# Optimal Simple Monetary Policy Rules with Nominal Rigidities and Asymmetric Information in the Credit Market*

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**Abstract**

This paper evaluates monetary policy rules in a business cycle model with staggered prices, preset nominal wages and asymmetric information in the credit market. Rules are compared in a utility based welfare metric, the effects of the model's nonlinear dynamics are captured by a quadratic approximation to the policy function. The firms net worth crucially affects the terms of obtaining outside finance. Financial frictions dampen the economies response to shocks and make them highly persistent. When prices are sticky, but wages flexible, these frictions do not matter for monetary policy: The optimal policy is strong inflation targeting. When both wages and prices are sticky, there exists a trade-off between inflation and output. Financial frictions increase the importance of stabilizing output, since they make the output variations more persistent.

**Keywords:** Monetary policy rules, nominal rigidities, welfare evaluation, quadratic approximation, agency costs, credit frictions.

**JEL Classification:** E32, E52, E58

## 1 Introduction

This paper makes two distinct contributions to research on optimal monetary policy. The first is to reconcile the findings of the empirical literature on monetary policy reaction functions, with the theoretical literature on optimal simple monetary policy rules. Empirically, central banks react not only to inflation, but also to the cyclical component of output, as shown by Judd and Rudebusch (1998) or Surico (2003). However, most welfare based analysis of optimal simple monetary policy rules in calibrated models finds that a reaction to the cyclical

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component of output is not warranted.[1] This is indicated by the finding of Rotemberg and Woodford (1999) for a closed economy and Kollmann (2002) for an open economy.

The standard model new Keynesian monetary model does not feature an unemployment-inflation trade-off, other than through an somewhat ad hoc cost-push shock.

This paper includes nominal wage rigidities into the model in a very simple manner, focusing on preset wages rather than asynchronous wage setting a la Calvo (1983). Furthermore, it adopts the simplest form of staggered price setting. The joint analysis of nominal wage stickiness and asynchronized price setting that this paper advocates introduces an inflation output trade-off. This trade-off generates a role for central banks to react to output on top of inflation.

A second novelty is the joint analysis of nominal rigidities in wages and prices with imperfections in the credit market. Whereas a lot of papers have examined the effect of such informational frictions on the business cycle and the propagation of shocks, no study as yet rigorously assessed what the implications of these frictions are for optimal monetary policy rules. The message of this paper is that these frictions are irrelevant for optimal policy, when the model does not feature an inflation output trade-off. However, if there is such a trade-off for the central bank, informational frictions affect the optimal weights in a simple Taylor.

This paper presents a framework for the analysis of monetary policy rules when borrowing and lending is subject to asymmetric information and costly state-verification. It extends Carlstrom and Fuerst (2000) to an environment with staggered price setting in the goods market and one period preset nominal wages. This model can be used to study how financial market frictions interact with nominal rigidities and how they change the trade-off between inflation and output for the central bank arising from sticky prices and sticky wages.

The outline of the paper is as follows. The subsequent section presents estimates of Taylor rules from the existing literature used to motivate this study. Next, the setup of the model is described. Section 4 illustrates the working of the model using impulse response functions.

Section 5 displays the welfare measure used for the analysis of optimal monetary policy and the solution method, namely a second order Taylor approximation to the policy function following Schmitt-Grohé and Uribe (2004). Subsequently in section 6, the optimized coefficients of a simple Taylor rule are presented for different model variants and different shocks. Section 7 concludes.

## 2 Empirical Taylor rule literature

This section documents the empirical regularities in central bank's interest rate setting that were mentioned to motivate the paper in the introduction. Instead of reviewing the large literature on empirical estimates of Taylor rules, I just present a very recent comparative publication in the field.

[1] Clearly, the appropriate variable to target in the New Keynesian model is the output gap, i.e. actual output minus output under flexible prise. In fact stabilizing the gap is equivalent to stabilizing inflation in the standard NK model. However, the gap and the cyclical component of output are quite different measures, they usually do not coincide.

Surico (2003) estimates a reaction function of the ECB, the FED and the Bundesbank. The analyzed interest rule is:

$$i_t = (1-\rho)(c_0 + c_1\pi_t + c_2 y_t + c_3\pi_t^2 + c_4 y_t^2 + c_5 y_t\pi_t) + \rho i_{t-1} + v_t \qquad (2.1)$$

The central bank reacts to a function of the deviation of inflation from target $\pi_t = \Pi_t - \Pi^*$ and a measure of the output gap, defined as HP detrended output, $y_t = \log(Y_t^{HP})$. The dependent variable is a 3 month money market rate.

Table 1: Estimates of Taylor rules taken from Surico (2003)

| | ECB 1997:7 -2002:10 | FED 1997:7 -2002:10 | BUBA 1992:2 - 1998:12 |
|---|---|---|---|
| $\rho$ | $0.701^{**}$ | $0.617^{**}$ | $0.917^{**}$ |
| c1 | $0.579^{**}$ | $1.033^{**}$ | $0.839^{**}$ |
| c2 | $0.602^{**}$ | $2.403^{**}$ | $0.856^{**}$ |
| c3 | n.s. | n.s | n.s. |
| c4 | $-0.270^{**}$ | $-0.549^{**}$ | $-0.244^{*}$ |
| c5 | $0.701^{**}$ | $0.722^{**}$ | $0.216^{*}$ |

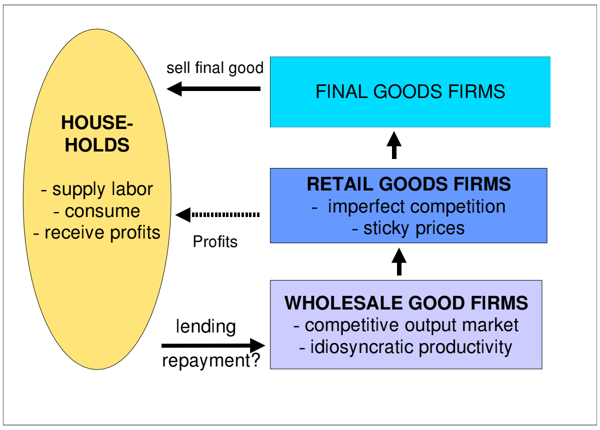
Table 1 shows that all three central banks react not only to inflation, but also to the cyclical component of output. The reaction is strongest for the FED, which reacts to output even more than to inflation. It is this widespread empirical regularity that this paper is motivated by: *Why do central banks react strongly to output?*

# 3 The model

The model is an extension of Carlstrom and Fuerst (2000) to include nominal wage and price stickiness. Carlstrom and Fuerst (2000) incorporate an asymmetric information problem in the borrower - lender relationship into a standard business cycle model with money. The agency cost set up is only slightly changed here. Before going into detail, figure 1 sketches the economy's structure and serves as a road map for the future exposition.

Households work, consume and lend funds to firms. The borrower-lender relationship is characterized by asymmetric information about idiosyncratic firm productivity. The production structure has three levels as shown in the graph. At the lowest level, a continuum of firms produce a homogeneous wholesale good. The asymmetric information problem takes place between entrepreneurs and households at this level. The wholesale good is used as input of production by a continuum of imperfectly competitive firms that set price in a staggered fashion at the second level. Finally, a competitive bundler aggregates these intermediate varieties into the final output goods, which can be used for consumption and investment. This structure is designed to keep the asymmetric information problem separate from the price stickiness, following Bernanke, Gertler, and Gilchrist (2000). The set up of the model will be explained in more details in the next subsections.

