

# Sales and Promotions and the Great Recession Deflation

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## Abstract

This paper investigates the effect of sales and promotions on the pricing decisions of firms. This study provides a theoretical model where firms face menu costs when adjusting their price and apply sales offers that decrease temporarily the listed price to attract higher demand, especially because households exert effort to locate the price deals. Thus, each period the final price is determined by the price set by the firm which is common knowledge to all agents and a sales deal that is a draw from a distribution with endogenous time-varying support. In a recession, even though prices in the economy look sticky, firms increase the frequency and the range of sales on their products substantially. This implies that traditional inflation measures are overstated in recessions, because they ignore the surge in sales and promotions and the consumers' tendency to hunt those limited time offers more actively. This framework can explain the mild deflation experienced during the Great Recession. Moreover, it is demonstrated that using traditional inflation measures can prolong recessions.

**Keywords:** *E31, E32, E58.*

**JEL Classification:** *Sales and promotions, deflation, menu cost, discounts.*

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# 1 Introduction

There is a large debate in the aftermath of the great depression on the response of prices and inflation during its course. Even though all other indices of macroeconomic activity such as unemployment, capital investment and asset prices have been affected significantly, inflation has not followed in the same direction during this time. As resources are underutilized under during depressions economic theory predicts that prices of goods and services should deteriorate ([Hall \(2011\)](#) and [King and Watson \(2012\)](#)). However, prices of goods and services have barely responded to the depression which is at odds with standard economic reasoning.

The phenomenon has been attributed to various sources. [Gilchrist et al. \(2017\)](#) demonstrate that firms facing financial frictions are less likely to decrease their prices in downturns. They provide evidence that those liquidity constrained firms have been increasing their price during the Great Recession. [Christiano et al. \(2015\)](#), report that the mild response of inflation in that same period is attributed on the fall in total factor productivity as well as on the rising cost of working capital. In addition, the phenomenon can be also explained by the rising share of sticky price firms as they tend to respond to inflation expectations than current shocks as documented in [Millard and O'Grady \(2012\)](#).

This paper attributes this phenomenon on the inability of inflation measures such as the CPI and PPI to account for temporary price changes such as sales and promotions. It is demonstrated using a theoretical model that during a recession, the frequency and magnitude of sales and promotion on items increases significantly while everyday consumers are more actively allocating their time to pinpoint such bargains. If the frequency and magnitude of sales and the effort to uncover good deals by consumers are not cyclical, then the traditional inflation measures are not biased. However, those variables are indeed cyclical as [Kryvtsov and Vincent \(2014\)](#) demonstrate and thus the inflation appear more sticky downwards and more flexible upwards. In an expansion, not all price increases can be replicated by decreasing the frequency and the magnitude of sales and thus it is expected that prices (without sales and other offers) are more responsive when the economy is expanding than when it is contracting. As price decreases can be more easily replicated by more frequent and more generous offers on different items, in downturns firms rely more heavily on sales than on permanent price decreases. Moreover, the increasing search intensity of bargain hunters in downturns makes the reliance on sales a better strategy than lowering prices, as the latter is subject to menu costs.

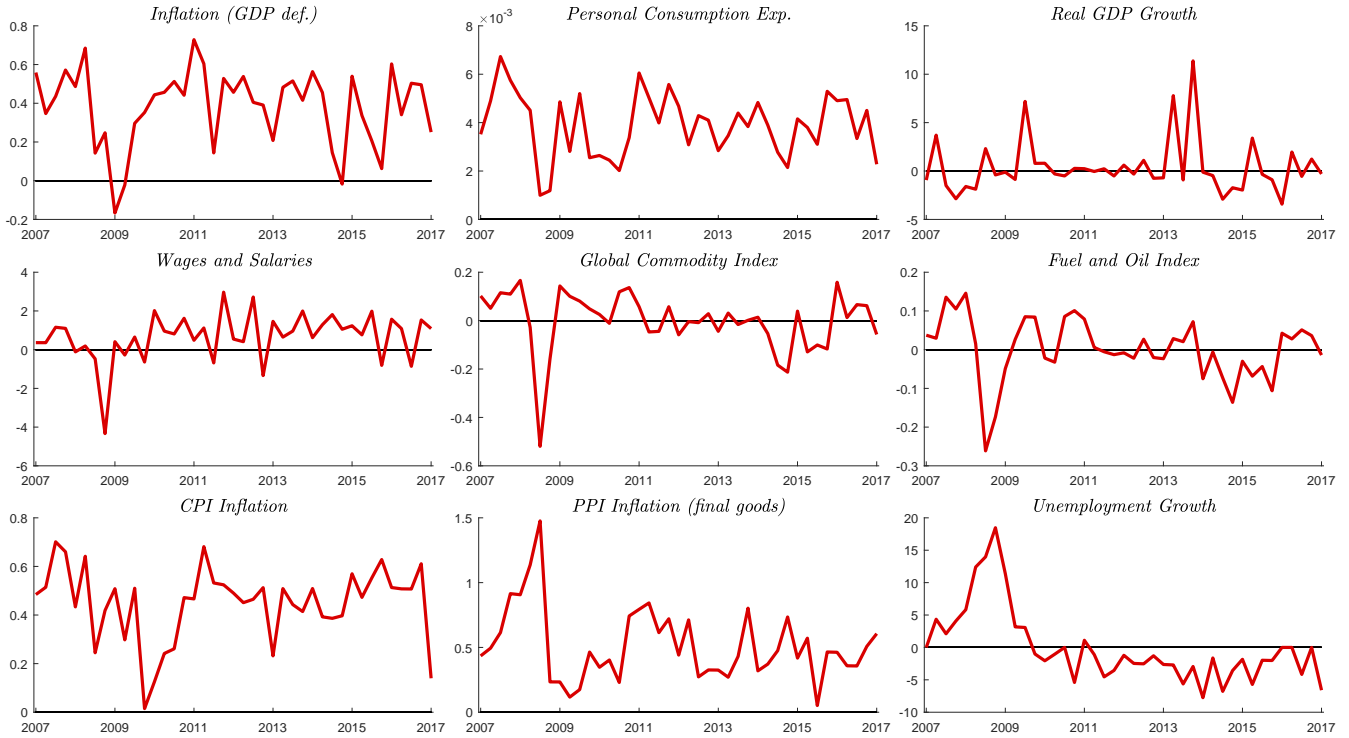
The price frequently listed on items is not necessarily what consumers pay because they are

subject to temporary sales and promotions as documented by [Nakamura and Steinsson \(2008\)](#). The way the Consumer Price Index (CPI) is calculated includes various products that are currently on sale such as direct decreases in the price of the product. However, alternative sale and promotion strategies are harder to quantify and are thus excluded from the CPI calculation. Such promotions include the buy 1 get the other half price or bundling items together and many others. Moreover, many items may include further discounts that have never been listed on the product. For example, consumers usually buy cars at a lower price than the one listed since salesmen use further discounts as a form of price discrimination. Other examples include the uses of loyalty cards and promotional coupons that are hard to include in the index. Nevertheless, what is important is that albeit sales and promotions are temporary, bargain hunters tend to purchase mostly items that are on sale and during depressions the search for good deals becomes more prominent. This substitution effect is not covered by any of the most popular inflation indices and thus prices seem more sticky than they actually are.

Moreover, studying the distribution of sales and promotions may be more informative than the listed prices on items as those are less prone to menu costs and stickiness in general. Inflation and GDP are sometimes not as correlated as they must be according to theory (for example price puzzle, see [Hanson \(2004\)](#)). It is possible that the sales distribution is more tuned with changes in the macroeconomic environment than prices. For this purpose this study estimates a Phillips curve that incorporates measures of sales and promotions to generate the inflation process. Results are pending.

In addition, the way CPI is measured affects monetary policy and also the agent's saving decisions. When agents form their expectations they might be tempted to use measures such as the CPI for inflation. If the inflation measures used for forecasting are as they are extracted from the CPI index but the real prices consumers pay appear to have a different cyclical, then it is interesting to investigate how such an economy responds to shocks. For example in such an environment, in recessions the CPI inflation is going to decrease to a lesser extent if the larger frequency of sales and the larger amount of bargain hunters are not included and thus the CPI inflation would seem less responsive. If agents make saving and investment decisions according to such an inflation measure, how can that affect the way the economy falls back to the long run level.

