EVALUATING THE MACROECONOMIC EFFECTS OF THE ECB'S UNCONVENTIONAL MONETARY POLICIES

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ABSTRACT. We quantify the macroeconomic effects of the European Central Bank's unconventional monetary policies using a DSGE model which includes a shadow Eonia rate. Extracted from the yield curve, this shadow rate provides an unconstrained measure of the overall stance of monetary policy. Without unconventional measures, the euro area would have suffered a cumulative loss of output of 19% of its pre-crisis level since the Great Recession, and deflation episodes in 2009Q1 and 2016Q1. This translates into year-on-year inflation and GDP growth that would have been on average about 0.25% and 0.56% below their actual levels over the period 2014Q1-2016Q1, respectively.

JEL: E32, E44, E52.

Keywords: Unconventional monetary policy, shadow policy rate, DSGE model, euro area.

1. INTRODUCTION

Decisions of central banks rely on an assessment of their monetary policy stance, i.e. the contribution made by monetary policy to real economic and financial developments. In the past, policymakers could compare the policy rate to the prescriptions of simple policy rules, to get a sense of whether their actions were appropriate in view of their objectives. However, the severity of the financial crisis in 2008 led many central banks to lower their key rates at levels close to their effective lower bound (ELB), limiting their ability to stimulate further the economy. Unable to move the short-end of the yield curve, central banks turned to a number of unconventional policies to provide additional monetary accommodation. In the context of the euro area, these policies included an increase in the average maturity of outstanding

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liquidity, the use of forward guidance, several asset purchase programs and negative deposit facility rates. With the implementation of such measures, there is no directly observable indicator that summarizes the stance of policy. How can one quantify the effects of these new policy measures from a macroeconomic perspective?

In this paper, we address this question by building a shadow policy rate and subsequently integrating it in a dynamic stochastic general equilibrium (DSGE) model to reveal the macroeconomic effects of unconventional measures implemented by the European Central Bank (ECB).

This shadow rate is the shortest maturity rate extracted from a term structure model, that would generate the observed yield curve (Kim and Singleton, 2012; Krippner, 2012; Christensen and Rudebusch, 2015, 2016). It incorporates both the effect of monetary policy measures on current economic conditions as well as market expectations of future policy actions. The shadow rate coincides with the policy rate in normal times and is free to go into negative territory when the policy rate is stuck at its lower bound. Claus, Claus and Krippner (2014), Francis, Jackson and Owyang (2014) and Van Zandweghe (2015) show that the shadow rate captures the stance of monetary policy during lower bound episodes in the same way the policy rate does in normal times. Hence, the dynamic relationships between macroeconomic variables and monetary policy are preserved, in any economic environment, by using a shadow rate. Particularly, exploiting the entire yield curve allows to account for the influence of direct and/or indirect market interventions on intermediate and longer maturity rates. It can therefore be used as a convenient indicator for measuring the total accommodation provided by both conventional and unconventional policies (Krippner, 2013; Wu and Xia, 2016).

In order to adequately quantify the macroeconomic effects of unconventional policies, we further need a macroeconomic model that is structural in the sense that *(i)* it formalises the behavior of economic agents on the basis of explicit micro-foundations and *(ii)* it can appropriately control for the effects of policy measures through expectations to respond to the Lucas (1976) critique. Hence, we consider a medium-scale DSGE model *à la* Smets and Wouters (2007) as it has been successful in providing an empirically plausible account of key macroeconomic variables. We deliberately choose to keep such a standard framework because *(i)* it has been shown to be useful in explaining a number of facts in the post crisis period (Del Negro et al., 2015b) and *(ii)* it is challenging to incorporate all the channels through which we

think unconventional measures can act (selecting only a few of these channels could distort the results). In addition, Wu and Zhang (2017) theoretically show that the impact of unconventional measures (particularly asset purchases and lending facilities) is identical to that of a negative shadow rate that enters directly into the IS curve, validating our approach.

Therefore, we propose to compare a Taylor rule based on a shadow rate with the usual Taylor rule based on the Eonia. Particularly, the shadow rate is used to extract the shocks stemming from all monetary policy actions within our DSGE model. Through a counterfactual exercise, those shocks can subsequently be compared to the monetary policy shocks obtained with the same model but substituting the shadow rate with the Eonia rate. Indeed, the latter shocks only account for the conventional part of monetary policy. This analysis enables us to isolate and gauge the effects of unconventional policies on economic activity and inflation.

We find that in the absence of such monetary policies, the euro area would have suffered a cumulative loss of output of around 19% of its pre-crisis level over the 2008Q1-2016Q1 period. Moreover, these measures have helped in avoiding (i) deflation episodes in 2009Q2 and 2016Q1, and (ii) a slowdown in price increases in 2015 and 2016. This translates into year-on-year (y-o-y) inflation and GDP growth differentials of 0.1% and 0.2% on average over the period 2008Q1-2016Q1, respectively. Drawing attention on the period 2014Q1-2016Q1, when public and private sector asset purchase programs have been announced and conducted, y-o-y inflation and GDP growth would have been lower by 0.25% and 0.56%, respectively. A robustness analysis suggests that our benchmark model's results are in line with those obtained using alternative shadow-rate measures. Our analysis also highlights that we can still use standard linear DSGE models in low interest rate environments when using an unconstrained proxy of the monetary policy stance such as the shadow rate.

Despite the growing interest in unconventional monetary policies, the literature has mainly concentrated on the financial market effects of FOMC-decisions, especially through event studies (see the survey by Bhattarai and Neely, 2016). There have been relatively few studies which have investigated the impact of unconventional monetary policies on macro variables, whether for the United States or the euro area.[1] In addition, these studies focus exclusively on

[1]These studies include Chen, Cúrdia and Ferrero (2012), Baumeister and Benati (2013), Gertler and Karadi (2013), Cova, Pagano and Pisani (2015), Andrade, Breckenfelder, De Fiore, Karadi and Tristani (2016), Sahuc (2016) and Weale and Wieladek (2016) on asset purchases, Del Negro, Eggertsson, Ferrero and Kiyotaki (2017) and Cahn, Matheron and Sahuc (forthcoming) on liquidity injections, and Del Negro, Giannoni and Patterson

the effects of large-scale asset purchases and do not consider all the measures implemented by central banks, with the notable exceptions of Engen, Laubach and Reifschneider (2015) and Wu and Xia (2016). The former evaluate the macroeconomic effects of both forward guidance and asset purchases in the United States by including private-sector forecasters' perceptions of monetary policy in a DSGE model. Nonetheless, survey data are not available at a sufficiently high frequency making the stance of monetary policy harder to gauge in real time. The latter assess the effects of the various measures adopted by the Fed after the Great Recession using their estimate of the shadow rate in a factor-augmented Vector Autoregression (VAR). However, VAR-based policy counterfactuals are sensitive to *(i)* unknown structural characteristics of the underlying data generating process and *(ii)* identification schemes (Benati, 2010). Especially, the VAR model by Wu and Xia (2016) displays a price puzzle (i.e. aggregate prices and the interest rate move in the same direction following a monetary policy shock) that leads to misleading interpretations when considering counterfactual monetary policy regimes.

