A note on the US/UK real exchange rate - real interest differential relation, 1921-2002

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Abstract

Using a multivariate regime switching framework and focusing on the period 1921-2002, which is characterized by different monetary regimes, we find supportive evidence of the US/UK real exchange rate – real interest differential relation, in terms of regime dependence between the two variables. The regime dependence is originated from the regime of the US/UK real exchange rate, namely the real exchange rate regime affects the real interest differential regime, and not vice versa. Thus, allowing for regime switching in the real exchange rate – real interest differential relation bridges the gap between popular theories of real exchange rate determination, which predict such a relation, and previous empirical studies, which failed to uncover such a relation for the US/UK real exchange rate.

Keywords: Real exchange rate, regime switching, real interest differential, US, UK.

JEL Classification: F30, F41.

1. Introduction

This note revisits the relation between the real exchange rate and the real interest differential for the US and the UK. Although the real exchange rate – real interest differential relation is theoretically justified by popular theories of real exchange rate determination (Dornbusch, 1976; Frenkel, 1976), empirical work addressing the existence of this relation between the US and the UK is controversial: Campbell and Clarida (1987), Meese and Rogoff (1988), Clarida and Gali (1994), and Edison and Pauls (1993) rejected the hypothesis that there is such a relation for the US/UK real exchange rate, while Baxter (1994) found some positive evidence.

In the present paper, we examine whether such a relation exists by testing for stochastic regime dependence between the US/UK real exchange rate and the real interest differential for the period 1921-2002. Previous work has indicated that both the US/UK real exchange rate, and the US and the UK real interest rates are characterized by univariate stochastic regime switching (Engel and Kim, 1999; Garcia and Perron, 1996). Using a bivariate regime switching framework, we explore whether there is a linkage between the regimes of the US/UK real exchange rate and the regimes of the real interest differential, namely whether the event of the real exchange rate being in one regime depends on the event of the real interest differential being in the same regime. We employ a Markov regime switching vector autoregression model, which captures regime switching in the bivariate relation, and find that the two variables are jointly characterized by regime switching in their volatility. Strong evidence is found that the regimes of the US/UK real exchange rate and the real interest differential are dependent. We find that this regime link is originated from the real exchange rate regime and not from the real interest differential regime, namely that the real exchange rate regime affects the real interest differential regime and not vice versa. Our findings provide supportive evidence of a US/UK real exchange rate – real interest differential relation in terms of stochastic regime switching characterizing the behavior of the two variables, and are in line with theoretical studies which content that nominal exchange rate regimes and monetary policy regimes do affect the behavior of real exchange rates and real interest differentials (Huizinga and Mishkin, 1986; Stockman, 1988; Bernanke and Gertler, 1989; Grilli and Kaminsky, 1991; Garcia and Perron, 1996; Hasan and Wallace, 1996; Canzoneri et al. 1997; de Haan and Spear, 1998; Bernanke et al. 1999). As we find that the regime of the real exchange rate affects the regime of the real interest differential, we can interpret our results as evidence that nominal exchange rate and monetary policy regime switching exercises originally an effect on the real exchange rate regime, and this regime effect is then transmitted to the real interest differential regime. Overall, allowing for regime switching in the relation between the two variables bridges the gap between popular theories of real exchange rates, which support such a relation, with previous empirical results, which failed to uncover such a relation for the US/UK real exchange rate.

The remainder of this note is as follows. The next section outlines some theoretical underpinnings in the real exchange rate – real interest differential relation, and previous empirical results for the US/UK real exchange rate. Section 3 outlines the data and discusses some features of the period under consideration. Sections 4 and 5 discuss the empirical methodology, and the results respectively. Finally, Section 6 concludes.

2. The real exchange rate - real interest differential relation and regime switching

The relation between the real exchange rate and the real interest differential is predicted by the Dornbusch (1976) model of exchange rate overshooting due to sluggish price adjustment, and by the Frenkel (1976) model which assumed that prices were flexible and stressed the link between expected depreciation of a currency and expected inflation differentials. Both theories begin with the assumption that the uncovered interest parity (UIP) holds. Here, we consider a more general uncovered interest rate parity condition permitting deviations from UIP by including an exchange risk premium:

$$E_t(s_{t+k}) - s_t = ({}_kR_t - {}_kR_t^*) + u_t \tag{1}$$

where s_t is the nominal exchange rate ($s per 1 pound), ${}_kR_t$ and ${}_kR_t^*$ denote the period t nominal yields to maturity on k-period US and UK bonds respectively, $E_t(s_{t+k}) - s_t$ is the expected change in the log nominal exchange rate between periods t and $t+k$, and u_t is the risk premium. An expression of the log real exchange rate, q_t, can be obtained by adding the term $\{[E_t(p_{t+k}) - p_{t+k}] - [E_t(p_{t+k}^*) - p_{t+k}^*]\}$ to both sides of (1):

