Macroeconomic sources of risk in the term structure

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Abstract

In this paper we develop a new way of modelling time variation in term premia. This is based on the stochastic discount factor (SDF) model of asset pricing, observable macroeconomic factors and the joint estimation of the no-arbitrage conditions for holding period returns of different maturity and the factors using multi-variate GARCH with conditional covariances in the mean. We estimate the contribution made to the term premia at different maturities by real and nominal macroeconomic sources of risk. From the estimated term premia we estimate the term structure of interest rates, show how it varies through time, and estimate the contributions made by the macroeconomic factors. Finally, we examine the implications for testing the rational expectations hypothesis of the term structure (REHTS) when allowance is made for time-varying term premia.

Keywords: Term structure, the stochastic discount factor model, term premia, GARCH

JEL Classification: G1, E4,C5

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1 Introduction

In this paper we develop a new way of modelling time variation in term premia. This is based on the stochastic discount factor (SDF) model of asset pricing, the use of observable macroeconomic factors and the joint estimation of the no-arbitrage conditions for holding period returns of different maturity and the factors through a multi-variate GARCH model with conditional covariances in the mean. We estimate the contribution made to the term premia at different maturities by real and nominal macroeconomic sources of risk. From the estimated term premia we estimate the term structure of interest rates, show how it varies through time, and estimate the contributions made by the macroeconomic factors. Finally, we examine the implications for testing the rational expectations hypothesis of the term structure (REHTS) when allowance is made for time-varying term premia.

There is a large literature on the empirical evidence on the term structure. Much of the early work was based on the rational expectations hypothesis of the term structure (REHTS) and came to the conclusion that the REHTS was rejected by the data. Campbell and Shiller (1984) concluded that the reason for the failure of the REHTS was that long rates under-reacted to short rates. Campbell and Shiller (1991) and Hardouvelis (1994) took a contrary view, that the main cause of a failure might be the over-reaction of long rates to short rates. An alternative opinion, first emphasized by Mankiw and Miron (1986), followed by Hardouvelis (1988) and Rudebusch (1995), and formalized in a theoretical model by McCullum (1994), is that short rates were responding to changes in monetary policy. More recently, Favero and Mosca (2001) found that the REHTS cannot be rejected for short-term bonds in periods of low uncertainty of monetary policy. Hsu and Kugler (1997) attributed similar findings to the use by the Fed, since the late 1980s, of the spread as an indicator for monetary policy. Another strand of the literature relates the failure of the REHTS to "noise traders" and various other forms of irrational expectations of at least some market participants, see Froot (1989), Bekaert, Hodrick and Marshall (1997a).

A more fundamental criticism of the REHTS is that it is theoretically incorrect. It assumes that investors are risk neutral, whereas they are well-known to be risk averse. The REHTS is therefore misspecified due to omitting a time-varying term premium. The early studies of Fama (1984), Keim and Stambaugh (1986), Mankiw (1986) and Hardouvelis (1988) supported the significance of time-varying risk premia in bond returns. More recent evidence is by Froot (1989), Fama (1990), Mishkin (1990), Tzavalis and Wickens (1997) and Cochrane and Piazzesi (2001). Froot (1989), using survey data, found that for the short end of the yield curve, the failure of the REHTS is due primarily to a time-varying risk premium, while on the very long end the principal reason is expectations error. Tzavalis and Wickens (1997) found that, once allowance is made for a term premium, long rates react to short rates as theory predicts. This suggests that omitted variables bias may be the cause of the failure of the REHTS. Unfortunately, the omitted variable - the term premium - is not directly observable, so the solution is not as simple as just including an additional observable variable.

Two approaches to resolving this problem have been used. One is to find an observable proxy for the term premium. Modigliani and Shiller (1973), Fama (1976), Mishkin (1982) and Jones and Roley (1983) used various measures of the variability of interest rates. Shiller, Campbell and Schoenholtz (1983) used a measure of the volume of trade in bonds. Campbell (1987) used a latent variable model of the returns on bills, bonds and common stocks to infer time-varying risk premia in all three markets. Engle, Lilian and Robins (1987) used a time-varying conditional variance of interest rates derived from an ARCH-M model. Simon (1989) used the square of the excess holding-period return to proxy the term premium. Tzavalis and Wickens (1997) used the ex-post holding-period return of one maturity to provide a proxy for the risk premia of other maturities, thereby allowing the yield curve to have a single factor representation. Cochrane and Piazzesi (2001) used a single factor, a tent-shaped function of forward rates to predict one-year excess holding-period returns.

A second way of trying to resolve the problem is to specify the term premium based on an appropriate model of the term structure. The usual way to do this is to use latent affine factor models of the term structure, see Campbell (1987), Knez, Litterman and Scheinkman (1994), Gong and Remolona (1997), Cochrane and Piazzesi (2001) and Dai and Singleton (2002). Piazzesi (2002) surveys this literature. Two widely-used affine models of bond prices are those of Vasicek (1977) and Cox, Ingersoll and Ross (1985). The general conclusions to emerge are that the CIR model provides a more flexible description of the yield curve than the Vasicek model, and that three latent factors are required.

Both the Vasicek and the CIR models are examples of the SDF model, see Cochrane (2001). Their attraction is that they provide linear models of yields and term premia. This also means that they are a restricted type of SDF model. In general, the risk premia implied by the SDF model are captured by the conditional covariances of the excess return over the risk-free rate. In both the Vasicek and CIR models these covariances are linear functions of the factors. In single factor affine models, for example, all bond yields are just a function of the short rate and the shape of the yield curve is fixed through time. The single factor Vasicek model implies that the shape of the yield curve depends just on the time to maturity and is the same for all time periods, whilst the single factor CIR model allows the yield curve to move over time, but fixes the shape. Multi-factor affine factor models are more flexible but, arguably, are still over-constrained.

Another problem is how to interpret the latent factors extracted from the data. Pearson and Sun (1994), for example, labelled their two factors "short rate" and "inflation". Such labels are, however, in general only loose ex-post descriptions. They give only rough guidance of the causes of changes in the shape of the yield curve. Using observations on the short rate and inflation would, in principle, be preferable for interpretation purposes.

To date, very little work has been done on observable factor models of the term structure. Ang and Piazzesi (2001) estimated a multifactor Vasicek model, allowing the SDF to depend on shocks to both observable and unobservable factors. They use 3 latent and 2 observable variables (measures of inflation and real activity), and draw on the literature on monetary policy rules to justify their choice of variables. They find that time variation in bond risk premia depends significantly on the macroeconomic factors. Lee (1995) used a multivariate model to generate conditional variances and covariances, and a cash-in-advance general equilibrium model to justify his choice of macroeconomic variables. However, by including conditional variances of the macroeconomic variables in the mean of the holding-period return instead of conditional covariances with the excess holding-period return arbitrage possibilities are not eliminated.

We address both of these criticisms of the standard approach to modelling the term structure. Although greater generality is obtained by using multi-factor affine models, an even more general approach, and one that deals with the issue of how to interpret the factors, is to estimate the conditional covariances directly from the data. This requires the use of observable factors and the specification of the joint distribution of excess holding-period returns and factors that has a time-varying conditional covariance matrix. In order to satisfy the no-arbitrage condition derived from the SDF model we must include conditional covariances of excess holding-period returns with the factors in the conditional mean equation of the excess returns. The multi-variate GARCH model has been widely used in empirical finance, and can be used here. In order to satisfy the no-arbitrage condition, however, it is essential to include conditional covariances in the mean. The failure of virtually all of the applications of multi-variate GARCH to do this means that, in general, they do not satisfy a no-arbitrage condition. This is the approach proposed for FOREX by Wickens and Smith (2000), and for equity by Smith, Sorensen and Wickens (2003). A general survey of the approach is Smith and Wickens (2002). In this paper, we develop this methodology for application to bond pricing and the term structure.

Our choice of factors is motivated from the well-known general equilibrium consumption-based intertemporal capital asset pricing model (C-CAPM) of Rubinstein (1976) and Lucas (1978) which, like most asset pricing models, can be shown to be a particular version of the SDF model. We estimate time-varying risk premia for different maturities within a no-arbitrage framework. We are then able to reconstruct the yield curve. Finally, we re-examine whether, by including our estimates of the term premia, it is possible to account for the lack of empirical support for the REHTS.

2 Theoretical framework

2.1 Basic concepts

We use the following notation. $P_{n,t}$= price of an n-period zero-coupon (pure discount) bond at t, where $P_{0,t}$= 1 as the pay-off at maturity is 1. $R_{n,t}$= yield to maturity of this bond, where the one-period, risk-free rate $R_{1,t}$= s_t. $h_{n,t+1}$ = return to holding an n-period bond for one period from t to $t+1$. It follows that

$$P_{n,t} = \frac{1}{[1+R_{n,t}]^n}$$

and

$$R_{n,t} \simeq -\frac{1}{n} \ln P_{n,t}$$

Thus

$$\begin{aligned} 1 + h_{n,t+1} &= \frac{P_{n-1,t+1}}{P_{n,t}} \\ &= \frac{(1+R_{n-1,t})^{-(n-1)}}{(1+R_{n,t})^{-n}} \end{aligned}$$

And if $p_{n,t} = \ln P_{n,t}$ then taking logs

$$h_{n,t+1} \simeq p_{n-1,t+1} - p_{n,t} = nR_{n,t} - (n-1)R_{n-1,t+1}$$

The no-arbitrage condition for bonds is that, after adjusting for risk, investors are indifferent between holding an n-period bond for one period and holding a risk-free 1-period bond. The risk is due to the price of the bond next period being unknown this period.

$$E_t[h_{n,t+1}] = s_t + \rho_{n,t} \tag{1}$$

where $\rho_{n,t}$ is the risk premium on an n-period bond at time t.

2.2 SDF model of the term structure

The SDF model relates the price of an n-period zero-coupon bond in period t to its discounted price in period $t+1$ when it has $n-1$ periods to maturity. Thus

$$P_{n,t} = E_t[M_{t+1}P_{n-1,t+1}]$$

where M_{t+1} is a stochastic discount factor, or pricing kernel. It follows that

$$E_t[M_{t+1}(1+h_{n,t+1})] = 1$$

and for $n=1$,

$$(1+s_t)E_t[M_{t+1}] = 1$$

If $P_{n,t}$ and M_{t+1} are jointly log-normally distributed and $m_{t+1} = \ln M_{t+1}$ then

$$p_{n,t} = E_t(m_{t+1}) + E_t(p_{n-1,t+1}) + \frac{1}{2}V_t(m_{t+1}) + \frac{1}{2}V_t(p_{n-1,t+1}) + Cov_t(m_{t+1}, p_{n-1,t+1}) \tag{2}$$

and as $p_{o,t} = 0$,

$$p_{1,t} = E_t(m_{t+1}) + \frac{1}{2}V_t(m_{t+1}) \tag{3}$$

Subtracting (3) from (2) and re-arranging gives the no-arbitrage equation

$$E_t(p_{n-1,t+1}) - p_{n.t} + p_{1,t} + \frac{1}{2}V_t(p_{n-1,t+1}) = -Cov_t(m_{t+1}, p_{n-1,t+1}) \tag{4}$$

This can be re-written in terms of yields as

$$-(n-1)E_t(R_{n-1,t+1}) + nR_{n,t} - s_t + \frac{(n-1)^2}{2}V_t(R_{n-1,t+1}) = (n-1)Cov_t(m_{t+1}, R_{n-1,t+1})$$

and, since $h_{n,t} \simeq -(n-1)R_{n-1,t+1} + nR_{n,t}$, as

$$E_t(h_{n,t+1} - s_t) + \frac{1}{2}V_t(h_{n,t+1}) = -Cov_t(m_{t+1}, h_{n,t+1}) \tag{5}$$

This is the fundamental no-arbitrage condition for an n-period bond, and each point on the yield curve must satisfy this no-arbitrage condition. The term on the right-hand side is the term premium and $\frac{1}{2}V_t(h_{n,t+1})$ is the Jensen effect. Comparing equations (1) and (5), we note that the SDF model implies that

$$\rho_{n,t} = -\frac{1}{2}V_t(h_{n,t+1}) - Cov_t(m_{t+1}, h_{n,t+1})$$

Empirical work on the term structure can be distinguished by the choice of $\rho_{n,t}$ and the discount factor m_t. The expectations hypothesis, which is the basis of most tests of the REHTS, assumes that $\rho_{n,t} = 0$. The evidence cited earlier rejects this assumption. We now consider the choice of m_t.

