

Sticky Prices and the Purchasing Power Parity Puzzle

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Abstract

This paper investigates the Purchasing Power Parity Puzzle in the context of a Vector Autoregressive (VAR) model, where the real exchange rate is Granger caused by macroeconomic variables, suggested by ‘sticky-price’ theories of exchange rate determination. By doing this, we are able to discern the relative effect of nominal price rigidities on the speed of adjustment of the real exchange rate toward the PPP level. We first show that the impulse response function of a variable participating in the VAR is not, in general, the same with the impulse response function obtained from the equivalent ARMA representation of this variable, if the latter is Granger caused by other variables in the system. The difference between the two functions captures the effects of the Granger-causing variables on the dynamic adjustment process of the variable of interest. Our empirical results for a set of four currencies suggest that price stickiness accounts for a substantial fraction of the persistence of deviations from PPP. In particular, we provide evidence that between 22% and 50% of the half-life of innovations in the real exchange rate is due to nominal price rigidities.

Keywords: Real exchange rate; persistence measures; nominal rigidities; VAR.

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1 Introduction

Purchasing Power Parity (PPP) is one of the central arbitrage relationships in international economics. When exchange rates started to float worldwide in 1973, it was widely believed that they would quickly adjust to changes in relative price levels. With the obvious failure of short-run PPP, attention turned to whether long-run PPP holds. The emerging consensus view in the literature is that real exchange rates are stationary, but highly persistent with half-lives of deviations from PPP between three and five years.¹ Studies using panel data, find only slightly shorter half-lives.²

The empirical evidence of an extremely slow speed of convergence towards PPP cannot be easily reconciled with the stylized fact that short-term deviations from PPP are both large and volatile. Indeed, the short-term volatility of real exchange rates is of the same order of magnitude as the volatility of nominal exchange rates. Combined with this stylized fact, the finding of high persistence of the real exchange rate constitutes a puzzle as to the nature of the shocks driving real exchange rates. Rogoff (1996) termed this the “PPP puzzle”. Following Rogoff, the PPP puzzle can be stated as follows: Given that a significant fraction of exchange rate volatility is caused by monetary shocks (e.g. Dornbusch (1976) and the related literature on short-term price-stickiness), one would expect deviations from PPP to die out at a much faster rate than the consensus view suggests, because monetary shocks can only have real effects over a time frame in which nominal wages and prices are sticky (in general, about one year – see Taylor (1999)). Of course, it is not difficult to rationalize slow adjustment of real exchange rates if real shocks – shocks to tastes and technology – are predominant. However, real shocks are not frequent enough to account for the high volatility of real exchange rates.

The majority of empirical studies compute half-lives of PPP deviations within a univariate framework, typically by estimating a first-order autoregressive model (AR(1))³. In such a specification, the error term, which accounts for the variation of the real exchange rate, can be thought of as a ‘composite shock’ that incorporates various individual shocks, such as monetary shocks or shocks to tastes and technology. As a result, impulse response analysis (IRA) within the univariate framework cannot identify the effect of each individual shock, but simply tells us how fast the real exchange rate adjusts to a disturbance of unknown origins. If the estimated adjustment

¹See, e.g. Frankel (1986, 1990), Abuaf and Jorion, (1990), Glen (1992), Froot and Rogoff (1995), Rogoff (1996) and Lothian and Taylor (1996), among others.

²See, e.g. Wu (1996), Oh (1996), Frankel and Rose (1997), Lothian (1997) and Papell (1997), among others.

³The half-life of a zero-mean stationary process, y_t , is defined as follows: Assume that before $t = 0$, $y_t = 0$. In period $t = 0$, a shock occurs which causes y_0 to take the value H . The half-life of y_t is defined as the number of periods required for y_t to reach the value of $H/2$ if no further shocks occur.

period is sufficiently short, with a half-life of, say, one year, then the shock can be interpreted as a monetary one. In the opposite case, the shock has to be real, since price stickiness alone cannot account for half-lives of three to six years that are usually reported in the literature.

Various studies have attempted to measure the contributions of monetary and real shocks to the variability of the real exchange rate by estimating structural Vector Autoregressive (VAR) models. The results are mixed. Clarida and Gali (1994) find monetary shocks to be unimportant in contrast to aggregate demand shocks, which appear to be the major determinant of real exchange rate variability. On the other hand, Rogers (1999) reports that 20% to 60% of the variability of real exchange rates is attributable to monetary shocks.

In this paper, we also employ a VAR methodology, although we take a different approach than decomposing the variance of the real exchange rate within a structural VAR framework. Rather than trying to identify the sources of structural shocks to the real exchange rate, we measure the relative contribution of price-stickiness on the persistence of deviations from PPP by comparing the half-life estimates obtained from univariate models of the real exchange rate with those obtained from multivariate models. The latter include apart from the real exchange rate, macroeconomic variables that determine the short-run dynamics of PPP deviations, as predicted by sticky-price theoretical models for the exchange rate. By doing so, we are able to isolate the effect of these variables on the speed of adjustment of the real exchange rate toward the PPP level. By comparing the half-life of the univariate model with the half-life of the multivariate model, we are able to measure directly the contribution of ‘price-stickiness’ on the persistence of PPP deviations. For example, if the multivariate half-life estimates are not statistically shorter than the univariate ones, then ‘price-stickiness’ cannot account, even partly, for PPP deviations. In such a case, productivity shocks should be called upon.

In order to clarify this point and motivate the discussion that follows, let us first define the real exchange rate, y_{1t} , as the relative price of foreign goods in terms of domestic goods. In log form:

$$y_{1t} \equiv s_t - (p_t - p_t^*)$$

where s_t is the nominal exchange rate, measured in units of domestic currency per unit of foreign currency, and p_t (p_t^*) is the domestic (foreign) price index. Furthermore, let \mathbf{y}_{2t} be an $(n-1)$ vector of macroeconomic variables, which, according to the sticky-price model, affect the dynamic adjustment of the real exchange rate towards the PPP level. Finally, let \mathbf{y}_{3t} be a $(m \times 1)$ vector of variables which capture shocks to tastes and technology and deter-

mine the long-run equilibrium level of the real exchange rate.⁴ Since shocks to tastes and technology are difficult to proxy in practice, we restrict our attention to the $(n \times 1)$ vector of observable variables $Y_t = [y_{1t}, \mathbf{y}_{2t}]^T$.

Let us further assume that Y_t follows a n -variate VAR(1) model. It is well known that each variable in the VAR(1) model (including y_{1t}) has an equivalent univariate ARMA($n, n - 1$) representation, where n and $n - 1$ are the maximum orders of the autoregressive and moving average parts, respectively (see Lutkepohl (1993)). In view of this ‘equivalence’, there is no specification error involved in one’s decision to employ the ARMA model for estimating the response of the real exchange to a unit shock in the error term, say e_t . The latter, however, is a combination of the errors in the VAR model, which in turn implies that the origins of this shock cannot be identified. Assume for simplicity that there is no contemporaneous correlation among the elements of Y_t , and consider the first equation of the VAR, that is the one for the real exchange rate. The error term in this equation, say ε_{1t} , describes the shocks in the real exchange rate not accounted for by the observed (and included in the model) variables \mathbf{y}_{2t} , that is it describes the effects of the unobservable variables, \mathbf{y}_{3t} plus those of any other random factors that affect the exchange rate not predicted by any theory (pure noise). The VAR-response, IR^V , of y_{1t} to a unit shock in ε_{1t} should now be faster than its equivalent ARMA-response, IR^A , to e_t if the variables \mathbf{y}_{2t} have actually a role to play. Indeed, the difference, $D = IR^A - IR^V$, describes the dynamic adjustment path of the real exchange rate which is solely due to the observed variables \mathbf{y}_{2t} . Obviously, the effects of the unobservable variables \mathbf{y}_{3t} (plus noise) are captured by IR^A itself. The bigger D is, the more (less) important the role of \mathbf{y}_{2t} (\mathbf{y}_{3t} +noise) for the persistence of the real exchange rate will be.

