f-IGEM: An extension of the Italian General Equilibrium Model to financial frictions*

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Abstract

This paper provides a full technical description of a variant of the Italian General Equilibrium Model (IGEM) through the introduction of an imperfect financial sector. Among the different approaches followed by the recent literature, we consider a mechanism of financial accelerator inspired by Gertler and Karadi (2011). Credit market imperfections work on the side of the relationships between consumers-savers and banks, as an effect of asymmetric information. To illustrate how the above model behaves, the paper presents some simulation scenarios of unexpected disturbances and announced fiscal reforms.

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1 Introduction

This paper extends the Italian General Equilibrium Model (IGEM)[1] in use at the Department of Treasury to include financial imperfections through the introduction of a financial sector characterized by the presence of endogenous financial frictions.

According to Brzoza-Brzezina *et al.* (2015), who evaluate the most popular approaches to implementing financial frictions into DSGE models, standard financial accelerator setups are not sufficient in explaining the recent financial crisis and more sophisticated frameworks of financial intermediation are requested. Specifically, Brzoza-Brzezina *et al.* (2015) take DSGE models where financial frictions are modelled through US data. They make use of different approaches employing price and quantity of loans as key variables. Comparison favors the price framework, but the model is not able to make a clear improvement over the New Keynesian benchmark model.[2] As a matter of fact, to improve the fit, they suggest to explicitly model financial intermediation (Gertler and Kiyotaki, 2011; Gertler and Karadi, 2011) or to include occasionally binding collateral constraints (Mendoza, 2010; Brunnermeier and Sannikov, 2014). We follow the first approach by augmenting IGEM with the explicit modeling of financial intermediation suggested by Gertler and Karadi (2011).

In our setup, the financial accelerator of external shocks works on the side of relationships between consumers-savers and banks featured by asymmetric informa-

[1] IGEM is a medium scale Dynamic General Equilibrium (DGE) model for the Italian economy developed at the Department of Treasury of the Italian Ministry of the Economy and Finance and currently managed at Sogei S.p.A. (IT Economia - Modelli di Previsione ed Analisi Statistiche). It has been developed by Annicchiarico *et al.* (2013a, 2013b). A stochastic version has been built and estimated by Acocella *et al.* (2016a). The model has been designed to study the impact and the propagation mechanism of temporary shocks and to evaluate alternative reform scenarios, single policy interventions and fiscal consolidation packages. It features a large assortment of nominal and real frictions. The main characterization of IGEM is the segmented labor market, which captures one of the main peculiarities of the Italian economy.

[2] See also Christiano *et al.* (2014).

tion.[3] Specifically, the agency problem introduces endogenous constraints on the leverage ratios of intermediaries. As a result, in the financial sector, credit flows are tied to the equity capital of intermediaries. A deterioration of intermediary capital raises credit costs, lowering lending and borrowing.

The specification of the banking sector, suggested by Gertler and Karadi (2011) and Gertler and Kiyotaki (2011), has become quite popular in the recent macroeconomic literature on the role of credit imperfections (Lendvai *et al.*, 2013; Andreasen *et al.*, 2013; Beqiraj *el al.*, 2016; Rannenberg, 2016) and the importance of unconventional monetary policy in financial crisis (e.g., Dedola *et al.*, 2013; Gertler and Karadi, 2013, 2015).

Alternative models have been suggested, as we specify in the next section. It is worth noting that an additional value added of the Gertler and Karadi (2011) model is that it can better deal with the interactions between the specific characteristics of the Italian labour market (a feature of IGEM) and the banking sector. In fact, a shock to the economy has a different impact on labour of different categories of workers. This, in turn, affects the level of consumers' income and the depositors' conduct towards banks.

The rest of the paper is organized as follows. Section 2 briefly reports a theoretical background of the related literature. Section 3 describes the model setup. Section 4 presents the model calibration. Section 5 provides some example of $f-$IGEM usage. Specifically, some simulations are designed to illustrate how the model behaves and how it can be used to evaluate fiscal reforms. We considers both unexpected disturbances (technology, capital quality, and fiscal shock) and an example of the evaluation of an announced fiscal reform. The reform is assumed to be implemented when a sovereign debt crisis hits the economy. Section 5 concludes.

[3] Results can be extended to the case of frictions on the side of interbank relations (see Gertler and Kiyotaki, 2011).

2 Related literature[4]

The idea that imperfections in the financial sphere amplify the business cycle is not new. The financial accelerator theory dates back to Fisher (1933). But the possibility that large fluctuations in aggregate economic activity may arise from small financial shocks has been fully developed in a DSGE model by Bernanke *et al.* (1996) starting from the work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), who focused on the costs of borrowing and lending associated with asymmetric information.

Although the research line of Bernanke *et al.* (1996) was not the core topic among academics in the 1990s and the early 2000s, after the financial crisis the possibility that adverse conditions in the real economy and in financial markets mutually reinforce has been revisited by a number of authors.

The current literature explicitly models financial intermediaries. Financial frictions are introduced by assuming an agency problem either between banks and borrowers or between banks and depositors. The former route focuses on the interaction between labour markets and the financial sector. The latter is relevant for understanding and evaluating unconventional monetary policies since it attempts to model the consequences of credit shortages in the credit market (bank–firm), in the interbank market (bank–bank), or both.

New Keynesian extensions to financial frictions are based on the principal–agent theory. The idea is that the lender cannot accurately ascertain the return on the borrower's investment ex post because of asymmetric information. Lenders will thus require that borrowers post collateral and/or pay a credit premium to obtain funding when the return can be monitored incurring some costs in doing so.

Movements in current asset prices will then determine agents' access to credit.

[4] This section is largely drawn from Acocella *et al.* (2016b: 6.3)

Either because assets already owned represent collateral, or because when asset values increase the borrower's stake in the project increases, and incentives to default fall. The direct link between asset prices and credit implies that borrowing costs are countercyclical, which will strengthen the effects of a given shock and gives rise to a financial accelerator mechanism.

The financial frictions in DSGE New Keynesian models are founded on one of two mechanisms (or both):

1. An external finance premium (Carlstrom and Fuerst, 1997; Bernanke *et al.*, 1999).

2. Collateral constraints (Kiyotaki and Moore, 1997).

The former operates through the link between asset values and the incentive to default, which, given the monitoring cost, determines a countercyclical external finance premium. The latter operates by the relation between the value of the collateral constraints and the availability of credit, which is rationed when the (collateral) asset value falls.

In line with the first mechanism, Bernanke *et al.* (1999) extended the canonical New Keynesian model to include an entrepreneurial sector. Entrepreneurs, who are identical up to an idiosyncratic shock, require external funding to invest in new risky projects (funding is drawn from household deposits). Asymmetric information and costly verification imply that the external finance premium is a decreasing function of the share of project financed by net worth (equity), which equals profits to surviving entrepreneurs. Financing costs are then counter–cyclical.

Augmenting their model with investment adjustment costs (delays), Bernanke *et al.* (1999) show that the presence of a financial accelerator explains the extent and persistence of fluctuations caused by demand and supply shocks.

Gertler and Karadi (2011) and Gertler and Kiyotaki (2011) develop environments that can replicate the high interest rate spreads and the fall in lending which have characterized the recent events. In particular, Gertler and Kiyotaki (2011) model the fall in interbank loans. They compare the relative efficiency of different kinds of financial market interventions implemented by the Fed during the crisis: direct lending to private firms; lending to banks; injection of equity into banks.

Gertler and Karadi (2011) simulate a financial crisis and evaluate the welfare effects of unconventional monetary policies in a New Keynesian DSGE world with financial intermediaries who face endogenous balance sheet constraints. They find that the credit policy can significantly moderate the contraction induced by the crisis. The policy intervention in fact dampens the rise in the spread, which – in turn – dampens the decline in investment. In performing their welfare analysis, they show how the benefits of credit policies are substantial. Moreover, in the case of a binding ZLB constraint, these benefits are significantly enhanced.

In general, introducing an external finance premium is necessary to improve the statistical properties of the DSGE models in fitting the data in terms of degree and persistence of the fluctuations. However, its effects strongly depend on the assumptions introduced (Christiano *et al.*, 2014; Brzoza–Brzezina *et al.*, 2015).

The collateral mechanism is summarized in the Kiyotaki and Moore (1997) model, where entrepreneurs use contracts with fully secured debt to obtain capital resources. The collateral good has then two different roles: i) it is a productive input and ii) it serves as collateral for debt.

Entrepreneurs must provide collateral goods if they wish to borrow. If the value of the collateral good declines, they remain credit constrained. The following forced decline in investment demand leads to a decrease in expected future output. This feeds back into the collateral goods market, driving its price down further. Thus, due to the expected and actual decline in the collateral price, fluctuations can be amplified.

Kiyotaki and Moore's idea has been used by Iacoviello (2005) to illustrate the asymmetric effects of financial shocks in an otherwise standard New Keynesian model. He assumes real estate is a collateral good. Negative price shocks in the housing market reduce the amount of household loans used to facilitate household consumption because of the collateral constraint. The model is validated by the empirical analysis of Iacoviello and Neri (2010).

Thus, a novelty of the recent literature is the explicit consideration of financial intermediaries in monetary models. The pioneering attempt to introduce banks into a DSGE model is to be found in Goodfriend and McCallum (2007), followed by Gertler and Karadi (2011, 2015) and Gertler and Kiyotaki (2011), and also by others – among them Gerali *et al.* (2010) and Cúrdia and Woodford (2011).

By merging Iacoviello (2005) and Christiano *et al.* (2008), Gerali *et al.* (2010) introduce a banking sector into the DSGE model assuming an imperfectly competitive banking sector in both the deposit market and the loan market. They explain how the decisions of the central bank are not instantaneously or entirely transmitted into decisions of households and firms as the monopolistic power of banks changes the pass–through of the policy rate.

Eggertsson and Woodford (2003, 2004) meanwhile describe how quantitative easing operating through the expansion and composition of balance sheets of the central bank are not effective at the ZLB in a standard model with complete financial markets, competitive pricing and a representative agent. In that case, the rational expectations equilibrium is independent in both the length and the composition of the balance sheet of the central bank. Therefore, quantitative easing does not represent an additional tool for monetary policy at the ZLB in this special case. But forward guidance, formalized as a credible commitment regarding future monetary policy, can still be used to affect the economy.

Cúrdia and Woodford (2011) develop a stylized New Keynesian model with a banking sector, where the irrelevance result of Eggertsson and Woodford (2003, 2004) does not apply. Here the size and composition of the balance sheet of the central bank become relevant. Among other assumptions, they show that one needs to assume some non–trivial heterogeneity among private agents in order for intermediaries to matter in allocation of resources (i.e., financial contracting is feasible only via specialized intermediaries). In their model, financial intermediation is realized between heterogeneous households (rather than between households and firms). Borrowers and lenders have a different marginal utility of consumption, the optimality conditions of the model contain two discount factors, and the model produces two different interest rates and a credit spread which is an endogenous function of the markup in the intermediary sector and the cost of the loans. Their model confirms the canonical prescriptions for monetary policy and rejects the irrelevance result of quantitative easing at a ZLB. Moreover, Ricardian equivalence no longer holds, opening up space for active fiscal policies.

3 The model

3.1 Population structure, workers and households

There is a continuum of households in the space $[0, 1]$. There are two types of households differing with respect to their ability to access financial markets:

1. non-Ricardian households or non–Ricardians (indexed by NR) in the interval $[0, s_{NR}]$, who simply consume their disposable income and supply differentiated labour services as atypical workers and unskilled employees;

2. Ricardian households or Ricardians (indexed by R) in the interval $(s_{NR}, 1]$, who are able to smooth consumption over time and supply differentiated labour services as skilled and unskilled employees and as self-employed.

Each household is endowed with one unit of time in each period and share it between leisure and working efforts. It is assumed that each type of household provides all differentiated labour inputs within each category it supplies. It follows that by denoting s_{N_A}, s_{N_S}, s_{L_L}, and s_{L_H}, respectively, the population shares of atypical workers, self-employed workers, unskilled and skilled employees, we have that the following identities must hold: $s_{NR} = s_{N_A} + \lambda_{L_L} s_{L_L}$ and $1 - s_{NR} = s_{N_S} + s_{L_H} + (1 - \lambda_{L_L}) s_{L_L}$, where λ_{L_L} is the share of unskilled labour inputs supplied by non-Ricardian households.

For the sake of readability, we define the following sets: $\ell = \{L_L, L_H, N_S, N_A\}$, $\ell^R = \{L_L, L_H, N_S\}$, $\ell^{NR} = \{L_L, N_A\}$, which defines all the labor kinds, all the kinds supplied by Ricardians, and all the kinds supplied by non–Ricardians, respectively.

3.1.1 Households

Households are either Ricardian (R) or non-Ricardian (NR), i.e., liquidity constrained. The representative households of kind $i \in \{R, NR\}$ derive utility from consumption C^i of the final good and experience disutility from supplying labour inputs. According to our assumptions, Ricardian households supply labour as unskilled employees L_L, skilled employees L_H, and self-employed N_S. Non-Ricardian supply labour inputs as atypical workers N_A, and unskilled employees L_L.

The representative household's preferences are defined by a period utility function:

$$\mathcal{U}_t^i(C_t^i, \ell_t^i) = \log\left(C_t^i - h_{C^i}\overline{C}_{t-1}^i\right) + \sum\nolimits_{\ell^i} \frac{\omega_{\ell^i}\tilde{s}_{\ell^i}\left(1 - \ell_t^i\right)^{1-v_{\ell^i}}}{1 - v_{\ell^i}} \quad i \in \{R, NR\} \tag{1}$$

where the term $h_{C^i} \in [0,1)$ measures the external habit coefficient, ω_{ℓ^i} and v_{ℓ^i} denote category-specific preference parameters and $\tilde{s}_{\ell^i}$ denotes the share of time devoted by the household to the working activity of kind ℓ^i.[5] Being each household endowed with one unit of time we have $\sum_{\ell^i} \tilde{s}_{\ell^i} = 1$.

