation for this paper is the conjecture that productivity differentials are at the origin of the dollar's appreciation at the end of the 1990s. We analyse the effect of productivity on the real euro-dollar exchange rate, using a structural VAR. Our identifying restrictions build on the NOEM-type model in Benigno and Thoenissen (2003), which suggests that productivity shocks in the tradable goods sector can lead to a real appreciation through the Balassa-Samuelson effect. We identify economy-wide and sector-specific productivity shocks, demand shocks and nominal shocks.

We find an insignificant contribution of the Balassa-Samuelson effect to the real appreciation of the dollar over the period 1981-2003. Indeed, our results show that productivity shocks, whether economy-wide or sector-specific, are much less important in explaining the variation in the euro-dollar exchange rate than are demand and nominal shocks. Especially at short horizons, nominal shocks are the dominant source of real exchange rate movements. While this finding contrasts with the contribution of Clarida and Gali (1994), it confirms the more recent result obtained in Peersman and Farrant (2004).

Shocks to uncovered interest parity (UIP) and monetary policy shocks are observationally equivalent since they have similar effects on output and prices. Peersman and Farrant (2004) distinguish between them by looking at the behaviour of the interest rate differential $(i - i^*)$. A monetary easing is characterised by a fall in $(i - i^*)$ while a "pure exchange rate shock" results in a rise in $(i - i^*)$. Peersman and Farrant find a substantial impact of pure exchange rate shocks in the short run. We therefore suspect that what we have labelled nominal shocks are mostly developments in the foreign exchange market that have nothing to do with monetary policy or indeed any other fundamentals. Such developments could be instead related to changes in perceived asset risk premia and speculative activity. Taking account of these potentially important factors represents a challenge for the NOEM

literature.

How can we then reconcile our result that productivity does not matter with the finding by Alquist and Chinn (2002) of a strong correlation between productivity measures and the euro-dollar exchange rate? One possibility is that productivity is important, but affects international prices in ways that are not captured by a standard NOEM model.

Firstly, anticipated future productivity improvements may give rise to wealth effects on consumption. People spend more today as they expect to earn more in the future. If the increased demand falls disproportionately on home-produced goods (due to home bias in consumption), prices rise relative to the foreign country and the real exchange rate appreciates. Secondly, an increase in productivity may attract new entrants to the home market, introducing new products. Consumers value product variety, and will switch their expenditure towards home-produced goods.

If these demand effects of productivity improvements are stronger than the price reducing effects, our empirical model is misspecified. This is because we identify a productivity shock as one that lowers prices. This may explain why in our specification, demand shocks have a significant effect on the real exchange rate, while productivity shocks do not. Further research is needed to assess the importance of wealth effects and product innovation.

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5 Appendix

5.1 Data sources

The sample period is 1981Q1 to 2003Q1. All indexed series have base year 1995.

For each euro legacy currency, the *nominal exchange rate* series (national currency per US dollar) is taken from the IFS (line rf) and divided by its fixed euro conversion rate with the euro. The synthetic euro-dollar exchange rate before 1999Q1 is computed by taking a geometrically weighted average of the euro legacy currencies' exchange rates vis-à-vis the dollar. The weights are those proposed in Schnatz, Vijselaar and Osbat (2003).

Taking the US as the home country, the *real exchange rate* is computed as the inverse of the nominal exchange rate, multiplied by the ratio of euro area to US consumer price index. Sources of the consumer prices series are given below.

Consumer prices before 1999Q1 are taken from the area-wide model (AWM), and after 1999Q1 from the IFS (line 64h). For the US, producer prices are from the IFS (line 64).

For the US, the euro member countries before 1999Q1 and the euro area thereafter, *producer or wholesale prices* are from the IFS (line 63). Euro area producer prices before 1999Q1 are computed as a geometrically weighted average of the producer price indices of the euro members, using the same trade weights as proposed in Schnatz, Vijselaar and Osbat (2003). Due to a lack of data, Portugal is not included in the euro area producer price index.

The *relative price* ratio between the nontraded and traded goods sector is proxied by consumer prices divided by producer prices.