2.3 Modelling the discount factor

2.3.1 Affine models

The affine multi-factor model is the most widely-used in empirical work. It has sufficient flexibility to be able to describe the term structure quite accurately. The Vasicek and CIR affine models are the most popular. They assume that m_t is a linear function of a vector of unobservable variables z_t and, as a result, so is $p_{n,t}$, the log price of an n-period bond. The following is a general representation of these models, see Ang and Piazzesi (2002) and Dai and Singleton (2000)

$$\begin{aligned}
p_{n,t} &= -[A_n + B_n' z_t] \\
-m_{t+1} &= \ell' z_{t+1} + \lambda' e_{t+1} \\
z_{t+1} - \mu &= \theta(z_t - \mu) + e_{t+1} \\
e_{t+1} &= \mathbf{S}_t^{\frac{\delta}{2}} \Sigma \varepsilon_{t+1} \\
S_{ii,t} &= \nu_i + \phi_i' z_t
\end{aligned}$$

where ℓ is a vector of ones, S_t is a diagonal matrix, ε_{t+1} is $i.i.d(0, I)$, and θ and Σ are both square matrices. The Vasicek model sets $\delta = 0$ and the CIR model sets $\delta = 1$. It is also possible to implement these models with observable variables. Ang and Piazzesi (2002) and Ang, Piazzesi and Wei (2003), using the Vasicek model, which assumes that the factors are generated by a VAR(1), have used a mixture of observable and unobservable variables.

2.3.2 General equilibrium models

Despite the practical advantages of affine models, a theoretically more attractive approach is to use general equilibrium theory to choose the factors and the functional form of m_t. The most widely-used general equilibrium model is based on power utility which has a constant coefficient of relative risk aversion σ. It can be shown that for *nominal* returns

$$m_{t+1} = \theta - \sigma \Delta c_{t+1} - \pi_{t+1}$$

where c_t= log real consumption. Thus, there are two observable factors, one real and one nominal. The resulting no-arbitrage condition is, see Smith and Wickens (2002) and Smith, Sorensen and Wickens (2003),

$$E_t(h_{n,t+1} - s_t) + \frac{1}{2} V_t(h_{n,t+1}) = \sigma Cov_t(h_{n,t+1}, \Delta c_{t+1}) + Cov_t(h_{n,t+1}, \pi_{t+1}) \tag{6}$$

A generalisation allows utility to be time non-separable. Smith, Sorensen and Wickens (2003) show that the resulting no-arbitrage condition can be written

$$E_t(h_{n,t+1} - s_t) = \beta_1 V_t(h_{n,t+1}) + \beta_2 Cov_t(h_{n,t+1}, \Delta c_{t+1}) + \beta_3 Cov_t(h_{n,t+1}, \pi_{t+1})$$

The β_i are functions of the deep structural parameters of the general equilibrium model, and β_2 is no longer equal to σ. This formulation is clearly more general than power utility.

2.3.3 SDF model

An alternative to the general equilbrium model is the SDF model. Assuming log-normality, this assumes that the logarithm of the pricing kernel is a linear function of a set of stochastic discount factors z_{it}

$$m_{t+1} = a + \sum_i b_i z_{i,t+1}.$$

The resulting no-arbitrage condition is

$$E_t(h_{n,t+1} - s_t) + \frac{1}{2}V_t(h_{n,t+1}) = -\sum_i b_i Cov_t(h_{n,t+1}, z_{i,t+1})$$

Although the SDF model places no restrictions on the coefficients of the conditional covariance terms, it does restrict the coefficient on the conditional variance. Unlike the general equilibrium model, SDF theory provides no guidance on the choice of factors z_{it}. They can be latent or observable factors.

2.3.4 Testable restrictions

Finally, we note that the coefficients in all of these no-arbitrage conditions are independent of the time to maturity. This does not imply that the term premia themselves are independent of the time to maturity. Time to maturity is reflected in the term premia through the conditional covariance terms, which vary through time and with n. The constancy of the coefficients in the no-arbitrage conditions across maturities provides a statistical test for the joint hypothesis of the specification of the SDF model and the no-arbitrage principle against the alternative that one or the other doesn't hold.

2.4 Yield curve

Having derived the term premia, we now consider how they affect the shape of the yield curve. Yields for different maturities can be obtained from the no-arbitrage condition by replacing holding period returns with yields. Equation (1) can be re-expressed as

$$E_t[h_{n,t+1}] = nR_{n,t} - (n-1)E_t[R_{n-1,t+1}] = s_t + \rho_{n,t}$$

Hence,

$$(n-1)[E_t[R_{n-1,t+1} - R_{n,t}] = (R_{n,t} - s_t) - \rho_{n,t}$$

where $R_{n,t} - s_t$ is the term spread and

$$\rho_{n,t} = -\frac{1}{2}\frac{(n-1)^2}{n}V_t(R_{n-1,t+1}) - \frac{n-1}{n}Cov_t(m_{t+1}, R_{n-1,t+1})$$

It follows that

$$\begin{aligned} R_{n,t} &= \frac{n-1}{n}E_t[R_{n-1,t+1}] + \frac{1}{n}(s_t + \rho_{n,t}) \\ &= \frac{1}{n}\sum\nolimits_{i=0}^{n-1} E_t[s_{t+i} + \rho_{n-i,t+i}] \\ &= \frac{1}{n}\sum\nolimits_{i=0}^{n-1} E_t s_{t+i} + \omega_{n,t} \end{aligned} \tag{7}$$

where

$$\omega_{n,t} = \frac{1}{n}\sum\nolimits_{i=0}^{n-1} E_t \rho_{n-i,t+i} \tag{8}$$

Thus the yield to maturity is the average of expected future short rates plus the average risk premium on the bond over the rest of its life, $\omega_{n,t}$.

>From the Fisher equation

$$s_t = r_t + E_t[\pi_{t+1}]$$

where r_t is the 1-period real rate and π_t is inflation, hence nominal yields can be decomposed into

$$R_{n,t} = \frac{1}{n}\sum\nolimits_{i=0}^{n-1} E_t[r_{t+i} + \pi_{t+i+1} + \rho_{n-i,t+i}]$$

Thus, each yield - and hence the shape of the yield curve - has three components: real, nominal and a risk component. All are affected by the time to maturity.

In order to re-construct yields it is necessary to estimate $\omega_{n,t}$. In principle, this requires having term premia at all maturities. But, in practice, only a few yields are observable at any time; other yields are estimated by linear interpolation.

3 Econometric methodology

The problem is to estimate the term premia for each maturity. The usual way to do this is to assume that the factors are affine and either latent or observable, see Ang and Piazzesi (2002), (2003). We adopt an approach that allows for a more general covariance structure, see Smith and Wickens (2002) and Sorensen, Smith and Wickens (2003). The aim is to model the joint distribution of the excess holding-period returns jointly with the macroeconomic factors in such a way that the mean of the conditional distribution of each excess holding-period return in period $t + 1$, given information available at time t, satisfies the no-arbitrage condition, equation (6) The conditional mean of the excess holding-period return involves selected time-varying second moments of the joint distribution. We therefore require a specification of the joint distribution that admits a time-varying variance-covariance matrix. A convenient choice is the multi-variate GARCH-in-mean (MGM) model. For a review of multivariate GARCH models see Bollerslev, Chou and Kroner (1997), and see Flavin and Wickens (1998) for a discussion of specification issues for their use in financial models.

Let $\mathbf{x}_{t+1} = (h_{n_1,t+1} - s_t,\ h_{n_2,t+1} - s_t,..., z_{1,t+1},\ z_{2,t+1},...)'$, where $z_{1,t+1},\ z_{2,t+1},...$ include the macroeconomic variables that give rise to the factors in the SDF through their conditional covariances with the excess holding-period returns. In principle, they may also include additional variables that are jointly distributed with these macroeconomic variables as this may improve the estimate of the joint distribution. The MGM model can then be written

$$\mathbf{x}_{t+1} = \boldsymbol{\alpha} + \boldsymbol{\Gamma}\mathbf{x}_t + \mathbf{B}\mathbf{g}_t + \varepsilon_{t+1}$$

where

$$\boldsymbol{\varepsilon}_{t+1} \mid I_t \sim D[0, \mathbf{H}_{t+1}]$$

$$\mathbf{g}_t = vech\{\mathbf{H}_{t+1}\}$$

The *vech* operator converts the lower triangle of a symmetric matrix into a vector. The distribution is the multivariate log-normal distribution. If there are K maturities (excluding the short rate) then the first K equations of the model are restricted to satisfy the no-arbitrage condition.

It will be noted that the theory requires that the macroeconomic variables display conditional heteroskedasticity. This is not something traditionally assumed in macro-econometrics, but seems to be present in our data. Ideally, we would like to use high frequency data for asset returns, but very little macroeconomic data are published for frequencies higher than one month, and then only a few variables are available. Although more macroeconomic variables are published at lower frequencies, they tend not to display conditional heteroskedasticity.

Whilst the MGM model is convenient, it is not ideal. First, it is heavily parameterised which can create numerical problems in finding the maximum of the likelihood function due to the likelihood surface being relatively flat, and hence uninformative. Second, asset returns tend to be excessively volatile. Assuming a non-normal distribution such as a $t-$distribution can sometimes help in this regard by dealing with thick tails. The main problem, however, is not thick tails, but a small number of extreme values. The coefficients of the variance process of the MGM model have a tendency to produce a near unstable variance process in their attempt to fit these extreme values. In principle, a stochastic volatility model, which includes an extra random term in the variance, could capture these extreme values. Unfortunately, as far as we are aware, no multivariate stochastic model with in mean effects in the conditional covariances has been proposed in the literature.