Figure 1: Graph of the model




## 3.1 Household

Households are infinitely lived, supply labor $N$ and receive real wage $w$, consume final goods $C$, rent capital goods $K$ to firms at the real rental rate $r$. Furthermore, they hold government bonds $B$, are subject to lump sum transfers $T$, hold nominal money balances $M$ and receive profits $\Pi$ from the monopolistic retailers. The utility function is assumed to be separable in consumption, real money balances and leisure. The representative household's problems is:

$$\max_{B_{t+1},K_{t+1},M_{t+1},C_t,N_t} E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[ \ln C_{t+i} + a \ln \left( \frac{M_{t+i}}{P_{t+i}} \right) + v \ln(1 - N_{t+i}) \right] \quad \text{s.t.} \tag{3.1}$$

$$C_t = \frac{R_t B_t - B_{t+1}}{P_t} + w_t N_t + T_t + \Pi_t + (1 + r_t) K_t - K_{t+1} - \frac{M_{t+1} - M_t}{P_t}. \tag{3.2}$$

The first order conditions are with respect to bonds, capital and money holdings are standard and need not further discussion:

$$C_t^{-1} = E_t \beta \left\{ \frac{P_t}{P_{t+1}} C_{t+1}^{-1} \right\} R_{t+1}^n \tag{3.3}$$

$$C_t^{-1} = E_t \beta \left\{ (1 + r_{t+1}) C_{t+1}^{-1} \right\} \tag{3.4}$$

$$\frac{M_t}{P_t} = a C_t \left( \frac{R_{t+1}^n - 1}{R_{t+1}^n} \right)^{-1} \tag{3.5}$$

Although households also lend to firms, lending does not appear in this budget constraint. It is assumed to be intra-period. This assumption is made in order to facilitate comparison with the model of Carlstrom and Fuerst (2000).

For some simulations, wage stickiness will be assumed. The literature on sticky wages by and large adopts the Calvo (1983) staggered price setting environment to model staggered wage setting. This paper, however argues that some European labor markets are characterized by centralized wage setting. One may then regard the inefficient level of the wages and not the dispersion of wages as the main distortion in the labor market. Therefore, this paper departs from staggered wage setting and focuses on synchronized but sticky wages.

The framework for motivating wage rigidities is standard. A continuum of households supply differentiated labor $N_t(h)$, which is aggregated according to the standard Dixit-Stiglitz form:

$$L_t = \left[ \int_0^1 [N_t(h)]^{\frac{\kappa-1}{\kappa}} \, dh \right]^{\frac{\kappa}{\kappa-1}}. \tag{3.6}$$

Here $W_t$ is the standard Dixit-Stiglitz index. The demand function for differentiated labor is:

$$N_t(h) = \left[ \frac{W_t(h)}{W_t} \right]^{-\kappa} L_t \tag{3.7}$$

A randomly chosen fraction $1 - \theta_L$ of households can freely set its nominal wage in every period.[2] Their wage setting implies the usual constant markup of the real wage over the marginal rate of substitution between leisure and consumption. The remaining random fraction of households set its nominal wage before the realization of period $t$ aggregate uncertainty subject to the budget constraint and the demand function for heterogeneous labor. The FOC for this problem is:

$$E_{t-1} \left\{ N_t(h) \left( \frac{v}{1 - N_t(h)} - \frac{\kappa - 1}{\kappa} \frac{W_t^{fx}(h)}{P_t C_t} \right) \right\} = 0 \tag{3.8}$$

The remaining fraction of flexible wage households choose the standard markup formula.

$$\frac{W_t^{fl}(h)}{P_t} = \frac{\kappa}{\kappa - 1} \frac{v C_t}{1 - N_t(h)} \tag{3.9}$$

There exists a symmetric equilibrium in which all sticky wage households set the same nominal wages and work the same hours. As is standard in the literature, the assumption of a complete contingent claims market [3] ensures that all household consume the same amount.

## 3.2 Production structure

The economy has three different layers of production. At the lowest level, a continuum of firms produce a homogeneous wholesale good. This good is used as input of production by a continuum of imperfectly competitive firms at the second level. These firms buy the wholesale good and costlessly differentiate it into an intermediate good of variety $j$. A competitive bundler aggregates

[2] This is a departure from the standard assumption of Calvo wage or price setting. Henderson and Kim (2002) shows how one can obtain analytical solutions in a simple model where *all* households set wages based on last periods information set.

[3] Such a market is not explicitly modeled in this paper. However, it could easily be incorporated

these intermediate varieties into the final output goods, which can be used for consumption and investment.

At the wholesale goods level, there is an asymmetric information problem between borrowers and lenders. At the intermediate goods level, there is imperfect competition and asynchronized price setting. Following Bernanke, Gertler, and Gilchrist (2000), this layered production structure is designed to separate the agency problem from price stickiness, which facilitates aggregation in this model.

### 3.2.1 The asymmetric information problem

Since the firms' problems are static, time subscripts are suppressed for the ease of exposition in this subsection. There is a continuum of firms with unit mass indexed by $i$. Each firm is owned by an infinitely lived entrepreneur, who has a probability $\gamma$ of dying in each period. The production function $F(K_i, L_i)$ for the wholesale good $Y^W$ displays constant returns to scale in labor $L_i$, and capital $K_i$. The idiosyncratic productivity $\omega_i$ has distribution function $\Phi(\omega_i)$ and density function $\phi(\omega_i)$ with nonnegative support and mean of unity.

$$Y_i^W = \omega_i \cdot A \cdot F(K_i, L_i) \tag{3.10}$$

**Assumption 1.** *Lender and borrower contract after the realization of aggregate uncertainty $A$ and before the realization of idiosyncratic uncertainty $\omega$. Once realized, $\omega$ is private information of the entrepreneur. The lender's cost of verifying $\omega$ is a fraction $\mu$ of the firm's output.*

$A$ denotes the level of technology, which is common across all entrepreneur, aggregate total factor productivity. It is assumed that this variable is known to both lenders and entrepreneurs at the time of the loan contract, which eliminates aggregate uncertainty for the contracting problem. For simplicity of the exposition, this term is normalized to unity in the subsequent paragraphs. It follows an AR(1) process in the simulations.

In order to generate a role for external finance, it is assumed that the firm has to lay out its production costs before receiving the payment for the output good. Let $X_i \equiv wL_i + rK_i$ denote the cost for the input bill, determined by the real wage $w$ and the real rental rate of capital $r$ as well as factor demands. The firm then borrows $X_i - N_i$ from a financial intermediary. Here, $N_i$ is the net worth of the firm $i$. For convenience, it is assumed that borrowing is intraperiod, so the relevant opportunity cost for the household per unit of lending is one.

**Assumption 2.** *There is enough anonymity in the credit market, such that agents can only sign one period contracts.*

The above assumption rules out reputation considerations and other aspects of long term credit relationships. This is done for the sake of model tractability. Dynamic credit relationships require the use of much more involved numerical techniques; they also challenge the conclusion from one period models, see Smith and Wang (2000).

Since the realization of the productivity shock is private information of the firm and verification of the true productivity is costly to the lender, the firm has an incentive to misreport its productivity.

The finding of Gale and Hellwig (1985) is that the optimal incentive compatible contract with costly state verification is risky debt. The authors characterized the contract that maximizes expected entrepreneurs payoff subject to the risk neutral lender breaking even in expectation.[4] The contract requires a fixed repayment when the firm is solvent and allows the creditor to recoup as much of the debt as possible in case of bankruptcy. The flat payoff schedule in case of solvency follows directly from incentive compatibility. Allowing the lender to seize all obtainable assets in case of default minimizes the number of states in which monitoring must occur for the lender to break even.

