On the strength of the US dollar: Can it be explained by output growth?*

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Abstract

One popular view on the current strength of the US dollar is that the higher growth in the US compared to Europe has stimulated foreigners to buy American assets, thereby driving up the exchange rate. In this paper a modified portfolio balance model is presented, in which it is shown that the impact of output growth on the exchange rate depends crucially on the origin of this growth. An improvement of the output gap is shown to actually depress the exchange rate whereas an increase in potential output growth leads to an appreciation, especially if this improvement is likely to be persistent. In an empirical example, it is shown that the equilibrium Dmark dollar rate is indeed positively affected by a positive trend growth differential between the US and Germany, whereas it is negatively affected by a positive output gap differential.

Keywords: rational expectations, portfolio balance model, Taylor rule, Kalman filter, foreign direct investment

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1 Introduction

Many observers have been surprised by the continuous strength of the US dollar. One popular explanation for this strength is the so called prosperous economy view. According to this view the dollar has been appreciating because international investors prefer American assets in order to profit from the presumed higher growth potential in the US compared to Europe or Japan.[1] This view lacks a sound theoretical foundation however, as the higher growth potential of the US should already be reflected in the price of US assets (Sinn and Westermann 2001). Moreover, the theory does not explain the ever increasing willingness of foreigners to hold American assets, necessary to finance the deficit on the US current account.

In this paper an attempt is made to theoretically justify the presumed link between output growth and real appreciation. For this purpose a modified portfolio-balance rational-expectations exchange rate model is developed. As usual, any surplus or deficit on the capital account has to be balanced by the current account. The capital account contains two elements in our model: net foreign direct investment and net portfolio investment. All of these elements are affected by developments in output growth, the extent to which depends crucially however on the composition of this growth. In the theoretical model a distinction is made between changes in the output gap on the one hand and changes in potential output on the other. With respect to the latter two different kind of shocks are modelled: one-time shocks and persistent shocks to the growth potential of the economy. The current account is assumed to depend on the real exchange rate and the output gap, net direct investment depends crucially on trend growth, whereas desired net portfolio investment depends on expected deviations from uncovered interest parity. After calibrating the model for the US economy, the model is solved using model-consistent forward looking expectations for the exchange rate. It turns out that positive demand shocks – that affect the output gap, but not potential output – result in a depreciation

[1] Corsetti and Pesenti (1999) were among the first to show the apparent link between the euro dollar rate and the expected Europe US growth differential. They conclude that cyclical divergence seems to be the root cause for the slide of the euro, but do not present a formal theory explaning this phenomenon.

of the dollar, whereas supply shocks, especially persistent ones, lead to an appreciation.

The theoretical model can thus offer two explanations for the persistent strength of the dollar despite the large deficit on the current account. First, the continuous outflow of dollars due to the deficit on the current account is partly compensated by the inflow due to net foreign direct investment. Second, as the output gap and trend growth are not observable, the prolonged period of high growth without much inflationary pressures might have resulted in a gradual update on the likely composition of output growth. Indeed, as the prosperous period of high growth prolonged, more and more people started to believe in a New Economy in which the output gap would be less pronounced and potential output growth would be permanently higher than before.

In an empirical application on the Dmark-dollar rate, it is shown that the equilibrium real value of the dollar is indeed positively affected by higher trend growth in the US compared to Germany, whereas a positive output gap differential has a negative impact. The explanatory power of the relationship is relatively weak however. The real exchange rate may occasionally deviate from its 'equilibrium value' by as much as 40%. Based on these results, the current weakness of the euro – according to our model the euro is about 15% undervalued relative to the dollar – is not exceptional.

The rest of the paper is structured as follows. Section 2 provides some background on recent developments, primarily in the US. In Section 3 the theoretical model will be presented. The model will be calibrated for the US and the results are shown by means of impulse response functions. In Section 4 the predictions of the theoretical model will be confronted with realisations of the Dmark dollar rate. Section 5 concludes, Appendix A describes the state space representation of the theoretical model, and Appendix B provides the data sources.

2 Stylised facts

Figure 1 shows the annual real growth in both the US and (up until 1991 West-) Germany. In historical perspective, the real growth rates in the US during the nineties were not exceptional.

Even higher growth rates were occasionally achieved in both the sixties, seventies and the eighties. What was exceptional though, was the length of the prosperous period. Moreover, at the same time, Germany was experiencing an exceptionally long period of slow growth in the aftermath of the German unification. For Japan, economic conditions during the nineties were even worse. Consequently, for a prolonged period the US outperformed Germany, and Japan, in terms of growth. Also during the mid eighties, growth in the US was substantially higher in the US than in Germany, especially in 1984. During this period the dollar seemed also highly overvalued (Figure 2). Therefore, there seems to be some empirical support for the prosperous economy view.

Figure 3 sheds some more light on this issue. It shows that the bulk of the large deficits on the current account resulting from an overvalued dollar is financed by portfolio investment. However, within this category, net portfolio investment in equity is relatively unimportant. For instance, during much of the nineties the net capital flow related to equity was actually negative for the US, meaning that US investors bought more foreign stocks that foreigners bought US ones. Foreign direct investment seems more directly related to growth differentials, although with a lag.

3 The theoretical model

As a starting point for our theoretical model the dynamics of output, inflation and interest rates is modelled similar to the closed economy models of Gerlach and Smets (1997), Rudebusch and Svensson (1999) and Smets (1999), supplemented by some open economy features:

$$y_t^r \equiv y_t^p + h_t \tag{1}$$

$$y_t^p = y_{t-1}^p + g_t + \varepsilon_t^{sup} \tag{2}$$

$$g_t = \alpha_0 + \alpha_1 g_{t-1} + \varepsilon_t^g \tag{3}$$

$$h_t = \phi_0 + \phi_1 h_{t-1} + \phi_2 h_{t-2} + \phi_i(i_{t-1} - \bar{\pi}_{t-1}) + \phi_q q_{t-1} + \phi_{sup}(\varepsilon_t^{sup} + \varepsilon_t^g) + \varepsilon_t^d \tag{4}$$

$$\pi_t = \beta_0 + \beta_1 \pi_{t-1} + \beta_2 \pi_{t-2} + \beta_3 \pi_{t-3} + \beta_4 \pi_{t-4} + \beta_h h_t + \beta_{\Delta s} \Delta s_t + \varepsilon_t^\pi \tag{5}$$

$$\Delta i_t = \gamma_{\Delta h} \Delta h_t + \gamma_{\Delta \pi} \Delta \bar{\pi}_t + \gamma_i (i_{t-1} - \gamma_0 - \gamma_{h1} h_{t-1} - \gamma_{\pi 1} \bar{\pi}_{t-1}) + \varepsilon_t^i \tag{6}$$

where y_t^r is log real GDP; y_t^p is log potential output; h_t is the output gap; g_t is trend growth of potential output; i_t is the quarterly average nominal short term interest rate; π_t is quarterly CPI inflation (not annualised), $\bar{\pi}_t \equiv \sum_{i=0}^{3} \pi_{t-i}$ is annual inflation; q_t is the log CPI-based real exchange rate; s_t is the quarterly average log nominal exchange rate expressed as the domestic currency value of foreign currency; Δ denotes the first difference operator $\Delta x_t \equiv x_t - x_{t-1}$.

Output is decomposed in potential output and the output gap (Equation 1). With respect to potential output the main difference between our model and the one in Smets (1999) is that the trend growth of potential output changes over time. The trend growth process (Equation 3) is supposed to be stationary ($\alpha_1 < 1$) as there is ample evidence of output growth being stationary.[2] Equation 4 describes the dynamics of the output gap as being related to its lagged values, the lagged real interest rate, the lagged real exchange rate, demand shocks (ε_t^d), and one-time or persistent supply shocks (ε_t^{sup} respectively ε_t^g). The influence of supply shocks on the output gap seems natural as the output gap is defined as the difference between demand and supply. If ϕ_{sup} is restricted to zero, any shock to supply has to result in an equal rise in demand. Moreover, supply shocks would not have any influence on inflation in that case. Equation 5 can be interpreted as a Phillips-curve which relates inflation to the output gap and to lags in inflation. Moreover, as this system is part of an open economy model, current depreciation is also supposed to increase inflation. Finally, Equation 6 represents the familiar

[2] Gerlach and Smets (1997) model a random walk process for trend growth.

Taylor rule (Taylor 1993), expressed in error correction form.

Our exchange rate model follows the tradition of the portfolio balance approach (Branson, Halttunen, and Masson 1977) in the sense that domestic and foreign bonds are viewed as imperfect substitutes, and in that the equilibrium on the balance of payments is given a central role. We distinguish three main elements on the balance of payments: the trade balance, net foreign direct investment and portfolio investment. The exchange rate related equations are the following:

$$\mathrm{FA}^*_t = \delta_{uip}\left(\frac{i^f_t - i_t}{4} + E_t\Delta s_{t+1}\right) + \nu^{fa}_t \qquad \text{with} \qquad \nu^{fa}_t = \delta_1\nu^{fa}_{t-1} + \varepsilon^{fa}_t \tag{7}$$

$$\mathrm{FDI}_t = \chi_0 + \chi_g\frac{1}{4}\sum_{i=1}^{4} g_{t-i} + \varepsilon^{fdi}_t \tag{8}$$

$$\Delta\mathrm{TB}_t = \tau_1(\mathrm{TB}_{t-1} - \tau_q q_{t-1}) + \tau_{\Delta h}\Delta h_t + \varepsilon^{tb}_t \tag{9}$$

$$\mathrm{FA}^{rev}_{t|t-1} = \left(\mathrm{FA}^{ass}_{t-1}\frac{S_t}{S_{t-1}}(1 + \frac{i^f_{t-1}}{4}) - \mathrm{FA}^{lia}_{t-1}(1 + \frac{i_{t-1}}{4} - \mu_{\varepsilon^i}\varepsilon^i_t)\right)\frac{\mathrm{GDP}^{us}_t}{\mathrm{GDP}^{us}_{t-1}} + \varepsilon^{rev}_t \tag{10}$$

$$\mathrm{FA}_t = \mathrm{FA}^{rev}_{t|t-1} + \mathrm{TB}_t + \mathrm{FDI}_t \tag{11}$$

$$\mathrm{FA}_t = \mathrm{FA}^*_t \tag{12}$$

$$\Delta s_t \equiv \pi_t - \pi^f_t - \Delta q_t \tag{13}$$

where FA^*_t and FA_t denote the desired respectively actual stock of net foreign portfolio assets, that is to say foreign assets (FA^{ass}_t) minus foreign liabilities (FA^{lia}_t), both measured in dollars and scaled by nominal US GDP (GDP^{us}_t); $\mathrm{FA}^{rev}_{t|t-1}$ denotes the revalued scaled dollar value, including interest or dividend payments, of the time $t-1$ stock of net foreign assets; FDI_t and TB_t represent the US net flow of foreign direct investment respectively the US trade balance, both measured in dollars and scaled by nominal GDP; i^f_t and π^f_t denote foreign short term interest rate and quarterly inflation rate respectively; E_t represents the expectations operator, conditional on time t information.