Figure 1: *Various macroeconomic variables' growth rates from previous quarter.*



Notes: St. Louis FRED. Inflation is the GDP deflator, Personal Consumption Exp. is the Personal Consumption Expenditures Excluding Food and Energy (Chain-Type Price Index), Wages and Salaries the Compensation of Employees, Received: Wage and Salary Disbursements, the Global Commodity Index the Global Price Index of All Commodities and the Fuel and Oil Index is the Consumer Price Index for All Urban Consumers: Fuel oil and other fuels, the CPI is the Consumer Price Index for All Urban Consumers: All Items Less Food and Energy, the PPI is the Producer Price Index by Commodity for Final Demand: Finished Goods Less Foods and Energy.

## 2 Motivation and Evidence

For the years following the crisis the deflation experienced has been too mild compared to what theory predicts. Figure 1 depicts various price indexes along with GDP growth and unemployment growth. The series spans the period from 2007 to 2017 using quarterly figures. "CPI Inflation", "PPI Inflation" and "Personal Consumption Expenditure" never reached negative territory during the period which is surprising. Although indexes such as "Global Commodity Index" and "Fuel and Oil Index" have decreases substantially during the period, the inflation indexes have been stubbornly unaffected. The same motivation can be found in [Gilchrist et al. \(2017\)](#) and [Christiano et al. \(2015\)](#).

Prices change infrequently, and such changes affect not only the demand for the good in scrutiny

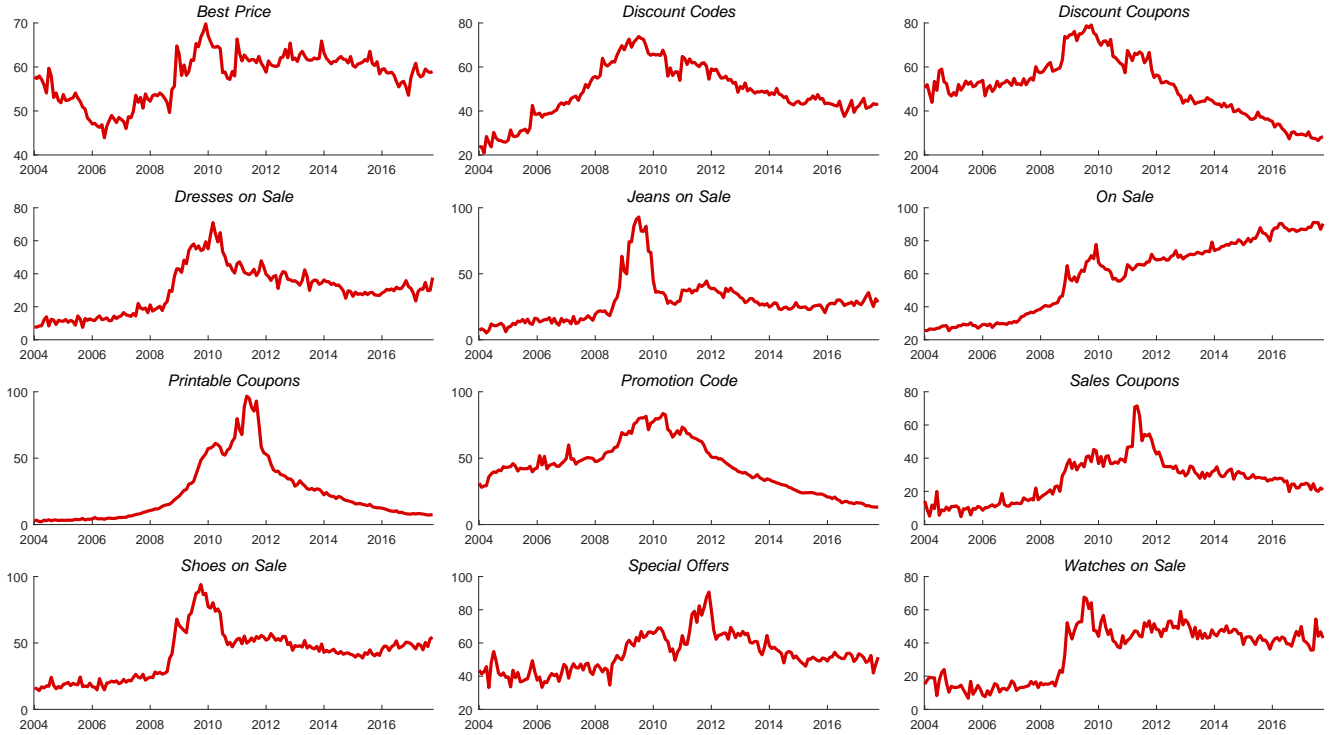
but also all other goods that are substitutes. Price changes can lead to price wars and fierce competition. During downturns, especially because firms are more vulnerable to price decreases due to the need to cover financing costs, it is more desirable to decrease prices using temporary sales and promotions rather than starting a price war. It is noticeable that prices during the crisis have not deteriorated fast enough but sales appear to be nearly on every item more frequently than they used to be. The tendency of attracting more demand by temporary sales is enticing to firms because decreasing the price is only going to induce more firms to follow with limited benefit to each competitor. Temporary sales can replicate price decreases without the cost associated with actual price changes such as menu costs or the cost of starting a price war.

The CPI inflation barely notices temporary sales and promotions and more importantly it ignores the effort by consumers to find these sales and their consequent preference for those low priced goods and services. The PPI completely ignores temporary sales and promotions as it measures the change in the price set by producers. It is possible that the CPI is more responsive than the PPI because it contains some measure of temporary sales. [Kryvtsov and Vincent \(2014\)](#) find that the sales are countercyclical in the United States, increasing in recessions and decreasing in expansions. Moreover, they find that the frequency of sales doubled during the Great Recession.

More importantly, the substitution effect is also more pronounced after the Great Recession. More consumers tend to actively seek for good price bargains in the post crisis period. Figure 2 presents the popularity of various searches on Google, acquired from the Google Trends per month, from 2004/01 to 2017/10. The title of each sub-plot is the actual search phrase under investigation. The measurement units are the number of searches over the number of searches of the most popular date. In the unadjusted series the most popular date received a 100% magnitude. The scales in the figure are due to seasonal adjustment and thus no series reaches a 100 percent popularity. For nearly every single series the popularity of searches started to increase after 2008 reached their peak and then started to deteriorate back to normal levels.

For example the fourth plot from the top of the left column, corresponds to the Google searches of the phrase "printable coupons". The phrase corresponds to searches for promotional coupons for discounts at various stores than can be printed. The series starts from around 5% popularity, becomes around 20 times more popular during the post crisis years and falls back to the initial level after 2015. Similar behavior is observed with other related searchers such as "discount codes" or "Jeans on sale" or "dresses on sale". The important thing is that the popularity of searches for sales or the effort to locate those good deals has more than doubled for nearly every series, having also incidences of more than 15-20 fold increases.

Figure 2: *Popularity in various Google searches across time.*



Notes: Google trends. Numbers represent search interest relative to the highest point on the chart for the given region and time. A value of 100 is the peak popularity for the term. A value of 50 means that the term is half as popular. Likewise a score of 0 means the term was less than 1% as popular as the peak. The series is seasonally adjusted.

The combination of temporary sales and larger effort to find those sales can severely bias any inflation index especially during a downturn and this is the main culprit for the mild deflation after the crisis according to this study. The following section develops a menu cost model that accounts for those stylized facts.

### 3 The Model

The model builds on the theory of sales developed by [Varian \(1980\)](#) where sales realistically retain their unpredictability and temporary nature by coming as a draw from a distribution of sales. Firms have a listed price on every item that is common knowledge to all consumers and costly to adjust as in [Rotemberg \(1982\)](#). They can temporarily offer a price reduction (sale) on each item that is modelled as a draw from a distribution. This distribution is endogenously determined as

a Nash equilibrium mixed strategy equilibrium between all the producers and the distribution's shape and support depend on the state of the economy. Households can find the cheapest deal by only putting effort. If they put the necessary effort they can identify the least costly alternative, otherwise they purchase goods from a random store. The monetary authority affects interest rates through a Taylor rule. The following sections present the details for each of the agents.

