## MULTINATIONAL CORPORATE FINANCING AND PARITY RELATIONS

M. Shahid Ebrahim\*
Ike Mathur\*\*

This draft: October 23, 2003

- \* University of Nottingham, UK.
- \*\* Southern Illinois University, Carbondale, IL, USA.

We thank Robert Berry, Kevin Dowd, Vince Hooper, Basant Kapur, Johannes Riyanto, Peter Swan and Steve Toms for their helpful comments on earlier drafts of the paper.

**Correspondence address:** M. Shahid Ebrahim

Nottingham University Business School

Jubilee Campus, Wollaton Road

Nottingham NG8 1BB United Kingdom

Tel: 44 (0) 115 846 7654 Fax: 44 (0) 115 846 6667

E-Mail: m.shahid.ebrahim@nottingham.ac.uk

### MULTINATIONAL CORPORATE FINANCING AND PARITY RELATIONS

#### **Abstract**

Given the increasing importance of financing in foreign currency denominated debt, researchers have shown the conditions under which raising capital denominated in a foreign currency becomes feasible. We extend this literature for multinational corporate financing by modeling the *agency perspective* of equity and debt. We use a modified general equilibrium approach to (i) derive conditions for *eight* equilibria under which lending would occur, (ii) demonstrate the *endogeneity* of bankruptcy, (iii) derive an *interior* optimal capital structure *contingent* on the risk of expropriation, and (iv) derive a set of pricing conditions *different* from the well-known foreign exchange parity relationships under *risk aversion* and market imperfections such as taxes. We show that irrespective of the form of debt used, *the optimal capital structure of a multinational corporation involves the pareto-efficient design of securities*.

JEL Classifications: G32; F23; F30

Keywords: Multinational financing; Pareto-optimal financial contracts

#### I. INTRODUCTION

Globalization of business has emerged as one of the dominant corporate trends in the past three decades. For many multinational corporations (MNCs), growth from overseas operations has outstripped the growth in their domestic markets. MNCs face major considerations regarding their investment and financing decisions. They are exposed to exchange rate risk, political instability, threats of expropriation and other types of risk that are unique to their multinational scope of operations. MNCs have to deal in a number of currencies, understand the intricacies of different money markets and cope with the laws of various foreign governments. Yet, they are willing to undertake these risks due to the potential for enhanced returns associated with expanding and operating in foreign markets.

An optimal financing mix for MNCs is a significant managerial consideration. In general, compared to purely domestic firms, MNCs can access more alternatives to finance themselves. They are also exposed to more risk, especially foreign exchange risk, than domestic firms. MNCs seek to obtain the best possible financing mix to attain their corporate objectives.

Several papers have addressed the financing and investment decisions of MNCs, generally using either the asset pricing or the market imperfections approaches. Early work incorporated the International Capital Asset Pricing Model (ICAPM). For example, Black (1974) models international capital market equilibrium under tax barriers to rationalize the bias towards domestic assets. Mehra (1978) prices securities in the presence of exchange rate risk. He finds that the ICAPM beta needs to take into account exchange risk in addition to the covariance of the security with the world market portfolio. If not, there will be a systematic bias in the capital budgeting decision process. Senbet (1979) shows that in the absence of differential taxes, the international financing mix is *irrelevant*. This is true even in the presence of foreign exchange risk and differential international interest rates.

However, in the presence of differential taxes, international financing mix is relevant and impacts on the value of an MNC.

Other authors have used the market imperfections approach. Lee and Zechner (1984) price risk in a *risk-neutral* framework by extending the well-known Miller (1977) analysis. They arrive at a knife-edge, i.e., a corner solution, for MNC financing. In other words, an MNC capital structure is either all debt or all equity if capital markets are integrated and tax subsidies differ across countries. However, in the presence of barriers to capital markets, Miller's (1977) theorem holds. That is, leverage is *indeterminate* for an individual firm while it is determinate for the aggregate economy. Shapiro (1984) discusses MNC financing by considering the presence or absence of corporate income taxes and flotation costs according to the following rule: Borrow in the weaker currency country in the presence of corporate taxes, and borrow in the stronger currency country in the presence of flotation costs. Rhee, et al (1985), using numerical analysis, extend Shapiro's (1984) work to the case where both taxes and flotation costs are present in a country. They conclude that the tax effect dominates the flotation effect in most economies. Hence, the MNC should still borrow in the country that has the weaker currency. 1 Madura and Fosberg (1990) argue that taxes should not be the sole criterion for an MNC's use of leverage as the risk of the project is impacted by the source of financing. Thus, there is a need for a thorough analysis of a project's payoff and risk incorporating the stochastic nature of a project's cash flow and currency exposure after debt repayment and taxes.

More recently, there is an emerging focus on developing *risk-neutral* models using variants of the Trade-off Hypothesis (see Myers, 1984). For instance, Chowdhry and Coval (1998) evaluate the optimal capital structure of an MNC subsidiary as involving intra-firm parent debt and/ or equity, while Singh and Hodder (2000) evaluate MNC capital structure

<sup>1</sup> The adjectives *weaker* or *stronger* used for currency by Shapiro (1984) and Rhee et al (1985) imply that the currency is expected to depreciate or appreciate, respectively.

using non-debt tax shields (DeAngelo and Masulis, 1980) and allowing for varying degrees of financial flexibility.

In general, any analysis of optimal MNC financing should cover three areas. First, it should incorporate the agency perspective for both the borrower (agent) and the lender (principal) to endogenously determine the necessary demand and supply functions and, hence, the equilibrium funding parameters under various financing schemes. This approach is a significant improvement over the ICAPM, which assumes exogeneity of funding parameters. Second, it should address the project selection and financing criteria separately under a risk-averse framework instead of a linear framework such as a risk-neutral model. This is because optimizing a linear objective function is similar to a linear programming model with an *indeterminate* solution in the absence of taxes and a *corner* (*knife-edge*) solution in the presence of taxes. Incorporation of bankruptcy costs in a risk-neutral framework does yield an *interior* solution as demonstrated in the Trade-off hypothesis. However, the debt ratio is still too high when contrasted with empirical evidence cited in Graham (2000).<sup>2</sup> Finally, it should segregate risk-free and risky debt to obtain the relevant mix in terms of domestic or foreign sources of financing.<sup>3</sup> This issue is important as it avoids the implicit assumption in the Trade-off Hypothesis that risky debt evolves as a continuum of risk-free debt and is pareto-optimal to it. It also helps demonstrate the endogeneity of bankruptcy.

We present a modified general equilibrium framework in this paper in which we model the conflict of interest between the *risk-averse* equity owner of an MNC (*agent*) and that of

<sup>2</sup> Recent research too finds inconsistencies with the Trade-off Hypothesis (see Korajczyk and Levy, 2003).

Risk-free debt denotes debt that is repaid fully at maturity and is independent of the state of the economy. In contrast, risky debt entails default in some states of the economy and full payment in the remaining. Thus, risky debt is state dependent.

its financiers (*principals*), while incorporating the deadweight costs of bankruptcy and taxes.<sup>4, 5</sup> In this regard we emulate Diamond (1989) and Hirshleifer and Thakor (1992) who have discussed *agency* issues stemming from the conflict of interest between equity and debt. We also develop a framework for MNC financing, based on the principle of pareto optimality, by segregating loans according to their status as risk-free or risky, unhedged or hedged and on their sources of funding (in terms of domestic or foreign or a combination of the two). The results obtained here are *contingent* on the risk of expropriation and complete project failure (henceforth termed as the *Risk*). Finally, we investigate international parity relationships under conditions of pareto optimality of risk-free financial facilities over their risky counterparts.