The contract is characterized by a non default value $\bar{\omega}_i$, loan size $X_i - N_i$ and an implicit interest rate $\tilde{r}$. A firm with productivity $\omega_i \geq \bar{\omega}_i$ is able to repay its obligation $(1+\tilde{r}_i)(X_i - N_i)$ and no monitoring takes place. The non default value is therefore defined by $(1+\tilde{r}_i)(X_i - N_i) = \bar{\omega}_i F(K_i, L_i)$. For lower productivity, the firm defaults, the lender monitors and seizes all of the projects output. The pair $(\bar{\omega}_i, X_i)$, is sufficient for the description of the optimal contract, the interest rate follows immediately as $1+\tilde{r}_i = \bar{\omega}_i F(K_i, L_i)/(X_i - N_i)$. The optimal financial contract maximizes the expected payoff for the borrower, subject to the participation constraint for the lender. Let $P^W$ denote the nominal price of the firms output good and $P$ the consumer price index. The problem of the firm can be written as follows.

$$\max_{K_i,\, L_i,\, \bar{\omega}_i} P^W F(K_i, L_i) \int_{\bar{\omega}_i}^{\infty} (\omega_i - \bar{\omega}_i)\phi(\omega_i)\mathrm{d}\omega_i \qquad \text{s.t.} \tag{3.11}$$

$$F(K_i, L_i)\left\{(1-\mu)\int_0^{\bar{\omega}_i} \omega_i \phi(\omega_i)\mathrm{d}\omega_i + [1-\Phi(\bar{\omega}_i)]\bar{\omega}_i\right\} \geq \frac{P}{P^W}(rK_i + wL_i - N_i) \tag{3.12}$$

The firm expects as residual income the conditional mean $\mathrm{E}[(\omega_i - \bar{\omega}_i)|\omega_i \geq \bar{\omega}_i]F(K_i, L_i)$ when it is solvent and nothing otherwise. The lender receives the negotiated repayment $\bar{\omega}_i F(K_i, L_i)$ in states of solvency and expects a conditional mean $\mathrm{E}[\omega_i|\omega_i \leq \bar{\omega}_i]F(K_i, L_i)$ corrected for monitoring costs in states of bankruptcy. The first order conditions for the firm's problem can be conveniently expressed as:

$$r\frac{P}{S_i P^W} = F_K(K_i, L_i) \tag{3.13}$$

$$w\frac{P}{S_i P^W} = F_L(K_i, L_i) \tag{3.14}$$

Here the term $S_i$ is given by:

$$S_i \equiv 1 - \mu\left\{\bar{\omega}_i \phi(\bar{\omega}_i)\left[\frac{\int_{\bar{\omega}_i}^{\infty} \omega_i \phi(\omega_i)\mathrm{d}\omega_i}{1-\Phi(\bar{\omega}_i)} - \bar{\omega}_i\right] + \int_0^{\bar{\omega}_i} \omega_i \phi(\omega_i)\mathrm{d}\omega_i\right\} \tag{3.15}$$

Finally, the break even constraint holds with equality. Inspecting (3.13) and (3.14) reveals the crucial role of monitoring costs for the inefficiency of the economy. When there are no monitoring costs ($\mu = 0$), the usual efficiency

[4] The household is not risk-neutral, however without aggregate risk, he can perfectly diversify all the risk. With aggregate risk, it remains to be shown that the firm absorbs all aggregate risk.

condition holds in this model: marginal product equals factor cost. $S_i$ is unity in this case. When monitoring costs are strictly positive however, the economy is distorted. Real marginal product is above the real wage, $S_i$ is below unity. Note, that $S_i$ must be strictly below one with positive monitoring costs, otherwise revenues would be exhausted by factor payments alone and monitoring costs could not be covered. However, the inefficiency goes further than that. Entrepreneurs are making positive profits, revenues are strictly higher than factor payments plus monitoring costs. The firm would like to expand production, but is not able to obtain further external finance.

### 3.2.2 A note on aggregation

**Remark 1.** *The models heterogeneity does not require to keep track of the distribution of net worth of the entrepreneurs, the means are a sufficient statistic.*

Entrepreneurs are identical ex-ante with respect to their expected productivity, but heterogeneous ex-post once idiosyncratic productivity has materialized. Therefore, even when starting with identical initial endowment of wealth, over time the entrepreneurs will have different levels of wealth.

Thanks to linear monitoring technology and constant returns to scale in production, this heterogeneity does not require to keep track of the distribution of wealth as a state of the system.

First note from (3.13) and (3.14) that the ratio of marginal products of capital and labor are equal for all firms. This implies for linear homogeneous production functions such as the Cobb-Douglas, that the capital-labor-ratios are equal across firms. But the capital-labor-ratio uniquely determines the marginal product of a factor with linear homogeneity, which implies that marginal products are equal across firms and therefore $S_i$ is independent of $i$. One can conclude from (3.15) that $\bar{\omega}$ must be independent of $i$. The lender break even constraint (3.12) can be rearranged to show that all firms must choose the same ratio of external finance to project size.

$$1 - \frac{N_i}{X_i} = \frac{(1-\mu)\int_0^{\bar{\omega}} \omega\phi(\omega)\mathrm{d}\omega + [1-\Phi(\bar{\omega})]\bar{\omega}}{1 - \mu\left\{\bar{\omega}\phi(\bar{\omega})\left[\frac{\int_{\bar{\omega}}^{\infty} \omega\phi(\omega)\mathrm{d}\omega}{1-\Phi(\bar{\omega})} - \bar{\omega}\right] + \int_0^{\bar{\omega}} \omega\phi(\omega)\mathrm{d}\omega\right\}} \tag{3.16}$$

The essential terms of the debt contract, leverage and cutoff value are thus equal across firms. It follows that the FOC for the optimal contract also hold for aggregate variables, defined as the integral over all $i$ variables. Subscripts $i$ are therefore dropped in what follows. Call the right hand side of (3.16) $\Psi(\bar{\omega})$, the projects leverage is given by:

$$\frac{X_i}{N_i} = [1 - \Psi(\bar{\omega})]^{-1} \tag{3.17}$$

### 3.2.3 The retailer

A continuum of retailers on the unit line, indexed by $z$ buy the entrepreneurs' wholesale good at price $P_t^W$ and costlessly differentiate it into a good of variety $z$.

The simplest formulation of price stickiness is adopted in this model. A fraction $\theta$ of the firms, the so called fixed-price firms, set their prices $P^{fx}$ before the realization of period $t$ uncertainty and the remaining fraction sets its price $P^{fl}$ after that realization. Prices are sticky for at most one period in this framework.