GDP series for the US is line 11199BVRZF... from the IFS. For the euro area, data before 1999Q1 is from the AWM, series YER. From 1999Q1, data from the IFS is used (line 16399BVRZF...). The GDP series are seasonally adjusted.

5.2 Methodology

A vector autoregression (VAR) is a system of equations in which every endogenous variable is a function of all lagged endogenous variables¹¹. Consider the vector of the n endogenous variables $\mathbf{x}_t = (x_{1t}, x_{2t}, \dots, x_{nt})'$ and the vector of n unobservable structural disturbances $\mathbf{z}_t = (z_{1t}, z_{2t}, \dots, z_{nt})'$. In its structural form, the VAR can be written

$$B\mathbf{x}_t = C(L)\mathbf{x}_{t-1} + \mathbf{z}_t \quad (10)$$

where $C(L) = C_0 + C_1L + C_2L^2 + \dots + C_qL^q$, L is the lag operator, the $(n \times n)$ matrix B comprises the parameters on the contemporaneous endogenous variables. A non-zero

¹¹This exposition follows Keating (1992). For simplicity, we do not consider deterministic variables such as constants, time trends or seasonal dummies.

element in B indicates that an endogenous variable has a contemporaneous effect on another. Assuming that B is invertible, we derive the reduced form VAR by multiplying the structural form by B^{-1}

$$\mathbf{x}_t = B^{-1}C(L)\mathbf{x}_{t-1} + B^{-1}\mathbf{z}_t \quad (11)$$

If we model all structural disturbances as unit root processes, then $\Delta\mathbf{z}_t = \mathbf{z}_t - \mathbf{z}_{t-1} = \varepsilon_t$. If the variables in \mathbf{x}_t are $I(1)$ and not cointegrated, it is valid to estimate the VAR in first differences. Applying the first difference operator $\Delta = (1 - L)$ to equation (11), we have

$$\Delta\mathbf{x}_t = \beta(L)\Delta\mathbf{x}_{t-1} + e_t \quad (12)$$

where $\beta(L) = \beta_0 + \beta_1L + \beta_2L^2 + \dots + \beta_qL^q$ and $e_t = B^{-1}\varepsilon_t$. The lag length q can be determined using, for example, a sequential likelihood ratio test or the Akaike information criterion. An equation-by-equation OLS regression of the reduced form (12) yields estimates of the coefficients, $\beta(L) = B^{-1}C(L)$ and the reduced form residuals $e_t = B^{-1}\varepsilon_t$, as well as the variance-covariance matrix of the residuals, Σ_e .

5.2.1 VAR identification with sign restrictions

We want to identify shocks that are mutually orthogonal and have unit variance, i.e. they should have $\Sigma_\varepsilon = E(\varepsilon_t\varepsilon_t') = I$. Define $A = B^{-1}$; then $\Sigma_e = AA'$. Without imposing any restrictions, there are infinitely many possible decompositions of Σ_e . For example, using the eigenvalue-eigenvector decomposition, we have $\Sigma_e = PDP'$, where D is a diagonal matrix of eigenvalues and P consists of the eigenvectors of Σ_e and thus $A = PD^{\frac{1}{2}}$. However, any decomposition of Σ_e , such that $\Sigma_e = AQQ'A'$, where Q is orthonormal (i.e. $QQ' = I$) is also valid. We use the decomposition $Q = \prod_{m,n} Q_{m,n}(\theta_i)$, where $Q_{m,n}(\theta_i)$ are rotation matrices of the following form.

$$Q_{m,n}(\theta_i) = \begin{bmatrix} 1 & \dots & 0 & \dots & 0 & \dots & 0 \\ \dots & \ddots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \cos\theta_i & \dots & -\sin\theta_i & \dots & 0 \\ \vdots & \vdots & \vdots & 1 & \vdots & \vdots & \vdots \\ 0 & \dots & \sin\theta_i & \dots & \cos\theta_i & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \ddots & \dots \\ 0 & \dots & \dots & 0 & \dots & \dots & 1 \end{bmatrix}$$

where the subscript (m, n) indicates that rows m and n are rotated by an angle θ_i , $0 < \theta_i < \pi$ and $i = 1, \dots, 6$. With four variables, the number of possible bivariate rotations is six