Two key results are derived in this paper. First, since leverage is uniquely determined under various financing schemes, optimal capital structure of an MNC involves searching for financing packages that are pareto-efficient and contingent on the Risk. In the absence of the Risk, risk-free debt (in unhedged or hedged form), consisting of combinations of domestic and foreign sources of funds, are the pareto-dominant forms of financing. In the presence of the Risk, risky debt consisting of the unhedged combination of domestic and foreign sources of financing is the only viable and, therefore, pareto-efficient solution. Bankruptcy is, thus, endogenously determined. Second, a set of pricing conditions different from the well-known foreign exchange parity relationships are derived in the absence of the Risk. These results demonstrate that our endogenously determined domestic and foreign (real) interest rates are not priced in accordance with the International Fisher Effect. The same is true for our

<sup>4</sup> Taxes constitute leakages or dissipation of resources in our model, as they are considered to be exogenous in financial economics (Finnerty, 1988). We classify our analysis as a *modified general equilibrium* one in the context of Auerbach and King (1983), who do not endogenize the tax rates as it yields an untenable result, i.e., optimal taxes solve out as a function of agent risk aversion.

For the purpose of mathematical tractability, we assume an MNC to be managed by an entrepreneur (shareholder) despite the fact that in the real world MNCs have a diversified shareholder base. This assumption does not impact on the quality of our results.

endogenously determined futures contract with respect to the exchange rate as it is not priced in accordance with the well-known expectations hypothesis. Given the substantial debate on MNC financing (see, e.g., Reeb et al, 2001) and deviations of basic parity conditions from actual observed conditions (see Cheung and Lai, 2000; Lothian and Taylor, 1996; Miffre, 2000; and Xu, 2003), our study reconciles seemingly conflicting results in the literature and provides the basis for future empirical investigations on these related issues. Our distinct results are attributed to the fact that our objective function is non-linear, where the value-additivity discussed in the capital structure theorems does not hold (Varian, 1987).

This paper is organized as follows: Section II derives the MNC owner's and lender's objective functions and market clearing conditions, while Section III evaluates the model solutions with the key results. Finally, Section IV provides the concluding remarks.

#### II. MODEL DEVELOPMENT

For simplicity and mathematical tractability, we assume a two-period model where there are three types of agents, namely an entrepreneur and two financiers. Two of these agents (the entrepreneur and a financier) reside in the domestic economy, while the third resides in the foreign economy. There are exogenous entities (in both economies) called Governments, who impose taxes on the three agents. The entrepreneur (also termed the MNC owner) has access to a foreign project with a net operating income  $(\widetilde{d_1})$  and a liquidating value  $(\widetilde{P_1})$  in period t=1, where both  $\widetilde{d_1}$  and  $\widetilde{P_1}$  are non-negative random first-order Markov processes. However, he lacks sufficient capital resources, which must be obtained from the financiers, who cannot store any excess capital resources (in the form of an initial endowment). They can either consume it or lend it out on the basis of a risk-free or a risky loan contract. Thus, the entrepreneur has the choice of borrowing from either the

domestic or the foreign lender or both. He can also hedge part of his payoffs (net of taxes in the foreign consumption units) by negotiating a futures contract with the foreign lender. However, he has limited liability. Therefore, in the event he cannot repay the full obligation of the loans, he defaults and forfeits the residual value of the project to the financiers.<sup>6</sup> The analysis is carried out by modeling the objective functions of the entrepreneur and the financiers, maximizing their respective expected utilities at t=0. The optimal contracts are then determined by evaluating their respective social welfare under the alternative financing schemes.

# II.a. Modeling the Entrepreneur as Agent of Financiers

The goal of the entrepreneur is to optimize the expected utility of consumption:<sup>7</sup>

Max. 
$$E_0 \{U(c_0) + \gamma U(c_1)\}$$
  
 $(in c_0, c_1, s, Q_d, Q_f)$ 

subject to the constraints

$$c_0 = w_0 - s e_0 P_0 + Q_d + e_0 Q_f$$
 (1)

$$\widetilde{c}_{1} = \widetilde{w}_{1} + s \left[\widetilde{e}_{1} \left(\widetilde{d}_{1} + \widetilde{P}_{1}\right)\right]_{AT} - Q_{d} \left(1 + \widetilde{r}_{d}\right)_{AT} - Q_{f} \left[\widetilde{e}_{1} \left(1 + \widetilde{r}_{f}\right)\right]_{AT}$$

$$(2)$$

$$\widetilde{c}_{1} = \widetilde{w}_{1} + \left\{ \widetilde{e}_{1} \left[ s \left( \widetilde{d}_{1} + \widetilde{P}_{1} \right) - Q_{f} \left( 1 + \widetilde{r}_{e} \right) \right] \right\}_{AT} - Q_{d} \left( 1 + \widetilde{r}_{d} \right)_{AT}$$

$$(2')$$

$$\widetilde{c}_{1} = \widetilde{w}_{1} + \{F[s(d_{1}^{*} + P_{1}^{*}) - Q_{f}(1 + r_{f}^{*})] + \widetilde{e}_{1}[s((\widetilde{d}_{1} - d_{1}^{*}) + (\widetilde{P}_{1} - P_{1}^{*})) - Q_{f}(\widetilde{r}_{f} - r_{f}^{*})]\}_{AT}$$

$$- Q_{d}(1 + \widetilde{r}_{d})_{AT}$$
(2a)

A domestic and/or foreign loan has been assumed in this model. We do not consider a foreign loan swapped into domestic loan as it avoids duplication of the analysis. Finally, there is also the possibility of a facility akin to a cross-currency loan in the real world. This latter alternative is indirectly evaluated in this paper by hedging the foreign exchange loan using a futures contract.

The consumption levels of the agents in our model at the two time periods are evaluated as the residual of endowments/ payoffs after payments towards purchase/ debt obligations. We abstract away from the fact that the consumption baskets of domestic agents differ from that of foreign agents, as this is not the focus of this paper.

where  $E_0\{\cdot\}$  is the expectation operator at time 0,  $U(\cdot)$  is a differentiable and quasi-concave utility function,  $c_0$  is the consumption of entrepreneur at t = 0 (in *real* units),  $\tilde{c_1}$  is the stochastic consumption of entrepreneur at t = 1 (in *real* units),  $w_0$  is the endowment at t = 0,  $\widetilde{w_1}$  is the risky endowment at t=1,8  $\gamma$  is the discount factor, s is the fractional investment in the foreign project,  $Q_d$  is the amount of resources borrowed domestically,  $Q_f$  is the amount of resources borrowed from a foreign source, e<sub>0</sub> is the value of the exchange rate (in domestic consumption units per foreign consumption units) at t = 0,  $e_1$  is the value of the exchange rate (in domestic consumption units per foreign consumption units) at t = 1,  $P_0$  is the price of the foreign project (in the foreign consumption units) at t = 0,  $\widetilde{r_d}$  is the *real* domestic interest rate,  $\widetilde{r_f}$  is the *real* foreign interest rate,  $r_f^*$  is the optimal real foreign interest rate hedged,  $\widetilde{d_1}$  is the net operating income (NOI) of the project received at t = 1 (in the foreign consumption units),  ${}^{9}d_{1}^{*}$  is the optimal NOI of the project hedged at t = 1,  $\widetilde{P}_{1}$  is the liquidating value of the project at t = 1 (in the foreign consumption units),  $P_1^*$  is the optimal liquidating value of the project hedged at t = 1, F is the price of the futures contract for hedging the optimal  $[s(d_1^* +$  $P_1^*$ )- $Q_f(1+r_f^*)$ ]<sub>AT</sub> amount of foreign consumption units,<sup>10</sup> and AT represents after-tax terms.

The budget constraint at t = 0 (Equation 1) illustrates the initial consumption utilizing the endowment  $(w_0)$  after deducting  $(se_0P_0)$  for the purchase of s fraction of the MNC

<sup>8</sup> A risky endowment enables us to incorporate systematic risk and reduce unsystematic risk.