There is a continuum of households with a unit measure, each having a continuum of members and also a continuum of good varieties albeit there are  $N$  producers for each good variety. Each consumer is responsible for purchasing a single good  $i \in [0, 1]$  and must therefore choose which one of the  $N$  stores to shop from. The household chooses the fraction  $V_t$  of their members searching for good deals while the rest  $1 - V_t$  shop from a random store. Searching gives the option to discard a part of the price distribution and focus only on the lower prices of the distribution. It basically truncates the distribution of sales on the right side. In this exercise searching enables the consumer to identify the cheapest price from the  $N$  available while non-searchers can find the cheapest store with probability  $1/N$  as they randomly choose a purchasing spot. The price of a good is  $p_t$  but the price charged to consumers is the sales fraction times the price  $s_t p_t$  where  $s_t \in [\bar{s}_t, 1]$  and  $\bar{s}_t$  is the minimum sale the firm can set<sup>1</sup>. Let the distribution function of  $s_t$  be  $f(s_t)$  and the cumulative distribution function  $F(s_t)$ . When markets are open, sales are determined as a draw from a distribution, thus the firm sets the distribution of sale rates every period. A store with no price posted is considered closed and no consumer can visit it.

### 3.1 Price of the Aggregate Good

Households solve the following cost minimization problem that implicitly defines the true cost of living or the price of the aggregate good:

$$\min_{c_{it}} \int_0^1 s_{it} p_{it} c_{it} di \quad (1)$$

subject to the aggregated good which is

$$C_t = \left[ \int_0^1 (c_{it})^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad (2)$$

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<sup>1</sup>If there was free entry and thus  $N$  time varying,  $\bar{s}_t$  would be the sale fraction consistent with zero profit.

When the household decides upon its purchases it means that it has sent its members to the stores and they observe with no uncertainty the prices and associated sales. However, once they enter a store and observe the prices and offers they cannot switch to another store<sup>2</sup>. A fraction of those goods are purchased from the cheapest stores and the others from random stores. However, the demand for each good variety depends on the sales and prices according to

$$c_{it} = \left( \frac{s_{it}p_{it}}{P_t} \right)^{-\theta} C_t$$

which comes from maximizing objective (1) subject to (2). The aggregate price which is ultimately the shadow price of the above constrained minimization problem is

$$P_t = \left[ \int_0^1 (s_{it}p_{it})^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (3)$$

The aggregate price (3) the households pay can be further expanded as it depends on the amount of searchers and non-searchers the household sends shopping. The above aggregate price thus becomes:

$$P_t = \left[ \int_{1-V_t}^1 (s_{it}p_{it})^{1-\theta} di + \int_{V_t}^1 (s_{jt}p_{jt})^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

Given the distribution of sales, the aggregate price depends on the current price which is the same for all stores, the number of searchers and the corresponding sales draw for each store. That is

$$P_t = \left[ (1 - V_t) (p_{it})^{1-\theta} \int_{\bar{s}_t}^1 (s_{it})^{1-\theta} f(s_{it}) ds_{it} + V_t (p_{it})^{1-\theta} \int_{\bar{s}_t}^1 (s_{jt})^{1-\theta} \frac{(1 - F(s_{jt}))^{N-1} f(s_{jt})}{I_V} ds_{jt} \right]^{\frac{1}{1-\theta}} \quad (4)$$

The first term of the right hand side is the average price (with sales) that is paid by the  $1 - V_t$  non-searchers. The term in the integral is the average sales fraction as non searchers enter randomly every store. The second term is the average price paid by the searchers that manage to identify the cheapest price. The relevant pdf should be  $\frac{(1-F(s_{jt}))^{N-1} f(s_{jt})}{I_V}$  as it is the pdf  $f(s_{jt})$  weighted by the probability the other firms to have a higher price  $(1 - F(s_{jt}))^{N-1}$ . For example, if  $s_{jt} = \bar{s}_t$

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<sup>2</sup>This can only be achieved by putting effort and this is the reason searchers identify the best sales offers by running to every single store. However this is costly.



then  $F(s_{jt}) = 0$  and thus this value gets a weight  $(1 - F(s_{jt}))^{N-1}$  of 1, as it is very likely to have the cheaper price from all  $N$  rivals if the draw of  $s_{jt}$  is the lowest possible. However, if the draw  $s_{jt} = 1$  then the weight is zero as it is very unlikely to be the cheapest from all stores if no sale is offered on the initial price. The term  $I_V$  is the necessary weight to guarantee the probability weight in the second integral in eq. (4) is a pdf. That is

$$I_V = \int_{\bar{s}_t}^1 (1 - F(s_{jt}))^{N-1} f(s_{jt}) dj$$

The difference between standard inflation measures and the true inflation measure can be inferred by examining eq. (4). Usually there is a basket of goods considered and only the listed price is included. However, sales and promotions are very important not only because they bias the index that much for those that enter the stores randomly but because there is a time varying number of people that hunt after those low prices. As evidence support that those deal-searchers are significantly changing their behavior along the business cycle, the traditional inflation measure can be biased. As long as the distribution of sales  $f(s_{it})$  and its support  $\bar{s}_t$  changes along the business cycle as well as the number of searchers  $V_t$ , measures of inflation such as CPI and PPI might be problematic especially when the economy is below its potential level.

### 3.2 Firms

Each firm produces  $y_{it}$  using labor  $L_{it}$  according to the following production function:

$$y_{it} = z_t L_{it}$$

where  $z_t$  is productivity. The demand from each consumer that enters the store is  $y_t = \left(\frac{s_t p_t}{P_t}\right)^{-\theta} Y_t$ . Stores receive customers depending on their relative price. The firm's demand is  $\left(V_t + \frac{1-V_t}{N}\right) y_t$  when they offer the best deal among competitors. This includes the  $V_t$  searchers and its share of non-searchers  $\frac{1-V_t}{N}$  as there are  $N$  stores. The demand is  $\left(\frac{1-V_t}{N}\right) y_t$  when there is at least one better deal out there which is simply the share of non-searchers. The demand for each firm depends on the number of customers entering the store and each persons demand which is actually identical for every consumer.

### 3.2.1 Price Setting

Firms face adjustment cost when they try to change their price as in [Rotemberg \(1982\)](#). However, they do not absorb such costs when using sales and promotions. The adjustment cost  $ADC_t$  is given by the following equation:

$$ADC_t = \frac{\phi}{2} \left( \frac{p_{it}}{\pi p_{it-1}} - 1 \right)^2 P_t$$

From this point and on the  $i$  subscript is dropped as all firms are identical. The expected profit of the firm to be maximized is

$$\max_{\{p_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \int_{\bar{s}_t}^1 \left[ \Pi_t^V (1 - F(s_t))^{N-1} + \Pi_t^{NV} \left( 1 - (1 - F(s_t))^{N-1} \right) \right] f(s_t) ds_t - \frac{\phi}{2} \left( \frac{p_t}{\pi p_{t-1}} - 1 \right)^2 \frac{P_t}{P_0} \quad (5)$$

where

$$\Pi_t^V = \left( V_t + \frac{1 - V_t}{N} \right) \left[ \frac{s_t p_t}{P_t} y_t - m_t y_t \right] \quad (6)$$

is the part of the profit earned when the firm manages to be the cheapest of all  $N$  competitors, and

$$\Pi_t^{NV} = \frac{1 - V_t}{N} \left[ \frac{s_t p_t}{P_t} y_t - m_t y_t \right] \quad (7)$$

the part of profit when there is at least one cheaper store among the competitors. The demand by each individual is:

$$y_t = \left( \frac{s_t p_t}{P_t} \right)^{-\theta} Y_t \quad (8)$$

The marginal cost is as usual the real wage per efficiency unit

$$m_t = \frac{w_t}{z_t}$$

The objective (5) states that firms get the  $\Pi_t^V$  searchers and non-searchers with probability  $(1 - F(s_t))^{N-1}$ , which is the productivity all the other firms to be more expensive and they get  $\Pi_t^{NV}$  from just the non-searchers with probability  $1 - (1 - F(s_t))^{N-1}$  which is the probability for at least one cheaper firm to exist. Maximizing the expected profit of the firm (5) yields the following foci:

Maximization with respect to price is:

$$\int_{\bar{s}_t}^1 \left[ \frac{d\Pi_t^V}{dp_t} (1 - F(s_t))^{N-1} + \frac{d\Pi_t^{NV}}{dp_t} \left( 1 - (1 - F(s_t))^{N-1} \right) \right] f(s_t) ds_t \quad (9)$$

$$- \phi \left( \frac{p_t}{\pi p_{t-1}} - 1 \right) \frac{1}{\pi p_{t-1}} + E_t \phi \beta_{t,t+1} \left( \frac{p_{t+1}}{\pi p_t} - 1 \right) \frac{p_{t+1}}{\pi} \left( \frac{1}{p_t} \right)^2 \frac{P_{t+1}}{P_t} = 0$$