By introducing a shadow rate within a consistent DSGE framework, our paper is the first to provide a tractable assessment of the macroeconomic effects of *all* unconventional policies implemented by a central bank since 2008. Our approach has the advantage of overcoming the computational issues associated with the presence of the ELB by using traditional estimation techniques.

In the remainder of the paper Section 2 introduces the zero lower bound consistent term structure model that generates our shadow policy rate for the euro area, Section 3 describes the dynamic stochastic general equilibrium model, Section 4 presents our empirical results on the quantification of the effects of unconventional monetary policy measures in the euro area, and Section 5 concludes.

2. A SHADOW EONIA RATE

In this section, we introduce the shadow-rate model by Christensen and Rudebusch (2015), which is a no-arbitrage term structure model that is consistent with the existence of an effective lower bound. The concept of a shadow rate as a modeling tool to account for the zero lower bound is attributed to Black (1995). He argued that the observed nominal short-term interest rate is non-negative because physical currency is an alternative asset to investors that

(2015a), McKay, Nakamura and Steinsson (2016), Sahuc (2016) and Kulish, Morley and Robinson (2017) on forward guidance.

carries a nominal interest rate of zero. Therefore, yields are bounded by zero with the existence of currency. Despite this theoretical bound set at zero, episodes of negative policy rates have occurred in the euro area since June 2014. Hence, with an objective of fit in mind, one could consider rendering the lower bound time varying. However, any variation of the lower bound can be problematic when gauging the monetary policy stance. Indeed, interest rates that enter into negative territory should be considered as an unconventional measure per se and thus ought to be reflected in the monetary policy stance. If the term structure model allows for a time-varying lower bound, the shadow rate can no longer be used as a proxy for the stance. Specifically, a decrease in the lower bound manifests itself in an increase in the shadow rate, which contradicts the accommodative nature of such a policy. We thus opt for a constant lower bound set at zero.

2.1. **A shadow-rate term structure model.** In a shadow-rate term structure model, the policy rate R_t, which is used for discounting cash flows when valuing securities, is equal to zero or to the shadow rate S_t, whichever is larger:

$$R_t = \max\left(0, S_t\right).$$

We assume that the shadow rate is an affine function of some state variables $\mathbf{F}_t$,

$$S_t = \rho_0 + \boldsymbol{\rho_1}' \mathbf{F}_t,$$

where ρ_0 is a scalar and $\boldsymbol{\rho_1}$ is an $n \times 1$ vector. The dynamics of the pricing factors under the risk-neutral measure $\mathbb{Q}$ follows a vector autoregressive process of order one:

$$\mathbf{F}_t = \tilde{\boldsymbol{\Phi}}\tilde{\boldsymbol{\mu}} + \left(\mathbf{I} - \tilde{\boldsymbol{\Phi}}\right)\mathbf{F}_{t-1} + \boldsymbol{\Sigma}\tilde{\boldsymbol{\xi}}_t,$$

where $\tilde{\boldsymbol{\xi}}_t \sim \mathcal{NID}\left(\mathbf{0}, \mathbf{I}\right)$. The mean level of the pricing factor is controlled by $\tilde{\boldsymbol{\mu}}$ of dimension $n \times 1$, while the persistence and the conditional volatility of the factors are determined by the $n \times n$ matrices $\tilde{\boldsymbol{\Phi}}$ and $\boldsymbol{\Sigma}$, respectively. The relationship between the physical measure $\mathbb{P}$ and the risk-neutral measure $\mathbb{Q}$ is given by $\tilde{\boldsymbol{\xi}}_t = \boldsymbol{\xi}_t + \wp\left(\mathbf{X}_t\right)$, and the factor dynamics under $\mathbb{P}$ are therefore

$$\mathbf{F}_t = \boldsymbol{\Phi}\boldsymbol{\mu} + \left(\mathbf{I} - \boldsymbol{\Phi}\right)\mathbf{F}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\xi}_t,$$

with $\boldsymbol{\xi}_t \sim \mathcal{NID}\left(\mathbf{0}, \mathbf{I}\right)$. In order to obtain an affine process for the pricing factors under $\mathbb{P}$ (see Duffee, 2002), we let the price of risk $\wp\left(\mathbf{F}_t\right) = \wp_{\mathbf{0}} + \wp_{\mathbf{1}}\mathbf{F}_t$, where $\wp_{\mathbf{0}}$ has dimension $n \times 1$ and

$\wp_1$ is an $n \times n$ matrix. This implies the following dynamics for the pricing factors under the real-world measure $\mathbb{P}$:

$$\mathbf{F}_t = \tilde{\mathbf{\Phi}}\tilde{\boldsymbol{\mu}} + \mathbf{\Sigma}\wp_0 + \left(\mathbf{I} - \tilde{\mathbf{\Phi}} + \mathbf{\Sigma}\wp_1\right)\mathbf{F}_{t-1} + \mathbf{\Sigma}\boldsymbol{\xi}_t.$$

At this point, it is important to note that all pricing formulas of this term structure model are derived in continuous time. Thus the states' vector $\mathbf{F}_t$ follows an Ornstein-Uhlenbeck process, which under the physical measure, takes the following form, once discretized:

$$\mathbf{F}_t = \left[\mathbf{I} - \exp\left(-\boldsymbol{\kappa}\right)\right]\boldsymbol{\mu} + \exp\left(-\boldsymbol{\kappa}\right)\mathbf{F}_{t-1} + \mathbf{\Sigma}\boldsymbol{\xi}_t.$$

Its $\mathbb{Q}$-measure analogue is of the same form and parameters are denoted with a tilde.

The measurement equation relates observed zero-coupon yields with maturity T at time t, $M(t,T)$, to the pricing factors as follows:

$$M(t,T) = \frac{1}{T-t}\int_t^T \underline{f}(t,s)ds,$$

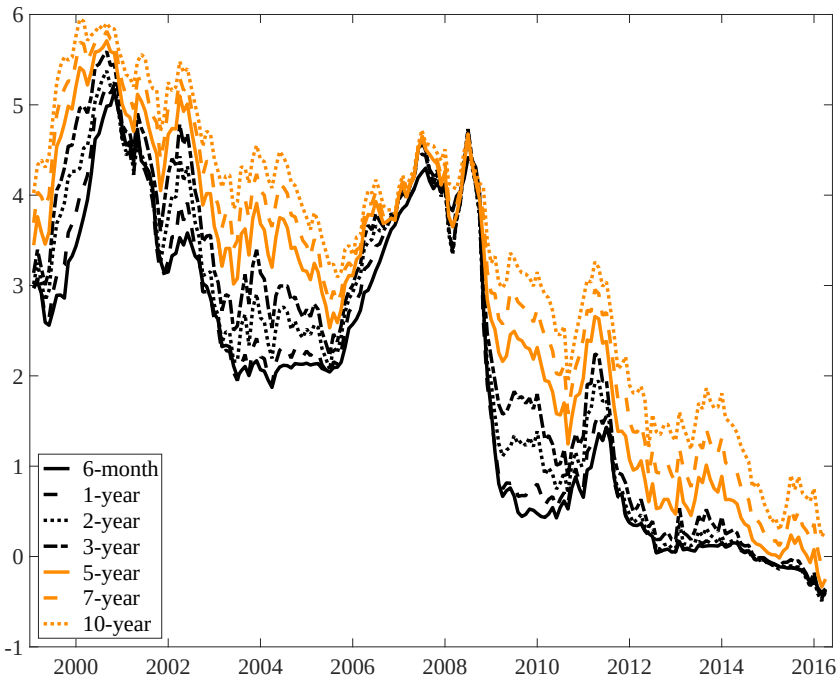
where $\underline{f}(t,s)$ is the ZLB instantaneous forward rate as derived in Christensen and Rudebusch (2015).