<sup>9</sup> The MNC faces transaction risk when the NOI and/or the liquidating value of the project are a function of exchange rates. However, this risk can be reduced with a futures contract as demonstrated in Equation (14) in Section III.

Both F and [s  $(d_1^* + P_1^*) - Q_f (1 + r_f^*)$ ] are evaluated *endogenously* in our model as demonstrated in Equations (14), (8) (10a), (10b) and (11b).

financed, respectively, by domestic and foreign loans of  $Q_d$  and  $e_0Q_f$ . The budget constraint at t=1 (Equations 2/2') incorporates consumption from the future risky endowment  $(\widetilde{w_1})$  in addition to the after-tax payoffs of the same s fraction of the MNC  $(s[\widetilde{e_1}(\widetilde{d_1}+\widetilde{P_1})]_{AT})$  after deducting the respective domestic  $(Q_d(1+\widetilde{r_d})_{AT})$  and foreign  $(Q_f[\widetilde{e_1}(1+\widetilde{r_f})]_{AT})$  loan payments with interest. Finally, Equation (2a) extends the above budget constraint to a special case, where the entrepreneur is able to hedge part of the net payoffs by negotiating a futures contract with the foreign lender.

The Lagrangian L using Equations (1) and (2) can be written as follows:

$$\begin{split} L &= E_0 \{ [U(c_0) + \gamma U(\widetilde{c_1})] + \lambda_0 [w_0 - s e_0 P_0 + Q_d + e_0 Q_f - c_0] \\ &+ \lambda_1 \gamma [\stackrel{\sim}{w_1} + s [\stackrel{\sim}{e_1} (\widetilde{d_1} + \stackrel{\sim}{P_1})]_{AT} - Q_d (1 + \stackrel{\sim}{r_d})_{AT} - Q_f [\stackrel{\sim}{e_1} (1 + \stackrel{\sim}{r_f})]_{AT} - \stackrel{\sim}{c_1}] \}, \end{split}$$

where  $\lambda_0$  and  $\lambda_1$  are the Lagrange multipliers for the two constraints, respectively.

The First-Order Necessary Conditions (FONCs) are as follows:

(i) At the margin, the entrepreneur will only bid for the fraction (s) of the MNC, which makes the net discounted benefit equal to zero. Similarly, the entrepreneur will avoid investing in the MNC if the net discounted benefit is less than zero. This simplifies to the *demand* function for the foreign project described as follows:

$$P_{0} = \gamma E_{0} \left\{ \left[ \frac{U'(c_{1})}{U'(c_{0})} \right] \left[ \left( \frac{\widetilde{e}_{1}}{e_{0}} \right) (\widetilde{d}_{1} + \widetilde{P}_{1}) \right]_{AT} \right\}$$
(3)

Thus, the price of the MNC bid by the entrepreneur is equal to the intertemporal marginal rate of substitution [MRS $_E = \gamma \ E_0(\frac{U'(\widetilde{c}_1)}{U'(c_0)})$ ] times the (after-tax) proceeds from the net operating income plus the liquidating value grossed up by the foreign exchange returns.

The entrepreneur can reduce his foreign exchange exposure by entering into a Futures contract. The price, thus, bid for the MNC is simplified further as follows:<sup>11</sup>

$$P_{0} = \gamma E_{0} \left\{ \left[ \frac{U'(\widetilde{c_{1}})}{U'(c_{0})} \right] \left[ \left( \frac{F}{e_{0}} \right) \left( d_{1}^{*} + P_{1}^{*} \right) + \left( \frac{\widetilde{e_{1}}}{e_{0}} \right) \left( \left( \widetilde{d_{1}} + \widetilde{P_{1}} \right) - \left( d_{1}^{*} + P_{1}^{*} \right) \right) \right]_{AT} \right\}$$
(3a)

Thus, Equations (3) and (3a) depict a *risk-averse* version of the NPV decision criterion used in the finance literature. In fact, they represent a two-period version of the well-known Lucas (1978) model incorporating all relevant taxes and exchange rates.

(ii) At the margin, the net discounted benefit of borrowing a single unit of domestic consumption unit equals zero. This result is also consistent with the NPV criterion and simplifies to the *demand function* for the domestic loan:

$$1 - \gamma E_0 \{ \left[ \frac{U'(c_1)}{U'(c_0)} \right] (1 + r_d)_{AT} \} = 0$$
 (4)

Thus, the expected  $MRS_E$  times the compound factor consisting of one plus the real interest rate (after-tax) is equal to the unit value of resources loaned.

(iii) At the margin, the net discounted benefit of borrowing a single unit of foreign consumption unit equals the cost of it. This NPV criterion also simplifies to the *demand function* for a foreign loan:

$$1 - \gamma E_0 \{ \left[ \frac{U'(c_1)}{U'(c_0)} \right] \left[ \left( \frac{e_1}{e_0} \right) (1 + r_f) \right]_{AT} \} = 0$$
 (5)

The entrepreneur can reduce the foreign exchange risk in equation (5) further by hedging with a futures contract, which simplifies to the following pricing condition:<sup>12</sup>

Equation (3a) is derived by using the same Lagrangian procedure described previously by substituting Equation (2a) for Equation (2).

<sup>12</sup> If the MNC is fully diversified, then hedging exchange risk partially reduces both systematic as well as the project specific (idiosyncratic) risk. However, if the MNC is not diversified, then hedging partially reduces the unsystematic risk.

$$1 - \gamma E_0 \{ \left[ \frac{U'(\widetilde{c_1})}{U'(c_0)} \right] \left[ \left( \frac{F}{e_0} \right) (1 + r_f^*) + \left( \frac{\widetilde{e_1}}{e_0} \right) (\widetilde{r_f} - r_f^*) \right]_{AT} \} = 0$$
 (5a)<sup>13</sup>

Thus, from the perspective of the entrepreneur, the foreign loan pricing condition distinguishes itself from that for the domestic loan (Equation (4)) as it incorporates the

gross cost of foreign exchange given by  $(\frac{e_1}{e_0})$  in Equation (5). Reduction of this exchange risk with the use of the Futures contract is incorporated in Equation (5a).

The above analysis separates the investment decision from the financing one as shown in Equations (3)/(3a) and (4)/ (5)/ (5a), respectively. Nonetheless, the two decisions impact on each other through the optimal consumption parameters  $(c_0, c_1)$ .

## IIb. Modeling the Financiers as Principals

We assume that the lenders are also *risk averse* and optimize the expected utility of consumption. Given these assumptions, we can derive the following profitability conditions, i.e., *supply functions* (FONCs) for the domestic and foreign lenders, respectively:

For the domestic lender:14

$$\gamma' E_0 \{ \left[ \frac{V_d'(\vec{c'}_1)}{V_d'(c'_0)} \right] (1 + \vec{r}_d)_{AT'} \} - 1 = 0$$
(4a)

In case of a risky loan, the expectation operator in the investment and financing pricing conditions are decomposed into two integrals representing the default and normal states of the economy. This is explained in the model solution.

Here  $V_d(\cdot)$  and  $V_f(\cdot)$  represent the utility functions for the domestic and foreign lenders, respectively; while the terms  $\gamma'$ ,  $\gamma''$ ,  $c_0'$ ,  $c_0''$ ,  $c_1'$  and  $c_1''$  have the same meaning as before. It should be noted that the consumption units are in *real* terms described as follows:  $c_0' = w_0' - Q_d$  [in domestic consumption units],  $\widetilde{c_1'} = w_1'' + Q_d(1 + \widetilde{r_d})_{AT'}$  [in domestic consumption units],  $c_0'' = w_0'' - Q_f$  [in foreign consumption units]  $\Rightarrow c_{0(DC)}'' = e_0'' - Q_f$  [in domestic consumption units], and  $\widetilde{c_1''} = w_1'' + Q_f(1 + \widetilde{r_f})_{AT''}$  [in foreign consumption units]  $\Rightarrow c_{0(DC)}'' = e_0'' - Q_f(1 + \widetilde{r_f})_{AT''}$  [in domestic consumption units]  $\Rightarrow c_{0(DC)}'' = e_0'' - Q_f(1 + \widetilde{r_f})_{AT''}$  [in domestic consumption units]  $\Rightarrow c_0'' = e_0'' - Q_f(1 + \widetilde{r_f})_{AT''}$  [in domestic consumption units].