For the flexible price firms, the usual markup formula holds:

$$P_t^{fl} = \frac{\epsilon_t}{\epsilon_t - 1} P_t^W \tag{3.18}$$

The fixed price firms set their price to satisfy:

$$\mathrm{E}_{t-1}\left\{P_t^{\epsilon-1} Y_t^{fi}\left[P_t^{fx}(\epsilon_t - 1) - \epsilon_t P_t^W\right]\right\} = 0 \tag{3.19}$$

The price index corresponding to the technology of the bundler is[5]:

$$P_t = \left[\int_0^1 P_t(z)^{1-\epsilon_t} dz\right]^{\frac{1}{1-\epsilon_t}} \tag{3.20}$$

The consumer-price index can therefore be written as:

$$P_t = \left\{\theta\left[P_t^{fx}\right]^{1-\epsilon_t} + (1-\theta)\left[P_t^{fl}\right]^{1-\epsilon_t}\right\}^{\frac{1}{1-\epsilon_t}}. \tag{3.21}$$

Goods market clearing in the wholesale goods market requires that the output net of monitoring costs of the wholesale sector be absorbed by fixed-price firms' input demand and the flexible-price firms' input demand:

$$Y_t^W(1 - \mu\int_0^{\bar{\omega}_i} \omega_i \phi(\omega_i)\mathrm{d}\omega_i) = \theta Y_t^{fx} + (1-\theta)Y_t^{fl} \tag{3.22}$$

### 3.2.4 Final goods production

Final goods are produced using the continuum of differentiated input goods using a standard technology.

$$Y_t^f = \left[\int_0^1 \left[y_t(z)\right]^{\frac{\epsilon_t - 1}{\epsilon_t}} dz\right]^{\frac{\epsilon_t}{\epsilon_t - 1}} = \left[\theta\left[y_t^{fx}\right]^{\frac{\epsilon_t-1}{\epsilon_t}} + (1-\theta)\left[y_t^{fl}\right]^{\frac{\epsilon_t-1}{\epsilon_t}}\right]^{\frac{\epsilon_t}{\epsilon_t-1}} \tag{3.23}$$

The last equality follows from the symmetry of all firms with fixed prices, and flexible prices respectively. The parameter governing the degree of monopolistic competition is assumed to follow an AR(1) process in logs:

$$log(\epsilon_t) = (1-\rho_\epsilon)log(\bar{\epsilon}) + \rho_\epsilon log(\epsilon_{t-1}) + u_t \tag{3.24}$$

The competitive firms choose $Y_t(z)$ as to maximize profits:

$$\Pi_t = P_t Y_t^f - \int_{z=0}^1 Y_t(z) P_t(z)\mathrm{d}z \tag{3.25}$$

The demand function for intermediate good $z$ resulting from this problem is:

$$Y_t(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon_t} Y_t^f. \tag{3.26}$$

The final output good may be consumed by households $C$, by entrepreneurs $C^E$, or be used for investment $I$, and government expenditures $G$.

$$Y_t^f = C_t + C_t^E + I_t + G_t \tag{3.27}$$

[5] The price index is defined as the minimum level of expenditures necessary to purchase one unit of the final good.

### 3.3 Entrepreneurs

Entrepreneurs have the same time preferences rate as the households. Their objective is to maximize

$$\max_{\{C^E_{t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \beta^{t+i} E_0 C^E_{t+i} \tag{3.28}$$

**Assumption 3.** *With constant probability* $1 - \gamma$*, an entrepreneur dies in a given period. Entrepreneurs can consume all their net worth just before death.*

This assumption allows to limit the size of aggregate net worth in an infinite horizon set up. Since the return to internal funds is higher than that to the external funds, risk neutral entrepreneurs want to postpone consumption, until they can self-finance their entire project. To avoid the irrelevance of financial frictions in the long run, exogenous death probabilities are introduced. When an individual entrepreneur receives a signal about his death he consumes all his net worth. Aggregate entrepreneurial consumption is thus given by:

$$C^E_t = \gamma F(K_t, L_t) \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}_t) \tag{3.29}$$

The evolution of the borrowers net worth plays a crucial role for the dynamics of the model. Net worth is determined as follows. Let $Z_{t-1}$ denote the units of physical capital owned by the entrepreneur at the beginning of period $t$. The entrepreneurs' net worth $N_t$ is then simply defined as the market value of $Z_{t-1}$, where $Z_{t-1}$ is equal to the entrepreneurs' last period's project share minus his consumption in last period $C^E_{t-1}$.

$$N_t = Z_{t-1} \left[ 1 - \delta + S_t \frac{P^W_t F_K(K_t, L_t)}{P_t} \right] \tag{3.30}$$

$$Z_t = F(K_t, L_t) \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}_t) \mathrm{d}\omega - C^E_t \tag{3.31}$$

### 3.4 The government

The government finances an exogenous stream of expenditures using a mix of lump-sum-taxes and seignorage revenue.

$$\frac{M_t - M_{t-1}}{P_t} = G_t + TR_t \tag{3.32}$$

For simplicity, $G_t = 0 \quad \forall t$.

## 4 Model calibration and analysis

Most of the model's parameter are calibrated to match certain time series averages or set to standard values in the literature.[6]

[6] I am currently working on an estimation of the models parameters using Euro area data and the Maximum Likelihood approach following Ireland (2003). Preliminary result suggest a larger degree of wage stickiness than price stickiness.

The capital share in output $\alpha$ is standard 0.36. The leisure parameter in the utility function $v$ is set such that workers spend 30 per cent of their time working. The discount factor $\beta$ is set to 0.99 as to induce a steady state real interest rate of roughly 4 per cent on an annual basis. The depreciation rate is set to 0.02 corresponding to an annual rate of 8 per cent. The fraction of sticky price firms $\theta$ is 0.5. As a benchmark, the fraction of sticky wages households is set to unity.[7] This is in line with the finding of Christiano, Eichenbaum, and Evans (2001) that wages are much more sticky than prices. The parameters governing monopolistic competition $\epsilon$ and $\kappa$ are set to 11 as to induce in markup of 10 per cent, roughly consistent with the empirical estimates. The autoregressive parameter in the law of motion for technology is set to the standard 0.95. In line with the empirical estimates by Ireland (2002) the persistence parameter of the markup $\epsilon$ is set to 0.95 as well.

The calibration of the key parameter characterizing the credit frictions chosen to emphasize the effects of such frictions. The cost of state verification $\mu$, which should be interpreted as a broad cost of bankruptcy is assumed to be 0.5 and taken from Harrison, Sussmann, and Zeira (1999). The parameter $\gamma$, governing entrepreneurs consumption, is set as to induce a steady state default probability of 25 per cent.

The working of the model can be illustrated by the use of impulse-response functions. Initially, a weak form of inflation targeting is assumed for the Taylor Rule. The nominal interest rate responds to current period inflation with an elasticity of $1 + \iota$, where $\iota$ is a small positive number. This is just enough to ensure a locally determinate rational expectations equilibrium.

$$\log(R_t) = \log(\bar{R}) + (1 + \iota)\log(\Pi_t) \tag{4.1}$$

## 4.1 Amplification and persistence?

Does the introduction of credit frictions lead to an amplified or dampened response of the economy to technology and monetary policy shocks? Does it create endogenous persistence into the model, i.e. autocorrelation functions of the models' variables which are not inherited from the exogenous shocks? The response of aggregate output to a technology shock is depicted in figure 2 for three different model settings. The benchmark is a model with monopolistic competition and flexible prices. Then credit frictions are added, finally credit frictions and sticky prices are added jointly.

Credit frictions dampen the response of output to a technology shock and increase persistence. This effect can be explained by the sluggish behavior of net worth as seen in figure 3 The firm would like to increase employment of labor and capital, but the loan size is related to net worth of the firm. However, net worth consists largely of profits accumulated in the past and cannot jump much on impact. It increases slowly and reaches its peak after about 2 years, explaining the strong persistence of shocks in the credit friction model. Output persistence in the credit friction model, measured by the half life of the initial shock, is roughly 7 quarters in the frictionless model and more than 20 quarters in the credit friction model.

[7] Sensitivity analysis with respect to these parameters are work in progress.