The desired stock of foreign assets is usually related to the stock of wealth in a portfolio balance model. For the US demand of foreign assets US wealth would probably indeed be the theoretically preferred scaling variable. However, for the foreign demand of US assets, the net flow of foreign direct investment or the US trade balance, US GDP seems more appropriate.

Moreover, using wealth would be problematic when calibrating the model as wealth is not easily observable. The choice of the scaling variable is not essential in our model.

The desired amount of net foreign assets (Equation 7) – which comprises the US demand for foreign assets minus foreign demand for US assets – is assumed only to depend systematically on expected deviations from uncovered interest parity (UIP). According to the prosperous-economy view the strength of the dollar is primarily due to the better growth potential for the US than for Europe or Japan. Consequently, international investors want to buy more US assets, thereby driving up the exchange rate. This would mean a direct influence of growth potential on foreign asset demand. However, as Sinn and Westermann (2001) rightfully argue, the presumed higher growth potential of the US would not only trigger foreigners to buy US stocks, but US investors as well. Therefore, the improved profit potential should already be priced in the US stocks. Given the already higher stock price it is not a priori clear why foreigners would like to buy more US stocks. In order to justify the relative increase in foreign demand for US assets it either has to be the case that the required discount rate used by foreigners is lower than the one for US investors or that expected future exchange rate changes make US assets more attractive for foreigners. With respect to expected future appreciations, it does not make much difference which US assets are bought. Regarding the required discount rate, a similar result seems plausible. Decomposing the required discount factor into the risk free rate plus the required risk premium, the latter is not likely to explain the lower rate for foreigners. Higher growth potential in the US could hardly alter the risk premium for them, whereas the risk premium for US citizens might fall due to lower precautionary saving. A lower risk free rate for foreigners is however quite likely, as the central bank controlled short term interest rates are likely to be lower if economic conditions are worse. This argument however leads to the conclusion that for portfolio investment, increased growth potential is primarily important because of its influence on interest rates or future exchange rates. The fact that the UIP relationship is central to the portfolio investment decision does not mean however that all portfolio investments are done in one period bonds. It merely states that the relative attractiveness for foreign assets depends on short term interest rate differential and expected exchange rate developments. In order to model persistent deviations of desired asset values from UIP-implied ones the disturbance term (ν_t^{fa}) in Equation 7 is given an AR(1) structure.[3]

The argument that higher growth potential as such does not make equity investment more interesting for foreigners than for domestic investors is less convincing when foreign direct investment is concerned. For foreign direct investment not only the profit opportunities of the acquired firm are important, but also the extent to which the investment improves the performance of the acquiring firm. Synergy effects and, for instance, improved marketing and distribution channels due to an acquisition are likely to be higher if the acquired firm is located in a fast growing economy. Therefore, it is assumed that net foreign direct investment is a function of the trend growth of the economy (Equation 8). As direct investment is a long term decision, the – relatively short lived – output gap is not considered a relevant variable. The one to four quarter lag in the influence of trend growth seems reasonable given information and decision lags. Apart from trend growth, other variables might have been included in the specification as well. One obvious candidate would be the real exchange rate as a highly overvalued dollar would make foreign direct investment to the US more expensive. On the other hand in as far as foreign direct investment is complementary to foreign trade, an overvalued dollar stimulates FDI if the overvaluation is likely to last. Campa and Goldberg (1999) reveal that the real exchange does have little or no effect on direct investment in high mark-up sectors which absorb much of the actual exchange rate changes in their mark-ups, whereas the reverse holds for low mark-up industries. We found the influence of the real exchange rate on FDI to be highly instable, with a significant positive effects during some periods, and a significant negative impact on others. Also the effects of interest rates, see Stokman and Vlaar (1996), primarily representing intra-firm capital transfers, turned out to be unstable.

Equation 9 describes the trade balance as being determined in the long run by the real

[3]Apart from financial considerations net foreign asset positions might also depend on real factors. Lane and Milesi-Ferretti (2001) find a significant impact of relative output levels, the stock of public debt and demographic factors. The fit of their model for the US is rather poor however.

exchange rate and in the short run by changes in the output gap. The real exchange rate only starts to affect imports and exports with a lag as trade volumes are only adjusted gradually, the so-called J-curve effect. The output gap is included to account for the increased import demand in economic upswings.[4] The capital income part of the current account is directly related to the lagged stocks of foreign assets. This part is modelled together with the revaluation of net foreign assets (Equation 10). The time t scaled dollar value of $t-1$ net foreign assets equals the time $t-1$ stock of foreign assets of US investors, subject to possible dollar depreciations and quarterly interest rate payments, minus the time $t-1$ US liabilities, subject to interest payments and possible revaluation due to unexpected interest rate changes.[5] Both US foreign assets and liabilities are moreover adjusted for nominal GDP changes as this is the denominator of the fraction. The revaluation effect of unexpected interest rate changes is included as investors do not only buy short term foreign bonds, but also long term bonds or stocks. Foreign variables are not supposed to be subject to unexpected changes.

The last three equations are identities. Equation 11 represents the equilibrium on the balance of payments, Equation 12 states that desired foreign asset demands are always fulfilled, and Equation 13 relates nominal to real depreciations.

The required equality of desired and actual foreign asset stocks is the main driving force for the exchange rate in the model. Given the expected future exchange rate, the solution for the current exchange rate is obvious. Following Dornbusch (1976) we will assume that expectations are formed fully rationally — that is to say model-consistent — and that the system will evolve towards a long-run steady state in which the real exchange rate reaches an equilibrium level

[4]According to the intertemporal theory of the current account (see Sachs (1981) and Obstfeld (1986)), temporarily high growth rates associated with the positive output gap should conversely lead to higher savings, and therefore a current account surplus. The empirical evidence on this issue is ambiguous. Backus and Kehoe (1992) find net export positions to be coutercyclical. Vredin and Warne (1991) on the other hand find the Swedish current account to be positively correlated with the transitory component of GDP.

[5]In this formula it is implicitly assumed that the average quarterly return on all foreign assets equals the interest rate. Under the assumption that domestic and foreign equity risk premia are equal, this simplifying assumption has no consequences for the solution of the model.

that is consistent with current account balance. In the portfolio balance literature the concept of model-consistent expectations is hardly pursued. An exception is Dooley and Isard (1982), who use an iterative procedure to derive model-consistent one month ahead predictions. Also in their model, expectations for two or more months ahead are not rational however. Our model is solved by means of the method of undetermined coefficients (Blanchard and Kahn 1980). See Appendix A for details.

3.1 Calibration

In order to find the parameters for the output and inflation equations of the model, we follow Kuttner (1994), Gerlach and Smets (1997) and Smets (1999) in using the unobserved components (UC) technique, see Harvey (1989). The main reason to estimate potential output and output gap equations as opposed to relying on published series is that it is easier to match our model characteristics. The potential output series published by the Congressional Budget Office, for instance, is very smooth and contains a unit root. Therefore, it would violate the steady state assumption for output growth, and one-time supply shocks would be absent. Moreover, by applying the one-sided Kalman filter to estimate the unobserved state variables, one can be sure that indeed no future information is used in calculating these variables.[6] Another advantage of estimating the output gap is that the same method can be used for other countries, in this case Germany (Section 4).

Some small modifications had to be made relative to the theoretical model in order to better account for the empirical regularities. Both output and inflation turned out not to be free of seasonal patterns. For inflation this was remedied by including seasonal dummies, whereas the seasonal pattern of output was modelled by means of three extra state variables (Harvey 1989) as it seemed to be changing over time. The inflation equation was extended to include

[6] In the UC methodology, the state variables refer to the unobserved components. This should not be confused with the endogenous state variables referred to in the previous chapter, as there – in the tradition of the real business cycle literature – state variables stand for the endogenous variables that drive the dynamics of the system.

the supply factors oil and import prices, and the real long term interest rate to account for forward looking aspects of inflation. These factors pick up systematic changes in inflation that are not due to the output gap. Finally, the system was extended with an unemployment rate equation, in order to be better able to identify the different components of output. As the unemployment rate itself does not influence any of the other variables in the model, there will be no implications for the theoretical model.[7] The estimated system has the following state space representation:

$$\begin{bmatrix} \Delta y_t^r \\ \pi_t \\ \Delta u_t \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & 1 & 1 & -1 & 0 \\ \beta_h & 0 & 0 & 0 & 0 & 0 & 0 \\ \psi_{\Delta h} & -\psi_{\Delta h} & \psi_g & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} h_t \\ h_{t-1} \\ g_t \\ f_t \\ s1_t^y \\ s2_t^y \\ s3_t^y \end{bmatrix} +$$

$$\begin{bmatrix} 0 \\ \left(\begin{array}{c} \beta_0 + \beta_1 \pi_{t-1} + \beta_2 \pi_{t-2} + \beta_3 \pi_{t-3} + \beta_4 \pi_{t-4} + \beta_{oil} \Delta p_t^{oil} + \beta_{imp1} \Delta p_{t-1}^{imp} \\ + \beta_{imp2} \Delta p_{t-2}^{imp} + \beta_{rrl} (i_t^l - \bar{\pi}_t) + \beta_{s1} s1_t + \beta_{s2} s2_t + \beta_{s3} s3_t \end{array} \right) \\ \psi_0 + \psi_1 u_{t-1} + \psi_{\Delta 1} \Delta u_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t^{\pi} \\ \varepsilon_t^{u} \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} h_t \\ h_{t-1} \\ g_t \\ f_t \\ s1_t^y \\ s2_t^y \\ s3_t^y \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} h_{t-1} \\ h_{t-2} \\ g_{t-1} \\ f_{t-1} \\ s1_{t-1}^y \\ s2_{t-1}^y \\ s3_{t-1}^y \end{bmatrix}$$

$$+ \begin{bmatrix} \phi_i (i_{t-1} - \bar{\pi}_{t-1}) + \phi_q q_{t-1} \\ 0 \\ \alpha_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & \phi_{sup} & \phi_{sup} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_t^d \\ \varepsilon_t^g \\ \varepsilon_t^{sup} \\ \varepsilon_t^{sy} \end{bmatrix} \quad (15)$$

[7] When both the output gap and (changes in the) unemployment rate were included in the inflation equation, only the former turned out to be significant.

where u_t denotes the log unemployment rate; f_t is an auxiliary variable representing the one-time supply shocks[8]; $s1_t^y, s2_t^y$ and $s3_t^y$ are state variables representing the seasonal pattern of y_t^r; p_t^{oil} and p_t^{imp} represent log oil and import prices respectively; i_t^l denotes the long term interest rate; $s1, s2$ and $s3$ are demeaned seasonal dummies; the real interest rate and real exchange rate are demeaned in order to derive a zero-mean output gap.