$$E(q_{t+k}) - q_t = ({}_kr_t - {}_kr_t^*) + u_t \tag{2}$$

where p_{t+k} and p_{t+k}^* are the log prices in the US and the UK respectively, $q_t \equiv s_t - p_t + p_t^*$, and ${}_kr_t \equiv {}_kR_t - (E_t p_{t+k} - p_t)$ denotes the real interest rate (with ${}_kr_t^*$ defined similarly). Following Dornbusch (1976), Frenkel (1976), and Meese and Rogoff (1988), we assume $E(q_{t+k}) = \overline{q}_t$, where $\overline{q}_t$ is the real exchange rate that would prevail at time t if prices were fully flexible. Thus, equation (2) is written as:

$$q_t = ({}_k r_t - {}_k r_t^*) + \overline{q}_t + u_t \qquad (3)$$

As discussed in Meese and Rogoff (1988), in the Dornbusch (1976), and Frenkel (1976) models, $\overline{q}_t$ is constant. Equation (3) provides the theoretical basis for various empirical studies which tested for a statistical association between the US/UK real exchange rate and the real interest differential. Campbell and Clarida (1987) find that the US/UK real exchange rate is so volatile that only a small fraction of its movement can be explained by the real interest differential. Meese and Rogoff (1988), using conventional regression analysis and cointegration tests, failed to establish a statistically significant relation between the two variables. Edison and Pauls (1993), using error correction models to detect a long-run relation between the two variables, yielded little encouraging results. Baxter (1994), who in contrast to the previous studies focused on low-frequency components of the data, found that the contemporaneous correlation between the US/UK real exchange rate and the real interest differential is 0.25 and 0.16 at a 2-5 and a 6-32 quarter frequency band respectively.

The present study departs from the previous empirical studies by allowing for regime switching in the real exchange rate and the real interest differential. A well-known feature of rational expectations models is that regime shifts can alter the stochastic behaviour of economic variables. In the present study, we allow for regime switching in the volatility of the real exchange rate and the real interest differential (volatility regime switching), which may be associated with monetary regime shifts. A monetary regime change is defined to occur when the monetary authority responds to economic shocks in a different way (Lucas, 1976). We focus on two types of monetary regime shifts. The first is the shift in the nominal exchange rate regime. Under fixed exchange rates, the monetary authority is constrained to maintain the exchange rate around its par value, while under floating, monetary policy can be used to pursue domestic objectives. Empirically, several studies have found evidence that nominal exchange rate regime shifting is associated with volatility regime switching in the US/UK real exchange rate. Specifically, the volatility of the US/UK real exchange has been found to be higher in the post-Bretton Woods period (Hasan and Wallace, 1996; Stockman, 1988; Caporale and Pittis, 1995), which is in line with many theoretical models which contend that the nature of economic fluctuations is related to the nominal exchange rate regime. In line with the previous findings, Engel and Kim (1999) have found evidence of volatility regime switching in the univariate series of the US/UK real exchange rate for the period 1885-1995, partly due to changes in the nominal exchange rate regime, and partly due to the occurrence of major monetary events. Grilli and Kaminsky (1991) found that the real exchange rate volatility changes substantially across historical periods.