The choice of specification of the conditional covariance matrix is a compromise between generality and feasibility. The latter requires that we limit the number of parameters to estimate. Accordingly, our specification of H_t is that of Ding and Engle (1994), the vector-diagonal multivariate GARCH-in-mean

$$H_t = H_0(ii' - aa' - bb') + aa' * \Sigma_{t-1} + bb' * H_{t-1}$$

where i is a vector of ones, $*$ denotes element by element multiplication (the Hadamard product) and $\Sigma_{t-1} = \varepsilon_{t-1}\varepsilon'_{t-1}$. This is a special case of the diagonal Vech model, in which each conditional covariance depends only on its own past values and on surprises. The restrictions implicitly imposed by this parameterisation of the multivariate GARCH process guarantee positive-definiteness and also substantially reduce the number of parameters to be estimated, thus facilitating computation and convergence. Stationarity conditions are imposed.

First we estimate the whole system as a standard homoskedastic VAR. This gives a consistent estimator of the long-run variance covariance matrix H_0. In the subsequent estimation step we constrain H_0 to this value. Our sample has 348 observations. The estimation is performed using quasi-maximum likelihood.

4 Data

4.1 Term structure data

The complete sample is monthly, from January 1970 to December 1998. Until 1991, the term structure data are those of McCulloch and Kwon. After this, and until 1998, we use the data extended by Bliss which is based on using the same technique. Excess holding-period returns are taken in excess of the one month risk free rate provided by K. French. This is essentially the one-month Treasury bill rate. Selected yields are plotted in Figure 9a.

McCulloch used tax-adjusted cubic splines to interpolate the entire term structure from data on most of the outstanding Treasury bills, bonds and notes, see McCulloch (1975) and McCulloch and Kwon (1993). These data have been used extensively in empirical work are considered very "clean", in that they are based on a broad spectrum of government bond prices and are corrected for coupon and special tax effects. Furthermore, spline-based techniques allow for a high degree of flexibility, since individual curve segments can move almost independently of each other (subject to continuity constraints). Hence they are able to accommodate large variations in the shape of the yield curve. Even so, fitting the yield curve is a complex procedure, and is itself subject to error. The main errors seem to arise at the long end of the term structure, see Deakon and Derry (1994) and McCulloch and Kochin (2000). In constructing holding-period returns we use the change in n-period yields $\Delta R_{n,t+1}$ instead of $R_{n-1,t+1} - R_{n,t}$. This will be a good approximation for medium and long yields.

4.2 Macroeconomic variables

The macroeconomic variables used are the month-on-month growth rates of: the consumer price index for all urban consumers (abbreviated here as CPI), real retail sales (RRETS), industrial production (IP), real total personal consumption (TPC) and real personal consumption on non-durables and services (PCNDS).[1] The USA is one of only two countries to keep monthly personal consumption data.[2] Hence, in order to allow for potential comparison with similar work on other countries, we also use real retail sales, a common proxy for consumption. The estimations including industrial production are exploratory rather than theoretically motivated.

4.3 Descriptive statistics

Table 1 presents some descriptive statistics for our dataset. All data series are in annualized percentages. Our yield curve data are characterised by some standard stylised facts. The average yield curve in our sample is upward sloping. Average excess holding-period returns are positive for all maturities and increase with time to maturity. Like most financial data, excess holding-period returns exhibit excess skewness and kurtosis, particularly for short maturities. Fitting a univariate GARCH(1,1) to them indicates there is also significant heteroskedasticity.[3] Their unconditional variance increases with maturity, and so do the unconditional covariances of the excess returns with the macroeconomic variables, in absolute terms. Among the consumption-measuring variables, real retail sales are by far the most variable, skewed and leptokurtic.

Table 2 presents the unconditional sample correlations. The unconditional correlations of all excess returns with the macro variables are negative and generally also increase with maturity. Industrial production is an exception to this, being most highly correlated with the 1-year bond's excess return in particular, and the shorter bonds in general. All excess returns at different

[1] Data on CPI were obtained from the U.S. Department of Labor's Bureau of Labor Statistics, IP data are obtained from Datastream, while all three measures of consumption, as well as RRETS were obtained from the Federal Reserve Bank of St Louis.

[2] The other is the U.K.

[3] Estimation results are available upon request.

maturities are highly correlated, especially at the short end of the yield-curve. For example, the 6-month and 1-year excess returns have a correlation of 0.95.

5 Empirical results

We provide estimates of a number of different models. The models differ in the number of holding-period returns included and in the choice of observable factors. Every holding-period equation is constrained to satisfy the no-arbitrage condition.

It is important to note that the model being estimated is non-linear and as a result the coefficient estimates are not independent of the units of measurement. According to the C-CAPM, the estimated coefficient of the covariance of consumption is equal to the coefficient of relative risk aversion σ, while that of inflation is 1. However, the theory was developed in absolute terms, and our data have been converted to annualized percentages. Consequently, the coefficient on the own variance of the holding-period return is constrained to be $\frac{1}{2400}$ and not $\frac{1}{2}$; the coefficient of the inflation covariance in the general equilibrium model is constrained to be $\frac{1}{1200}$ and not 1; and the estimated consumption covariance coefficient should be interpreted as $\frac{1}{1200}\sigma$ and not σ.

5.1 Coefficient estimates

Our results are reported in Tables 3a and 3b. First, we estimate C-CAPM, equation (6). We then relax the assumptions imposed by C-CAPM and test the theoretical restrictions of C-CAPM. The general alternative model (with the own variance constrained) is the SDF model. We consider two combinations of maturities using 3 and 5 maturities:

(i) 6 months, 2 years and 10 years

(ii) 6 months, $1, 2, 5$ and 10 years

We note that, in general, the most reliable results are obtained when the short end of the yield curve is adequately represented, i.e. when the 6-month excess return is included. Although the statistical significance and the explanatory power of the model vary slightly, the signs of the estimated coefficients and the trends along the yield curve are almost identical for all other combinatins we examined.

For the sake of brevity, estimates of the covariance structure are not reported. All of the ARCH and GARCH coefficients are highly significant for the excess holding-period return equations and for the macroeconomic variables; typically, the t-statistics have more than two significant places. The GARCH coefficients are always much larger than the ARCH coefficients. Typically estimates are close to 0.9 and 0.3, respectively. Further, the GARCH parameter shows a tendency to increase with maturity. As a result, the conditional covariance structure of excess holding-period returns depends almost entirely on the lagged conditional covariance matrix, and much less on lagged innovations. In the equations for the macroeconomic variables, the constant and own lagged values are always highly significant.

Table 3a is based on using 3 maturities and C-CAPM with total personal consumption and inflation. The first row contains the estimates of coefficients in the conditional mean of the excess returns; the coefficient of inflation is constrained to its theoretical value and, as C-CAPM predicts that the coefficients of the consumption covariances are the same for all maturities, we impose this as an additional constraint. Thus, in effect, only one parameter is being estimated in each system of equations for the conditional means of the excess holding-period returns.

The coefficient of the consumption covariance is estimated to be -0.03, implying an estimate of σ of 36. This is implausibly large. It is, however, typical of estimates of the coefficient of relative risk aversion found in the literature. We also note that this estimate is not statistically significant at conventional significance levels, and that the consumption covariances of all maturities are also jointly insignificant (see column 7), implying that the term premia are not significant. The explanatory power of the models is very low (see columns $12-14$), suggesting that excess holding-period returns are due almost entirely to innovations in the price of bonds.

If we relax the constraint on the coefficient of inflation and drop the interpretation that the coefficient of the conditional covariance of consumption is the coefficient of relative risk aversion then our model becomes an unrestricted SDF model. The estimation results for the same combination of bonds are given in the second row of Table 3a. An entirely different picture emerges. The coefficient on the consumption covariance has changed considerably; it is now positive and significant. The estimate of 0.09 would implying an estimate of σ of 108, if this were still an appropriate interpretation. The estimate of the coefficient of the inflation covariance is -0.41 and is highly significant. This is very far from its theoretical value under C-CAPM. The conditional covariances included in the conditional mean of the excess returns are now jointly significant, see column 7. A likelihood ratio test of the C-CAPM restrictions strongly rejects the model in favour of the more general SDF model. Finally, we now find that the explanatory power of the model has increased dramatically. 19% of the variance of the excess holding-period return on the shortest bond and 6% on the longest bond are now explained.

A key prediction of the SDF model is that the coefficients in the no-arbitrage equations are the same, no matter the time to maturity. We examine this prediction by allowing the coefficients to differ for each maturity. Thus we remove the cross-equation constraints. Table 3a row 3 retains the cross-equation restriction on the coefficient of the consumption covariance but removes it on the inflation covariances, row 4 is the reverse, it retains the cross-equation restriction on the coefficient of the inflation covariance but removes it on the consumption covariances. Row 5 removes the restrictions on both.

The main finding is that the coefficient estimates decline in absolute size as time to maturity of the holding-period returns increases. All but the coefficient on the consumption covariance for the 10-year maturity in row 5 are highly significant. These results suggest that investors' perception of risk is better informed at shorter than longer time horizons. As a result, term premia are adjusted more often, and hence are more volatile at short horizons than at longer horizons. Comparing the unrestricted estimates, row 5, with the C-CAPM models of rows 3 and 4, we find that coefficient restrictions on the inflation covariances are rejected at the 5% level, but those on the consumption covariances are not, see row 5, column 10.

Table 3b reports the corresponding results for the model with 5 maturities. The estimates are remarkably similar to those of Table 3a. The main difference is that all of the cross-equation restrictions are now rejected, including those of the consumption covariances. A possible explanation for this difference is that without the 1-year bond we are unable to capture the curvature of the yield curve at the short end as well as when we include it. In all subsequent calculation we use the estimates of the unrestricted model in Table 3b.

Looking at the estimated coefficients in greater detail, we observe that all of the inflation covariance coefficients are negative, whilst all of the consumption covariance coefficients are positive. We note, however, from Table 2, that the unconditional covariances and the average conditional covariances between every excess holding-period return and each macroeconomic variable are negative. Hence, to make a positive contribution to the term premium the coefficient estimates must be negative. The inflation terms all satisfy this, and hence make a positive contribution to the term premia, but the consumption terms do not. They seem to be making a negative contribution to the term premia. One possible explanation might be that the choice of macroeconomic factors is incorrect. Relevant factors may have been omitted, thereby introducing biases in the coefficients.

5.2 Estimated Term Premia

Figure 1 plots the total time-varying term premia (plus Jensen effect) and the separate contributions from the two macroeconomic factors for the unrestricted version of the model with five maturities. Table 4 gives summary statistics for the estimated risk premia. The total term premium is positive in nearly every period, it is due almost entirely to inflation risk and the contribution of consumption risk is very small. The "incorrect" sign of the consumption coefficient has hardly any effect. Figure 2 plots the total term premium together with the excess holding-period return (which is scaled). The graphs reveal the large noise component in excess holding-period returns.

Inspecting the term premia as they change through time, we note that investors required a large term premium in order to be willing to hold government bonds during the 1979-82 period, which corresponds to the "monetary experiment" by the Fed. This temporary change of monetary policy to strict money-base targeting led to high and volatile short rates. It was also accompanied by very high and volatile inflation and a US recession. We would therefore expect that both real and nominal factors would contribute to the term premium during this period. It is, however, the inflation risk premium that dominates. Term premia are also relatively high in the early 1970's, during the first oil crisis, and throughout the 1980's. In both periods inflation volatility was still relatively high. The 1990's was a decade of macroeconomic stability and low inflation (the Greenspan era). Term premia are much lower and more stable as a result.

