

# Optimal Real Exchange Rate Targeting: A Stochastic Analysis

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## Abstract

This paper extends the literature on real exchange rate targeting inside a stochastic optimization framework where the real exchange rate displays long run mean reversion while temporarily reflecting a “liquidity effect”. When real exchange rate volatility is constant, an active stabilization rule is welfare increasing with respect to non intervention only beyond a given volatility threshold. Moreover, the welfare gains are larger the lower is the degree of mean reversion. Under a stochastic volatility assumption, the policy maker’s intertemporal discount rate has instead a major influence, and real exchange rate targeting is welfare increasing only if the policy-maker is sufficiently farsighted.

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# 1 Introduction

Real exchange rate targeting is one of the most popular economic policy strategies among developing countries. A glance at the historical records of these countries reveals that this targeting rule has usually been implemented in two different ways. A former approach, commonly known as “purchasing power parity (PPP) rule”, aims at preserving a constant level of external competitiveness: the nominal exchange rate is thus continuously depreciated in order to offset any positive difference between domestic and foreign inflation rates. In a latter, more aggressive version, this policy rule prescribes instead a more depreciated level of the real exchange rate, letting the nominal exchange rate to overreact in the face of domestic or external shocks.

Typical examples of real exchange rate targeting policies are offered by many Latin American countries starting with the early Brazilian experience since the late 60’s; the empirical evidence, on the other hand, provides apparently robust support to active exchange rate management policies of this kind.<sup>1</sup>

Although enjoying a considerable popularity, real exchange rate targeting is clearly at odds with some traditional policy prescriptions. According to the current consensus view, either the nominal exchange rate or the money supply should act as “nominal anchors” in an open economy, in order to provide a satisfactory degree of macroeconomic stability. Given a nominal anchor, and assuming that policy makers are credible and that the appropriate structural policies are implemented, price stability should ensure convergence to a long run growth rate consistent with the economy’s potential.

Under a real exchange rate targeting rule, by contrast, both the nominal exchange rate and the money supply become endogenous variables, thus removing any nominal anchor to tie down the domestic price level. In this perspective, starting from Dornbusch (1982) seminal paper, many authors have pointed out that PPP exchange rate rules (as well as policies designed to achieve a more depreciated level of the real exchange rate) are likely to generate serious destabilizing effects, while the goal of preserving a satisfactory level of external competitiveness can only temporarily be achieved.

This paper takes a fresh perspective on the current debate about real exchange rate targeting revisiting this topic inside an infinite-horizon, continuous time, dynamic optimization model. Our theoretical framework shares some basic insights with previous work. First, we assume that monetary policy can only temporarily affect the real exchange rate, while retaining monetary neutrality as a long run equilibrium condition. Second, we recognize that an active exchange rate management policy is bound to raise various destabilizing effects,

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<sup>1</sup>Calvo *et al.* (1995) provide a comprehensive account of real exchange rate targeting policies in various Latin American countries (Brazil, Chile, and Colombia). Turning to the empirical evidence, the relevance of PPP rules to prevent negative effects of real exchange rate volatility on export performance has been widely documented. Sekkat and Varoudakis (2000), for instance, detect a favourable impact of an active exchange rate management policy for a panel of major Sub-Saharan African countries over the period 1970-92. A similar conclusion is reached in Arize *et al.* (2000), applying cointegration techniques to a sample of thirteen developing countries over the period 1973-86.

and explicitly account for this instability in our theoretical model.

Although reflecting some distinctive features outlined in previous research, this paper innovates upon the existing literature in two main respects.

A former innovation is that we draw explicit attention to the policy-maker's incentive to preserve a satisfactory degree of external competitiveness. We model this incentive inside a dynamic optimization framework, where deviations of the current real exchange rate from a pre-specified equilibrium target appear as a relevant argument in the policy-maker's social welfare function. This modelling approach significantly departs from previous research, which either lacks explicit microeconomic foundations or solves an optimization problem from the standpoint of a representative consumer maximizing his lifetime utility over a basket of (tradeable and nontradeable) goods. We regard our modelling approach as more promising since real exchange rate targeting, rather than being artificially superimposed on a given macroeconomic model, is endogenously derived as an optimal policy strategy for a small open economy largely dependent on external demand. A further advantage of this approach is that it allows a more consistent evaluation of the basic policy tradeoff implied by a PPP rule, where the gain in real flexibility involves substantial macroeconomic instability.

A latter innovation of this paper regards the overall analytical framework to evaluate a policy of real exchange rate targeting. Whereas most theoretical contributions addressing the macroeconomic effects of PPP rules hinge upon deterministic models,<sup>2</sup> this paper relies on a stochastic optimal control approach. More specifically, we provide a rich characterization of the stochastic disturbances impinging upon the real exchange rate, assuming that this variable is governed by a mean-reverting process with state dependent volatility. This assumption has strong motivations on empirical grounds. A large body of empirical evidence supports mean reversion in real exchange rates (see, for instance, Froot and Rogoff, 1995), while recent contributions suggest that half-lives of PPP deviations are actually lower than was previously thought (see, among others, Taylor *et al.*, 2001). As regards stochastic volatility, this assumption reflects some empirical regularities exhibited by PPP deviations (high degree of persistence and high short run volatility) while being supported by a large strand of econometric work applying GARCH models to real exchange rate data.

The remainder of this paper is structured as follows. Section 2 sets the stage for the subsequent analysis, providing an overview of theoretical models addressing the macroeconomic implications of real exchange rate targeting. Section 3 outlines our theoretical model, assuming that the real exchange rate can temporarily be controlled through variations in the nominal interest rate. Section 4 solves the policy-maker's optimization problem, deriving an active monetary policy rule and characterizing its main properties. Drawing on this analysis, Section 5 performs a welfare comparison between this active stabilization rule and a non intervention scenario where the policy-maker keeps the nominal interest rate constant over all periods. Section 6 concludes.

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<sup>2</sup>Two relevant exceptions include Adams and Gros (1986) (Section 2), where the role of stochastic shocks in the inflationary process is examined, and Uribe (2003) (Section 4) where the theoretical model is extended to allow for stochastic PPP rules.

## 2 Real exchange rate targeting: an overview of the theoretical literature

This section provides a brief overview about the theoretical literature on real exchange rate targeting. The main purpose is to set the stage for our subsequent analysis, where the scope for a policy of this kind will be discussed inside a stochastic optimal control approach.

The literature on real exchange rate targeting focuses on two basic topics: the former relates to its potentially inflationary effects, while the latter concerns the extent to which a tight credit policy may provide a nominal anchor under imperfect asset substitutability.<sup>3</sup>

The consensus view on the former issue is quite strong, although the actual channels through which an inflationary upsurge is generated widely differ across various theoretical models. Turning to the latter issue, capital controls are generally shown to be powerful in offsetting the monetary implications of foreign exchange intervention, thus curbing inflationary pressures. However, under capital controls, other kinds of destabilizing influences are likely to emerge, so that real exchange rate targeting may ultimately prove to be unsustainable in this case.

Dornbusch (1982) seminal paper paved the way to subsequent theoretical work inside a rational expectations model with overlapping wage contracts. Nominal exchange rate indexing operates through both a demand-side channel (external competitiveness) and a supply-side channel (costs of imported intermediate goods). An increase in the degree of nominal exchange rate indexation always increases the variability of prices. The effects on output variability are instead crucially affected by the relative size of the above influences: when the supply-side (demand-side) channel dominates the variability of output is increased (reduced).

Building upon the above paper, the macroeconomic implications of real exchange rate targeting have been explored in a variety of theoretical models. This work broadly falls into two categories:

1. A former group of contributions usually relying upon a comparative static approach. Although displaying a richer macroeconomic structure with respect to Dornbusch (1982), this work relies on *ad hoc* macroeconomic models lacking rigorous microeconomic foundations.
2. A latter group of more recent contributions developed inside an intertemporal optimization framework. This research provides the analysis with explicit microeconomic foundations, making up for a relevant drawback in the previous literature.

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<sup>3</sup>Theoretical contributions in this area address both the effects of policies aimed at stabilizing external competitiveness (PPP rules) and the macroeconomic consequences induced by a real exchange rate depreciation. Since our theoretical model assumes that the policy-maker aims at stabilizing the real exchange rate at a (constant) long run equilibrium level, this section will predominantly focus on the former group of policies.

We start surveying earlier research in this field, drawing a basic distinction between models assuming perfect or imperfect capital mobility.

Perfect capital market integration is retained in Adams and Gros (1986) where, assuming a single traded good and nominal price stickiness, a policy adjusting the nominal exchange rate to inflation differentials generates serious destabilizing effects on domestic inflation. Since the nominal exchange rate fully accommodates current price shocks, the absence of a nominal anchor implies that domestic inflation follows a random walk pattern, failing to settle down to any steady state level.<sup>4</sup> Montiel and Ostry (1991) obtain more optimistic conclusions in the context of a dynamic, dependent-economy model, where uncovered interest parity holds and price flexibility ensures that full employment is continuously maintained. This model exhibits multiple equilibria (conditional on the interest elasticity of money demand) and, quite interestingly, a low-inflation equilibrium may arise in this set up. In sharp contrast with the fixed exchange rate version of the model, however, various kinds of real shocks have always inflationary implications when the government pursues a real exchange rate target.<sup>5</sup>

A relevant issue addressed in this literature is whether money supply can provide the needed nominal anchor if capital is less than perfectly mobile. Assuming zero capital mobility, Adams and Gros (1986) show that the authorities can successfully sterilize the monetary consequences of foreign exchange market intervention, stabilizing the money supply and hence domestic inflation. However, under a real exchange rate rule, this policy gives rise to a permanent disequilibrium in the current account and therefore does not appear to be sustainable in the long run. An equally pessimistic conclusion is reached in Montiel and Ostry (1992). This paper models capital controls specifying a dual exchange rate regime: the Central Bank operates on the market for commercial transactions (following a real exchange rate targeting rule), while it refrains from engaging in foreign exchange transactions for financial purposes. The main conclusion is that, under capital controls, a money supply targeting rule can successfully be implemented to stabilize domestic inflation in the face of negative external shocks. However, the only way this can happen is by creating an ever-widening gap between commercial and financial exchange rates. In strict analogy with Adams and Gros (1986), the basic policy implication is that, under real exchange rate targeting, capital controls cannot be permanently employed as a powerful anti-inflationary device.<sup>6</sup>

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<sup>4</sup>This key insight of the paper is basically confirmed in many variants of the model incorporating nontraded goods, a surprise supply function, or an overshooting process à la Dornbusch (1976). See, respectively, Adams and Gros (1986), Sections 3, 4, and 5.

<sup>5</sup>The shocks analyzed in this paper are a change in the terms of trade, in the composition of government spending, in import tariffs, and in world real interest rates: see Montiel and Ostry (1991), Section 2. Note, moreover, that the choice of an overdepreciated real exchange rate target would unambiguously lead to a permanent rise in the economy's inflation rate.

<sup>6</sup>Lizondo (1993) explores the macroeconomic implications of real exchange rate targeting inside a flexible-price model where foreign and domestic assets are assumed to be imperfect substitutes. Imperfect asset substitutability provides the authorities with some degree of control over domestic interest rates and hence on domestic inflation (via a temporary expenditure-

We now turn to more recent theoretical work in this field. In order to facilitate a comparison with earlier research, we retain the basic distinction between models assuming perfect and imperfect capital mobility (or, alternatively, a risk premium component in the uncovered interest parity condition).

Calvo *et al.* (1995) analyze a small open economy with perfect foresight, in which a representative agent maximizes his lifetime utility consuming a basket of traded and nontraded goods. The representative consumer faces a cash-in-advance constraint, while the nominal exchange rate is exogenously set by the monetary authority. In this setting, the (unique) steady-state real exchange rate associated with the social optimum is constant and independent from permanent changes in the rate of nominal devaluation. Assuming perfect capital mobility, a PPP rule stabilizing the real exchange rate in the face of external shocks is inflationary. Although this result displays some analogies with earlier contributions, the nature and the direction of the shocks are quite different in the present set up.<sup>7</sup> The same inflationary result is obtained assuming that the monetary authority pursues an overdepreciated real exchange rate target, although the welfare effects are opposite in this case.<sup>8</sup> The basic assumptions of Calvo *et al.* (1995) (optimizing representative agent, uncovered interest parity) are retained in Uribe (2003), where a real exchange rate targeting rule potentially yields some novel aggregate costs with respect to the previous literature. In line with Calvo *et al.* (1995), this model admits a Pareto-optimal, perfect foresight equilibrium, involving a constant consumption of traded goods. However, since the asset market is incomplete<sup>9</sup>, there exists an infinite number of (welfare decreasing) equilibria, where the path followed by the consumption of traded goods does not necessarily converge to the social optimum. The main insight is that a real exchange rate targeting rule may give rise to endogenous fluctuations originating from self-fulfilling revisions in private agents' expectations. Although the occurrence of multiple equilibria has already been emphasized in some previous

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reducing effect). Nevertheless, as shown in this paper, this positive anti-inflationary effect is short-lived. A restrictive monetary policy (open market sale of domestic bonds) leads to a higher domestic real interest rate. As time goes by, private wealth accumulates tending to appreciate the real exchange rate. Under a real exchange rate targeting rule, the authorities are forced to intervene, accelerating the rate of nominal devaluation and thus generating a higher inflation rate. Differently from Adams and Gros (1986) and Montiel and Ostry (1992), an anti-inflationary monetary policy does not destabilize other macroeconomic variables. However, in this setting, this policy leads only to a temporary decrease in the inflation rate, followed by higher inflation in the future.

<sup>7</sup>As underlined before, Montiel and Ostry (1991) focus on the macroeconomic implications of real shocks, whereas Calvo *et al.* (1995), Section 2.1, address the consequences of a nominal shock. Moreover, Montiel and Ostry (1991) assume a rise in the world real interest rate, whereas the inflationary outcome obtained in Calvo *et al.* (1995) stems from a temporary fall in foreign inflation (and hence in the world nominal interest rate).