The second type of monetary regime shifts is the adoption of specific monetary targets, namely the adoption of targets for the growth rate of domestic monetary aggregates or targets for monetary variables (inflation or interest rate). The adoption of monetary targets is a regime shift because the targets, if binding, impose a constraint on the conduct of monetary policy. Thus, this regime shift is a monetary *policy* regime shift. In the US, examples of a monetary policy regime shifts include three sub-periods during the recent float, namely the period prior to October 1979, the period from October 1979 to October 1982, and the post-October 1982 period (Mishkin, 1992). The period prior to October 1979 corresponds to the Federal Reserve's targeting of interest rates, while the subsequent period through October 1982 is associated with a shift in operating procedures from targeting interest rates to targeting nonborrowed reserves in order to improve monetary control. The final period after October 1982 reflects another regime switching from focusing on nonborrowed reserves to borrowed reserves. (Huizinga and Mishkin, 1986; Mishkin, 1992; Malliaropulos, 2000; Thornton, 2004). Furthermore, Huizinga and Mishkin (1986) argue that another case of a shift in the monetary policy regime with consequences on the stochastic process of the real interest rate was in 1920s when the Federal Reserve sharply raised its discount rate twice. Several studies have shown that shifts in the monetary policy regime do affect the volatility of the US real interest rate (Bernanke et al. 1999; Bernanke and Gertler, 1989; Canzoneri et al. 1997; de Haan and Spear, 1998; Evans and Lewis, 1995; Huizinga and Mishkin, 1986). Garcia and Perron (1996) have shown that the US real interest rate is characterized by volatility regime switching over the period 1961-1986, partly due to the shifts in the monetary policy in the previously described three sub-periods during the float, and partly due to the shift to floating from fixed exchange rates. In the UK, an example of a monetary policy regime shift, which might have affected the volatility of the real interest rate, is the adoption of the inflation targeting policy in late 1992, following the exit of the pound from the Exchange Rate Mechanism (Bowen, 1995). Several authors have argued that inflation targeting is consistent with changing (reducing) the UK real interest rate volatility (Dehejia and Rowe, 2001; and Siklos and Skoczylas, 2002).[1] A second example of monetary policy regime switching in the UK was the announcement of new monetary growth targets in July 1976 by the Bank of England.[2] A third example occurred in September 1931, when the UK opted to suspend the convertibility of the pound into gold in response to financial pressures occasioned by the international crisis.[3] Finally, the volatility of real interest rates could have been affected not

[1] See also, Koedijk et al. (2000)
[2] The first formal target for monetary growth was announced on 22 July 1976 (Bank of England Quarterly Bulletin, September 1976, page 307).
[3] Evans et al. (1994), and Aziz and Prisman (2000) offer additional empirical results on the behaviour of the US and the UK real interest rates, while Bonser-Neal (1990) explores the behaviour of ex-ante

only by monetary policy regime switching but also by the nominal exchange rate regime switching. Frankel and MacArthur (1988), Johnson (1992), and Garcia and Perron (1996) have shown that the shift to flexible rates is associated with an increase of the volatility of real interest rates in the US and the UK, as well as the volatility of the real interest differential.

As there is both theoretical justification and empirical evidence of volatility regime switching in the univariate series of the US/UK real exchange rate and the real interest differential (univariate regime switching), it would be appropriate to explore whether there is a volatility regime switching in the relation of the two variables (bivariate regime switching). If a bivariate regime switching does exist, then we could explore the statistical association of the regimes of the two variables by testing whether the regime of one variable depends on the regime of the other variable. Evidence of regime dependence is consistent with the existence of a statistical association between the two variables in terms of their unobserved regimes which characterize the dynamics of each variable.

3. Data and period characteristics

The data set comprises monthly observations over the period January 1921-December 2002 (1921:1-2002:12), giving a total of 984 monthly observations. This relatively large number of monthly observations ensures high power of the statistical tests (Lothian and Taylor, 1997). In addition, this period is characterized by sub-periods of different nominal exchange rate regimes (floating and fixed exchange rates), and various major monetary and political events. Floating exchange rates prevailed over the periods 1921:1-1925:4, 1931:9-1939:8, and 1973:3-present. The Gold Exchange Standard and the Bretton Woods System of fixed exchange rates applied during the periods 1925:5-1931:8, and 1949:10-1972:5, while during the period 1939:9-1949:9 there were wartime controls on the pound exchange rates. In addition to these regimes, the following major monetary events took place:

1. Mid- to late 1933: The US ceases stabilizing the price of gold.
2. September 1939: The UK devalues the pound.
3. July 1946: Rapid US inflation as price controls are removed.
4. September 1949, and late 1967: The pound is devalued.
5. June 1984-February 1985: This period is characterized as a bubble in the dollar.
6. September 1992: The UK leaves the Exchange Rate Mechanism.

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real interest rates across different monetary regimes. Finally, Lioui and Poncet (2004), in a recent contribution, explore real interest dynamics in a monetary economy.

Therefore, the period under consideration is characterized by conditions which, according to the previous studies discussed above, may affect the volatility of the US/UK real exchange rate and the volatility of the real interest differential.