($C_2^4 = 6$). We have

$$Q = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \theta_2 & -\sin \theta_2 \\ 0 & 0 & \sin \theta_2 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} \cos \theta_3 & 0 & -\sin \theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ \sin \theta_3 & 0 & \cos \theta_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} \cos \theta_4 & 0 & 0 & -\sin \theta_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin \theta_4 & 0 & 0 & \cos \theta_4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_5 & -\sin \theta_5 & 0 \\ 0 & \sin \theta_5 & \cos \theta_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_6 & 0 & -\sin \theta_6 \\ 0 & 0 & 1 & 0 \\ 0 & \sin \theta_6 & 0 & \cos \theta_6 \end{bmatrix}$$

Any rotation can be produced by varying the angles θ_i in the range $[0, \pi]$. A "rotation" amounts to drawing numbers for θ_i from a uniform distribution: the probability of drawing one particular number is constant over the range. Because the six θ_i 's can take on infinitely many values in this range, we divide it into intervals separated by $M = 12$ equally spaced points, such that we have a finite number of rotations over which to search. With six θ_i 's and twelve possible values for each θ_i , i.e. twelve possible values for Q_i , there are 12^6 possible rotation matrices Q .

5.2.2 Computing error bands with Monte Carlo simulations

In the following¹², we explain how to compute standard error bands using Monte Carlo integration. This is a Bayesian method in which we make draws from the posterior distribution of the impulse response functions.

In Bayesian statistics, we try to improve upon our estimates by incorporating prior information (i.e., information we have before observing the sample) into our analysis. The true value of our estimator, θ , is regarded as a random variable. Reflecting the uncertainty we have about θ , inference therefore takes the form of a probability statement. Any prior information about θ is represented by the prior density function, $f(\theta)$. In this framework, the sample likelihood is the density of y conditional on a particular on the value of the random variable θ , denoted $f(y|\theta)$. The marginal density multiplied by the prior density equals the joint density.

$$f(y, \theta) = f(y|\theta) \cdot f(\theta)$$

Probability statements about θ , once the data y have been observed, are made on the basis of the posterior density function, given by

$$f(\theta|y) = \frac{f(y, \theta)}{f(y)}$$

In the case of a linear regression model given by $y_t = \mathbf{x}_t' \boldsymbol{\beta} + e_t$, prior information about $\boldsymbol{\beta}$ can be represented by a $N(m, \sigma^2 M)$ distribution, where m is the best guess of the true coefficient vector $\boldsymbol{\beta}$ and $\sigma^2 M$ is the uncertainty surrounding this guess. This assumes that

¹²Discussion based on Hamilton (1994) and Uhlig (1999).

we know the true variance σ^2 . However, since σ^2 is unknown, we need to assume a prior distribution for it. The gamma distribution¹³ lends itself to this application.

Let's look at Bayesian inference in a VAR framework. Write the vector autoregression with n variables as

$$\mathbf{y}_t = (\mathbf{I}_n \otimes \mathbf{X}_t) \boldsymbol{\beta} + \mathbf{u}_t \quad t = 1, \dots, T.$$

where \mathbf{y}_t is $(n \times 1)$, \mathbf{X}_t is $(1 \times k)$, $\boldsymbol{\beta}$ is $(kn \times 1)$, the coefficient matrix in its columnwise vectorised form, and k is the number of coefficients per equation. Assume that the errors are independently and identically distributed as $\mathbf{u}_t \sim N(0, \Sigma)$, where Σ is $(n \times n)$, and the likelihood function is conditional upon the values of \mathbf{y}_t for t less than 1. Let \mathbf{b} and \mathbf{S} be the OLS estimates of $\boldsymbol{\beta}$ and Σ . With a joint prior distribution for $\boldsymbol{\beta}, \Sigma$ given by

$$f(\boldsymbol{\beta}, \Sigma) \propto |\Sigma|^{-(n+1)/2}$$

the posterior distribution of Σ is Normal-inverse Wishart¹⁴, with

$$\Sigma^{-1} \sim \text{Wishart}\left((T\mathbf{S})^{-1}, T\right) \text{ and, given } \mathbf{S}, \quad (13)$$

$$\boldsymbol{\beta}|\Sigma \sim N\left(\mathbf{b}, \Sigma \otimes (\mathbf{X}\mathbf{X})^{-1}\right) \quad (14)$$

where \mathbf{X} is $(T \times k)$.