<sup>8</sup>As discussed in Calvo *et al.* (1995), Section 2.1, a PPP rule stabilizing the real exchange rate, although producing a temporary increase in domestic inflation, is welfare-improving since the monetary authority fully offsets an exogenous distortion affecting the economy. Since this is not the case when a policy of overdepreciation is pursued, this latter strategy is instead welfare-decreasing.

<sup>9</sup>Households are assumed to allocate their wealth only between domestic money and an internationally traded bond denominated in foreign currency: see Uribe (2003), Section 2.2.

work (Montiel and Ostry, 1991), such an aggregate “instability cost” represents a new result in this literature. Quite interestingly, this result turns out to be more likely the more elastic the PPP rule, while being robust to some relevant extensions of the theoretical model.<sup>10</sup>

We finally turn to the recent strand of optimizing models where the assumption of perfect capital mobility is explicitly or implicitly removed. The macroeconomic implications of PPP rules in this setting reveal various analogies with the theoretical literature analyzed so far. Some models predict that, under no capital mobility, a domestic inflationary upsurge can be avoided, although other macroeconomic variables will significantly be destabilized.<sup>11</sup> According to other models, under imperfect capital mobility not only will domestic inflation tend anyway to rise, but the whole macroeconomic stability might seriously be jeopardized raising, once again, the likelihood of multiple equilibria. These negative implications are obtained in Nana Davies (2002) inside an optimizing framework including a country-specific risk premium in the uncovered interest parity condition.<sup>12</sup>

To sum up, the literature surveyed in this section points out that a real exchange rate targeting policy will unavoidably generate not negligible macroeconomic costs. The nature of these costs is crucially affected by the underlying theoretical model and by its specific assumptions concerning the degree of capital mobility. Under perfect capital mobility, a PPP rule implies, in most theoretical models, a destabilizing effect on domestic inflation. Under imperfect capital mobility, the destabilizing effects are instead more likely to involve other macroeconomic variables (current account, domestic interest rates, commercial and financial exchange rates). A further relevant insight emerging from more recent contributions is that, under certain conditions, real exchange rate targeting may also generate some endogenous aggregate instability, driven by private agents’ self-fulfilling expectations.

<sup>10</sup>These extensions include the introduction of stochastic PPP rules or of price stickiness in the benchmark theoretical model: see, Uribe (2003), Sections 4 and 5, respectively.

<sup>11</sup>This result is reached in Calvo *et al.* (1995), Section 2.2, examining the (polar) case of zero capital mobility through the same optimizing approach. Similarly to Montiel and Ostry (1992), the model formally becomes, under this assumption, a dual exchange rate regime with no leakages. Assuming the same external shock (see Footnote 2 above), a PPP rule now implies the absence of inflationary pressures, although the economy experiences an upward jump and a subsequent rise in the domestic real interest rate. In close analogy with the perfect capital mobility case (see Footnote 2 above), this PPP rule is again welfare-improving, in sharp contrast with a policy pursuing an overdepreciation of the real exchange rate.

<sup>12</sup>Nana Davies (2002) does not disaggregate between traded and nontraded goods, outlining a perfect foresight, single good model, where both money and consumption enter the representative agent’s utility function. Since instantaneous PPP holds, an increase in the rate of nominal devaluation leads to a correspondent increase in domestic inflation. Differently from the constant risk premium case, monetary neutrality breaks off when the risk premium is assumed to be inversely related to the nominal devaluation rate. Thus, under certain conditions, an increase in the devaluation rate has a permanent positive effect on external balance. This paper, however, strongly warns against the potentially destabilizing effects associated to a policy of this kind. If the risk premium exceeds a given threshold, the macroeconomic equilibrium becomes unstable, while further destabilizing effects may stem from the nonlinearity of the risk premium function or from shifts in this function driven by exogenous factors.

Although the development of microfounded theoretical models has marked a significant progress, some specific issues have not yet received, in our opinion, an adequate attention.

Theoretical models addressing the macroeconomic effects of PPP rules have almost exclusively been developed inside a deterministic framework, where the effect of exogenous disturbances is explored through a comparative static approach. This represents a relevant limitation since, as documented by a large body of empirical evidence, stochastic shocks continuously impinging upon the real exchange rate significantly affect both the deviations from its equilibrium level and its conditional volatility. Recent optimizing theoretical models (e.g. Calvo *et al.*, 1995) predict a unique steady-state equilibrium level of the real exchange rate, independent of monetary disturbances. Although this prediction is consistent with the empirical evidence (supporting mean reversion in real exchange rates), the absence of a stochastic framework clearly oversimplifies the theoretical analysis.

A further drawback of existing work is that it appears poorly equipped to provide concrete policy prescriptions. Although this literature points out a basic policy tradeoff associated to a PPP rule (i.e. a stabilization in external competitiveness versus a destabilization of the domestic economy) it cannot exactly qualify under which conditions the above policy actually represents an optimal choice. More recent contributions outline the macroeconomic implications of PPP rules in models with infinitely lived and optimizing private agents. From a normative standpoint, however, a model with an optimizing policy-maker would do a better job when searching, inside a dynamic stochastic setting, the optimal degree of real exchange rate targeting.

The above shortcomings provide the main motivation for the theoretical analysis carried out in the next section.

### 3 The model

In line with the previous theoretical literature addressing the macroeconomic implications of real exchange rate targeting, our model focuses on a small open economy. This economy operates under a floating exchange rate regime, facing an exogenous (and constant) foreign interest rate. As emphasized in the previous section, the main distinctive feature of this paper is that it relies on a stochastic optimization framework, providing a rich characterization of real exchange rate dynamics and explicit microeconomic foundations to policy-maker's choices.

As regards real exchange rate dynamics, our modelling approach reflects the latest developments both in the theoretical and in the empirical literature. In line with theoretical models based on intertemporal utility maximization, we assume the existence of a unique steady-state value of the real exchange rate, independent from nominal disturbances. This equilibrium value can be thought of as associated to a constant consumption of traded goods, therefore defining a



social optimum for the domestic small open economy analyzed in this paper.<sup>13</sup> Following a large body of evidence stemming from current empirical research, we further assume that purchasing power parity does not instantaneously hold, so that the real exchange rate exhibits persistent deviations from its long run equilibrium value. As widely recognized in the literature, alternative explanations can be put forward to motivate the above pattern: either a significant degree of price stickiness (our preferred explanation in the present set up), or a pricing-to-market behavior by a representative international firm, setting different prices for the same good across segmented national markets. Consistently with the above discussion, the real exchange rate is modelled as a stochastic process displaying mean reversion towards a long run (optimal) equilibrium value.<sup>14</sup>

A further assumption of our theoretical model is that monetary policy has real effects in the short run, providing the policy-maker with some degree of control over the external competitiveness of the economy. In other words, although the steady-state real exchange rate is independent of nominal disturbances, we posit that domestic interest rate variations may significantly affect its short run dynamics. A restrictive (expansionary) monetary policy generates a massive capital inflow (outflow) which tends to instantaneously appreciate (depreciate) the nominal exchange rate: given some degree of domestic price stickiness, the real exchange rate will tend to follow an analogous short-term pattern. The rationale behind this “liquidity effect” is very close to Dornbusch (1976) seminal paper where short run monetary nonneutralities, producing relevant co-movements between nominal and real exchange rates, hinge crucially upon the different speed of adjustment of assets and goods markets.<sup>15</sup> Although sharing some basic assumptions with Dornbusch (1976) (such as the existence of nominal rigidities on the output market and the exogeneity of the foreign interest rate), our model yields a richer real exchange rate dynamics since, differently

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<sup>13</sup> As discussed in Section 2, these assumptions are quite common in the more recent literature relying on explicit microeconomic foundations: see, at this purpose, Calvo *et al.* (1995) and Uribe (2003).

<sup>14</sup> Mean reversion in real exchange rates is currently strongly supported by the empirical evidence although, up to about a decade ago, the consensus view in the literature suggested a very slow rate of convergence towards long run equilibrium (Froot and Rogoff, 1995). More recent empirical works shed novel insights into this “PPP puzzle” (Rogoff, 1996), namely the apparently too high degree of persistence recorded in real exchange rates. As emphasized in this research, various market frictions in international trade produce significant nonlinearities in the adjustment process: real exchange rates are typically strongly mean reverting for sizable deviations from PPP, while becoming closer to a random walk process as the magnitude of the disequilibrium decreases. On the whole, the latest contributions applying nonlinear econometric techniques have revised upwards the estimated speed of mean reversion, pointing out that average half-lives of PPP deviations are lower than it was previously thought (see, among others, Baum *et al.*, 2001, and Taylor *et al.*, 2001). Further research relying on complementary approaches, such as multivariate or fractionally integrated unit root tests, provides additional strong evidence supporting mean reversion in real exchange rates (Sarno and Taylor, 1998, and Gil and Alana, 2000).

<sup>15</sup> The existence of a significant correlation, on a short run basis, between nominal and real exchange rates is an outstanding stylized fact in the applied international finance literature: see, among others, MacDonald (1999).

from the above paper, it relies on a stochastic rather than on a deterministic framework.<sup>16</sup>

In line with the above discussion, the evolution of the real exchange rate is described by the following stochastic differential equation:

$$dq = \alpha(\beta - q)dt - \gamma(i - i_F)dt + \sigma(x, t)' dW, \quad q(t_0) = q_0, \quad (1)$$

$\begin{matrix} 1 \times k & k \times 1 \end{matrix}$

where  $q$  is the real exchange rate,<sup>17</sup>  $\beta$  is its steady-state equilibrium value,  $(i - i_F)$  is the nominal interest rate differential between domestic and foreign financial assets,  $\alpha$  and  $\gamma$  are two non-negative constant parameters.

The first component on the right-hand side of Equation (1) captures the mean reverting behavior of  $q$ , with the  $\alpha$  coefficient measuring the strength of this effect. The second component refers instead to short run monetary non-neutralities, inducing a temporary influence of domestic monetary policy on external competitiveness (the strength of this influence is measured by  $\gamma$ ).<sup>18</sup> Finally, the third additive component on the right-hand side of Equation (1), represents the stochastic source of changes in the real exchange rate. In particular,  $dW$  is the differential of a  $k$ -dimensional Wiener process and summarizes the composite influence of various real and monetary shocks (see Footnote 3). The volatility of  $q$  is given by  $\sigma(x, t)' \sigma(x, t)$ , where  $\sigma(x, t)$  is a vector (hereafter the prime denotes transposition).

One relevant assumption underlying Equation 1 is that this volatility is not constant over time and space. Stochastic volatility models are quite common in the finance literature exploring nominal exchange rate dynamics (see, for instance, Bates, 1996), while many studies using high-frequency data point out a significant degree of persistence in the volatility pattern of major bilateral exchange rates (see, among others, Mahieu-Schotman, 1998). Quite interestingly, moreover, stochastic volatility models have recently found strong empirical support in real exchange rates series as well, with many papers documenting a significant degree of persistence in their conditional variance. On the whole, the highly significant GARCH estimates reported in this research point out that time-varying volatility and “volatility clustering” are prominent features in real exchange rate data.<sup>19</sup> Consistently with this empirical literature, we model the

<sup>16</sup>In Dornbusch (1976), once the adjustment to a monetary shock is completed, domestic and foreign interest rates are equalized, while the economy attains a new long run equilibrium where money is neutral and the real exchange rate is back to its steady-state level. In the stochastic set up of the present paper, by contrast, besides domestic monetary policy, a wide set of real and nominal shocks is allowed to create large and persistent deviations from PPP. Therefore, even when domestic and foreign interest rates coincide, the real exchange rate is not necessarily at its steady-state equilibrium value.

<sup>17</sup>The real exchange rate  $q$  is given by  $EP^*/P$ , where  $E$  is the nominal exchange rate and  $P^*$  and  $P$  are, respectively, the foreign and the domestic price level. Since  $E$  is defined as the number of domestic currency units necessary to buy one unit of foreign currency, an increase (decrease) in  $q$  corresponds to a real exchange rate depreciation (appreciation).

<sup>18</sup>The policy-maker (i.e. the domestic Central Bank) has full control over the domestic interest rate  $i$  whereas, as underlined before, the small open economy assumption implies that the foreign interest rate  $i_F$  is exogenous in the present set up.

<sup>19</sup>The literature applying GARCH models to real exchange rates focuses primarily on the

volatility of  $q$  as time-varying and, more specifically, as dependent on a stochastic variable  $x$  whose behavior is given by the following stochastic differential equation:

$$dx = f(x, t) dt + g(x, t)' dW, \quad x(t_0) = x_0, \quad (2)$$

$1 \times k \quad k \times 1$

where the drift and diffusion terms  $f(x, t)$  and  $g(x, t)$  are supposed to satisfy the usual Lipschitzian conditions guaranteeing that Equation (2) has a unique strong solution (see Karatzas and Shreve, 1991). Furthermore,  $f$  and  $g$  are  $\mathcal{F}_t$  measurable, where  $\mathcal{F}_t$  is the  $\sigma$ -algebra through which the Wiener processes are measured on the complete probability space  $(\Theta, \mathcal{F}, \mathbb{P})$ .

We close our theoretical model with a further equation describing the policy-maker's objective function. As underlined in section 2, the focus on the policy-maker's optimizing behavior represents an innovative feature with respect to the existing literature. The policy-maker is assumed to minimize, in each period, the costs associated to real exchange rate and interest rate deviations from their targets; the instantaneous loss function can thus be specified as:

$$\Lambda(q, i) = \frac{\phi}{2} (q - q_T)^2 + \frac{\eta}{2} (i - i_T)^2, \quad (3)$$

where  $q_T$  and  $i_T$  represent the real exchange rate and the domestic interest rate target, respectively and  $\phi$  and  $\eta$  are two non-negative parameters measuring their relative weights in the loss function.<sup>20</sup>

In the theoretical literature on monetary policy, the objective function is usually specified in terms of output gap, while including a preference of the policy-maker towards inflation stabilization (see, for instance, Clarida *et al.*, 1999, Section 2.2). As shown by Equation (3), neither the output gap nor the domestic inflation enter the objective function as direct arguments. The reason is that our loss function is specified in order to account for some peculiar features of the economy we are considering and for some relevant macroeconomic implications highlighted in the literature on real exchange rate targeting. Although departing from more conventional specifications of the objective function, Equation (3) closely reflects a policy-maker's concern for output and inflation stabilization.