To further clarify our point, assume that the half-life of PPP deviations, estimated within the ARMA model for the real exchange rate is 20 quarters. On the other hand, assume that the half-life estimate obtained from the VAR model, which includes y_{1t} and \mathbf{y}_{2t} is only 12 quarters. This means that the half-life of the real exchange rate due to price stickiness (which is captured by \mathbf{y}_{2t}) is $20 - 12 = 8$ quarters. The remainder 12 quarters is the number of periods required for the real exchange rate to adjust (by half) to shocks in \mathbf{y}_{3t} , i.e. shocks to tastes and technology. In such a scenario, both monetary and technology shocks are important. Monetary shocks (which are the main source of exchange rate volatility in the sticky price monetary model) account for 40% ($= 8/20$) of the persistence of the real exchange rate. ‘Being there’, the monetary shocks can also explain the observed volatility of the real exchange rate.

⁴See, e.g. Balassa (1964) and Samuelson (1964). According to the so-called “Balassa-Samuelson hypothesis”, the long-run equilibrium real exchange rate is determined by the share of nontradable goods in the consumer basket (i.e. by consumer preferences) and relative total factor productivity in the tradables and non-tradables sector.

Our methodology has three main advantages over the structural VAR approaches previously employed in the literature. First, it does not require to impose identifying restrictions in order to decompose innovations in the real exchange rate into structural shocks. Second, it allows to recover directly the effect of price-stickiness on the persistence of PPP deviations from the impulse-response function, as opposed to the structural VAR approach, which focuses on the effect of some type of structural shock on the variance of the real exchange rate. Third, it provides simple testable conditions on the estimates of the VAR which allow the researcher to assess the role of nominal price rigidities in determining the persistence of PPP deviations.

Our work is also related to a number of recent studies which try to assess whether calibrated equilibrium business cycle models with nominal wage and price rigidities are able to replicate the observed volatility and persistence of real exchange rates.⁵ The results from this literature suggest that stochastic business cycle models with nominal rigidities can reproduce real exchange rates which are appropriately volatile and quite persistent, although they generally fail to reproduce the consensus estimate of persistence under reasonable parameter values.⁶

The remainder of the paper is structured as follows. Section 2 focuses on the econometric methodology. In the context of a first-order bivariate VAR model, it compares the impulse response function (IRF) of the first variable of the VAR with the IRF obtained from the univariate ARMA representation of this variable. It also derives conditions under which these two IRFs are identical. Section 3 reviews several versions of the sticky-price model, which suggest potential macroeconomic variables affecting the dynamic adjustment of the real exchange rate towards the PPP level. Such variables include real or nominal interest rate differentials and relative business cycle positions of the economies, proxied by output gap or GDP growth differentials. Section 4 reports the univariate and multivariate half-life estimates for the real exchange rate of some industrialized countries vis-a-vis the US dollar. Our analysis provides clear evidence that the macroeconomic variables suggested by sticky-price theories account for a substantial fraction of the total adjustment process of the real exchange rate towards PPP. We find that between 22% and 50% of the half-life of PPP deviations can be accounted for by these variables. Section 5 briefly concludes the paper.

⁵See Kollman (2001), Bergin and Feenstra (2001) and Chari et al.(2002), among others.

⁶For example, Chari et al. (2002) find that when prices are exogenously fixed for one year, the first-order autocorrelation coefficient of the real exchange rate is in the range 0.62 to 0.69, corresponding to a half-life of real exchange rate innovations of about one year.

2 Impulse Response Analysis: Multivariate Models and their Equivalent Univariate Representations

This section highlights our main methodological point, namely that the impulse response analysis within a VAR model differs in general from that conducted within the equivalent univariate ARMA models. For illustrative purposes and in order to avoid unnecessary complications, we focus on the simplest possible case, namely that of a zero-mean bivariate VAR(1) model. The results extend to the case of a k -variate VAR(p) model in a straightforward way.

Let $Y_t = (y_{1t}, y_{2t})'$ follow a stable VAR(1) process:

$$Y_t = AY_{t-1} + U_t \quad (1)$$

where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $a_{ij} \in R$. The error vector $U_t = (u_{1t}, u_{2t})'$ is a white noise process, that is, $E(U_t) = 0$, $E(U_t U_t) = \Sigma_u = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$ and $E(U_t U_s) = 0$ for $t \neq s$. The covariance matrix Σ_u is assumed to be non-singular. The following proposition holds:

Proposition 1 *Let $Y_t = (y_{1t}, y_{2t})'$ satisfy (1). Then, each component series y_{it} , $i = 1, 2$, has a univariate ARMA(p, q) representation, where $p \leq 2$ and $q \leq 1$.*

Proof. See Corollary 6.1.1. in Lutkepohl (1993), page 232. ■

To be specific, the ARMA(2,1) representation of y_{1t} is as follows:

$$y_{1t} - (a_{11} + a_{22})y_{1t-1} + (a_{11}a_{22} - a_{21}a_{12})y_{1t-2} = e_{1t} + \gamma_1 e_{1t-1} \quad (2)$$

where $Var(e_{1t}) = \sigma_1^2$, $\gamma_1 = \frac{S \pm \sqrt{Q+R}}{F}$ and $\sigma_1^2 = \frac{G_1}{\gamma_1}$.⁷

Furthermore,

$$S = (1 + a_{22}^2)\sigma_{11} - 2a_{12}a_{22}\sigma_{12} + a_{12}^2\sigma_{22},$$

$$Q = (1 + a_{22}^4 - 2a_{22}^2)\sigma_{11}^2 + a_{12}^4\sigma_{22}^2 + (4a_{12}^2a_{22}^2 - 4a_{12}^2)\sigma_{12}^2 - 4(a_{12}a_{22}^3 - a_{22}a_{12})\sigma_{11}\sigma_{12},$$

$$R = (2a_{12}^2 + 2a_{22}^2a_{12}^2)\sigma_{11}\sigma_{22} - 4a_{12}^3a_{22}\sigma_{12}\sigma_{22},$$

$$F = 2(a_{12}\sigma_{12} - a_{22}\sigma_{11}),$$

$$G_1 = a_{12}\sigma_{12} - a_{22}\sigma_{11}.$$

⁷Note that we have to choose the invertible solution for γ_1 . This means that we have to choose the value of γ_1 that satisfies $|\gamma_1| < 1$.