Step 1: Draws for Σ , \mathbf{S}_{MC} , can therefore be obtained by drawing from a Wishart distribution centred on the identity matrix, inverting and pre- and postmultiplying by the factor matrices for Σ .

Step 2: To get draws for $\boldsymbol{\beta}$, the covariance matrix in (14) is factored into

$$(P_\Sigma \otimes P_{XX})(P_\Sigma \otimes P_{XX})' \text{ where } P_\Sigma P'_\Sigma = S \text{ and } P_{XX} P'_{XX} = (\mathbf{X}\mathbf{X})^{-1}$$

where P_Σ is $(n \times n)$ and P_{XX} is $(k \times k)$. Premultiplying a $(kn \times 1)$ draw of random Normals, $\text{vec}(V)$, by $(P_\Sigma \otimes P_{XX})$ gives the desired deviation from the OLS coefficients. The structure of the Kronecker product can be exploited¹⁵ to simplify this to $P_{XX}VP'_\Sigma$ where V is a $k \times n$ matrix of Normal draws. This produces a $k \times n$ coefficient matrix with the distribution we want. Draws for the VAR coefficients, \mathbf{b}_{MC} , are then computed as the sum of the OLS coefficients \mathbf{b} and the deviations from the OLS coefficients, $P_{XX}VP'_\Sigma$.

¹³Let $\{Z_i\}_{i=1}^N$ be a sequence of i.i.d. $N(0, \tau^2)$ variables. Then $W = \sum_{i=1}^N Z_i^2$ is said to have a gamma distribution with N degrees of freedom and scale parameter λ , indicated $W \sim \Gamma(N, \lambda)$, where $\lambda = 1/\tau^2$. W has the distribution of τ^2 times a $\chi^2(N)$ variable.

¹⁴If X_i for $i = 1, \dots, m$ has a multivariate normal distribution with mean vector $\mu = 0$ and covariance matrix Σ , and X denotes the $m \times p$ matrix composed of the row vectors X_i , then the $p \times p$ matrix $X'X$ has a Wishart distribution with scale matrix Σ and degrees of freedom parameter m . The Wishart distribution is most typically used when describing the covariance matrix of multinormal samples.

¹⁵ $(P_\Sigma \otimes P_{XX}) \text{vec}(V) = \text{vec}(P_{XX}VP'_\Sigma)$

Step 3: With draws for the covariance matrix, \mathbf{S}_{MC} , and the coefficients, \mathbf{b}_{MC} , and choosing a particular decomposition matrix A , we can obtain impulse response functions, variance decompositions and historical decompositions.

Steps 1 to 3 constitute one Monte Carlo draw. Repeating these three steps, say, 1000 times, we obtain posterior distributions of the parameters of interest. The problem we face is that for each Monte Carlo replication, we ought to try 12^6 possible rotations for the decomposition matrix A , check the impulse response functions and save the ones that meet our sign restrictions. This would imply computing and checking $12^6 \cdot 1000$ decompositions. Since this is computationally too demanding, we use the method proposed by Peersman (2003). For each Monte Carlo draw, we try one possible rotation for the decomposition matrix A and check the signs of the impulse responses, saving the solutions that match our restrictions. We continue until we have 1000 valid decompositions.