The log of the real exchange rate, q, is defined as $q=e-p_{US}+p_{UK}$, where e is the log of the nominal rate ($s per pound), and p_{US} and p_{UK} are the logs of the US and the UK producer prices (in line with Grilli and Kaminsky, 1991). The nominal interest rates for constructing the real rates are the three-month Treasury Bill yields. The real interest differential is defined as $r=r_{US}-r_{UK}$. Data sources are reported in the Appendix. Table 1 reports descriptive statistics. As seen in the Table, the real exchange rate and the real interest differential are not normally distributed. The sample mean of both variables is positive, suggesting a real pound appreciation, and that that the US real interest rates were higher on average, over the period 1921-2002. Using the ADF test, the real exchange rate is found to be nonstationary. This indicates that PPP does not hold as a long-run equilibrium relation for the US/UK real exchange rate over the period 1921:1-2002:12, echoing the findings of many recent empirical studies.[4] Importantly, there is no cointegration between the two variables. The real interest differential and the first difference of the log real exchange rate are stationary. Thus, in empirical analysis, the first difference of the log real exchange rate and the real interest differential should be used.

4. Methodology

As the period under consideration is characterized by switching in the nominal exchange rate regimes and in the monetary policy regimes, which may affect the volatility of the US/UK real exchange rate and the real interest differential, allowing for volatility regime switching is of paramount importance in the empirical analysis. Thus, in modelling the bivariate relation between the two variables, proper allowance should be made for volatility regime switching. Importantly, regime switching should be allowed for not only in each univariate series but also in the bivariate relation between the variables. To achieve this objective, we employ a bivariate Markov Switching Vector Autoregressive (MS-VAR) model. The MS-VAR model, introduced in Krolzig (1997), is a multivariate generalisation of the univariate Markov Switching autoregressive model introduced by Hamilton (1989). In this study, we employ the following MS-VAR model with regime-dependent variance-covariance matrix:

[4] See Taylor et al. (2001) for a scholarly review.

$$\Delta q_t = a_{\Delta q} + \sum_{k=1}^{p} a_{\Delta q,\Delta q,k} \Delta q_{t-k} + \sum_{k=1}^{p} a_{\Delta q,r,k} r_{t-k} + \Sigma(s_t) u_{\Delta q,t} \quad u_{\Delta q,t} \sim \text{NID}(0,\ 1)$$

$$r_t = a_r + \sum_{k=1}^{p} a_{r,\Delta q,k} \Delta q_{t-k} + \sum_{k=1}^{p} a_{r,r,k} r_{t-k} + \Sigma(s_t) u_{r,t} \qquad u_{r,t} \sim \text{NID}(0,1) \tag{4}$$

where s_t is the unobservable regime, assumed to follow an irreducible ergodic m-regime Markov process with constant transition probabilities p_{ij} given by

$$p_{ij} = \Pr(s_{t+1} = j \mid s_t = i), \quad \sum_{j=1}^{m} p_{ij} = 1, \quad \forall i, j \in \{1,...,m\} \tag{5}$$

These probabilities are gathered in a transition probability matrix **P**, with a typical element given by (5). So, **P** is given by

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix} \tag{6}$$

In model (4), we allow for regime switching in the variance-covariance matrix, namely we allow for the variances and the covariance of the two variables to vary across regimes.[5] This is in line with the theoretical and empirical studies, which document volatility regime switching in the real exchange rate and the real interest differential across different nominal exchange rate regimes, and different regimes of monetary policy. In estimating (4), the number of volatility regimes m was set equal to 2, using the Hansen (1992) test. Further, the lag length p was set equal to 3 on the basis of LR tests.[6] Maximum likelihood estimation is based on the EM algorithm. From the maximum likelihood estimation, the transition probabilities, p_{ij}, are obtained, namely the probabilities that the real exchange rate changes and the real interest differential will *jointly* move from regime i to regime j over two subsequent periods. Also,

[5] In model (4), one could allow for the autoregressive parameters to be regime-dependent too. In the present study, we refrain from allowing for regime-dependent autoregressive parameters firstly because our primary objective is to capture volatility regime switching, as suggested by previous studies, and secondly, for ensuring parsimony in the model specification.
[6] Empirical results are available upon request.

the smoothed probabilities are obtained, representing the ex-post inference about the system being in regime s_t at date t.

To test for regime dependence between the two variables, we proceed as follows. In this bivariate formulation with two regimes, the number of primitive regimes, s_t^*, is four:

s_t^*=1: Δq —low volatility, r —low volatility

s_t^*=2: Δq —low volatility, r —high volatility

s_t^*=3: Δq —high volatility, r —low volatility

s_t^*=4: Δq —high volatility, r—high volatility

As discussed in Hamilton and Lin (1996), the transition probabilities p_{ij}^* could be restricted to fit the independence hypothesis. For example, focusing on p_{24}^*, if the volatility regimes of Δq and r are independent, then $p_{24}^* = p_{12}^{\Delta q} p_{22}^{r}$. To test the null hypothesis of independence, we estimate the MS-VAR model (4), imposing no restriction on the transition probabilities, and obtain the log likelihood function of the unrestricted model, L_U. We next estimate the model by imposing the restricted transition probability matrix **P**, with elements such as $p_{14}^* = p_{12}^{\Delta q} p_{12}^{r}$, and obtain the log likelihood of the restricted model, L_R. Then, we calculate a Likelihood Ratio test, LR=-2*(L_R-L_U). Under the null, this test has a $\chi^2(d)$ distribution, where d is the number of additional parameters under the alternative hypothesis.