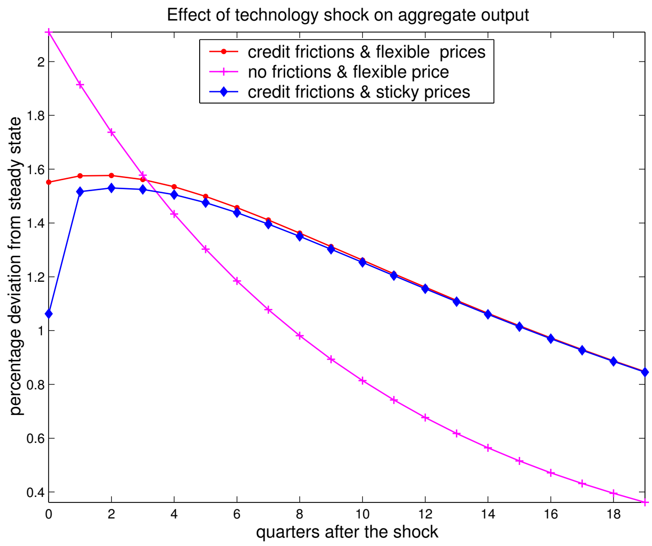



Figure 2: Response of output to technology shock

Adding price stickiness strengthens this persistence and induces a more hump shaped effect of the technology shock. The lower impact effect is due to preset prices being too high given an unanticipated shock. This drop in output relative to the flexible price world implies that the initial increase in net worth of the entrepreneurs is much lower. Through this effect on net worth, the negative effect of price stickiness on output is propagated for about 4-6 quarters, although prices are sticky only for one period.

## 4.2 Implications for monetary policy

Two scenarios are compared: Sticky prices only and sticky prices and sticky wages. The policy conclusion are very different.

### 4.2.1 Sticky prices only

In the model set up described so far, the implications of credit frictions for monetary policy are simple. Price stability remains the appropriate goal. Since the loss in output due to sticky price can be completely eliminated if the general price level remains stable, no loss in net worth accrues under price stability. Therefore, the effect of one period price stickiness cannot propagate via net worth as in the example of very weak inflation targeting. This conclusion is illustrated in figure 4, which plots the response of output to the technology shock under strong inflation targeting (coefficient of 3 in the Taylor rule).

Strong inflation targeting eliminates the distortions arising from staggered price setting and achieves output levels almost identical as in the flexible price case. Achieving the flexible price allocation is the best allocation the central bank can induce. The case for strong inflation targeting may be substantially weakened, however, when one considers stickiness in nominal wages as well.

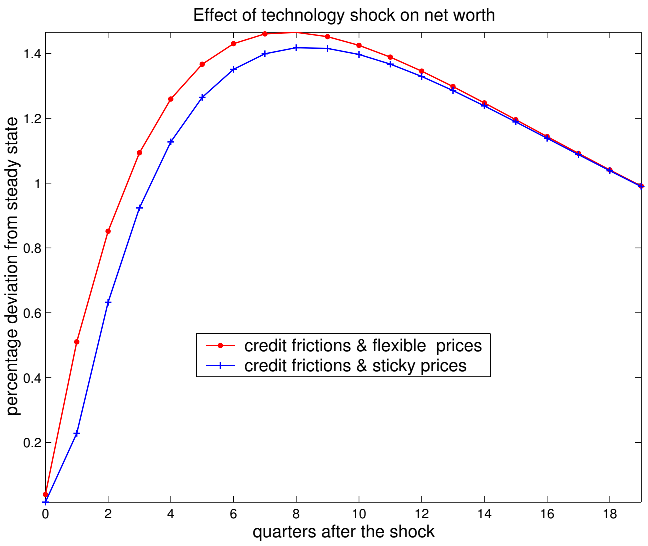



Figure 3: Response of net worth to technology shock

#### 4.2.2 Sticky prices and sticky wages

The model is altered slightly, nominal wages are set one period in advance. This is to capture in a simple way the idea that wage contracts are changed less frequently than the central bank adjusts nominal rates. The central bank can therefore respond to events that nominal wages do not respond to, such as the business cycle. Figure 5 shows the effect of a negative technology shock on output under strong inflation targeting, the optimal policy in the sticky price world.

There is now a trade off between output and inflation for the central bank. Strong inflation targeting is successful at keeping inflation low, but at the cost of incurring a loss in output that is much stronger than in world of flexible prices and wages. This is due to the fact that the real wage should fall with a negative productivity shock. But with nominal wages fixed at too high a level, this requires inflation. Inflation, however is contained at almost zero. Therefore, the central bank cannot achieve simultaneously price stability, i.e. minimization of the inefficient allocation of resources, and output gap stability. The role of credit frictions is then simply to propagate the negative effect of sticky wages on output beyond the period of wage stickiness. That is, output is below its value in the reference world of flexible wages and prices for another 4-6 quarters although all stickiness lasts for one period only. Again this effect is due to the decrease in net worth of the entrepreneur. While the propagation effect is modest in size, it does increase the importance of output gap stabilization in the implicit trade-off between inflation and output.

So far it has been argued that solely stabilizing consumer price inflation cannot be the optimal policy rule with sticky prices and sticky wages. There exists a trade-off between stabilizing the consumer price index and implementing an efficient level of the ex post real wage. The question then is, which coefficients

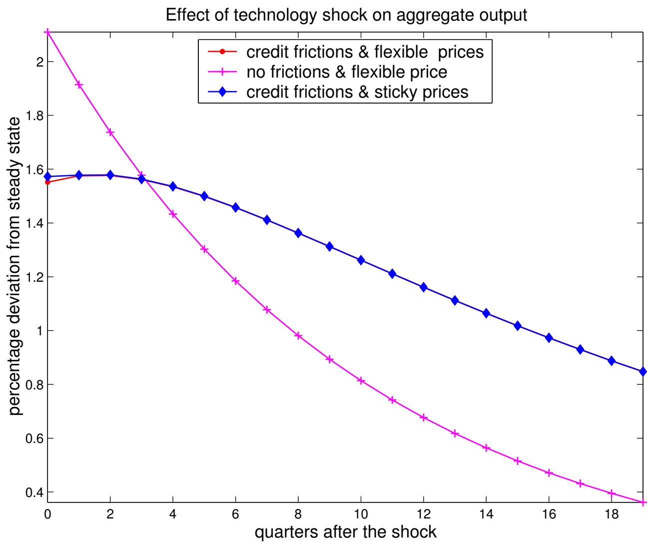



Figure 4: Response of output to technology shock: strong inflation targeting

in a monetary policy rule that relates the nominal interest linearly to consumer price inflation and deviations of output from trend maximize aggregate welfare.

The next sections perform a rigorous welfare analysis that derives the constrained optimal policy rule as the best linear rule responding to output deviations from trend and to current period consumer price inflation.

# 5 Welfare measure and solution method

Since the model is built from first principles, one can use the utility of the agents[8] to construct a welfare measure rather than resorting to an ad hoc loss function as in non-structural models.

Several authors have pointed to the pitfalls of using a linear approximation to the policy function when evaluating a second order approximation to the utility function. Kim and Kim (2003) have shown that such a procedure can result in a spurious reversal of welfare comparison in international economics: Autarky appears preferable to full risk sharing.

Essentially the problem is that some second order terms are included in the welfare criterion while other are neglected. A second order approximation to the policy function is used in this paper in order to avoid the bias in welfare comparisons of policy rules. The following subsections briefly summarize the solution methods and the employed welfare measure.

[8]The subsequent optimal policy analysis neglects the utility of entrepreneurs and maximizes only the utility of the households. This is done in order to facilitate comparison with a version of the model in which are no credit frictions.