Consider, at this purpose, the first additive term in Equation (3). The focus of this paper is on a developing, small open economy, where the real exchange rate is a primary source of output fluctuations. Following the theoretical literature surveyed in Section 2, keeping the real exchange rate as close as possible to a given equilibrium target is therefore a basic policy goal in this context.

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effects of volatility on international trade flows. As shown in this literature, the strong persistence in the conditional variance of real exchange rates is a robust empirical regularity, holding both during the first half of the last century (Pozo, 1992) and after the collapse of the Bretton Woods system (Caporale and Doroodian, 1994). The existence of time-varying volatility in real exchange rates, finally, has recently been documented both for developing and for industrialized countries (see Singh, 2002, and Sukar and Hassan, 2001, respectively).

<sup>20</sup> The loss function  $\Lambda(q, i)$  is supposed to be additive with respect to its arguments in order to make easier the solution of the stochastic optimization problem.

Although the real exchange rate displays a long run mean reverting tendency, deviations from PPP are quite large and persistent, as implied by the empirical evidence and by our stochastic volatility assumption. The policy-maker will thus exploit short run monetary nonneutralities in order to foster the long run convergence of the real exchange rate, thus avoiding excessive and costly real output fluctuations.<sup>21</sup>

Turning to the latter quadratic component in Equation (3), the loss arising from interest rate level fluctuations around a given target can be motivated on the basis of the usual interest rate smoothing argument appearing both in theoretical and empirical work on monetary policy (see Woodford, 2002, and Surico, 2002, respectively). In the specific context of this paper, a complementary motivation can be offered assuming that the policy-maker dislikes deviations of domestic inflation from a predetermined long run equilibrium target.<sup>22</sup> Suppose that  $i_T$  reflects, on average, a monetary policy stance consistent with a long run inflation target: large and persistent discrepancies between  $i$  and  $i_T$  will then be costly since they imply a monetary policy potentially inconsistent with inflation stabilization. Viewed in this perspective, the latter quadratic term in Equation (3) highlights a pervasive macroeconomic implication of the literature surveyed in Section 2: namely that a policy of real exchange rate targeting may potentially destabilize domestic inflation.<sup>23</sup>

Faced with large and persistent deviations from PPP, the policy-maker can manipulate the domestic interest rate in order to speed up the adjustment of the real exchange rate towards its steady-state equilibrium level. This stabilization of external competitiveness may however involve not negligible macroeconomic costs, with monetary policy potentially clashing with other internal stabilization goals.

This relevant policy tradeoff will be formally addressed in the next section, solving the policy-maker's problem inside a stochastic optimal control framework.

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<sup>21</sup>Note that the literature surveyed in Section 2 provides a further rationale for real exchange rate targeting. As remarked in this section, recent theoretical models based on an intertemporal optimization approach imply that the equilibrium corresponding to a steady-state level of the real exchange rate is Pareto optimal (Calvo *et al.*, 1995, Uribe, 2003).

<sup>22</sup>Inflation targeting is usually regarded as a useful monetary policy framework for advanced countries. However, as pointed out in some recent contributions, this strategy may be appropriate for some emerging market countries as well. Mishkin (2000), and Masson *et al.* (1997) discuss the scope for inflation targeting in developing countries, underlining how the applicability of this approach heavily relies on the implementation of adequate institutional changes.

<sup>23</sup>As discussed in Section 2, the literature on real exchange rate targeting points out various destabilizing effects, other than on domestic inflation, depending on the degree of capital mobility or on arbitrary revisions in agents expectations. In a wider perspective, therefore, deviations of  $i$  from  $i_T$  can also be seen as capturing the costs associated to the destabilization of other macroeconomic variables (as, for instance, in Montiel and Ostry, 1992) or those associated to endogenous and welfare decreasing aggregate fluctuations (as emphasized in Uribe, 2003).

## 4 Optimal interest rate policy and real exchange rate dynamics

### 4.1 Optimal interest rate policy and its properties

Given the theoretical model outlined in the previous section, the stochastic optimization problem analyzed in this paper may compactly be written as follows:

$$\left\{ \begin{array}{l} \min_i \mathbb{E}_{t_0} \left[ \int_{t_0}^H e^{-\rho(t-t_0)} \left( \frac{\phi}{2} (q - q_T)^2 + \frac{\eta}{2} (i - i_T)^2 \right) dt \right] \\ \left[ \begin{array}{c} dx \\ dq \end{array} \right] = \left[ \begin{array}{c} f(x, t) \\ \alpha(\beta - q) - \gamma(i - i_F) \end{array} \right] dt + \left[ \begin{array}{c} g(x, t)' \\ \sigma(x, t)' \end{array} \right] dW \\ x(t_0) = x_0, \quad q(t_0) = q_0, \quad \forall t_0 \leq t \leq H \end{array} \right. \quad (4)$$

where the domestic interest rate  $i$  is the control variable, the real exchange rate  $q$  and the determinant of its stochastic volatility  $x$  are two state variables,  $H$  is the time horizon of the policy-maker, and  $\rho$  is its (constant) intertemporal discount rate.

The optimal value of the domestic interest rate in this set up ( $i^*$ ) is shown in the following proposition (see Appendix A for technical details).

**Proposition 1** *The optimal domestic interest rate solving Problem (4) is given by*

$$\begin{aligned} i^* &= i_T + 2\frac{\gamma}{\eta} a(t) q \\ &+ \frac{\gamma}{\eta} \int_t^H (2(\alpha\beta - \gamma(i_T - i_F)) a(s) - \phi q_T) e^{-\int_t^s (\rho + \alpha + 2\gamma^2 \frac{1}{\eta} a(\tau)) d\tau} ds, \end{aligned} \quad (5)$$

where

$$\begin{aligned} a(t) &= \frac{\phi \tanh\left(\frac{1}{2}\sqrt{\Delta}(H-t)\right)}{\sqrt{\Delta} + (2\alpha + \rho) \tanh\left(\frac{1}{2}\sqrt{\Delta}(H-t)\right)}, \\ \Delta &\equiv (2\alpha + \rho)^2 + 4\gamma^2 \frac{\phi}{\eta}. \end{aligned}$$

**Proof.** See Appendix A. ■

Although Equation (5) is apparently quite complex, a closer inspection of this expression reveals that all the influences upon optimal interest rate dynamics have a strong economic intuition.

Note first that, differently from  $q$ , stochastic volatility  $\sigma(x, t)$  does not enter Equation (5). As formally shown in Appendix A, the optimal value of the objective function depends on  $\sigma(x, t)$  whereas the optimal value of the interest rate does not. The underlying intuition is that all the effects arising from stochastic volatility are conveyed through real exchange rate variations: in other words, the

optimal interest rate reacts to changes in  $\sigma(x, t)$  through its direct dependence on  $q$ . Consider the second additive component in Equation (5) and note that  $a(t) > 0, \forall t_0 \leq t \leq H$ . Whenever  $q$  increases (decreases), the optimal interest rate must increase (decrease) in order to counterbalance a real exchange rate depreciation (appreciation) through international capital flows. As discussed in the previous section, this liquidity effect *à la* Dornbusch (1976) provides the basic rationale for real exchange rate targeting in the present framework.

Consider, first, the response of the optimal interest rate to changes in  $\alpha$ , i.e. the coefficient measuring the speed of real exchange rate mean reversion. When  $\alpha$  tends to infinity the real exchange rate never deviates from its long run equilibrium level and there is obviously no scope for real exchange rate targeting. As revealed by Equation (5), the model is consistent with the above prediction suggesting that, in this limiting case, the policy-maker should simply keep the domestic interest rate at a constant target value.<sup>24</sup>

Let us now turn to the influence of the parameters expressing the weights of the policy targets in the objective function.

When the policy-maker does not care at all about stabilizing external competitiveness ( $\phi = 0$ ), an easy substitution in Equation (5) shows that  $a(t) = 0$  so that the optimal policy simply prescribes  $i^* = i_T$ .

In line with economic intuition, variations in the latter policy parameter ( $\eta$ ) involve reverse effects on the optimal interest rate. As  $\eta$  approaches zero (i.e. when the weight assigned to domestic interest rate stabilization becomes negligible), the Hamiltonian tends to become linear in the control variable (see Appendix A, Equation (13)), implying that the policy-maker is ready to tolerate large interest rate variations in order to stabilize the real exchange rate.

Consider, now, the role of the  $\gamma$  parameter, capturing the sensitivity of the real exchange rate to the interest rate differential between domestic and foreign assets. Lacking this “liquidity effect” on real exchange rate dynamics, there is clearly no scope in pursuing an active monetary policy. Equation (5) supports the above conclusion: actually, when  $\gamma = 0$ , the optimal policy reduces once again to the simple rule  $i^* = i_T$ .

A further important influence on the optimal interest rate policy is finally associated with the policy-maker’s time horizon. As shown by Equation (5), two relevant features stand out in this regard. First, assuming a finite time horizon, the optimal monetary policy in the last period (i.e. when  $t = H$ ) collapses, once again, to the simple rule  $i^* = i_T$ .<sup>25</sup> Actually, whatever the value of  $q$  in the last period, there is obviously no need to further manipulate the domestic interest rate. Second, when the policy-maker’s time horizon tends to infinity, the optimal policy becomes an affine transformation of Equation (5). More specifically, the  $a(t)$  component reduces to a constant nonlinear combination of model’s parameters, while the current level of the real exchange rate continues to directly affect the optimal interest rate pattern. This result is described in

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<sup>24</sup>The limit of  $a(t)$  for  $\alpha \rightarrow \infty$  is zero. Therefore, in this case, Equation (5) reduces simply to  $i^* = i_T$ .

<sup>25</sup>We recall that  $\tanh(0) = 0$  so that, setting  $t = H$ , both the second and the third component vanish in Equation (5).

the following Corollary to Proposition 1.

**Corollary 1** *The optimal domestic interest rate solving Problem for a policy maker with an infinite time horizon (i.e.  $H \rightarrow \infty$ ) is given by*

$$i^* = i_T + 2\frac{\gamma}{\eta}aq + \gamma \frac{2(\alpha\beta - \gamma(i_T - i_F))a - \phi q_T}{\eta\rho + \eta\alpha + 2\gamma^2 a}, \quad (6)$$

where

$$a \equiv \frac{\phi}{\sqrt{\Delta} + 2\alpha + \rho}, \quad \Delta \equiv (2\alpha + \rho)^2 + 4\phi\frac{\gamma^2}{\eta}.$$

**Proof.** By recalling that  $\lim_{y \rightarrow +\infty} \tanh y = 1$ , it is sufficient to compute the limit of the third component of  $i^*$  in (5) where  $a(t)$  is now a constant. ■

Thus, the optimal interest rate in (6) is a simplified version of those presented in (5). This allows us to analyze in further details the role of the parameters in affecting the optimal stabilization policy. In fact, these parameters entered Equation (5) in a highly nonlinear fashion and a visual inspection of those equation was not sufficient to fully characterize its properties. This taken into account, our analysis is supplemented with some numerical simulations displaying the sensitivity of the optimal policy to different values of each parameter and allowing, in every case, for different degrees of real exchange rate misalignment. The results of these numerical simulations are summarized in Figure 1.<sup>26</sup>

[Fig. 1 here]

The reaction of the optimal policy when  $\alpha$  takes finite values is strongly in line with economic intuition (as for the case with  $\alpha$  tending to infinity). As shown by the numerical simulation (Figure 1, top-right panel), the deviations of the domestic interest rate from its target are larger the lower the coefficient of mean reversion. A slow rate of convergence towards PPP (relatively low  $\alpha$ ) implies a stronger monetary policy reaction since, under these circumstances, it becomes more important to hasten the adjustment process through interest rate variations. Note moreover that, in line with our previous discussion, the above tendency is stronger the larger the distance of the real exchange rate from its steady-state level (i.e. the larger the gap between  $q$  and 1), as shown by the upward bending of the surface in the  $(i, \alpha, q)$  space.

When the value of  $\phi$  is progressively increased (Figure 1, top-left panel) the optimal policy response, as intuitively expected, is strictly increasing in the

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<sup>26</sup>The targets for the real exchange rate ( $q_T$ ) and for the domestic interest rate ( $i_T$ ) were respectively set at 1 and 0.05. Different degrees of real exchange rate misalignment (over-depreciation) are considered in Figure 1, progressively increasing the value of  $q$  from its equilibrium level to 1.25 – 1.50. For additional technical details and the calibration of remaining parameters, see Appendix D. All simulations refer to the infinite horizon case. However, since the optimal policy in this case corresponds to an affine transformation of Equation (5) (see Corollary 1 and Equation (6)), the properties of the optimal interest rate policy over a finite time horizon are qualitatively identical.

weight assigned to real exchange rate targeting. Differently from the previous case, moreover, the concave surface in the space  $(i, \phi, q)$  is symmetric, suggesting that increases in  $\phi$  and in the degree of real exchange rate misalignment exert approximately the same quantitative effect on interest rate variations.

The effect of the parameter  $\eta$  is shown in Figure 1, low- right panel. The surface in the  $(i, \eta, q)$  space exhibits a pronounced upward bending as  $\eta$  tends towards zero. Conversely, a large weight assigned to interest rate stabilization and a low degree of real exchange rate misalignment imply. This behaviour is clearly in line with what we have already argued in the general case of Equation (5).