It is interesting to note that the MA error term, $w_{1t} \equiv e_{1t} + \gamma_1 e_{1t-1}$, is related to the original VAR errors as follows:

$$w_{1t} = u_{1t} - a_{22}u_{1t-1} + a_{12}u_{2t-1} \quad (3)$$

This relationship shows that the error in the univariate representation of y_{1t} can be thought of as an aggregation of the original errors in the VAR. As a result, the variation of w_{1t} is due to the variation of either u_{1t} or u_{2t} or both. Furthermore the above relationships show that the variance, σ_1^2 , of the error term, e_{1t} , is a complicated function of the VAR parameters. This means that the shock e_{1t} of y_{1t} in the context of the ARMA model is determined by the structure of the intertemporal interactions between y_{1t} and y_{2t} and the second moments of u_{1t} and u_{2t} . As a consequence, its ‘origins’ are far from clear.

Let us now examine the response of y_{1t} to a unit shock in its innovations, in the context of both the VAR(1) and the ARMA(2,1) models. Before we proceed any further, it is important to emphasize the role of $\sigma_{12} \neq 0$ on the interpretation of the errors in the VAR model. If $\sigma_{12} \neq 0$, then the error, u_{1t} , in the first equation of the VAR, cannot be interpreted as the innovations driving y_{1t} . On the other hand, if $\sigma_{12} = 0$, then u_{1t} regains its status as ‘the innovations’ of y_{1t} in the VAR model and can be thought of as summarizing the factors that contribute to the variability of y_{1t} , other than y_{1t-1} and y_{2t-1} . We are interested in examining the impulse response function, IRF_u , of y_{1t} , from the univariate model with the impulse response function, IRF_m , of y_{1t} from the multivariate model. Note that IRF_m refers to the response of y_{1t} to a unit shock in u_{1t} .⁸ The cases $\sigma_{12} = 0$ and $\sigma_{12} \neq 0$ are analyzed in subsections 2.1 and 2.2 respectively.⁹

2.1 The Case of a Diagonal Covariance Matrix, $\sigma_{12} = 0$

Throughout this subsection we assume $\sigma_{12} = 0$. The impulse response functions under consideration, IRF_u and IRF_m , are defined as follows:

$$IRF_u(k) = \gamma_k + \sum_{j=1}^k a_j IRF_u(k-j)$$

where $k = 1, 2, 3, \dots$, $IRF_u(0) = 1$, $\gamma_k = 0$ for $k > 1$, $a_1 = (a_{11} + a_{22})$, $a_2 = (a_{21}a_{12} - a_{11}a_{22})$ and $a_k = 0$ for $k > 2$. On the other hand, IRF_m is

⁸In the case of the VAR model, a response in y_{1t} may be caused by an impulse in u_{2t} , even if $\sigma_{12} = 0$.

⁹The diagonality restrictions on the covariance matrix are tested in the empirical part of the paper for all the countries under consideration.

usually defined in the context of the infinite moving average representation of Y_t , that is $Y_t = \sum_{i=0}^{\infty} \Phi_i U_{t-i}$ where $\Phi_i = A^i$. Then, it is easy to show that

$$IRF_m(k) = \phi_{11,k}$$

where $\phi_{11,k}$ is the upper left element of Φ_k .

We are interested in comparing $IRF_u(k)$ with $IRF_m(k)$. We present our results in the form of the following propositions.

Proposition 2 $IRF_u(k)$ is in general not equivalent to $IRF_m(k)$ for some $k < \infty$.¹⁰

Proof. It is easy to show that in the context of (1), $IRF_m(1) = a_{11}$. On the other hand, $IRF_u(1) = a_{11} + a_{22} + \gamma_1$. Similarly, $IRF_m(2) = a_{11}^2 + a_{12}a_{21}$, whereas $IRF_u(2) = (a_{11} + a_{22})(\gamma_1 + a_{11} + a_{22}) - a_{11}a_{22} + a_{21}a_{12}$. Similar results are obtained for $k > 2$. Therefore, in general, $IRF_u(k) \neq IRF_m(k)$. ■

Due to the presence of γ_1 in $IRF_u(k)$, it is analytically impossible to identify all the cases where $IRF_u(k) > IRF_m(k)$. If, however, we restrict our attention to the case that the univariate representation of y_{1t} is purely autoregressive, that is $\gamma_1 = 0$, then it is easy to show that for $a_{11}, a_{22} > 0$, $IRF_u(k) > IRF_m(k)$ for $k = 1, 2$.¹¹ However, there is one case where $IRF_u(k) = IRF_m(k)$ for every k . Specifically, this case arises when y_{2t} does not Granger cause y_{1t} . Before we prove this result, we need to take an intermediate step, as described in the following remark:

Remark 1 Let $A = \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix}$, where $a_{ij} \in R$. Then, for every integer $d > 0$, $A^d = \begin{bmatrix} a_{11}^d & 0 \\ q_1 & a_{22}^d \end{bmatrix}$, where q_1 is a function of a_{ij} .

Proof. See Appendix. ■

Lemma 1 When $a_{12} = 0$, $IRF_u(k) = IRF_m(k)$ for every $k \geq 0$.

Proof. We have defined $IRF_m(k)$ to be equal to the upper left element, $\phi_{11,k}$, of $\Phi_k = A^k$. By means of the previous remark, we have that Φ_k is of the form: $\begin{bmatrix} a_{11}^k & 0 \\ q_1 & a_{22}^k \end{bmatrix}$, where q_1 is a function of a_{ij} . Therefore, $IRF_m(k) = a_{11}^k$. Next, it is easy to show that when $a_{12} = 0$, i.e. y_{2t} does not Granger cause y_{1t} , the univariate representation of y_{1t} is the following AR(1) model: $y_{1t} = a_{11} * y_{1t-1} + e_{1t}$, which in turn implies that $IRF_u(k) = a_{11}^k$. Thus, $IRF_u(k) = IRF_m(k)$ for every k . ■

¹⁰ Given the stability of (1), both IRF_u and IRF_m tend to zero as $k \rightarrow \infty$.

¹¹ We suspect that $IRF_u(k) > IRF_m(k)$ holds for $k > 2$, although it is impossible to prove it analytically for every k . However, extensive numerical simulations seem to confirm this conjecture.

It is important to note that only when $a_{12} = 0$, the AR(1) is the correct univariate specification for y_{1t} . In the opposite case, the AR(1) is a misspecified model, thus producing misleading results in every aspect of statistical inference. This has direct implications on the wide application of the AR(1) model as the univariate representation of the real exchange rate. In the presence of even a single Granger-causing variable for the real exchange rate, the AR(1) model is clearly inappropriate.