2.2. **Financial data.** The model described above is typically used to price zero-coupon sovereign bonds. However, no such bonds are issued on euro denominated public debt. We consequently need a proxy for risk-free rates within the euro area. Our analysis is based on Eonia overnight indexed swap (OIS) rates. These swap rates cover several maturities and their market has become increasingly liquid in recent years, rendering them a popular substitute for zero-coupon sovereign yields in the euro area. These OIS rates become available starting January 1999 for short maturities, while longer maturities become available progressively and data prior to their availability is proxied using Euribor swap rates.

The data set therefore consists of monthly zero-coupon OIS yields spanning from January 1999 to March 2016 and includes a set of seven maturities, namely 6, 12, 24, 36, 60, 84 and 120 months, and is depicted in Figure 1.

2.3. **Number of factors and identification.** Before proceeding to the identification and estimation of the model, we first conduct a principal component analysis to determine how many pricing factors are required to explain the cross-sectional variation of nominal yields.

Figure 1. Monthly yield curve data



It is widely accepted in the literature that three pricing factors are typically considered sufficient (see Litterman and Scheinkman, 1991; Ang and Piazzesi, 2003). This is further confirmed via a principal component analysis. Table 1 displays the loadings from the principal component analysis for the set of maturities and the percentage of variation of yields that is being captured by each component. We notice that the first component is characteristic of a level factor due to its stability across all maturities, the second component incorporates a sign switch between shorter and longer maturities hence featuring a slope-like behavior and finally the third component, being parabolic, has the shape of a curvature factor. Additionally, the first three components explain 99.98% of the cross-sectional yield variation.

The principal component analysis results corroborate our use of three factors bearing the level, slope and curvature interpretation. Thus, we set $n = 3$ and let $\mathbf{F}_t = [\mathbf{F}_{1,t}, \mathbf{F}_{2,t}, \mathbf{F}_{3,t}]'$ denote the state variables, which can be interpreted as level, slope and curvature factors (see Nelson and Siegel, 1987).

The pricing factors are considered to be latent (i.e. unobserved) and a set of normalization restrictions are therefore needed to identify the model. We require *(i)* $\rho_0 = 0$ and $\boldsymbol{\rho_1} = [1, 1, 0]'$,

(ii) $\tilde{\boldsymbol{\mu}} = [0,0,0]'$, *(iii)* $\boldsymbol{\Sigma}$ to be diagonal, and iv) $\tilde{\boldsymbol{\kappa}}$ to be given by

$$\tilde{\boldsymbol{\kappa}} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \omega & -\omega \\ 0 & 0 & \omega \end{pmatrix},$$

where ω is a mean-reversion parameter and $\epsilon = 10^{-6}$ to obtain a near unit root behavior for the level factor. This identification scheme constraints the $\mathbb{Q}$ dynamics for the pricing factors, whereas the $\mathbb{P}$ dynamics are unrestricted.

Table 1. First three principal components (PC)

Maturity (months)	First PC	Second PC	Third PC
6	0.36	-0.51	0.61
12	0.38	-0.44	0.01
24	0.39	-0.21	-0.46
36	0.40	-0.01	-0.45
60	0.39	0.26	-0.16
84	0.38	0.41	0.12
120	0.36	0.52	0.41
% explained	97.56	2.29	0.13

Note: We provide the loadings of the yields of the set of maturities on the first three principal components. The percentage of all yields' cross-sectional variation accounted for by each component is displayed on the final row. The data comprise of monthly yields from January 1999 to March 2016.

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2.4. **Estimation and model specification.** The model has a state-space representation, whereby the transition and measurement equations are given by:

$$\mathbf{F}_{t+1} = \boldsymbol{\Phi}\boldsymbol{\mu} + (\mathbf{I} - \boldsymbol{\Phi})\,\mathbf{F}_t + \Sigma\boldsymbol{\xi}_{t+1},$$
$$\mathbf{M}_{t+1} = \mathcal{G}(\mathbf{F}_{\mathbf{t+1}}) + \boldsymbol{\eta}_{t+1},$$

where $\mathcal{G}(.)$ is a non-linear function and $\boldsymbol{\eta}_{t+1} \sim \mathcal{N}(\mathbf{0}, \Xi)$.

The estimation of a shadow-rate term structure model requires the use of the extended Kalman filter. Unlike the standard Kalman filter algorithm, the extended procedure relies on a first-order Taylor expansion of the measurement equation around the current predicted state. The conditional distribution of $\mathbf{F}_t$ is approximated as a Normal distribution with mean $\check{\mathbf{F}}_{t|t}$ and covariance matrix $\mathbf{P}_{t|t}$. The extended Kalman filter recursion begins with initial conditions $\check{\mathbf{F}}_{0|0}$ and $\mathbf{P}_{0|0}$, which are set to the unconditional mean and covariance matrix, respectively. The prediction step consists of the following system:

$$\check{\mathbf{F}}_{t+1|t} = \boldsymbol{\Phi}\boldsymbol{\mu} + (\mathbf{I} - \boldsymbol{\Phi})\,\check{\mathbf{F}}_{t|t},$$
$$\mathbf{P}_{t+1|t} = (\mathbf{I} - \boldsymbol{\Phi})\,\mathbf{P}_{t|t}\,(\mathbf{I} - \boldsymbol{\Phi})' + \Sigma\Sigma'.$$

The update of $\check{\mathbf{F}}_{t+1|t+1}$ and $\mathbf{P}_{t+1|t+1}$ are given as follows:

$$\check{\mathbf{F}}_{t+1|t+1} = \check{\mathbf{F}}_{t+1|t} + \mathbf{K}^G_{t+1}\left(\mathbf{M}_{t+1} - \check{\mathbf{M}}_{t+1|t}\right),$$
$$\mathbf{P}_{t+1|t+1} = \left(\mathbf{I} - \mathbf{K}^G_{t+1}\mathbf{H}'_{t+1}\right)\mathbf{P}_{t+1|t},$$

with:

$$\check{\mathbf{M}}_{t+1|t} = \mathcal{G}(\check{\mathbf{F}}_{\mathbf{t+1|t}}), \mathbf{H}_{t+1} = \left(\frac{\partial F(\mathbf{F}_{t+1})}{\partial \mathbf{F}'_{t+1}}|_{\mathbf{F}_{t+1}=\hat{\mathbf{F}}_{t+1|t}}\right)', \mathbf{K}^G_{t+1} = \mathbf{P}_{t+1|t}\mathbf{H}_{t+1}\left(\mathbf{H}'_{t+1}\mathbf{P}_{t+1|t}\mathbf{H}_{t+1} + \Xi\right)^{-1}.$$

We use a general-to-specific method in order to impose the relevant restrictions to our model, this allows us to find the best specification for the $\boldsymbol{\kappa}$ matrix. The procedure consists of estimating an unrestricted model and setting the least significant element of $\boldsymbol{\kappa}$ to zero. This process is repeated until we are left with a diagonal $\boldsymbol{\kappa}$. Two criteria, the Akaike Information Criterion (AIC) and Bayes Information Criterion (BIC), are provided on Table 2, and our decision is ruled by minimizing the AIC (when the AIC and BIC decision rules do not coincide). The preferred specification is thus given by specification (3).