Finally, let us turn to the parameter  $\gamma$ . Whenever  $\gamma$  is greater than zero, one would a priori expect a case for an active monetary policy, exploiting short run monetary nonneutralities and giving rise to a gap between the domestic interest rate and its target level. The numerical simulation (Figure 1, low-left panel) confirms the above intuition, pointing out that the optimal interest rate is positively related to increases in  $\gamma$  and in the degree of real exchange rate misalignment. The asymmetric shape of the convex surface in the  $(i, \gamma, q)$  space reveals that the optimal monetary policy reacts more strongly to an increase in the degree of misalignment than to an increase in the  $\gamma$  coefficient. The underlying economic intuition is straightforward. Whenever  $\gamma$  is greater than zero the optimal response prescribes an active interest rate policy; however, as the sensitivity of  $q$  to  $(i - i_F)$  gets higher (i.e. as  $\gamma$  increases) a smaller interest rate increase is required in order to stabilize the real exchange rate around its steady-state level.

## 4.2 Implications of real exchange rate dynamics on the optimal interest rate policy

The analysis carried out in the previous section has solved the policy-maker intertemporal optimization problem deriving a policy rule, in terms of domestic interest rate variations, both over a finite and over an infinite time horizon. Moreover, with the help of some numerical simulations, we have characterized the main properties of the above rule, describing how domestic monetary policy is affected by changes in structural model's parameters.

The optimal policy rule derived so far (see Equations (5) and (6)) treats the real exchange rate target ( $q_T$ ) and the domestic interest rate target ( $i_T$ ) as exogenous parameters. However, under this exogeneity assumptions, there is no guarantee that, once the stabilization policy starts being implemented, the real exchange rate will univocally converge towards its “right” long run equilibrium value.

Recall that, according to the theoretical model outlined in Section 3, the basic motivation behind real exchange rate targeting is to foster the speed of the adjustment process towards the (unique) steady-state value of the real exchange rate consistent with a social optimum for the domestic economy. As we



will formally show in the present section, this goal can actually be reached inside our theoretical framework, provided that either  $q_T$  or  $i_T$  are appropriately endogenized.

Consider, at this purpose, Equation (6) defining the optimal interest rate policy when the time horizon tends to infinity. After substituting the value of  $i^*$  from this equation into Equation (1) we have:

$$dq^* = \left( \alpha + 2\frac{\gamma^2}{\eta}a \right) \left( \frac{\alpha\beta\eta(\rho + \alpha) + 2\gamma^3a(i_T - i_F) + \gamma^2\phi q_T}{(\eta\rho + \eta\alpha + 2\gamma^2a)(\eta\alpha + 2\gamma^2a)} \eta - q^* \right) dt + \sigma(x, t)' dW, \quad (7)$$

$\begin{matrix} 1 \times k & k \times 1 \end{matrix}$

where the two policy targets  $q_T$  and  $i_T$  remain exogenous and  $dq^*$  indicates the dynamic path followed by the real exchange rate under the optimal policy rule.

Focusing on Equation (7), two relevant observations are in order:

1. under the optimal intervention policy the stochastic process followed by the real exchange rate is still mean reverting, although the strength of mean reversion is higher (in fact,  $\eta$  is positive by assumption and  $a$  by construction);
2. under the optimal intervention policy the real exchange rate does not necessarily converge to the preferred steady-state equilibrium level ( $\beta$ ), unless one of the policy targets ( $q_T$ ,  $i_T$ ) is appropriately endogenized.

As regards the former effect, since shocks to long run PPP are neutralized at a quicker rate under an active monetary policy, the optimal rule defined in this paper satisfies the basic motivation for real exchange rate targeting.

Turning to the latter point, Equation (7) highlights that, once the optimal stabilization policy is implemented, the steady-state value of the real exchange rate ( $q^*$ ) becomes a highly nonlinear function of structural parameters and policy targets. This latter effect has a clear economic intuition along the lines suggested by the well known Lucas critique.

Think of  $\beta$  as an exogenous long run equilibrium value generated by a model where a representative consumer maximizes his lifetime utility over an infinite time horizon (Calvo *et al.*, 1995, Uribe, 2003). In line with this theoretical literature, we assume that this constant steady-state value is associated with a social optimum for the domestic economy. As revealed by Equation (1), the real exchange rate would spontaneously converge towards  $\beta$  under a scenario in which the policy-maker completely abstains from any active monetary policy. Consider, in this perspective, the implications of an active stabilization policy. Since this represents a structural policy change, the real exchange rate will respond to a different set of macroeconomic influences and, as a consequence, will no more necessarily converge towards  $\beta$ .

Although a spontaneous convergence towards  $\beta$  does not necessarily occur, a policy-maker endowed with rational expectations can ensure that this will actually be the case. Assume that the policy-maker knows the underlying theoretical model describing this economy and, more specifically, the feedback effects

induced by the active real exchange rate targeting policy outlined above (Equation (1)). Given this assumption, the policy-maker will be able to account for these feedback effects, by appropriately setting his policy targets in order to guarantee that the real exchange rate will converge towards the unique socially optimum equilibrium value.

Consider, at this purpose, Equation (7) which may be rewritten as:

$$dq^* = \left( \alpha + 2\frac{\gamma^2}{\eta}a \right) \left( \frac{\alpha(\rho + \alpha)\beta\eta + \gamma^2\phi q_T + \gamma\eta(i_F - i_T)(\rho + \alpha)}{(\eta\rho + \eta\alpha + 2\gamma^2a)(\eta\alpha + 2\gamma^2a)} \eta - q^* \right) dt + \sigma(x, t)'_{1 \times k} dW,_{k \times 1}$$

therefore, in order to ensure that the real exchange rate will converge to  $\beta$  we must have:

$$\frac{\alpha(\rho + \alpha)\beta\eta + \gamma^2\phi q_T + \gamma\eta(i_F - i_T)(\rho + \alpha)}{(\eta\rho + \eta\alpha + 2\gamma^2a)(\eta\alpha + 2\gamma^2a)} \eta = \beta.$$

This equation can be solved either for  $q_T$  or for  $i_T$ . Actually, the policy maker needs only one tool in order to make the controlled exchange rate converge towards its equilibrium level. The level of the exchange rate target  $q_T$  solving the above equation is

$$q_T^* = (i_T - i_F) \frac{\eta(\rho + \alpha)}{\gamma\phi} + 2\beta a \frac{\eta(\rho + 2\alpha) + 2\gamma^2a}{\phi\eta}, \quad (8)$$

while the level of the domestic interest rate target is

$$i_T^* = i_F + q_T \frac{\gamma\phi}{\eta(\rho + \alpha)} - 2\beta\gamma a \frac{\eta(\rho + 2\alpha) + 2\gamma^2a}{\eta^2(\rho + \alpha)}. \quad (9)$$

When either  $q_T$  or  $i_T$  are properly endogenized as above, the optimal interest rate is given by:

$$i^* = i_F + 2\frac{\gamma}{\eta}a(q - \beta).$$

As shown by this equation, the main properties of the optimal monetary policy remain unaffected when a rational policy-maker explicitly accounts for the feedback effects induced by an active stabilization policy. In line with Section 4.1, real exchange rate volatility does not enter the equation for the optimal interest rate. Moreover, the optimal policy prescribes again to exploit the liquidity effect induced by domestic interest rate variations in order to obtain a faster real exchange rate adjustment. More specifically, whenever  $q$  is higher (lower) than its desired long run equilibrium level ( $\beta$ ), the optimal policy prescribes a monetary policy more restrictive (expansionary) than in the rest of the world.<sup>27</sup>

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<sup>27</sup> Note that the equation describing the optimal interest rate policy when the policy-maker accounts for the feedback effects is just an affine transformation of Equation (6). Accordingly, numerical simulations performed in Section 4.1 are generally enough to fully characterize the sensitivity of the optimal policy to changes in structural model's parameters.

When one of the two targets  $(q_T, i_T)$  are appropriately endogenized as above, the optimal intervention policy over an infinite time horizon generates the following dynamic path for the real exchange rate:

$$dq^* = \left( \alpha + 2\frac{\gamma^2}{\eta}a \right) (\beta - q^*) dt + \sigma \begin{pmatrix} x, t \end{pmatrix}' \begin{pmatrix} 1 \times k & k \times 1 \end{pmatrix} dW.$$

Comparing this expression with Equation (7), it can immediately be recognized that the strength of the mean reversion effect is identical whereas the policy-maker has now attained its goal of setting the steady-state value at the desired equilibrium level.

## 5 Real exchange rate targeting versus non intervention: a welfare analysis

### 5.1 Preliminary remarks

The previous section has addressed a policy-maker's intertemporal optimization problem inside a small open economy where the real exchange rate is mean-reverting and may exhibit a significant degree of persistence in its conditional variance. In this context, we have shown how an active monetary policy rule can accelerate the convergence of the real exchange rate towards its steady-state (and socially optimal) equilibrium value. The analysis conducted so far has not envisaged any alternative to an active stabilization policy. This assumption, however, is not an innocuous one, particularly if the welfare implications of real exchange rate targeting are explicitly considered.

As underlined in Section 1, a strategy relying on real exchange rate targeting involves a significant departure from more conventional policy prescriptions. According to these prescriptions, a policy-maker should retain the control of some nominal anchor in a small open economy (either in terms of nominal exchange rate or in terms of money supply). Inside a "real target approach", by contrast, any nominal anchor is implicitly removed, while the theoretical literature surveyed in Section 2 reveals that this policy is likely to raise serious destabilizing effects. The above considerations suggest that bypassing any nominal target might not always represent the best available policy option, since the gains from a greater real flexibility to cope with various stochastic shocks might not necessarily outweigh the costs associated to a wider macroeconomic instability. In other words, since the real exchange rate displays an autonomous mean-reverting tendency, the policy-maker might actually be better off by abstaining from any active intervention aimed at stabilizing external competitiveness.

This relevant criticism against a "real target approach" underlines the relevance of performing a welfare-based analysis in order to properly evaluate the case for real exchange rate targeting. On the whole, notwithstanding a large body of theoretical contributions, the existing literature has not yet provided a comprehensive treatment of this issue.

As discussed in Section 2, any kind of welfare comparison is prevented in earlier research, since the underlying theoretical models lack explicit microeconomic foundations. Turning to more recent work, the main insight from Calvo *et al.* (1995) is that a policy where the real exchange rate is kept constant in the face of various shocks is welfare improving (since it corrects an initial distortion) as opposed to a policy where the authorities temporarily induce a real over-depreciation. However, this paper does not pursue any comparison between an active PPP rule and a non-intervention scenario. Uribe (2003) takes an important step in this direction inside an optimizing model of a small open economy. The main message here is that the equilibrium under a constant monetary rule Pareto dominates those associated to real exchange rate targeting. This strong conclusion, however, hinges heavily upon the indeterminacy of real exchange rate targeting equilibria (as a result of self-fulfilling expectations), while this paper neglects other structural factors which might as well exert a significant influence.

In line with the above remarks, the present section contributes to the current literature providing an explicit welfare comparison between two alternative economic policy strategies. The former replicates the “real target approach” outlined in the previous section, where the policy-maker implements an active monetary policy rule in order to ensure a faster correction of real exchange rate disequilibria. The latter strategy, by contrast, does not prescribe any active stabilization policy, relying exclusively on the long run self-adjusting properties characterizing real exchange rate dynamics.

We stress that, for various reasons, our theoretical framework is particularly suited for an analysis of this kind. First, differently from existing contributions, the optimization problem is solved with respect to a social loss function (taking as exogenous the steady-state real exchange rate) rather than from the perspective of an infinitely lived representative agent. This difference makes our approach more relevant from the normative standpoint, since the welfare effects stemming from alternative economic strategies can easily be translated into concrete policy prescriptions. A further advantage of our framework is that, albeit highly stylized, it allows to pinpoint the welfare effects associated to various structural features of the domestic economy. As underlined above, the existing literature is rather elusive on this topic whereas, as we will document in this section, both the degree of mean reversion and real exchange rate volatility play a crucial role in this regard. Last but not least, our theoretical model allows to perform a welfare-based analysis under alternative assumptions regarding real exchange rate volatility (see Equation (2)). More specifically, we will first compare alternative policy strategies under a constant volatility assumption (Section 5.2) and then assess the robustness of our results extending the analysis to the stochastic volatility case (Section 5.3). This comprehensive approach is quite profitable since, as discussed in Section 3, the existence of stochastic volatility has usually received a strong support from real exchange rate data.

## 5.2 Welfare analysis: the constant volatility case

The welfare analysis performed in the present section assumes a constant volatility in the stochastic process driving real exchange rate dynamics. Following the previous discussion, we will compare, over an infinite time horizon, the welfare effects associated to two alternative policies:

1. the optimal real exchange rate targeting rule outlined in Section 4, where  $q_T$  is appropriately endogenized in order to ensure a convergence of external competitiveness towards its desired steady-state level ( $\beta$ );
2. a non-intervention scenario, where the policy-maker abstains from any active stabilization rule, permanently setting  $i = i_T = i_F$ .<sup>28</sup>

Since the rationale underlying an active stabilization policy is to induce a faster correction of real exchange rate misalignments, we expect both  $\alpha$  and  $\sigma$  to play a significant influence in our welfare comparisons. Intuitively, a higher coefficient of mean reversion ( $\alpha$ ) should make a non-intervention policy more attractive because, by exploiting a more efficient self-adjustment mechanism, the policy-maker avoids costly domestic interest rate variations. In the same vein, a higher volatility ( $\sigma$ ) should make a targeting rule more attractive because, other things being equal, an active stabilization policy allows to dampen large and costly real exchange rate fluctuations.

A welfare analysis requires, in the present context, to derive a formal expression for the value function under the alternative policy strategies described above. The analytical steps to perform such computations are described in full detail in Appendix B. As shown there, the value function depends in a complicated and highly nonlinear way on all model's parameters, the two policy targets ( $q_T, i_T$ ), and the initial real exchange rate level  $q(t_0)$ . This taken into account, many numerical simulations were carried out in order to highlight the influence of various factors, thus providing meaningful welfare comparisons among alternative policy strategies. Our main results are summarized in Figures 2, 3, and 4.<sup>29</sup>

[Fig. 2 here]

The graphs reproduced in Figure 2 focus on the sensitivity of the value function with respect to the initial real exchange rate level  $q(t_0)$  and its volatility  $\sigma$ . The value function is always measured on the vertical axis: an upward movement along this axis denotes, therefore, an increasing loss for the policy-maker.