Equation 14 is the measurement or observation equation, which states how the vector of observed variables is related to the vector of unobserved components. Equation 15 is the transition equation, which specifies the time-series process that governs the evolution of the unobservable variables. The disturbances in both the measurement and the transition equations are assumed to be uncorrelated.

The unemployment rate is supposed to be affected by changes in the output gap and the trend growth, but not by one-time shocks to potential output. The reason for including trend growth is that high potential output growth is likely due to a more flexible and optimistic business environment. In such an environment, the natural rate of unemployment is likely to be lower as search costs for a new job reduce, and the opportunities to start up new businesses improve. One-time shocks on the other hand do not influence the structural situation and thereby are not believed to influence unemployment.

In order to be optimally able to identify the output gap and changes in trend output, the maximal available data set is used to estimate the parameters as preferably several cycles should be included. For the US this means data for 1957:I until 2001:II. Table 1 gives the estimated maximum likelihood parameters, except seasonal dummies, for the US.[9]

The estimate of ϕ_{sup} turned out to be very imprecise and quite sensitive for slight changes in the model. For none of the specifications a value of -0.5 could be rejected. Also, no significant effect of the real exchange rate could be found. Moreover, as the free estimate resulted in a

[8] The supply shocks are modelled as state variables as they influence the output gap, which is also a state variable.

[9] The model is estimated in Gauss, with the use of the Kalman filter procedure written by Paul Söderlind. This procedure follows the syntax of Harvey (1989).

slightly positive coefficient – meaning a positive impact of overvaluation on the output gap – it was restricted to zero.

Figure 4 shows the resulting output gap (straight line), using only current and lagged data, together with a plus or minus two standard deviation confidence band. Also, the output gap according to the Congressional Budget Office (CBO) is shown (dashed line) as a reference value. Although the overall pattern of the two series is similar, substantial differences also arise. Especially during the high inflation period from the mid seventies to the beginning of the eighties, the UC-method predicts a significantly higher output gap. Overall, the fluctuations according to the UC-method are somewhat smaller, with only occasionally an output gap that is significantly different from zero. These results are comparable to Smets (1999).

Regarding potential output, our estimates indicate that the persistence of growth shocks is very significant, though fairly limited in size as the half-time of growth shocks is less than one year. Moreover, one-time shocks to potential output seem to dominate persistent shocks as σ^2_{sup} is about ten times larger than σ^2_g. This contrasts with the Congressional Budget Office estimate of potential output, which is smooth and seems integrated of order two. Both series are plotted in Figure 5. The UC-estimate of potential growth (straight line) is much more volatile, and seems to lead the CBO-indicator (dashed line) by at least a year. The two standard deviation confidence band is very large however.

The coefficient for the inflation and unemployment equations all have the expected sign and most are very significant. With respect to the exchange rate effect on inflation ($\beta_{\Delta s}$ in the theoretical model), the estimated equation does not give direct guidance. Exchange rate effects pass trough via the oil and imports prices. Due to pricing to market however, the relationship between the two is probably not one to one. Based on these results we calibrate $\beta_{\Delta s} = 0.06$ in the baseline model, and investigate the results for $\beta_{\Delta s} = 0$ as well.

Table 2 provides the estimates of the interest rate (6), foreign direct investment (8) and trade balance (9) equations. The estimation sample starts in 1980 in order to account for changes in interest rate policy and especially trade and capital liberalisation. The direct inflation effect on the interest rate turned out to be negative, though not significantly so, and was restricted to zero.[10] In the long run the inflation effect is significantly positive, although the estimated coefficient (1.1) is somewhat smaller than the standard value (1.5). Equivalence of the short- and long-run effects had to be strongly rejected. Therefore this restriction, which is most common in the literature, was not imposed. The sensitivity with respect to these parameters will be investigated later on.

Both the foreign direct investment and the trade balance equation were extended with seasonal dummies. Although all coefficients have the expected sign and are highly significant, both equations seem to be somewhat misspecified, as higher order serial correlation remains. The trade balance equation is estimated by two stage least squares as simultaneity problems between the trade balance and the output gap seem likely. The short term interest rate, which is used as an instrument, is not expected to be influenced by the trade balance contemporaneously.

The remaining parameters to be calibrated are $\delta_{uip}, \delta_1, \mu_{\varepsilon i}$ and μ_{fa}. These parameters have to do with the (composition of the) stock of foreign assets. Unfortunately, data on these matters are scarce and probably not very reliable. Figure 6 shows the stock of US assets and liabilities as reported in the international investment position of the IFS. We include both portfolio and other investment stocks as the latter form a non-negligible part of the balance of payments. The other investments are dominated by banks, for which portfolio considerations might also play a significant role. The stock of assets and liabilities as a percentage of GDP hardly converges to a steady state as it is constantly increasing over time. Moreover, it seems that the stock of liabilities is growing faster than the stock of assets. Based on these results, the 'steady state' value for the stock of assets and liabilities (μ_{fa}) was calibrated at 0.5 as current values are most representative. With respect to the composition of foreign assets nothing much can be said. Equity comprises a non-negligible part of foreign assets (about one third), but nothing is known about the maturity of bonds. The interest rate shock elasticity of foreign

[10] If the sample for the interest rate equation was restricted to 1989:IV-2001:II, serious autocorrelation problems occurred and the long-run inflation coefficient turned out to be negative as well.

assets ($\mu_{\varepsilon i}$) is calibrated at 1.

As can be seen from Figure 6, the net foreign assets of the US display a clear negative trend. Unfortunately, the impact of interest rate differentials or future depreciations could hardly be detected. When a time trend was included in the regression, the estimate of the interest rate effect became as small as 0.36 and not significant at all. When the trend was not included or when the ex-post deviation from uncovered interest parity was modelled, no positive sign was detected.[11] Therefore, δ_{uip} was calibrated at 0.36 in the standard scenario, whereas an alternative was investigated with $\delta_{uip} = 100$. In order to model the very persistent unexplained residual, δ_1 was given the value 0.99.[12]

3.2 Impulse response analysis

Figure 7 gives the responses of a one percent demand shock on several key variables. The output gap initially improves before declining after the second quarter. It takes just over three years for the output gap to reach zero again. The interest rate immediately reacts to the demand shock, almost on a one to one basis. Consequently, the impact on inflation is relatively small, although it should be realised that the inflation rate in the model is not annualised. Given the firm rise in interest rates it is surprising that the real exchange rate is depreciating in first instance. The expected future depreciations more than compensate for the higher interest rate, leading to a net increase in foreign assets held by US investors. The expected depreciation might partly be explained by the trade deficit. The real exchange rate reaches its lowest value of -1.8% after almost two years. In nominal terms (not shown here) the maximum depreciation is about 3%, also reached after two years.

The responses of a one percent trend growth shock are given in Figure 8. This shock results in a substantial real appreciation, about 18% after two and a half years (22% in nominal terms). This is primarily due to a substantial increase of net foreign direct investment. Also

[11] The results were slightly worse if the net other investment positions were excluded.

[12] Formally, δ_1 needs to be smaller than one in order for the steady state to exist.

the demand of foreigners for US portfolio assets exceeds the US demand for foreign assets as the net foreign asset position of the US declines. Again, the expected appreciation effect more than compensates for the somewhat lower interest rate. Inflation declines, both due to the appreciation and the negative output gap. In the longer run, the real appreciation deteriorates the trade balance (after five years the deficit is about 0.9% of GDP), which in turn leads to an outflow of foreign assets and, in as far as this is not desired, to a gradual depreciation.

The impact of a one-time supply shock is not shown as the only impact of this shock is via the negative impact on the output gap. Consequently, the impulse responses are equivalent to the ones for a demand shock, except for the sign and the size.

Figure 9 provides the impulse responses of an inflation shock. As the interest rate only responds to inflation with a lag, real interest rates decline and the output gap improves. Nevertheless, the inflation rate quickly returns to low level. The real exchange rate initially rises only slightly less than inflation leaving the nominal exchange rate almost unchanged. In the longer run the nominal exchange rate depreciates up to 3.5% after five years.

The responses to an interest rate shock are shown in Figure 10. An interest rate shock leads to an immediate appreciation of the dollar. This appreciation has two causes. Firstly, the higher interest rate induces foreigners to hold more dollar assets, thereby reducing US net foreign assets. Here the interest rate effect is big enough to compensate for the expected future depreciation. Second, the interest rate rise depreciates the current stock of US assets held by foreigners. With a lag of one quarter the output gap starts to deteriorate leading to an improved trade balance and lower inflation.

The final impulse responses shown are those of a desired foreign assets shock. As shown in Figure 6, foreign asset positions can deviate substantially and persistently from UIP-induced values. The gap between net foreign assets and liabilities can be as much as 15% of GDP, whereas our model assumes equality in steady state. A one percent of GDP increase in net foreign asset demand leads to an immediate real depreciation of about 1.8% (Figure 11). This depreciation leads to an improvement of the trade balance, thereby accumulating foreign assets over time. Consequently, despite the fact that the increased extra demand for foreign assets is very persistent ($\delta_1 = 0.99$), the real exchange rate returns to zero within five years.

Taken together, one may wander whether the model can explain the large shifts is the real exchange rate observed in reality (Figure 2). A one percent shock in trend growth ultimately leads to a 18% real appreciation. However, a one percent change in quarterly trend growth is highly unrealistic as the standard deviation of these shocks is only 0.18%. Consequently, one can hardly justify real exchange rate changes of more than say 8% by these shocks.[13] The impact of demand shocks is limited to at most 2.5%, given the standard deviation of 0.66%. Inflation or interest rate shocks can explain even less. The desired foreign asset shocks can explain some deviation, but in order to explain sustained periods of overvaluation, a growing foreign demand for US assets seems necessary.

3.3 Sensitivity analysis

Not all the parameters in the model are without doubt. In this section some alternative scenarios will be presented. Thereby, we will concentrate on the real exchange rate. The first scenario considered is the one in which the assets of different countries are almost perfect substitutes. In the model, this is reflected in a calibrated value for δ_{uip} of 100. Figure 12 shows the impact of a one percent demand, trend respectively interest rate shock on the real exchange rate. Surprisingly enough, the real exchange rate response to an interest rate shock is not very much affected by the abandonment of this crucial assumption of the portfolio balance model. The only difference for the exchange rate is that its equilibrium level is reached already after two year instead of after three and a half for the standard model.