5. Results

Table 2 reports the results from estimating the (unrestricted) MS-VAR model (4). The starting point is to test the null hypothesis of no volatility regime switch (i.e. one volatility regime) against the alternative of a volatility regime switching (two volatility regimes). The null hypothesis is equivalent to homoscedasticity and thus, to the linear VAR. The non-standard LR test and the approach proposed in Davies (1987) are employed to test this hypothesis. As shown in Table 2, the log likelihood value of the MS-VAR is significantly higher than the log likelihood value of the linear VAR. The LR test statistic is 1137.6, which suggests that the null is rejected, even by invoking the upper bound of Davies (1987). In addition, the Akaike Information Criterion (AIC) is also in favour of the MS-VAR. Thus, the US/UK real exchange rate changes and the real interest differential are jointly characterized by volatility regime switching. The standard deviation of real exchange rate changes ($\sigma_{\Delta q}$) and of the real interest differential (σ_r) are both higher in regime 1 than in regime 2. In regime 1, $\sigma_{\Delta q} = 0.029$ and $\sigma_r = 0.016$, while in regime 2, $\sigma_{\Delta q} = 0.009$ and $\sigma_r = 0.008$. This suggests that regime 1 is identified as the 'high' volatility regime, and regime 2 as the 'low' volatility regime. The probability that a month of high volatility will be followed by a month of high volatility (i.e. the transition probability from regime 1 to regime 1, p_{11}) is 0.9335. The transition probability p_{22} is 0.9344. Thus, both regimes are quite persistent.

The regime classification is represented in Figure 1, which plots the smoothed probabilities of both regimes. The periods of the high volatility regime roughly coincide with the periods of floating exchange rates, while the periods of the low volatility regime roughly coincide with the Gold Exchange Standard and Bretton Woods periods of fixed exchange rates. This result is in line with previous evidence suggesting that real exchange rate variability is higher in periods of floating exchange rates.[7] During the periods of low volatility, there are some spikes in the first panel, indicating short-lived jumps in the high volatility regime. These jumps correspond to major monetary events, namely the 1946 US inflation, and the two pound devaluations in 1949 and 1967, and provide empirical evidence that important monetary events do affect the volatility regime of the two variables even during a period of fixed nominal exchange rates. Thus, the MS-VAR model is capable of capturing monetary events which caused the two variables to temporarily move into the high volatility regime during the Bretton Woods period.

The contemporaneous correlation between the two variables is significantly different across regimes, with the correlation in regime 2 (0.872) being more than 8 times higher than the correlation in regime 1 (0.104). Thus, the strength of the relation between the two variables is regime-dependent, with the strength being much higher in regime 2. This result is plausible, since in periods of fixed exchange rates (regime 2), the exchange risk premium is relatively small and thus, the contemporaneous link between real exchange rate changes and the real interest differential becomes stronger. In contrast, during periods of floating exchange rates (regime 1), the risk premium increases entailing a loosening in the relation between the two variables. This finding of a regime-dependent contemporaneous correlation may be seen as bridging the gap between Meese and Rogoff (1988) who found no correlation between the two variables during the post-Bretton Woods period, and several theoretical monetary models of exchange rate determination. Baxter (1994), using band-pass filters and data for the recent float, found evidence of positive correlation between the ex-post short-term interest differential and the US/UK real exchange rate, with correlation values of 0.25 and 0.16 for 2-5 and 6-32 quarters frequency bands respectively. Our measure of contemporaneous

[7] See Baxter and Stockman (1989), and Hasan and Wallace (1996).

correlation, based on a much shorter frequency band, is also positive in regime 1 (0.104) but lower than that of Baxter (1994).

We next test for the volatility regime independence hypothesis. As Table 2 shows, the log likelihood value of the restricted model is 5840.50, yielding a LR test statistic of LR=35.80 with a p-value=0.00. This suggests that the null of independence is strongly rejected in favor of the alternative of dependence. Thus, there is a volatility regime link between the US/UK real exchange rate changes and the real interest differential: the volatility regimes of the US/UK real exchange rate changes and the real interest differential are dependent. This finding is supportive of the existence of a relation between the US/UK real exchange rate and the real interest differential, which is in contrast to most previous empirical studies, but in line with popular real exchange rate theories. Consequently, allowing for bivariate regime switching in the real exchange rate – real interest differential relation bridges the gap between popular real exchange rate theories and empirical studies regarding the existence of a relation between the US/UK real exchange rate and the real interest differential.