The graphs also show that the term premia increase in magnitude with maturity. As the coefficients decline in absolute value with maturity, this is attributable to an increase in the average size of the conditional covariances with maturity. This finding supports the conclusions of McCulloch (1987) and Wickens and Tzavalis (1997) who found that risk premia increase with maturity, but at a decreasing rate. Heston's (1992) conjecture that risk premia on excess holding-period returns of different maturities are related to each other also finds some support here, as the estimated risk premia seem to follow a similar pattern in all maturities and are highly correlated with each other, see Table 5.

We also find that the explanatory power of the model decreases with maturity. This result is consistent with that of Ang and Piazzesi (2001) who also find that their two macroeconomic factors primarily explain movements at the short end and middle of the yield curve, while their unobservable factors account for most of the movement at the long end. This suggests that additional factors might be needed in our model.

Although the results are not reported, we experimented with different definitions of consumption and additional variables. We found that using retail sales instead of total personal consumption allowed us to explain a maximium of $12 - 13\%$ of the variance of the excess holding-period returns on the shortest bond. Using personal consumption on non-durables and services gave similar results, but using industrial production gave much worse results. We conclude that there is little to choose between the alternative measures of consumption.

6 Reconstructing the Yield Curve

6.1 Estimating the rolling risk premia

In order to estimate the yield curve from the estimated term premia we require an estimate each period of the average term premium over the rest of the life of each bond. We refer to this as the rolling risk premium, see equation (8). We consider two estimates of this. The first uses estimates of the current term premia and a linear interpolation of missing maturities. The second uses estimated future term premia plus linear interpolation. Thus the first method assumes static expectations, and the second assumes perfect foresight. All calculations are based on the unrestricted model with five maturities.

A time-series plot of the rolling risk premium for each maturity is provided in Figure 4 based on static expectations, and in Figure 6 based on perfect foresight. Figure 5 expresses the rolling premia based on static expectations as a percentage of the total yield, and Figure 7 gives the corresponding information based on perfect foresight. Being averages, the rolling premia show much more persistence than the term premia. Otherwise, they reflect previous findings, with rolling premia being highest during the early 1980's, later in the 1980's and during the recession of the early 1990's. Tables 6a and 6b present some descriptive statistics for the two sets of rolling premia.

Considering first the results based on static expectations, the average estimated rolling risk premium included in the 6-month yield during the 1970's is 0.27 (which is 4.3% of the yield); in the period from 1979-1982 the corresponding figure is 0.92 (7.6%); the average until the end of the 1980's is 0.20 (2.5%). During the 1990's, a decade associated with low and very stable inflation as well as positive growth rates, estimated rolling risk premia are very low. For 6-month bonds the average is 0.09 (1.8%). These estimates seem plausible, at least for short and medium term maturities, and are comparable to those found in other studies. The rolling risk premia are larger the longer the time to maturity. On a few occasions our estimates become negative. Sometimes, particularly during the period 1979-1982, they are clearly excessively large as in April 1980 when our estimate for the 10-year bond is 114% of the actual yield.

The rolling premia based on perfect foresight are clearly very different from those based on static expectations, particularly in the 1970's. The reason is that they assume that the market anticipates the high term premia of the period of the Fed's monetary experiment. These estimates are clearly not plausible. This becomes obvious if we reconstruct the yields implied by the estimated rolling premia.

6.2 Reconstructing the yield curve

To estimate the yields we combine the estimated rolling term premia with estimates of expected future short rates as in equation (7). Again we consider two estimates: static expectations and perfect foresight of expected future short rates. Under static expectations expected future short rates are set equal to the current short rate s_t. This implies that the shape of the yield curve and of the term structure reflects nothing more than changes in the rolling risk premia at different maturities.

Figures 8 and 9 give actual and estimated yields under static expectations for short rates and term premia and perfect foresight for three maturities: 1-year, 2-year and 10-year bonds. The panels in Figure 8 are for each maturity, and those in Figure 9 are for each type of estimate. The two Figures show that yields are better estimated at shorter than longer maturities, and that static expectations provide better estimates than perfect foresight. The estimation errors for 6-month and 2-year yields based on static expectations are remarkably small. Even for 10-year bonds they are small from 1984 onwards. Perfect foresight does not introduce large errors for 6-month bonds, but for 2-year bonds they are larger. And for 10-year bonds prior to 1984 the errors are huge.

Inflation is well-known to be an important factor affecting short rates. And we have shown inflation volatility is the major factor in term premia. When inflation is low it appears to be both more predictable and to have lower and more predictable volatility. It is in this case that we are able to estimate yields best. The larger estimation errors under perfect foresight suggest that markets are more likely to mis-price bonds when inflation is more volatile.

7 Testing the REHTS

We now return to consider the question raised earlier of whether the empirical failures of the REHTS are due to the omission of a time-varying risk premium. We can include our estimate of the risk premium as an additional observable variable in the model. The model based on returns can be written

$$h_{n,t+1} = \alpha + \beta s_t + \gamma \varphi_{n,t} + \varepsilon_{n,t+1} \tag{9}$$

where $\varepsilon_{n,t+1}$ is a zero-mean disturbance. According to the REHTS $\alpha = 0$ and $\beta = 1$. If we have estimated the risk premium reasonably accurately then we also expect that $\gamma = 1$. Alternatively, we can express the model in terms of yields as

$$(n-1)(R_{n-1,t+1} - R_{n,t}) = \delta + \mu(R_{n,t} - s_t) + \nu \varphi_{n,t} + \epsilon_{n,t+1} \tag{10}$$

where we expect $\delta = 0$, $\mu = 1$, $\upsilon = -1$ and $\epsilon_{n,t+1} = -\varepsilon_{n,t+1}$.

OLS estimates of these equations are reported in Table 7 for each of the five maturities. The columns *a* and *d* are estimates of the standard tests of the REHTS without a risk premium, and columns *b* and *e* include the estimated risk premium derived from the unrestricted model with five maturities. In the models without the risk premium the estimates of β are close to unity for each maturity, but those for μ are not. This is the usual result. The difference is due to the holding-period returns and the short rate in equation (9) being I(1), or close to I(1), variables. This results in super-consistent estimates of β even if there is an omitted term premium, provided it is I(0). Whereas, in equation (10), the change in the yield and the term spread are I(0) variables, and so an omitted term premium is likely to give biased estimates of μ.

Including the risk premium is expected to have little effect on the estimate of β but ought, if the risk premium is well estimated, to remove the bias in the estimate of μ. The results in column two confirm this for equation (9), but not for equation (10). The bias is increased for most maturities in equation (10). Except for the 10-year maturity, the estimate of γ is close to unity and the estimate of υ is not far from -1. This suggests, once more, that the estimates of the term premia are less accurate at long maturities.

8 Conclusion

We have shown how it is possible to estimate time-varying term premia derived from observable factors whilst satisfying the condition of no-arbitrage along the yield curve. Based on the stochastic discount factor model, this requires the specification of the joint distribution of excess holding-period returns and observable factors in which the conditional mean of the excess returns is the no-arbitrage condition. Although multi-variate GARCH is widely used in empirical finance, it is rarely specified with conditional covariances in the mean. We show that, in general, this is essential if a no-arbitrage condition is to be satisfied. An exception is when a Vasicek model is used, but this has the drawback of not having a time-varying risk premium.

This paper is the first application of this methodology to the term structure. We find that the main source of risk at all maturities is nominal risk due to inflation. We also find that consumption is a significant, but less important, source of risk. Term premia at shorter maturities are better explained than at longer maturities. The magnitude of the term premia tends to increase with maturity due to the size of the conditional covariances, not the coefficients on the conditional covariances. Contrary to SDF theory, these coefficients appear to alter with time to maturity.

We show how it is possible to reconstruct yields, and hence estimate the term structure, using term premia. Our estimates seem to be more accurate for the shorter maturities and for longer maturities when inflation is predictable and has low volatility. The larger errors when estimating yields under perfect foresight suggest that markets appear to mis-price bonds more when inflation is hard to predict and is volatile.

Finally, we re-examine whether or not the main cause of the familiar finding of the failure of the rational expectations hypothesis of the term structure is an omitted term premium. Using our estimates of the term premia to proxy the term premium, we find that there is some reduction in bias at shorter maturities, but we are unable to attribute the failure of the REHTS solely to an omitted term premium.

9 References

Anderson N., Sleath J., 2001, "New estimates of the UK real and nominal yield curves" Bank of England, Working paper

Ang A., Piazzesi M., 2001, "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables" manuscript

Bekaert G., Hodrick R.J., Marshall D.A., 1997a, "On biases in tests of the Expectations Hypothesis of the term structure of interest rates" Journal of Financial Economics, 44, 309-348

Bekaert G., Hodrick R.J., Marshall D.A., 1997b, "'Peso problem' explanations for term structure anomalies" Federal Reserve Bank of Atlanta, Working paper 97-13,

Bollerslev T., Chou R.Y., Kroner K.F., 1992, "ARCH modelling in finance: a selective review of the theory and selective evidence" Journal of Econometrics, 52, 5-59

Campbell J.Y., 1997, "Some lessons from the yield curve" Journal of Economic Perspectives vol.9 no.3, 129-152

Campbell J.Y., 1987, "Stock returns and the term structure" Journal of Financial Economics, 18, 373-399

Campbell J.Y., 1986, "Bond and stock returns in a simple exchange model" Quarterly Journal of Economics, 101, 785-804

Campbell J.Y., Shiller R.J., 1984, "A simple account of the behaviour of long-term interest rates"

American Economic Review vol.7 issue 2, 44-48

Campbell J.Y., Shiller R.J., 1991, "Yield spreads and interest rate movements: a bird's eye view" Review of Economic Studies, 58, 495-514

Cochrane J.H., 2001 "Asset pricing" Princeton University Press,

Cochrane J.H., Piazzesi M., 2001, "Bond risk premia" manuscript

Cox, J.C., J. Ingersoll and S. Ross, 1985, "A theory of the term structure of interest rates, Econometrica", 53, 385-408.

Dai, Q. and K. Singleton, 2000, "Specification analysis of affine term structure models, Journal of Finance", 55,1943-1978.