All households decide over consumption plans and on the amounts of labour inputs to supply (labour supply will be discussed in the next subsection). Ricardian households are assumed to own three assets: government bonds, $B^{G,R}$ and bank deposits $B^{D,R}$ paying the gross nominal interest rate R and foreign financial assets, B_F^R, paying the gross rate equal to R^* adjusted for the risk premium ρ^F (increasing in the aggregate level of foreign debt). Thus, Ricardian households need to solve an inter–temporal problem since, differently from the non–Ricardian households, they can smooth consumption. Capital producers hold capital, so they also need to choose investment plans. We assume that investment is subject to convex adjustment costs (see below).

Formally, the representative Ricardian household chooses consumption plans to maximize the expected present discount value of (1):

$$E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}_t^R(C_t^R, \ell_t^R) \tag{2}$$

where E_0 is the expectations operator conditional on information available at time 0 and $\beta \in (0,1)$ denotes the subjective discount factor.

Defining $P_{C,t}$ as the price of a unit of consumption good, S_t as the nominal exchange rate,[6] P_t as the domestic production price level, the nominal budget con-

[5]The functional form specification in (1) implies that the Frisch elasticity of labor supply is decreasing in the level of hours worked.

[6]Units of domestic currency per unit of foreign currency.

straint of the Ricardian household in maximizing (2) is:

$$(1+\tau_t^C)P_{C,t}C_t^R + B_{t+1}^R + S_t B_{F,t+1}^R = P_t \mathcal{L}_t^R + R_t B_t^R + (R_t^* + \rho_t^F) S_t B_{F,t}^R + P_t \mathcal{T}_t^R \quad (3)$$

where τ_t^C is a tax on consumption; $B_t^R = B_t^{G,R} + B_t^{D,R}$ is the household government bond and bank deposits holdings, $\mathcal{L}_t^R$ is the after–tax labour nominal income and T_t^R are other real transfers, including profits from firms' ownership.

The after–tax labour real income $\mathcal{L}_t^R$ is

$$\mathcal{L}_t^R = \textstyle\sum_{\ell^R} \left(1 - \vartheta_t^{\ell^R}\right) \tilde{s}_{\ell^R} W R_t^{\ell^R} \ell_t^R - \sum_{\ell^R} \tilde{s}_{\ell^R} \Gamma_{W^{\ell^R},t} \quad (4)$$

where $WR_t^{\ell^R} = W_t^{\ell^R}/P_t$ is the ℓ_t^R type real wage ($W_t^{\ell^R}$ is the nominal wage); $\vartheta_t^{\ell^R} = \tau_t^{\ell^R} - \tau_{h,t}^{W^{\ell^R}}$, $\tau_t^{\ell^R}$ are taxes on wage incomes, $\tau_{h,t}^{W^{\ell^R}}$ are contributions paid by the households to social security; $\Gamma_{W^{\ell^R},t}$ denotes real adjustment costs of wage changes (defined in Section 3.2). Other real transfers are defined as $\mathcal{T}_t^R = PRO_t^R + Tr_t^R - TAX_t^R$, where PRO_t^R are dividends received from the non-financial (intermediate goods) and financial firms, T_t^R and TAX_t^R are lump sum net transfer and tax, respectively.

The resulting first–order conditions are:

$$\varrho_t^R \left(1 + \tau_t^C\right) = \frac{P_t}{P_{C,t}} \frac{1}{C_t^R - h_{C^R} C_{t-1}^R} \quad (5)$$

$$\beta E_t \Lambda_{t,t+1} \frac{R_{t+1}}{\Pi_{t+1}} = 1 \quad (6)$$

$$\beta E_t \Lambda_{t,t+1} \frac{R_{t+1}^* + \rho_{t+1}^F}{\Pi_{t+1}} \frac{S_{t+1}}{S_t} = 1 \quad (7)$$

where $\Lambda_{t,t+1} = \varrho_{t+1}^R / \varrho_t^R$ denotes the discount factor and Π_t is the inflation rate. The equation (5) denotes the marginal utility of consumption; equation (6) is the Euler equation for Ricardian households; equation (7) denotes the Euler equation related to foreign assets.

The supplied capital (K_t) accumulates according to the following law of motions:

$$K_{t+1} = (1 - \delta_K)\psi_t K_t + I_t. \tag{8}$$

where I_t denotes investments and δ_K is the capital depreciation rate and ψ_t is a capital quality disturbance.[7]

Investment decisions are subject to a convex adjustment cost of $\Gamma_I\left(I_t, K_t\right) K_t$ where $\Gamma_I\left(I_t, K_t\right) = \frac{\gamma_I}{2}\left(\frac{I_t}{K_t} - \delta_K\right)^2$, with $\gamma_I > 0$.

Capital producers maximize the present discounted value of the following period profit function:

$$\max_{I_t} E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_{0,t}^R \frac{P_t^I}{P_t}\left[Q_t - (1 - tcr_t^K)\right]\left(I_t - \delta_K \psi_t K_t\right) - \frac{P_t^I}{P_t}\Gamma_I\left(I_t, K_t\right) K_t.$$

The optimal investment decision results in the following first–order condition

$$Q_t = (1 - tcr_t^K) + \gamma_I\left(\frac{I_t}{K_t} - \delta_K\right) \tag{9}$$

where Q_t denotes the Tobin's Q and tcr_t^K denotes a tax credit on investments.

The representative non-Ricardian household chooses consumption to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}_t^{NR}(C_t^{NR}, \ell_t^{NR}) \tag{10}$$

[7] Later we will assume that it follows an $AR(1)$ process in logs, setting $\exp(\zeta_t) = \psi_t$, $\zeta_t = (1-\rho_\zeta)\zeta_{t-1} + \epsilon_t^\zeta$ with $\epsilon_t^\zeta \sim N(0, \sigma_\zeta^2)$.

subjected to the following constraint

$$(1+\tau_t^C)\frac{P_{C,t}}{P_t}C_t^{NR} = \sum_{\ell^R}(1-\vartheta_t^{\ell^{NR}})\tilde{s}_{\ell^{NR}} WR_t^{\ell^{NR}} \ell_t^{NR} - \tilde{s}_{L_L}\Gamma_{W^{L_L},t} + \mathcal{T}_t^{NR} \tag{11}$$

where $WR_t^{\ell^{NR}} = W_t^{\ell^{NR}}/P_t$ and $\mathcal{T}_t^{NR} = T_t^{NR} - TAX_t^{NR}$.

As non-Ricardian households do not access financial markets, they cannot smooth consumption so their problem is static. Their consumption just equates their income that only derives from labour activities adjusted for taxation.

According to the budget constraint (*11*), the non-Ricardian consumption is

$$C_t^{NR} = \frac{P_t}{P_{C,t}}\frac{\sum_{\ell^{NR}}(1-\vartheta_t^{\ell^{NR}})\tilde{s}_{\ell^{NR}} WR_t^{\ell^{NR}} \ell_t^{NR} - \tilde{s}_{L_L}\Gamma_{W^{L_L},t} + \mathcal{T}_t^{NR}}{1+\tau_t^C} \tag{12}$$

and, in the symmetric equilibrium, their marginal utility of consumption is

$$\varrho_t^{NR}\left(1+\tau_t^C\right) = \frac{P_t}{P_{C,t}}\frac{1}{C_t^{NR} - h_{C^{NR}}C_{t-1}^{NR}} \tag{13}$$

3.2 Wage equations and labour supplies

The labour market setup is the core of IGEM. Monopolistic trade unions set wages of skilled and unskilled workers, which exhibit stable contracts and strong protection. Atypical workers are instead price takers and have flexible working patterns and weak labour protection. Self-employed workers and professionals supply labour under contracts for services. They have also some market power, due to the existence of professional order or to their limited number. Employees and self-employed are price makers and face both price (nominal wages).

The degree of nominal wage stickiness and hiring and firing costs, as well as their market power, are assumed to be much higher for employees. Market power introduces a wedge between the real wage rate and the marginal rate of substitution between leisure and consumption. By contrast, atypical workers, who often fail to qualify for labour protection rights and no market power.

Formally, labour supply decisions for the self–employed are taken by professional orders which supply labour services monopolistically to a continuum of labour markets of measure 1 indexed by $h_{N_S} \in [0,1]$. In each market the professional order is assumed to face a demand for labour, given by $N_{S,t}(h_{N_{S,t}}) = \left(\frac{W_t^{N_S}(h_{N_S})}{W_t^{N_S}}\right)^{-\sigma_{N_S}} N_{S,t}$, where $\sigma_{N_S} > 1$ is the elasticity of substitution between labour inputs, $W_t^{N_S}(h_{N_S})$ denotes the h_{N_S}–market–specific nominal wage and $N_{S,t} = \int_0^1 N_{S,t}(h_{N_S}) dh_{N_S}$.

Professional orders set $W_t^{N_S}(h_{N_{S,t}})$ to maximize households' expected utility, given the demand for its differentiated labour services and subject to a convex adjustment costs function: $\Gamma_{W^{N_S},t} = \frac{\gamma_{W^{N_S}}}{2}\left(\frac{1}{\Pi_{t-1}^{\kappa_W}\overline{\Pi}^{1-\kappa_W}} \frac{W_t^{N_S}(h_{N_s})}{W_{t-1}^{N_S}(h_{N_s})} - 1\right)^2 Y_t$, where $\gamma_{W^{N_S}} > 0$ and $\Pi_{t-1}^{\kappa_W}\overline{\Pi}^{1-\kappa_W}$ is a geometric average of past (gross) and long-run inflation, where the weight of past inflation is determined by the indexation parameter $\kappa_W \in [0,1]$.

The first-order condition in the symmetric equilibrium leads to the following wage equation:

$$\frac{\gamma_{W^{N_S}}\left[\left(\Omega_t^{N_S} - 1\right) Y_t \Omega_t^{N_S} - \beta\Lambda_{t+1}\left(\Omega_{t+1}^{N_S} - 1\right) Y_{t+1}\Omega_{t+1}^{N_S}\right]}{\left(\sigma_{N_S} - 1\right) N_{S,t}} = \frac{\sigma_{N_S}\omega_{N_S}\left(1 - N_{S,t}\right)^{-v_{N_S}}}{\left(\sigma_{N_S} - 1\right)\varrho_t^R} - \left(1 - \vartheta_t^{N_S}\right) WR_t^{N_S} \quad (14)$$

where $\Omega_t^{N_S} = WR_t^{N_S}\Pi_t / \Pi_{t-1}^{\kappa_W}\overline{\Pi}^{1-\kappa_W} WR_{t-1}^{N_S}$.

The first-order condition in the steady-state implies:

$$\left(1 - \vartheta^{N_S}\right)\frac{W^{N_S}}{P} = \frac{\sigma_{N_S}}{\sigma_{N_S} - 1}\frac{\omega_{N_S}}{\varrho^R\left(1 - N_s\right)^{v_{N_S}}}, \quad (15)$$

i.e., the market power in the labour market introduces a wedge between the net real remuneration of self-employed workers, $(1-\vartheta^{N_S})W^{N_S}/P$, and the marginal rate of substitution between leisure and consumption, $\omega_{N_S}/\varrho^R\,(1-N_s)^{v_{N_S}}$. This markup, $\sigma_{N_S}/(\sigma_{N_S}-1)$, is decreasing in the elasticity of substitution between differentiated labour services, σ_{N_S}, and reflects the degree of imperfect competition characterizing the labour market.

Monopolistic trade unions set wages of skilled and unskilled workers.

1. Skilled labour services are only provided by Ricardian households. Centralized unions supply them to a continuum of labour markets of measure 1 indexed by $h_{L_H} \in [0,1]$. In each market, the union faces a demand for labour given by $L_{H,t}(h_{L_H}) = \left(W_t^{L_H}(h_{L_H})/W_t^{L_H}\right)^{-\sigma_{L_H}} L_{H,t}$ where $\sigma_{L_H} > 1$ is the elasticity of substitution between differentiated labour services, $W_t^{L_H}(h_{L_H})$ is the market-specific nominal wage, $W_t^{L_H}$ denotes the wage index and $L_{H,t} = \int_0^1 L_{H,t}(h_{L_H})dh_{L_H}$.

2. Unskilled labour services are provided by both Ricardian and non-Ricardian households. As for skilled employees, a continuum of differentiated labour inputs indexed by $h_{L_L} \in [0,1]$ are supplied monopolistically by unions. We assume that households are distributed uniformly across unions. Hence, aggregate demand of h_{L_L}–labour $L_{L,t}(h_{L_L}) = \left(W_t^{L_L}(h_{L_L})/W_t^{L_L}\right)^{-\sigma_{L_L}} L_{L,t}$, is evenly distributed between all households, where $\sigma_{L_L} > 1$ is the elasticity of substitution between differentiated labour services, $W_t^{L_L}(h_{L_L})$ is the nominal wage of type h_{L_L}, and $L_{L,t} = \int_0^1 L_{L,t}(h_{L_L})dh_{L_L}$.

Both high-skilled and low-skilled workers face costly nominal wages adjustments of the form: $\Gamma_{W^i,t} = \frac{\gamma_{W^i}}{2}\left(\frac{W_t^i(h_i)}{\Pi_{t-1}^{\kappa_W}\overline{\Pi}^{1-\kappa_W}W_{t-1}^i(h_i)}-1\right)^2 Y_t,\ i \in \{L_H, L_L\}$, where $\gamma_{W^{L_H}} > 0$ and $\gamma_{W^{L_L}} > 0$.