For the foreign lender:

$$\gamma'' E_0 \{ \left[ \frac{V_f'(c_1)}{V_f'(c_0)} \right] (1 + \widetilde{r_f})_{AT''} \} - 1 = 0$$
 (5b)<sup>15</sup>

$$\gamma'' E_0 \{ \left[ \frac{V_f'(c''_{1(DC)})}{V_f'(c''_{0(DC)})} \right] \left[ \left( \frac{\widetilde{e_1}}{e_0} \right) (1 + \widetilde{r_f}) \right]_{AT''} \} - 1 = 0$$
 (5bc)

$$\Rightarrow \gamma'' E_0 \{ \left[ \frac{V_f'(c''_{1(DC)})}{V_f'(c''_{0(DC)})} \right] \left[ \left( \frac{F}{e_0} \right) (1 + r_f^*)_{AT''} + \left( \frac{\tilde{e}_1}{e_0} \right) (\tilde{r}_f - r_f^*)_{AT''} \right] \} - 1 = 0$$
 (5c)

The first term on the left side of the above equations equals the discounted proceeds of a unit amount of resources loaned. Equations (4a), (5b)/ (5bc) and (5c) have the same economic intuition as their respective counterparts, namely Equations (4), (5) and (5a). However, they are evaluated from the perspective of the lenders. These results state that the intertemporal marginal rate of substitution (MRS) times the grossed up factors, consisting of the after-tax payoffs, equals a unit of resource loaned. For the debt market to be in equilibrium, the MRS for both the borrower and the lender has to adjust to a *unique* cost of resources as discussed in the loan pricing condition in Propositions 1 and 2 in Section III.

### II.b1. Domestic Risk-Free Loan

The *supply function* for the domestic risk-free loan is determined by substituting  $r_d^{RF}$  for  $r_d^{\sim}$  in Equation (4a). However, in this case we assume no default risk. The maximum amount of the domestic loan plus interest is constrained by the asset payoffs (translated in the domestic consumption units) in the worst scenario:

Min 
$$\{[(e_1) s (d_1 + P_1)] - (Q_d^{RF})_{Max} (1 + r_d^{RF})\}_{AT} > 0$$

<sup>15</sup> Equation (4a) evaluates the *supply* functions by maximizing the Lagrangian function for the domestic lender, while Equations (5b) and (5bc) do the same for the foreign lender with respect to the foreign and domestic consumption units, respectively. The period one consumption for the foreign lender translated to the domestic consumption units (c"<sub>1(DC)</sub>) in Equation (5bc) *differentiates* it from Equation (5b). Finally, Equation (5c) evaluates the hedged version of Equation (5bc).

$$\Rightarrow (Q_d^{RF})_{Max} < \frac{Min \{(\widetilde{e_1}) s (\widetilde{d_1} + \widetilde{P_1})\}_{AT}}{(1 + r_d^{RF})_{AT}}$$
(6)

The left hand side of the above inequality (Equation (6)) represents the maximum amount of the domestic resources loaned, while the right hand side represents the *risk-neutral* version of discounted minimum terminal payoffs from the project (after-tax).

### II.b2. Foreign Risk-Free Loan

The *supply function* for the domestic risk-free loan is determined by substituting  $r_f^{RF}$  for  $r_f^{RF}$  in Equation (5b). Here too, we assume no default risk. The maximum amount of the foreign loan plus interest is constrained by the asset payoffs (in foreign consumption units) in the worst scenario:

$$\operatorname{Min} \left\{ s \left( \widetilde{d}_{1} + \widetilde{P}_{1} \right) - \left( Q_{f}^{RF} \right)_{Max} \left( 1 + r_{f}^{RF} \right) \right\}_{AT} > 0$$

$$\Rightarrow \left( Q_{f}^{RF} \right)_{Max} < \frac{\operatorname{Min} \left\{ s \left( \widetilde{d}_{1} + \widetilde{P}_{1} \right)_{AT} \right\}}{\left( 1 + r_{f}^{RF} \right)_{AT}} \tag{7}$$

The left hand side of the above inequality (Equation (7)) represents the maximum amount of the foreign resources loaned, while the right hand side represents the *risk-neutral* version of discounted minimum terminal payoffs of the project (after-tax).

## II.b3. Domestic Risky Loan

With this alternative, we remove the condition that the domestic loan has to be default free and re-evaluate the *supply function* given by Equation (4a) in the following manner: 16

$$\gamma' \int_{0}^{Z_{d}} \left[ \frac{V_{d}'(\widetilde{c'_{1}})}{V_{d}'(c'_{0})} \right] \left[ \frac{k_{d} \left[ (\widetilde{d_{1j}} + \widetilde{P_{1j}})\widetilde{e_{1j}} \right]}{Q_{d}^{R}} \right] \delta j' + \gamma' \int_{Z_{d}}^{\infty} \left[ \frac{V_{d}'(\widetilde{c'_{1}})}{V_{d}'(c'_{0})} \right] \left[ 1 + r_{d}^{R} \right]_{AT'} \delta j = 1.$$
(4b)

 $<sup>\</sup>delta j' = f_1 \big( (d_{1j} + P_{1j}) e_{1j} \big) \delta ((d_{1j} + P_{1j}) e_{1j}) \text{ and } \delta j = f_2 (d_{1j} + P_{1j}) \delta (d_{1j} + P_{1j}), \text{ where } f_1 (\cdot) \text{ and } f_2 (\cdot) \text{ represent probability density functions.}$ 

The first integral involves the default alternative (below the critical state  $z_d$ ) when the lender receives a fraction  $k_d$  of the terminal cash flow  $(\widetilde{d}_{1j} + \widetilde{P}_{1j})$  in the foreign consumption units. In this equation,  $(1-k_d)$  is the sum of direct and indirect bankruptcy costs (see Kim, 1978), and  $\widetilde{e}_{1j}$  is the stochastic exchange rate in period t = 1.17 The second integral represents payment in full in the normal states of the economy.

## II.b4. Foreign Risky Loan

This alternative re-evaluates the *supply function* for the foreign risky loan given by Equation (5b) as follows:

$$\gamma'' \int_{0}^{Z_{f}} \left[ \frac{V_{f}'(\tilde{c}_{0})}{V_{f}'(c_{0})} \right] \left[ \frac{k_{f}(\tilde{d}_{1j} + \tilde{P}_{1j})}{Q_{f}^{R}} \right] \delta j + \gamma'' \int_{z_{f}}^{\infty} \left[ \frac{V_{f}'(\tilde{c}_{0})}{V_{f}'(\tilde{c}_{0})} \right] \left[ 1 + r_{f}^{R} \right]_{AT''} \delta j = 1$$
(5d)

Here too, the first integral involves default (below the critical state  $z_f$ ), while the second incorporates full payment in the normal states of the economy. No exchange rate variable is incorporated in the first integral since the default occurs in foreign consumption units.<sup>18</sup>

### **II.c. Market Clearing Conditions**

The following conditions are necessary for equilibrium:

(i) For the asset (foreign project market) to be in equilibrium:

The fractional investment (s) by the entrepreneur must equal one, i.e., s = 1. (8)

In Equations (4b) and (5d) the costs of bankruptcy are bounded in the interval [0, 1] constraining the values of k<sub>d</sub> and k<sub>f</sub> in the same interval.