Dai Q., Singleton K., 2002, "Term structure dynamics in theory and reality" manuscript

Deacon M., Derry D., 1994, "Estimating the term structure of interest rates" Bank of England, Working paper 24

den Haan W.J. 1995, "The term structure of interest rates in real and monetary economies" Journal of Economic Dynamics and Control, 19, (5-7), 909-940

Ding Z., Engle R.F., 2001, "Large scale conditional covariance matrix modelling, estimation and testing" manuscript

Duffie D., Kan R., 1996, "A yield factor of interest rates" Mathematical Finance, 6, 379-406

Engle R.F., Lilien D,M., Robins R.P. 1987, "Estimating the time-varying risk premia in the term structure: the ARCH-M model" Econometrica, 55, (2), 391-407

Evans M.D.D., Lewis K.K., 1994, "Do stationary risk premia explain it all? Evidence from the term structure" Journal of Monetary Economics, 33, 285-318

Fama E.F., 1984, "The information in the term structure" Journal of Financial Economics, 13, 509-528

Fama E.F., 1976, "Inflation uncertainty and expected return on Treasury bills" Journal of Political Economy, 84, 427-448

Fama E.F., 1990, "Term structure forecasts of interest rates, inflation and real returns" Journal of Monetary Economics, 25, 59-76

Fama E.F., Bliss R.R., 1987, "The information in long-maturity forward rates" American Economic Review vol.77, 680-692

Favero C.A., Mosca F., 2001, "Uncertainty on monetary policy and the expectations model of the term-structure of interest rates" Economic Letters, 71, 369-375

Fisher M., Gilles C., 1998, "Around and around: the expectations hypothesis" Journal of Finance, 53, 1, 365-383

Flavin T.J., Wickens M.R., 1998, "A risk management approach to optimal asset allocation" manuscript

Froot K.A., 1989, "New hope for the expectations hypothesis of the term structure of interest rates" Journal of Finance, 44, 283-305

Gong, F.F. and E.M. Remolona, 1997, "Two factors along the yield curve", Papers in Money, Macroeconomics and Finance: The Manchester School Supplement, 65, 1-31.

Hardouvelis G.A., 1988, "The predictive power of the term-structure during recent monetary regimes" Journal of Finance, vol.43, issue 2, 339-356

Hardouvelis G.A., 1994, "The term structure spread and future changes in long and short rates in the G7 countries: Is there a puzzle?" Journal of Monetary Economics, 33, 255-283

Hardouvelis G.A., Barnhart S.W., 1989, "The evolution of Federal Reserve credibility: 1978-1984" The Review of Economics and Statistics, vol.71, issue3, 385-393

Heston S., 1992, "Testing continuous time models of the term structure of interest rates" School of Organisation and Management, Yale University, Working paper

Hsu C., Kugler P., 1997, "The revival of the expectations hypothesis of the US term structure of interest rates" Economic Letters, 55, 115-120

Jones D.S., Roley V.V., 1983, "Rational expectations and the expectations model of the term structure: a test using weekly data" Journal of Monetary Economics, 12, 453-465

Kan R., Zhou G., 1999, "A critique of the stochastic discount methodology" Journal of Finance, 54, 1221-1248

Keim D.B., Stambaugh R.F., 1986, "Predicting returns in the stock and bond markets" Journal of Financial Economics, 17, 357-390

Knez P., Litterman R., Scheinkman A., 1994, "Explorations into factors explaining money market returns" Journal of Finance, 49, 5, 1861-1882

Lee S.-S., 1995, "Macroeconomic sources of time-varying risk premia in the term structure of interest rates" Journal of Money, Credit and Banking, vol.27, no.2

Longstaff F.A., 2000, "The term structure of very short term rates: New evidence for the expectations hypothesis" Journal of Financial Economics,

Lucas R.E., 1978, "Asset prices in an exchange economy" Econometrica, 46, 1429-1446

Mankiw N.G., 1986, "The term structure of interest rates revisited" Brookings papers on Economic Activity, 1, 61-96

Mankiw N.G., Miron J.A., 1986, "The changing behaviour of the term structure of interest rate" Quarterly Journal of Economics, 101, 211-228

McCallum B., 1994, "Monetary policy and the term structure of interest rates" NBER, Working paper 4938

McCulloch J.H., 1975, "The tax-adjusted yield curve" Journal of Finance 30, 811-30

McCulloch J.H., 1987, "The monotonicity of the term premium" Journal of Financial Economics, 18, 185-192

McCulloch J.H., Kochin L.A., 2000, "The Inflation Premium Implicit in the U.S. Real and Nominal Term Structures of Interest Rates" "Ohio State University, Working Paper 98-12, revised 9/2000

McCulloch J.H., Kwon H.C., 1993, "US term structure data, 1947-1991" Ohio State University, Working paper 93-6

Mishkin F., 1990, "The information in the longer-maturity term structure about inflation" Quarterly Journal of Economics, 55, 815-828

Mishkin F.S., 1982, Monetary policy and short-term interest rates: an efficient markets, rational expectations approach" Journal of Monetary Economics, 37, 63-72

Modigliani F., Shiller R.J., 1973, “Inflation, rational expectations and the term structure of interest rates” Economica, 40, 12-43

Pearson N., Sun T.S., 1994, “Exploiting the conditional density in estimating the term structure: An application to the Cox, Ingersoll and Ross model: Journal of Finance, 54, 1279-1304

Piazzesi M., 2002, “Affine term structure models” forthcoming in the Handbook of Financial Econometrics

Roberds W., Runkle D., Whiteman C.H., 1996, “Resurrecting the term-structure: a daily view of yield spreads and short-term interest rate movements” Journal of Money, Credit and Banking, 28, 34-53

Rubinstein M., 1976, “The valuation of uncertain income streams and the pricing of options” Bell Journal of Economics and Management Science” 7, 407-425

Rudebusch G.D., 1995, “Federal Reserve interest rate targeting, rational expectations and the term structure” Journal of Monetary Economics, 35, 245-274

Shiller R.J., Campbell J.Y., Schoenholtz K.L., 1983, “Forward rates and future policy: interpreting the term-structure of interest rates” Brookings papers on Economic Activity, 173-217

Simon D.P., 1989, “Expectations and risk in the Treasury bill market: An instrumental variable approach” Journal of Financial and Quantitative Analysis, 24, 357-368

Smith P.N., 1993, “Modelling risk premia in international asset markets” European Economic Review, 37, 159-176

Smith P.N., Sorensen S., Wickens M.R., 2003 “Macroeconomic sources of equity risk” CEPR Discussion Paper no. 4070.

Smith P.N., Wickens M.R., 2002, “Asset pricing with observable stochastic discount factors” University of York, Working paper 2002/03

Tzavalis E., Wickens M.R., 1997, “Explaining the failures of the term spread models of the rational expectations hypothesis of the term structure” Journal of Money, Credit and Banking, vol.29, no.3, 364-380

Vasicek, O., 1977, “An equilibrium characterization of the term structure”, Journal of Financial Economics, 5, 177-188.

Wickens M.R., Smith P.N., 2001, “Macroeconomic sources of FOREX risk” University of York, Working paper 2001/13

Table 1: Descriptive Statistics

		Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Risk Free	1 month	6.803	6.041	17.459	2.549	2.848	1.274	4.847
Returns on bonds	6 months	7.101	6.430	16.511	2.878	2.737	1.138	4.217
	1 year	7.343	6.846	16.345	3.090	2.675	1.057	3.942
	2 years	7.640	7.071	16.145	3.803	2.517	1.067	3.852
	5 years	8.041	7.534	15.696	4.352	2.307	1.094	3.649
	10 years	8.315	7.802	15.065	4.514	2.144	1.028	3.397
Excess holding period returns on bonds	6 months	0.354	0.189	24.813	-14.404	3.188	1.227	15.792
	1 year	0.646	0.739	47.242	-33.230	6.719	0.558	12.495
	2 years	1.066	1.017	82.478	-71.008	12.602	0.198	11.300
	5 years	1.770	2.172	111.494	-128.107	26.015	-0.224	6.076
	10 years	2.361	2.554	152.263	-163.351	42.952	0.020	4.619
Month-on-month growth rates of macro variables	CPI	5.285	4.180	23.958	-5.353	4.338	0.996	4.254
	TPC	3.523	3.352	29.870	-25.594	7.160	0.138	4.756
	PCNDS	3.930	3.933	37.254	-26.126	7.234	0.140	5.644
	RRETS	3.177	2.317	89.913	-57.910	15.458	0.797	8.111
	IP	3.314	4.059	49.395	-39.928	9.897	-0.073	5.598

Notes

1. All series are in annualised percentages

Table 2: Sample Correlations

		6 months	1 year	2 years	5 years	10 years	CPI	TPC	PCNDS	RRETS	IP
Excess returns on bonds of maturity:	6 months	1.00									
	1 year	0.95	1.00								
	2 years	0.88	0.96	1.00							
	5 years	0.76	0.87	0.94	1.00						
	10 years	0.67	0.76	0.84	0.94	1.00					
Growth rate of macroeconomic variables	CPI	-0.10	-0.12	-0.15	-0.17	-0.22	1.00				
	TPC	-0.05	-0.07	-0.06	-0.09	-0.07	-0.26	1.00			
	PCNDS	-0.06	-0.06	-0.06	-0.06	-0.05	-0.20	0.89	1.00		
	RRETS	-0.06	-0.07	-0.08	-0.12	-0.10	-0.20	0.80	0.65	1.00	
	IP	-0.20	-0.22	-0.21	-0.17	-0.14	-0.08	0.26	0.26	0.21	1.00

Table 3a: Estimation results
Bond maturities: 6 months, 2 & 10 years
Macroeconomic variables: CPI Inflation & Total Personal Consumption

Estimation no.	Constraints imposed	In mean coefficients: 6 month		2 year		10 year		Likelihood ratio tests[1]: Significance	C-CAPM	2	3A	3B	Share of variance explained in each maturity		
		CPI	TPC	CPI	TPC	CPI	TPC								
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	C-CAPM	0.00	-0.03	0.00	-0.03	0.00	-0.03	0.36					0.00	0.00	0.00
		.	-0.63	.	-0.63	.	-0.63								
2	2	-0.41	0.09	-0.41	0.09	-0.41	0.09	12.34	12.03				0.19	0.13	0.06
		-11.41	2.73	-11.41	2.73	-11.41	2.73								
3	3A	-0.42	0.06	-0.35	0.06	-0.25	0.06	30.43	30.12	18.10			0.14	0.06	0.02
		-11.16	2.14	-7.71	2.14	-5.70	2.14								
4	3B	-0.41	0.08	-0.41	0.09	-0.41	0.13	14.21	13.90	1.87			0.18	0.12	0.06
		-10.68	2.13	-10.68	2.65	-10.68	2.64								
5	Unrestr.	-0.49	0.16	-0.35	0.08	-0.23	0.06	30.53	30.22	18.20	0.10	16.33	0.12	0.04	0.01
		-10.51	4.19	-6.68	2.52	-4.24	1.42								

Notes

1. Likelihood ratio tests:
   The column title corresponds to the restriction tested.
   The first column tests the joint significance of all covariance terms in the estimation
2. * denotes significance at the 5% level
3. T-statistics are below the estimated parameters.