The impact of a demand shock on the other hand changes radically under perfect substitution. The real exchange rate now appreciates initially 0.8% after which it gradually depreciates to zero in four and a half years. The initial appreciation is due to the fact that foreigners

[13]Larger effects can probably be generated by distinguishing tradables from non-tradables. If productivity changes are concentrated in the tradables sector, the real exchange rate is permanently affected by supply shocks. This is the well known Balassa-Samuelson effect (see Balassa (1964) and Samuelson (1964)).

demand much more US assets in response to the implied interest rate rise in the US. The outflow of funds due to the trade balance deficit is less important under perfect substitution. The impact on the real exchange rate of a trend growth shock also changes substantially as it is no longer hump-shaped. The initial appreciation of almost four percent is still more substantial than the one for a demand shock. In contrast to the standard model, the real exchange rate does not appreciate further as the flow of capital due to net foreign direct investment is easily absorbed. However, also the pace of subsequent depreciation is much slower under perfect substitution, as not even half of the initial appreciation is made up after ten years. Concluding one might say that allowing for perfect substitution, the variability of real exchange rates is even less understood.

The second scenario investigates the impact of a different Taylor rule parameterization. The absence of a direct interest rate effect of inflation changes might have consequences for the impulse responses. Therefore, we also computed them under the more common parameterization: $\gamma_{\Delta\pi} = 0.36; \gamma_{\pi 1} = 1.5; \gamma_{\Delta h} = 0.12; \gamma_{h1} = 0.5; \gamma_i = -0.24$. This calibration implies a combination of interest rate smoothing and similar short- and long-run targets. The results of this exercise are shown in Figure 13. Apart from the somewhat longer persistence of the interest rate shock, hardly any changes are visible. Consequently, the results are robust in this respect.

The final two scenarios check the sensitivity with respect to the direct price effect of depreciations ($\beta_{\Delta s} = 0$, see Figure 14) and the influence of supply shocks on the output gap ($\phi_{sup} = 0$, see Figure 15). The real exchange rate dynamics turns out not to be sensitive for these adjustments.

4 The D-mark dollar rate

The empirical support for the portfolio balance model has not been very positive (Taylor 1995). One of the reasons for the poor results is probably the lack of reliable data on wealth and, especially, foreign investment positions. Usually, accumulated current account balances are used to approximate foreign asset holdings. This procedure however, does not take into account direct investment and revaluations of assets other than due to exchange rate developments.

We will not try to derive an exact testable equation from our theoretical model. Data limitations, especially with respect to the stock of foreign assets, or even the bilateral capital flows, would make this almost impossible. Moreover, for an empirical model the convenient assumption that the rest of the world is constantly in steady state is clearly not realistic. Consequently, a multi-country model needs to be derived first. This is clearly beyond the scope of this paper. Also, as is quite clear from Figure 6, the relationships modelled in this paper can not tell the full story, and therefore can not be expected to forecast well. The purpose of this section is more modest. It merely seeks empirical evidence for our model predictions with respect to the influence of different kind of output shocks on the exchange rate. Therefore, an ad hoc empirical model on the Dmark dollar rate will be estimated.

Two of the ingredients of the empirical exchange rate equation will be the German trend growth and output gap. Therefore, the unobserved components model (Equations 14 and 15), was also estimated for Germany. The results are shown in Table 3. The measurement equation for output was augmented with a dummy for 1991:I in order to take account of the German unification. The parameters are broadly similar to the ones for the US. The variance parameters of the trend growth and the unemployment shocks had to be fixed as they would converge to zero otherwise.

Based on these results the following equation was estimated over the period 1973:II – 2001:II:

$$\begin{aligned}\Delta s_t^{dm} = & \underset{(3.1)}{-0.11}\left(s_{t-1}^{dm} + p_{t-1}^{dif} - \underset{(1.5)}{2.99}\; i_{t-1}^{dif} - \underset{(2.0)}{68.09}\; g_{t-1}^{dif} + \underset{(2.1)}{6.04}\; h_{t-1}^{dif} - \underset{(2.1)}{0.21}\; p_{t-1}^{oil} + \underset{(0.8)}{24.43}\right) \\ & \underset{(2.2)}{+0.21}\; \Delta s_{t-1}^{dm} + \underset{(2.2)}{13.2}\; \Delta g_t^{dif} + \underset{(2.1)}{2.71}\; s1_t + \underset{(2.0)}{2.52}\; s2_t + \underset{(1.3)}{1.65}\; s3_t \end{aligned} \tag{16}$$

$$\bar{R}^2 = 0.17 \quad LM(1) = \underset{[0.20]}{1.69} \quad LM(4) = \underset{[0.10]}{2.01} \quad \text{Chow}_{99-01} = \underset{[0.39]}{1.08}$$

where s_t^{dm} denotes the log dollar value in Dmarks; the *dif*-superscript refers to the difference between the US and the German value; $LM(1)$ and $LM(4)$ are Breusch-Godfrey serial correlation LM tests in F-form; Chow_{99-01} denotes the Chow forecast test performed over the EMU-period 1999:I–2001:II; Absolute t-values in parenthesis, p-values in brackets.

In order to get a reasonable fit, including the oil price in the equation seemed necessary. Higher oil prices increase the value of the dollar. This might be due to the fact that the US is less dependent on oil imports than Germany is. One of the main predictions of the theoretical model is confirmed by this empirical exercise: Trend growth differentials significantly increase the value of the dollar, whereas output gap differentials have the opposite (and much smaller) effect. Also, the impact of interest rate differentials is in accordance with the theoretical model.

The Chow forecast test does not detect a structural break at the start of the euro (January 1999). As a matter of fact, the parameter estimates are remarkably stable over time. Recursive parameter estimates (not shown) do not show any major movements from the beginning of the eighties on. This is also reflected in Figure 16, where the actual Dmark dollar rate is compared to 'equilibrium' rate based on the error correction term in our model. Three different parameter sets are used to calculate the equilibrium rate: based on all data (as shown in 16), based on 73:II-83:IV, and based on 73:II-92:IV.

The equilibrium rate is remarkably similar according to the three parameter estimates. The actual rate follows a similar pattern, but at times deviates considerably. Especially the large appreciation of the dollar in 1985 is not foreseen by the empirical model. At the moment (2001:III) these results suggest that the euro is about 15% undervalued relative to the dollar. Compared to the huge deviations during the mid eighties these percentages can hardly be called exceptional.

5 Conclusions

One of the main purposes of this exercise was to find out whether the prolonged strength of the dollar can be explained by real output growth in the US. Based on the theoretical model, the answer to this question must probably be: partly. Trend growth indeed can explain real appreciation, primarily due to the expected net foreign direct investment it attracts. A positive output gap on the other hand seems to depress the real exchange rate as the resulting deficit on the trade balance depletes foreign asset holdings.

The size of these effects are however quite small compared to real life fluctuations in real exchange rates. This should not come as a complete surprise as the bulk of the changes in the capital account are due to portfolio investments, whereas our empirical success in explaining desired foreign asset holdings was disappointing. In order to better understand exchange rate developments, a better understanding of foreign asset holdings seems essential. This would probably imply not only modelling investment decisions (where to invest), but also savings decisions (when to invest). The current theoretical framework might be a useful starting point for such an analysis.[14]

So far, our empirical successes are limited. Although we do find significant effects of trend growth and output gaps, as predicted by the theoretical model, major deviations between predicted and actual exchange rates remain. Nevertheless, the stability of the investigated relationship, and its theoretical appeal, make this effort worth pursuing further. This will be left for future research.

[14]A related relevant literature is the one on the natural real exchange rate (NATREX), see Stein and Allan (1995). As in our model, the real exchange rate brings about equilibrium on the balance of payments, and productivity shocks are an important source for real exchange rate fluctuations. Moreover, the approach explicitly models savings and investment decisions. However, the NATREX approach is only concerned with the medium to long run. Moreover, as the NATREX model does not assume a constant steady state, the methods used in our model are not directly applicable.

A Solving the model

We will use the toolkit of Uhlig (1999), which is based on Blanchard and Kahn (1980), in order to derive a fully consistent path for the variables in our model in response to shocks. Within this method, all variables are expressed in deviation from steady state. The variables are classified in three groups: exogenous variables (denoted z_t), endogenous state variables (x_t), and jump variables (y_t). The exogenous variables are not influenced by any endogenous variable in the system. The distinction between the state and jump variables is that lagged jump variables do not enter the system. The system has to be rearranged such that it can be expressed as follows:

$$0 = Ax_t + Bx_{t-1} + Cy_t + Dz_t \tag{17}$$

$$0 = E_t\left(Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right) \tag{18}$$

$$z_{t+1} = Nz_t + \varepsilon_{t+1} \qquad E_t\varepsilon_{t+1} = 0 \tag{19}$$

One of the necessities of this system is its linearity. This is clearly not fulfilled for the revaluation equation (10). This equation can be log-linearised under the assumption that there exists a steady state stock of scaled foreign assets and that this steady state value equals the one for foreign liabilities (denoted here μ_{fa}):

$$\begin{aligned}
\mathrm{FA}^{rev}_{t|t-1} &= \left(\mathrm{FA}^{ass}_{t-1}\frac{S_t}{S_{t-1}}(1+\frac{i^f_{t-1}}{4}) - \mathrm{FA}^{lia}_{t-1}(1+\frac{i_{t-1}}{4} - \mu_{\varepsilon i}\varepsilon^i_t)\right)\frac{\mathrm{GDP}^{us}_t}{\mathrm{GDP}^{us}_{t-1}} + \varepsilon^{rev}_t \\
&\approx \mu_{fa}\left((1+\mathrm{fa}^{ass}_{t-1})(1+\Delta s_t)(1+\frac{i^f_{t-1}}{4}) - (1+\mathrm{fa}^{lia}_{t-1})(1+\frac{i_{t-1}}{4} - \mu_{\varepsilon i}\varepsilon^i_t)\right)(1+\Delta y^r_t + \pi_t) + \varepsilon^{rev}_t \\
&\approx \mu_{fa}\left(\mathrm{fa}^{ass}_{t-1} - \mathrm{fa}^{lia}_{t-1} + \Delta s_t + \frac{i^f_{t-1} - i_{t-1}}{4} + \mu_{\varepsilon i}\varepsilon^i_t\right) + \varepsilon^{rev}_t \\
&\approx \mathrm{FA}_{t-1} + \mu_{fa}\left(\Delta s_t + \frac{i^f_{t-1} - i_{t-1}}{4} + \mu_{\varepsilon i}\varepsilon^i_t\right) + \varepsilon^{rev}_t
\end{aligned} \tag{20}$$

where fa^{ass}_{t-1} and fa^{lia}_{t-1} denote the log deviations of foreign assets and liabilities from steady state values. The main approximation in the first step is replacing actual stock values by log deviations from steady state: $\mathrm{FA} \equiv \mu_{fa}e^{\mathrm{fa}} \approx \mu_{fa}(1 + \mathrm{fa})$. In the second step all second order effects are supposed to be negligible, and in the last step the first step is reversed.