As shown by numerical simulations, the policy-maker's loss is strictly increasing in real exchange rate volatility, both under a targeting rule (top-left plot) and without any active stabilization policy (top-right plot). The initial

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<sup>28</sup>Note that, on the basis of our theoretical framework, this is the only way to formally characterize a non-intervention policy.

<sup>29</sup>The values of the parameters are chosen accordingly to what argued in Appendix D.

real exchange rate level, on the other hand, does not exert any appreciable influence.

As volatility increases, the value function under the optimal targeting rule ( $J$ ) increases less rapidly than that under no active stabilization policy ( $J_0$ ). This relevant feature can more easily be grasped from the lower graphs, plotting the difference between the value functions with and without intervention ( $J - J_0$ ). Focusing on the lower left graph we observe that, while the initial real exchange rate level is again irrelevant, the concave surface is steadily decreasing as volatility increases, with  $J - J_0$  turning from positive to negative values. Therefore, in line with our previous economic intuition, these numerical simulations reveal that when real exchange rate volatility exceeds a given threshold (corresponding to  $J - J_0 = 0$ ) an active targeting rule is welfare increasing with respect to a non intervention policy.

The bidimensional graph on the lower right hand side of Figure 2 displays the relationship between  $(J - J_0)$  and  $\sigma$ , assuming that  $q(t_0)$  corresponds to its steady-state equilibrium level (i.e. setting  $q(t_0) = \beta = 1$ ). Under these conditions, the trigger value of real exchange rate volatility approximately corresponds to 1.24. Since the estimates of  $\sigma$  reported in the empirical literature are usually lower than this value,<sup>30</sup> our numerical simulations imply that the case for active real exchange rate targeting is not likely to find a pervasive support in real world data.

We turn now to Figure 3, addressing the sensitivity of the value function with respect to  $\sigma$  and to  $\alpha$ . Although numerical simulations represented in this figure assume no initial real exchange rate misalignment (i.e.  $q(t_0) = \beta$ ), these results are robust to different values for  $q(t_0)$ .

[Fig. 3 here]

Focusing on the two upper graphs, the influence of  $\sigma$  mimics the results emerging from the previous discussion. The upward bending of the two convex surfaces shows moreover that, under both policy strategies, the value function is higher the lower is the mean reversion parameter ( $\sigma$ ). This result makes economic sense because, independently of the strategy which is actually pursued, a slower self-adjustment process involves anyway a greater policy-maker's loss.

Consider, now, the joint influence of  $\sigma$  and  $\alpha$  on the policy-maker's alternative whether to actively intervene or not. The lower graph in Figure 3 shows how the difference between the corresponding value functions ( $J - J_0$ ) is affected by these parameters. The concave surface reproduced in Figure 3 displays a close similarity with its counterpart in Figure 2. More specifically, real exchange rate volatility turns out to play again a crucial role on the welfare effects associated to alternative policy strategies. In line with previous results, real exchange rate targeting is welfare increasing only when volatility exceeds a given threshold.

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<sup>30</sup> See, among others, Devereux (1997) where volatility estimates ranging between 0.03 and 0.09 are documented for some major real exchange rates. Note, however, that these values refer to long run estimates, covering a period of more than twenty years. Considering shorter time horizons, real exchange rate volatility would obviously exhibit significantly higher values.

Note moreover that, given relatively high values of  $\sigma$ , the negative difference  $(J - J_0)$  becomes greater the lower is  $\alpha$ . In other words, whenever real exchange rate targeting is welfare increasing with respect to a non intervention policy, the gains from active stabilization are greater the lower is the mean reversion parameter. Intuitively, the lower is  $\alpha$ , the more profitable is real exchange rate targeting in speeding up the convergence towards long run equilibrium. On the whole, therefore, the numerical simulations underlying Figure 3, while revealing that  $(J - J_0)$  is more sensitive to  $\sigma$  than to  $\alpha$ , are again strongly in line with a priori economic intuition.

[Fig. 4 here]

We turn finally to Figure 4, addressing the sensitivity of the value function with respect to  $\sigma$  and to  $\gamma$ . The surface reproduced in the upper left graph plots  $(J - J_0)$  as a function of a wide set of the above parameters. The remaining bidimensional graphs plot instead  $(J - J_0)$  as a function of  $\gamma$  alone, in correspondence of three selected values of real exchange rate volatility. More specifically, we distinguish among a “low volatility” case ( $\sigma = 0.5$ ), an “intermediate volatility” case ( $\sigma = 1.2$ ), and a “high volatility” case ( $\sigma = 1.5$ ).

In close line with previous results, these simulations confirm the major role exerted by real exchange rate volatility in shaping the policy-maker’s choice between active stabilization and non intervention. Actually, under a “low volatility” scenario (top right plot)  $(J - J_0)$  lies always above zero independently of the value of  $\gamma$ , implying that active stabilization is unambiguously welfare decreasing in this case. Focusing on the polar “high volatility” scenario this conclusion is reversed. As revealed by the lower plot on the right-hand side of Figure 4, when  $\sigma = 1.5$ ,  $J - J_0$  turns out to be always negative for all  $\gamma$  values, implying that real exchange rate targeting becomes now unambiguously welfare increasing.

Assuming an intermediate value for real exchange rate volatility, a different picture stands out. As shown in the lower plot on the left-hand side of Figure 4, setting  $\sigma = 1.2$  active stabilization is welfare increasing either when  $\gamma$  exceeds an upper threshold or when it lies under a lower threshold. This last result has an equally nice economic intuition. When  $\gamma$  is particularly high (exceeding 0.8 in our numerical example), the real exchange rate is very sensitive to the nominal interest rate differential, thus making the case for an active stabilization policy. The same result, however, holds even when  $\gamma$  gets very low since, in this case, the policy-maker has a larger freedom to pursue a monetary policy consistent with internal targets, independently of external influences. Conversely, for intermediate values of  $\gamma$ , the above effects vanish making non intervention the most profitable policy strategy.

### 5.3 Welfare analysis: the stochastic volatility case

In order to address the case of stochastic real exchange rate volatility, we provide Equation (2) with an explicit functional form. We consider therefore a more

general optimization problem including the following state variables:

$$\begin{aligned} dq &= \alpha(\beta - q)dt - \gamma(i - i_F)dt + \sigma dW_q, \\ d\sigma &= \alpha_\sigma(\beta_\sigma - \sigma)dt + \sigma_q dW_q + \sigma_\sigma dW_\sigma. \end{aligned} \quad (10)$$

The latter equation relative to  $d\sigma$  assumes that volatility exhibits some degree of persistence before reverting to its long run equilibrium level ( $\beta_\sigma$ ), while  $\alpha_\sigma$  is the mean reverting coefficient and  $dW_q, dW_\sigma$  are two independent Wiener processes. Note, moreover, that  $\sigma_q$  captures the reaction of volatility to real exchange rate shocks, while  $\sigma_\sigma$  represents an autonomous volatility component.

As shown in Appendix B.1, a specific constraint ( $\rho > 2\alpha_\sigma$ ) must be satisfied, over an infinite time horizon, to ensure the convergence of the semidefinite integral expressing the policy-maker's loss. In words, convergence is satisfied either if the intertemporal discount rate is sufficiently high or if the degree of mean reversion in stochastic volatility is sufficiently low (i.e. if there is significant persistence in real exchange rate volatility).

Figure 5 here

Figure 5 summarizes the main results from our numerical simulations.<sup>31</sup> The upper graphs in this figure display the sensitivity of the value function with respect to  $\sigma_q$  and  $\alpha_\sigma$ , and to  $\sigma_\sigma$  and  $\sigma_q$ , under active real exchange rate targeting. The two intermediate graphs display the same information when the policy-maker does not follow any active stabilization rule. The lower graphs, finally, plot the sensitivity of  $(J - J_0)$  with respect to the same set of parameters.

Focusing on the two intermediate graphs we observe that, under a non intervention scenario, the value function is strictly increasing in both volatility components (right-hand side graph); moreover, at relatively high levels of  $\sigma_q$ , the value function is increasing in the degree of mean reversion of stochastic volatility (left-hand side graph). While the former result is fully in line with economic intuition, the latter is slightly counter-intuitive since one would a priori expect the policy-maker's loss to be higher the higher is the degree of volatility persistence. The two upper graphs in Figure 5 display identical response surfaces, while their height on the vertical axis is consistently lower. Therefore, although the sensitivity of the value function to these parameters is unaffected by the policy strategy, the policy-maker's loss turns out to be unambiguously lower under real exchange rate targeting (see, at this purpose, also the lower graphs plotting  $J - J_0$  as a function of both sets of parameters).

The case for real exchange rate targeting is not so strong as Figure 5 would seem to suggest. All numerical simulations reproduced in this figure assume a relatively low value for the policy-maker's intertemporal discount rate ( $\rho = 0.02$ ). In sharp contrast with the constant volatility case, however, these results are not robust to alternative values for this parameter. Actually, selecting higher values for the discount rate (for instance  $\rho = 0.2$ ) yields a radical change in the

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<sup>31</sup>The numerical values of the parameters are chosen accordingly to what presented in Appendix D.



above results. While the slope of the response surfaces is unaffected,  $(J - J_0)$  turns out to be consistently positive, revealing that non intervention becomes now welfare increasing.

The crucial role played by the intertemporal discount rate under stochastic volatility has a strong economic intuition. As  $\rho$  gets relatively low, expected future outcomes have an increasingly large influence on total policy-maker's loss. Faced with an unpredictable volatility pattern, a farsighted policy-maker will thus opt for active real exchange rate targeting, since this allows to smooth potentially large real exchange rate fluctuations. The converse argument holds when  $\rho$  gets relatively high. The more the policy-maker is myopic, the less he will be concerned with negative outcomes occurring very late in the future. Therefore, beyond a given discount rate threshold, non intervention becomes welfare increasing.

Taken as a whole, numerical simulations carried out in the present section suggest that, once different parameters configurations are considered, neither active intervention nor a purely passive rule emerge as first-best strategies under all circumstances. The underlying assumption about real exchange rate volatility, moreover, makes a great difference when searching for structural influences affecting the optimal policy choice.

In the constant volatility case, a critical volatility threshold and the degree of mean reversion play a crucial role. Whenever volatility is particularly high and strong nominal rigidities prevent a rapid self-adjusting process, real exchange rate targeting is welfare increasing. Conversely, if volatility is not particularly high and low nominal rigidities lead to a faster absorption of real exchange rate misalignments, active stabilization becomes welfare decreasing.

Allowing volatility to be driven by a stochastic process, the relevance of the above factors is instead significantly reduced, while the intertemporal discount rate displays a major influence in shaping the net benefits associated to alternative policy options.

## 6 Concluding remarks

Active policies of real exchange rate management, either in terms of PPP rules or in terms of more aggressive strategies aimed at increasing external competitiveness, have been quite common in many developing countries since the late 60's. The diffusion of real exchange rate targeting rules has motivated a large number of theoretical contributions, starting with Dornbusch (1982) seminal paper, addressing the macroeconomic consequences of these policies.

The main point raised by earlier work in this area was to highlight many destabilizing influences induced by the removal of a nominal anchor, emphasizing how the nature of these effects is crucially affected by the degree of capital mobility. Although displaying a rich macroeconomic structure, this work suffered from various shortcomings, since it relied on a comparative static approach and lacked explicit microeconomic foundations. More recent contributions in this lit-

erature overcame these drawbacks, addressing the macroeconomic implications of real exchange rate targeting inside dynamic intertemporal optimizing models and outlining new potentially destabilizing effects driven by self-fulfilling expectations. Although marking a significant progress, this research has still been rather elusive as regards a detailed welfare comparison between active stabilization and alternative non-intervention strategies. Moreover, since the optimization problem is typically solved from the standpoint of a representative agent, this work appears to be poorly equipped to yield concrete policy prescriptions.

This paper contributes to the real exchange rate targeting literature focusing on a PPP rule where the policy-maker aims at stabilizing external competitiveness at a pre-specified equilibrium level. In line with some recent theoretical contributions, we assume that this level corresponds to a social optimum for the domestic economy. Moreover, following recent empirical work on PPP, we assume that, independently from an active stabilization policy, the real exchange rate would anyway converge towards this steady-state equilibrium level.

While sharing some relevant similarities with latest research, this paper innovates upon the existing literature in two main respects. First, we address the optimization problem inside a stochastic rather than in a deterministic framework. This feature is particularly relevant in the present context since, as widely documented, real exchange rates typically exhibit a high degree of persistence and volatility. Second, we focus on the policy-maker's incentive to pursue an active stabilization policy. This allows an accurate evaluation of the basic policy tradeoff implied by real exchange rate targeting, where the gains from a greater flexibility to cope with various stochastic shocks are bound to raise significant destabilizing effects on other macroeconomic variables.

The main results achieved in this paper may be summarized as follows

Assuming a “liquidity effect” à la Dornbusch (1976), we show that the policy-maker is able to induce a faster real exchange rate adjustment when various stochastic shocks generate persistent departures from long run PPP. Given the policy-maker's loss function, we solve his dynamic optimization problem and derive a policy rule exploiting short run monetary nonneutralities. We then characterize the main properties of the above rule, both over a finite and over an infinite time horizon. As discussed in Section 4, the main properties of this stabilization policy fully reflect economic intuition. More specifically, deviations of the domestic interest rate from its target are larger (implying a more intense stabilization policy), the lower the degree of autonomous convergence to long run PPP. The intensity of real exchange rate stabilization, moreover, is strictly increasing (decreasing) in the weight assigned by the policy-maker to his external (internal) targets. Finally, lacking a “liquidity effect”, namely when real exchange rate dynamics is insensitive to the nominal interest rate differential, the optimal policy collapses to a simpler rule prescribing a constant domestic interest rate over all periods. As formally documented in Section 4, although under active stabilization the real exchange rate does not necessarily converge to its socially optimal value, the policy-maker can ensure that this will actually be the case by appropriately endogenizing its real exchange rate target.