Table 2. Evaluation of alternative specifications of the shadow-rate model

Alternative specifications	$\log \mathcal{L}$	τ	p-value	AIC	BIC
(1) Unrestricted κ	8081.86	23		-16117.72	-16041.07
(2) $\boldsymbol{\kappa_{32}} = 0$	8081.85	22	0.88	-16119.69	-16046.38
(3) $\boldsymbol{\kappa_{32}} = \boldsymbol{\kappa_{13}} = 0$	8081.46	21	0.68	**-16120.91**	-16050.93
(4) $\boldsymbol{\kappa_{32}} = \boldsymbol{\kappa_{13}} = \boldsymbol{\kappa_{31}} = 0$	8078.99	20	0.18	-16117.98	-16051.33
(5) $\boldsymbol{\kappa_{32}} = ... = \boldsymbol{\kappa_{23}} = 0$	8078.15	19	0.80	-16118.30	-16054.98
(6) $\boldsymbol{\kappa_{32}} = ... = \boldsymbol{\kappa_{21}} = 0$	8078.00	18	1.00	-16120.01	-16060.02
(7) $\boldsymbol{\kappa_{32}} = ... = \boldsymbol{\kappa_{12}} = 0$	8077.23	17	0.96	-16120.46	**-16063.80**

Note: We estimate and evaluate seven alternative specifications of the shadow-rate model. For each specification, we record its log-likelihood $(\log \mathcal{L})$, number of parameters (τ) and the p-value of a likelihood ratio test of the hypothesis that a specification with $(\tau - i)$ parameters is different from the one with $(\tau - i + 1)$ parameters. The information criteria (AIC and BIC) are reported and we display their minimum in bold.

Table 3. Shadow-rate model estimates

$\boldsymbol{\kappa}$	$\boldsymbol{\kappa_{.,1}}$	$\boldsymbol{\kappa_{.,2}}$	$\boldsymbol{\kappa_{.,3}}$	$\boldsymbol{\mu}$	$\boldsymbol{\Sigma_{i,i}}$
$\boldsymbol{\kappa_{1,.}}$	0.038	-0.304	0.000	0.039	0.007
	(0.079)	(0.056)		(0.004)	(0.000)
$\boldsymbol{\kappa_{2,.}}$	0.691	0.402	-0.282	-0.022	0.009
	(0.287)	(0.132)	(0.143)	(0.004)	(0.001)
$\boldsymbol{\kappa_{3,.}}$	-0.273	0.000	0.250	-0.046	0.020
	(0.134)		(0.069)	(0.006)	(0.002)

Note: The estimated parameters of the $\boldsymbol{\kappa}$ matrix, $\boldsymbol{\mu}$ vector, and diagonal diffusion matrix $\boldsymbol{\Sigma_{i,i}}$ are given for our preferred shadow-rate model. The estimated value of ω is 0.468 with standard deviation of 0.014. The numbers in parentheses are the standard deviations of the estimated parameters.

Table 3 indicates the parameter estimates and their respective standard errors while Table 4 provides measures of the in-sample fit of the model. The Root Mean Squared Error (RMSE)

varies between 1.7 and 9.4 basis points depending on the maturity of the yields and the fit is particularly good at longer maturities. On average the RMSE amounts to 5 basis points conveying our model provides a good fit for the entire term structure, one which is comparable to those found in the literature.

Table 4. Measures of fit for the shadow-rate model

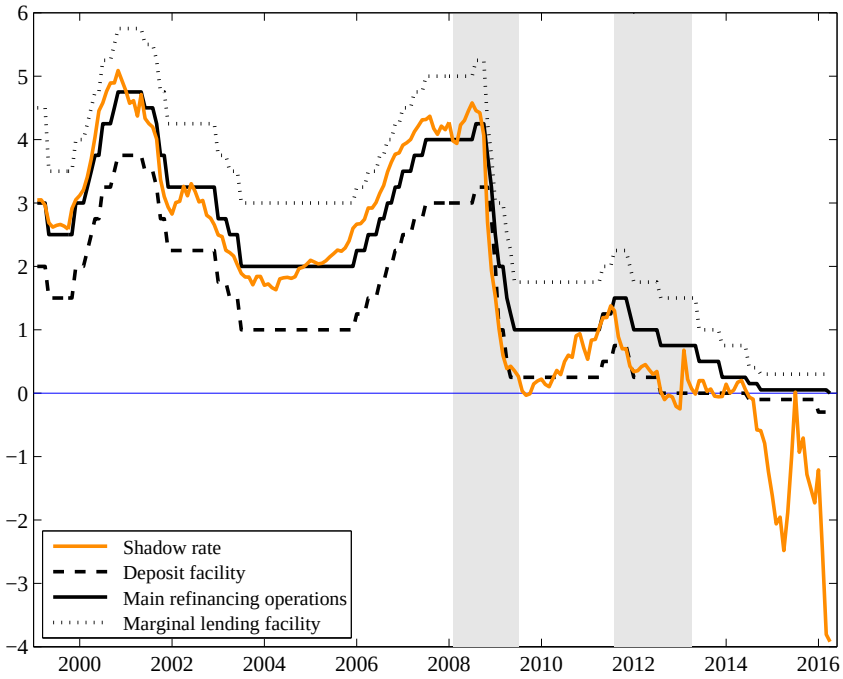
Maturity (months)	Mean (in bp)	RMSE (in bp)
6	1.81	9.03
12	2.52	8.10
24	2.99	9.41
36	0.39	8.14
60	0.74	4.36
84	1.06	2.54
120	0.21	1.73

Note: The mean and RMSE of fitted errors of the preferred shadow-rate model are given. All values are measured in basis points. The nominal yields span from January 1999 to March 2016.

Figure 2 displays the three key rates of the ECB, i.e. the rates on the main refinancing operations (MRO), the deposit facility (DF) and the marginal lending facility, along with our estimate of the shadow Eonia rate. The estimated shadow rate displays time variation. We notice that it tracks relatively well the rate on the MRO prior to the Great Recession.[2] Notably, with the advent of unconventional monetary policies, the shadow rate first converges towards the DF and then turns significantly negative reaching levels of up to nearly -400 basis points. This dissociation from the MRO is what allows the shadow rate to continue serving as a proxy for the policy stance even when short-term maturities are stuck at the ELB. In particular, the changes in the shadow rate correspond to the various monetary policy measures implemented by the ECB (see Figure A1 of Appendix A).