To further explore this link, we examine if this link has originated from the volatility regime of the real exchange rate or from the volatility regime of the real interest differential. To address this issue, we estimate a univariate Markov autoregressive model for each variable:

$$\Delta q_t = a_{\Delta q} + \sum_{k=1}^{3} a_{\Delta q,k} \Delta q_{t-k} + \Sigma(s_t) u_{\Delta q,t}, \; u_{\Delta q,t} \sim \text{NID}(0,1). \quad (7)$$

$$r_t = a_r + \sum_{k=1}^{3} a_{r,k} r_{t-k} + \Sigma(s_t) u_{r,t}, \quad u_{r,t} \sim \text{NID}(0,1) \quad (8)$$

Each of these models is nested within (4). From estimating (7), we obtain the smoothed probabilities that the real exchange rate is in the high volatility regime, $p_{h,t}^{\Delta q}$, and in the low volatility regime, $p_{l,t}^{\Delta q}$, as well as the squared residuals $\hat{u}_{\Delta q,t}^2$ reflecting the volatility of the real exchange rate changes after having filtered away any own volatility regime switching effects. From estimating (8), we obtain the smoothed probabilities that the real interest differential is in the high volatility regime, $p_{h,t}^{r}$, and in the low volatility regime, $p_{l,t}^{r}$, as well as the squared residuals, $\hat{u}_{r,t}^2$ reflecting the volatility of the real interest differential after having filtered away any own volatility regime switching effects.

Then, following the procedure of Coe (2002), we run a regression in which the dependent variable is $\hat{u}^2_{\Delta q,t}$, and the independent variable is $p^r_{h,t}$, representing the probabilistic inference that the real interest differential is in the high volatility regime at any month within the sample. The corresponding regression model is given by (9a):

$$\hat{u}^2_{\Delta q,t} = b_0 + b_r p^r_{h,t} + w_t \ , \quad w_t \sim \text{NID}(0,1) \tag{9a}$$

We also run a second regression in which the independent variable is $p^r_{l,t}$, given by (9b):

$$\hat{u}^2_{\Delta q,t} = b'_0 + b'_r p^r_{l,t} + w'_t \ , \quad w'_t \sim \text{NID}(0,1) \tag{9b}$$

In model (9a), we seek to assess whether the probability of the real interest differential being in the high volatility regime, $p^r_{h,t}$, affects the volatility of the real exchange rate changes as reflected by the squared residuals $\hat{u}^2_{\Delta q,t}$. Similarly, in model (9b), we examine if the probability of the real interest differential being in the low volatility regime, $p^r_{l,t}$, affects the volatility of the real exchange rate changes as reflected by the squared residuals $\hat{u}^2_{\Delta q,t}$. Statistical significance of b_r and b'_r suggests that the volatility of real exchange rate changes is affected by the real interest differential being in the high and low volatility regimes.

In addition, we run two regressions in which the dependent variable is $\hat{u}^2_{r,t}$ and the independent variable is $p^{\Delta q}_{h,t}$ in one model, and $p^{\Delta q}_{l,t}$ in the other, namely

$$\hat{u}^2_{r,t} = b_1 + b_{\Delta q} p^{\Delta q}_{h,t} + e_t \ , \quad e_t \sim \text{NID}(0,1) \tag{10a}$$

$$\hat{u}^2_{r,t} = b'_1 + b'_{\Delta q} p^{\Delta q}_{l,t} + e'_t \ , \quad e'_t \sim \text{NID}(0,1) \tag{10b}$$

In model (10a), we seek to assess whether the probability of the real exchange rate changes being in the high volatility regime affects the volatility of the real interest differential reflected in $\hat{u}^2_{r,t}$, and in model (10b) whether the probability of the real exchange rate changes being in the low volatility regime affects the volatility of the real interest differential. Statistical significance of $b_{\Delta q}$ and $b'_{\Delta q}$ suggests that the volatility of the real interest differential is affected by the real exchange rate being in the high and low volatility regime.