2.2 The Case of a Non-Diagonal Covariance Matrix, $\sigma_{12} \neq 0$

In this case, the error term, u_{1t} , in the first equation of the VAR(1) does not coincide with the innovations driving y_{1t} . Following standard practice, we restore the orthogonality of the errors by utilizing the Cholesky decomposition of Σ_u , that is $\Sigma_u = PP'$, where P is a lower triangular matrix. After some algebra, we obtain the following representation for Y_t :

$$\begin{aligned} y_{1t} &= a_{11}y_{1t-1} + a_{12}y_{2t-1} + v_{1t} \\ y_{2t} &= \frac{\sigma_{12}}{\sigma_{11}}y_{1t} + (a_{21} - \frac{\sigma_{12}}{\sigma_{11}}a_{11})y_{1t-1} + (a_{22} - \frac{\sigma_{12}}{\sigma_{11}}a_{12})y_{2t-1} + v_{2t} \end{aligned} \quad (4)$$

where $V_t = \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} = \begin{bmatrix} u_{1t} \\ u_{2t} - \frac{\sigma_{12}}{\sigma_{11}}u_{1t} \end{bmatrix}$ with covariance matrix

$$\Sigma_V = \begin{bmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}} \end{bmatrix}. \text{ This particular representation was obtained}$$

by assuming that the y_{1t} is causally prior to y_{2t} . This means that the current values of y_{1t} do not react contemporaneously to changes in y_{2t} . The error term, v_{1t} , in the first equation of (4) is orthogonal to y_{1t-1} and y_{2t-1} , that is it can be thought of as summarizing all the other factors that contribute to the variability of y_{1t} , apart from y_{1t-1} and y_{2t-1} . Based on (4), we obtain the following infinite MA representation of Y_t :

$$Y_t = \sum_{i=0}^{\infty} \Theta_i W_{t-i}$$

where $\Theta_i = \Phi_i P$ and $W_t = (w_{1t} \ w_{2t})' = P^{-1}U_t$ ¹².

We now define the Impulse Response Function, IRF_{mo} , of y_{1t} to be:

$$IRF_{mo}(k) = \frac{\theta_{11,k}}{\sqrt{\sigma_{11}}}$$

¹²By construction, the variance-covariance matrix of W_t is $\Sigma_W = I_2$.

where $\theta_{11,k}$ is the upper left element of Θ_k . By definition, $IRF_{mo}(k)$ is the response of y_{1t} to a unit shock in its innovations, v_{1t} , after k periods. Therefore, $IRF_{mo}(k)$ is directly comparable to $IRF_u(k)$. The following proposition holds:

Proposition 3 *In general, $IRF_{mo}(k) \neq IRF_u(k)$ for some finite k .*

Proof. It is straightforward to show that $IRF_{mo}(1) = a_{11} + a_{12}\frac{\sigma_{12}}{\sigma_{11}}$, which is in general different than $IRF_u(1) = a_{11} + a_{22} + \gamma_1$. Similarly, $IRF_{mo}(2) = a_{11}^2 + a_{11}a_{12}\frac{\sigma_{12}}{\sigma_{11}} + a_{12}a_{21} + a_{12}a_{22}\frac{\sigma_{12}}{\sigma_{11}}$, whereas $IRF_u(2) = (a_{11} + a_{22})(\gamma_1 + a_{11} + a_{22}) - a_{11}a_{22} + a_{21}a_{12}$. Similar results are obtained for $k > 2$. Therefore, in general, $IRF_u(k) \neq IRF_{mo}(k)$. ■

The following lemma provides the sufficient condition to obtain equivalence of $IRF_{mo}(k)$ and $IRF_u(k)$.

Lemma 2 *When $a_{12} = 0$, $IRF_u(k) = IRF_{mo}(k)$ for every $k \geq 0$.*

Proof. We have already shown that when $a_{12} = 0$, $IRF_u(k) = a_{11}^k$, $k \geq 0$. In addition, Φ_k is of the form: $\begin{bmatrix} a_{11}^k & 0 \\ q_1 & a_{22}^k \end{bmatrix}$ (see lemma 1), where q_1 is a function of a_{ij} . Given that P is lower triangular, it is easy to show that $\Theta_k = \Phi_k P$ has the following form: $\Theta_k = \begin{bmatrix} a_{11}^k \sqrt{\sigma_{11}} & 0 \\ q_1 & q_2 \end{bmatrix}$, where q_1 and q_2 are functions of a_{ij} and σ_{ij} , $i, j = 1, 2$. Thus, $IRF_{mo}(k) = \frac{\theta_{11,k}}{\sqrt{\sigma_{11}}} = a_{11}^k = IRF_u(k)$. ■

3 Sticky prices and fundamental determinants of PPP deviations

3.1 Relative output gaps and nominal interest rate differentials

While real disturbances, such as changes in tastes and technology, are likely to explain long-term changes in the real exchange rate, medium- and short-term changes are more likely to reflect monetary or aggregate demand shocks. Such shocks can have substantial effects on the real economy in the presence of short-term nominal price rigidities. This is a central feature of the sticky-price monetary model, such as Dornbusch (1976). In this model, monetary disturbances lead to overshooting of the exchange rate due to short-term price stickiness. During the adjustment to long-term equilibrium, deviations from PPP are related to excess demand in the goods market. This excess demand arises both from (a) the decline in domestic interest rates relative to world interest rates (both in real and nominal terms), which fuel domestic demand, and (b) the depreciation of the real exchange rate, which lowers the relative price of domestic goods. The lower interest rate and the lower relative price of domestic goods lead to excess demand, causing

domestic prices to rise and real money balances to decline. As a result, interest rates increase and the currency appreciates in real terms, restoring the initial real equilibrium of the economy.

Sticky price models of exchange rate determination combine the uncovered interest parity relationship with the assumption that inflation is related to excess demand (output gap) in the goods market:

$$E_t s_{t+\tau} - s_t = i_{t,t+\tau} - i_{t,t+\tau}^* \quad (5)$$

$$E_t p_{t+\tau} - p_t = \varphi^\tau x_t \quad (6)$$

$$E_t p_{t+\tau}^* - p_t^* = \varphi^\tau x_t^* \quad (7)$$

where $i_{t,t+\tau}$ ($i_{t,t+\tau}^*$) is the domestic (foreign) nominal interest rate at time t of a pure discount bond with $\tau \geq 1$ terms to maturity, x_t (x_t^*) is the domestic (foreign) output gap, defined as the deviation of output from potential, E_t is the conditional expectations operator, given information up to time t and $\varphi > 0$ is a constant.¹³

Assuming adaptive expectations as in Dornbusch (1976, 1989) and Meese and Rogoff (1988), i.e.

$$E_t y_{1t+\tau} = b^\tau y_{1t+1} \quad (8)$$

where $0 < b < 1$, we obtain from equations (5)-(8):

$$y_{1t} = -\frac{1}{(1-b^\tau)}(i_{t,t+\tau} - i_{t,t+\tau}^*) + \frac{\varphi^\tau}{(1-b^\tau)}(x_t - x_t^*) \quad (9)$$

Equation (9) describes the disequilibrium dynamics of the sticky-price monetary model after a monetary (or aggregate demand) shock. According to equation (9), deviations from PPP are negatively related to the nominal interest rate differential and positively related to the relative business cycle position of the economy, measured by the relative output gap between the domestic and the foreign economy. Since output gaps are difficult to measure in practice, we use in our empirical analysis the real GDP growth differential

¹³Note that in the original Dornbusch (1976) model, the output gap determines realized inflation. However, since the Dornbusch model is deterministic, realized and expected inflation are identical. Furthermore, we have added an inflation equation for the foreign country, equation (7), in order to endogenize the foreign price level, which is typically treated as exogenous in the standard small-country model.

between the domestic and the foreign economy instead of $(x_t - x_t^*)$. If both the domestic and the foreign economy converge to a long-run equilibrium $x_t = x_t^* = 0$, then real GDP growth ought to be negatively related to the output gap. As a result, deviations from PPP will be negatively correlated with the GDP growth differential between the domestic and the foreign economy. In other words, the currency will tend to appreciate in real terms in countries with higher GDP growth.