The derivative of equation (6) after plugging in the demand, equation (8) is

$$\frac{d\Pi_t^V}{dp_t} = \left( V_t + \frac{1 - V_t}{N} \right) [(1 - \theta) s_t \Omega_t + \theta m c_t] (s_t)^{-\theta} (\Omega_t)^{-\theta-1} \frac{Y_t}{P_t} \quad (10)$$

and in the same way the derivative of equation (7) is and

$$\frac{d\Pi_t^{NV}}{dp_t} = \frac{1 - V_t}{N} [(1 - \theta) s_t \Omega_t + \theta m c_t] (s_t)^{-\theta} (\Omega_t)^{-\theta-1} \frac{Y_t}{P_t} \quad (11)$$

where  $\Omega_t \equiv \frac{p_t}{P_t}$  in both eq. (10) and (11). Plug eq. (10) and (11) in equation (9) to get

$$\int_{\bar{s}_t}^1 \left[ V_t (1 - F(s_t))^{N-1} + \frac{1 - V_t}{N} \right] [(1 - \theta) (s_t) (\Omega_t) + \theta m c_t] (s_t \Omega_t)^{-\theta} Y_t f(s_t) ds_t$$

$$- \phi \left( \frac{\pi_t}{\pi} \frac{\Omega_t}{\Omega_{t-1}} - 1 \right) \frac{\pi_t}{\pi} \frac{\Omega_t}{\Omega_{t-1}} + E_t \phi \beta_{t,t+1} \left( \frac{\pi_{t+1}}{\pi} \frac{\Omega_{t+1}}{\Omega_t} - 1 \right) \frac{\pi_{t+1}}{\pi} \frac{\Omega_{t+1}}{\Omega_t} \pi_{t+1} = 0$$

where  $\pi_t \equiv \frac{P_t}{P_{t-1}}$  is the gross inflation and  $\frac{p_t}{P_t} \equiv \Omega_t$ .

### 3.2.2 Sales and Promotions

Varian (1980) demonstrates that there is no equilibrium in pure strategies for the determination of the sales fraction  $s_t$  which implies that there exists an equilibrium in mixed strategies. It is assumed that the number of competitors in each sector is  $N$  which is fixed. Every period the firm determines the distribution from which its sales percentage  $s_t$  is drawn. The support of the distribution, the range at which it takes non zero probability values is  $s_t \in [\bar{s}_t, 1]$ . A mixed strategy equilibrium dictates that the firm must earn the same ex ante profit for any sales offer within the interval  $[\bar{s}_t, 1]$ . This signifies that if a firm sets  $s_t = 1$ , the expected profit should be the same at any state.

This information guides us to pin down the most generous sale fraction  $\bar{s}_t$ , the lower bound of  $s_t$ . For every possible  $s_t$ , net of price adjustment costs<sup>3</sup>, the firm expects to make:

$$\Xi(s_t) = \left[ \Pi_t^V(s_t) (1 - F(s_t))^{N-1} + \Pi_t^{NV}(s_t) \left( 1 - (1 - F(s_t))^{N-1} \right) \right] \quad (12)$$

which as before states that the firm earns  $\Pi_t^V$  if it is the cheapest<sup>4</sup> and  $\Pi_t^{NV}$  when at least one competitor has better offers. If there is no sale on the listed price, the probability to have the best offer is zero and thus the probability another competitor to offer a better deal is  $F(1) = 1$ . The firm in such case expects to get

$$\Xi(1) = \frac{1 - V_t}{N} (\Omega_t - mc_t) (\Omega_t)^{-\theta} Y_t \quad (13)$$

where  $\Pi_t^{NV}$  is as defined in eq. (7) and  $\Omega_t$  the relative price  $p_t/P_t$ .

Alternatively, when the sales fraction is the most generous, i.e.  $s_t = \bar{s}_t$  and  $F(\bar{s}_t) = 0$  then,

$$\Xi(\bar{s}_t) = \Pi_t^V(\bar{s}_t) = \left( V_t + \frac{1 - V_t}{N} \right) \left( (\Omega_t \bar{s}_t)^{1-\theta} Y_t - mc_t (\Omega_t \bar{s}_t)^{-\theta} Y_t \right) \quad (14)$$

where  $\Pi_t^V$  is as defined in eq. (6). Equation (12) must hold for any sales price within the closed interval  $[\bar{s}_t, 1]$  and thus the expected profit from setting the most generous sale  $\Xi(\bar{s}_t)$  must equal  $\Xi(1)$ . Using (13) and (14) this condition becomes

$$\left( V_t + \frac{1 - V_t}{N} \right) \left( (\Omega_t \bar{s}_t)^{1-\theta} Y_t - mc_t (\Omega_t \bar{s}_t)^{-\theta} Y_t \right) = \frac{1 - V_t}{N} (\Omega_t - mc_t) (\Omega_t)^{-\theta} Y_t \quad (15)$$

The above defines the support of the distribution of sales as it pins down the upper bound  $\bar{s}_t$ .

The Nash equilibrium in mixed strategies and thus the underlying distribution of sales can be identified in a similar exercise. In mixed strategy equilibrium the expected payoff of every deterministic sales choice  $\Xi(s_t)$  must be equal to each other to induce the firm to randomly pick the sales fraction. If any  $\Xi(s_t)$  implies a higher profit than the rest of the choices then the equilibrium would be a pure strategy one. As long as firms are ex ante identical<sup>5</sup>, then the distribution of sales

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<sup>3</sup>By equating the profits under different sales fractions, the adjustment costs for the price cancel out.

<sup>4</sup>Being the cheapest is having picked a sales deal  $s_t$  lower than the rest which occurs with probability  $(1 - F(s_t))^{N-1}$ , the probability for all the rest  $N - 1$  competitors to pick a less generous sales offer.

<sup>5</sup>The sales draw from the distribution makes the firms ex post different.

must be determined by

$$\Xi(s_t) = \Xi(1)$$

From (12) and (13) this implies that

$$\Pi_t^V (1 - F(s_t))^{N-1} + \Pi_t^{NV} \left(1 - (1 - F(s_t))^{N-1}\right) = \frac{1 - V_t}{N} (\Omega_t - m_{c_t}) (\Omega_t)^{-\theta} Y_t \quad (16)$$

Plug in eq. (16) eq. (6) and (7) to get an expression for the cumulative distribution function

$$F(s_t) = 1 - \left[ \frac{1 - V_t}{V_t} \frac{1}{N} \left( \frac{\frac{p_t}{P_t} - m_t}{s_t \frac{p_t}{P_t} - m_t} s_t^\theta - 1 \right) \right]^{\frac{1}{N-1}} \quad (17)$$

The associated pdf can be derived by differentiating (17) with respect to  $s_t$

$$f(s_t) = -\frac{1}{N-1} [1 - F(s_t)]^{2-N} \frac{1 - V_t}{V_t} \frac{1}{N} \left( \theta \frac{\frac{p_t}{P_t} - m_t}{s_t \frac{p_t}{P_t} - m_t} s_t^{\theta-1} - s_t^\theta \frac{p_t}{P_t} \frac{\frac{p_t}{P_t} - m_t}{\left(s_t \frac{p_t}{P_t} - m_t\right)^2} \right) \quad (18)$$

To get a picture on how this distribution looks like, a simulation example follows. Figure 3 depicts the probability distribution function of sales, eq. (18) along with the cumulative distribution function eq. (17) over their support,  $[\bar{s}_t, 1]$ . According to the parameterization in table 1, sales vary from 0.72 to 1 over the price. This pins down the relative price  $\Omega$  in steady state which is at 1.25. The number of firms in each good variety is 5. The number of searchers in each household is also pinned down from other steady state values as this number cannot easily be calibrated. The value employed in this exercise is 0.37. The Nash equilibrium in mixed strategies gives rise to the pdf in the left panel of figure 3 where with a high probability firms charge either the full price or nearly a 30% discount.

The shape of the distribution of sales depends a lot on the number of competitors. If the number of competitors increases, then the benefit of sales is diminished and the distribution becomes a strictly increasing function from  $\bar{s}_t$  going up to 1. Even if there is vast competition for some goods and services, consumers are located at certain areas and their only consideration is to find the lower price for the goods they pursue around the area they live. This restricts the competition further and setting  $N$  to 5 might be a reasonable value.