[2]The correlation between shadow and Eonia rates is 0.94 over the 1999-2007 period.

Figure 2. Shadow rate and key ECB interest rates



Note: Gray bars denote CEPR-defined recessions.

Shadow rates have become increasingly popular in summarizing the stance of monetary policy due to their maintained correlation with macroeconomic variables, even when key policy rates are kept at the ELB (see, Krippner, 2013; Hakkio and Kahn, 2014; Doh and Choi, 2016; Wu and Xia, 2016). This desirable property comes from the fact that shadow rates typically stem from term structure models which exploit the entire yield curve, including long-term yields which are highly informative on expectations of future short-term yields. Specifically, Claus, Claus and Krippner (2014), Francis, Jackson and Owyang (2014) and Van Zandweghe (2015) show that the shadow rate captures the stance of monetary policy during lower bound episodes in the same way the policy rate does in normal times. Hence, the dynamic relationships between macroeconomic variables and monetary policy are preserved, in any economic environment, by using a shadow rate.[3]

[3]Claus, Claus and Krippner (2014) find that the shadow rate is a reasonable approximation of both conventional and unconventional monetary policy shocks in the US. Since the Federal Reserve began to use unconventional methods, the impact of monetary policy surprises on asset markets is estimated to have been larger compared to the prior conventional period. Francis, Jackson and Owyang (2014) find that, when using a dataset

3. Estimating a Macroeconomic Model Using the Shadow Rate

This section presents the structural model used for quantifying the macroeconomic effects of unconventional policies, and discusses the estimation results on the 1980Q1-2016Q1 period.

3.1. **The structural model.** The model combines a neoclassical growth core with several shocks and frictions (see Smets and Wouters, 2007; Justiniano et al., 2010). It includes features such as habit formation, investment adjustment costs, variable capital utilization, monopolistic competition in goods and labor markets, and nominal price and wage rigidities. The economy is populated by five classes of agents: producers of a final good, intermediate goods producers, households, employment agencies and the public sector (government and monetary authorities).[4] The nominal interest rate is assumed to be the shadow rate. Obviously, no transactions are taking place at the shadow rate, but various borrowing/lending rates that private agents face co-move with it, with correlations of about 0.9 (see Figure A2 of Appendix A). We observe, in particular, the same sharp decline in 2014, followed by a rebound in 2015 and a further decline in 2016 (behavior that the Eonia cannot reproduce). This strong link between bank rates and the shadow rate has also been documented by Wu and Zhang (2017) in the case of the United States. This indicates that the shadow rate has comparable dynamics to the borrowing/lending rates (notably to the 3-month government bond rate, the underlying counterpart in the model, which becomes negative from mid-2014) and that the difference in levels results from the additional easing of the financing conditions provided by the non-standard measures.

3.1.1. *Household sector.*

Employment agencies–. Each household indexed by $j \in [0,1]$ is a monopolistic supplier of specialized labor $N_{j,t}$. At every point in time t, a large number of competitive "employment agencies" combine households' labor into a homogenous labor input N_t sold to intermediate firms, according to $N_t = \left[\int_0^1 N_{j,t}^{\frac{1}{\varepsilon_{w,t}}} \mathrm{d}j\right]^{\varepsilon_{w,t}}$. Profit maximization by the perfectly competitive

that spans both the pre-ZLB and ZLB periods in the US, the shadow rate acts as a fairly good proxy for monetary policy by producing impulse responses of macro indicators similar to what we would expect based on the post-WWII, non-ZLB benchmark and by displaying stable parameter estimates when compared to this benchmark. Finally, Van Zandweghe (2015) implements formal statistical tests that cannot reject the hypothesis that macroeconomic variables have the same relationship with a lagged shadow federal funds rate, since the start of the current recovery, as they had with the effective federal funds rate before the recession.

[4] In the following, we let variables without a time subscript denote steady-state values.

employment agencies implies the labor demand function $N_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{-\frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1}} N_t$, where $W_{j,t}$ is the wage paid by employment agencies to the household supplying labor variety j, while $W_t \equiv \left(\int_0^1 W_{j,t}^{\frac{1}{\varepsilon_{w,t}-1}} \mathrm{d}j\right)^{\varepsilon_{w,t}-1}$ is the wage paid by intermediate firms for the homogenous labor input sold to them by the agencies. The exogenous variable $\varepsilon_{w,t}$ measures the substitutability across labor varieties and its steady-state is the desired steady-state wage mark-up over the marginal rate of substitution between consumption and leisure.

Household's preferences–. The preferences of the jth household are given by

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{b,t+s} \left(\log\left(C_{t+s} - hC_{t+s-1}\right) - \frac{N_{j,t+s}^{1+\nu}}{1+\nu} + \mathcal{V}\left(G_{t+s}\right) \right),$$

where E_t denotes the mathematical expectation operator conditional upon information available at t, $\beta \in (0,1)$ is the subjective discount factor, $h \in [0,1]$ denotes the degree of habit formation, and $\nu > 0$ is the inverse of the Frisch labor supply elasticity. C_t denotes consumption, $N_{j,t}$ is labor of type j, and $\varepsilon_{b,t}$ is a preference shock. Finally, $\mathcal{V}(.)$ is a positive concave function.

Household j's period budget constraint is given by

$$P_t\left(C_t + I_t\right) + T_t + B_t \leq S_{t-1}B_{t-1} + A_{j,t} + D_t + W_{j,t}N_{j,t} + \left(R_t^k u_t - P_t \vartheta\left(u_t\right)\right)\bar{K}_{t-1},$$

where I_t is investment, T_t denotes nominal lump-sum taxes (transfers if negative), B_t is the one-period riskless bond, S_t is the nominal interest rate on bonds, $A_{j,t}$ is the net cash flow from household's j portfolio of state contingent securities, D_t is the equity payout received from the ownership of firms. The capital utilization rate u_t transforms physical capital $\bar{K}_t$ into the service flow of effective capital K_t according to $K_t = u_t\bar{K}_{t-1}$, and the effective capital is rented to intermediate firms at the nominal rental rate R_t^k. The costs of capital utilization per unit of capital is given by the convex function $\vartheta\left(u_t\right)$. We assume that $u = 1$, $\vartheta\left(1\right) = 0$, and we define $\eta_u \equiv \left[\vartheta''\left(1\right)/\vartheta'\left(1\right)\right]/\left[1 + \vartheta''\left(1\right)/\vartheta'\left(1\right)\right]$.[5] The physical capital accumulates according to

$$\bar{K}_t = \left(1-\delta\right)\bar{K}_{t-1} + \varepsilon_{i,t}\left(1 - \Psi\left(\frac{I_t}{I_{t-1}}\right)\right) I_t,$$

where $\delta \in [0,1]$ is the depreciation rate of capital, and $\Psi\left(.\right)$ is an adjustment cost function which satisfies $\Psi\left(\gamma_z\right) = \Psi'\left(\gamma_z\right) = 0$ and $\Psi''\left(\gamma_z\right) = \eta_k > 0$, γ_z is the steady-state (gross)

[5]Later, we estimate η_u rather than the elasticity $\vartheta''\left(1\right)/\vartheta'\left(1\right)$ to avoid convergence issues.

growth rate of technology, and $\varepsilon_{i,t}$ is an investment shock. Households set nominal wages according to a staggering mechanism. In each period, a fraction θ_w of households cannot choose its wage optimally, but adjusts it to keep up with the increase in the general wage level in the previous period according to the indexation rule $W_{j,t} = \gamma_z \pi^{1-\gamma_w} \pi_{t-1}^{\gamma_w} W_{j,t-1}$, where $\pi_t \equiv P_t / P_{t-1}$ represents the gross inflation rate, π is steady-state (or trend) inflation and the coefficient $\gamma_w \in [0,1]$ is the degree of indexation to past wages. The remaining fraction of households chooses instead an optimal wage, subject to the labor demand function $N_{j,t}$.