5.3 Figures

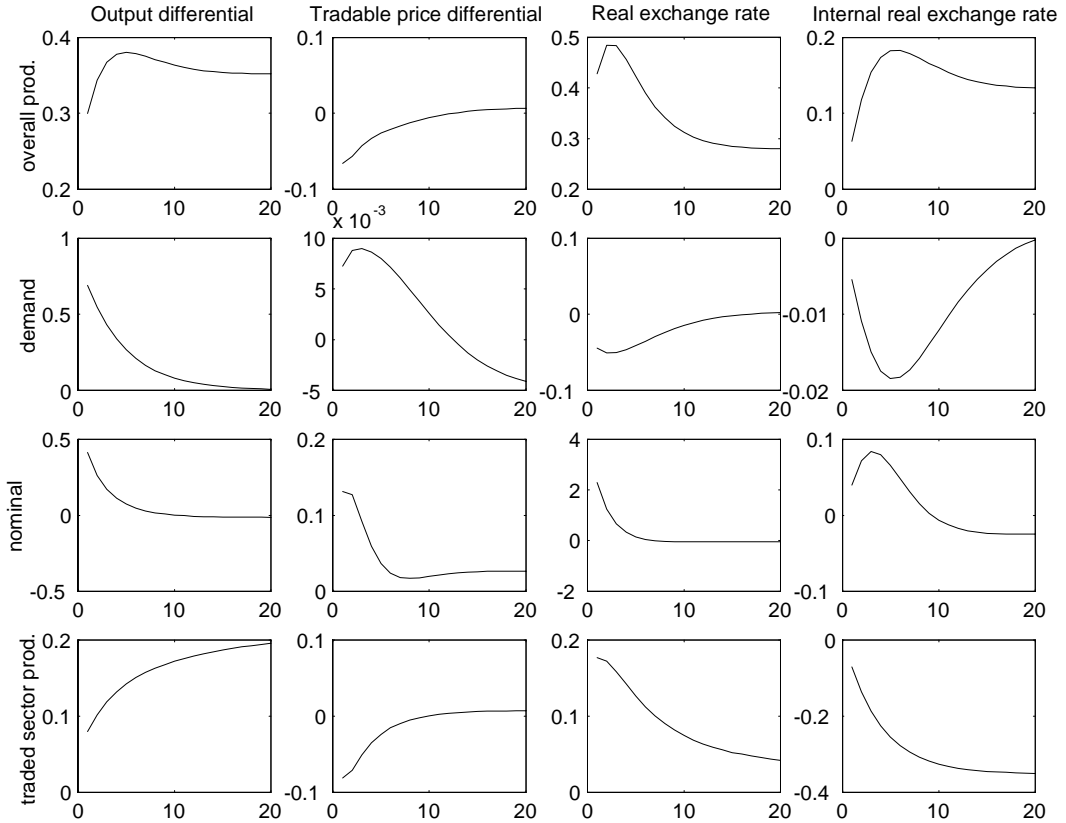


Figure 1: Impulse responses from theoretical model

Data used in VAR

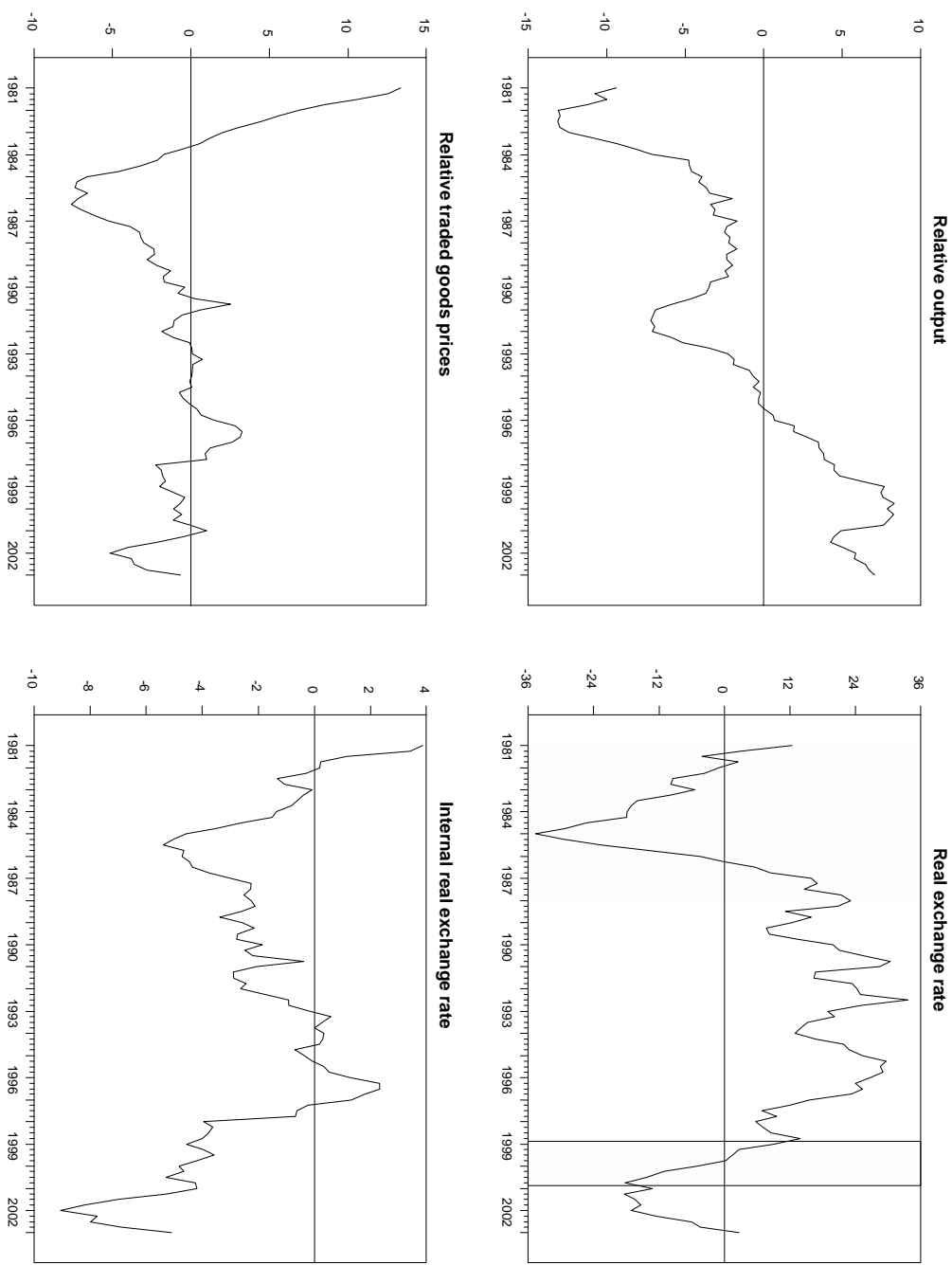


Figure 2: Data series

Impulse responses

prod=economy-wide productivity shock, dem=demand shock, nom=nominal shock, sec=sectoral productivity shock