The next thing to do is to get rid of variables that do not converge to a steady state. In our model these are actual and potential output, and the nominal exchange rate. The latter can be replaced by the real exchange rate (Equation 13), whereas the former two are not essential for the dynamics of the system. The steady state deviations for the remaining variables is simply given by subtracting their unconditional mean. In order to reduce the number of variables further, the revalued net foreign assets (20) can be substituted in (11), and the desired foreign assets (7) can be replaced by the actual foreign assets (12). As only one period lagged variables are allowed in the system (17) to (19), additional variables need to be introduced for every variable that enters an equation with more than one lag.

The final system contains one jump variable, ten state variables, and fourteen exogenous variables:[15]

$$
\begin{aligned}
y_t &= [\,\mathrm{FDI}_t\,] \\
x_t &= \begin{bmatrix} h_t & \pi_t & i_t & q_t & \mathrm{FA}_t & \mathrm{TB}_t & h_{t-1} & \pi_{t-1} & \pi_{t-2} & \pi_{t-3} \end{bmatrix}' \\
z_t &= \begin{bmatrix} g_t & g_{t-1} & g_{t-2} & g_{t-3} & g_{t-4} & \varepsilon_t^d & \varepsilon_t^{sup} & \varepsilon_t^{\pi} & \varepsilon_t^i & \varepsilon_t^{tb} & \varepsilon_t^{fdi} & \varepsilon_t^{rev} & \nu_t^{fa} & \nu_{t-1}^{fa} \end{bmatrix}'
\end{aligned}
$$

The matrices for the non-forward-looking equations in the system (Equation 17) are:

$$
A = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_h & -1+\beta_{\Delta s} & 0 & -\beta_{\Delta s} & 0 & 0 & 0 & 0 & 0 & 0 \\
\gamma_{\Delta h} & \gamma_{\Delta \pi} & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mu_{fa} & 0 & -\mu_{fa} & -1 & 1 & 0 & 0 & 0 & 0 \\
\tau_{\Delta h} & 0 & 0 & 0 & 0 & -1 & -\tau_{\Delta h} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
\phi_1 & -\phi_i & \phi_i & \phi_q & 0 & 0 & \phi_2 & -\phi_i & -\phi_i & -\phi_i \\
0 & \beta_1 & 0 & \beta_{\Delta s} & 0 & 0 & 0 & \beta_2 & \beta_3 & \beta_4 \\
-\gamma_{h1}\gamma_i - \gamma_{\Delta h} & -\gamma_i\gamma_{\pi 1} & 1+\gamma_i & 0 & 0 & 0 & 0 & -\gamma_i\gamma_{\pi 1} & -\gamma_i\gamma_{\pi 1} & -\gamma_{\Delta \pi} - \gamma_i\gamma_{\pi 1} \\
0 & 0 & -\frac{\mu_{fa}}{4} & \mu_{fa} & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\tau_1\tau_q & 0 & 1+\tau_1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}'
$$

[15] As the subtraction of steady state values is not material for the results, the same notation will be pursued for the deviations from steady state. Alternatively one could add vectors of intercepts related to the steady states to the system (17) to (19).

$$D = \begin{bmatrix} \phi_{sup} & -\alpha_1\phi_{sup} & 0 & 0 & 0 & 1 & \phi_{sup} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_{fa}\mu_{\varepsilon i} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\chi_g}{4} & \frac{\chi_g}{4} & \frac{\chi_g}{4} & \frac{\chi_g}{4} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The matrices of the forward-looking equations (Equation 18) have the following structure:

$$\begin{aligned} F &= \begin{bmatrix} 0 & \delta_{uip} & 0 & -\delta_{uip} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ G &= \begin{bmatrix} 0 & 0 & -\frac{\delta_{uip}}{4} & \delta_{uip} & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ H &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ J &= [\,0\,] \\ K &= [\,0\,] \\ L &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ M &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

The AR(1) matrix for the exogenous process (Equation 19) is:

$$N = \begin{bmatrix} \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

What one is looking for in order to solve the system is the recursive law of motion:

$$x_t = Px_{t-1} + Qz_t \tag{21}$$

$$y_t = Rx_{t-1} + Sz_t \tag{22}$$

The solution is given in Theorem 3.2 of Uhlig (1999). The tedious part of this solution concerns the P-matrix which is shown to satisfy the following (matrix) quadratic equations:

$$0 = C^0AP + C^0B \tag{23}$$

$$0 = (F - JC^+A)P^2 - (JC^+B - G + KC^+A)P - KC^+B + H \tag{24}$$

where C^+ is the pseudo-inverse of the matrix C, and C^0 denotes a matrix whose rows form a basis for the null space of C'.

Uhlig (1999) solves this system by means of a generalised eigenvalue problem. In our case, the solution can also be found directly. The linear equations (23) are related to the non-forward-looking equations in the model (17) and can be solved analytically. The quadratic equations (24) refer to the forward looking part of the system (Equations 18), which in our case is only the UIP relationship (7). Given the ten state variables in our model, this results in ten quadratic equations in ten elements of P, which can be solved numerically with the help of Mathematica (Wolfram 1999). The system results in eleven solutions, only one of which is stationary. This is the only relevant solution as the other (explosive) solutions do not converge to a steady state.

B Data sources

Most data come from the September 2001 International Financial Statistics cd-rom of the IMF. The following lines were used:

00176AAZZF...	Spot oil price
11160B..ZF...	Federal Funds Rate US
11161...ZF...	Government Bond Yield US: 10 Year
11164...ZF...	Consumer Price Index US
11175..DZF...	Import Prices US
11178AFDZF...	Balance of Payments: Balance on goods and services US
11178AIDZF...	Balance of Payments: Balance on goods, services and income US
11178BDDZF...	Balance of Payments: Direct Investment abroad US
11178BEDZF...	Balance of Payments: Direct Investment in rep. econ., N.I.E. US
11178BFDZF...	Balance of Payments: Portfolio Investment Assets US
11178BGDZF...	Balance of Payments: Portfolio Investment Liabilities US
11178BKDZF...	Balance of Payments: Portfolio Investment Equity Assets US
11178BMDZF...	Balance of Payments: Portfolio Investment Equity Liabilities US
11179ACDZF...	International Investment Position: Portfolio Investment Assets US
11179ADDZF...	International Investment Position: Portfolio Investment Assets US, Equity
11179AFDZF...	International Investment Position: Other Investment Assets US
11179LCDZF...	International Investment Position: Portfolio Investment Liabilities US
11179LDDZF...	International Investment Position: Portfolio Investment Liabilities US, Equity
11179LFDZF...	International Investment Position: Other Investment Liabilities US
11199B.CZF...	Gross Domestic Product US
11199BVRZF...	GDP Volume (1995=100) US
134..AF.ZF...	Quarterly average Dmark dollar rate
13460B..ZF...	Call Money Rate Germany
13461...ZF...	Government Bond Yield Germany: 10 Year
13464...ZF...	Consumer Price Index Germany
13475...ZF...	Import Unit Values Germany
13499BVRZF...	GDP Volume (1995=100) Germany

The US unemployment rate, potential output and corresponding real output series are taken from the Federal Reserve Bank of St. Louis databank. The seasonally adjusted civilian unemployment rate originates from the U.S. Department of Labor, Bureau of Labor Statistics. The real potential GDP series is by the US Congress, Congressional Budget Office. The corresponding seasonally adjusted real GDP series is by the US Department of Commerce, Bureau of Economic Analysis.

For Germany, the unemployment rate was taken from Datastream (Code WGTOTUN%E). We chose to work with West-German unemployment rates only. The break in the German CPI series due to the German unification was remedied by imposing the growth rate of the total German series in 1991:I to be the same as the one for Western German CPI, taken from the BIS database. The seasonal patterns of the total German and Western German series were comparable.

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Figure 1: Annual real growth US and Germany (%)

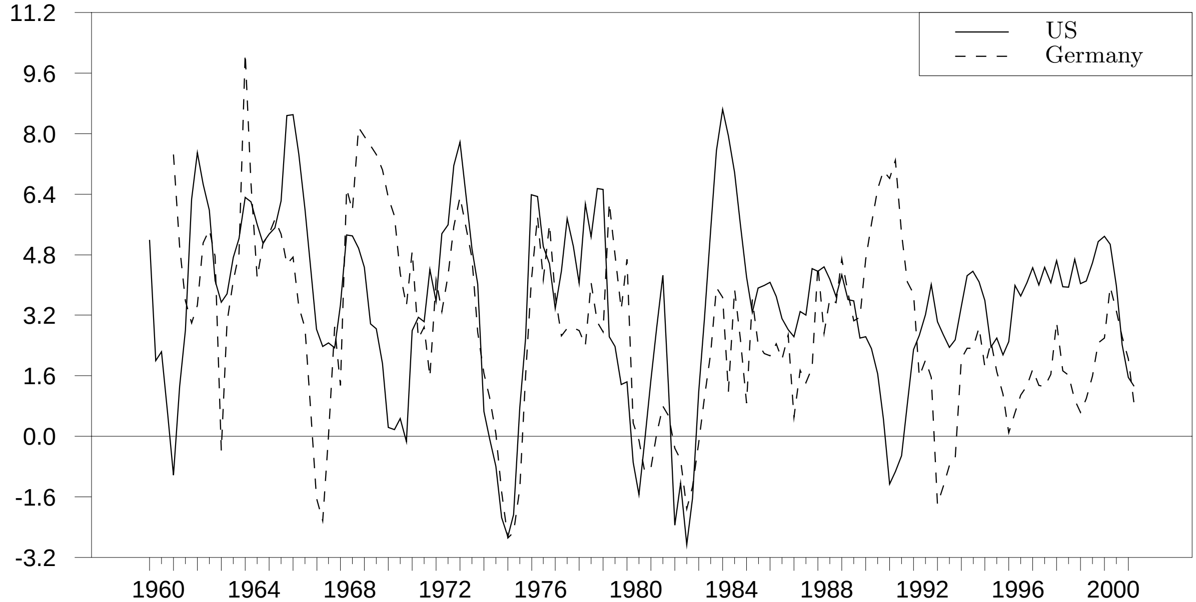

Figure 2: Bilateral real CPI-based Dmark dollar rate (% deviation from average)