Likewise, Equation (5bc) can be decomposed into two integrals similar to Equation (5d) after incorporating the (i) respective domestic consumption terms in the MRS, (ii) gross returns on the exchange as  $(\frac{e_1}{e_0})$  and (iii) the joint probability distribution between the states spanned by  $(d_{1i}+p_{1i})$  and  $e_{1i}$ .

This condition is equivalent to stating that the *supply* of the foreign project in equilibrium is one.

(ii) For the bond market (in each country) to be in equilibrium:

Resources borrowed (
$$Q_{Borrowed}$$
) must equal resources lent ( $Q_{Lent}$ ) (9)

(iii) For the foreign exchange market to be in equilibrium:

The exchange rate  $(e_0)$  adjusts to (i) endowments (in each country); (ii) demand for foreign asset and bonds (in each country).

(iv) For the foreign exchange futures market to be in equilibrium:

The optimal amount of foreign exchange futures *demanded* by the entrepreneur (after-tax) must be strictly positive and equal to the optimal amount *supplied* by the foreign lender (after-tax).

First, since the optimal amount of the foreign exchange hedged is strictly positive:

$$\Rightarrow [s(d_1^* + P_1^*) - Q_f(1 + r_f^*)]_{AT} = [(d_1^* + P_1^*) - Q_f(1 + r_f^*)]_{AT} > 0 \text{ (using Equation (8))}.$$

This result implies that futures contracting can only be used for risk-free debt and not for risky debt. This is due to the fact that the entrepreneur would not be able to honor the obligations of futures contracting under risky debt as  $[(d_{1j}+P_{1j})-Q_f(1+\stackrel{\sim}{r_f})]<0\ \forall\ j< z_f.$ 

This result implies that under foreign exchange futures hedging  $\tilde{r_f} = r_f^{RF}$  (10a) Furthermore, the optimal amount of foreign exchange futures *demanded* = optimal amount *supplied*.

$$\Rightarrow [(d_1^* + P_1^*) - Q_f(1 + r_f^*)]_{AT} = Q_f(1 + r_f^*)_{AT"}$$

$$\Rightarrow (d_1^* + P_1^*)_{AT} = Q_f(1 + r_f^*)_{AT} + Q_f(1 + r_f^*)_{AT"}$$
(10b)

#### III. MODEL SOLUTIONS

Assuming competitive markets and no initial capital constraints, *eight distinct interior* solutions (discussed below) are feasible for *risk-averse* agents under risk-free and risky debt financing if their necessary conditions are satisfied. The optimal capital structure of an MNC is contingent on the *Risk* and entails searching for the pareto-optimal financing package that minimizes the endogenous agency costs of debt and market imperfections such as taxes. This result is different from the *knife-edge* solutions discussed in the literature stemming from *risk-neutral* models. It should be noted that our solutions involve only risk-free or risky debt. Combinations of the two are not feasible as they contradict each other by their very definition. This is because one (risk-free) is assumed free of default, while the other (risky) *implicitly* involves default. Bankruptcy in our model is, thus, *endogenous* as financial contracting prices default-free loans separately from those involving default. This result is different from the one discussed in the Trade-off Hypothesis and its extensions as the hypothesis *does not* demarcate between risk-free and risky debt, i.e., risky debt is assumed to evolve as a continuum of risk-free debt with an implicit assumption of its pareto optimality.

## III.a. Necessary Conditions For Model Solutions

## **Proposition 1.**

A modified general equilibrium for *risk-free loan* financing involves *five* distinct solutions, i.e., domestic, foreign (unhedged), foreign (hedged), combination of domestic and foreign (unhedged), and combination of domestic and foreign (hedged). These solutions require satisfaction of the following necessary conditions.

(i) <u>Basic Condition</u>: The terminal payoff of a foreign project (composed of the sum of its NOI plus liquidating value) still retain some value even in the worst state of the economy in the following period. That is, Min.  $(d_{1j}+P_{1j}) > 0 \,\forall j$ .

- (ii) <u>Debt (Loan) Pricing Condition</u> requires equality between the *demand* and *supply* functions for debt financing.
  - (a) For the domestic risk-free loan:

$$\gamma E_{0} \{ [\frac{U'(\widetilde{c_{1}})}{U'(c_{0})}] (1+r_{d}^{RF})_{AT} \} = \gamma' E_{0} \{ [\frac{V_{d}'(\widetilde{c_{1}})}{V_{d}'(c_{0}')}] (1+r_{d}^{RF})_{AT'} \} = 1$$
(11)

(b) For the unhedged foreign risk-free loan:

$$\gamma E_{0}\{\left[\frac{U'(\widetilde{c_{1}})}{U'(c_{0})}\right]\left[(\frac{\widetilde{e_{1}}}{e_{0}})(1+r_{f}^{RF})\right]_{AT}\} = \gamma'' E_{0}\{\left[\frac{V_{f}'(\widetilde{c''_{1}})}{V_{f}'(c''_{0})}\right](1+r_{f}^{RF})_{AT''}\} = 1$$
(11a)

(c) For the hedged foreign risk-free loan:

$$\gamma E_{0} \{ \left[ \frac{U'(c_{1})}{U'(c_{0})} \right] \left[ \left( \frac{F}{e_{0}} \right) (1 + r_{f}^{*}) + \left( \frac{\widetilde{e}_{1}}{e_{0}} \right) (r_{f}^{RF} - r_{f}^{*}) \right]_{AT} \} \\
= \gamma'' E_{0} \{ \left[ \frac{V_{f}'(c''_{1(DC)})}{V_{f}'(c''_{0(DC)})} \right] \left[ \left( \frac{F}{e_{0}} \right) (1 + r_{f}^{*}) + \left( \frac{\widetilde{e}_{1}}{e_{0}} \right) (r_{f}^{RF} - r_{f}^{*}) \right]_{AT''} \} = 1$$
(11b)

- (d) For the combination of domestic and foreign (unhedged) risk-free loans both Equations (11) and (11a) need to be satisfied simultaneously.
- (e) For the combination of domestic and foreign (hedged) risk-free loans both Equations (11) and (11b) need to be satisfied simultaneously.
- (iii) Asset (MNC) Pricing Condition requires equality between the (a) demand function for the foreign project (Equations (3) and (3a)) and with its supply in equilibrium (Equation (8)) and the (b) demand function for the foreign exchange futures contract with its supply (Equations (10a) and (10b)).

### **Proof:**

(i) In the absence of the *Risk*, we realize  $(d_{1j}+P_{1j}) > 0 \ \forall$  state j of the economy in the following period. This condition facilitates the loan to be default-free, i.e., the

borrower is able to pay it back fully with interest even in the worst future state of the economy. This satisfies the prerequisite conditions required for domestic and foreign risk-free loans as described in Equations (6) and (7), respectively.

- (iia) Equation (11) is derived using Equations (4), (4a) and (9).
- (iib) Equation (11a) is derived using Equations (5), (5b) and (9).
- (iic) Equation (11b) is derived using Equations (5a), (5c), (9) and (10a). Q.E.D.

# Proposition 2.

A modified general equilibrium for a *risky non-recourse loan* involves *three* distinct solutions, i.e., domestic, foreign and combination of domestic-foreign. These require satisfaction of the following necessary conditions.