Section 5 extends our theoretical analysis to a welfare comparison between two alternative policy strategies. The former corresponds to the optimal monetary policy rule outlined in the previous section, while the latter is represented by a non intervention scenario where the policy-maker abstains from any active stabilization rule. The motivation for this welfare analysis is twofold. On the one hand, real exchange rate targeting departs from more traditional policy prescriptions since, given long run monetary neutrality, external competitiveness can only temporarily be affected by interest rate changes. As emphasized before, on the other hand, although recognizing the unconventional character of real exchange rate targeting, the existing literature has been rather elusive in terms of welfare comparisons. Drawing on our theoretical set up, our analysis encompasses, over an infinite time horizon, both the constant and the stochastic volatility case.

Numerical simulations assuming a constant volatility reveal that, while the initial real exchange rate level is irrelevant, many structural parameters exert a significant influence. Our basic result is that, beyond a given volatility threshold, active stabilization is welfare increasing with respect to non intervention. These simulations, however, do not seem to establish a pervasive case for active targeting in the real world since this threshold is quite high compared with historical volatility values. Whenever real exchange rate targeting is welfare increasing (as a result of particularly high volatility), the gains from active stabilization are greater the lower is the mean reversion parameter.

Allowing real exchange rate volatility to be modelled as a stochastic process complements the above findings with additional relevant insights. The policy-maker's intertemporal discount rate turns now to play a major influence on the welfare effects associated to alternative policies. More specifically, if the discount rate is relatively low (i.e. assuming that the policy-maker is sufficiently farsighted), active real exchange rate targeting is unambiguously welfare increasing. On the other hand, if the discount rate is relatively high, non intervention becomes welfare increasing independently from other parameters values. Once again, these results have a strong economic intuition. A farsighted policy-maker, giving a large weight to the negative effects implied by future unpredictable volatility patterns, will anyway opt for real exchange rate targeting, at the expense of some destabilization in the domestic interest rate. The converse argument holds for a myopic policy-maker, rendering in this case non intervention welfare increasing with respect to an active monetary policy rule.

## A Minimization problem

The Hamiltonian of Problem (4) is

$$\begin{aligned} \mathcal{H} = & e^{-\rho(t-t_0)} \left( \frac{\phi}{2} (q - q_T)^2 + \frac{\eta}{2} (i - i_T)^2 \right) + J_x f \\ & + J_q (\alpha (\beta - q) - \gamma (i - i_F)) + \frac{1}{2} g' g J_{xx} + \frac{1}{2} \sigma' \sigma J_{qq} + g' \sigma J_{qx}, \end{aligned} \quad (11)$$

where  $J(x, q, t)$  is the value function and the subscripts on it indicate partial derivatives. The first order condition on  $\mathcal{H}$  for the optimal interest rate policy gives

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial i} &= e^{-\rho(t-t_0)} \eta (i - i_T) - J_q \gamma = 0 \\ \implies i^* &= i_T + e^{\rho(t-t_0)} J_q \frac{\gamma}{\eta}. \end{aligned} \quad (12)$$

Consequently, the Hamilton-Jacobi-Bellman equation can be written as

$$\begin{aligned} 0 &= J_t + \frac{\phi}{2} e^{-\rho(t-t_0)} (q_T - q)^2 - \frac{1}{2} \frac{\gamma^2}{\eta} e^{\rho(t-t_0)} J_q^2 + J_x f + J_q \alpha (\beta - q) \\ &\quad - J_q \gamma (i_T - i_F) + \frac{1}{2} g' g J_{xx} + \frac{1}{2} \sigma' \sigma J_{qq} + g' \sigma J_{qx}, \end{aligned} \quad (13)$$

whose boundary condition is

$$J(x, q, H) = 0.$$

This condition is also valid as a transversality condition when the time horizon  $H$  tends towards infinity. Since the value function often inherits its functional form from the objective function, then we try the following guess function

$$J(t, x, q) = e^{-\rho(t-t_0)} (a(x, t) q^2 + b(x, t) q + c(x, t)),$$

where  $a(x, t)$ ,  $b(x, t)$ , and  $c(x, t)$  are three functions that must be determined and whose boundary conditions are

$$a(x, H) = b(x, H) = c(x, H) = 0.$$

After substituting the guess function into Equation (13) we obtain

$$\begin{aligned} 0 &= -\rho (a q^2 + b q + c) + (a_t q^2 + b_t q + c_t) + \frac{\phi}{2} (q_T - q)^2 - \frac{1}{2} \frac{\gamma^2}{\eta} (2a q + b)^2 \\ &\quad + (a_x q^2 + b_x q + c_x) f + (2a q + b) \alpha (\beta - q) - (2a q + b) \gamma (i_T - i_F) \\ &\quad + \frac{1}{2} g' g (a_{xx} q^2 + b_{xx} q + c_{xx}) + \sigma' \sigma a + g' \sigma (2a_x q + b_x). \end{aligned}$$

Since the equality must hold for each value of  $q$ , then we can split this equation in three different equations: one for each power of  $q$ . Thus, we can write

$$\begin{cases} 0 = -\rho a q^2 + a_t q^2 + \frac{\phi}{2} q^2 - 2 \frac{\gamma^2}{\eta} a^2 q^2 + a_x f q^2 - 2 \alpha a q^2 + \frac{1}{2} g' g a_{xx} q^2, \\ 0 = -\rho b q + b_t q - \phi q_T q - 2 \frac{\gamma^2}{\eta} a b q + b_x f q + 2 \alpha a \beta q - \alpha b q - 2 a \gamma (i_T - i_F) q \\ \quad + \frac{1}{2} g' g b_{xx} q + 2 g' \sigma a_x q, \\ 0 = -\rho c + c_t + \frac{\phi}{2} q_T^2 - \frac{1}{2} \frac{\gamma^2}{\eta} b^2 + c_x f + \alpha b \beta - b \gamma (i_T - i_F) \\ \quad + \frac{1}{2} g' g c_{xx} + \sigma' \sigma a + g' \sigma b_x. \end{cases} \quad (14)$$

The value of  $a(x, t)$  can be obtained from the first equation

$$0 = a_t - (2\alpha + \rho)a + \frac{\phi}{2} - 2\frac{\gamma^2}{\eta}a^2 + a_x f + \frac{1}{2}g'ga_{xx}.$$

Since  $g$  is the only variable depending on  $x$ , then there exists a solution to this equation having the form  $a(t)$ . In this case  $a_x = a_{xx} = 0$  and the function  $a(t)$  must solve

$$\begin{aligned} 0 &= a_t + \frac{\phi}{2} - (2\alpha + \rho)a - 2\frac{\gamma^2}{\eta}a^2, \\ 0 &= a(H), \end{aligned}$$

where all parameters take non-negative values and do not depend on  $x$ .

This is a Riccati equation whose solution is given by

$$a(t) = \frac{\phi \tanh\left(\frac{1}{2}\sqrt{\Delta}(H-t)\right)}{\sqrt{\Delta} + (2\alpha + \rho) \tanh\left(\frac{1}{2}\sqrt{\Delta}(H-t)\right)},$$

as shown in Appendix C, where  $\Delta \equiv (2\alpha + \rho)^2 + 4\phi\frac{\gamma^2}{\eta}$ .

Once the value of  $a(t)$  has been determined, the second equation of System (14) gives the value of  $b(x, t)$ . It is sufficient to solve the differential equation<sup>32</sup>

$$0 = b_t - \left(\rho + \alpha + 2\frac{\gamma^2}{\eta}a(t)\right)b + 2(\alpha\beta - \gamma(i_T - i_F))a(t) - \phi q_T + b_x f + \frac{1}{2}g'gb_{xx}.$$

Another time, since  $g$  is the only function depending on  $x$ , then the solution of this equation has the form  $b(t)$  with  $b_x = b_{xx} = 0$ . Thus, we obtain

$$\begin{aligned} 0 &= b_t - \left(\rho + \alpha + 2\frac{\gamma^2}{\eta}a(t)\right)b + 2(\alpha\beta - \gamma(i_T - i_F))a(t) - \phi q_T, \\ 0 &= b(H), \end{aligned}$$

whose solution is

$$b(t) = \int_t^H (2(\alpha\beta - \gamma(i_T - i_F))a(s) - \phi q_T) e^{-\int_t^s (\rho + \alpha + 2\frac{\gamma^2}{\eta}a(\tau))d\tau} ds.$$

Given the value of  $a(t)$  and  $b(t)$  the result stated in Proposition 1 follows.

Finally, the value of  $c(x, t)$  must solve the following equation:

$$0 = c_t + c_x f + \frac{1}{2}g'gc_{xx} - \rho c + \frac{\phi}{2}q_T^2 - \frac{1}{2}\frac{\gamma^2}{\eta}b(t)^2 + \alpha\beta b(t) - \gamma(i_T - i_F)b(t) + \sigma'\sigma a(t),$$

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<sup>32</sup>The term containing  $a_x$  disappears since  $a(t)$  depends only on time.

which can be solved through the Feynman-Kač theorem as follows:

$$\begin{aligned}
c(x, t) &= \mathbb{E}_t \left[ \int_t^H \left( \frac{\phi}{2} q_T^2 - \frac{1}{2} \frac{\gamma^2}{\eta} b(s)^2 + \alpha \beta b(s) \right. \right. \\
&\quad \left. \left. - \gamma (i_T - i_F) b(s) + \sigma' \sigma a(s) \right) e^{-\rho(s-t)} ds \right] \\
&= \int_t^H \left( \frac{\phi}{2} q_T^2 - \frac{1}{2} \frac{\gamma^2}{\eta} b(s)^2 + \alpha \beta b(s) \right. \\
&\quad \left. - \gamma (i_T - i_F) b(s) + \mathbb{E}_t [\sigma(x, s)' \sigma(x, s)] a(s) \right) e^{-\rho(s-t)} ds.
\end{aligned}$$

## B Value function

Let us define

$$\begin{aligned}
A &\equiv \alpha + 2 \frac{\gamma^2}{\eta} a, \\
B &\equiv \frac{(\rho + \alpha) \eta (\alpha \beta - \gamma (i_T - i_F)) + \gamma^2 \phi q_T}{(\eta \rho + \eta \alpha + 2 \gamma^2 a) (\eta \alpha + 2 \gamma^2 a)} \eta,
\end{aligned}$$

and

$$\begin{aligned}
h_0 &\equiv i_T + \gamma \frac{2 (\alpha \beta - \gamma (i_T - i_F)) a - \phi q_T}{\eta \rho + \eta \alpha + 2 \gamma^2 a}, \\
h_1 &\equiv 2 \frac{\gamma}{\eta} a.
\end{aligned}$$

Then, the behaviour of the optimal exchange rate ( $q^*$ ) follows the stochastic differential equation

$$dq^* = A (B - q^*) dt + \sigma dW, \quad (15)$$

while the optimal interest rate for an infinite horizon is

$$i^* = h_0 + h_1 q^*.$$

Accordingly, the value function can be written as

$$\begin{aligned}
&\mathbb{E}_{t_0} \left[ \int_{t_0}^H e^{-\rho(t-t_0)} \left( \frac{\phi}{2} (q^* - q_T)^2 + \frac{\eta}{2} (h_0 + h_1 q^* - i_T)^2 \right) dt \right] \\
&= \frac{1}{2} (\phi + \eta) \int_{t_0}^{\infty} e^{-\rho(t-t_0)} h_1^2 \mathbb{E}_{t_0} [q^{*2}] dt \\
&\quad + (\phi q_T + \eta h_1 (h_0 - i_T)) \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \mathbb{E}_{t_0} [q^*] dt \\
&\quad + \frac{\phi}{2\rho} q_T^2 + \frac{\eta}{2\rho} (h_0 - i_T)^2,
\end{aligned}$$

where we just have to compute the expected value of  $q^*$  and  $q^{*2}$ . It is easy to show that the solution of Equation (15) is

$$q^*(t) = (q(t_0) - B)e^{-A(t-t_0)} + B + \int_{t_0}^t e^{-A(t-s)} \sigma' dW_s,$$

where  $q(t_0)$  is a deterministic variable. Then, the expected value of  $q^*$  and its variance are given by

$$\begin{aligned}\mathbb{E}_{t_0}[q^*] &= (q(t_0) - B)e^{-A(t-t_0)} + B, \\ \mathbb{V}_{t_0}[q^*] &= \mathbb{V}_{t_0}\left[\int_{t_0}^t e^{-A(t-s)} \sigma' dW_s\right] = \int_{t_0}^t \mathbb{E}_{t_0}\left[e^{-2A(t-s)} \sigma' \sigma\right] ds,\end{aligned}$$

and, since  $\mathbb{V}_{t_0}[q^*] = \mathbb{E}_{t_0}[q^{*2}] - \mathbb{E}_{t_0}^2[q^*]$  then we can write

$$\begin{aligned}\mathbb{E}_{t_0}[q^{*2}] &= \mathbb{V}_{t_0}[q^*] + \mathbb{E}_{t_0}^2[q^*] \\ &= \int_{t_0}^t e^{-2A(t-s)} \mathbb{E}_{t_0}[\sigma' \sigma] ds + (q(t_0) - B)^2 e^{-2A(t-t_0)} \\ &\quad + B^2 + 2B(q(t_0) - B)e^{-A(t-t_0)}.\end{aligned}$$

Now, we are able to compute the value function:

$$\begin{aligned}J &= \frac{\phi + \eta}{2} h_1^2 \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left( \int_{t_0}^t e^{-2A(t-s)} \mathbb{E}_{t_0}[\sigma' \sigma] ds \right) dt + \frac{1}{2} \frac{\phi + \eta}{\rho + 2A} h_1^2 (q(t_0) - B)^2 \\ &\quad + \frac{\phi + \eta}{\rho + A} h_1^2 B (q(t_0) - B) + \frac{q(t_0) - B}{A + \rho} (\phi q_T + \eta h_1 (h_0 - i_T)) + \frac{\phi + \eta}{2\rho} h_1^2 B^2 \\ &\quad + \frac{B}{\rho} (\phi q_T + \eta h_1 (h_0 - i_T)) + \frac{\phi}{2\rho} q_T^2 + \frac{\eta}{2\rho} (h_0 - i_T)^2,\end{aligned}$$