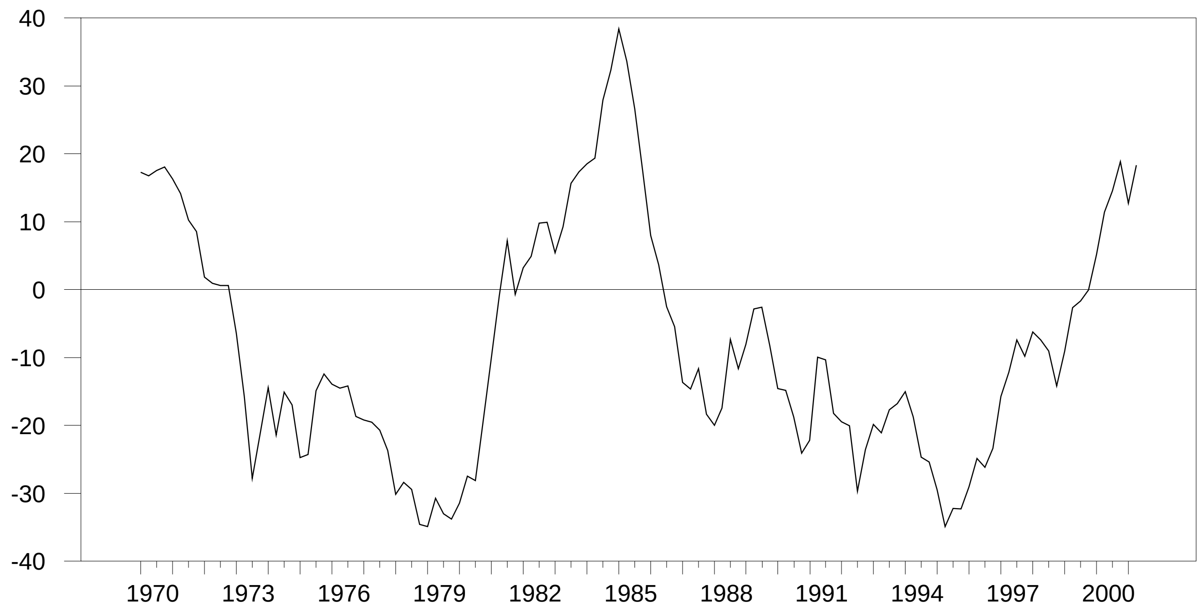

Figure 3: US balance of payment flows (% of GDP)

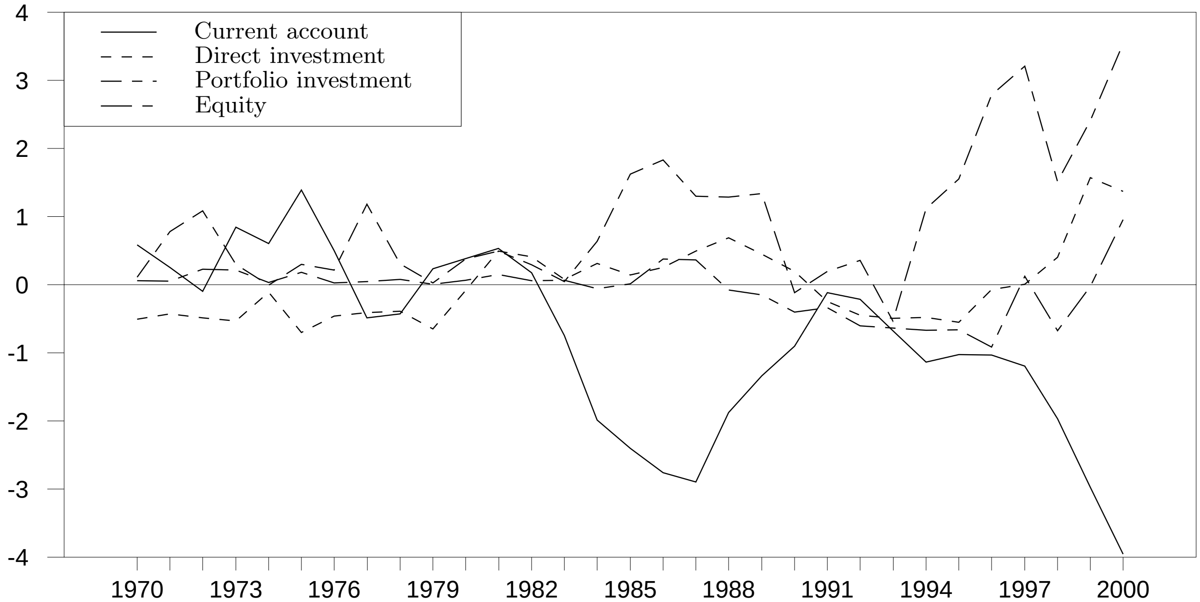

Figure 4: US output gap according to UC method and Congressional Budget Office

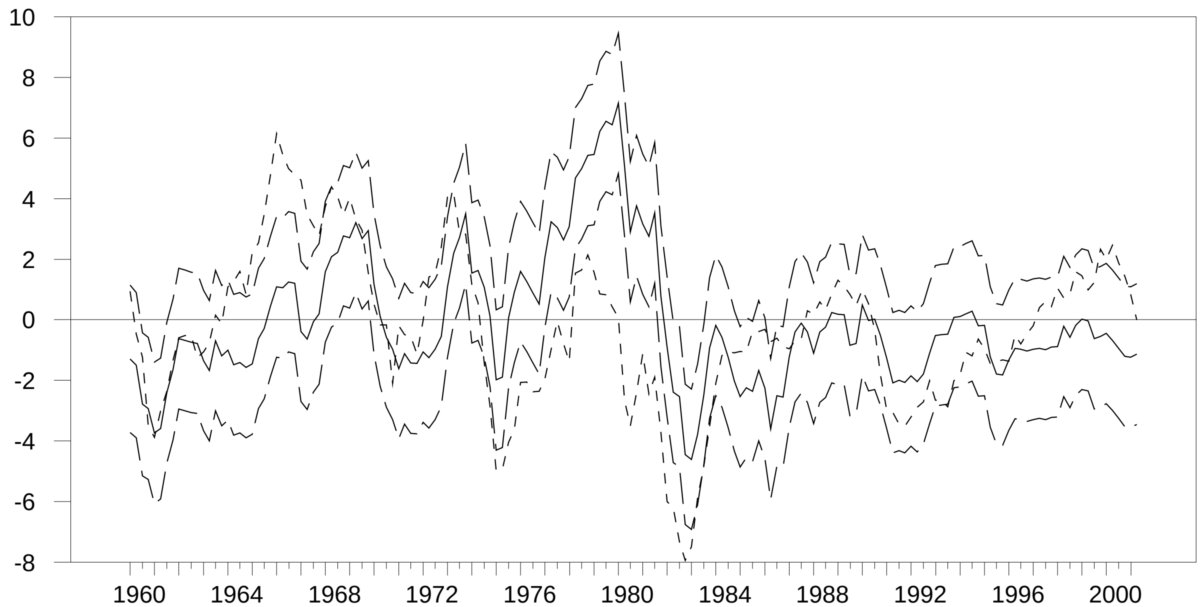

Figure 5: US growth potential according to UC method and Congressional Budget Office

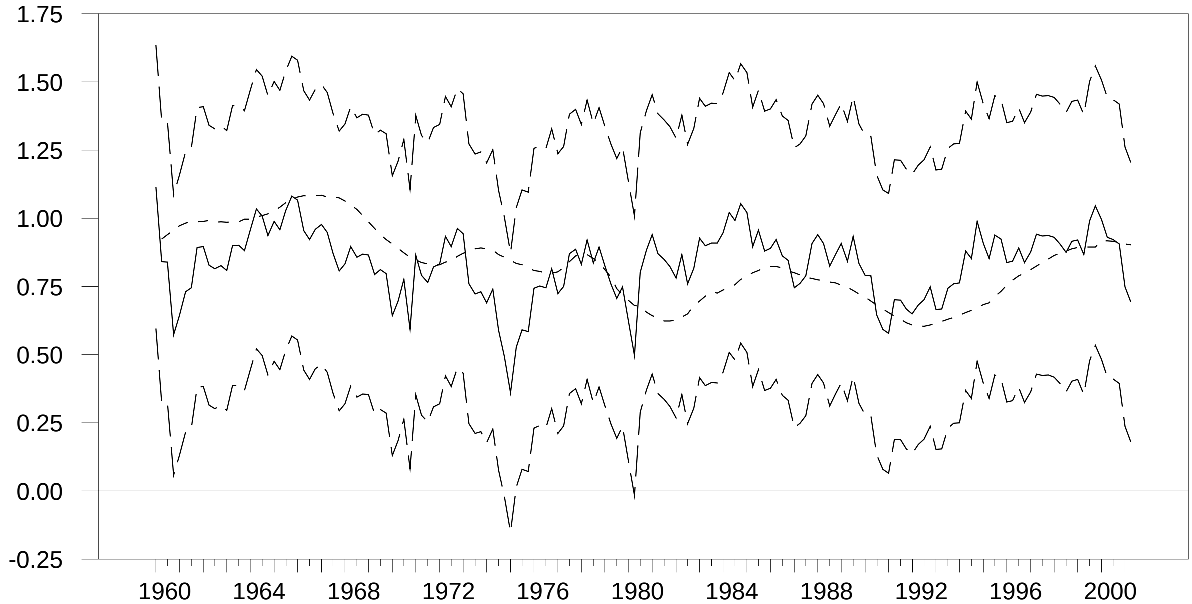

Figure 6: US foreign investment position (% of GDP)