Table 3b: Estimation results
Bond maturities: 6 months, 1, 2, 5 & 10 years
Macroeconomic variables: CPI Inflation & Total Personal Consumption

Estimation no.	Constraints imposed	In mean coefficients: 6 month		1 year		2 year		5 year		10 year		Likelihood ratio tests[1]: Significance	C-CAPM	2	3A	3B	Share of variance explained in each maturity				
		CPI	TPC	CPI	TPC	CPI	TPC	CPI	TPC	CPI	TPC										
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
6	C-CAPM	0.00	-0.06	0.00	-0.06	0.00	-0.06	0.00	-0.06	0.00	-0.06	3.23					0.00	0.00	0.00	0.00	0.00
		.	-1.95	.	-1.95	.	-1.95	.	-1.95	.	-1.95										
7	2	-0.15	-0.06	-0.15	-0.06	-0.15	-0.06	-0.15	-0.06	-0.15	-0.06	3.85	0.71				0.00	0.00	0.00	0.00	0.00
		-4.21	-1.85	-4.21	-1.85	-4.21	-1.85	-4.21	-1.85	-4.21	-1.85										
8	3A	-0.42	0.00	-0.42	0.00	-0.39	0.00	-0.38	0.00	-0.30	0.00	6.10	2.96	2.25			0.14	0.10	0.07	0.04	0.02
		-12.13	0.10	-10.85	0.10	-10.32	0.10	-8.15	0.10	-7.17	0.10										
9	3B	-0.43	0.11	-0.43	0.04	-0.43	0.07	-0.43	0.07	-0.43	0.12	12.57	9.43	8.72			0.12	0.09	0.08	0.05	0.05
		-10.73	3.19	-10.73	1.28	-10.73	2.05	-10.73	1.99	-10.73	2.60										
10	Unrestr.	-0.46	0.15	-0.37	0.03	-0.34	0.04	-0.27	0.01	-0.18	-0.02	40.39	37.25	36.54	34.29	27.82	0.06	0.03	0.02	0.01	0.01
		-9.44	3.37	-7.09	0.89	-6.62	1.12	-4.37	0.33	-2.79	-0.33										

Notes

1. Likelihood ratio tests:
   The column title corresponds to the restriction tested.
   The first column tests the joint significance of all covariance terms in the estimation
2. * denotes significance at the 5% level
3. T-statistics are below the estimated parameters.

Table 4: Descriptive statistics of estimated risk premia

	6 months	1 year	2 years	5 years	10 years
Mean	0.574	1.094	1.814	2.891	4.676
Median	0.321	0.785	1.341	2.211	3.740
Maximum	5.174	6.243	11.891	13.498	15.139
Minimum	-1.433	-0.360	-2.567	-4.065	-0.177
Std. Dev.	0.767	1.112	1.842	2.720	2.789
Skewness	2.270	2.282	2.283	1.645	1.482
Kurtosis	10.240	8.098	8.787	5.645	4.700
Unit root test[1]	-2.259	-1.913	-1.860	-1.742	-1.877

Notes

1. Augmented Dickey-Fuller Test statistics, allowing for a trend, intercept and 12 lags of first differences. The unit root hypothesis cannot be rejected at conventional significance levels.
2. Estimated risk premia from estimation 10 have been used

Table 5: Correlations of estimated risk premia

	6 months	1 year	2 years	5 years	10 years
6 months	1.00				
1 year	0.90	1.00			
2 years	0.87	0.99	1.00		
5 years	0.72	0.91	0.94	1.00	
10 years	0.62	0.85	0.88	0.96	1.00

Notes

1. Estimated risk premia from estimation 10 have been used

Table 6a: Reconstructed rolling risk premia and yields (static expectations)						
Maturities	Observed Yields (A)	Reconstructed Yields (B)	Rolling Risk Premium	Rolling Risk Premium (% of A)	Error (A-B)	Error (as a % of A)
Average of period 1970-1998						
6 months	7.098	7.086	0.287	4.044	0.012	0.173
1 year	7.342	7.381	0.582	7.931	-0.039	-0.531
2 years	7.640	7.832	1.033	13.520	-0.192	-2.507
5 years	8.044	8.633	1.834	22.800	-0.589	-7.322
10 years	8.322	9.880	3.081	37.020	-1.558	-18.720
Average of period 1970-1978						
6 months	6.283	6.119	0.273	4.338	0.164	2.614
1 year	6.510	6.356	0.510	7.831	0.154	2.364
2 years	6.716	6.662	0.815	12.140	0.054	0.807
5 years	7.051	7.002	1.156	16.390	0.049	0.696
10 years	7.281	7.646	1.799	24.710	-0.365	-5.007
Average of period 1979-1982						
6 months	12.060	12.660	0.915	7.587	-0.600	-4.977
1 year	12.110	13.510	1.767	14.590	-1.400	-11.550
2 years	11.980	14.790	3.039	25.360	-2.803	-23.390
5 years	11.770	16.860	5.117	43.470	-5.091	-43.240
10 years	11.650	19.170	7.418	63.650	-7.511	-64.450
Average of period 1983-1989						
6 months	7.855	7.729	0.195	2.483	0.126	1.604
1 year	8.237	8.001	0.467	5.670	0.236	2.863
2 years	8.725	8.471	0.937	10.740	0.254	2.908
5 years	9.296	9.577	2.043	21.980	-0.281	-3.020
10 years	9.626	11.430	3.895	40.460	-1.802	-18.720
Average of period 1990-1998						
6 months	5.103	5.056	0.094	1.835	0.047	0.920
1 year	5.342	5.179	0.217	4.053	0.163	3.046
2 years	5.774	5.393	0.430	7.450	0.381	6.605
5 years	6.386	5.839	0.877	13.730	0.546	8.554
10 years	6.848	6.741	1.779	25.970	0.107	1.566

Notes

1. Estimated risk premia from estimation 10 have been used

Table 6b: Reconstructed rolling risk premia and yields (perfect foresight of short rates and risk premia)						
Maturities	Observed Yields (A)	Reconstructed Yields (B)	Rolling Risk Premium	Rolling Risk Premium (% of A)	Error (A-B)	Error (as a % of A)
Average of period 1970-1988						
6 months	7.988	7.996	0.392	4.501	-0.007	0.482
1 years	8.242	8.410	0.774	8.723	-0.168	-1.700
2 years	8.495	9.058	1.336	14.891	-0.562	-7.352
5 years	8.820	10.166	2.371	27.523	-1.345	-19.868
10 years	9.026	11.500	3.902	46.866	-2.474	-36.676
Average of period 1970-1978						
6 months	6.283	6.198	0.280	4.317	0.085	1.632
1 years	6.510	6.562	0.538	8.135	-0.052	-0.901
2 years	6.716	7.336	0.971	14.057	-0.620	-9.467
5 years	7.051	9.944	2.390	32.703	-2.893	-39.773
10 years	7.281	13.458	4.981	68.183	-6.177	-85.184
Average of period 1979-1982						
6 months	12.062	12.624	0.953	7.907	-0.561	-5.559
1 years	12.115	13.464	1.874	15.567	-1.349	-13.324
2 years	11.984	14.448	3.062	26.188	-2.464	-24.734
5 years	11.773	13.485	3.611	33.109	-1.711	-20.746
10 years	11.655	12.527	4.195	37.820	-0.872	-11.814
Average of period 1983-1988						
6 months	7.783	7.557	0.182	2.500	0.226	2.817
1 years	8.209	7.760	0.388	5.027	0.449	4.875
2 years	8.789	7.998	0.723	8.587	0.790	7.350
5 years	9.457	8.280	1.517	16.174	1.177	10.021
10 years	9.843	7.932	2.118	21.515	1.911	18.164

Notes

1. Estimated risk premia from estimation 10 have been used

Table 7: Tests of the REHTS with and without risk premia

Holding period equation				Term spread equation			
	a	b	c		d	e	f
6-month bond							
Constant	-0.183	0.415	-0.243	Constant	-0.173	0.332	0.243
	-0.244	0.686	-1.437		-0.825	1.410	1.437
Short rate	1.076	0.904	1.000	Spread	0.471	0.402	1.000
	8.468	8.886	.		1.443	1.333	.
Risk Premium		0.995	1.000	Risk Premium		-0.842	-1.000
		2.078	.			-1.660	.
R-squared	0.483	0.501	0.000	R-squared	0.020	0.061	0.000
Adj. R-squared	0.482	0.498	0.000	Adj. R-squared	0.017	0.056	0.000
Durbin-Watson stat	2.103	2.146	2.146	Durbin-Watson stat	1.887	1.912	2.146
1-year bond							
Constant	-0.726	0.052	-0.483	Constant	-0.411	0.639	0.483
	-0.490	0.043	-1.349		-0.783	1.059	1.349
Short rate	1.197	0.917	1.000	Spread	0.632	0.546	1.000
	4.788	4.465	.		1.102	1.029	.
Risk Premium		1.029	1.000	Risk Premium		-0.917	-1.000
		1.356	.			-1.269	.
R-squared	0.207	0.219	0.000	R-squared	0.010	0.032	0.000
Adj. R-squared	0.205	0.214	0.000	Adj. R-squared	0.007	0.027	0.000
Durbin-Watson stat	1.845	1.856	1.855	Durbin-Watson stat	1.774	1.786	1.855
2-year bond							
Constant	-0.838	0.468	-0.820	Constant	-0.537	1.315	0.820
	-0.319	0.211	-1.217		-0.478	1.068	1.217
Short rate	1.269	0.787	1.000	Spread	0.457	0.269	1.000
	2.890	1.997	.		0.536	0.359	.
Risk Premium		1.095	1.000	Risk Premium		-0.933	-1.000
		1.188	.			-1.144	.
R-squared	0.077	0.089	0.000	R-squared	0.002	0.020	0.000
Adj. R-squared	0.074	0.084	0.000	Adj. R-squared	-0.001	0.014	0.000
Durbin-Watson stat	1.733	1.737	1.735	Durbin-Watson stat	1.694	1.693	1.735
5-year bond							
Constant	1.115	2.647	-1.221	Constant	0.477	2.915	1.221
	0.232	0.583	-0.874		0.197	1.304	0.874
Short rate	1.082	0.396	1.000	Spread	-0.721	-0.878	1.000
	1.368	0.500	.		-0.553	-0.729	.
Risk Premium		1.083	1.000	Risk Premium		-0.775	-1.000
		1.242	.			-0.947	.
R-squared	0.014	0.021	0.000	R-squared	0.002	0.008	0.000
Adj. R-squared	0.011	0.015	0.000	Adj. R-squared	-0.001	0.002	0.000
Durbin-Watson stat	1.777	1.781	1.775	Durbin-Watson stat	1.745	1.746	1.775
10-year bond							
Constant	4.647	4.223	-2.460	Constant	2.430	4.838	2.460
	0.624	0.560	-1.063		0.615	0.946	1.063
Short rate	0.643	0.167	1.000	Spread	-2.044	-2.142	1.000
	0.531	0.128	.		-1.137	-1.251	.
Risk Premium		0.784	1.000	Risk Premium		-0.483	-1.000
		0.610	.			-0.413	.
R-squared	0.002	0.003	0.000	R-squared	0.006	0.007	0.000
Adj. R-squared	-0.001	-0.002	0.000	Adj. R-squared	0.003	0.002	0.000
Durbin-Watson stat	1.765	1.766	1.759	Durbin-Watson stat	1.746	1.746	1.759

Notes

1. Estimated risk premia from estimation 10 have been used
2. T-statistics using White heteroskedasticity-consistent standard errors are below the estimated parameters.