3.1.2. *Business sector.*

Final good producers–. At every point in time t, a perfectly competitive sector produces a final good Y_t by combining a continuum of intermediate goods $Y_t(\varsigma)$, $\varsigma \in [0,1]$, according to the technology $Y_t = \left[\int_0^1 {Y_{\varsigma,t}}^{\frac{1}{\varepsilon_{p,t}}} \mathrm{d}\varsigma\right]^{\varepsilon_{p,t}}$. Final good producing firms take their output price, P_t, and their input prices, $P_{\varsigma,t}$, as given and beyond their control. Profit maximization implies $Y_{\varsigma,t} = \left(\frac{P_{\varsigma,t}}{P_t}\right)^{-\frac{\varepsilon_{p,t}}{\varepsilon_{p,t}-1}} Y_t$ from which we deduce the relationship between the price of the final good and the prices of intermediate goods $P_t \equiv \left[\int_0^1 {P_{\varsigma,t}}^{\frac{1}{\varepsilon_{p,t}-1}} \mathrm{d}\varsigma\right]^{\varepsilon_{p,t}-1}$. The exogenous variable $\varepsilon_{p,t}$ measures the substitutability across differentiated intermediate goods and its steady state is then the desired steady-state price markup over the marginal cost of intermediate firms.

Intermediate-goods firms–. Intermediate good ς is produced by a monopolist firm using the following production function

$$Y_{\varsigma,t} = {K_{\varsigma,t}}^{\alpha} \left[Z_t N_{\varsigma,t}\right]^{1-\alpha} - Z_t \Omega,$$

where $\alpha \in (0,1)$ denotes the capital share, $K_{\varsigma,t}$ and $N_{\varsigma,t}$ denote the amounts of capital and effective labor used by firm ς, Ω is a fixed cost of production that ensures that profits are zero in steady state, and Z_t is an exogenous labor-augmenting productivity factor whose growth-rate is denoted by $\varepsilon_{z,t} \equiv Z_t / Z_{t-1}$. In addition, we assume that intermediate firms rent capital and labor in perfectly competitive factor markets.

Intermediate firms set prices according to a staggering mechanism. In each period, a fraction θ_p of firms cannot choose its price optimally, but adjusts it to keep up with the increase in the general price level in the previous period according to the indexation rule $P_{\varsigma,t} = \pi^{1-\gamma_p} \pi_{t-1}^{\gamma_p} P_{\varsigma,t-1}$,where the coefficient $\gamma_p \in [0,1]$ indicates the degree of indexation to past prices. The remaining fraction of firms chooses its price $P_{\varsigma,t}^{\star}$ optimally, by maximizing

the present discounted value of future profits

$$\mathrm{E}_t \sum_{s=0}^{\infty} \left(\beta\theta_p\right)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left\{ \Pi_{t,t+s}^{p} P_{\varsigma,t}^{\star} Y_{\varsigma,t+s} - \left[W_{t+s} N_{\varsigma,t+s} + R_{t+s}^{k} K_{\varsigma,t+s} \right] \right\},$$

where

$$\Pi_{t,t+s}^{p} = \begin{cases} \prod_{\nu=1}^{s} \pi^{1-\gamma_p} \pi_{t+\upsilon-1}^{\gamma_p} & s > 0 \\ 1 & s = 0, \end{cases}$$

subject to the demand from final goods firms and the production function. Λ_{t+s} is the marginal utility of consumption for the representative household that owns the firm.

3.1.3. *Public sector.* Fiscal policy is fully Ricardian. The government finances its budget deficit by issuing short-term bonds. Public spending is determined exogenously as a time-varying fraction of output

$$G_t = \left(1 - \frac{1}{\varepsilon_{g,t}}\right) Y_t,$$

where $\varepsilon_{g,t}$ is a government spending shock.

The monetary authority follows a shadow-rate Taylor rule by gradually adjusting the nominal interest rate in response to inflation, and output growth:

$$\frac{S_t}{S} = \left(\frac{S_{t-1}}{S}\right)^{\varphi_s} \left[\left(\frac{\pi_t}{\pi}\right)^{\varphi_\pi} \left(\frac{Y_t}{\gamma_z Y_{t-1}}\right)^{\varphi_y} \right]^{(1-\varphi_s)} \varepsilon_{s,t},$$

where $\varepsilon_{s,t}$ is a monetary policy shock.

3.1.4. *Market clearing and stochastic processes.* Market clearing conditions on final goods market are given by

$$Y_t = C_t + I_t + G_t + \vartheta\left(u_t\right) \bar{K}_{t-1},$$

$$\Delta_{p,t} Y_t = \left(u_t \bar{K}_{t-1}\right)^{\alpha} \left(Z_t N_t\right)^{1-\alpha} - Z_t \Omega,$$

where $\Delta_{p,t} = \int_0^1 \left(\frac{P_{\varsigma,t}}{P_t}\right)^{-\frac{\varepsilon_{p,t}}{\varepsilon_{p,t}-1}} \mathrm{d}\varsigma$ is a measure of the price dispersion.

Regarding the properties of the stochastic variables, productivity and monetary policy shocks evolve according to $\log\left(\varepsilon_{x,t}\right) = \zeta_{x,t}$, with $x \in \{z, s\}$. The remaining exogenous variables follow an AR(1) process $\log\left(\varepsilon_{x,t}\right) = \rho_x \log\left(\varepsilon_{x,t-1}\right) + \zeta_{x,t}$, with $x \in \{b, i, g, p, w\}$. In all cases, $\zeta_{x,t} \sim i.i.d.\mathcal{N}\left(0, \sigma_x^2\right)$.