By imposing symmetry across differentiated skilled labour services, the wage equations are:

$$\frac{\gamma_{W^{L_H}}\left[\left(\Omega_t^{L_H}-1\right)\Omega_t^{L_H}Y_t-\beta\Lambda_{t+1}\left(\Omega_t^{L_H}-1\right)\Omega_t^{L_H}Y_{t+1}\right]}{\left(\sigma_{L_H}-1\right)L_{H,t}}= \frac{\omega_{L_H}\sigma_{L_H}\left(1-L_{H,t}\right)^{-\upsilon_{L_H}}}{\left(\sigma_{L_H}-1\right)\varrho_t^R}-\left(1-\vartheta_t^{L_H}\right)WR_t^{L_H} \quad (16)$$

$$\frac{\gamma_{W^{L_L}}\left[\left(\Omega_t^{L_L}-1\right)\Omega_t^{LL}Y_t-\beta\Lambda_{t+1}\left(\Omega_t^{L_L}-1\right)\Omega_t^{L_L}Y_{t+1}\right]}{\left(\sigma_{L_L}-1\right)L_{L,t}}= \frac{\omega_{L_L}\sigma_{L_L}\left(1-L_{L,t}\right)^{-\upsilon_{L_L}}}{\left(\sigma_{L_L}-1\right)\varrho_t}-\left(1-\vartheta_t^{L_L}\right)WR_t^{L_L} \quad (17)$$

where $\Omega_t^{L_H} = WR_t^{L_H}\Pi_t/\Pi_t^{\kappa_W}\overline{\Pi}^{1-\kappa_W}WR_{t-1}^{L_H}$, $\Omega_t^{L_L} = WR_t^{L_L}\Pi_t/\Pi_t^{\kappa_W}\overline{\Pi}^{1-\kappa_W}WR_{t-1}^{L_L}$, CORRETTO and $\varrho_t = (1-\lambda^{L_L})\varrho_t^R + \lambda^{L_L}\varrho_t^{NR}$.

By imposing symmetry across differentiated skilled labour services, in steady-state the wage equations are

$$\left(1-\vartheta^{L_H}\right)\frac{W^{L_H}}{P}=\frac{\sigma_{L_H}}{\sigma_{L_H}-1}\frac{\omega_{L_H}}{\varrho^R\left(1-L_H\right)^{\upsilon_{L_H}}} \quad (18)$$

$$\left(1-\vartheta^{L_L}\right)\frac{W^{L_L}}{P}=\frac{\sigma_{L_L}}{\sigma_{L_L}-1}\frac{\omega_{L_L}}{\varrho\left(1-L_L\right)^{\upsilon_{L_L}}} \quad (19)$$

where we have used the fact that given the population structure, the weights assigned by the union to Ricardian and non–Ricardian households are given by $(1-s_{NR})$ and s_{NR}, respectively, and given the allocation of time within each household, the effective weights are $(1-\lambda_{L_L})$ and λ_{L_L}, respectively.[8]

By assumption, only non–Ricardian households supply labour services as atypical workers. For this category of workers, with no union coverage, their labour

[8]Given the population structure and the allocation of time within each household, the weights attached by the union to Ricardian and non–Ricardian households are, in fact, given by $(1-s_{NR})\frac{1-\lambda_{L_L}}{1-s_{NR}}s_{L_L}$ and $s_{NR}\frac{\lambda_{L_L}}{s_{NR}}s_{L_L}$.

supply then equates the net real wage to marginal rate of substitution between leisure and consumption:

$$\left(1-\vartheta_t^{N_A}\right)\frac{W_t^{N_A}}{P_t}=\frac{\omega_{N_A}}{\varrho_t^{NR}\left(1-N_{A,t}\right)^{\upsilon_{N_A}}}. \tag{20}$$

3.3 Banking sector

The representation of the financial sector is borrowed from Gertler and Karadi (2011) and Beqiraj *et al.* (2016). Banks are owned by households. Each period a fraction θ of bankers survives while a fraction $1-\theta$ exits and is replaced.[9]

The balance sheet of a representative bank j is

$$Q_t S_{j,t}^F = NW_{jt} + B_{jt+1}^D \tag{21}$$

where NW_{jt} denotes the amount of wealth (net worth) that a j-banker has at the end of period t, B_{jt+1}^D represents the deposit that the intermediary obtains from households, $S_{j,t}^F$ is the quantity of financial claims on non-financial firms that the intermediary holds, and Q_t is the relative price.

The banker's equity capital evolves as the difference between the earnings on assets, $R_{kt+1}Q_tS_{j,t}^F$, and interest payments on liabilities, $R_{t+1}B_{jt+1}^D$. Thus:

$$NW_{jt+1}=\left[R_{kt+1}-R_{t+1}-\left(R_{kt+1}-1\right)\tau_t^K\right]Q_tS_{j,t}^F+R_{t+1}NW_{jt} \tag{22}$$

The term $\left[R_{kt+1}-R_{t+1}-\left(R_{kt+1}-1\right)\tau_t^K\right]$ represents the premium that the banker earns on his assets.

Each j-banker's objective is then to maximize V_{jt}, i.e., the expected discounted

[9] New bankers are endowed with a fraction $\zeta/(1-b)$ of the value of the assets intermediated by the existing bankers. Indeed, there are different ways to model bankers turnover. See, e.g., Gertler and Kiyotaki (2011) for a discussion.

present value of its future flows of net worth NW_t, that is:

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} NW_{jt+1+i}. \tag{23}$$

In order to limit the ability of the intermediary to expand its assets indefinitely a moral hazard enforcement problem (derived from agency cost) is introduced. In particular, at the beginning of each period the banker can choose to divert a fraction λ of available funds from the project and transfer them back to the household.

The incentive constraint faced by the lender that wants to supply funds to the banker can be expressed as:

$$V_{jt} \geq \lambda Q_t S_{j,t}^F \tag{24}$$

The term V_{jt} represents the loss supported by the banker when he diverts a fraction of assets, whereas the right side is the gain from diverting funds. We can express V_{jt} as:

$$V_{jt} = \nu_t Q_t S_{j,t}^F + \eta_t NW_{jt} \tag{25}$$

where:

$$\begin{aligned} \nu_t &= E_t \left\{ (1-\theta)\, \beta \Lambda_{t+1} \left[R_{kt+1} - R_{t+1} - (R_{kt+1} - 1)\, \tau_t^K \right] + \beta \Lambda_{t+1} \theta x_{t,t+1} \nu_{t+1} \right\} \\ \eta_t &= E_t \left\{ (1-\theta) + \beta \Lambda_{t+1} \theta z_{t,t+1} \eta_{t+1} \right\}. \end{aligned}$$

Here τ_t^K represents the capital taxes, ν_t is the value of bank's capital and represents the expected discounted marginal gain to the banker of expanding assets $Q_t S_{j,t}^F$ by a unit holding net worth constant. The term η_t denotes the value of banks' net worth and represents the expected discounted value of having another unit of NW_{jt}, holding $K_{j,t}$ constant. Instead, $x_{t,t+1} \equiv Q_{t+i} S_{j,t+1}^F / Q_t S_{j,t}^F$ indicate the gross growth rate in assets between t and $t+i$, whereas $z_{t,t+1} \equiv NW_{jt+1}/NW_{jt}$ denotes the gross growth rate of net worth.

Moreover, $\left[R_{kt+1} - R_{t+1} - (R_{kt+1} - 1)\,\tau_t^K\right]$ is the risk premium suggesting the presence of financial frictions. Given the equation for the banker's loss, V_{jt}, (25) then rewriting the incentive constraint (24) yields

$$\nu_t Q_t S_{j,t}^F + \eta_t NW_{jt} \geq \lambda Q_t S_{j,t}^F. \tag{26}$$

If this rewritten incentive constraint binds, then the assets the banker can acquire will depend positively on his equity:

$$Q_t S_{j,t}^F = \frac{\eta_t}{\lambda - \nu_t} NW_{jt} = \phi_t NW_{jt} \tag{27}$$

where ϕ_t is the ratio of privately intermediated assets to equity (private leverage ratio).

The evolution of the growth rate of banks' capital, $x_{t,t+1}$, and the growth rate of banks' net wealth, $z_{t,t+1}$, can be written as:

$$z_{t,t+1} = \frac{NW_{jt+1}}{NW_{jt}} = \left[R_{kt+1} - R_{t+1} - (R_{kt+1} - 1)\,\tau_t^K\right]\phi_t + R_{t+1} \tag{28}$$

$$x_{t,t+1} = \frac{Q_{t+1} S_{j,t+1}^F}{Q_t S_{j,t}^F} = \frac{\phi_{t+1}}{\phi_t}\frac{NW_{jt+1}}{NW_{jt}} = \frac{\phi_{t+1}}{\phi_t} z_{t,t+1}. \tag{29}$$

The law of motion of NW_t is given by the sum of the net worth of existing bankers, NW_{et}, and the net worth of entering bankers, NW_{nt}:

$$NW_t = NW_{et} + NW_{nt}. \tag{30}$$

The existing banks' net worth accumulation is

$$NW_{et} = \theta \left[\left(R_{kt} - R_t - \left(R_{kt} - 1 \right) \tau_{t-1}^{K} \right) \phi_{t-1} + R_t \right] NW_{t-1} \tag{31}$$

The new banks' net worth is given by

$$NW_{nt} = \omega Q_t S_{j,t-1}^{F} \tag{32}$$

where ω is the proportional starting up funds that each period the household transfers to the entering bankers.

3.4 The domestic production sector

The intermediate goods sector is made by a continuum of competitive producers indexed by $j \in [0,1]$. The typical firm j uses labour inputs and capital to produce intermediate goods $Y_t(j)$ according to the following technology:

$$Y_t(j) = A_t \left[\mathcal{L}_t^{\alpha_L} \mathcal{N}_t^{\alpha_N} \left(u_t^k \psi_t K_{j,t} \right)^{1-\alpha_L-\alpha_N} \right]^{1-\alpha_G} \left(K_t^G \right)^{\alpha_G} \tag{33}$$

where α_L, $\alpha_N > 0$ and $\alpha_L + \alpha_N < 1$, A_t denotes the total factor productivity, K_t^G is the public capital; $\mathcal{L}_t = L_{CES,t} - OH_t^L$, $\mathcal{N}_t = N_{CES,t} - OH_t^N$, with $L_{CES,t}$ and $N_{CES,t}$ denoting CES aggregates of labour inputs hired, whereas OH_t^L and OH_t^N stand for overhead labour.

The bundles $L_{CES,t}$ and $N_{CES,t}$ are defined as follows:

$$L_{CES,t} = \left[sx_{L_L}^{\frac{1}{\sigma_L}} \left(ef_{L_L} LY_{L,t} \right)^{\frac{\sigma_L - 1}{\sigma_L}} + sx_{L_H}^{\frac{1}{\sigma_L}} \left(ef_{L_H} LY_{H,t} \right)^{\frac{\sigma_L - 1}{\sigma_L}} \right]^{\frac{\sigma_L}{\sigma_L - 1}} \tag{34}$$

$$N_{CES,t} = \left[sx_{N_S}^{\frac{1}{\sigma_N}} \left(ef_{N_S} NY_{S,t} \right)^{\frac{\sigma_N - 1}{\sigma_N}} + sx_{N_A}^{\frac{1}{\sigma_N}} \left(ef_{N_A} NY_{A,t} \right)^{\frac{\sigma_N - 1}{\sigma_N}} \right]^{\frac{\sigma_N}{\sigma_N - 1}} \tag{35}$$

where σ_L, $\sigma_N > 1$ measure the elasticity of substitution between the categories of workers of each CES aggregator, the coefficients ef_{L_L}, ef_{L_H}, ef_{N_S}, and ef_{N_A} measure efficiency, the terms sx_{L_L}, sx_{L_H}, sx_{N_S}, and sx_{N_A} represent the shares of each category of workers; $LY_{L,t} = s_{L_L} L_{L,t}$, $LY_{H,t} = s_{L_H} L_{H,t}$, $NY_{S,t} = s_{N_S} N_{S,t}$, and $NY_{A,t} = s_{N_A} N_{A,t}$ denote the labour inputs.

The stock of government capital depends on the public infrastructure investment decisions I_t^G and evolves according to the following law of motion:

$$K_{t+1}^G = (1 - \delta_G) K_t^G + I_t^G$$

with δ_G being the depreciation rate.

Producing firms are also assumed to control the rate of utilization at which this factor is utilized, u_t^K. As in Christiano *et al.* (2005), using the stock of capital at a rate u_t^K entails a cost in terms of the final good equal to $\Gamma_{u^K}\left(u_t^K\right)\psi_t K_t(i)$, where $\Gamma_{u^K}\left(u_t^K\right) = \gamma_{u_1^K}\left(u_t^K - 1\right) + \frac{\gamma_{u_2^K}}{2}\left(u_t^K - 1\right)^2$, with $\gamma_{u_1^K}, \gamma_{u_2^K} > 0$.

The objective of each firm j is to maximize the sum of expected discounted real profits by setting the optimal price $P_t(j)$ and making choices about labour inputs and physical capital. In setting its decisions, each firm is constrained by the available technology (33), the demand schedule for variety j, $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta_Y} Y_t$,[10] quadratic adjustment costs on price setting $\Gamma_{P,t} = \frac{\gamma_P}{2}\left(\frac{1}{\Pi_{t-1}^{\kappa_P}\Pi^{1-\kappa_P}}\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t$, (with $\gamma_p > 0$ and $\kappa_P \in [0,1]$ denoting weight of past inflation in the indexation). Due to the existence of hiring and firing costs, each firm is also constrained by quadratic adjustment costs in changing labor demands; formally, these costs are: $\Gamma_{NY_i} = \frac{\gamma_{N_i}}{2}\left(\frac{NY_{i,t}(j)}{NY_{i,t-1}(j)} - 1\right)^2 Y_t$ for $i \in \{A, S\}$ and $\Gamma_{LY_A} = \frac{\gamma_{L_i}}{2}\left(\frac{LY_{i,t}(j)}{LY_{i,t-1}(j)} - 1\right)^2 Y_t$ for $i \in \{H, L\}$.

[10] The intermediate good j is demanded by final good firms to produce consumption and investment goods, $Y_{H,t}$, and by exporters to produce tradable goods, EX_t.

At the optimum and under symmetry, the optimal pricing decision equation describes the time path of domestic inflation:

$$\frac{1-\theta_Y+MC_t\theta_Y}{\gamma_p}=\left(\Omega_t-1\right)\Omega_t-\beta E_t\Lambda^R_{t+1}\left(\Omega_{t+1}-1\right)\Omega_{t+1}\frac{Y_{t+1}}{Y_t} \tag{36}$$

where $\Omega_t = \Pi_t/\Pi_{t-1}^{\kappa_P}\overline{\Pi}^{1-\kappa_P}$. In the steady state, equation (36) implies $MC = (\theta_Y-1)/\theta_Y$ (the steady state marginal cost equals the inverse markup).