After accounting for autocorrelation in the residuals, the estimated values of $b_{\Delta q}$ and $b'_{\Delta q}$ are $b_{\Delta q}$=0.0012, and $b'_{\Delta q}$ =0.0011, with t-statistics of 6.125 and 5.952 respectively. The estimated values of b_r and b'_r are b_r=0.0014 and b'_r =0.0011, with t-statistics of 1.401 and 1.702 respectively. This result suggests that $b_{\Delta q}$ and $b'_{\Delta q}$ are statistically significant, while b_r and b'_r are not at the 5% level, thereby indicating that the US/UK real exchange rate regime affects the volatility of the real interest differential, while the regime of the real interest differential does not affect the volatility of the US/UK real exchange rate. Consequently, the volatility regime link between the two variables is originated from the volatility regime of the US/UK real exchange rate. This result is in line with Johnson (1992), and Frankel and MacArthur (1988), who found that the volatility of the real interest differential has increased in the recent float. Overall, our findings indicate that a nominal exchange rate or monetary policy regime switching originally exercises an effect on the volatility regime of the US/UK real exchange rate, and this effect is then transmitted to the volatility of the real interest differential.

6. Conclusion

This note has presented empirical evidence supporting the existence of a relation between the US/UK real exchange rate and the real interest differential over the period 1921-2002. The relation holds in terms of regime dependence between the two variables: the event of the US/UK real exchange rate being in the high (low) volatility regime is dependent upon the event of the real interest differential being in the high (low) volatility regime. This dependence is originated from the real exchange rate regime, and not from the real interest differential regime. Our findings contend that monetary regime switching originally exercises an effect on the volatility regime of the US/UK real exchange rate, and this effect is then transmitted to the volatility regime of the real interest differential, thereby establishing a regime link between the two variables. Thus, allowing for regime switching in the US/UK real exchange rate – real interest differential relation is important in uncovering a statistical association between the two variables, as suggested by popular real exchange rate theories.

Appendix: Data sources:

Dollar/Pound nominal exchange rate:

1921-1985: Federal Reserve (tape)

1986-2002: IMF Financial Statistics

US producer price index:

1921-1930: Warren and Pearson (1932, table I, pp.6-10).

1931-1945: US Bureau of Labor Statistics, various issues.

1945-2002: Citibase (tape)

UK producer price index:

1921-1931: Annual Abstract of Statistics

1932-1938: Board of Trade Journal, various issues.

1939-2002: Monthly Digest of Statistics, various issues.

UK interest rate (Treasury Bills):

1921-1982: A Monetary History of the United Kingdom, 1870-1982, Volume I.

1983-2002: Datastream.

US interest rate (Treasury Bills):

1921-1923: International Abstract of Economics Statistics. ICES, UK, 1934.

1924-1946: League of Nations: Monthly Bulletin of Statistics.

1947-2002: Monthly Bulletin of Statistics, United Nations.

Figure 1: Real exchange rate – real interest differential: Smoothed probabilities