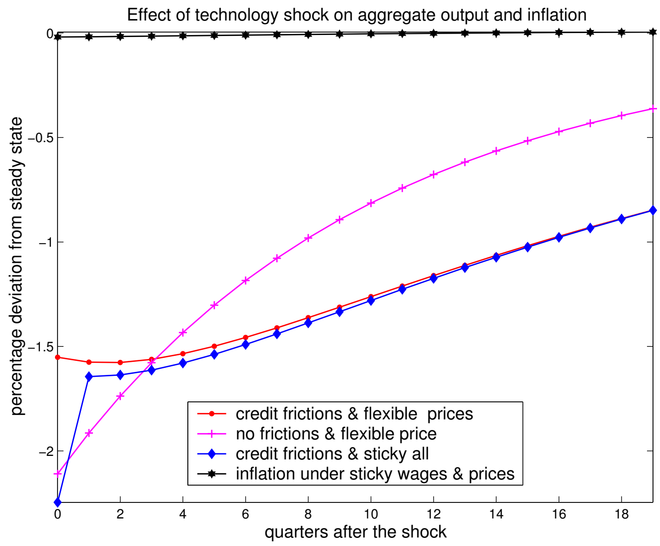



Figure 5: Response to technology shock: strong inflation targeting

## 5.1 Model representation and form of the solution

To fix notation, consider the generic representation for rational expectations models introduced by Schmitt-Grohé and Uribe (2004)

$$E_t\, f(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, \boldsymbol{x}_{t+1}, \boldsymbol{x}_t) = 0. \tag{5.1}$$

$f$ is a known function describing the equilibrium conditions of the model economy, $\boldsymbol{y}_t$ is a vector of co-state variables and $\boldsymbol{x}_t$ a vector of state variables partitioned as $\boldsymbol{x}_t = [\boldsymbol{x}_{1,t}; \boldsymbol{x}_{2,t}]$. $\boldsymbol{x}_{1,t}$ is a vector of endogenous state variables and $\boldsymbol{x}_{2,t}$ a vector of state variables following an exogenous stochastic process

$$\boldsymbol{x}_{2,t+1} = \boldsymbol{L}\boldsymbol{x}_{2,t} + \tilde{\boldsymbol{N}}\sigma\boldsymbol{\epsilon}_t. \tag{5.2}$$

$\boldsymbol{L}$ and $\tilde{\boldsymbol{N}}$ are known coefficient matrices, $\boldsymbol{\epsilon}_t$ is a vector of innovations with bounded support, independently and identically distributed with mean zero and covariance matrix $I$. $\sigma$ is a parameter scaling the standard deviation of the innovations. The solution to the model described by (5.1) is of the form

$$\boldsymbol{y}_t = g(\boldsymbol{x}_t, \sigma), \tag{5.3}$$

$$\boldsymbol{x}_{t+1} = h(\boldsymbol{x}_t, \sigma) + \boldsymbol{N}\sigma\boldsymbol{\epsilon}_{t+1}, \qquad \text{with: } \boldsymbol{N} = \begin{bmatrix} \boldsymbol{0} \\ \tilde{\boldsymbol{N}} \end{bmatrix}. \tag{5.4}$$

Schmitt-Grohé and Uribe (2004) derive the second-order Taylor approximation to the policy functions $g(\cdot)$ and $h(\cdot)$ and provide MATLAB codes for the numerical implementation. The approximate model dynamics obtained from their second-order approximation can be compactly expressed as

$$\boldsymbol{y}_t = \boldsymbol{G}\boldsymbol{x}_t + \frac{1}{2}\boldsymbol{G}^*(\boldsymbol{x}_t \otimes \boldsymbol{x}_t) + \frac{1}{2}\boldsymbol{g}\sigma^2, \tag{5.5}$$

$$\boldsymbol{x}_{t+1} = \boldsymbol{H}\boldsymbol{x}_t + \frac{1}{2}\boldsymbol{H}^*(\boldsymbol{x}_t \otimes \boldsymbol{x}_t) + \frac{1}{2}\boldsymbol{h}\sigma^2 + \sigma\boldsymbol{N}\boldsymbol{\epsilon}_{t+1}. \tag{5.6}$$

Here, the vectors $\boldsymbol{y}_t$ and $\boldsymbol{x}_t$ denote deviation or log-deviation from the steady state. $\boldsymbol{G}$ and $\boldsymbol{H}$ are coefficient matrices representing the linear part of the Taylor approximation. The matrices $\boldsymbol{G^*}$ and $\boldsymbol{H^*}$ form the second-order part jointly with the vectors $\boldsymbol{g}$ and $\boldsymbol{h}$.

## 5.2 Unconditional welfare

A natural welfare measure for rankings of fiscal or monetary policy that can be easily constructed from the second-order approximation is the unconditional expectation of period utility.

The second-order approximation to an arbitrary utility function $u(\boldsymbol{y}_t)$ of co-states $\boldsymbol{y}_t$ is

$$u(\boldsymbol{y}_t) \approx u(\overline{\boldsymbol{y}}) + \nabla u(\overline{\boldsymbol{y}})\boldsymbol{y}_t + \frac{1}{2} vec\left(\nabla^2 u(\overline{\boldsymbol{y}})\right)' (\boldsymbol{y}_t \otimes \boldsymbol{y}_t), \tag{5.7}$$

such that upon taking expectations

$$E(u(\boldsymbol{y}_t)) \approx u(\overline{\boldsymbol{y}}) + \nabla u(\overline{\boldsymbol{y}})\boldsymbol{\mu_y} + \frac{1}{2} vec\left(\nabla^2 u(\overline{\boldsymbol{y}})\right)' vec(\boldsymbol{\Sigma_y} + \boldsymbol{\mu_y \mu_y'}). \tag{5.8}$$

Here, $\boldsymbol{\mu_y}, \boldsymbol{\Sigma_y}$ denote unconditional mean and covariance matrix of $\boldsymbol{y}$, respectively. To construct first and second moments of the co-state variables assume covariance stationarity and take expectation of (5.5) and (5.6)

$$\boldsymbol{\mu_y} = \boldsymbol{G}\boldsymbol{\mu_x} + \frac{1}{2}\boldsymbol{G}^* vec(\boldsymbol{\Sigma_x} + \boldsymbol{\mu_x}\boldsymbol{\mu_x'}) + \frac{1}{2}\boldsymbol{g}\sigma^2, \tag{5.9}$$

$$\boldsymbol{\mu_x} = \boldsymbol{H}\boldsymbol{\mu_x} + \frac{1}{2}\boldsymbol{H}^* vec(\boldsymbol{\Sigma_x} + \boldsymbol{\mu_x}\boldsymbol{\mu_x'}) + \frac{1}{2}\boldsymbol{h}\sigma^2. \tag{5.10}$$

Note that while under the linear approximation unconditional means do not differ from the steady state values, the second-order approximation is able to capture the effect of variances on means. Since variances can be computed accurately up to second-order from the linear part of the policy function, it is sufficient to approximate $vec(\boldsymbol{\Sigma_x} + \boldsymbol{\mu_x}\boldsymbol{\mu_x'}) \approx vec(\boldsymbol{\Sigma_x})$ and $vec(\boldsymbol{\Sigma_y} + \boldsymbol{\mu_y}\boldsymbol{\mu_y'}) \approx vec(\boldsymbol{\Sigma_y})$. It is then possible to construct these using the simple formulas

$$vec(\boldsymbol{\Sigma_y}) = (\boldsymbol{G} \otimes \boldsymbol{G}) vec(\boldsymbol{\Sigma_x}), \tag{5.11}$$

$$vec(\boldsymbol{\Sigma_x}) = \sigma^2 (\boldsymbol{I} - \boldsymbol{H} \otimes \boldsymbol{H})^{-1} (\boldsymbol{N} \otimes \boldsymbol{N}) vec(\boldsymbol{I}). \tag{5.12}$$