Without the intervention of the policy maker (i.e.  $i^* = i_F = i_T$ ) the value function has the following form:

$$\mathbb{E}_{t_0} \left[ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \frac{\phi}{2} (q^* - q_T)^2 dt \right]$$

with

$$dq^* = \alpha(\beta - q^*) dt + \sigma' dW.$$

Thus, the previous computations are modified as follows

$$\begin{aligned}&\mathbb{E}_{t_0} \left[ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \frac{\phi}{2} (q^* - q_T)^2 dt \right] \\ &= \frac{\phi}{2} \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \mathbb{E}_{t_0}[q^{*2}] dt + \frac{\phi}{2} \frac{1}{\rho} q_T^2 - \phi q_T \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \mathbb{E}_{t_0}[q^*] dt \\ &= \frac{\phi}{2} \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left( \int_{t_0}^t e^{-2\alpha(t-s)} \mathbb{E}_{t_0}[\sigma' \sigma] ds \right) dt + \frac{\phi}{2\rho} \beta^2 - \frac{\phi}{\rho} q_T \beta + \frac{1}{2} \frac{\phi}{\rho + 2\alpha} (q(t_0) - \beta)^2 \\ &\quad + \frac{\phi}{2\rho} q_T^2 - \frac{\phi}{\rho + \alpha} (\beta - q_T) (\beta - q(t_0)).\end{aligned}$$

When  $\sigma$  is constant we obtain the following simplification

$$\begin{aligned}
& \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left( \int_{t_0}^t e^{-2\alpha(t-s)} \mathbb{E}_{t_0} [\sigma' \sigma] ds \right) dt \\
&= \sigma' \sigma \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left( \int_{t_0}^t e^{-2\alpha(t-s)} ds \right) dt \\
&= \frac{1}{2\alpha} \sigma' \sigma \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left( 1 - e^{-2\alpha(t-t_0)} \right) dt \\
&= \frac{1}{\rho + 2\alpha} \frac{1}{\rho} \sigma' \sigma.
\end{aligned}$$

### B.1 The case with stochastic volatility

We refer to Equation (10) and, as show in the previosu section of this appendix, we just have to compute the expected value, in  $t_0$ , of the square term  $\sigma' \sigma$ . In Equation (10)  $\sigma$  is a scalar and, from what we have already presented in this appendix, it is easy to show that we can write

$$\begin{aligned}
\mathbb{E}_{t_0} [\sigma] &= (\sigma_0 - \beta_{\sigma}) e^{-\alpha_{\sigma}(t-t_0)} + \beta_{\sigma}, \\
\mathbb{V}_{t_0} [\sigma] &= \mathbb{V}_{t_0} \left[ \int_{t_0}^t e^{-\alpha_{\sigma}(t-s)} \sigma_q dW_q \right] + \mathbb{V}_{t_0} \left[ \int_{t_0}^t e^{-\alpha_{\sigma}(t-s)} \sigma_{\sigma} dW_{\sigma} \right] \\
&= \sigma_q^2 \mathbb{E}_{t_0} \left[ \int_{t_0}^t e^{-2\alpha_{\sigma}(t-s)} dt \right] + \sigma_{\sigma}^2 \mathbb{E}_{t_0} \left[ \int_{t_0}^t e^{-2\alpha_{\sigma}(t-s)} dt \right] \\
&= (\sigma_q^2 + \sigma_{\sigma}^2) \frac{e^{-2\alpha_{\sigma}(t_0-s)} - e^{-2\alpha_{\sigma}(t-s)}}{2\alpha_{\sigma}},
\end{aligned}$$

and so

$$\begin{aligned}
\mathbb{E}_{t_0} [\sigma^2] &= \mathbb{V}_{t_0} [\sigma] + \mathbb{E}_{t_0}^2 [\sigma] \\
&= (\sigma_q^2 + \sigma_{\sigma}^2) \frac{e^{-2\alpha_{\sigma}(t_0-s)} - e^{-2\alpha_{\sigma}(t-s)}}{2\alpha_{\sigma}} + \beta_{\sigma}^2 \\
&\quad + (\sigma_0 - \beta_{\sigma})^2 e^{-2\alpha_{\sigma}(t-t_0)} + 2\beta_{\sigma} (\sigma_0 - \beta_{\sigma}) e^{-\alpha_{\sigma}(t-t_0)}.
\end{aligned}$$



Now, as shown in the previous section, we compute the following integral<sup>33</sup>

$$\begin{aligned}
& \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left( \int_{t_0}^t \mathbb{E}_{t_0} \left[ e^{-2A(t-s)} \sigma' \sigma \right] ds \right) dt \\
= & \frac{\sigma_q^2 + \sigma_\sigma^2}{2\alpha_\sigma} \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left( \int_{t_0}^t e^{-2A(t-s)} \left( e^{-2\alpha_\sigma(t_0-s)} - e^{-2\alpha_\sigma(t-s)} \right) ds \right) dt \\
& + \beta_\sigma^2 \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left( \int_{t_0}^t e^{-2A(t-s)} ds \right) dt \\
& + (\sigma_0 - \beta_\sigma)^2 \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left( \int_{t_0}^t e^{-2A(t-s)} e^{-2\alpha_\sigma(t-t_0)} ds \right) dt \\
& + 2\beta_\sigma (\sigma_0 - \beta_\sigma) \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left( \int_{t_0}^t e^{-2A(t-s)} e^{-\alpha_\sigma(t-t_0)} ds \right) dt.
\end{aligned}$$

While the last three integral terms in the right hand side always converge, the first one converges if and only if  $\rho - 2\alpha_\sigma > 0$  (i.e., if the discount rate of the policy maker is sufficiently high). If this condition holds, then we obtain

$$\begin{aligned}
& \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left( \int_{t_0}^t e^{-2A(t-s)} \mathbb{E}_{t_0} [\sigma' \sigma] ds \right) dt \\
= & \frac{2(\sigma_q^2 + \sigma_\sigma^2)(\rho + A)}{\rho(\rho - 2\alpha_\sigma)(\rho + 2\alpha_\sigma + 2A)(\rho + 2A)} + \frac{\beta_\sigma^2}{(\rho + 2A)\rho} \\
& + \frac{(\sigma_0 - \beta_\sigma)^2}{(\rho + 2\alpha_\sigma + 2A)(\rho + 2\alpha_\sigma)} + \frac{2\beta_\sigma(\sigma_0 - \beta_\sigma)}{(\alpha_\sigma + \rho + 2A)(\rho + \alpha_\sigma)}.
\end{aligned}$$

## C Riccati differential equation with constant coefficients

In this appendix we show how to solve a Riccati differential equation having the following form:

$$\frac{\partial f(t)}{\partial t} + \gamma_0 + \gamma_1 f(t) + \gamma_2 f(t)^2 = 0, \quad (16)$$

where  $\gamma_i \in \mathbb{R}, i \in \{0, 1, 2\}$ , and with the boundary condition

$$f(H) = \gamma_H \in \mathbb{R}.$$

Since the coefficients are constant, we know two particular solutions of this equation:

$$f^*(t) = \frac{-\gamma_1 \pm \sqrt{\Delta}}{2\gamma_2},$$

---

<sup>33</sup>We recall that  $A$  must be substituted with  $\alpha$  when there is no intervention on the interest rate.

where  $\Delta \equiv \gamma_1^2 - 4\gamma_2\gamma_0$ . Nevertheless, because we want our general solution to be valid even when  $\gamma_2 = 0$ , then we chose the solution with the positive sign. In fact, in this case

$$\lim_{\gamma_2 \rightarrow 0} f^*(t) = \lim_{\gamma_2 \rightarrow 0} -\frac{\gamma_0}{\sqrt{\gamma_1^2 - 4\gamma_2\gamma_0}} = -\frac{\gamma_0}{\gamma_1},$$

which is a particular solution of the differential equation

$$\frac{\partial f(t)}{\partial t} + \gamma_1 f(t) + \gamma_0 = 0.$$

Now we consider the following transformation:

$$\phi(t) = \frac{1}{f(t) - f^*(t)} \Leftrightarrow f(t) = f^*(t) + \frac{1}{\phi(t)},$$

and, after substituting it into (16), we have:

$$\gamma_2 f^*(t)^2 + \gamma_1 f^*(t) + \gamma_0 - \frac{1}{\phi(t)^2} \frac{\partial \phi(t)}{\partial t} + \gamma_2 \frac{1}{\phi(t)^2} + 2\gamma_2 f^*(t) \frac{1}{\phi(t)} + \gamma_1 \frac{1}{\phi(t)} = 0.$$

The first three terms vanish and we have a linear first order differential equation:

$$\frac{\partial \phi(t)}{\partial t} - \sqrt{\Delta} \phi(t) - \gamma_2 = 0,$$

whose boundary condition is

$$\phi(H) = \frac{1}{f(H) - f^*(H)} = \frac{2\gamma_2}{2\gamma_2\gamma_H + \gamma_1 - \sqrt{\Delta}}.$$

The solution of the ODE in  $\phi(t)$  is

$$\phi(t) = -\frac{\gamma_2}{\sqrt{\Delta}} \left( 1 - \frac{\gamma_1 + 2\gamma_2\gamma_H + \sqrt{\Delta}}{\gamma_1 + 2\gamma_2\gamma_H - \sqrt{\Delta}} e^{-\sqrt{\Delta}(H-t)} \right),$$

and so we can use our initial transformation for obtaining the final result:

$$f(t) = \frac{-\gamma_1 + \sqrt{\Delta}}{2\gamma_2} - \frac{\sqrt{\Delta}}{\gamma_2} \left( 1 - \frac{\gamma_1 + 2\gamma_2\gamma_H + \sqrt{\Delta}}{\gamma_1 + 2\gamma_2\gamma_H - \sqrt{\Delta}} e^{-\sqrt{\Delta}(H-t)} \right)^{-1},$$

which can be simplified to<sup>34</sup>

$$f(t) = \frac{\gamma_H + (2\gamma_0 + \gamma_1\gamma_H) \frac{1}{\sqrt{\Delta}} \tanh\left(\frac{1}{2}\sqrt{\Delta}(H-t)\right)}{1 - (\gamma_1 + 2\gamma_2\gamma_H) \frac{1}{\sqrt{\Delta}} \tanh\left(\frac{1}{2}\sqrt{\Delta}(H-t)\right)}. \quad (17)$$

---

<sup>34</sup>We recall that

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

We underline that this solution is asymptotically valid when  $\Delta$  tends to zero. In this case, it is easy to compute the following limit thanks to De L'Hôpital's rule:

$$\lim_{\Delta \rightarrow 0} \frac{\tanh \frac{1}{2} \sqrt{\Delta} (H-t)}{\sqrt{\Delta}} = \lim_{\Delta \rightarrow 0} \frac{\frac{1}{4} \left(1 - \tanh^2 \frac{1}{2} \sqrt{\Delta} (H-t)\right) \frac{H-t}{\sqrt{\Delta}}}{\frac{1}{2\sqrt{\Delta}}} = \frac{1}{2} (H-t).$$

Furthermore, Solution (17) is valid even for negative values of  $\Delta$ . In this case, the function  $f(t)$  can be written as follows

$$f(t) = \frac{\gamma_H + (2\gamma_0 + \gamma_1 \gamma_H) \frac{1}{i\sqrt{-\Delta}} \tanh\left(\frac{1}{2} i\sqrt{-\Delta} (H-t)\right)}{1 - (\gamma_1 + 2\gamma_2 \gamma_H) \frac{1}{i\sqrt{-\Delta}} \tanh\left(\frac{1}{2} i\sqrt{-\Delta} (H-t)\right)},$$

and, since we know, from the Euler's formulae, that

$$\frac{1}{i} \tanh(iy) = \tan y, \quad \forall y \in \mathbb{R}$$

then we can conclude

$$f(t) = \frac{\gamma_H + (2\gamma_0 + \gamma_1 \gamma_H) \frac{1}{\sqrt{-\Delta}} \tan\left(\frac{1}{2} \sqrt{-\Delta} (H-t)\right)}{1 - (\gamma_1 + 2\gamma_2 \gamma_H) \frac{1}{\sqrt{-\Delta}} \tan\left(\frac{1}{2} \sqrt{-\Delta} (H-t)\right)}.$$

## D Numerical values

In order to carry out the simulations presented in the main text, we need to establish the range of parameter values. For this purpose we underline what follows.

1. The parameters  $q_T$  and  $i_T$  represent two targets for the central bank and they heavily depend on the policy maker's objectives. We have already shown that, when the policy maker wants to keep the real exchange equilibrium value to its optimal value  $\beta$ , it can freely operate only on one of these two targets. The results of the simulations are unaffected whether this free target is  $q_T$  or  $i_T$ . Thus, we chose to put  $q_T$  at the level shown in Equation (8) and  $i_T = 0.05$ .
2. The value of the foreign interest rate  $i_F$  is put equal to the target level of the interest rate ( $i_T$ ). The differences between the simulations carried out for the cases with  $i_F \neq i_T$  do not seem to be relevant.
3. The parameter  $\beta$  indicates the equilibrium value of the real exchange rate and, without loss of generality, it can be put equal to 1. Rescaling this parameter does not lead to relevant changes in the shape of the graphs but simply in a changing in their levels.