In the symmetric equilibrium, profit maximization implies that

$$R_{kt+1}=\frac{\left(1-\alpha_L-\alpha_N\right)\frac{MC_{t+1}Y_{t+1}}{K_{t+1}\psi_{t+1}}+\left[Q_{t+1}-\delta_K-\Gamma_{u^K}\left(u^K_{t+1}\right)\right]\psi_{t+1}}{Q_t} \tag{37}$$

The utilization rate of capital is also optimally set and satisfies:

$$\left(1-\alpha_L-\alpha_N\right)MC_t\frac{Y_t}{u_t}=\frac{\partial\Gamma_{u^K}\left(u^K_t\right)}{\partial u_t}\psi_t K_t \tag{38}$$

where $\partial\Gamma_{u^K}\left(u^K_t\right)/\partial u_t=\gamma_{u^K_1}+\gamma_{u^K_2}\left(u^K_t-1\right)$.[11]

In the symmetric equilibrium, then, the firm's cost minimization implies the following implicit labour demands for $LY_{L,t}$, $LY_{H,t}$, $NY_{S,t}$ and $NY_{A,t}$:

$$\begin{aligned}\left(1-\vartheta^{N_i}_{f,t}\right)WR^{N_i}_t=&\frac{\alpha_N\left(1-\alpha_G\right)MC_tY_tsx^{1/\sigma_N}_{N_i}ef^{(\sigma_N-1)/\sigma_N}_{N_i}}{\left(N_{CES,t}-OH^N_t\right)}\left(\frac{N_{CES,t}}{NY_{i,t}}\right)^{1/\sigma_N}+\\&-\gamma_{N_i}\left(\frac{NY_{i,t}}{NY_{i,t-1}}-1\right)\frac{Y_t}{NY_{i,t-1}}+\beta\Lambda_{t+1}\gamma_{L_i}\left(\frac{NY_{i,t+1}}{NY_{i,t}}-1\right)\frac{Y_{t+1}NY_{i,t+1}}{NY^2_{i,t}}\end{aligned} \tag{39}$$

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[11] $u_t=-\frac{1}{2}\frac{\left(\gamma_{u^K_1}-\gamma_{u^K_2}\right)\psi_tK_t(i)+\sqrt{\left[\gamma_{u^K_1}\psi_tK_t(i)-\gamma_{u^K_2}\psi_tK_t(i)\right]^2+4\gamma_{u^K_2}(1-\alpha_L-\alpha_N)MC_tY_t\psi_tK_t(i)}}{\gamma_{u^K_2}\psi_tK_t(i)}$

for $i \in \{A, S\}$, and

$$\left(1-\vartheta_t^{L_i}\right) WR_t^{L_i} = \frac{\alpha_L\left(1-\alpha_G\right) MC_t Y_t sx_{L_i}^{1/\sigma_L} ef_{L_i}^{(\sigma_L-1)/\sigma_L}}{\left(L_{CES,t}-OH_t^L\right)}\left(\frac{L_{CES,t}}{LY_{i,t}}\right)^{1/\sigma_L} + \\ -\gamma_{L_i}\left(\frac{LY_{i,t}}{LY_{i,t-1}}-1\right)\frac{Y_t}{LY_{i,t-1}}+\beta\Lambda_{t+1}\gamma_{L_i}\left(\frac{LY_{i,t+1}}{LY_{i,t}}-1\right)Y_{t+1}\frac{LY_{i,t+1}}{LY_{i,t}^2} \quad (40)$$

for $i \in \{H, L\}$, where $\vartheta_{f,t}^j = sub_t^j + \tau_{f,t}^{Wj}$ and $\tau_{f,t}^{Wj}$ is the social contributions paid by the firms to workers of each kind $j \in \ell$, we assume $sub_t^{N_S} = 0$.

3.5 Foreign sector

The open economy aspects are captured by two channels: i) The presence of exporting and importing firms; ii) the ability of the agents to save or borrow on foreign financial assets. A continuum of monopolistically competitive exporting (importing) firms transform domestic (foreign) intermediate goods into exportable (importable) goods using a linear technology. Exporters and importers seek to maximize profits by setting prices. Then the development of the net foreign asset position depends on the current account surplus and on the decisions of firms, households and government. The net external position will depend on conditions in both financial and goods markets.

In the export sector, there exist a continuum of monopolistically competitive firms transforming domestic intermediate goods into exportable goods using a linear technology. This implies that exporters are able to set the price for their product at a markup over their marginal cost. However, there are costs of adjusting prices $\Gamma_{P_X.t} = \frac{\gamma_{EX}}{2}\left(\frac{P_{X,t}(j)/P_{X,t-1}(j)}{\Pi_{t-1}^{*\kappa_{EX}}\overline{\Pi}^{*1-\kappa_{EX}}}-1\right)^2 EX_t$, where $P_{X,t}(j)$ is the price set by the exporter in foreign currency for the good j, $\gamma_{EX} > 0$, $\Pi_{t-1}^{*\kappa_{EX}}\overline{\Pi}^{*1-\kappa_{EX}}$ denotes a geometric average of past (gross) and long-run inflation prevailing in the foreign market, where the weight of past inflation is determined by the indexation parameter $\kappa_{EX} \in [0,1]$.

The typical exporting firm will set the exporting price $P_{X,t}(j)$, so as to maximize the expected discounted value of future profits, taking as given the adjustment cost, the exchange rate S_t and the world demand for good j given by $EX_t(j) = (P_{X,t}(j)/P_{X,t})^{-\theta_{EX}}\ EX_t$, where $\theta_{EX} > 1$ is the elasticity of substitution between tradeable goods, EX_t denotes the total demand of export and $P_{X,t}$ is the aggregate export price index, given by $P_{X,t} = \left(\int_0^1 P_{X,t}(j)^{1-\theta_{EX}}\, dj\right)^{\frac{1}{1-\theta_{EX}}}$.[12]

Similarly, importers set prices in local currency as a markup over the import prices of intermediate goods produced abroad and facing a demand $IM_t(j) = \left(P_{M,t}(j)/P_t^M\right)^{-\theta_{IM}}\ IM_t$, where $\theta_{IM} > 1$ is the elasticity of substitution between imported goods. The term IM_t denotes the total demand of imported goods, $P_{M,t}(j)$ is the price of the imported good expressed in domestic currency and $P_{M,t}$ is the aggregate import price index, given by $P_{M,t} = \left(\int_0^1 P_{M,t}(j)^{1-\theta_{IM}}\, dj\right)^{\frac{1}{1-\theta_{IM}}}$. The quadratic cost function of adjusting prices is $\Gamma_{P_M,t} = \frac{\gamma_{IM}}{2}\left(\frac{P_{M,t}(j)/P_{M,t-1}(j)}{\Pi_{t-1}^{\kappa_{IM}}\overline{\Pi}^{1-\kappa_{IM}}} - 1\right)^2 IM_t$, where $\gamma_{IM} > 0$ and $\kappa_{IM} \in [0,1]$.

Resulting export and import price inflation dynamics are described by the following expressions:

$$\Omega_t^{EX} = \frac{S_t P_{X,t}(1-\theta_{EX}) + P_t\theta_{EX}}{\gamma_{EX} P_t} + \beta E_t \Lambda_{t+1} \Omega_{t+1}^{EX} \frac{EX_{t+1}}{EX_t} \tag{41}$$

$$\Omega_t^{IM} = \frac{P_{M,t}(1-\theta_{IM}) + S_t P_t^* \theta_{IM}}{\gamma_{IM} P_t} + \beta E_t \Lambda_{t+1} \Omega_{t+1}^{IM} \frac{IM_{t+1}}{IM_t} \tag{42}$$

where $\Omega_t^{EX} = \frac{\left(\Pi_t^{EX} - (\Pi_{t-1}^*)^{\kappa_{EX}}(\overline{\Pi}^*)^{1-\kappa_{EX}}\right)\Pi_t^{EX}}{\left((\Pi_{t-1}^*)^{\kappa_{EX}}(\overline{\Pi}^*)^{1-\kappa_{EX}}\right)^2}$, $\Omega_t^{IM} = \frac{\left(\Pi_t^{IM} - \Pi_{t-1}^{\kappa_{IM}}\overline{\Pi}^{1-\kappa_{IM}}\right)\Pi_t^{IM}}{\left(\Pi_{t-1}^{\kappa_{IM}}\overline{\Pi}^{1-\kappa_{IM}}\right)^2}$, and $\overline{\Pi}^*$ is the rest of the world gross inflation rate in the steady state.

To close the foreign sector, we have to determine the evolution of net foreign

[12] We assume that evolves according to $EX_t = \alpha_{EX}\left(P_{X,t}/P_{C,t}^*\right)^{-\sigma_{EX}} WD_t$, where WD_t is the world demand and α_{EX} and σ_{EX} are parameters.

assets. The economy's net foreign asset position denominated in domestic currency evolves as:

$$S_t B_{F,t} = \left(R_t^* + \rho_t^F\right) S_t B_{F,t-1} + S_t P_{X,t} EX_t - P_{M,t} IM_t, \tag{43}$$

where the risk premium ρ_t^F is assumed to be increasing in the aggregate level of foreign debt. The following functional form for the risk premium is used: $\rho_t^F = -\varphi^F(e^{B_t^{r,F} - \overline{B}^{r,F}} - 1)$, where φ^F is a positive parameter, $B_{F,t}^R = S_t B_{F,t+1}/P_t$ and $\bar{B}_F^R$ is the steady state level of net foreign assets in real terms and home currency (see Schmitt-Grohé and Uribe, 2003). Clearly, in the steady-state $\rho^F = 0$.[13]

3.6 Aggregation and market clearing

Only Ricardian households hold financial assets and own domestic firms. Therefore, equilibrium requires that the following aggregation conditions must be satisfied: $B_t = (1 - s_{NR})B_t^R$, $B_{F,t} = (1 - s_{NR})B_{F,t}^R$, $PRO_t = (1 - s_{NR})PRO_t^R$. By contrast, aggregate consumption is $C_t = (1 - s_{NR})C_t^R + s_{NR}C_t^N$.

In the banking sector, market clearing implies that on aggregate:

$$Q_t S_t^F = Q_t K_{t+1} \tag{44}$$

Since the final good can be used for private and public consumption and for private and public investment, we have

$$P_{C,t} = P_{I,t} = \left[(1 - \alpha_{IM})P_t^{1-\sigma_{IM}} + \alpha_{IM} P_{M,t}^{1-\sigma_{IM}}\right]^{\frac{1}{1-\sigma_{IM}}} \tag{45}$$

[13]Note that (43) implies $B_{F,t}^R = \frac{(R_t^* + \rho_t^F)}{\Pi_t} \frac{S_t}{S_{t-1}} B_{F,t-1}^R + \frac{S_t P_{X,t}}{P_t} EX_t - \frac{P_{M,t}}{P_t} IM_t$.

and

$$P_{M,t}IM_t + P_tY_{H,t} = P_{C,t}\left(C_t + C_t^G + I_t + I_t^G\right) \tag{46}$$

Market clearing in the intermediate goods market requires

$$Y_t = Y_{H,t} + S_t\frac{P_{X,t}}{P_t}EX_t \tag{47}$$

From (46) and (47), the resource constraint of the economy immediately follows:

$$Y_t = \frac{P_t^C\left(C_t + C_t^G + I_t + I_t^G\right)}{P_t} + \frac{S_tP_{X,t}EX_t - P_{M,t}IM_t}{P_t} + \Gamma_t \tag{48}$$

where Γ_t is the sum of all the Γ–adjustment costs present in the economy.

3.7 Fiscal and monetary authorities

The government issues nominal debt in the form of interest-bearing bonds. Public consumption and investment, interest payments on outstanding public debt, transfers to households and subsidies to firms are financed by taxes on capital, labour and consumption and/or by issuance of new bonds. To ensure that the fiscal budget constraint is met, the fiscal authority is assumed to adopt a fiscal rule responding to public debt. The external monetary authority (ECB) controls the nominal interest rate, which, to some extent, responds to domestic conditions.

The fiscal sector is characterized by two equations defining the public deficit dynamics and the tax rule. Specifically, the government purchases final goods for consumption C_t^G and investment I_t^G, makes transfers to households Tr_t, gives subsidies to intermediate goods producers SUB_t, receives lump-sum taxes T_t and tax payments on labour income, consumption and capital, and issues nominal bonds B_t^G (in real terms $B_t^{G,r}$).

The real deficit (D_t) dynamics is then given by:

$$D_t = (R_t - 1)\frac{B_t^{G,r}}{\Pi_t} + \frac{P_{C,t}G_t + P_{I,t}I_t^G}{P_t} + Tr_t - T_t - \sum_{X_t \in TX} X_t + SUB_t, \tag{49}$$

where $TX = \{L^{TAX},\ C^{TAX},\ K^{TAX}\}$ are the revenues determined by the given tax rates on labour, consumption and capital. Note that $L_t^{TAX} = \sum_{i\in\ell} s_i L_{i,t} WR_t^i(\tau_t^i + \tau_{h,t}^{W_i} + \tau_{f,t}^{W_i})$, $C_t^{TAX} = \tau_t^C \frac{P_{C,t}}{P_t} C_t$, $K_t^{TAX} = \frac{P_{t-1}}{P_t}(R_{k,t} - \delta_k - 1)\tau_t^K \frac{Q_{t-1}}{P_{t-1}} K_t - tcrk_t \frac{P_{I,t}}{P_t} I_t$. Public (consumption and investment) expenditure and transfers evolve according to $AR(1)$ processes in their log deviations from the steady states.

The tax dynamics is described by a fiscal rule, which captures the tax response to the public finance indicators (debt and deficit) and the stance of stabilization policies. Formally, the fiscal rule in deviations from the steady state is:

$$\frac{T_t}{\bar{T}} = T_B \frac{B_t^{G,r}}{\bar{B}^{G,r}} \frac{1}{\Pi_t} + T_D \frac{D_t}{\bar{D}} + T_Y \frac{Y_t}{\bar{Y}} \tag{50}$$

where T_B, T_D and T_Y are policy parameters. The tax is equally imposed to Ricardian and non–Ricardian, i.e., $T_t^R = \left(1 - s_{TAX}^{NR}\right) T_t$ and $T_t^{NR} = s_{TAX}^{NR} T_t$.