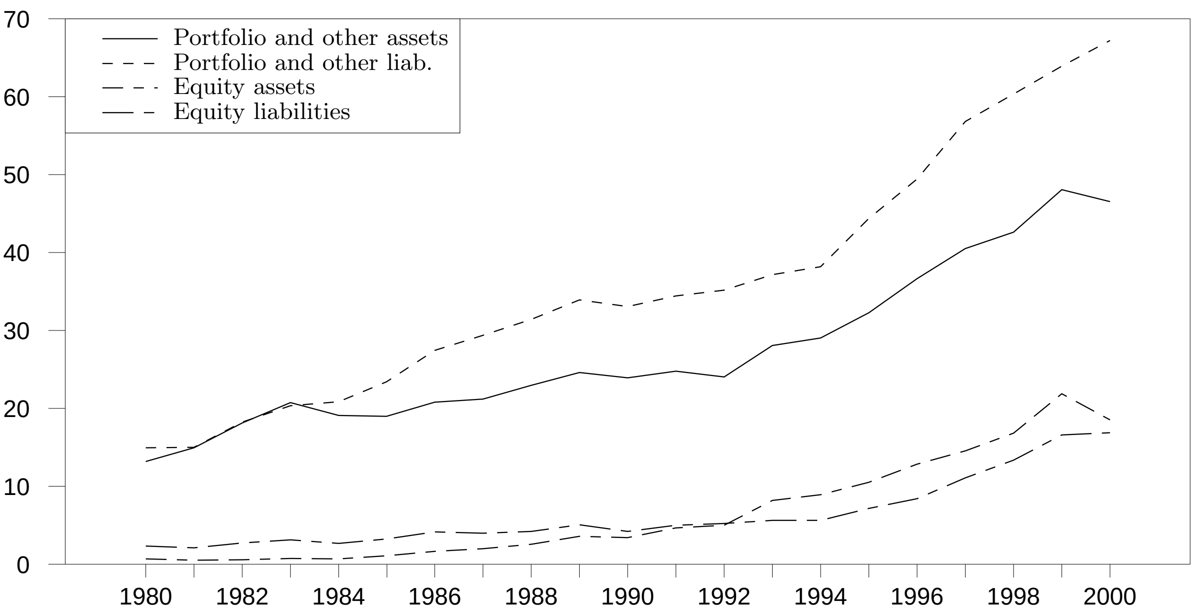

Figure 7: Impact of a demand shock

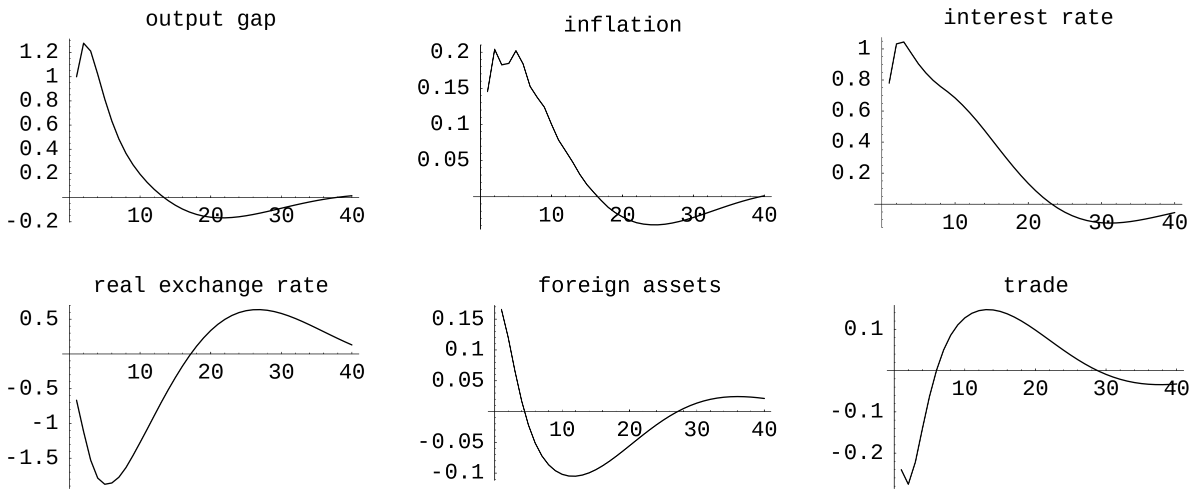

Figure 8: Impact of a trend growth shock

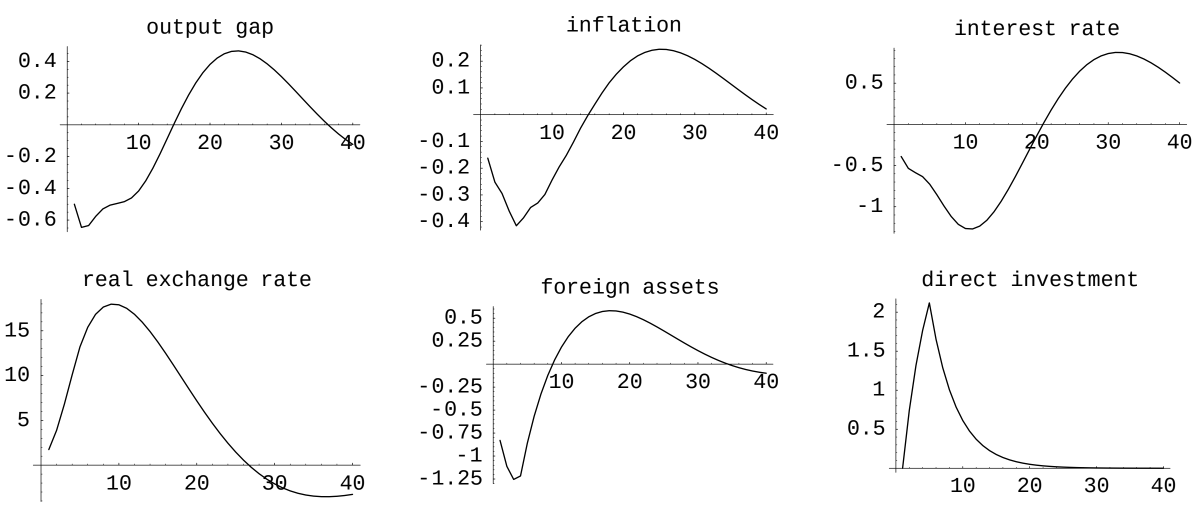

Figure 9: Impact of an inflation shock

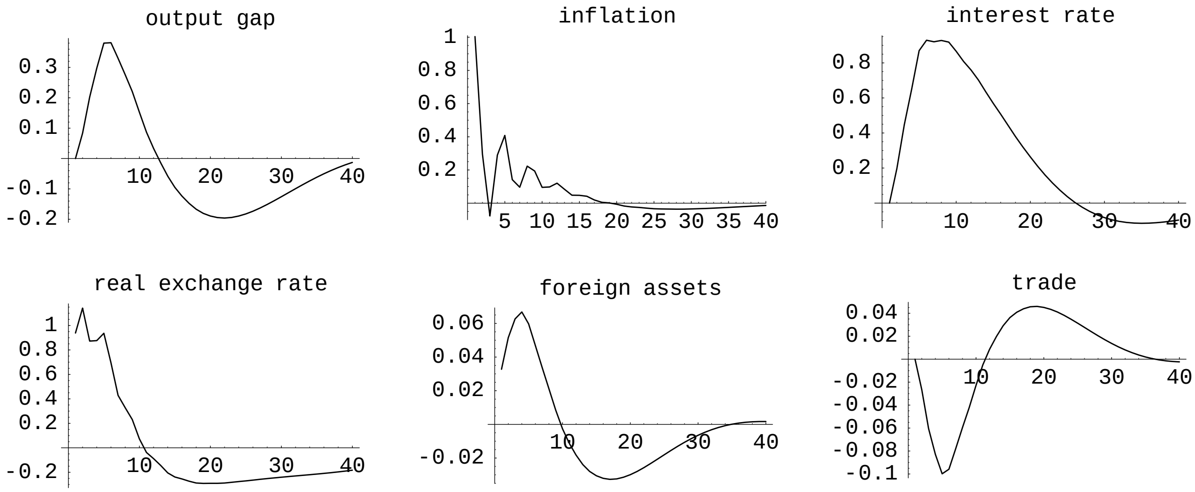

Figure 10: Impact of an interest rate shock

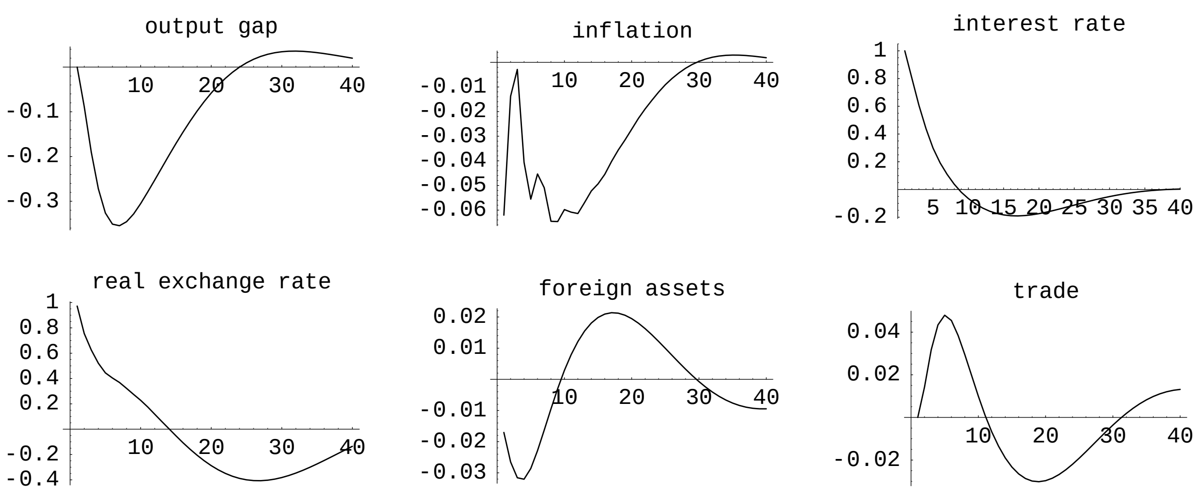

Figure 11: Impact of a shock to the desired foreign assets

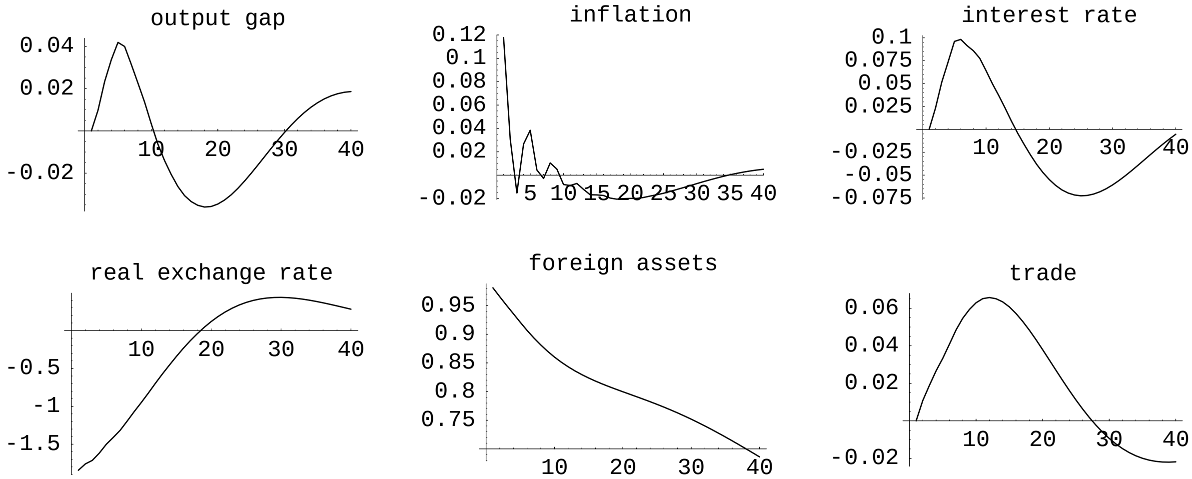

Figure 12: Response of real exchange rate under almost perfect substitution

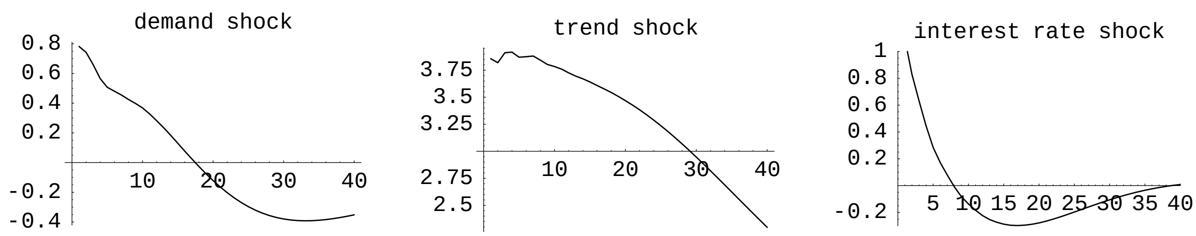

Figure 13: Response of real exchange rate under conventional Taylor rule parameters