- (i) <u>Basic Conditions</u>: (a) The loan is structured in such a way that it involves default in some state of the economy (in the following period); (b) The interest rate contracted for risky debt is greater than that for risk-free debt; (c) The debt ratio for risky debt is greater than that for risk-free debt.
- (ii) <u>Debt (Loan) Pricing Condition</u> requires equality between the *demand* and *supply* functions for risky debt financing.
  - (a) For the domestic risky loan:

$$\gamma \int_{0}^{Z_{d}} \left[\frac{U'(\widetilde{c_{1}})}{U'(c_{0})}\right] \left[\frac{(\widetilde{d_{1j}} + \widetilde{P_{1j}}) \ \widetilde{e_{1j}}}{Q_{d}^{R}}\right] \delta j' + \gamma \int_{Z_{d}}^{\infty} \left[\frac{U'(\widetilde{c_{1}})}{U'(c_{0})}\right] \left[1 + r_{d}^{R}\right]_{AT} \delta j$$

$$= \gamma' k_{d} \int_{0}^{Z_{d}} \left[\frac{V_{d'}(\widetilde{c_{1}})}{V_{d'}(c_{0}')}\right] \left[\frac{\left[(\widetilde{d_{1j}} + \widetilde{P_{1j}}) \ \widetilde{e_{1j}}\right]}{Q_{d}^{R}}\right] \delta j' + \gamma' \int_{Z_{d}}^{\infty} \left[\frac{V_{d'}(\widetilde{c_{1}})}{V_{d'}(c_{0}')}\right] \left[1 + r_{d}^{R}\right]_{AT'} \delta j = 1 \tag{12}$$

(b) For the unhedged foreign risky loan:

$$\gamma \int_{0}^{Z_{f}} \left[\frac{U'(\widetilde{c_{1}})}{U'(c_{0})}\right] \left[\frac{[(\widetilde{d_{1j}}+\widetilde{P_{1j}})\ \widetilde{e_{1j}}]}{Q_{f}^{R}}\right] \delta j' + \gamma \int_{0}^{\infty} \frac{U'(\widetilde{c_{1}})}{U'(c_{0})} \left[\left[(\frac{\widetilde{e_{1}}}{e_{0}})\ (1+r_{f}^{R})\right]_{AT}\right] \delta j}$$

$$= \gamma'' k_{f} \int_{0}^{Z_{f}} \left[\frac{V'_{f}(c''_{1(DC)})}{V'_{f}(c''_{0(DC)})}\right] \left[\left(\frac{\widetilde{d_{1j}}+\widetilde{P_{1j}}}{Q_{f}^{R}}\right)\left(\frac{\widetilde{e_{1j}}}{e_{0}}\right)\right] \delta j' + \gamma'' \int_{Z_{f}} \left[\frac{V'_{f}(c''_{1(DC)})}{V'_{f}(c''_{0(DC)})}\right] \left[\left(1+r_{f}^{R}\right)\left(\frac{\widetilde{e_{1j}}}{e_{0}}\right)\right]_{AT''} \delta j'$$

$$= \gamma'' k_{f} \int_{0} \left[\frac{V'_{f}(\widetilde{c''_{1}})}{V'_{f}(c''_{0})}\right] \left[\frac{\widetilde{d_{1j}}+\widetilde{P_{1j}}}{Q_{f}^{R}}\right] \delta j + \gamma'' \int_{Z_{f}} \left[\frac{V'_{f}(\widetilde{c''_{1}})}{V'_{f}(c''_{0})}\right] \left[1+r_{f}^{R}\right]_{AT''} \delta j = 1 \tag{12a}$$

- (c) For the combination of domestic and unhedged foreign risky loans both Equations (12) and (12a) must be satisfied simultaneously, with the critical state  $z = z_d = z_f$  and  $(k_d + k_f) \le 1$ .
- (iii) <u>Asset (MNC) Pricing Condition</u> requires satisfaction of the *demand* function for a foreign project with the *supply* of it in equilibrium. That is,

$$P_{0} = \gamma \int_{Z_{d/f}}^{\infty} \left[ \frac{U'(\widetilde{c_{1}})}{U'(c_{0})} \right] \left[ (\frac{\widetilde{e_{1}}}{e_{0}}) (\widetilde{d_{1}} + \widetilde{P_{1}}) \right]_{AT} \delta j'$$
(13)

### **Proof:**

- (i) (a) In the presence of risk of expropriation and complete project failure, the borrower defaults in some state of the economy in the future; (b and c) The reasons why the debt ratio and the contract rate of interest for risky debt are higher than that for risk-free debt are due to the fact that the supply curve is upward sloping. The borrower prefers a high debt ratio, while the lender seeks extra compensation for it and for the states of default.
- (iia) Equation (12) is derived using Equations (4), (4b) and (9).

- (iib) Equation (12a) is derived using Equations (5), (5bc), (5d) and (9).
- (iic) The costs of bankruptcy are bounded in the interval [0, 1], thereby constraining  $(k_d + k_f)$ .
- (iii) Equation (13) is derived using Equations (3) and (8) for the case of risky domestic/unhedged foreign loan. Here, the expectations operator consists of only the integral in the normal states of the economy as the entrepreneur defaults in the states of the economy  $j < z_{d/f}$  and loses the project to the foreign government or the financiers.

Q.E.D.

# III.b. Key Results

#### Theorem 1.

MNC financing is undertaken in a pareto-efficient financing package (contingent on the *Risk* of expropriation and complete business failure) that minimizes the *endogenous* agency costs of debt and market imperfections such as taxes. The following general results can be inferred from the model: In the absence of the *Risk*, the risk-free loans comprising of the unhedged and hedged combination of domestic-foreign facilities are pareto optimal over all other facilities. However, in the presence of the *Risk*, the combination of domestic-foreign risky debt is the only viable and pareto-optimal solution.

### **Proof:**

The *eight* equilibria described in Propositions 1 and 2 are derived using optimization techniques and thus constitutes optimal financing packages. However, the equilibria are impacted differentially by the *endogenous* agency costs of debt along with market imperfections such as taxes. A *sequential search process* is employed to determine the *pareto-efficient* packages as explained below.

(i) The Pareto-optimality of risk-free facilities over risky ones in the absence of the Risk.

The risky loan pricing conditions (Equations (12) and (12a)) consist of two parts -- one in the default state of the economy (below the critical  $z_{d/f}$  state of the economy) and the other in the normal state. In the default state, the entrepreneur loses tax credits and pays the lender the residual NOI and the liquidating value of the project. But due to the direct and indirect costs of bankruptcy, the lender receives a fraction  $(k_{d/f})$  of the proceeds in contrast to the normal state of the economy, where he receives the full proceeds of the loan. Thus, in equilibrium, the bankruptcy costs are incorporated in such a way that the lender does not face them. It is the entrepreneur (MNC Owner) who bears these costs in the form of higher interest rates.

The absence of the *Risk* implies that  $(d_{1j}+P_{1j})>0 \ \forall j$ . Here, risky debt is subject to bankruptcy costs in addition to taxes, while risk-free debt is only subject to market imperfections such as taxes. Since bankruptcy costs are passed on by the lender to the entrepreneur, his welfare is lower with risky debt than with risk-free debt. Furthermore, equilibrium with risky debt is feasible only when the bankruptcy costs are capped or under a certain limit. In contrast, equilibrium with risk-free debt is feasible even when that with risky debt is unfeasible due to excessive bankruptcy costs. In this context, risk-free facilities obtained from domestic or foreign or both sources of debt are pareto-optimal to their risky counterparts, respectively.<sup>19</sup> This result is consistent with the finance literature as Myers (1977) attributes the agency cost of debt restraining firms from investing in positive NPV projects leading to the *underinvestment* issue. One way to alleviate this agency cost of debt is to collaterize debt with the tangible assets of a firm as discussed in Stulz and Johnson (1985). This is precisely what our

<sup>19</sup> See Ebrahim and Mathur (2000) for a numerical illustration of pareto-optimality of risk-free debt over risky debt in the context of domestic capital structure.

model entails in our design of the risk-free debt facility by ensuring that borrowers pledge adequate security to lenders as required by Equations (6) and (7).

The above result has the capacity to resolve the seemingly conflicting results cited in Reeb *et al* (2001) stating that U.S. based MNCs' cost of financing and debt ratio are simultaneously *lower* then their domestic counterparts (DC). The resolution to this contradiction is suggested by Bartov *et al.* (1996) and Reeb *et al.* (1998), who attribute the riskiness of U.S. based MNCs to their internationalization.<sup>20</sup> In a modified general equilibrium framework such as ours, an increase in the riskiness of the MNC leads to a low debt ratio as it reduces the assets pledged as security to lenders, while simultaneously retaining its ability to arbitrage imperfect capital markets to reduce the cost of debt.