Figure 1: Estimated Risk Premia (including Jensen effect) and their macroeconomic components

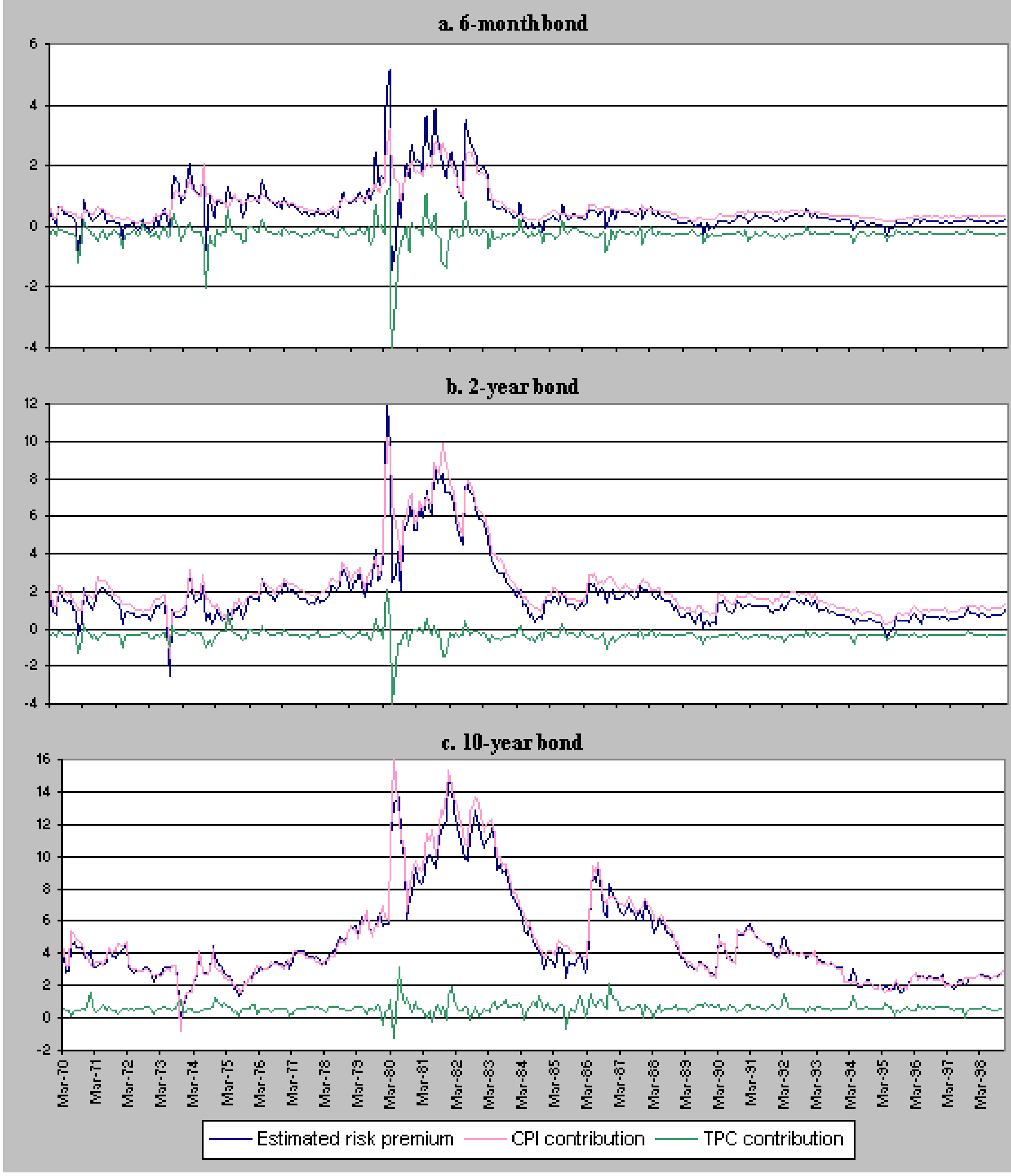

Figure 2: Estimated Risk Premia (including Jensen effect) and Excess Returns

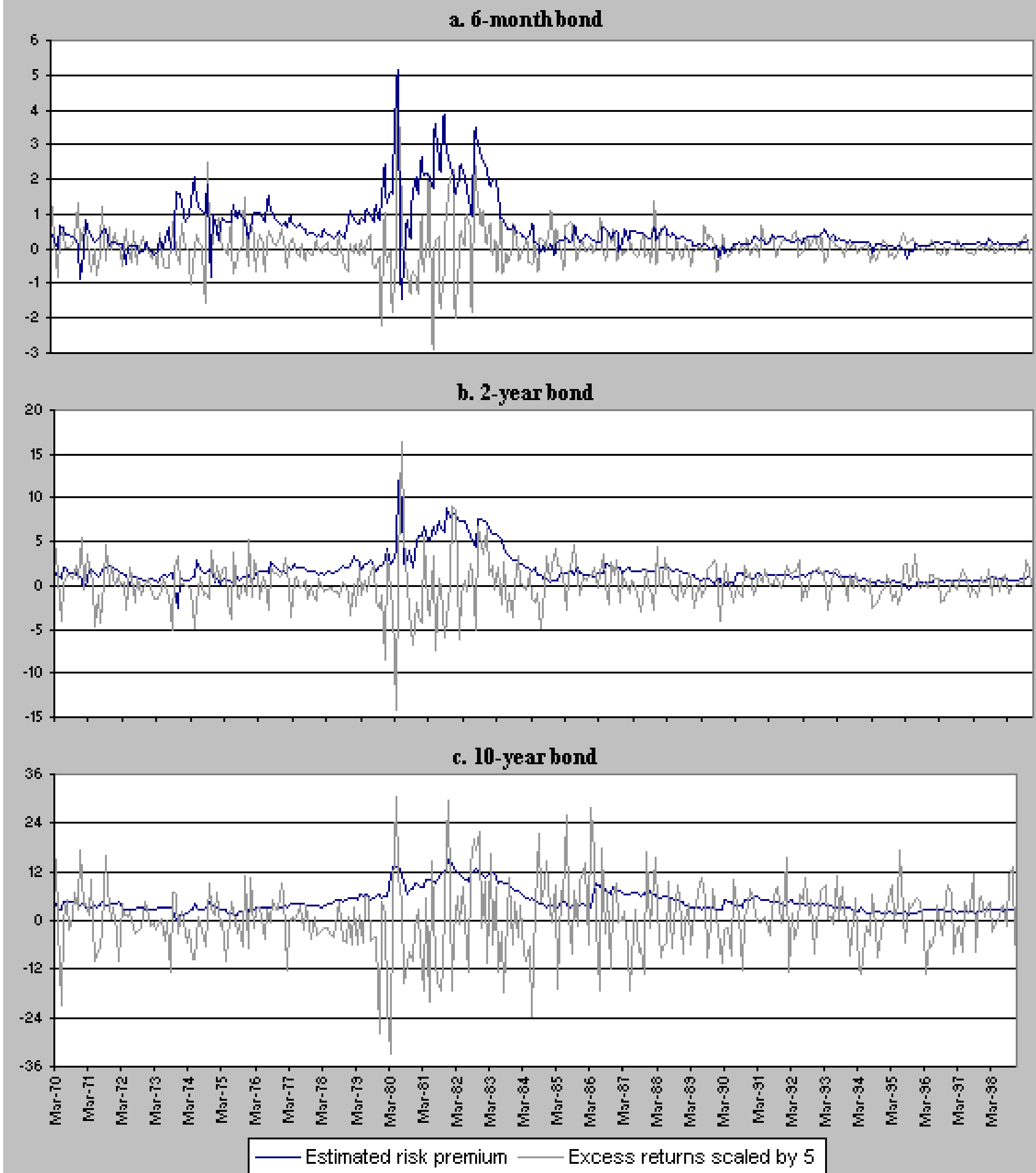

Figure 3: Estimated Risk Premia (including Jensen effect)

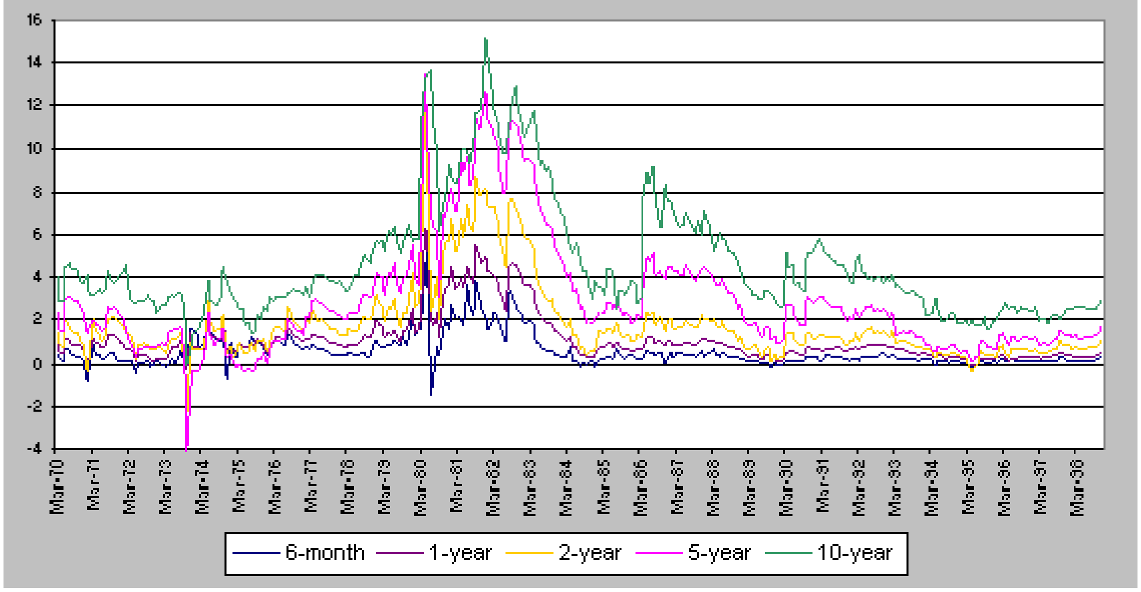

Figure 4: Rolling risk premia in bonds of different maturities
Static expectations

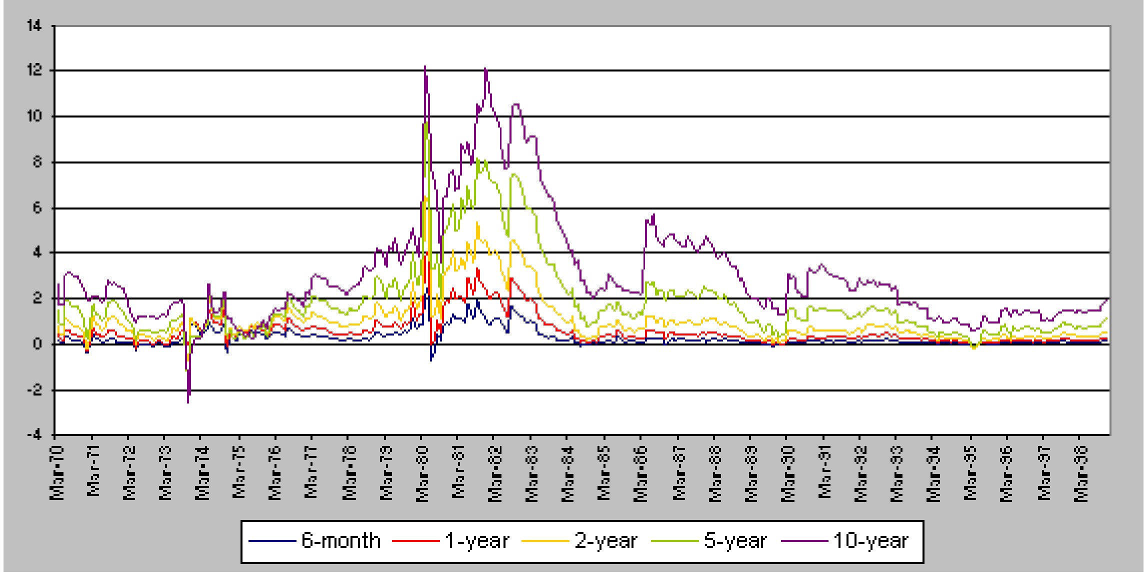

Figure 5: Rolling risk premia in bonds of different maturities as % of yield
Static expectations