The gross nominal interest rate paid by the one-period nominal bond issued the government in the domestic market may deviate from the Euro Area risk–free nominal interest rate by a spread factor, $spread_t$, i.e., it is formalized as:[14]

$$R_t = spread_t R_t^{EU} \tag{51}$$

where R_t^{EU} is the Euro Area risk–free nominal interest rate. In the steady state, we assume no spread so $\bar{R} = \bar{R}^{EU}$.[15]

[14] We use the same formalization used by Gerali *et al.* (2016) in their experiment on the Italian sovereign debt crisis. See also Corsetti *et al.* (2012).

[15] The spread is modelled as a stochastic anticipated or not anticipated disturbance,

The currency area is modeled through a monetary authority that sets short-term nominal interest rates by following a Taylor rule:

$$\frac{R_t^{EU}}{\overline{R}} = \left(\frac{R_{t-1}^{EU}}{\overline{R}}\right)^{\iota_r} \left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\iota_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\iota_y} \left(\frac{S_t}{\overline{S}}\right)^{\iota_s}\right]^{1-\iota_r} \tag{52}$$

4 Calibration

The f-IGEM model is calibrated on a quarterly basis in order to match steady-state ratios and some specific features of the Italian economy. The private consumption share $\overline{C}/\overline{Y}$ is set at 0.57, the investment share $\overline{I}/\overline{Y}$ at 0.18, the public consumption share $\overline{C}^G/\overline{Y}$ at 0.20. The tables below describe the parameters. Table 1 reports some deep parameters, whereas Table 2 focuses on parameters related to labour markets. Table 3 reports fiscal parameters. Table 4 and 5 summarize parameters characterizing nominal and financial frictions. Apart from financial frictions, the calibration mainly follows that of IGEM in Annicchiarico *et al.* (2013a), to whom we refer for more details.[16]

[16] If not differently stated, we use the same calibration of IGEM (Annicchiarico *et al.*, 2013a).

Table 1 – Calibration of deep parameters

Parameter	Description	Value
β	Discount factor	0.99
δ_K	Depreciation rate of capital	0.025
δ_{KG}	Depreciation rate of pubic capital	0.025
α_L	Production function parameter, LL and LH workers	0.35
α_N	Production function parameter, NS and NA workers	0.35
α_G	Production function parameter, public capital	0
α_{IM}	Share of foreign goods in total consumption	0.26
α_{EX}	Share of foreign goods in total consumption for the rest of the world	0.26
θ_Y	Elasticity of substitution between domestic intermediate goods	5
θ_{EX}	Elasticity of substitution between exported intermediate goods	5
θ_{IM}	Elasticity of substitution between imported intermediate goods	5
σ_{IM}	Elasticity of substitution between domestic and foreign intermediate varieties	1.1
κ_P	Price backward indexation	1
κ_W	Wage backward indexation	1
$\overline{\Pi}$	Steady-state inflation	1
$\overline{\Pi}^*$	Steady-state inflation (foreign sector)	1
ι_r	Taylor rule parameter, interest	0.8
ι_π	Taylor rule parameter, inflation	1.7
ι_Y	Taylor rule parameter, output	0.125
ι_s	Taylor rule parameter, exchange rate	0.00

The discount factor β is equal to 0.99, implying an annual real interest rate of 4%. The rates of depreciation of private and public physical capital δ_K and δ_G are set to 0.025 implying a 10% annual depreciation rate of capital. The capital share in the intermediate goods production is equal to 0.3, hence $1 - \alpha_L - \alpha_N = 0.3$. The labour shares are such that $\alpha_L = \alpha_N = 0.35$. The CES parameters σ_L and σ_N are set at 1.4 according to Katz and Murphy (1992) estimates, as in QUEST III for Italy. As in the baseline IGEM, the contribution of public capital to production is neglected (i.e., $\alpha_G = 0$).

The elasticities of substitution between domestic goods in the intermediate sector, θ_Y, is set equal to 5 so to have a steady-state level of net markup equal to 25%, which is consistent with the value set in the Italian version of QUEST III with *R&D* (see D'Auria *et al.*, 2009). Since in IGEM tradable goods are produced in the intermediate sector, we also set the elasticities of substitution between imported and exported varieties, θ_{IM} and θ_{EX}, at 5. The contribution of imported intermediate goods to the final good production, summarized by the parameter α_{IM} is equal to 0.26, consistently with Annicchiarico *et al.* (2013a), while the elasticity of substitution between domestic and foreign intermediate varieties σ_{IM} is set at 1.1.

The steady-state inflation is set equal to zero, $\Pi = 1$, and we assume full backward indexation of prices and wages, $\kappa_P = \kappa_W = 1$. The same holds for foreign inflation. As in Annicchiarico *et al.* (2013a), we consider a standard Taylor rule ($\iota_R = 0$, $\iota_\Pi = 1.5$, $\iota_Y = 0.125$, $\iota_S = 0$).

Table 2 is based on the RCFL-ISTAT 2008 data and microestimations.

Table 2 – Calibration of deep parameters of labour markets (IGEM)

Parameter	Description	Value
s_H	Share of skilled employees	0.11
s_L	Share of unskilled employees	0.42
s_S	Share of self-employed	0.21
s_A	Share of atypical workers	0.26
s_N	Share of non-Ricardian Households	0.26
σ_{HL}	Elasticity of substitution, skilled and unskilled employees	1.4
σ_{NA}	Elasticity of substitution, atypical and self-employed workers	1.4
σ_H	Elasticity of substitution, skilled employees	2.65
σ_L	Elasticity of substitution, unskilled employees	2.65
σ_S	Elasticity of substitution, self-employed workers	2.65
v_H	Preference parameter, skilled employees	8.01
v_L	Preference parameter, unskilled employees	8.36
v_A	Preference parameter, atypical workers	12.76
v_S	Preference parameter, self-employed workers	8.00

Drawing on RCFL–ISTAT 2008 data set, labour categories are defined as follows. Employees are identified with those workers with a stable labour contract and eligible of labour protection, so belonging to the primary labour market. In the available data, this category amounts to 53% of the whole workforce, within this category the share of the employees with tertiary education corresponds to the skilled workers and accounts for 11% of the workers (i.e., $s_H = 0.11$). The remaining share is identified with the unskilled employees (i.e., $s_L = 0.42$). The share of self-employed workers older than 35, is 21% and the model share s_S is set accordingly. As a matter of fact, we exclude from this category of workers the young, since at early stages of their careers they tend to be precarious and face the same difficulties of the workers with atypical contracts. Hence, the last category of workers labeled as "atypical" includes young self-employed, apprentices, temporary workers and other workers with atypical contracts characterized by weak security protection and low firing costs, so belonging to the secondary market. This residual fraction of workers amounts to 26% (i.e., $s_A = 0.26$).

According to the estimates based on *EconLav* microsimulation model, the Frisch elasticity of labour supply for the employees is 0.30, while for the atypical component of the labour force the Frisch elasticity is equal to 0.35. For the self-employed workers we set the Frisch elasticity at 0.30, since it is conjectured that the reactivity of their labour supply to changes in their remuneration is closer to that experienced by workers with stable contracts. In line with the literature, the elasticities of substitution between different varieties of labour σ_L, σ_H and σ_S are all set at 2.65 (see Forni *et al.*, 2010), reflecting the limited competition protecting the insiders.

The calibration of the tax system points to heavy taxation on capital and labour income, where different rates are considered for each labour category. Fiscal parameters are taken from Annicchiarico *et al.* (2013a) and reported in Table 3.

Table 3 – Tax system calibration

Parameter	Description	Value
τ^C	Tax rate of consumption	0.17
τ^K	Tax rate on physical capital	0.33
τ_l^L	Average legal tax rate on unskilled employees	0.24
τ_h^L	Social contributions on unskilled employees	0.09
τ_f^L	Contributions levied on firms, unskilled employees	0.33
τ_l^H	Average legal tax rate on skilled employees	0.27
τ_h^H	Social contributions on skilled employees	0.09
τ_f^H	Contributions levied on firms, skilled employees	0.33
τ_l^S	Average legal tax rate on self-employed	0.26
τ_h^S	Social contributions on self-employed	0.09
τ_f^S	Contributions levied on firms, self-employed	0.00
τ_l^A	Average legal tax rate on atypical workers	0.24
$\tau_h^{W_{N_A}}$	Social contributions on atypical workers	0.09
τ_f^A	Contributions levied on firms, atypical workers	0.27

The tax rate on consumption τ^C is equal to 0.17, while the tax rate on physical capital τ^K is 0.33, consistently with the calibration used in the Italian version of QUEST III (D'Auria *et al.*, 2009). For the tax rates on wage income, the calibration is based on data taken from RFCL-ISTAT 2008. In particular, the average legal tax rate on labour income paid by skilled employees τ_l^H is equal to 0.27, that for the unskilled, τ_l^L is set at 0.24, for the self-employed τ_l^S is 0.26 and for the atypical workers τ_l^A is 0.24. The social contribution rates paid by firms and workers are set, respectively, at 0.33 and 0.09 as legal rates of contribution. The tax rates (τ^H, τ^L, τ^S, and τ^A) are obtained by summing the average tax rate on labour income and the legal rates of contribution.

Table 4 reports the parameters characterizing the nominal rigidities of the model.[17]

Table 4 – Calibration adjustment costs and nominal rigidities (IGEM)

Parameter	Description	Value
h_{C^R}	Habit (Ricardian households)	0.7
$h_{C^{NR}}$	Habit (non-Ricardian households)	0.3
γ_S^W	Wage adjustment cost self-employed workers	10
γ_H^W	Wage adjustment cost high-skilled workers	71
γ_L^W	Wage adjustment cost low-skilled workers	71
γ_p	Price adjustment cost	10
γ_I	Investment adjustment cost	5

The parameters representing the financial sector (θ, ω, and λ) are chosen to match the following two steady-state targets: i) a leverage ratio equal to four ($\phi =$

[17] They are taken from Annicchiarico *et al.* (2013a). We assume no demand adjustment costs ($\gamma_{N_A} = \gamma_{N_S} = \gamma_{L_H} = \gamma_{L_L} = 0$).

4); ii) an average horizon of bankers of a decade ($\theta = 0.972$). Table 5 summarizes the parameters characterizing the financial sector.

Table 5 – Financial sector calibration

Parameter	Description	Value
θ	Fraction of bankers who survives in each period	0.972
ω	Proportional of starting up funds for entering bankers	0.0046
λ	Fraction of divertable funds	0.25
ϕ	Leverage ratio	4
	Banks' average horizon (years)	10

5 Simulation of policy scenarios

Several simulations are designed to illustrate how the model behaves and how it can be used to evaluate fiscal reforms. In the first subsection, we show how the model behaves by looking at its response to unexpected disturbances. We consider a technology, capital quality, and fiscal shock. We show the impulse response functions associated to these shocks for three main variables (output, investments, and the credit premium). The second subsection provides an example of evaluation of fiscal reforms. Specifically, we consider a consolidation plan implemented during a sovereign debt crisis. The reform plan is a combination of permanent fiscal policies, which involves increases in consumption and labor taxes and reductions in government consumption. We further assume that the plan is implemented when a sovereign debt crisis hits the economy, which is modelled as a temporary increase in the government debt in *f*-IGEM.

5.1 The financial accelerator at work

In this section, some experiments are designed to illustrate how the model behaves. We look at the response of the model economy to three disturbances: A negative technology shock; a decline in capital quality; a positive fiscal shock. The technology shock is a negative one percent innovation in TFP, with a quarterly autoregressive factor of 0.85. The capital quality shock aims to mimic some features of the recent banking crisis. It is an unanticipated negative five percent decline in the capital value, with a quarterly autoregressive factor of 0.66. Finally, the public consumption shock is an unanticipated positive one percent increase in the government budget, with a quarterly autoregressive factor of 0.85.

Results are described in Figure 1. Disturbances are reported by rows: (a) technology; (b) capital quality; (c) public consumption shock. The figure shows the responses of three key variables (by columns): output, investment and the premium. In each case the solid line shows the response of f–IGEM, dashed lines those of IGEM (i.e., the same model where financial frictions have been removed).

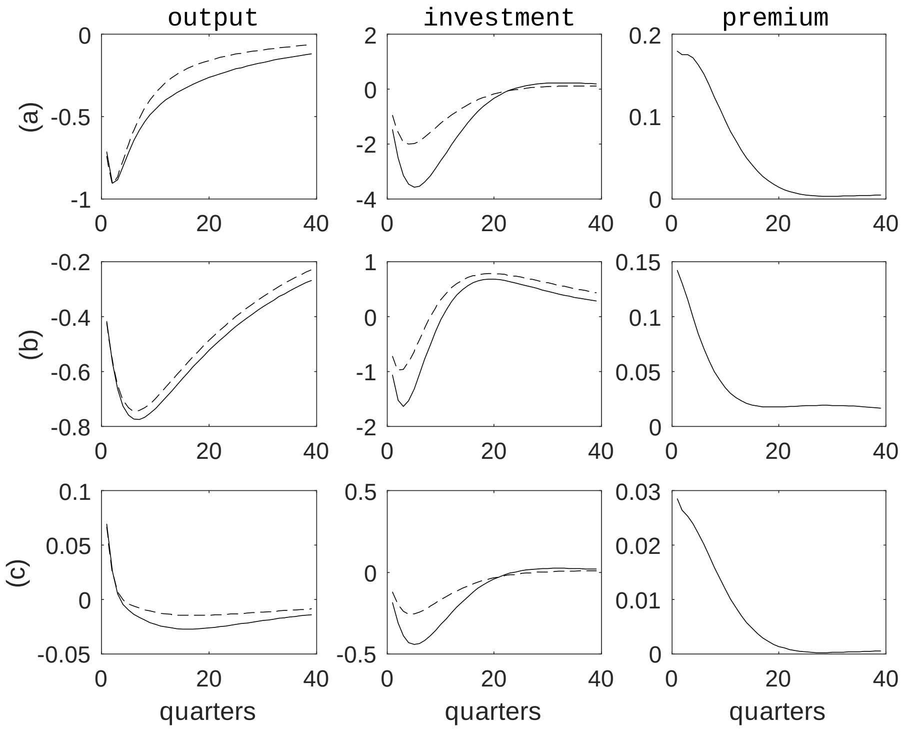

Figure 1 – The financial accelerator at work (solid lines = f-IGEM/dashed lines = IGEM). Note: (a) productivity; (b) capital quality; (c) fiscal (public consumption increase) shock.