4. The parameter  $\alpha$  measures the force of the mean reversion effect. We underline that the economic literature finds the half-recovery period<sup>35</sup> varying between 2 and 4 years. In our case, after assuming the domestic nominal interest rate to equate the foreign nominal interest rate ( $i = i_F$ ), the expected value of the exchange rate in  $t$  is given by (with  $t_0 = 0$ )

$$\mathbb{E}_0 [q(t)] = \beta + (q_0 - \beta) e^{-\alpha t}.$$

In this case, an initial shock  $q_0 - \beta$  is half absorbed when

$$\mathbb{E}_0 [q(t)] = \frac{q_0 + \beta}{2},$$

which means that the half-recovery period ( $t^*$ ) is given by

$$t^* = \frac{1}{\alpha} \ln 2.$$

Accordingly, since we know that  $t^* \in [2, 4]$ , it must be true that  $\alpha \in [0.17, 0.35]$ . During the simulations we will use the mean value of  $\alpha = 0.2$ .

5. The parameter  $\gamma$  on the interest rate differential plays the same role as  $\alpha$  does on the exchange rate. Thus, without any empirical work helping us to set the value of  $\gamma$  we have put  $\gamma = \alpha = 0.2$ .
6. The parameter  $\rho$ , measuring the policy maker's discount rate, is put equal to 0.02. The simulations carried out for different values of  $\rho$  show that the shapes of the graphs does not change while the optimal value of the interest rate is lower for higher values of  $\rho$ . We have shown in the main text that  $\rho$  plays a crucial role when the case of a stochastic volatility is implemented.
7. The parameters  $\phi$  and  $\eta$  measuring relative weights of exchange rate and domestic interest rate, respectively, are put initially equal  $\phi = \eta = 1$  while, during the simulations, we make them vary in the set  $[0, 1.5]$ .

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<sup>35</sup> The half-recovery period is defined as the time at which an initial shock on the exchange rate is half absorbed.

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Figure 1: Behaviour of the optimal interest rate with respect to the exchange rate  $q$  and parameters  $\phi$ ,  $\alpha$ ,  $\gamma$ , and  $\eta$ .

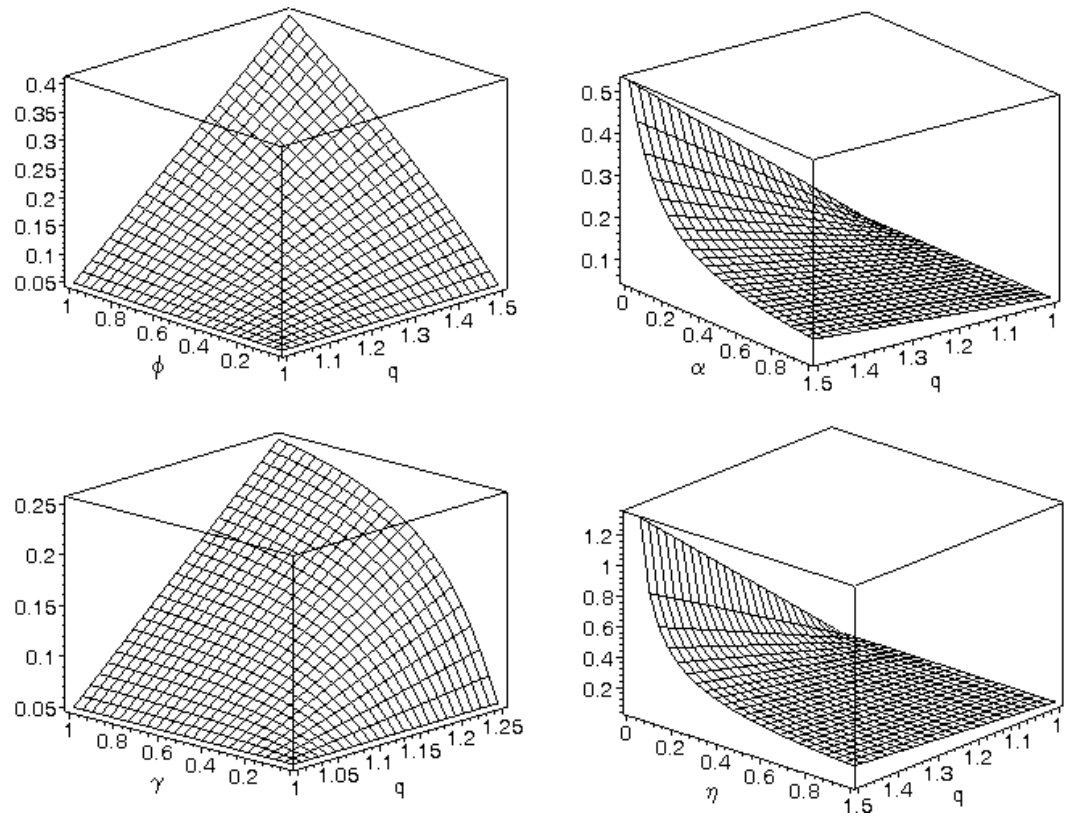
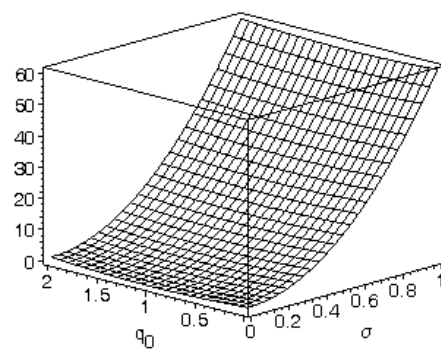
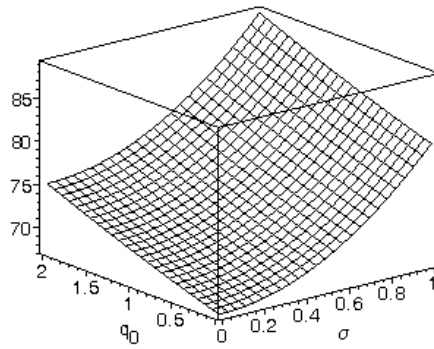


Figure 2: The value function, with respect to  $q_0$  and  $\sigma$ , with and without intervention

Value function, with intervention      Value function, without intervention



Difference between value functions with and without intervention

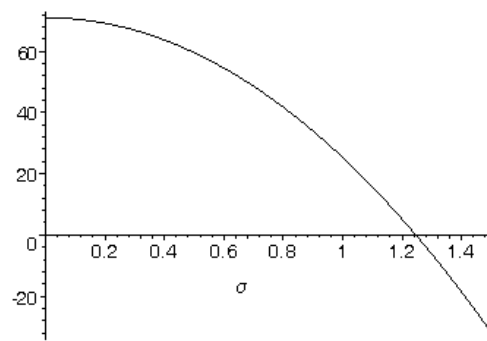
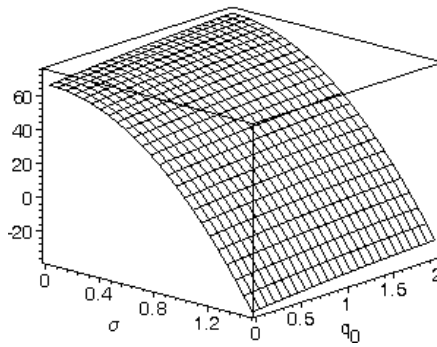
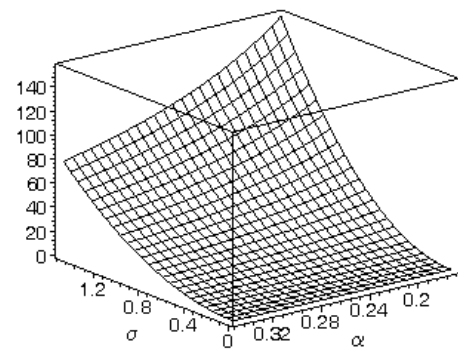
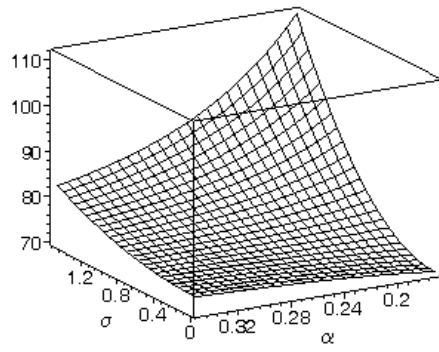




Figure 3: The value function, with respect to  $\alpha$  and  $\sigma$ , with and without intervention

Value function, with intervention      Value function, without intervention



Difference between value functions  
with and without intervention

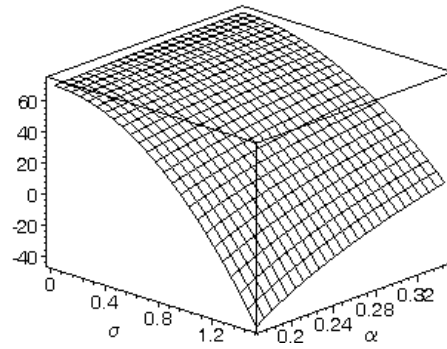


Figure 4: Difference between the value functions with and without intervention with respect to  $\sigma$  and  $\gamma$

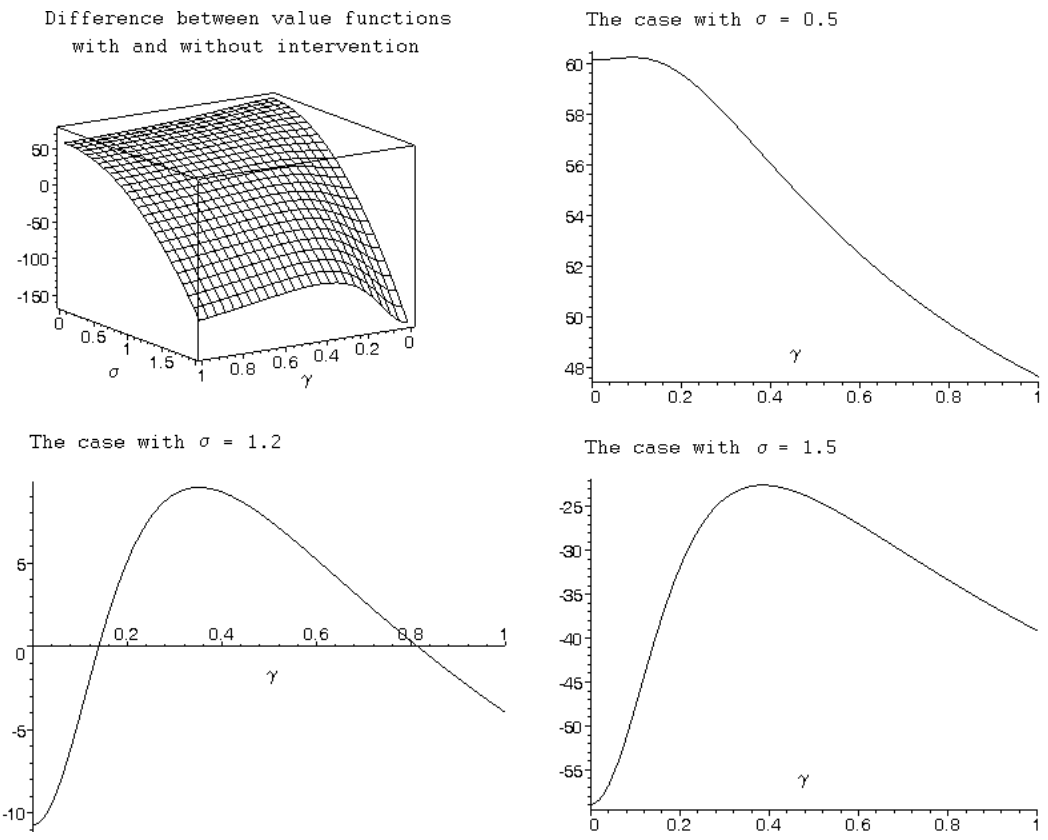
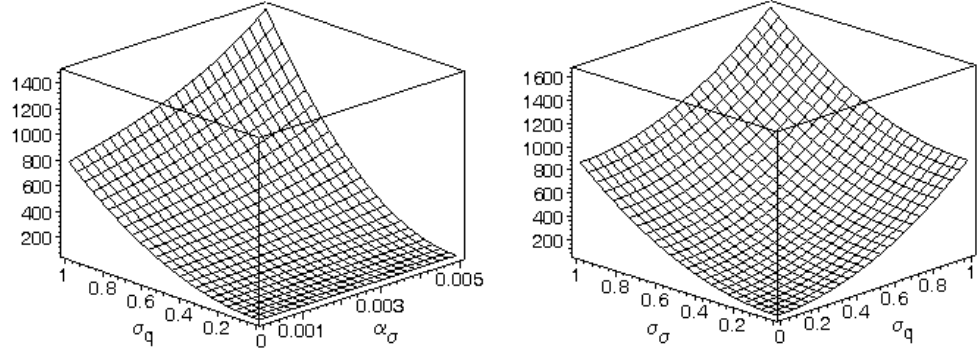
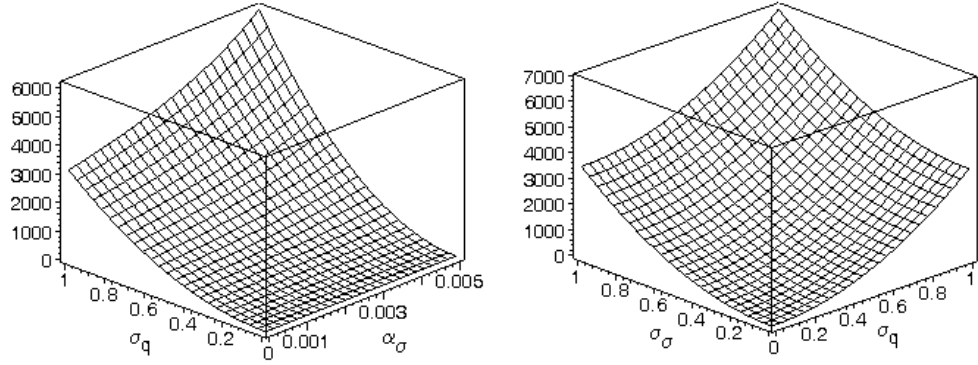


Figure 5: The value function computed with and without intervention, with respect to the parameters of the stochastic volatility

The case of stochastic volatility, the value function with intervention



The case of stochastic volatility, the value function without intervention



The difference between the value function with and without intervention