3.2. **Bayesian inference.**

3.2.1. *Macroeconomic data and econometric approach.* The quarterly euro area data run from 1980Q1 to 2016Q1 and are extracted from the AWM database compiled by Fagan, Henry and Mestre (2005) and the ECB Statistical Warehouse, except hours worked and the working age population. Inflation π_t is measured by the first difference of the logarithm of the GDP deflator (YED), and real wage growth $\Delta \log (W_t / P_t)$ is the first difference of the logarithm of the nominal wage (WRN) divided by the GDP deflator. Output growth $\Delta \log Y_t$ is obtained as the first difference of the logarithm of real GDP (YER), consumption growth $\Delta \log C_t$ is the first difference of the logarithm of real consumption expenditures (PCR), investment growth $\Delta \log I_t$ is the first difference of the logarithm of real gross investment (ITR). The shadow rate S_t is first transformed into quarterly averages over the 1999Q1-2016Q1 period and then merged with the Euribor (STN) over the 1980Q1-1998Q4 period. Real variables are divided by the working age population, extracted from the OECD Economic Outlook. Ohanian and Raffo (2012) constructed a new dataset of quarterly hours worked for 14 OECD countries. We then derived a weighted (by country size) average of their series of hours worked for France, Germany and Italy to obtain a series of total hours for the euro area. Interestingly, the series thus obtained is very close to that provided by the ECB on the common sample, *i.e.* 1995Q1-2016Q1. Total hours worked N_t are taken in logarithms. We use growth rates for the non-stationary variables in our data set (GDP, consumption, investment and the real wage) and express gross inflation, gross interest rates and the first difference of the logarithm of hours worked in percentage deviations from their sample means.

After normalizing trending variables by the stochastic trend component in labor factor productivity, we log-linearized the resulting systems in the neighborhood of the deterministic steady state (see Appendix B). Let $\boldsymbol{\theta}$ denote the vector of structural parameters and $\mathbf{v}_t$ be the r-dimensional vector of model variables. Thus, the state-space form of the different model specifications is characterized by the state equation $\mathbf{v}_t = \mathbb{A}(\boldsymbol{\theta})\mathbf{v}_{t-1} + \mathbb{B}(\boldsymbol{\theta})\boldsymbol{\zeta}_t$, where $\boldsymbol{\zeta}_t \sim i.i.d.N\left(0, \Sigma_\zeta\right)$ is the q-dimensional vector of innovations to the structural shocks, and $\mathbb{A}(\boldsymbol{\theta})$ and $\mathbb{B}(\boldsymbol{\theta})$ are complicated functions of the model's parameters $\boldsymbol{\theta}$. The measurement equation is given by $\mathbf{x}_t = \mathbb{C}(\boldsymbol{\theta}) + \mathbb{D}\mathbf{v}_t$, where $\mathbf{x}_t$ is an n-dimensional vector of observed variables, $\mathbb{D}$ and $\mathbb{E}$ are selection matrices, and $\mathbb{C}(\boldsymbol{\theta})$ is a vector that is a function of the structural parameters.

We follow the Bayesian approach to estimate the model (see An and Schorfheide, 2007, for an overview). The posterior distribution associated with the vector of observables is computed numerically using a Monte Carlo Markov Chain (MCMC) sampling approach.

Specifically, we rely on the Metropolis-Hastings algorithm to obtain a random draw of size 1,000,000 from the posterior distribution of the parameters. The likelihood is based on the following vector of observable variables:

$$\mathbf{x}_t = 100 \times [\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, \Delta \log (W_t / P_t), \log N_t, \pi_t, S_t]. \quad (1)$$

where Δ denotes the temporal difference operator.

3.2.2. *Estimation results.* The benchmark model contains eighteen structural parameters, excluding the parameters relative to the exogenous shocks. We calibrate six of them: The discount factor β is set to 0.99, the capital depreciation rate δ is equal to 0.025, the parameter α in the Cobb-Douglas production function is set to 0.30 to match the average capital share in net (of fixed costs) output (McAdam and Willman, 2013), the steady-state price and wage markups ε_p and ε_w are set to 1.20 and 1.35 respectively (Everaert and Schule, 2008), and the steady-state share of government spending in output is set to 0.20 (the average value over the sample period). The remaining twelve parameters are estimated. The prior distribution is summarized in the second column of Table 5. Our choices are in line with the literature, especially with Smets and Wouters (2007), Sahuc and Smets (2008) and Justiniano, Primiceri and Tambalotti (2010). The estimation results are displayed in the right-hand side columns of Table 5, where the posterior mean and the 90% confidence interval are reported for the full sample 1980Q1-2016Q1 and a pre-crisis sample 1980Q1-2007Q4. Based on the posterior mean, several results are worth commenting on. First, the estimated model parameters associated with the full sample are very close to those associated with the pre-crisis sample, suggesting that one can apply a DSGE model to a low-interest rate environment without observing any significant structural change. An impulse response analysis corroborates that the responses to a monetary policy shock of the macroeconomic variables estimated in the full sample are consistently similar to those based on the shorter sample ending in 2007Q4 (see Figure C1 of Appendix C).

Table 5. Prior densities and posterior estimates

Parameter	Prior	Posterior			
		1980Q1-2016Q1		1980Q1-2007Q4	
		Mean	90% CI	Mean	90% CI
Habit in consumption, h	$\mathcal{B}$ [0.50,0.15]	0.93	[0.91,0.95]	0.89	[0.85,0.93]
Elasticity of labor, ν	$\mathcal{G}$ [2.00,0.75]	2.67	[1.36,3.90]	2.66	[1.37,3.88]
Capital utilization cost, η_u	$\mathcal{B}$[0.50,0.10]	0.80	[0.72,0.88]	0.78	[0.68,0.87]
Investment adj. cost, η_k	$\mathcal{G}$[4.00,1.00]	5.14	[3.57,6.64]	4.92	[3.28,6.52]
Growth rate of technology, $\log(\gamma_z)$	$\mathcal{G}$[0.40,0.05]	0.31	[0.24,0.39]	0.33	[0.26,0.40]
Calvo price, θ_p	$\mathcal{B}$ [0.66,0.10]	0.83	[0.78,0.88]	0.82	[0.76,0.87]
Calvo wage, θ_w	$\mathcal{B}$ [0.66,0.10]	0.79	[0.72,0.86]	0.71	[0.61,0.81]
Price indexation, γ_p	$\mathcal{B}$[0.50,0.15]	0.14	[0.05,0.22]	0.18	[0.06,0.30]
Wage indexation, γ_w	$\mathcal{B}$[0.50,0.15]	0.28	[0.12,0.44]	0.34	[0.15,0.54]
Monetary policy-smoothing, ϕ_s	$\mathcal{B}$[0.75,0.15]	0.86	[0.84,0.88]	0.86	[0.82,0.89]
Monetary policy-inflation, ϕ_π	$\mathcal{G}$[2.00,0.30]	1.46	[1.27,1.65]	1.64	[1.34,1.93]
Monetary policy-output growth, ϕ_y	$\mathcal{G}$[0.125,0.05]	0.22	[0.10,0.34]	0.15	[0.06,0.23]
Wage markup shock persistence, ρ_w	$\mathcal{B}$[0.75,0.15]	0.94	[0.91,0.97]	0.93	[0.90,0.97]
Intertemporal shock persistence, ρ_b	$\mathcal{B}$[0.75,0.15]	0.25	[0.15,0.35]	0.27	[0.14,0.41]
Investment shock persistence, ρ_i	$\mathcal{B}$[0.75,0.15]	0.98	[0.96,0.99]	0.86	[0.77,0.94]
Price markup shock persistence, ρ_p	$\mathcal{B}$[0.75,0.15]	0.96	[0.93,0.99]	0.76	[0.61,0.92]
Government shock persistence, ρ_g	$\mathcal{B}$[0.75,0.15]	0.99	[0.97,0.99]	0.98	[0.97,0.99]
Wage markup shock (MA part), ϱ_w	$\mathcal{B}$[0.40,0.20]	0.76	[0.65,0.87]	0.64	[0.48,0.81]
Price markup shock (MA part), ϱ_p	$\mathcal{B}$[0.40,0.20]	0.71	[0.57,0.85]	0.52	[0.34,0.71]
Wage markup shock volatility, σ_w	$\mathcal{IG}$[0.25,2.00]	0.14	[0.11,0.16]	0.14	[0.11,0.17]
Intertemporal shock volatility, σ_b	$\mathcal{IG}$[0.25,2.00]	0.09	[0.07,0.12]	0.10	[0.07,0.12]
Investment shock volatility, σ_i	$\mathcal{IG}$[0.25,2.00]	0.30	[0.21,0.39]	0.28	[0.22,0.34]
Price markup shock volatility, σ_p	$\mathcal{IG}$[0.25,2.00]	0.12	[0.10,0.15]	0.14	[0.11,0.17]
Productivity shock volatility, σ_z	$\mathcal{IG}$[0.25,2.00]	0.71	[0.64,0.78]	0.68	[0.60,0.77]
Government shock volatility, σ_g	$\mathcal{IG}$[0.25,2.00]	0.36	[0.32,0.40]	0.35	[0.31,0.40]
Monetary policy shock volatility, σ_s	$\mathcal{IG}$[0.25,2.00]	0.15	[0.13,0.16]	0.13	[0.11,0.14]