The technology shock is described in row (a). The intermediary balance sheet mechanism produces an amplification of the decline in output in f-IGEM relative to IGEM. The amplification is mainly the product of a substantially enhanced decline in investment implied by the rise in the premium. The premium rises as the result of the deterioration in intermediary balance sheets driven by the unanticipated decline in investment reduces asset prices. The downturn in investment and asset prices is further fueled by the increase in the cost of capital, which reduces capital demand by non-financial firms.

We now turn to the decline in capital quality. As argued by Gertler and Karadi (2011), it aims to roughly capture the recent sub-prime crises. The deterioration in the quality of intermediary assets produces an enhanced decline in the net worth of banks, due to their high degree of leverage. The shock generates an exogenous and endogenous decline in asset values. After the initial decline in asset values due to the reduction of the effective quantity of capital, in fact, driven by the leverage ratio constraint, a drop in asset demand is induced by the weakening of intermediary balance sheets. The price per effective unit of capital and investment then falls, further shrinking the intermediary balance sheets and magnifying the overall contraction through the degree of leverage. The decline in capital quality also provides an amplification of the decline in output in f–IGEM relative to IGEM

Results from the technology and the capital quality shocks are similar to those obtained by Gertler and Karadi (2011).

The public consumption shock is an unanticipated positive one percent increase in the government budget, with a quarterly autoregressive factor of 0.85. Fiscal policy is less effective. It leads to an increase in the premium that reduces investments. In IGEM, the shock produces only a modest decline in investments. By contrast, in the model in f–IGEM, there is a sharp fall in investments, public consumption crowded out investments.

Summarizing, important differences derive for all variables when considering or not financial frictions. Their inclusion in the model results in a significant accentuation of all the negative effects. The differences are explained by the increase in the premium, which only occurs when the financial sector and the frictions related to it are considered.

5.2 Sovereign debt crises and fiscal consolidation

A reform plan is announced at time $t = 1$. The plan consists of a mix of reductions in public expenditure and increases in taxes. Specifically, we assume a reduction of 1% in public consumption and a 1% increase in consumption and labor taxes.

Simulations are carried out under the assumption that the fiscal interventions are fully credible and that all policy changes are permanent, as common practice in applied economic modeling when exploring the effects of policy interventions. Households and firms have perfect foresight. Therefore, any possible source of uncertainty about the underlying path of policy changes is ruled out. As a result, forward looking agents adjust their behavior accordingly, fully anticipating the long–run effects of the reforms.[18]

At time $t = 1$, when the reform is implemented, however, the economy is not on the steady state, but it hit by a spread over the Euro Area risk–free nominal interest rate. We assume that the sovereign debt crisis consists of a transitory shock on the spread, holding for 2 years, which implies a spread equal to 200 basis points.

The analysis of the permanent fiscal reform coupled with the temporary sovereign debt crisis is numerically simulated by using the non-linear version of *f*-IGEM.[19] To conduct our simulation exercise, we examine the deterministic response of the economy to unexpected permanent and temporary changes in the exogenous policy variables taking place at the beginning of the time horizon of the simulation. It should be noted that the analysis of the effects of permanent shocks requires solving a two–point boundary–problem, specifying the initial conditions for the predetermined variables and the terminal conditions for the forward looking variables.

[18] Similar experiments are described, e.g., in Vogel (2012, 2013, 2014), Annicchiarico *et al.* (2013a, 2013b, 2017), Lorenzani and Varga. (2014), Varga and in 't Veld (2014), Gerali *et al.* (2016), Ministero dell'Economia e delle Finanze (2017), Ferrara and Tirelli (2017).

[19] The model is solved by using both TROLL and Dynare, which rely on a Newton-type algorithm to solve non–linear deterministic models. Results are the same.

A rigorous approach to solve this problem would make it necessary to derive the new steady state of the model and use the theoretical equilibrium values as terminal conditions. However, when dealing with a large scale model this solution strategy can be very taxing. Alternatively, one may opt to reformulate the problem so that the terminal conditions are invariant to policy changes, as proposed by Roeger and in't Veld (1999). In this simulation, we have opted for this less demanding strategy.

The results of our simulations are reported in Figure 2. The figure describes the path of the output associated with a fiscal reform starting from the steady state (no sovereign debt crisis) or during the financial turmoil (sovereign debt crisis).

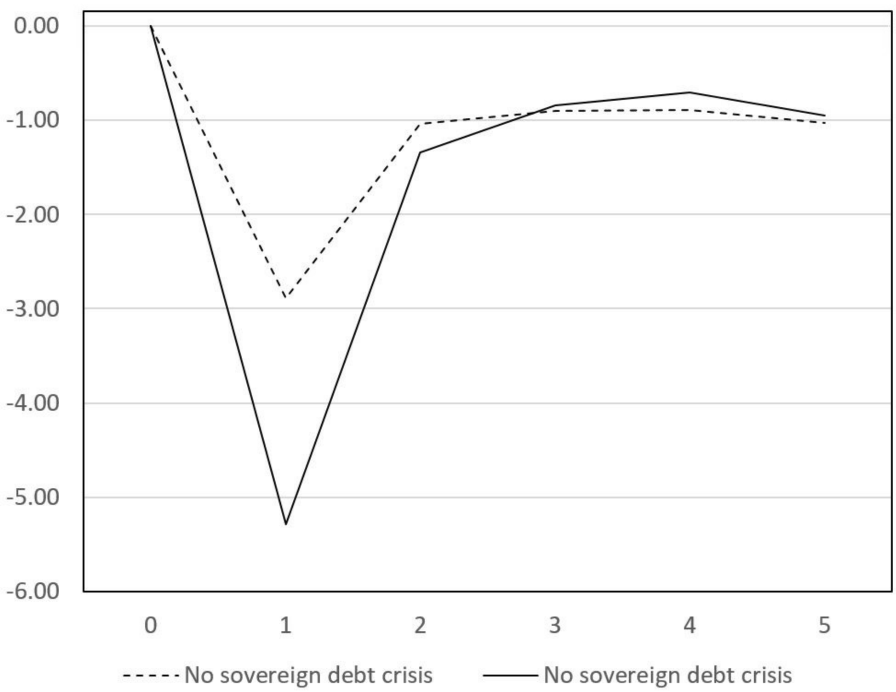

Figure 2 – Sovereign debt crisis and fiscal consolidation (annualized real output deviations from the initial steady state).

Implementing the consolidation plan during the debt sovereign crisis exhibits strong additional costs, a larger decline in GDP of about 2.5% in the first year, and about 1.3% on average during the two years of the crisis.

The overall impact of the fiscal reform on many economy variables is described in Table 6. The table measures the additional effect of the debt sovereign crisis on the implementation path of the fiscal reform, i.e., it measures the difference between a reform implemented from the steady state to one implemented during the debt sovereign crisis. Differences clearly emerge.

Table 6 – Difference between scenarios

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
GDP	-2.40	-0.30	0.06	0.18	0.08
Consumption	-1.82	-0.72	-0.28	-0.10	-0.06
Ricardian	-2.28	-0.84	-0.34	-0.10	-0.06
Non Ricardian	0.44	-0.08	-0.02	-0.02	-0.02
Investment	-0.28	-0.08	0.04	0.08	0.08
Labour	-0.78	-0.44	-0.04	0.12	0.12
Labour (unskilled)	-1.04	-0.64	-0.10	0.16	0.18
Labour (skilled)	-0.44	-0.30	-0.08	0.06	0.08
Labour (self–employed)	-0.94	-0.40	0.04	0.18	0.12
Labour (atypical)	-0.18	-0.06	0.00	0.02	0.02
Terms of trade	3.28	0.10	-0.10	-0.24	-0.20
Trade balance	-4.04	-0.10	0.14	0.30	0.22
Import	1.50	-0.36	-0.26	-0.24	-0.18
Real deficit	-15.24	-11.28	-4.46	-0.68	0.02

6 Conclusions

In the wake of the global financial crisis, the relevance of imperfections in the financial markets has been reconsidered as they can strongly affect the macroeconomic performance and effectiveness of economic policies.

Our paper developed a quantitative DSGE model with financial intermediaries who face endogenously determined balance sheet constraints. We introduced financial frictions in IGEM, which is a medium scale Dynamic General Equilibrium model for the Italian economy developed at the Department of Treasury of the Italian Ministry of the Economy and Finance.

The labor market setup is the core of IGEM, and thus of f–IGEM. A segmented labor market mimics the peculiarity of the Italian economy. Monopolistic trade unions set wages of skilled and unskilled workers, who exhibit stable contracts and strong protection. Atypical workers are instead price takers and have flexible working patterns and weak labor protection. Self–employed workers and professionals supply labor under contracts for services. They have also some market power, due to the existence of professional orders or to their limited number.

In the financial sector, banks hold long–term assets from non–financial firms (investors) and fund these assets with short–term liabilities from households (savers) and their own equity capital. Imperfections assume the form of an agency problem between banks and households that introduces endogenous constraints on the leverage ratios of the former. As a result, credit flows are tied to the equity capital of intermediaries. A deterioration of bank capital raises credit costs, lowering lending and borrowing.

The open economy aspects are captured by two channels: The presence of exporting and importing firms; the ability of the agents to save or borrow on foreign financial assets. A continuum of monopolistically competitive exporting (importing) firms transform domestic (foreign) intermediate goods into exportable (importable) goods using a linear technology. Exporters and importers seek to maximize profits by setting prices. Then the development of the net foreign asset position depends on the current account surplus and on the decisions of firms, households and government. The net external position will depend on conditions in both financial and goods markets. The currency area is modelled through a monetary authority that sets short–term nominal interest rates by a simple Taylor rule.

We performed several simulations to illustrate how the above model behaves. We also showed how it can be used to evaluate fiscal reforms by an example of consolidation package.

We illustrated the economy response to unexpected disturbances. The impulse response functions of f–IGEM were compared to those of the same model where financial frictions have been removed (IGEM). We accounted for a technology shock, capital quality decline (capturing several features of the recent sub-prime crises), and a fiscal shock. The inclusion of financial frictions in our model results in a significant accentuation of all the negative effects. Overall contractions due to technology and capital quality shock are magnified by the presence of financial frictions. The differences are mainly explained by the financial accelerator operating through increases in the credit premium. In the case of the positive shock in fiscal expenditure, financial frictions imply that public expenditure crowds out investments, making the expansionary policy less effective.

We then considered a consolidation plan implemented during a sovereign debt crisis. The reform is a combination of permanent increases in consumption and labor taxes and reductions in government consumption (a reduction of 1% in the public consumption and a 1% increase in the consumption and labor taxes). The plan is implemented when a sovereign debt crisis hits the economy and its outcome compared to those stemming from the same plan announced when the economy is on the steady state. The sovereign debt crisis is a transitory spread shock that implies a spread equal to 200 basis points for 2 years. Our results are as follows. In the first year, the additional decline in GDP involved in the consolidation plan implemented during the debt sovereign crisis is about 2.5% larger compared to an equal reform announced in the steady state. Considering the two–year average, the difference is about 1.3%.

Finally, a word of caution is needed since the quantification of the economic impact of economic reforms and disturbances represents an extremely difficult exercise. All results must be interpreted in the light of the model used. Thus, although built up with the purpose of assessing the effects of structural reforms and fiscal interventions, only a stylized representation of the economy under study is provided.

Appendix – The equations of the model

In this section all the equations of the model are listed.