3.2 Real interest rate differentials

An alternative representation of the deviation from PPP in the sticky price monetary model is in terms of the real interest rate differential.¹⁴ Sticky price models of exchange rate determination combine the uncovered interest parity relationship, equation (5), with the assumption that deviations of the real exchange rate from PPP are only temporary, equation (8). Under these assumptions, shocks to the real exchange rate are expected to reverse themselves over time. During the adjustment of real exchange rates to PPP, real interest rate differentials must be equal to the expected real depreciation of the currency in order to guarantee arbitrage equilibrium between domestic and foreign bond markets. For example, if the British pound real exchange rate is below its long-run level vis-a-vis the US dollar (i.e. the pound is overvalued relative to PPP), the pound is expected to depreciate in real terms in the future. Arbitrage equilibrium in the bond market requires that the expected real yield differential between UK and US bonds with the same term to maturity should be equal to the expected real depreciation of the pound over the term of the bonds. If the expected real depreciation is related to the deviation of the real exchange rate from PPP, then real interest rate differentials must also be related to deviations from PPP. This relationship can be easily obtained from equations (5) and (8). Subtracting expected inflation, $E_t p_{t+\tau} - p_t$, from both sides of equation (5), gives the real interest parity condition:

$$E_t y_{1t+\tau} - y_{1t} = r_{t,t+\tau} - r_{t,t+\tau}^* \quad (10)$$

where $r_{t,t+\tau}(r_{t,t+\tau}^*)$ is the expected real interest rate between time t and time $t + \tau$. Then, from (8) and (10), we obtain:

$$y_{1t} = -\frac{1}{(1 - b\tau)}(r_{t,t+\tau} - r_{t,t+\tau}^*) \quad (11)$$

The theory predicts that the relationship between y_{1t} and $r_{t,t+\tau} - r_{t,t+\tau}^*$ is negative: a positive deviation from PPP is related to a negative real interest rate differential. In other words, when domestic real interest rates

¹⁴See Dornbusch (1989), Meese and Rogoff (1988) and Baxter (1994). A similar relationship is also derived in Frankel (1979).

are lower than foreign ones, investors must anticipate a real appreciation of the domestic currency in order to be indifferent between domestic and foreign bonds. If deviations from PPP are transitory, as implied by equation (8), then the real exchange rate must exceed its PPP equilibrium, i.e. $y_{1t} > 0$, in order to generate the expectation of a real appreciation in the future.

In summary, economic theory suggests that when prices are sticky, deviations from PPP are related to economic variables such as real and nominal interest rate differentials and the relative business cycle position of the economy, proxied by the output gap or the real GDP growth differential.

4 Empirical results

4.1 Data

Our empirical analysis is based on post-1973, quarterly, real exchange rates for a number of major industrialized countries. Data for nominal exchange rates, consumer prices, long-term interest rates and real GDP are collected from International Financial Statistics (IFS CD-Rom, March 2002).¹⁵ The business cycle position relative to the US is proxied by the 4-quarter real GDP growth differential between the home country and the US.

We consider four country pairs, with the US serving as the foreign country. The domestic country is represented by France, Germany, Italy and the UK. The bilateral real exchange rate is measured as the nominal exchange rate, defined in units of domestic currency per dollar, multiplied by the ratio between the US and the domestic consumer price index. Figures 1a-1d present the relevant series.

Long-term interest rates are yields to maturity of 10-15 year government bonds. Ex ante returns on long-term bonds are difficult to compute since this requires a measure of expected inflation over the term of the bond. While these long-term inflation forecasts can be easily generated from time series models or filtering techniques, a drawback of these methods is that they produce time series for expected inflation that are very smooth, compared to realized inflation. An alternative method of computing real interest rates is to use past year realized inflation. In order to compute the real interest rate, we subtract consumer price inflation over the past four quarters from the nominal yield. Although this method of computing real interest rates is not entirely satisfactory, since inflation is not measured over the term of the bond, it avoids problems of overlapping observations, compared with the method of computing true ex post real interest rates.

¹⁵Nominal exchange rate: line ae.zf, long-term interest rate: line 61...zf, CPI: line 64...zf, real GDP: line 99BVRZF.

4.2 Unit root tests

The presence of a unit root in the real exchange rate seems to terminate the discussion on measuring the half-life of PPP, since in such a case there is no convergence toward PPP at all. However, inferences on the presence of unit roots in real exchange rates depend heavily on both the testing strategy and the sample employed. For example, Huizinga (1987) and Meese and Rogoff (1988) fail to reject the unit root null by means of standard unit root tests for the post-1973 period. The notorious low power of these tests may of course be the sole reason for not rejecting the null.¹⁶ On the other hand, when longer-run time series are employed, blending fixed and floating rate data, the unit root hypothesis is rejected.¹⁷ Similar evidence is obtained when the post-1973 data are expanded cross-sectionally, by means of panel data methods.¹⁸ In the present case, the results from a variety of unit-root tests are, as usual, mixed.¹⁹ When the null hypothesis of stationarity is tested, the KPSS test fail to reject the null for the real exchange rates as well as the other macroeconomic variables for all the countries under consideration. When the null hypothesis of a unit root is tested, the standard Dickey-Fuller (DF) or Phillips-Perron (PP) tests typically fail to reject the null. The GLS versions of the DF tests, however, being more powerful than the standard DF tests, reject the unit root null in many cases.

The general picture emerging from the empirical literature and our own tests suggests treating the real exchange rates and the macroeconomic variables as having a highly persistent but ultimately stationary univariate representation. As already mentioned, in the opposite case there would be no point in estimating half-lives of real exchange rate innovations since they would be infinite by definition. Our aim is to examine whether the real exchange rate is less persistent when we account for the effect of macroeconomic variables which proxy for nominal price rigidities.

¹⁶Some recent results by Taylor (2001) forcefully point toward the ‘low-power’ interpretation of not rejecting the unit root null. Specifically, sampling the data at low frequencies makes it impossible to identify an adjustment process occurring at high frequencies, thus producing the false impression of long or even infinite half-lives. In another recent paper, Imbs et al. (2002) show that estimates of persistence of real exchange rates suffer from a positive cross-sectional aggregation bias.

¹⁷See, for example, Abuaf and Jorion (1990), Frankel (1990), Cheung and Lai (1994), and Lothian and Taylor (1996).

¹⁸See, for example, Wei and Parsley (1995), Frankel and Rose (1996), Higgins and Zakrajsek (1999).

¹⁹The unit root null is tested by means of the following tests: the standard Dickey-Fuller tests, the Dickey-Fuller test with GLS detrending (Elliott *et al.* (1996)), the Point Optimal test (Elliott *et al.* (1996)), the Phillips-Perron test (Phillips and Perron (1988)) and the Ng-Perron test (Ng and Perron (2001)). The stationary null hypothesis is tested by means of the KPSS test (Kwiatkowski *et al.* (1992)). Results are available upon request.

4.3 Univariate models

The majority of studies employ the simplest univariate model, that is an AR(1), to estimate the half-life of deviations from PPP. Taylor (2001) refers to this as the ‘basic model’ in order to highlight the unanimity concerning the choices of models for the real exchange rate. However, there are neither theoretical nor statistical reasons that dictate this choice, over more general ARMA(p,q) models. In order, however, to relate our results to those of the existing literature, we begin our analysis by estimating an AR(1) model for each country under consideration. The estimation results along with the half-life estimates and their confidence intervals are reported in Table 1.