Given these approximations for the variances, the means can be computed from (5.9) and (5.10). The described welfare measure has the following compact representation, which can easily be verified by applying the rules of the partitioned inverse.

$$E(u(\boldsymbol{y}_t)) \approx u(\overline{\boldsymbol{y}}) + \left[\nabla u(\overline{\boldsymbol{y}}), \frac{1}{2} vec\left(\nabla^2 u(\overline{\boldsymbol{y}})\right)'\right] \times \\ \left( \begin{bmatrix} \boldsymbol{G} & \frac{1}{2}\boldsymbol{G}^* \\ \boldsymbol{0} & \boldsymbol{G} \otimes \boldsymbol{G} \end{bmatrix} \left[ \boldsymbol{I} - \begin{bmatrix} \boldsymbol{H} & \frac{1}{2}\boldsymbol{H}^* \\ \boldsymbol{0} & \boldsymbol{H} \otimes \boldsymbol{H} \end{bmatrix} \right]^{-1} \begin{bmatrix} \frac{1}{2}\boldsymbol{h} \\ \boldsymbol{N} \otimes \boldsymbol{N} vec(\boldsymbol{I}) \end{bmatrix} \sigma^2 + \begin{bmatrix} \frac{1}{2}\boldsymbol{g} \\ \boldsymbol{0} \end{bmatrix} \sigma^2 \right) \tag{5.13}$$

The task of computing optimal monetary policy then amounts to numerically optimizing this welfare measure through choice of the coefficients in the policy rule.

**Remark 2.** *If the second order approximation to the welfare function can be rewritten as to involve quadratic terms only, then linear and quadratic approximations to the policy functions will yield the same level of welfare. I.e. up to second order, there is no bias in welfare calculations based on linear policy rules.*

The effect of the higher order terms of the models dynamics on the first order terms in the welfare criterion is the origin of the bias. Woodford (2003) carefully rearranges the second order expansion to the utility function as to eliminate the first order terms. Whenever such a procedure is possible, using a first order approximation to the models dynamics does not introduce a bias in welfare comparison.

When the model involves endogenous state variables or is not efficient in the steady state, it is often not possible to eliminate the first order terms in the welfare criterion and a higher order solution to the models dynamics is needed.

## 5.3 Accuracy check: Inspection of Euler residuals

Judd (1998) advocates the inspection of Euler equations residuals to assess the accuracy of the obtained solution. This procedure has recently been employed and developed further in an extensive comparison of linear and nonlinear solution methods by Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2003). One plots the residual in the Euler equation as a function of the state variables of the system. Let $x_{t+1} = h^s(x_t)$ denote transition function for the state variables obtained under solution method $s$ and $y_t = g^s(x_t)$ the policy function for time $t$ decision variables. The residual arising from the Euler equation for capital holdings is:

$$R^s(x_t) = 1 - \frac{\left\{\beta \mathrm{E}_t \left[C\left(h^s(x_t)\right)^{-\sigma} \left\{\alpha P^W\left(h^s(x_t)\right) Y\left(h^s(x_t)\right) / K\left(h^s(x_t)\right) + 1 - \delta\right\}\right]\right\}^{-\frac{1}{\sigma}}}{C(x_t)} \tag{5.14}$$

Compared to the more formal methods such as the $\chi^2$ accuracy test developed by den Haan and Marcet (1994), this approach is easily interpretable: It gives the error from following the approximated policy rule as a fraction of current period current consumption. Under certain conditions the approximation error of the policy function is of the same order of magnitude as the Euler equation residual as pointed out by Santos (2000).

Figure (6) plots the consumption Euler equation residual obtained from the second-order accurate solution method over a range of deviations of the state variables from $-10\%$ to $+10\%$ from their steady state levels. *state*1 refers to capital, *state*2 to the predetermined price of the sticky price firms, all other state variables are fixed at their steady state values. [9] The Euler residual is on the order of magnitude of $10^{-4}$. This may be interpreted as 10 cents error for every 100 $ spent. Assuming that the error in approximating the policy function is of the same order of magnitude as the Euler equation residual, the achieved accuracy seems sufficient.

[9] The expectation is computed using Gaussian quadrature with 10 nodes.

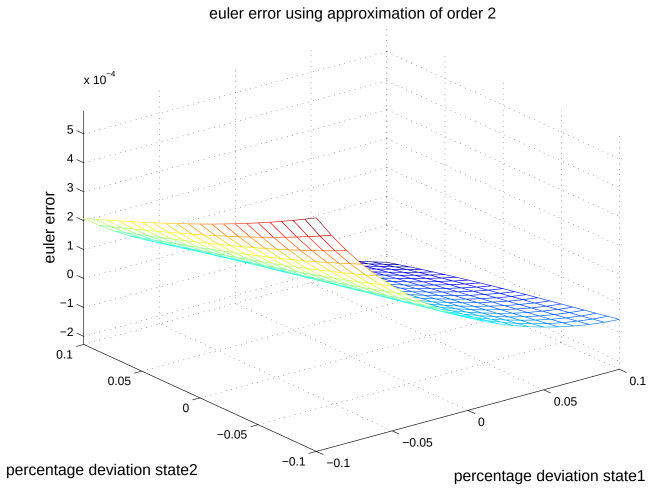



Figure 6: Euler equation residual for the credit friction model

# 6 Optimized simple policy rules

This section reports the optimized coefficients in the simple Taylor rule for different variants of the model. The two types of shocks that are considered are the technology shock and a markup shock. Monetary policy follows a simple Taylor type rule[10].

$$\log(R_t) = \log(\bar{R}) + c_1 \log(\Pi_t) + c_2 \log\left(\frac{y_t}{y_{ss}}\right) \tag{6.1}$$

The table 2 displays the results.

Table 2: Optimized coefficients in a simple monetary policy rule

| model | shock | c1 | c2 |
|---|---|---|---|
| sticky wages & prices | technology | 3.0016 | 0.3403 |
| sticky wages & prices & frictions | technology | 2.9849 | 0.3886 |
| sticky wages & prices | markup | 2.7059 | 2.2356 |
| sticky wages & prices | markup & technology | 3.2238 | 0.9671 |

The model with sticky wages and sticky prices involves a trade-off between eliminating the inefficiencies arising from price stickiness and from wage stickiness. Price stability achieves the efficient allocation of flexible price and fixed price output, however it stabilizes the real wage at an inefficient level.

[10]This papers neglects the fact the optimal policy with commitment and forward looking behavior of agents is clearly history dependent. Taking this feature into account would amount to adding a lagged value of the nominal interest rate to the Taylor rule. The conjecture is that adding inertia to Taylor rule does not change the qualitative insights on optimal weights on inflation and output gained from an optimal non-inertial rule

The table 2 indicates that this trade-off implies a substantial response to the deviation of output from steady state. The reason is simply that output deviation is strongly correlated with the real wage gap, defined as actual real wage minus flexible wage. Whereas the magnitudes of the coefficients presented in this table are not directly comparable to the ones in the empirical estimates[11] in section 2, the qualitative pictures is clear.

When technology shocks are the only source of fluctuation, there is a small but significant response to output in the optimal simple Taylor rule. The addition of financial frictions[12] increase the weight on output, since it makes loss in output due to inappropriate real wages more persistent. This effect is quantitatively quite small, the coefficient on output increases from 0.34 to 0.38 in the model with technology shocks only.