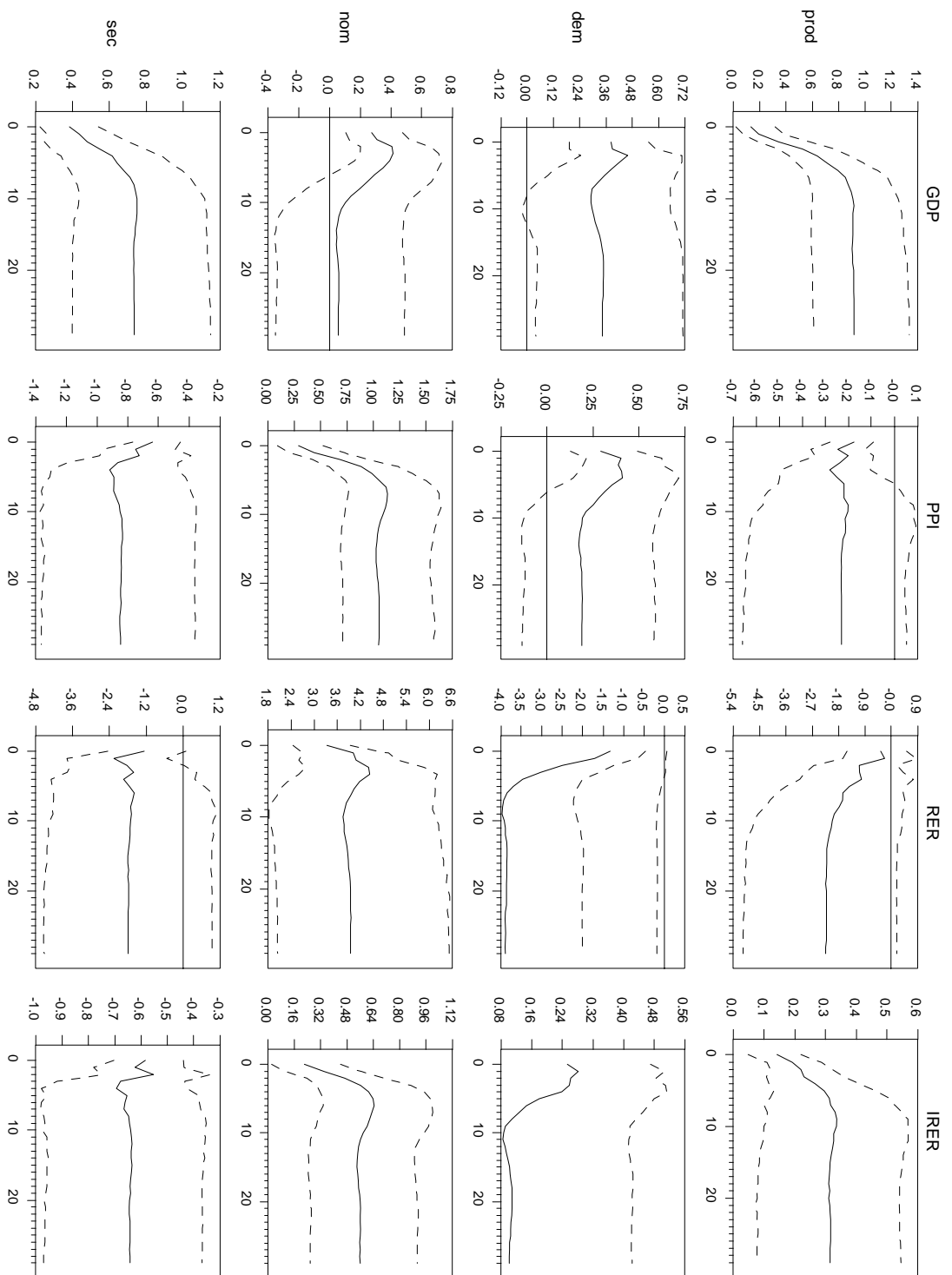


Figure 3: Impulse responses from the empirical model

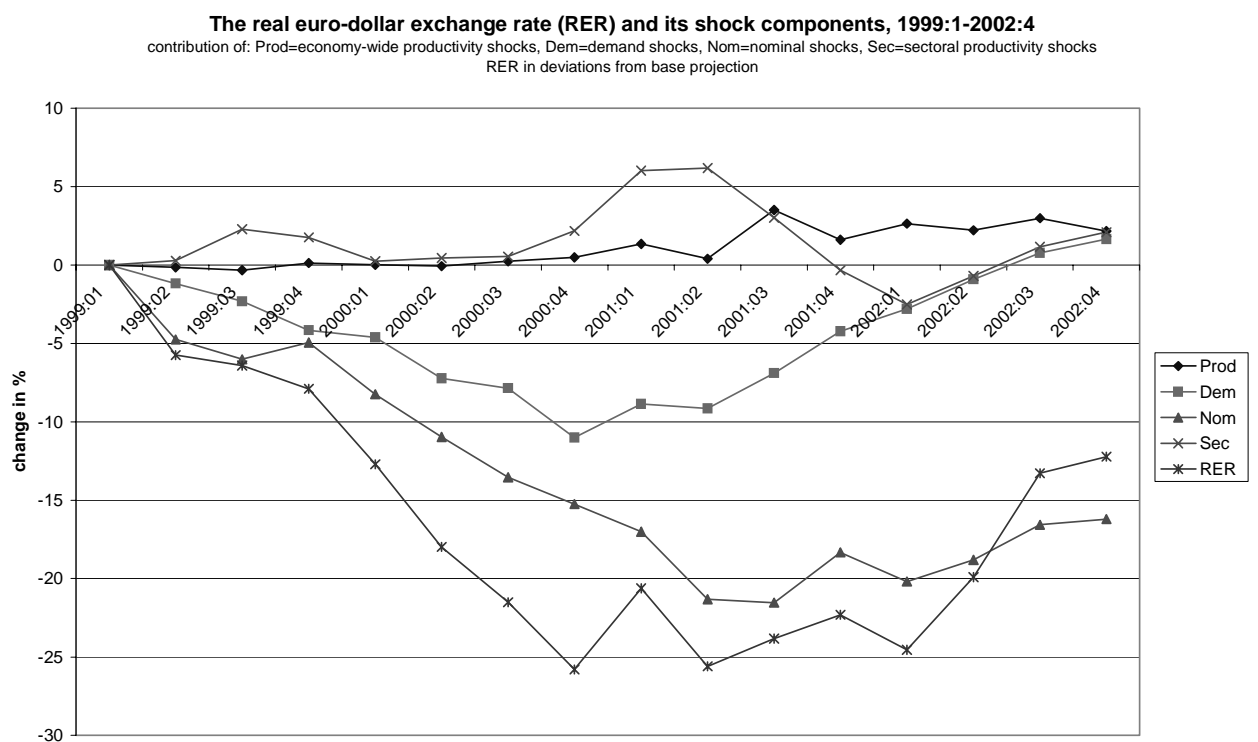


Figure 4: **Historical decomposition, 1999:1-2002:4**

5.4 Tables

Forecast error variance decomposition for relative output

	Overall productivity shocks			Demand shocks			Nominal shocks			Sectoral productivity shocks		
<i>horizon</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>
1 quarter	0,036	0,204	0,001	0,306	0,615	0,075	0,153	0,447	0,023	0,290	0,590	0,106
1 year	0,142	0,359	0,061	0,221	0,503	0,062	0,156	0,439	0,042	0,275	0,549	0,090
5 years	0,343	0,564	0,173	0,107	0,319	0,027	0,086	0,317	0,027	0,302	0,535	0,109
10 years	0,438	0,675	0,236	0,077	0,265	0,015	0,054	0,191	0,014	0,311	0,540	0,104

Forecast error variance decomposition for relative traded goods prices

	Overall productivity shocks			Demand shocks			Nominal shocks			Sectoral productivity shocks		
<i>horizon</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>
1 quarter	0,045	0,117	0,011	0,123	0,355	0,023	0,119	0,385	0,011	0,599	0,831	0,312
1 year	0,040	0,093	0,013	0,111	0,289	0,032	0,303	0,598	0,128	0,464	0,701	0,178
5 years	0,031	0,099	0,009	0,059	0,200	0,015	0,456	0,740	0,222	0,367	0,632	0,104