[insert Table 1]

It can be seen that the half-life estimates range from 9 to 14 quarters (2.25 years to 3.5 years). The shortest half-life corresponds to the UK pound, while the longest one to the German mark. The mean half-life for the four pairs of countries examined is 12.25 quarters which is very close to the estimate of Abuaf and Jorion (1990) for eight series of real exchange rates.

However, all the four AR(1) models, presented above are not adequate representations of the corresponding real exchange rates, since serial correlation problems are encountered. In such cases, the half-life estimates are inconsistent since they are based on inconsistent estimates of the autoregressive coefficients. This in turn implies that if we wish, for some unknown reasons, to remain within the univariate framework, we must at least choose the correct model, by means of standard model-selection criteria. We consider fourteen ARMA(p,q) models with $p=1,\dots,4$, $q=1,2$ and select p and q by means of the Schwartz Information Criterion (SIC). Table 2 reports the half-life estimates, calculated from the impulse response function of the selected model, along with their confidence intervals.

[insert Table 2]

The results suggest that half-lives are generally lower than in the AR(1) case, though not considerably: the mean half-life for the five pairs of countries is 10.75 quarters, compared to an estimate of 12.25 quarters from the AR(1) models.

So far, we have estimated half-lives of real exchange rate innovations based on univariate models, thus ignoring the interactions of real exchange rates with other macro-economic variables. The results of Section 2 have shown that the impulse response analysis within univariate models is, in general, not equivalent to the impulse response analysis within multivariate models, even if the univariate models are correctly specified. Therefore,

we proceed to estimate the half-life of PPP deviations within multivariate models.

4.4 Multivariate models

In order to assess the role of price-stickiness in determining the degree of persistence of the real exchange rate, we estimate VAR models in the real exchange rate and the set of macroeconomic models suggested by the sticky-price model. These variables include real or nominal interest rate differentials and GDP growth differentials between the home country and the US. To select the appropriate multivariate model for each country, we proceed along the lines of the ‘general-to-specific’ methodology. Specifically, we start with a general VAR(1) model containing all the candidate variables and then we end up with a parsimonious VAR(1) specification by excluding insignificant variables, i.e. variables that do not ‘Granger cause’ the real exchange rate.²⁰ The estimated VAR models for each country are presented in Tables 3-6.

[Insert Tables 3-6]

Our estimates suggest that deviations from PPP are significantly related to the set of macroeconomic variables, proposed by standard sticky-price models of the exchange rate. With the exception of Germany – where real GDP data are highly distorted due to the effect of unification in 1990 –, GDP growth differentials are in all countries significant determinants of real exchange rates. An increase in real GDP growth relative to the US is related to a real appreciation of the home currency both in the short-term and the long-term, in line with the theoretical predictions. Long-term interest rate differentials with the US are also an important determinant of real exchange rates. Our estimates suggest that in three out of four countries (France, Italy and UK), an increase in the real interest rate differential with the US is related to a real appreciation of the domestic currency. In Germany, we find that nominal long-term interest rate differentials are important in explaining deviations from PPP. As predicted by theory, an increase in the German nominal interest rate relative to the US is related to a real appreciation of the deutchmark.

As shown in Section 2 (Proposition 1 and Lemma 1), estimates of impulse response functions, and, hence, half-lives of deviations from PPP, are, in general, different in the context of a VAR, compared to estimates of univariate models. A condition for this to occur, is that (at least one of) the variables

²⁰The first-order VAR was found to be statistically adequate for all the countries under consideration.

included in the VAR Granger cause(s) the real exchange rate. This condition can be tested using the standard t-test to assess the significance of the coefficients of macro-variables in the real exchange rate equation. The results reported in Tables 3-6 suggest that this condition is satisfied in all countries, providing evidence that estimates of half-life in multivariate models will differ from those in univariate models. Given that the macro-variables included in the VAR are those proposed by standard sticky-price models of the exchange rate, our results provide evidence that price-stickiness can partly account for the persistence of the real exchange rate.

Before proceeding to the calculation of the half-life of deviations from PPP in the VAR model, we test whether the contemporaneous correlation between innovations in the real exchange rate and other variables in each multivariate model is statistically different from zero. The importance of this condition was already discussed in Section 2. In the case of a zero correlation, we can compute half-life using the original VAR innovations, otherwise our calculations should be based on the orthogonal transformation of the VAR innovations. In order to test this assumption, we estimated both a restricted and an unrestricted model and computed the Likelihood Ratio (LR) statistic. The results, reported in Table 7, suggest that the orthogonality restriction, i.e. zero contemporaneous correlation between innovations in the real exchange rate and other variables included in the VAR holds in all countries, but France.

[Insert Table 7]

We now proceed to examine the dynamic characteristics of the system by examining the impulse response functions. We employ responses to a unit shock in the cases of Germany, Italy and the UK, where the orthogonality restriction between innovations in the real exchange rate and other variables is satisfied. In the case of France, we employ orthogonal impulse responses, since the orthogonality restriction was rejected. It is important to note that when orthogonal IRFs are considered, these are dependent on the ordering of the variables. To ensure comparability of multivariate IRFs with univariate IRFs, the real exchange rate is the first variable in the VAR. The IRFs for each of the countries are displayed in Figures 2a-2d. Estimated half-lives along with their 95% asymptotic confidence intervals are presented in Table 8. In order to account for small sample effects, we also report Monte Carlo estimates of confidence intervals along with asymptotic ones.

[insert table 8]

Our results reported in Table 8 suggest that estimates of half-lives of deviations from PPP in the context of multivariate models are substantially lower than those of univariate models for all the countries considered. For example, the half-life for Germany reduces to 6 quarters from 12 quarters and in Italy to 7 quarters from 11 quarters. The mean half-life across the four country pairs is 7 quarters, compared with an average of 12.25 quarters from the AR(1) models and 10.75 quarters from the ARMA models. This suggests that macroeconomic variables proposed by the sticky-price model account for a substantial fraction of the half-life of PPP deviations. The difference between the ARMA estimate of half-life, HL_u (as reported in Table 2), and the VAR estimate of half-life, HL_m , is 3.75 quarters, in line with estimates of persistence of real exchange rates from calibrated international business cycle models with nominal price rigidities such as Chari et al. (2002). The remaining seven quarters of the half-life of deviations from PPP can be attributed to other (unspecified) sources of persistence, such as shocks to tastes and technology, which are less frequent but more persistent than monetary shocks. By comparing the half-life estimates of the multivariate models with the half-life estimates of their equivalent univariate representations, we can compute the fraction of half-life attributable to the set of macroeconomic variables included in the VAR as $(HL_u - HL_m)/HL_u$. As reported in the last column of Table 8, the fraction of half-life due to price-stickiness ranges from 22% in the UK to 50% in Germany, with an average across the four country-pairs of 33%.