<i>Variable</i>	<i>Value</i>	<i>Description</i>
$\bar{s}$	0.7190	<i>Lower bound for sales</i>
$N$	5	<i>Number of firms in each good variety</i>
$V$	0.37	<i>Measure of bargain searchers</i>
$\Omega$	1.25	<i>Relative price <math>\frac{P_t}{P_t}</math></i>
$\theta$	3	<i>Elasticity of substitution</i>
$v$	2	<i>Search cost curvature param.</i>
$h$	9.4	<i>Search cost parameter</i>
$mc$	0.86	<i>Marginal cost</i>

Table 1: The table presents the parameterization of the model

### 3.3 Household Problem

Households get utility from consumption, leisure and also suffer disutility from the effort to search for the best deals around. The household maximization problem is:

$$\max_{\{C_t, B_t, L_t, V_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[ U(C_t) - h_L \frac{L_t^{1+\ell}}{1+\ell} - h_v \frac{V_t^{1+\nu}}{1+\nu} \right] \quad (19)$$

where  $C_t$  is consumption  $B_t$  bond holdings  $L_t$  labor effort and  $V_t$  the bargain searchers in the household. The above objective is subject to the following budget constraint:

$$P_t(V_t) C_t + B_t = I_{t-1} B_{t-1} + W_t L_t + I_t$$

where  $W_t$  is the nominal wage and  $I_{t-1}$  the gross nominal interest rate. However, there is another constraint in the problem because the aggregate price  $P_t$  the household pays depends on the number of searchers it sends out for bargain hunting  $V_t$ . Eq. (4) determines how the aggregate price is affected by the number of searchers which is a decision variable for the household.

The first order condition for search effort  $V_t$  is:

$$h_v V_t^\nu = -\lambda_t \frac{dP_t}{dV_t} C_t \quad (20)$$

where  $\lambda_t$  is the Lagrange multiplier. The above condition states that the marginal cost of an extra bargain searcher (right hand side) must equal the benefit which is the reduction in the aggregate price (left hand side) since  $\frac{dP_t}{dV_t} < 0$ .

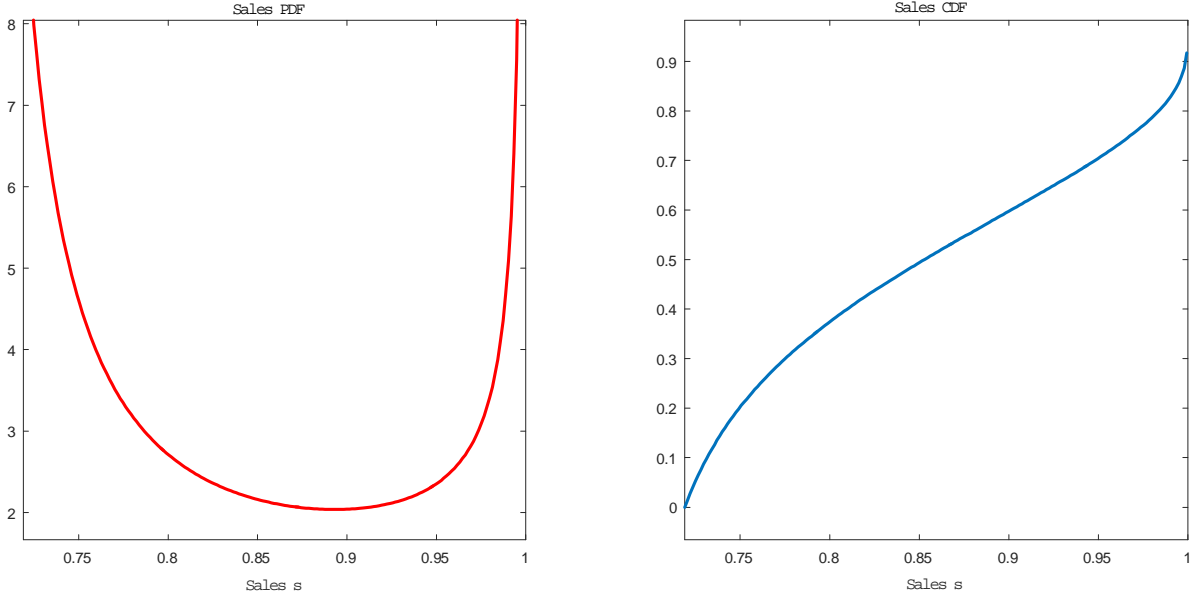


Figure 3: The left panel is the distribution of sales while the right is the cumulative distribution function.

For consumption  $C_t$  the first order condition is

$$U'(C_t) = \lambda_t P_t \quad (21)$$

For bond holdings  $B_t$  is

$$\lambda_t = \beta E_t \lambda_{t+1} R_t \quad (22)$$

where  $R_t = \frac{I_t}{E_t \pi_{t+1}}$  is the real interest rate. For labor  $L_t$  the first order condition is

$$\frac{G'(L_t)}{U'(C_t)} = w_t$$

where  $w_t$  is the real interest rate  $W_t/P_t$ . By altering their deal chasing efforts, households can affect prices. Take the derivative of eq. (4) with respect to  $V_t$ :

$$\frac{dP_t}{dV_t} = \frac{(p_{it})^{1-\theta}}{(\theta-1)(P_t)^{-\theta}} \int_{\bar{s}_t}^1 (s_{it})^{1-\theta} \left( 1 - \frac{(1-F(s_{it}))^{N-1}}{I_V} \right) f(s_{it}) di < 0$$

Plug the above in (20) to get

$$\lambda_t C_t \frac{(p_{it})^{1-\theta}}{(1-\theta)(P_t)^{-\theta}} \int_{\bar{s}_t}^1 (s_{it})^{1-\theta} \left( 1 - \frac{(1-F(s_{it}))^{N-1}}{I_V} \right) f(s_{it}) di = h_v V_t^\nu$$

Use (21) in the above to get

$$\frac{U'(C_t) C_t}{1-\theta} \left( \frac{p_{it}}{P_t} \right)^{1-\theta} \int_{\bar{s}_t}^1 (s_{it})^{1-\theta} \left( 1 - \frac{(1-F(s_{it}))^{N-1}}{I_V} \right) f(s_{it}) di = h_v V_t^\nu \quad (23)$$

After some algebra on eq. (4), it can be transformed to:

$$\int_{\bar{s}_t}^1 (s_{it})^{1-\theta} \left[ 1 - \frac{(1-F(s_{it}))^{N-1}}{I_V} \right] f(s_{it}) ds_{it} = \frac{1}{V_t} \left( \int_{\bar{s}_t}^1 (s_{it})^{1-\theta} f(s_{it}) ds_{it} - (\Omega_t)^{\theta-1} \right) \quad (24)$$

Use (24) in (23) to get

$$\frac{(1-\theta) h_v V_t^{1+\nu}}{U'(C_t) C_t} = (\Omega_t)^{1-\theta} \int_{\bar{s}_t}^1 (s_{it})^{1-\theta} f(s_{it}) ds_{it} - 1 \quad (25)$$

Eq. (25) solves for the optimal number of searchers  $V_t$  the household sends to the market.

### 3.4 Equilibrium and Inflation Indexes

So far  $P_t$  is the the true cost of living in this economy as it represents the true aggregated price, using the correct weights on prices given the number of searchers, non searchers and the sales distribution. Inflation measure stemming from this measure of the cost of living are ideal. However, in reality the commonly used measures of inflation might be far from this measure not when the economy is around its long run state but especially when it is far below its long run average. Below, a few standard measures of inflation are constructed using the model equilibrium variables.



The Consumer Price Index (CPI) without considering the possibility of sales is

$$\tilde{P}_t = \left[ \int_0^1 (p_{it})^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

which is simply the aggregation of prices from all good varieties. Every good variety is equally desirable to the households hence the weight of each good in the index is identical. Prices are all equal due to the menu cost assumption<sup>6</sup>  $\tilde{P}_t = p_t$  and thus

$$\tilde{P}_t = p_t = \Omega_t P_t \tag{26}$$

Divide with (26) a period in the past to get

$$\frac{\tilde{P}_t}{\tilde{P}_{t-1}} = \frac{\Omega_t P_t}{\Omega_{t-1} P_{t-1}}$$

which implies that if  $\tilde{\pi}_t$  is the CPI inflation measure, then it is related to the true inflation measure  $\pi_t$  according to

$$\tilde{\pi}_t = \frac{\Omega_t}{\Omega_{t-1}} \pi_t \tag{27}$$

The gross inflation without the sales  $\tilde{\pi}_t$  is related to the actual inflation (sales included)  $\pi_t$  according to eq. (27). It is evident that in the steady state, sales do not affect the inflation measure. However, the cyclicity of the two is different and this can give rise to important dynamics. In later sections the dynamics of the model are investigated under the assumption that households and/or policymakers form expectations using inflation measures such as (27) instead of  $\pi_t$ .