(ii) The Pareto optimality of the combination of domestic-foreign loans over the remaining three from a single source of debt (in the absence of the Risk).

Among the five *risk-free* facilities described in Proposition 1, the unhedged and hedged combinations of loans are less restrictive than their counterparts derived from a single (domestic or foreign) source of debt. Since the welfare of agents in an unconstrained optimization is higher than in a constrained one, we can deduce that the unhedged and hedged combinations of risk-free loans are pareto optimal over the other three. We cannot distinguish between the economic efficiency of the unhedged versus hedged combination as they involve tradeoff of foreign exchange risk priced in the futures contract as demonstrated in Equation (14) below.

<sup>20</sup> Kwok and Reeb (2000) demonstrate that when firms from developed economies make international investments, they increase their risk and reduce their debt utilization. In contrast, when firms from less developed economies make international investments, they decrease their risk and increase their debt utilization.

The unhedged-hedged model is completely solved as we have *eight primary* endogenous parameters ( $e_0$ , F,  $Q_d$ ,  $r_d^{RF}$ ,  $Q_f$ ,  $r_f^{RF}$ ,  $r_f^*$ ,  $P_0$ ) and exactly *eight independent* debt and asset pricing conditions (Equations (11), (11a), (11b), (3) and (3a)).

(iii) The Pareto optimality of risky facilities over risk-free ones in the presence of the Risk. The presence of the Risk, i.e.,  $(d_{1j}+P_{1j})=0$  for some state j in the economy, implies the infeasibility of risk-free loan facilities. This is due to the violation of the Basic Condition of Proposition 1. In this situation, the risky loan is the only viable alternative as long as the bankruptcy costs do not deter the feasibility of equilibrium as expressed in Myers (1977). Thus, bankruptcy in our model is *endogenous* as risky debt is priced differently from risk-free debt in contrast to the Trade-off Hypothesis.

Finally, the unrestricted risky combination of unhedged facilities is pareto optimal over the remaining two derived from a single source of debt. The model is again completely determined as there six *primary endogenous* variables ( $e_0$ ,  $Q_d^R$ ,  $r_d^R$ ,  $Q_f^R$ ,  $r_f^R$ ,  $P_0$ ) and six *independent* pricing conditions (Equations (12), (12a) and (13)). Q.E.D.

### Theorem 2.

The arbitrage-free foreign exchange parity relationships *different* from those typically discussed in the international finance literature are deduced from the pareto-optimal risk-free financing package consisting of a combination of domestic-foreign and unhedged-hedged facilities in the absence of the *Risk* of expropriation and total project failure.

### **Proof:**

### Foreign Exchange Expectations:

Solving Equations (5), (5a), (5bc) and (5c) simultaneously yields the *bias* (deviation) of foreign exchange futures from the expectations hypothesis:

$$F - E_0(\widetilde{e_1}) = \left\{ \frac{Cov_0[U'(\widetilde{c_1}); \widetilde{e_1}]}{E_0(U'(\widetilde{c_1}))} \right\} = \left\{ \frac{Cov_0[V_f'(c"_{1(DC)}); \widetilde{e_1}]}{E_0(V_f'(c"_{1(DC)}))} \right\}$$
(14)

Thus, Equation (14) provides a non-linear pricing mechanism for a futures contract to reduce transaction risk. It is consistent with the empirical evidence that foreign exchange futures trade at a premium to the expected spot price (see Miffre, 2000). Further assumption of *risk neutrality* and the absence of taxes yield the well-known expectations hypothesis:

$$F - E_0(\widetilde{e_1}) = 0 \Rightarrow F = E_0(\widetilde{e_1})$$
 (14a)

## International Fisher Relation:

Solving Equations (4) and (5) simultaneously yields the following relationship:

$$\gamma E_0 \left\{ \left( \frac{U'(\tilde{c}_1)}{U'(c_0)} \right) \left[ \left[ \left( \frac{\tilde{e}_1}{e_0} \right) (1 + r_f^{RF}) \right] - \left[ 1 + r_d^{RF} \right] \right]_{AT} \right\} = 0, \tag{15}$$

Equation (15) provides a non-linear relationship between the stochastic exchange rate at t = 1, and domestic and foreign *real* interest rates. These rates are not necessarily equal, in contrast to the existing theoretical literature that assumes equality of real interest rates across the globe, but are in agreement with the empirical evidence.

The assumption of *risk neutrality* leads to a solution similar to that of Shapiro (1984):

$$E_0\{\left[\left(\frac{e_1}{e_0}\right)(1+r_f^{RF})\right]_{AT}\} = \left[1+r_d^{RF}\right]_{AT}$$
(15a)

Applying the stringent condition of no taxes yields a relationship similar to the well-known International Fisher Effect as given below:

$$E_0\left\{\left(\frac{\widetilde{e_1}}{e_0}\right)(1+r_f^{RF})\right\} = \frac{F}{e_0}(1+r_f^{RF}) = (1+r_d^{RF})$$
(15b)

### Interest Rate Parity (IRP):

Solving Equations (4a), (5b), (5bc) and (5c) simultaneously yields the following relationship:

$$\frac{F}{e_0} = \left[ \frac{(1+r_d^{RF})_{AT'}}{(1+r_f^{RF})_{AT''}} \right] \left( \frac{MRS_{dL}}{MRS_{fL(DC)}} \right)$$
(16)

where MRS represents the intertemporal marginal rate of substitution of the domestic (dL) and foreign lenders (fL), respectively, and the remaining symbols have the same meaning as before. Equation (16) indicates that futures pricing relative to the spot rate is a function of real (after tax) interest rates and the MRS of domestic and foreign lenders. This result is different from the existing literature where the relative pricing of futures and spot rates is given as a function of nominal interest rates only. The difference between our result and the existing literature is due to the fact that investors in our model optimize the utility of consumption derived from assets whose payoffs are evaluated in real terms (after taxes) instead of nominal terms. Furthermore, since tax policy, wealth (in the form of endowments), and the risk aversion of agents varies across domestic and foreign economies, Equation (16) shows that the IRP in terms of real interest rates does not hold, in general. The deviations from IRP (in terms of real interest rates) are due to the impact of differential taxes and the MRS of the agents in the two economies (which is further impacted by the wealth and risk aversion parameters across economies). Our theoretical results are supported by empirical literature indicating that real interest rates vary across the globe.

The uniqueness of our results stems from the fact that we take the exchange rate as an *exogenous* stochastic variable and evaluate the interest rates *endogenously*, whereas existing literature assumes that interest rates are *exogenous* and evaluates their impact on *endogenous* exchange rates.

### Relative Purchasing Power Parity (RPPP):

Substituting the Fisher Relation (see Fisher, 1930) in Equation (16) provides the following relationship:

$$\frac{F}{e_0} = \left(\frac{1+\pi_f}{1+\pi_d}\right) \left[\frac{(1+i_d^{RF})_{AT'}}{(1+i_f^{RF})_{AT''}}\right] \left(\frac{MRS_{dL}}{MRS_{fL(DC)}}\right)$$
(17)

where i and  $\pi$  are the nominal interest rates and expected inflation rates in domestic (d) and foreign (f) countries, respectively. Here too, our results are different from the existing RPPP

literature as explained in the case of IRP due to the optimization of utility of consumption stemming from the payoffs of assets in real terms net of market imperfections such as taxes.

Q.E.D.