The 95% confidence intervals of half-lives are considerably tighter than in the univariate context, suggesting that our estimates of half-life are more precise. The lower bound of the confidence intervals is estimated at four quarters for all country pairs, compared with 9-12 quarters in the univariate models. The upper bounds range from 11 to 22 quarters, compared to 16-23 in the univariate models. Interestingly, the Monte Carlo confidence intervals are tighter than those based on the asymptotic distribution of the impulse response function. It is important to note that our estimates break the consensus view at the lower end of its range without accounting for a series of potential econometric pitfalls, such as temporal aggregation bias,²¹ nonlinear adjustment²² or cross-sectional aggregation bias.²³ Correcting for these econometric issues would certainly reduce estimated half-lives even further.

²¹For an extensive analysis of temporal aggregation bias in half-life estimates see Taylor (2001).

²²See, for instance, Michael et al. (1997), Taylor and Peel (2000) and Taylor (2001).

²³See, for instance, Imbs et al. (2002).

5 Conclusions

In this paper, we address the Purchasing Power Parity Puzzle in the context of a Vector Autoregressive model, where the real exchange rate is allowed to interact with a set of macroeconomic variables, suggested by ‘sticky-price’ theories of exchange rate determination. By doing this, we are able to discern the relative effect of these variables on the speed of adjustment of the real exchange rate towards long-run PPP. We first show that the impulse response function of a variable participating in the VAR is not, in general, the same with the impulse response function obtained from the equivalent ARMA representation of this variable, if the latter is Granger caused by other variables in the system. The difference between the two impulse response functions captures the effects of the Granger-causing variables on the dynamic adjustment process of the variable of interest. We investigate the implications of our analytical results for the speed of adjustment of four real exchange rates vis-a-vis the US dollar (French franc, German mark, Italian lira and UK pound) during the post Bretton Woods period. In order to capture the effect of price stickiness on the speed of adjustment of the real exchange rate towards long-run PPP, we include in the VAR macroeconomic variables such as real or nominal interest rate differentials and GDP growth differentials, as suggested by the sticky-price monetary model. Our empirical results suggest that real exchange rates are in fact Granger caused by these variables. As a consequence of this causality, the adjustment horizons of deviations from PPP decrease to levels that hardly produce a puzzle. The average half-life estimate across the four pairs of real exchange rates is below two years, suggesting that price stickiness accounts for a substantial fraction of deviations from PPP. Comparing the half-life estimates of the univariate models with the half-life estimates of the VAR, we conclude that between 22% and 50% of the half-life of deviations from PPP is due to nominal price rigidities.

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Appendix
Proof of Remark 1

prove the remark by induction.

For $d = 1$, $A^d = A = \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix}$, which is of the form: $\begin{bmatrix} a_{11}^d & 0 \\ q_1 & a_{22}^d \end{bmatrix}$
with $q_1 = a_{21}$.

Assume that $A^d = \begin{bmatrix} a_{11}^d & 0 \\ q_1 & a_{22}^d \end{bmatrix}$, where q_1 is a function of a_{ij} . Then, we
must show that $A^{d+1} = \begin{bmatrix} a_{11}^{d+1} & 0 \\ q_1' & a_{22}^{d+1} \end{bmatrix}$. Now,

$$A^{d+1} = A^d A = \begin{bmatrix} a_{11}^d & 0 \\ q_1 & a_{22}^d \end{bmatrix} \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11}^{d+1} & 0 \\ a_{11}q_1 + a_{21}a_{22}^d & a_{22}^{d+1} \end{bmatrix},$$

which is of the form: $\begin{bmatrix} a_{11}^{d+1} & 0 \\ q_1' & a_{22}^{d+1} \end{bmatrix}$.

Table 1: Estimated Half-lives and 95% Confidence Intervals of AR(1) Models

	ar(1)	HL _u	95% Confidence Intervals				Adj. R ²
			Asymptotic		Monte Carlo		
			Lower	Upper	Lower	Upper	
France	0.947	13	6	35	5	27	0.898
Germany	0.948	14	6	37	5	30	0.900
Italy	0.945	13	5	34	5	26	0.893
UK	0.924	9	5	21	4	18	0.861
Average	0.944	12.25	5.5	33.4	5	27.2	

Notes: ar(1): estimate of autoregressive coefficient. HL_u: estimate of half-life. Data are quarterly from 1973:Q1 to 1998:Q4 for France, Germany and Italy and 1973:Q1 to 2001:Q4 for the UK.

Table 2: Estimated Half-lives and 95% Confidence Intervals of ARMA(p,q) Models

	Model	HL _u	95% Confidence Intervals				Adj. R ²
			Asymptotic		Monte Carlo		
			Lower	Upper	Lower	Upper	
France	AR(2)	11	6	21	5	18	0.911
Germany	AR(4)	12	7	21	6	19	0.918
Italy	ARMA(1,1)	11	5	23	5	20	0.903
UK	ARMA(4,4)	9	5	16	4	19	0.883
Average		10.75	6.2	22.8	5.4	21	

Notes: HL_u: estimate of half-life.

Table 3: VAR Estimates (France)

Variable	<i>c</i>	$y_1(-1)$	$y_2(-1)$	$y_3(-1)$	Adj. R ²
y_1	0.242 (0.071)	0.861 (0.040)	-0.859 (0.320)	-0.389 (0.194)	0.917
y_2	0.001 (0.013)	-0.0004 (0.008)	0.891 (0.060)	0.013 (0.037)	0.808
y_3	-0.006 (0.020)	0.002 (0.012)	0.032 (0.092)	0.838 (0.056)	0.728

Notes: y_1 : real exchange rate, y_2 : real long term interest rate differential, y_3 : real GDP growth differential. Standard errors in parentheses below coefficient estimates.

Table 4: VAR Estimates (Germany)

Variable	c	$y_1(-1)$	$y_2(-1)$	Adj. R^2
y_1	0.048 (0.018)	0.891 (0.035)	-0.881 (0.286)	0.909
y_2	-0.004 (0.002)	0.005 (0.004)	0.974 (0.031)	0.928

Notes: y_1 : real exchange rate, y_2 : nominal long-term interest rate differential. Standard errors in parentheses below coefficient estimates.

Table 5: VAR Estimates (Italy)

Variable	c	$y_1(-1)$	$y_2(-1)$	$y_3(-1)$	Adj. R^2
y_1	0.793 (0.316)	0.892 (0.042)	-0.285 (0.218)	-0.565 (0.193)	0.904
y_2	0.241 (0.075)	-0.033 (0.010)	0.784 (0.052)	-0.143 (0.046)	0.854
y_3	-0.181 (0.095)	0.024 (0.013)	0.161 (0.065)	0.831 (0.058)	0.699

Notes: y_1 : real exchange rate, y_2 : real long term interest rate differential, y_3 : real GDP growth differential. Standard errors in parentheses below coefficient estimates.

Table 6: VAR Estimates (UK)

Variable	c	$y_1(-1)$	$y_2(-1)$	$y_3(-1)$	Adj. R^2
y_1	-0.047 (0.016)	0.904 (0.035)	-0.445 (0.188)	-0.426 (0.226)	0.871
y_2	-0.047 (0.005)	-0.006 (0.009)	0.849 (0.051)	-0.183 (0.061)	0.736
y_3	0.008 (0.004)	0.024 (0.010)	0.183 (0.055)	0.659 (0.066)	0.543

Notes: y_1 : real exchange rate, y_2 : real long term interest rate differential, y_3 : real GDP growth differential. Standard errors in parentheses below coefficient estimates.