Suppose that the inflation measure is corrected for temporary sales but the number of searchers that favor the most generous of those bargains is ignored. The cost of living (price aggregator) if sales are included but there is no distinction between searchers and non-searchers is

$$(P_t^S)^{1-\theta} = (p_{it})^{1-\theta} \int_{\bar{s}_t}^1 (s_{it})^{1-\theta} f(s_{it}) ds_{it}$$

---

<sup>6</sup>Calvo pricing make the model very hard to track without additional assumptions.

Divide both sides with  $(P_t)^{1-\theta}$  to get

$$\left(\frac{P_t^S}{P_t}\right)^{1-\theta} = \left(\frac{p_{it}}{P_t}\right)^{1-\theta} \int_{\bar{s}_t}^1 (s_{it})^{1-\theta} f(s_{it}) ds_{it} \quad (28)$$

Rearrange the equilibrium equation for the optimal number of searchers eq. (25) to get

$$\left(\frac{p_{it}}{P_t}\right)^{1-\theta} \int_{\bar{s}_t}^1 (s_{it})^{1-\theta} f(s_{it}) ds_{it} = 1 + \frac{(1-\theta) V_t H_V(V_t)}{U'(C_t) C_t} \quad (29)$$

Substitute eq. (29) in eq. (28). That is

$$\left(\frac{P_t^S}{P_t}\right) = \left(1 + \frac{(1-\theta) V_t H_V(V_t)}{U'(C_t) C_t}\right)^{\frac{1}{1-\theta}}$$

In equilibrium, consumption equals output and also the total labor demand is

$$L_t = (1 - V_t) Y_t \int_{\bar{s}_t}^1 \left(\frac{s_{it} p_{it}}{P_t}\right)^{-\theta} f(s_{it}) ds_{it} + \frac{V_t}{I_V} Y_t \int_{\bar{s}_t}^1 \left(\frac{s_{it} p_{it}}{P_t}\right)^{-\theta} (1 - F(s_{jt}))^{N-1} f(s_{it}) ds_{it}$$

because the labor demand also depends on the sales draws, the support of the sales distribution and the number of searchers and non-searchers.

### 3.5 Simulations

For the purposes of simulations from the model and specifically for the impulse response function analysis, the parameters are calibrated according to the table 1 as in the steady state example above. The model is kept as simple as possible to be able to analyze the behavior of sales on the macroeconomy. First the study analyzes the dynamics of the model equilibrium variables and also the dynamics of the sales distribution function. Following this section, the impulse responses of various models are examined where different inflation expectations are considered. Specifically the different scenarios depend on whether the agents in the model form inflation expectations by using traditional measures of inflation or the actual inflation.

### 3.5.1 Impulse Responses: Benchmark Model

Figure 5 depicts the impulse responses after a 1% shock that increases the federal funds rate. The calibration is based on table 1 as before. The increase in the federal funds rate puts the economy into a recession. Income decreases on impact as the rise in the interest rate decreases consumption today. The decrease in the demand for the final good induces a drop in the demand for labor and the real wage. In an attempt to lower the cost of its consumption basket, the household is investing more time into searching for good deals and thus the number of deal seekers  $V_t$  increases during the downturn. The firms respond by offering more generous sales as they increase the range of the sales distribution which is replicated in the model by lowering  $\bar{s}_t$ .

Interestingly, the aggregate price with the sales included (second plot of the first row), which corresponds to the change in the price of the aggregate good the households consume, is perfectly flexible. However, simply aggregating the listed prices on goods without the sale, the inflation response is much different. It replicates the persistence experienced empirically for the commonly used inflation indexes such as the CPI. Therefore, the response of Inflation (No Sale) in Figure 5 increases on impact and then gradually falls to the steady state after around 20 periods. The inflation implied by the CPI measure in the figure (inflation no sale), is more mild and more smooth even though the real measure of aggregate inflation in the economy responds more aggressively.

The increase in the federal funds rate affects also the distribution of sales. Using the responses obtain by the simulation to derive the response of the distribution of sales in equation (18) the response in figure 4 is generated. On impact the distribution jumps to the red and thicker line which allows a more generous sale on the price and gradually shifts to the steady state distribution depicted as the blue and thinner line.

### 3.5.2 Inflation Expectations

If the inflation measures commonly used to infer the cost of living are biased along the business cycle then it is interesting to investigate how such mismatches can affect the way the economy responds to shocks. For example, consumers interact with different prices and seek for sales and promotions when purchasing goods but they may rely on biased inflation measures such as the CPI for their saving decisions. The central bank may also conduct its interest setting policy by relying on the CPI or PPI index. The purpose of this section is to investigate how the economy responds to a monetary shock when agents use different measures of inflation to form their expectations or policy response.

To motivate this, take the consumption-savings first order condition that is consisted of equations (21) and (22) and log-linearize around the steady state. Consumption equals output in this model and thus the log-linearized equation is

$$\hat{y}_t = E_t \hat{y}_{t+1} - (i_t - E_t \pi_{t+1}) \quad (30)$$

where  $\hat{y}_t$  is the log deviation of output  $Y_t$  from its steady state,  $\pi_t$  the inflation and  $i_t$  the log linearized gross nominal rate from its steady state. The log-linearized Taylor rule according to which the central bank sets the interest rate is

$$i_t = \rho^i i_{t-1} + (1 - \rho^i) (\rho^\pi \pi_t + \rho^y \hat{y}_t) \quad (31)$$

Solving equation (30) forward implies

$$\hat{y}_t = - \sum_{i=0}^{\infty} E_t (i_{t+i} - E_t \pi_{t+i+1}) \quad (32)$$

Output in every period depends on the sum of all future real interest rates.

In Figure 5 the impulse responses after a 1 standard deviation shock that increases the federal funds rate are considered. In that simulation, the inflation measure employed by every agent (households and central bank) is the true inflation measure (Inflation Sales in the graph) that includes the true weights, prices and sales offers. For the impulse responses in Figure 5, the persistence in the federal funds rate response implies that the current and future nominal rates affect output according to eq. (32). The true inflation (with sales) adjusts instantly to the shock and its lack of persistence implies that the inflation expectations are zero. Hence, the decrease in the response of Income is due to the current increase in the interest rate and all the expected federal funds rates forward.

The next scenario investigated in this study is the one where households forecast inflation by using as an aggregate measure of inflation the change in the prices (Inflation no sales in graph). This inflation measure disregards the sales and promotions on items and more importantly the number of consumers that switch to those items. Those lower prices due to sales offers should get more weight in the aggregate price level but a price level comprised of only listed prices completely ignores those details. The impulse responses after a monetary shock that increases the federal funds rate in such an environment is depicted in Figure 6. The difference with the benchmark

case is that in this case, the household forecasts inflation using the aggregated prices (Inflation no sale in figure). Inflation (no sale) in figure 6 decreases and returns to steady state very slowly because the adjustment cost in prices requires small price adjustments each period. The future inflation  $E_t\pi_{t+i+1}$  in equation (32) is no longer zero and thus the household expects deflation in the future which dictates that the real interest rate is even higher. Even though the true inflation is not persistent, as long as the households are basing their savings decisions on just the aggregated prices, the expected real interest rates becomes even larger as inflation seems more persistent than it actually is. Even though the deflation is actually milder in the "no sales" inflation measure, because of its persistence, it changes the expectation for the real interest rate and through eq. (32) it induces a deeper recession.

Figure 7 depicts the impulse responses of the same shock as above, but with the central bank employing a biased measure of inflation while the households use the correct change in the cost of living for their savings decisions. As expected the differences with the benchmark model are not dramatic as the monetary authority simply guarantees determinacy of the equilibrium.

In the following figure, both the monetary authority and the consumers use the biased measure of inflation that ignores both sales and substitution effects. The effect is a more pronounced drop in income on impact. The inflation anticipated by consumers is exaggerated as argued above, but on top of that, the policy rate in eq. (31) increases because the Central bank responds to a more persistent measure of inflation instead of the actual inflation in the economy. Therefore, the expectations for the real rate is even more pronounced than in Figure 6.