The table also shows that markup shocks warrant a much stronger response to output, which is consistent with the finding of Woodford (2003), that these induce and inflation output trade-off even in a world with price stickiness only. When technology and markup shock jointly cause aggregate fluctuations, the magnitude of the optimal reaction to the cyclical component of output is reduced.

# 7 Conclusion

This paper has assessed whether central banks' Taylor rule should include a significant response to a cyclical measure of output in a model with nominal rigidities in both goods and labor market as well as asymmetric information in the credit market .[13]

It is shown that a particular form of nominal wage stickiness can render such a Taylor rule optimal. Nominal wages being predetermined one period in advance imply that solely stabilizing inflation implies an inefficient level of the ex-post real wage. In a recession the central should sacrifice some inflation in order to lower the ex-post real wage and bring it closer to the fully flexible real wage. Monetary policy has a role in stabilizing output over the cycle on top of stabilizing inflation. It should be noted that such a policy does not require full knowledge of the output gap as defined in Woodford (2003). A simple deviation of output from trend provides enough information on the real wage gap.

This result is contrary to the one in Amato and Laubach (2003), who find that reaction to output rather than the gap, is not optimal. The difference lies in the modeling of nominal wage stickiness. This paper adopts no staggering in wage setting, whereas the former authors apply the Calvo (1983) price setting framework to nominal wage setting. With staggered wage setting generating CPI inflation will lead to wage inflation which then increases the dispersion of

[11] Since the model is not estimated to match the data, one cannot compare the magnitude of these coefficients to the ones from Table 1. The coefficients may differ simply because of the scaling of output in the data and in the model.

[12] It should be noted that the utility of the entrepreneurs was neglected in the calculation of optimal policy. That is, in the models with and without credit friction, the policy maker's objective remains constant and equal to an approximation of expected household utility. Including the utility of the entrepreneur would probably not change the results much, since their utility is linear. Up to first order policy does not affect them.

[13] The question is posed here is somewhat similar to the one in Walsh and Ravenna (2003) who also analyze an inflation - unemployment trade-off.

nominal wages and is thus not optimal. Which assumption about wage stickiness is more realistic? In a country with centralized wage setting by unions and employers organizations, the main inefficiency may not be the dispersion in nominal wages, but rather the inappropriate level over the cycle.

The second contribution of the paper is to show how the optimal coefficient on output in the Taylor rule affected by adding another distortion to the model which is plainly realistic: frictions in the credit market. As modeled in this paper, these frictions imply a dampened but much more persistent response of the economy to exogenous shocks. Failing to respond to output implies that exogenous shocks are propagated beyond the period of price stickiness as shown in figure 5. This endogenous persistence generated through the effect of borrowers net worth warrants a stronger response to output than in the case of no credit frictions. The magnitude of this effect however is relatively small. However, the general message from this exercise is that credit friction can affect the output inflation trade off and that they may increase the weight that policymakers may put on output stabilization. Whereas in this model, the magnitude of this effect is very small, this may not be the case under a model variant similar to Bernanke, Gertler, and Gilchrist (2000), where the effects of credit frictions on the cycle are more amplified and propagated rather than dampened and propagated.

On a numerical side, the papers has used a second order approximation to the policy function in order compute welfare correctly. Such an approach is crucial for an accurate welfare comparison.

Future extensions of this research should include an estimation of the models identified parameters, rather than the simple calibration procedure used here. Furthermore, it appears promising to include a model variant into the analysis which features the financial accelerator as in Bernanke, Gertler, and Gilchrist (2000). This would amplify the economies response to shocks rather than dampen them and may lead to stronger policy conclusions.

# A Appendix: The model with sticky prices, sticky wages and credit frictions

Households:

$$E_{t-1}\left\{L_t\left(\frac{v}{1-L_t}-\frac{\kappa-1}{\kappa}\frac{P_t^W}{P_t}S_tA_tF_L(K_t,L_t)\frac{1}{C_t}\right)\right\}=0$$

$$C_t^{-1}=\beta E_t\left\{C_{t+1}^{-1}\left[1-\delta+\frac{P_{t+1}^W}{P_{t+1}}S_{t+1}A_{t+1}F_K(K_{t+1},L_{t+1})\right]\right\}$$

$$C_t^{-1}=E_t\beta\left\{\frac{P_t}{P_{t+1}}C_{t+1}^{-1}\right\}R_{t+1}^n$$

Optimal contract:

$$S_t = 1 - \mu \left\{ \bar{\omega}_t \phi(\bar{\omega}_t) \left[ \frac{\int_{\bar{\omega}_t}^{\infty} \omega_t \phi(\omega_t) \mathrm{d}\omega_t}{1 - \Phi(\bar{\omega}_t)} - \bar{\omega}_t \right] + \int_0^{\bar{\omega}_t} \omega_t \phi(\omega_t) \mathrm{d}\omega_t \right\}$$

$$\frac{P_t}{P_t^W}(rK_t + wL_t - N_t) = A_t F(K_t, L_t) \left\{ (1-\mu) \int_0^{\bar{\omega}_t} \omega_t \phi(\omega_t) \mathrm{d}\omega_t + [1 - \Phi(\bar{\omega}_t)]\bar{\omega}_t \right\}$$

Entrepreneurs:

$$N_t = Z_t \left[ 1 - \delta + \frac{P_t^W}{P_t} S_t A_t F_K(K_t, L_t) \right]$$

$$Z_{t+1} = \frac{P_t^W}{P_t} A_t F(K_t, L_t) \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}_t) - C_t^E$$

$$C_t^E = \gamma \frac{P_t^W}{P_t} A_t F(K_t, L_t) \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}_t)$$

Market clearing:

$$Y_t^f = C_t + C_t^E + K_{t+1} - (1-\delta)K_t$$

$$Y_t^W \left( 1 - \mu \int_0^{\bar{\omega}_i} \omega_i \phi(\omega_i) \mathrm{d}\omega \right) = \theta Y_t^{fx} + (1-\theta) Y_t^{fl}$$

Production:

$$Y_t^W = A_t F(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$$

$$Y_t^f = \left[ \theta \left[ y_t^{fx} \right]^{\frac{\epsilon_t - 1}{\epsilon_t}} + (1-\theta) \left[ y_t^{fl} \right]^{\frac{\epsilon_t - 1}{\epsilon_t}} \right]^{\frac{\epsilon_t}{\epsilon_t - 1}}$$

Price setting and demand function:

$$\frac{Y_t^{fx}}{Y_t^{fl}} = \left[ \frac{P_t^{fx}}{P_t^{fl}} \right]^{-\epsilon_t}$$

$$P_t^{fl} = \frac{\epsilon_t}{\epsilon_t - 1} P_t^W$$

$$\mathrm{E}_{t-1} \left\{ P_t^{\epsilon - 1} Y_t^{fi} \left[ P_t^{fx}(\epsilon_t - 1) - \epsilon_t P_t^W \right] \right\} = 0$$

$$P_t = \left\{ \theta \left[ P_t^{fx} \right]^{1-\epsilon_t} + (1-\theta) \left[ P_t^{fl} \right]^{1-\epsilon_t} \right\}^{\frac{1}{1-\epsilon_t}}$$

Monetary policy rule:

$$R_t = \bar{R} + c_0(\Pi_t - 1) + c_1 \mathrm{log}(y_t - y_{ss})$$

Exogenous stochastic processes:

$$log(A_{t+1}) = (1-\rho_a) log(\bar{A}) + \rho_a log(A_t) + u_{t+1}$$

$$log(\epsilon_{t+1}) = (1-\rho_e) log(\bar{\epsilon}) + \rho_e log(\epsilon_t) + v_{t+1}$$

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