#### V. CONCLUSIONS

As Eiteman *et al* (2004) point out, foreign currency capital is becoming an increasingly important source of financing for MNCs. Different authors, including Mehra (1978), Senbet (1979), Shapiro (1984), Chowdhry and Coval (1998) and Singh and Hodder (2000) have shown or derived the conditions under which it becomes feasible for an MNC to raise capital denominated in a foreign exchange. We extend the literature on this important topic of multinational financing by deriving a modified general equilibrium model for an MNC in which we (i) incorporate the conflict of interest (*agency issue*) between equity and debt, (ii) separate the investment decision from the financing decision, (iii) consider financing by both risk-free versus risky and domestic versus foreign debt and (iv) investigate the international parity relationships.

Our starting point is the consideration by the MNC of a project that is acceptable if the project's net present value is zero at the margin. The MNC will borrow either domestically or in a foreign currency provided the discounted benefits of borrowing equals the costs of it. These basic results are consistent with actual MNC investment and financing practices.

Next, we consider the conditions under which lenders would be willing to lend money to the MNC. *Eight* alternatives are considered, and conditions under which lending would occur are derived. We demonstrate the *endogeneity* of bankruptcy by segregating the risk-free and risky loans in contrast to the Trade-off Hypothesis. In the absence of the *Risk* of expropriation and total project failure, a combination of unhedged and hedged risk-free debt

from domestic and foreign sources are shown to pareto dominate all other facilities.<sup>21</sup> However, presence of the *Risk* leads to the viability and pareto optimality of the unhedged combination of risky debt from both domestic and foreign sources (as long as bankruptcy costs do not deter the equilibrium).

Next, we show that if capital markets are competitive, then opportunities for arbitrage do not exist. That is, there is no additional advantage for the MNC in seeking *only* foreign currency debt financing. Finally, we evaluate a set of pricing conditions that are *different* from the typical foreign exchange parity relationships (under the pareto optimality of a combination of risk-free facilities) to demonstrate their equivalence with the theoretical literature under a stringent set of conditions.

The model presented here holds important implications for MNCs. It demonstrates the optimal domestic and foreign currency denominated financing mix for MNCs based on the principle of Pareto optimality. Although this paper discusses risky debt in the form of junk bonds, other forms of debt such as income bonds, preferred stock and those involving esoteric features such as options can be brought into the realm of the current analysis. Irrespective of the form of debt specified in the model, the following statement holds true: Optimal capital structure of an MNC entails the Pareto-efficient design of securities.

\_

<sup>21</sup> The pareto optimality of risk-free debt over risky debt reconciles the seemingly conflicting results in empirical studies such as Reeb *et al* (2001).

#### REFERENCES

- Auerbach, A. J., and M.A. King. (1983). 'Taxation, Portfolio Choice, And Debt-Equity Ratios: A General Equilibrium Model,' *Quarterly Journal of Economics* 98(4), 587-609.
- Bartov, E. Bodnar, G.M. and Kaul, A. (1996). 'Exchange rate variability and the riskiness of U.S. Multinationals: Evidence from the breakdown of the Bretton Woods system', *Journal of Financial Economics* 42, 105-132.
- Black, F. (1974). 'International capital market equilibrium with investment barriers', *Journal of Financial Economics* 1, 337-352.
- Cheung, Y.W. and Lai, K.S. (2000). 'On the purchasing power parity puzzle', *Journal of International Economics* 52(2), 321-330.
- Chowdhry, B. and Coval, J.D. (1998). 'Internal financing of multinational subsidiaries: Debt vs. equity', *Journal of Corporate Finance* 4(1), 87-106.
- DeAngelo, H. and Masulis, R.W. (1980). 'Optimal capital structure under corporate and personal taxation', *Journal of Financial Economics* 8, 3-29.
- Diamond, D.W. (1989). 'Reputation acquisition in debt markets', *Journal of Political Economy* 97, 828-862.
- Ebrahim, M.S., and Mathur, I., (2000). 'Optimal Entrepreneurial Financial Contracting', Journal of Business Finance and Accounting 27 (9/10), 1349-1374.
- Eiteman, D.K., Stonehill, A.I. and Moffet, M.H. (2004). *Multinational Business Finance*, 10th ed., Addison-Wesley: Reading, MA.
- Finnerty, J.D. (1988). 'Financial engineering in corporate finance: An overview', *Financial Management* (Winter), 14-33.
- Fisher, I. (1930). The Theory of Interest, Macmillan: New York.
- Graham, J.R. (2000). 'How big are the tax benefits of debt'? *Journal of Finance* 55(5), 1901-1940.
- Hirshleifer, D. and Thakor, A.V. (1992). 'Managerial conservatism, project choice and debt', *Review of Financial Studies* 5(3), 437-470.
- Kim, E.H. (1978). 'A mean variance theory of optimal capital structure and corporate debt capacity', *Journal of Finance* 33(1), 45-63.
- Korajczyk, R.A. and Levy, A. (2003). 'Capital structure choice: Macro-economic conditions and financial constraints', *Journal of Financial Economics* 68, 75-109.
- Kwok, C.C.Y. and Reeb, D.M. (2000). 'Internationalization and Firm Risk: An upstream-downstream hypothesis', *Journal of International Business Studies* 31(4), 611-629.
- Lee, M. H., and Zechner, J. (1984). 'Debt, Taxes and International Equilibrium', *Journal of International Money and Finance* 3, 343-355.

- Lothian, J., and Taylor, M. (1996). 'Real exchange rate behavior: The recent float from the perspective of the past two centuries', *Journal of Political Economy* 104, 488-509.
- Lucas, R.E. (1978). 'Asset prices in an exchange economy', *Econometrica* 46(6), 1426-1445.
- Madura, J. and Fosberg, R.H. (1990). 'The impact of financing sources on multinational projects', *Journal of Financial Research* 13, 61-69.
- Mehra, R. (1978). 'On the financing and investment decisions of multinational firms in the presence of exchange risk', *Journal of Financial and Quantitative Analysis* 13, 227-244.
- Miffre, J. (2000). 'Normal backwardation is normal', *Journal of Futures Markets* 20(9), 803-821.
- Miller, M.H. (1977). 'Debt and taxes', Journal of Finance 32, 261-275.
- Myers, S. (1977). 'Determinants of corporate borrowing', *Journal of Financial Economics* 5, 147-175.
- \_\_\_\_\_\_, (1984). 'Presidential Address: The capital structure puzzle', *Journal of Finance* 39(3), 575-592.
- Reeb D.M., Mansi, S.A. and Allee, J.M. (2001). 'Firm internationalization and the cost of debt financing: Evidence from non-provisional publicly traded debt', *Journal of Financial and Quantitative Analysis* 36(3), 395-414.
- \_\_\_\_\_, Kwok, C.C.Y. and Baek, H.Y. (1998). 'Systematic risk of the multinational corporation', *Journal of International Business Studies* 29(2), 263-279.
- Rhee, S.G., Chang, R.P. and Koveos, P.E. (1985). 'The currency-of-denomination decision of debt financing', *Journal of International Business Studies* 16, 143-150.
- Senbet, L. (1979). 'International capital market equilibrium and the multinational firm financing and investment policies', *Journal of Financial and Quantitative Analysis* 14, 455-480.
- Shapiro, A.C. (1984). 'The impact of taxation on the currency-of-denomination decision for long-term foreign borrowing and lending', *Journal of International Business Studies* 15, 15-25.
- Singh, K. and Hodder, J.E. (2000). 'Multinational capital structure and financial flexibility', *Journal of International Money and Finance* 19, 853-884.
- Stulz, R. and Johnson, H. (1985). 'An analysis of secured debt', *Journal of Financial Economics* 14, 501-521.
- Varian, H.R. (1987). 'The Arbitrage Principle in Financial Economics,' *Journal of Economic Perspectives* 1(2), 55-72.
- Xu, Z.H. (2003). 'Purchasing power parity, price indices, and exchange rate forecasts', *Journal of International Money and Finance* 22, 105-130.