To check the relative magnitude of each scenario, Figure 9 depicts all impulse responses, the benchmark model and the other scenarios above in a single figure for each variable to visualize the relative impact more easily. The benchmark model corresponds to the black solid line

## 4 Conclusion

This study demonstrates a novel theoretical model to investigate the effect of sales on common inflation measures and the effect on the economy when alternative measures are employed instead. This framework provides an alternative explanation to the mild deflation after the great recession despite the deterioration in other economic variables such as GDP or unemployment. The weak link between unemployment and inflation in this study is due to the inability of common inflation measures to account for temporary sales and promotions and the associated preference for such offers by the households especially during downturns. Common inflation measures are not inac-

curate when the economy is at the long run average or even above the long run average. They are inadequate to account for the traditional Phillips curve relation when the economy is deteriorating. In crisis periods such as the great recession significantly more consumers are hunting for good deals and also the firms prefer to respond by more generous and more frequent sales on their items rather than decreasing their actual price. This make traditional inflation measures appear overstated even though the frequency of sales and the preference of consumer for items on sales imply that the cost of living is lower than what it appears to be.

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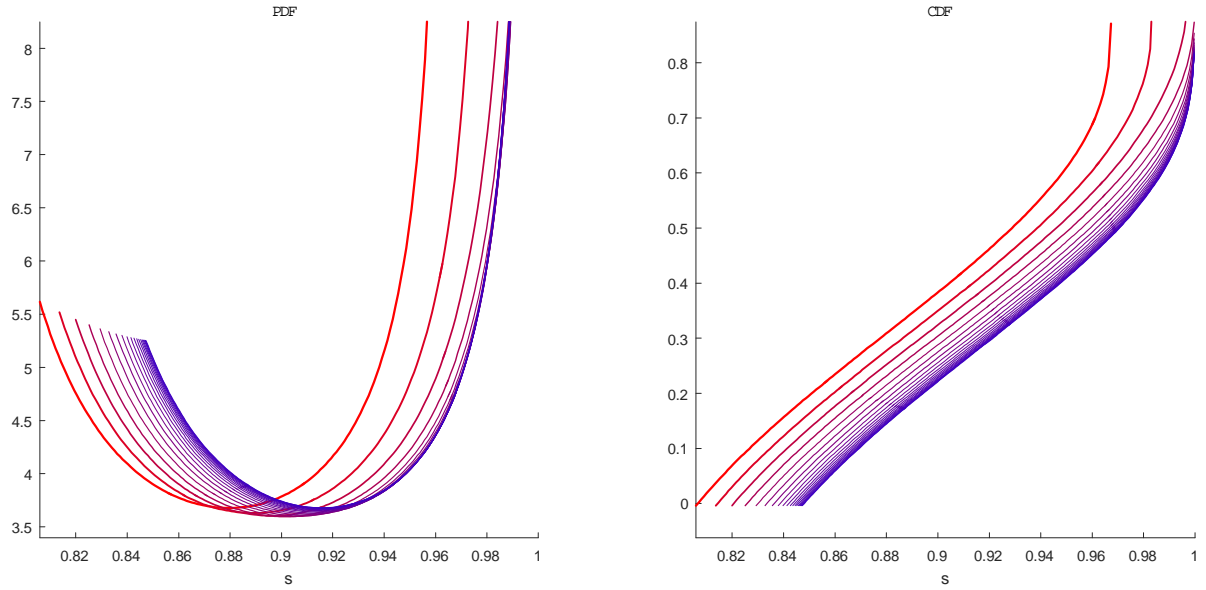


Figure 4: The responses of the pdf and the cdf of sales after a 1% federal funds rate shock. On impact, the distribution jumps to the red and thicker line and gradually shifting towards the blue and thin responses as the shock fades away.

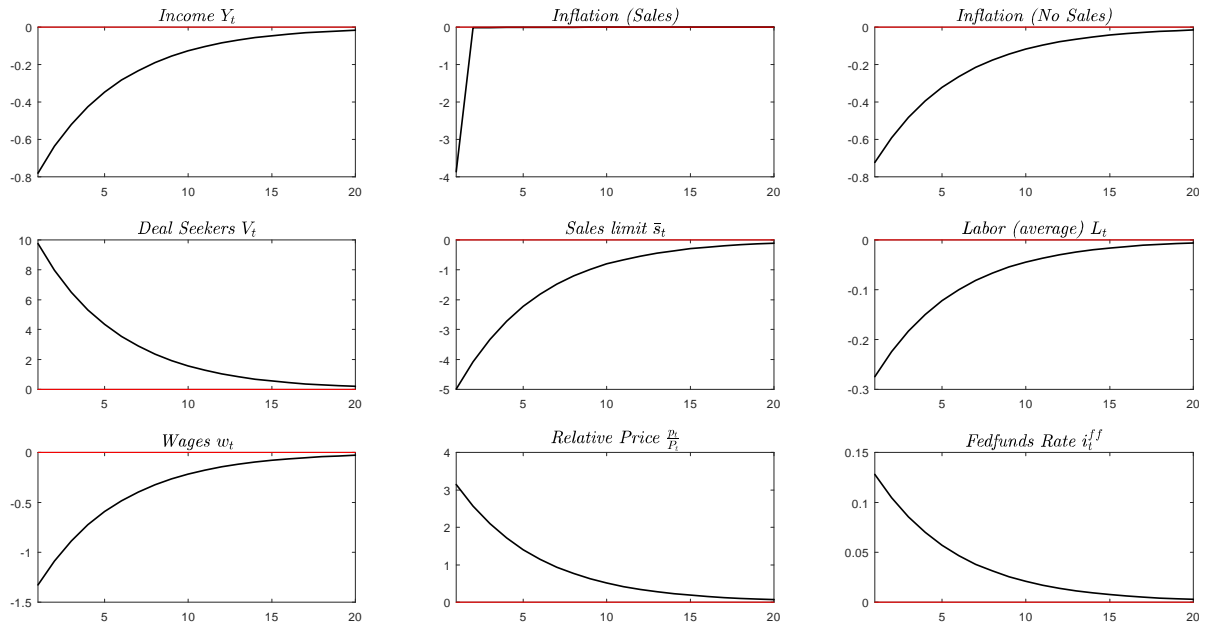


Figure 5: The impulse responses after a 1% federal funds rate shock.



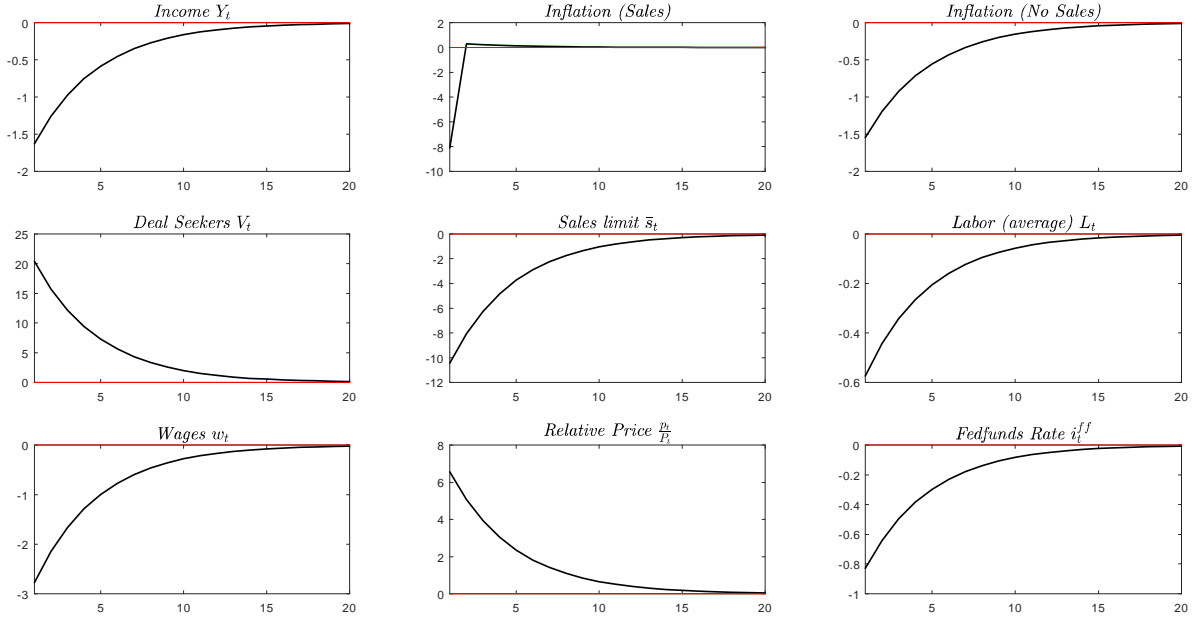


Figure 6: The impulse responses after a 1% federal funds rate shock when households make decisions measuring inflation using only prices and thus disregarding sales.