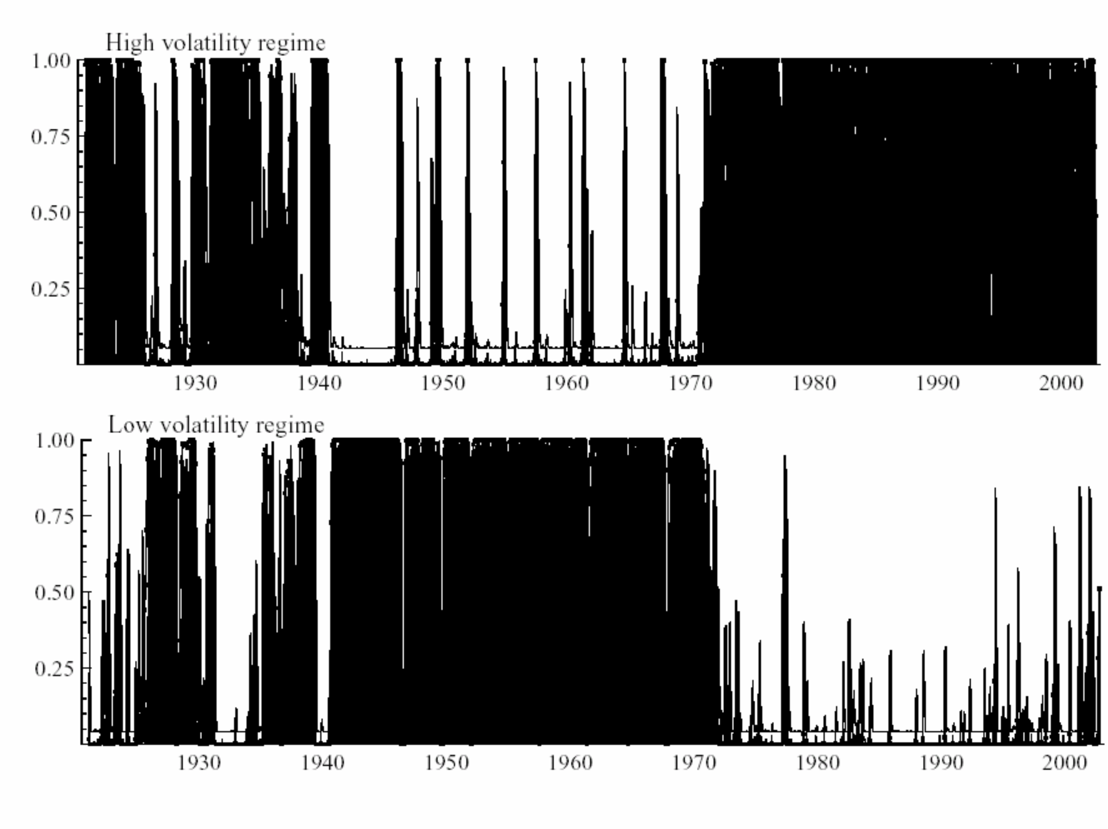

Table 1
Descriptive statistics, January 1921- December 2002

	Mean	Variance	Skewness	Kurtosis	JB	ADF	COINT
q	1.05 *	0.033	0.65 *	-0.37 *	75.62 *	-2.14 (3)	15.00
r	0.001*	0.001*	1.16 *	4.69 *	339.62 *	-4.37 * (3)	
Δq	0.0003	0.0005	-0.95 *	9.79 *	4085.6 *	-14.83 * (3)	

Notes:

1. q denotes the logarithmic real exchange rate, Δq denotes the first difference of the log real exchange rate, r denotes the real interest differential.
2. JB denotes the Jarque-Bera normality test. This is distributed as χ_2^2 .
3. ADF denotes the augmented Dickey Fuller unit root test. In parentheses next to the ADF test statistic is the number of lags in the ADF regression.
4. * denotes rejection of the null at the 5% level. For the ADF test, * denotes rejection of the null of nonstationarity. For the JB, * denotes rejection of the null of normality hypothesis.
5. For q, the null of nonstationarity cannot be rejected using the ADF test with 3 lags. Alternative lag lengths yield the same result.
6. COINT stands for the Johansen's trace statistic for cointegration. The 5% and 1% critical values are 15.41 and 20.04 respectively.

Table 2: MS-VAR model estimation

Parameters	Regime 1		Regime 2
$a_{\Delta q}$		0.0004 (1.26)	
$a_{\Delta q,\Delta q,1}$		0.570 * (16.20)	
$a_{\Delta q,\Delta q,2}$		-0.091 * (-2.73)	
$a_{\Delta q,\Delta q,3}$		0.067 * (2.35)	
$a_{\Delta q, r,1}$		-0.178 * (-4.54)	
$a_{\Delta q,r,2}$		0.131 * (2.81)	
$a_{\Delta q,r,3}$		0.031 (0.87)	
a_r		0.006 (1.63)	
$a_{r,\Delta q,1}$		-0.196 * (-7.68)	
$a_{r,\Delta q,,2}$		-0.013 (-0.53)	
$a_{r,\Delta q,3}$		-0.052 * (-2.31)	
$a_{r,r,1}$		0.708 * (20.08)	
$a_{r,r,2}$		0.083 * (2.09)	
$a_{r,r,3}$		0.169 * (5.12)	
$\sigma_{\Delta q}$	0.029		0.016
σ_r	0.009		0.008
Contemporaneous correlation	0.104		0.872
p_{11}		0.9335	
p_{22}		0.9344	
Null hypothesis: The variance and autoregressive parameters are equal across regimes (Linear VAR)			
Alternative: The variance and autoregressive parameters are different across regimes (MS-VAR)			
Likelihood		5858.4 {5289.6}	
LR		1137.6 [0.000]	
AIC		-11.89 {-10.75}	
Null: The volatility regimes of the real exchange rate and the real interest rate differential are *independent*			
Alternative: The volatility regimes of the real exchange rate and the real interest rate differential are *not independent*			
Likelihood (Restricted model)		5840.5	
LR		35.80 * (p-value:0.00)	

Notes:

1. LR denotes the likelihood ratio test. The value in squared brackets next to LR is the marginal significance level of this test, based on Davies (1987).
2. The values in curly brackets report the respective values from the linear VAR(3).
3. t-statistics are reported in parentheses. * denotes statistical significance at the 5% level.

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