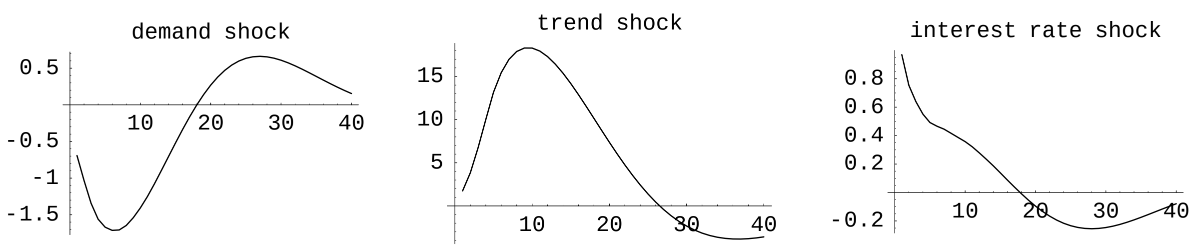

Figure 14: Response of real exchange rate under absence of direct price effect depreciation

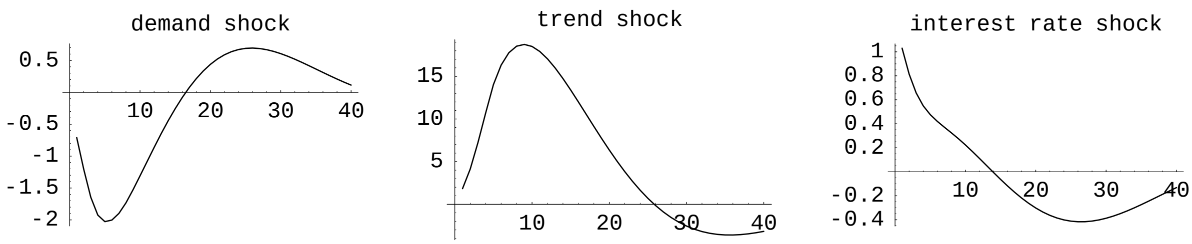

Figure 15: Impact of trend shock absent direct gap effect

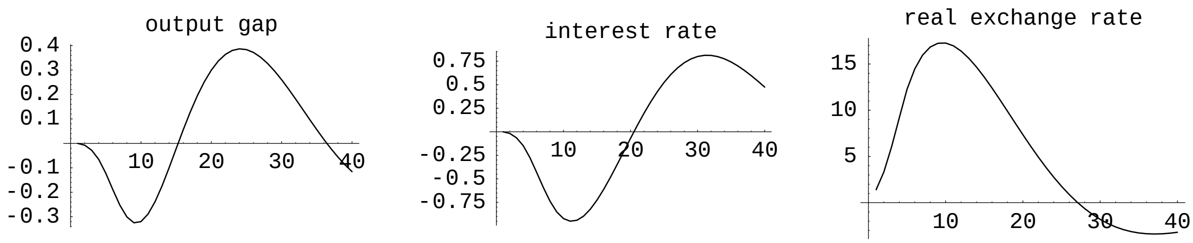

Figure 16: Actual and 'equilibrium' Dmark dollar rates

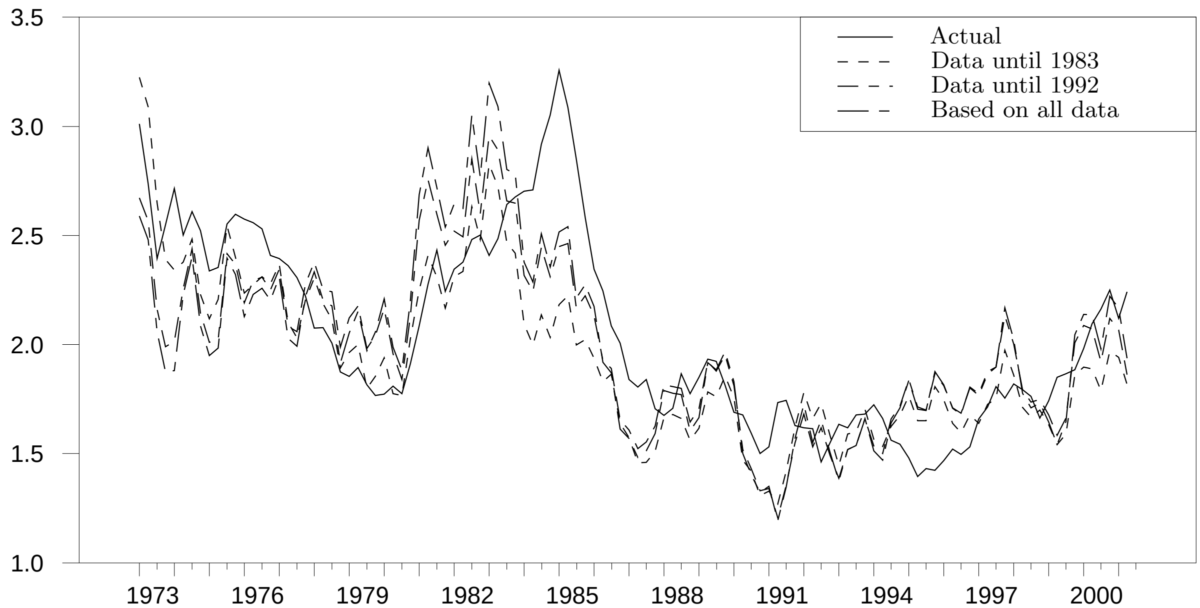

Table 1: State Space results US

	ϕ_1	ϕ_2	ϕ_i	ϕ_q	ϕ_{sup}	α_1	α_0	σ_h^2	σ_g^2	σ_{sup}^2	σ_{sy}^2
y_t^r	1.33	-.43	-0.083	0	-0.5	.78	0.18	0,43	0.032	0.32	0.001
	(11.9)	(4.0)	(2.4)	(-)	(-)	(3.5)	(1.0)	(6.6)	(0.6)	(3.6)	(0.8)
	β_h	β_0	β_1	β_2	β_3	β_4	β_{oil}	β_{imp1}	β_{imp2}	β_{rrl}	σ_π^2
π_t	0.097	0.20	0.28	-0.19	0.32	0.16	0.009	0.046	0.050	0.0005	0.073
	(4.6)	(1.1)	(3.3)	(2.5)	(3.9)	(2.5)	(4.0)	(3.0)	(3.0)	(1.7)	(7.4)
	$\psi_{\Delta h}$	ψ_g	ψ_0	ψ_1	$\psi_{\Delta 1}$	σ_u^2					
Δu_t	-4.27	-4.42	6.9	-0.019	0.35	4.2					
	(10.0)	(3.6)	(3.9)	(2.1)	(6.7)	(2.5)					

Notes: Effective sample 1958:II - 2001:II; Absolute t-values in parenthesis.

Table 2: Empirical results interest rate, foreign direct investment and trade balance

Equation	Method	Parameters							$\bar{R}^2$	DW	$LM(4)$
		$\gamma_{\Delta h}$	$\gamma_{\Delta\pi}$	γ_i	γ_0	γ_{h1}	$\gamma_{\pi 1}$				
Δi_t	OLS	0.78	0	-0.12	3.02	0.92	1.10		0.28	1.95	1.11
		(5.2)	(-)	(2.2)	(1.2)	(1.0)	(2.4)				[0.36]
		χ_0	χ_g	χ_{s1}	χ_{s2}	χ_{s3}					
FDI_t	OLS	-2.28	2.96	-0.87	-0.25	-0.54			0.18	1.79	3.70
		(2.8)	(3.1)	(3.3)	(0.9)	(2.0)					[0.01]
		τ_1	τ_q	$\tau_{\Delta h}$	τ_0	τ_{s1}	τ_{s2}	τ_{s3}			
ΔTB_t	2SLS	-0.11	-0.065	-0.24	0.18	0.08	-0.65	-0.69	0.63	2.15	4.74
		(3.0)	(3.2)	(2.6)	(1.6)	(0.9)	(6.8)	(7.5)			[0.00]

Notes: Effective sample 1980:I - 2001:II; Absolute t-values in parenthesis, p-values in brackets; DW and $LM(4)$ denote the Durbin Watson statistic and the F-version of the serial correlation LM-test respectively; Instruments in the trade balance equation are $\text{TB}_{t-1}, q_{t-1}, \Delta i_t, s1, s2$ and $s3$.

Table 3: State Space results Germany

	ϕ_1	ϕ_2	ϕ_i	ϕ_q	ϕ_{sup}	α_1	α_0	σ^2_h	σ^2_g	σ^2_{sup}	σ^2_{sy}
y^r_t	1.27	-.33	-0.063	0	-0.5	.91	0.057	0.55	0.01	0.62	0.083
	(9.9)	(3.3)	(1.7)	(-)	(-)	(17.0)	(1.6)	(3.2)	(-)	(2.9)	(1.9)
	β_h	β_0	β_1	β_2	β_3	β_4	β_{oil}	β_{imp1}	β_{imp2}	β_{rrl}	σ^2_π
π_t	0.056	-0.21	0.17	0.01	0.19	0.38	0.007	0.022	0	0.0010	0.130
	(3.4)	(1.0)	(2.0)	(0.2)	(2.4)	(5.8)	(2.2)	(2.0)	(-)	(2.8)	(1.9)
	$\psi_{\Delta h}$	ψ_g	ψ_0	ψ_1	$\psi_{\Delta 1}$	σ^2_u					
Δu_t	-9.81	-12.44	0.14	-0.031	0.19	0.01					
	(5.5)	(3.0)	(3.1)	(2.4)	(2.0)	(-)					

Notes: Effective sample 1960:II - 2001:II; Absolute t-values in parenthesis.