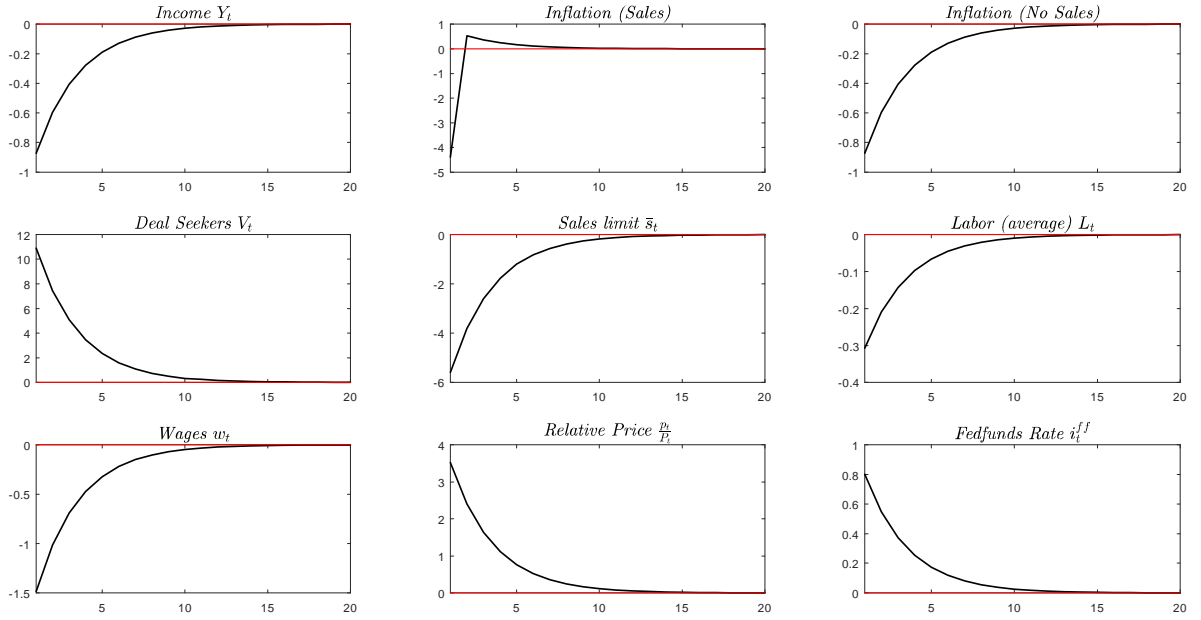


Figure 7: The impulse responses after a 1% federal funds rate shock when the Taylor rule of the central bank uses as a measure of inflation just the prices, disregarding sales completely.

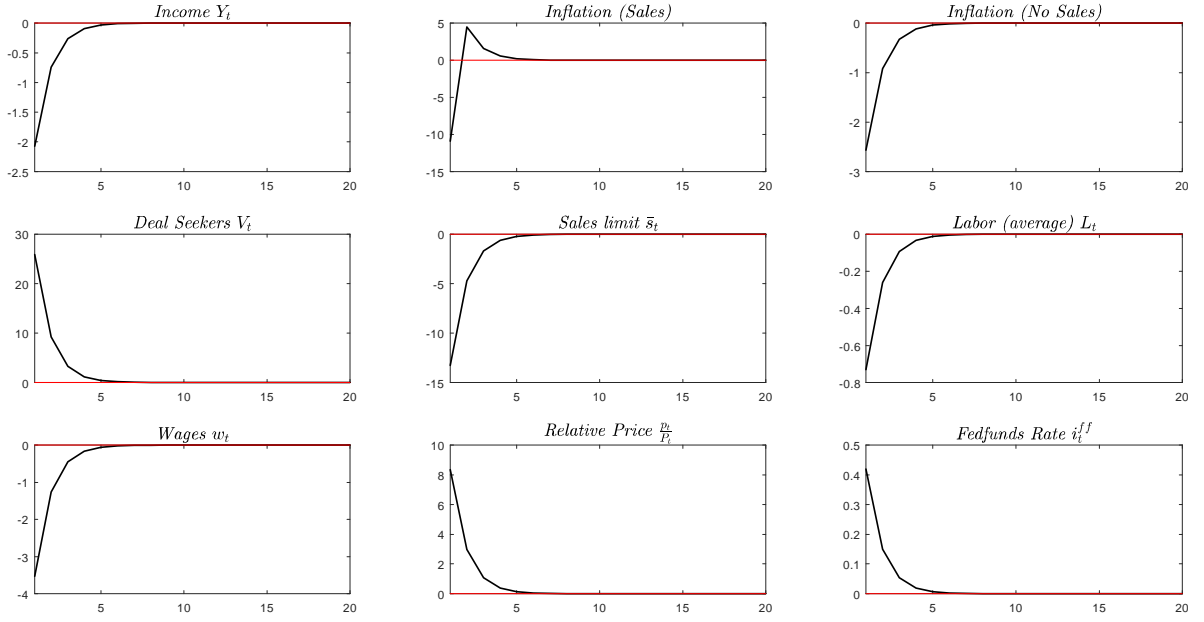


Figure 8: The impulse responses after a 1% federal funds rate shock when both households and the central bank use as a measure of inflation just the variation in listed prices, disregarding sales.

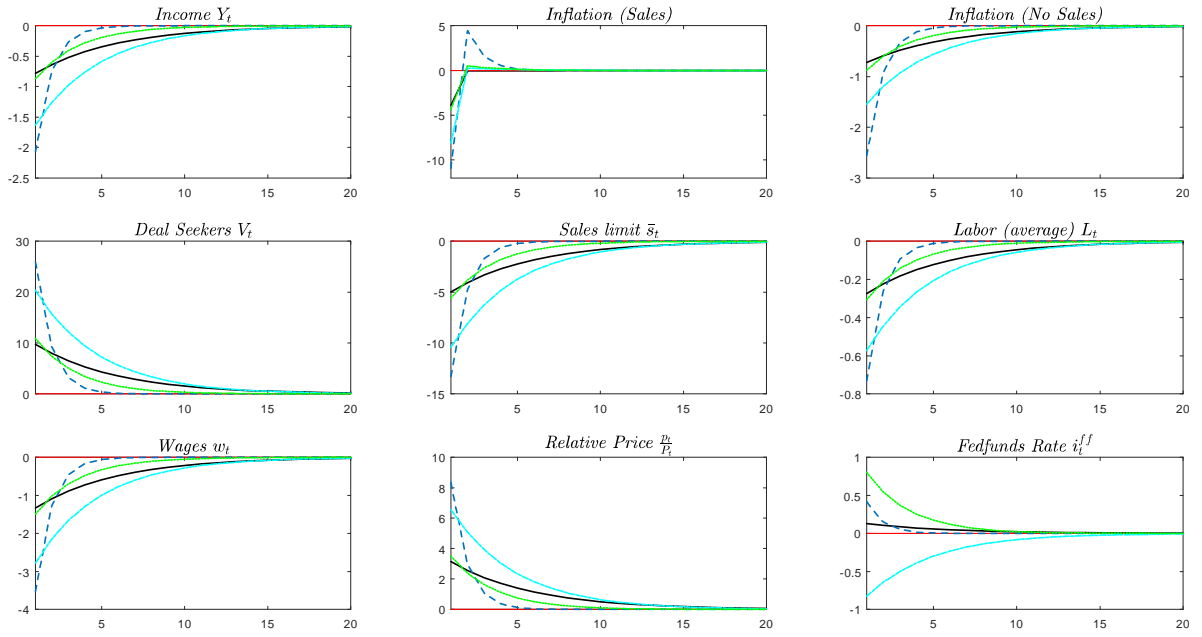


Figure 9: The impulse responses after 1 sd shock that increases the federal funds rate for the benchmark model and all the different scenarios analyzed.