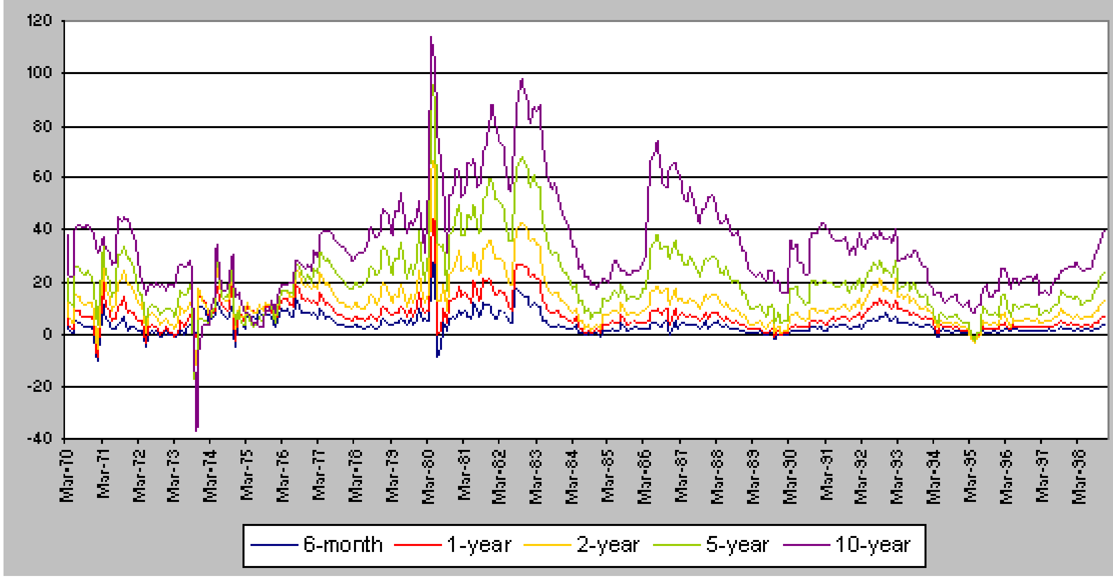

Figure 6: Rolling risk premia in bonds of different maturities
Perfect foresight

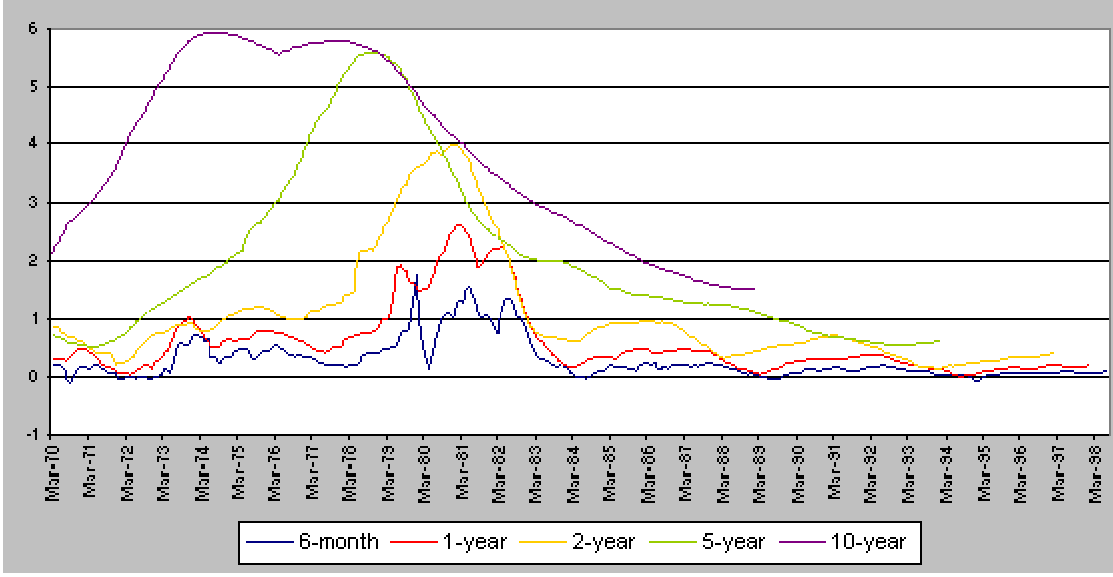

Figure 7: Rolling risk premia in bonds of different maturities as % of yield
Perfect foresight

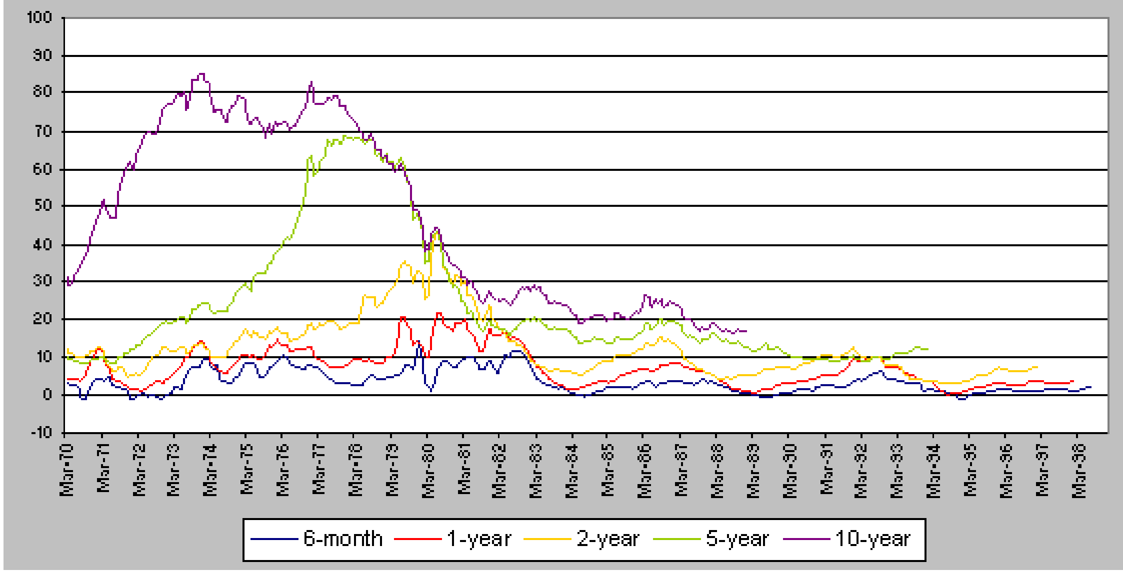

Figure 8: Yields

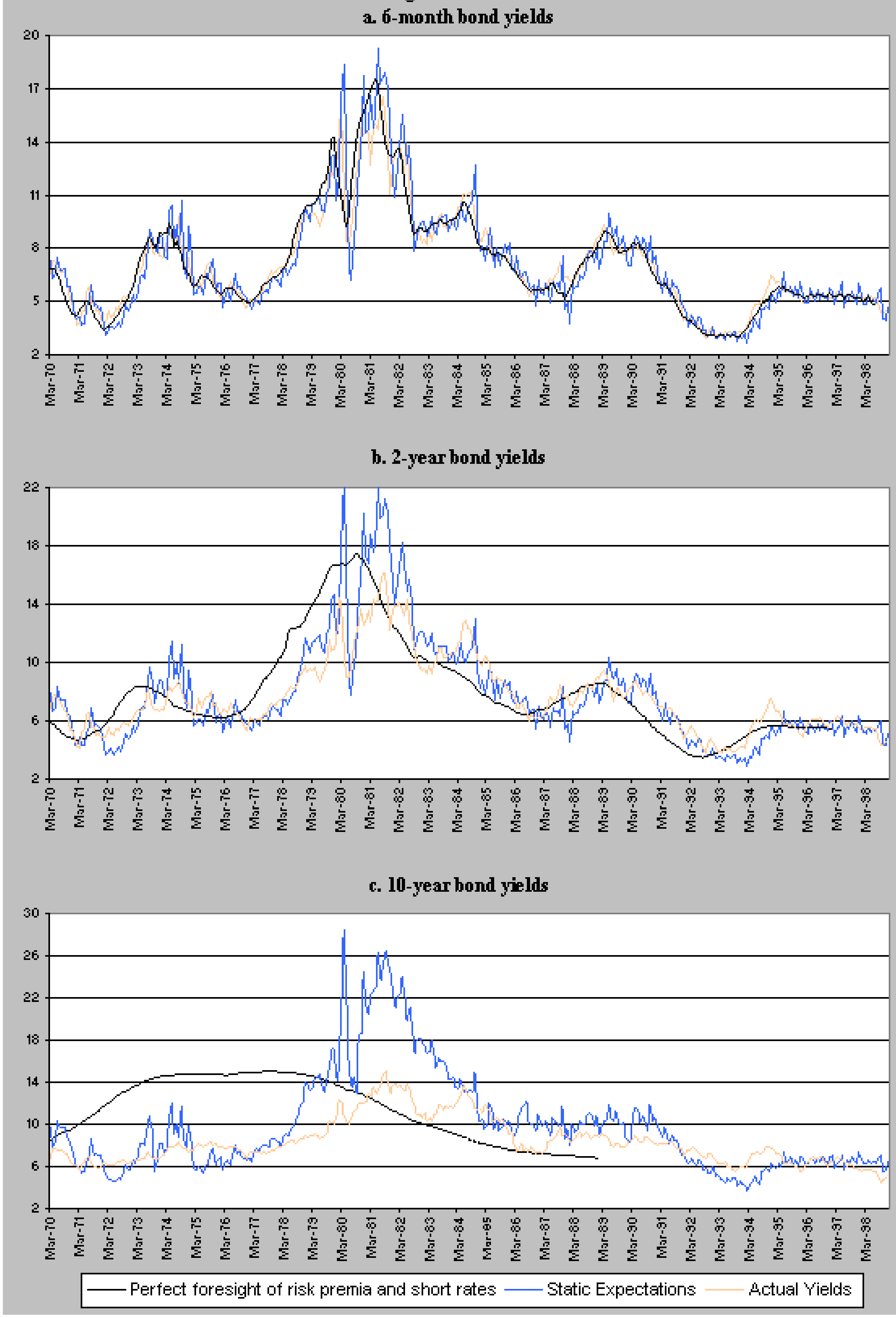

Figure 9: Term Structures