1. Euler equation of the Ricardian households ok

$$\varrho_t^R = \beta E_t \varrho_{t+1}^R \frac{R_{t+1}}{\Pi_{t+1}}$$

2. Lagrangian multiplier of the Ricardian households ok

$$\varrho_t^R = \frac{P_t}{P_{C,t}} \frac{1}{(1+\tau_t^C)\left(C_t^R - h_{C^R} C_{t-1}^R\right)}$$

3. Consumption of the non–Ricardian households ok

$$C_t^{NR}\left(1+\tau_t^C\right)\frac{P_{C,t}}{P_t} = \left(1 - \tau_t^{N_A} - \tau_{h,t}^{W^{N_A}}\right)\frac{s_{N_A}}{s_{NR}} WR_t^{N_A} N_{A,t} - TAX_t^{NR} +$$

$$+Tr_t^{NR} + \left(1 - \tau_t^{L_L} - \tau_{h,t}^{W^{L_L}}\right) \frac{\lambda^{L_L} s_{L_L}}{s_{NR}} WR_t^{L_L} L_{L,t} -$$
$$\frac{\lambda^{L_L} s_{L_L}}{s_{NR}} \frac{\gamma_{W^{L_L}}}{2} \left(\frac{\Pi_t}{\Pi_{t-1}^{\kappa_W} \overline{\Pi}^{1-\kappa_W}} \frac{WR_t^{L_L}}{WR_{t-1}^{L_L}} - 1 \right)^2 Y_t$$

4. Lagrangian multiplier of non–Ricardian households OK (BAR SOPRA)

$$\varrho_t^{NR} = \frac{P_t}{P_{C,t}} \frac{1}{(1+\tau_t^C)\left(C_t^{NR} - h_{C^{NR}} C_{t-1}^{NR}\right)}$$

5. Aggregate consumption

$$C_t = s_{NR} C_t^{NR} + (1 - s_{NR}) C_t^R$$

6. Wage equation (self–employed)

$$(\sigma_{N_S} - 1)\, \varrho_t^R \left(1 - \tau_t^{N_S} - \tau_{h,t}^{W^{N_S}}\right) WR_t^{N_S} N_{S,t} = \omega_{N_S} \sigma_{N_S} \left(1 - N_{S,t}\right)^{-v_{N_S}} N_{S,t} +$$
$$-\varrho_t^R \gamma_{W^{N_S}} \left(\frac{WR_t^{N_S} \Pi_t}{\Pi_{t-1}^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_{t-1}^{N_S}} - 1 \right) Y_t \frac{WR_t^{N_S} \Pi_t}{\Pi_{t-1}^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_{t-1}^{N_S}} +$$
$$+\beta \varrho_{t+1}^R \gamma_{W^{N_S}} \left(\frac{WR_{t+1}^{N_S} \Pi_{t+1}}{\Pi_t^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_t^{N_S}} - 1 \right) Y_{t+1} \frac{WR_{t+1}^{N_S} \Pi_{t+1}}{\Pi_t^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_t^{N_S}}$$

7. Wage equation (skilled employees)

$$(\sigma_{L_H} - 1)\, \varrho_t^R \left(1 - \tau_t^{L_H} - \tau_{h,t}^{W^{L_H}}\right) WR_t^{L_H} L_{H,t} = \omega_{L_H} \sigma_{L_H} \left(1 - L_{H,t}\right)^{-v_{L_H}} L_{H,t} +$$
$$-\varrho_t^R \gamma_{W^{L_H}} \left(\frac{WR_t^{L_H} \Pi_t}{\Pi_t^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_{t-1}^{L_H}} - 1 \right) Y_t \frac{WR_t^{L_H} \Pi_t}{\Pi_{t-1}^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_{t-1}^{L_H}} +$$
$$+\beta \varrho_{t+1}^R \gamma_{W^{L_H}} \left(\frac{WR_{t+1}^{L_H} \Pi_{t+1}}{\Pi_t^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_t^{L_H}} - 1 \right) Y_{t+1} \frac{WR_{t+1}^{L_H} \Pi_{t+1}}{\Pi_t^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_t^{L_H}}$$

8. Wage equation (unskilled employees)

$$(\sigma_{L_L} - 1) \left(1 - \tau_t^{L_L} - \tau_{h,t}^{W^{L_L}}\right) WR_t^{L_L} L_{L,t} =$$
$$+\frac{\omega_{L_L} \sigma_{L_L} \left(1 - L_{L,t}\right)^{-v_{LL}} L_{L,t}}{(1 - I^{NR} \lambda^{L_L}) \varrho_t^R + I^{NR} \lambda^{L_L} \varrho_t^{NR}} - \gamma_{W^{L_L}} \left(\frac{WR_t^{L_L} \Pi_t}{\Pi_{t-1}^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_{t-1}^{L_L}(h_{L_L})} - 1 \right) Y_t \frac{WR_t^{L_L} \Pi_t}{\Pi_{t-1}^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_{t-1}^{L_L}} +$$
$$+\beta \frac{(1 - I^{NR} \lambda^{L_L}) \varrho_{t+1}^R + I^{NR} \lambda^{L_L} \varrho_{t+1}^{NR}}{(1 - I^{NR} \lambda^{L_L}) \varrho_t^R + I^{NR} \lambda^{L_L} \varrho_t^{NR}} \gamma_{W^{L_L}} \left(\frac{WR_{t+1}^{L_L} \Pi_{t+1}}{\Pi_t^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_t^{L_L}} - 1 \right) Y_{t+1} \frac{WR_{t+1}^{L_L} \Pi_{t+1}}{\Pi_t^{\kappa_W} \overline{\Pi}^{1-\kappa_W} WR_t^{L_L}}$$

9. Labor supply equation for atypical labor services

$$\frac{1}{\varrho_t^{NR}} = WR_t^{N_A}\left(1-N_{A,t}\right)^{\upsilon_{N_A}}\frac{1-\tau_t^{N_A}-\tau_{h,t}^{W^{N_A}}}{\omega_{N_A}}$$

deve essere

$$(1-\tau_t^{N_A}-\tau_{h,t}^{W^{N_A}})WR_t^{N_A} = \frac{\omega_{N_A}}{\left(1-N_{A,t}\right)^{\upsilon_{N_A}}\varrho_t^{NR}}$$

CORRETTO IN TUTTE 4: $sx_{L_H}^{\frac{1}{\sigma_L}} => s_{L_H}^{\frac{1}{\sigma_L}}$

PRIME 3: $(1-I_X\alpha_G) => (1-\alpha_G)$

PRIME 2: $MC_tX_t => MC_tY_t$

VANNO TOLTE LE PARTI CON: γ_{L_L}

CHE PER NOI DOVREBBE ESSERE ZERO

10. Demand for skilled labor as employees

$$WR_t^{L_H}\left(1-sub_t^{L_H}+\tau_{f,t}^{W^{L_H}}\right) = \frac{\alpha_L(1-\alpha_G)MC_tX_t}{L_{CES,t}-OH^L}sx_{L_H}^{\frac{1}{\sigma_L}}ef_{L_H}^{\frac{\sigma_L-1}{\sigma_L}}\left(\frac{L_{CES,t}}{LY_{H,t}}\right)^{\frac{1}{\sigma_L}} + \\ -\gamma_{L_H}\left(\frac{LY_{H,t}}{LY_{H,t-1}}-1\right)\frac{Y_t}{LY_{H,t-1}} + \beta\Lambda_{t+1}^R\gamma_{L_H}\left(\frac{LY_{H,t+1}}{LY_{H,t}}-1\right)Y_{t+1}\frac{LY_{H,t+1}}{LY_{H,t}^2}$$

DOVREBBERO ESSERE TIPO *si se consideriamo zero gli aggiustamenti*

$$WR_t^{L_H}\left(1-sub_t^{L_H}+\tau_{f,t}^{W^{L_H}}\right) = \frac{\alpha_L\left(1-\alpha_G\right)MC_tX_t}{L_{CES,t}-OH^L}sx_{L_H}^{\frac{1}{\sigma_L}}ef_{L_H}^{\frac{\sigma_L-1}{\sigma_L}}\left(\frac{L_{CES,t}}{LY_{H,t}}\right)^{\frac{1}{\sigma_L}}$$

11. Demand for unskilled labor as employees

$$WR_t^{L_L}\left(1-sub_t^{L_L}+\tau_{f,t}^{W^{L_L}}\right) = \frac{\alpha_L(1-\alpha_G)MC_tX_t}{L_{CES,t}-OH^L}sx_{L_L}^{\frac{1}{\sigma_L}}ef_{L_L}^{\frac{\sigma_L-1}{\sigma_L}}\left(\frac{L_{CES,t}}{LY_{L,t}}\right)^{\frac{1}{\sigma_L}} + \\ -\gamma_{L_L}\left(\frac{LY_{L,t}}{LY_{L,t-1}}-1\right)\frac{Y_t}{LY_{L,t-1}} + \beta\Lambda_{t+1}^R\gamma_{L_L}\left(\frac{LY_{L,t+1}}{LY_{L,t}}-1\right)Y_{t+1}\frac{LY_{L,t+1}}{LY_{L,t}^2}$$

12. Demand for self–employed labor

$$WR_t^{N_S}\left(1+\tau_{f,t}^{W_{N_S}}\right) = \frac{\alpha_N(1-\alpha_G)MC_tY_t}{N_{CES,t}-OH^N}sx_{N_S}^{\frac{1}{\sigma_N}}ef_{N_S}^{\frac{\sigma_N-1}{\sigma_N}}\left(\frac{N_{CES,t}}{NY_{S,t}}\right)^{\frac{1}{\sigma_N}} + \\ -\gamma_{N_S}\left(\frac{NY_{S,t}}{NY_{S,t-1}}-1\right)\frac{Y_t}{NY_{S,t-1}} + \beta\Lambda_{t+1}^R\gamma_{N_S}\left(\frac{NY_{S,t+1}}{NY_{S,t}}-1\right)Y_{t+1}\frac{NY_{S,t+1}}{NY_{S,t}^2}$$

13. Demand for atypical labor

$$WR_t^{N_A}\left(1 - sub_t^{N_A} + \tau_{f,t}^{W^{N_A}}\right) = \frac{\alpha_N(1-\alpha_G)MC_tY_t}{N_{CES,t}-OH^N} sx_{N_A}^{\frac{1}{\sigma_N}} ef_{N_A}^{\frac{\sigma_N-1}{\sigma_N}} \left(\frac{N_{CES,t}}{NY_{A,t}}\right)^{\frac{1}{\sigma_N}} + \\ -\gamma_{N_A}\left(\frac{NY_{A,t}}{NY_{A,t-1}} - 1\right)\frac{Y_t}{NY_{A,t-1}} + \beta\Lambda_{t+1}^R\gamma_{N_A}\left(\frac{NY_{A,t+1}}{NY_{A,t}} - 1\right)Y_{t+1}\frac{NY_{A,t+1}}{NY_{A,t}^2}$$

NON E' Equilibrium 14-17, ma una definizione (si toglie facilmente ridef la f di p)

14. Equilibrium in the labor market (unskilled employees)

$$LY_{L,t} = s_{L_L}L_{L,t}$$

15. Equilibrium in the labor market (skilled employees)

$$LY_{H,t} = s_{L_H}L_{H,t}$$

16. Equilibrium in the labor market (self–employed workers)

$$NY_{S,t} = s_{N_S}N_{S,t}$$

17. Equilibrium in the labor market (atypical workers)

$$NY_{A,t} = s_{N_A}N_{A,t}$$

SI PUO' TOGLIERE

18. Labor aggregate

$$LN_t = s_{L_L}L_{L,t} + s_{L_H}L_{H,t} + s_{N_S}N_{S,t} + s_{N_A}N_{A,t}$$

19. Production function of the intermediate–goods producers

$$Y_t = A_t\left[\left(L_{CES,t} - OH_t^L\right)^{\alpha_L}\left(N_{CES,t} - OH_t^N\right)^{\alpha_N}\left(u_t^K\psi_tK_t\right)^{1-\alpha_L-\alpha_N}\right]^{1-\alpha_G}\left(K_t^G\right)^{\alpha_G}$$

CHECK sy_{L_L}

20. Employees labor CES aggregate

$$L_{CES,t} = \left[sx_{L_L}^{\frac{1}{\sigma_L}} \left(ef_{L_L} LY_{L,t} \right)^{\frac{\sigma_L - 1}{\sigma_L}} + sx_{L_H}^{\frac{1}{\sigma_L}} \left(ef_{L_H} LY_{H,t} \right)^{\frac{\sigma_L - 1}{\sigma_L}} \right]^{\frac{\sigma_L}{\sigma_L - 1}}$$

21. Self–employed and atypical labor CES aggregate

$$N_{CES,t} = \left[sx_{N_S}^{\frac{1}{\sigma_N}} \left(ef_{N_S} NY_{S,t} \right)^{\frac{\sigma_N - 1}{\sigma_N}} + sx_{N_A}^{\frac{1}{\sigma_N}} \left(ef_{N_A} NY_{A,t} \right)^{\frac{\sigma_N - 1}{\sigma_N}} \right]^{\frac{\sigma_N}{\sigma_N - 1}}$$

22. Physical capital accumulation equation

$$K_{t+1} = (1 - \delta_K)\, \psi_t K_t + I_t$$

23. Investment equation – Tobin's Q

$$Q_t = (1 - tcr_t^K) + \gamma_I \left(\frac{I_t}{K_t} - \delta_K \right)$$

NO E' EQULIBRIUM

6.0.1 24. Claims issued by firms

$$Q_t K_{t+1} = Q_t S_t^F$$

25. Demand of capital

$$\frac{R_{t+1}^K}{P_{t+1}} = \frac{\left[(1-\alpha_G)(1-\alpha_L-\alpha_N) MC_{t+1} \frac{Y_{t+1}}{\psi_{t+1} K_{t+1}} + \frac{Q_{t+1}}{P_{t+1}} - \delta_k \right] \psi_{t+1}}{Q_t}$$

26. Capital utilization

$$\frac{P_t^I}{P_t} \left[\gamma_{u_1^K} + \gamma_{u_2^K} \left(u_t^K - 1 \right) \right] K_t = MC_t \,(1 - \alpha_G)\,(1 - \alpha_L - \alpha_N)\, \frac{Y_t}{u_t^K}$$

ELIMINARE ADJ COSTS

27. Real profits of intermediate goods producers

$$PRO_t =$$

$$Y_t - WR_t^{L_L} \left(1 - sub_t^{L_L} + \tau_{f,t}^{W^{L_L}} \right) LY_{L,t} - WR_t^{L_H} \left(1 - sub_t^{L_H} + \tau_{f,t}^{W^{L_H}} \right) LY_{H,t} +$$

$$-WR_t^{N_S}\left(1+\tau_{f,t}^{W_{N_S}}\right)NY_{S,t}-WR_t^{N_A}\left(1-sub_t^{N_A}+\tau_{f,t}^{W^{N_A}}\right)NY_{A,t}+$$
$$-\frac{P_{t-1}}{P_t}R_t^K\frac{Q_{t-1}}{P_{t-1}}K_t+\left(\frac{Q_t}{P_t}-\delta\right)\psi_tK_t-\frac{P_t^I}{P_t}\left[\gamma_{u_1^K}\left(u_t^K-1\right)+\frac{\gamma_{u_2^K}}{2}\left(u_t^K-1\right)^2\right]K_t$$
$$-\frac{\gamma_p}{2}\left(\frac{\Pi_t}{\Pi_{t-1}^{\kappa_P}\overline{\Pi}^{1-\kappa_P}}-1\right)^2Y_t-\frac{\gamma_{L_H}}{2}\left(\frac{LY_{H,t}}{LY_{H,t-1}}-1\right)^2Y_t+$$
$$-\frac{\gamma_{L_L}}{2}\left(\frac{LY_{L,t}}{LY_{L,t-1}}-1\right)^2Y_t-\frac{\gamma_{N_S}}{2}\left(\frac{NY_{S,t}}{NY_{S,t-1}}-1\right)^2Y_t-\frac{\gamma_{N_A}}{2}\left(\frac{NY_{A,t}}{NY_{A,t-1}}-1\right)^2Y_t$$