10 years	0,032	0,138	0,006	0,036	0,161	0,007	0,504	0,777	0,233	0,329	0,620	0,066

Forecast error variance decomposition for the real exchange rate

	Overall productivity shocks			Demand shocks			Nominal shocks			Sectoral productivity shocks		
<i>horizon</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>
1 quarter	0,030	0,144	0,000	0,101	0,274	0,015	0,652	0,855	0,358	0,101	0,349	0,010
1 year	0,040	0,181	0,008	0,169	0,356	0,046	0,554	0,768	0,259	0,121	0,383	0,022
5 years	0,066	0,250	0,012	0,270	0,486	0,087	0,411	0,679	0,149	0,102	0,347	0,020
10 years	0,097	0,347	0,014	0,279	0,536	0,083	0,340	0,625	0,100	0,098	0,334	0,013

Forecast error variance decomposition for the internal real exchange rate

	Overall productivity shocks			Demand shocks			Nominal shocks			Sectoral productivity shocks		
<i>horizon</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>	<i>median</i>	<i>upper</i>	<i>lower</i>
1 quarter	0,034	0,087	0,004	0,115	0,400	0,011	0,090	0,347	0,006	0,621	0,856	0,343
1 year	0,053	0,113	0,020	0,099	0,293	0,013	0,236	0,540	0,056	0,518	0,748	0,238
5 years	0,076	0,175	0,019	0,066	0,205	0,014	0,321	0,644	0,085	0,441	0,703	0,173
10 years	0,095	0,253	0,014	0,044	0,182	0,010	0,322	0,656	0,072	0,415	0,692	0,147

Table 2: Variance decomposition