Table 7: Orthogonality Restrictions

	Log Likelihood			
	Unrestricted	Restricted	LR-statistic	p-value
France	-109.617	-114.902	10.571	0.005
Germany	86.33	85.66	1.341	0.247
Italy	-154.555	-156.557	4.003	0.135
UK	-191.866	-194.109	4.487	0.106

Note: The Table tests the restriction that the covariance between innovations of the real exchange rate and innovations of the other variables included in the VAR is zero.

Table 8: Estimated Half-lives and 95% Confidence Intervals of VAR(1) Models

HL _m	95% Confidence Intervals				Ratio	
	Asymptotic		Monte Carlo			
	Lower	Upper	Lower	Upper		
France	8	4	21	4	17	0.27
Germany	6	4	13	3	12	0.50
Italy	7	4	30	3	22	0.36
UK	7	4	22	4	17	0.22
Average	7.4	4	20.2	3.6	16	0.34

Notes: HL_m: half-life. Ratio: fraction of half-life due to price stickiness, computed as $(HL_u - HL_m)/HL_u$, where HL_m is the half-life estimate of the VAR model and HL_u is the half-life estimate of the univariate ARMA(p,q) model, as reported in Table 2. Estimates for France are based on orthogonalized innovations (see text).

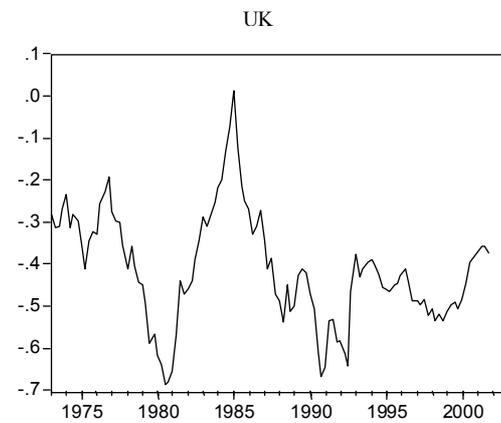
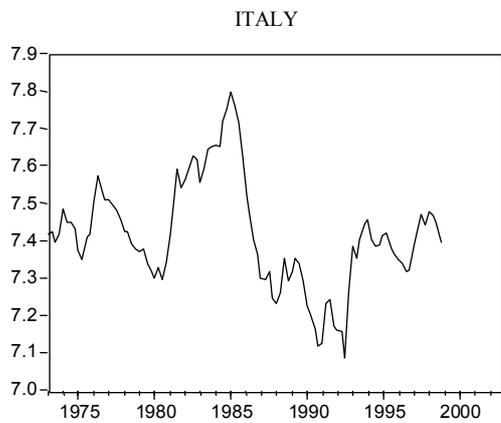
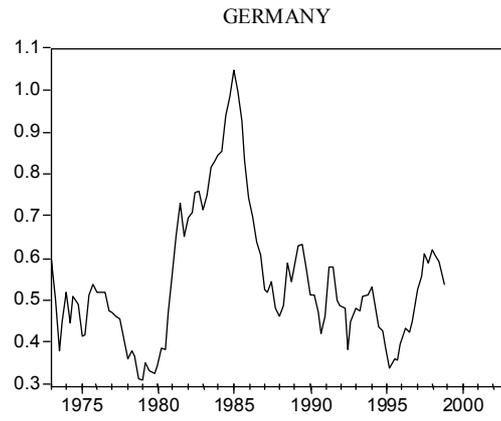
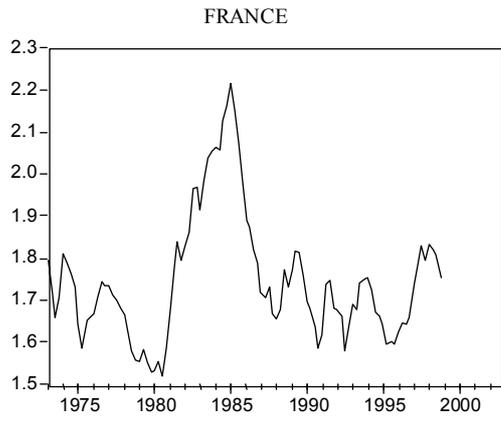


Figure 1: Real Exchange Rates

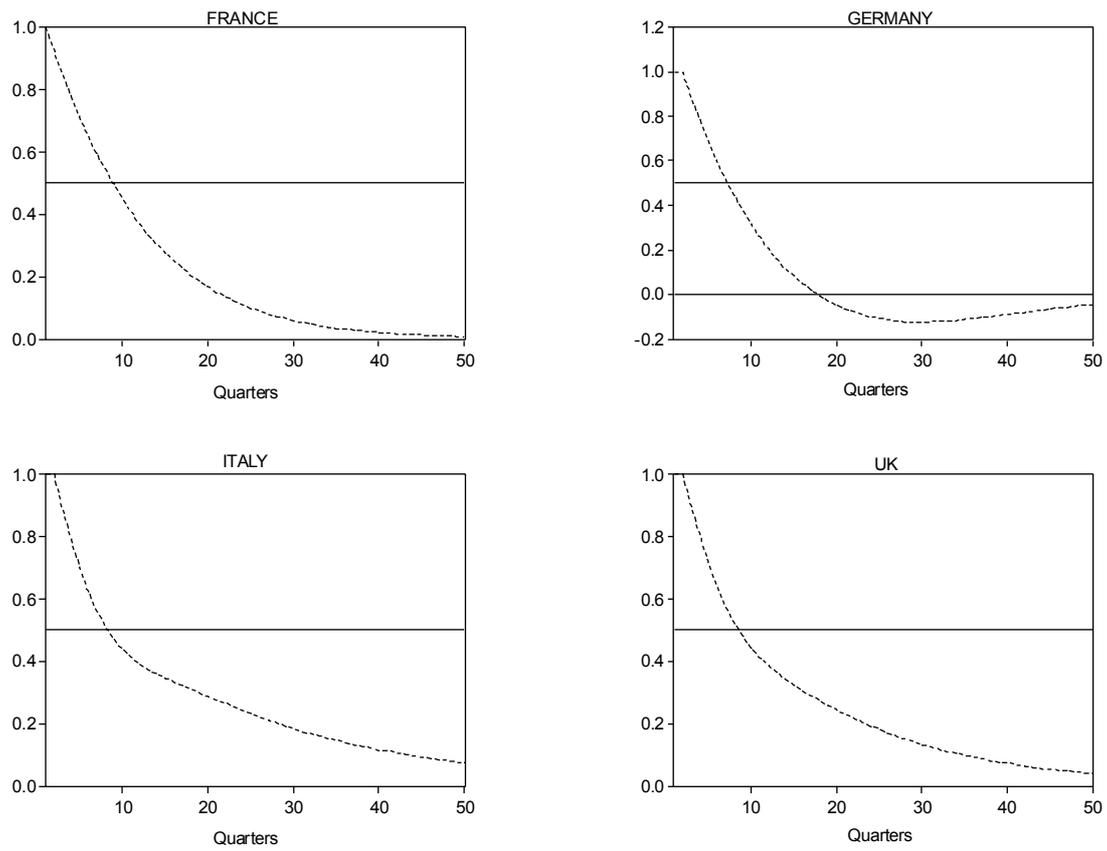


Figure 2: Impulse Responses to a Unit Shock