28. Inflation equation

$$Y_t-\gamma_p\left(\frac{\Pi_t}{\Pi_{t-1}^{\kappa_P}\overline{\Pi}^{1-\kappa_P}}-1\right)Y_t\frac{\Pi_t}{\Pi_{t-1}^{\kappa_P}\overline{\Pi}^{1-\kappa_P}}+\beta\gamma_pE_t\Lambda_{t+1}^R\left(\frac{\Pi_{t+1}}{\Pi_t^{\kappa_P}\overline{\Pi}^{1-\kappa_P}}-1\right)Y_{t+1}\frac{\Pi_{t+1}}{\Pi_t^{\kappa_P}\overline{\Pi}^{1-\kappa_P}}=$$
$$(1-MC_t)\,\theta_YY_t$$

29. Accumulation of public capital

$$K_{t+1}^G=I_t^G+(1-\delta_G)\,K_t^G$$

30. Flow budget constraint of the government

$$B_{t+1}^G=$$
$$R_tB_t^G+P_{C,t}C_t^G+P_{I,t}I_t^G+P_tTr_t-P_tTAX_t-P_t\left(L_t^{TAX}+C_t^{TAX}+K_t^{TAX}\right)+P_tSUB_t$$

31. Transfers

$$Tr_t=s_{NR}Tr_t^{NR}+(1-s_{NR})Tr_t^R$$

32. Labor taxes and social contributions (revenues)

$$L_t^{TAX}=$$
$$s_{L_L}L_{L_L,t}WR_t^{L_L}\left(\tau_t^{L_L}+\tau_{h,t}^{W_{L_L}}+\tau_{f,t}^{W_{L_L}}\right)+s_{L_H}L_{L_H,t}WR_t^{L_H}\left(\tau_t^{L_H}+\tau_{h,t}^{W_{L_H}}+\tau_{f,t}^{W_{L_H}}\right)+$$
$$+s_{N_S,t}L_{N_S,t}WR_t^{N_S}\left(\tau_t^{N_S}+\tau_{h,t}^{W_{N_s}}+\tau_{f,t}^{W_{N_S}}\right)+s_{N_A}L_{N_A,t}WR_t^{N_A}\left(\tau_t^{N_A}+\tau_{h,t}^{W_{N_A}}+\tau_{f,t}^{W_{N_A}}\right)$$

33. Consumption tax revenues

$$C_t^{TAX}=\tau_t^C\frac{P_{C,t}}{P_t}\left[s_{NR}C_t^{NR}+(1-s_{NR})C_t^R\right]$$

34 Capital tax revenues (net of tax credit)

$$K_t^{TAX} = \frac{P_{t-1}}{P_t}(R_{k,t} - \delta_k - 1)\tau_t^K \frac{Q_{t-1}}{P_{t-1}} K_t - tcrk_t \frac{P_{I,t}}{P_t} I_t$$

35. Fiscal rule

$$TAX_t = \overline{TAX} + T_B \frac{B_{t-1}}{P_t} + T_D \frac{D_t}{P_t} + T_Y (Y_t - Y_{t-1})$$

36. Lump–sum taxes levied on Ricardian households

$$TAX_t^R = \left(1 - s_{TAX}^{NR}\right) TAX_t$$

37. Lump–sum taxes levied on non–Ricardian households

$$TAX_t^{NR} = s_{TAX}^{NR} TAX_t$$

38. Labor subsidies

$$SUB_t = sub_t^{L_L} s_{L_L} L_{L,t} WR_t^{L_L} + sub_t^{L_H} s_{L_H} L_{H,t} WR_t^{L_H} + sub_t^{N} s_{N_A} N_{A,t} WR_t^{N_A}$$

39. Government (real) deficit

$$\frac{D_t}{P_t} = \frac{(R_t - 1)B_t^G}{P_t} + \frac{P_{C,t}G_t}{P_t} + \frac{P_{IG,t}I_t^G}{P_t} + Tr_t - TAX_t - L_t^{TAX} - C_t^{TAX} - K_t^{TAX} + SUB_t$$

VANNO TOLTE LE PARTI CON $\gamma_{L_L} = 0$ E SIMILI, NOI NON ABBIAMO ASSUNTO COSTI DI AGGIUSTAMENTO DELLA DOMANDA

40. Resource constraint of the economy

$$Y_t = \frac{P_t^C}{P_t}(G_t + C_t) + \frac{P_t^I}{P_t}\left(I_t + I_t^G\right) + \frac{S_t P_{X,t}}{P_t} EX_t - \frac{P_{M,t}}{P_t} IM_t +$$
$$+ \frac{\gamma_p}{2}\left(\frac{\Pi_t}{\Pi_{t-1}^{\kappa_P}\overline{\Pi}^{1-\kappa_P}} - 1\right)^2 Y_t + \frac{\gamma_I}{2}\frac{P_t^I}{P_t}\left(\frac{I_t}{K_t} - \delta_K\right)^2 K_t + \frac{\gamma_{L_H}}{2}\left(\frac{LX_{H,t}}{LX_{H,t-1}} - 1\right)^2 Y_t +$$
$$+ \frac{\gamma_{L_L}}{2}\left(\frac{LY_{L,t}}{LY_{L,t-1}} - 1\right)^2 Y_t + \frac{\gamma_{N_S}}{2}\left(\frac{NY_{S,t}}{NY_{S,t-1}} - 1\right)^2 Y_t + \frac{\gamma_{N_A}}{2}\left(\frac{NY_{A,t}}{NY_{A,t-1}} - 1\right)^2 Y_t +$$

$$+\frac{\gamma_{W^{L_L}}}{2}\left(\frac{1}{\Pi_{t-1}^{\kappa_W}\overline{\Pi}^{1-\kappa_W}}\frac{WR_t^{L_L}}{WR_{t-1}^{L_L}}\Pi_t-1\right)^2 Y_t+\frac{\gamma_{W^{L_H}}}{2}\left(\frac{1}{\Pi_{t-1}^{\kappa_W}\overline{\Pi}^{1-\kappa_W}}\frac{WR_t^{L_H}}{WR_{t-1}^{L_H}}\Pi_t-1\right)^2 Y_t+$$
$$+\frac{\gamma_{W^{N_S}}}{2}\left(\frac{1}{\Pi_{t-1}^{\kappa_W}\overline{\Pi}^{1-\kappa_W}}\frac{WR_t^{N_S}}{WR_{t-1}^{N_S}}\Pi_t-1\right)^2 Y_t+$$
$$+\frac{\gamma_{IM}}{2}\frac{P_{M,t}}{P_t}\left(\frac{\Pi_t^{IM}}{\Pi_{t-1}^{\kappa_{IM}}\overline{\Pi}^{1-\kappa_{IM}}}-1\right)^2 IM_t+\frac{\gamma_{EX}}{2}\frac{S_tP_{X,t}}{P_t}\left(\frac{\Pi_t^{EX}}{\left(\Pi_{t-1}^*\right)^{\kappa_{EX}}\left(\overline{\Pi}^*\right)^{1-\kappa_{EX}}}-1\right)^2 EX_t$$
$$+\frac{P_t^I}{P_t}\left[\gamma_{u_1^K}\left(u_t^K-1\right)+\frac{\gamma_{u_2^K}}{2}\left(u_t^K-1\right)^2\right]K_t$$

CORRETTA

41. Interest rate rule

$$\frac{R_t^{EU}}{\overline{R}}=\left(\frac{R_{t-1}^{EU}}{\overline{R}}\right)^{\iota_r}\left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\iota_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\iota_y}\left(\frac{S_t}{\overline{S}}\right)^{\iota_s}\right]^{1-\iota_r}u_t^R$$

SPREAD R=spREU

42. Imports demand

$$IM_t=\alpha_{IM}\left(\frac{P_{M,t}}{P_{C,t}}\right)^{-\sigma_{IM}}\left(C_t+I_t+G_t+I_t^G\right)$$

WD_t ESCE QUI PER LA PRIMA VOLTA

43. Exports demand

$$EX_t=\alpha_{EX}\left(\frac{P_{X,t}}{P_{C,t}^*}\right)^{-\sigma_{EX}}WD_t$$

44. Import price inflation

$$\frac{P_{M,t}(1-\theta_{IM})}{P_t}\,IM_t-\gamma_{IM}\left(\frac{\Pi_t^{IM}}{\Pi_{t-1}^{\kappa_{IM}}\overline{\Pi}^{1-\kappa_{IM}}}-1\right)\frac{IM_t\Pi_t^{IM}}{\Pi_{t-1}^{\kappa_{IM}}\overline{\Pi}^{1-\kappa_{IM}}}+\frac{S_tP_t^*\theta_{IM}}{P_t}\,IM_t+$$
$$+\beta\gamma_{IM}E_t\Lambda_{t+1}^R\left(\frac{\Pi_{t+1}^{IM}}{\Pi_t^{\kappa_{IM}}\overline{\Pi}^{1-\kappa_{IM}}}-1\right)\frac{IM_{t+1}\Pi_{t+1}^{IM}}{\Pi_t^{\kappa_{IM}}\overline{\Pi}^{1-\kappa_{IM}}}=0$$

45. Export price inflation

$$\frac{S_tP_{X,t}(1-\theta_{EX})}{P_t}\,EX_t-\gamma_{EX}\left(\frac{\Pi_t^{EX}}{\left(\Pi_{t-1}^*\right)^{\kappa_{EX}}\left(\overline{\Pi}^*\right)^{1-\kappa_{EX}}}-1\right)\frac{EX_t\Pi_t^{EX}}{\left(\Pi_{t-1}^*\right)^{\kappa_{EX}}\left(\overline{\Pi}^*\right)^{1-\kappa_{EX}}}+\theta_{EX}\,EX_t+$$
$$+\beta E_t\Lambda_{t+1}^R\gamma_{EX}\left(\frac{\Pi_{t+1}^{EX}}{(\Pi_t^*)^{\kappa_{EX}}\left(\overline{\Pi}^*\right)^{1-\kappa_{EX}}}-1\right)\frac{EX_{t+1}\Pi_{t+1}^{EX}}{(\Pi_t^*)^{\kappa_{EX}}\left(\overline{\Pi}^*\right)^{1-\kappa_{EX}}}=0$$

46. Domestic consumption price index

$$P_{C,t} = \left[(1-\alpha_{IM})P_t^{1-\sigma_{IM}} + \alpha_{IM}P_{M,t}^{1-\sigma_{IM}}\right]^{\frac{1}{1-\sigma_{IM}}}$$

47. Euler equation related to foreign assets OK

$$S_t = \beta E_t \Lambda_{t+1}^R \frac{R_{t+1}^* + \rho_{t+1}^F}{\Pi_{t+1}} S_{t+1}$$

48. Foreign assets net position in real terms

$$BR_t^F = \frac{\left(R_t^* + \rho_t^F\right)}{\Pi_t} \frac{S_t}{S_{t-1}} BR_{t-1}^F + \frac{S_t P_{X,t}}{P_t} EX_t - \frac{P_{M,t}}{P_t} IM_t$$

49. Risk premium OK

$$\rho_t^F = -\varphi^F \left(e^{BR_t^F - BR^F} - 1\right)$$

51. Investment goods price level OK

$$P_{I,t} = P_{C,t}$$

53. Imported good price level OK

$$P_{M,t} = \Pi_t^{IM} P_{M,t-1}$$

54. Domestic final good price level OK

$$P_t = \Pi_t P_{t-1}$$

55. Foreign final good price level OK

$$P_t^* = \Pi_t^* P_{t-1}^*$$

57. Export price OK

$$P_{X,t} = \Pi_t^{EX} P_{X,t-1}$$

58. Value of banks' capital

$$\nu_t = E_t \left\{ (1-\theta)\, \beta \tfrac{\Lambda_{t+1}}{\Pi_{t+1}} \left(\widetilde{R}_{k,t+1} - R_{t+1} \right) + \beta \tfrac{\Lambda_{t+1}}{\Pi_{t+1}} \theta x_{t,t+1} \nu_{t+1} \right\}$$

59. Value of banks' net wealth

$$\eta_t = E_t \left\{ (1-\theta) + \beta \tfrac{\Lambda_{t+1}}{\Pi_{t+1}} \theta z_{t,t+1} \eta_{t+1} \right\}$$

60. Leverage

$$\phi_t = \tfrac{\eta_t}{\varrho - \nu_t}$$

61. Growth rate of banks' capital

$$z_{t,t+1} = \tfrac{NW_{t+1}}{NW_t} = \left(\widetilde{R}_{k,t+1} - R_{t+1} \right) \phi_t + R_{t+1}$$

62. Growth rate of banks' net wealth

$$x_{t,t+1} = \tfrac{Q_{t+i} S^F_{t+1}}{Q_t S^F_t} = \tfrac{\phi_{t+1}}{\phi_t} \tfrac{NW_{t+1}}{NW_t} = \tfrac{\phi_{t+1}}{\phi_t} z_{t,t+1}$$

63. Assets that banks can acquire

$$Q_t S^F_t = \phi_t NW_t$$

64. Banks' net worth

$$NW_t = NW_{e,t} + NW_{n,t}$$

65. Existing banks' net worth accumulation

$$NW_{e,t} = \theta \left[\left(\widetilde{R}_{k,t} - R_t \right) \phi_{t-1} + R_t \right] NW_{t-1}$$

66. New banks' net worth

$$NW_{n,t} = \omega Q_t S^F_{t-1}$$

67. Risk Premium

$$premium_t = \widetilde{R}_{k,t} - R_t$$

68. Balance sheet of a representative bank j

$$Q_t S_t^F = NW_t + B_{t+1}^D$$

70. Net return on loans

$$\widetilde{R}_{k,t} = R_{k,t} - (R_{k,t} - 1)\tau_t^K$$

TOGLIEREI 71 e 72

71. Capital quality shock

$$\psi_t = \exp \zeta_t$$

72. Capital quality shock - Innovation

$$\zeta_t = (1 - \rho_\zeta)\zeta_{t-1} + \epsilon_t^\zeta$$

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