Note: This table reports the prior distribution, the mean and the 90 percent confidence interval of the estimated posterior distribution of the structural parameters.

Second, all estimated values are consistent with the bulk of contributions in the medium-scale DSGE literature. For instance, the probability that firms are not allowed to re-optimize their price is $\theta_p \approx 0.83$, implying an average duration of price contracts of about 17 months. With respect to wages, the probability of no change is $\theta_w \approx 0.79$, implying an average duration of wage contracts of about 14 months. These two probabilities are slightly above those on the pre-crisis sample indicating that the degree of nominal rigidities has increased with the crisis. All these figures are consistent with the results reported in the survey conducted by Druant, Fabiani, Kezdi, Lamo, Martins and Sabbatini (2012). Monetary policy parameters $(\phi_s, \phi_\pi, \phi_y) \approx (0.86, 1.46, 0.22)$ indicate that the systematic part of monetary policy displays gradualism and a smaller weight on inflation when focusing on the full sample than on the pre-crisis sample.

4. Quantifying the macroeconomic effects of the ECB's unconventional measures

This section presents our quantitative assessment of the actual stimulus to real activity and price and wage inflation provided by the ECB's policies since 2008, based on counterfactual simulation analysis. We find that the ECB's actions provided two boosts to the real economy, one during the recession period and the other one since 2014, with effects that have been substantial.

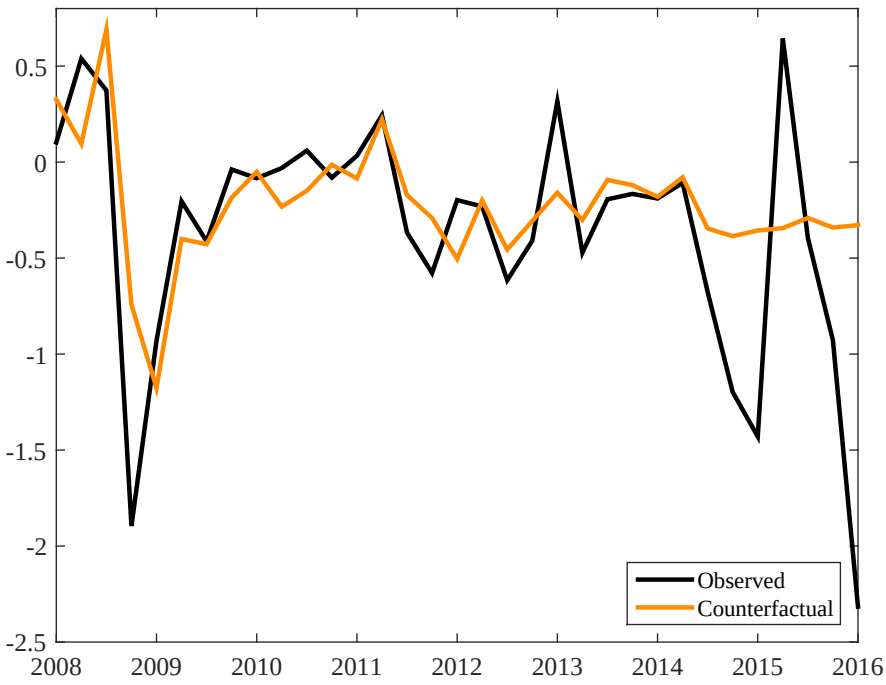
4.1. **Simulation design.**

In order to assess the state of the economy in the absence of the ECB's policies, we must build counterfactual scenarios. To this end, we proceed as follows:

(1) We take the mean of the posterior estimates of the structural parameters and compute the associated estimates of monetary policy shocks using the Kalman filter. These shocks are those from all monetary policy decisions ("observed").
(2) We then re-estimate the standard deviation of monetary policy shocks by replacing the shadow rate S_t by the usual Eonia rate R_t, all other parameters held fixed at their value obtained in step 1. These shocks are those that only come from the conventional part of monetary policy ("counterfactual").
(3) We then compute the simulated time-paths for the observed variables from the full estimated model with shadow rate using the first and second sets of monetary policy shocks.

4.2. **Baseline evaluation.** The average observed and counterfactual paths of the monetary policy shocks are illustrated in Figure 3. Major differences between the two series are visible in the early years of the financial crisis and then from 2014. Indeed, in response to the 2008-2009 crisis, faced with distressed financial intermediaries, the ECB embarked in longer-term refinancing operations (LTROs) with full allotment, with maturities of three, six, and finally twelve months in July 2009. The amounts borrowed at these facilities were substantial, roughly 5% of annual euro area GDP for 3-month LTROs, slightly less than 2% for 6-month LTROs, and about 6.5% for 12-month LTROs. Through these operations, the average maturity of outstanding liquidity was increased, from approximately 20 days before the crisis to more than 200 days in the second half of 2009. This policy was addressing funding concerns in the banking sector, in an attempt to allow banks to keep lending in spite of an acute confidence crisis. Cahn, Matheron and Sahuc (forthcoming) find that such liquidity injections have played a key role in adverting a major credit crunch in the euro area.

Figure 3. Monetary policy shocks



Since 2014, the macroeconomic climate in the euro area has been characterized by increased risks threatening price stability and the anchoring of inflation expectations. In this context, the ECB adopted a threefold response (see, Marx et al., 2016). First, there was a succession

of cuts in the deposit facility rate, from 0% in early 2014 to -0.40% in March 2016. The negative rate on the deposit facility puts a strain on the excess liquidity that banks deposit with the Eurosystem, which tends to encourage banks to lend to each other, thereby improving the flow of liquidity among banks in the euro area. These rate-cuts complemented the forward guidance policy already in place since July 2013. This forward guidance corresponds to a commitment on the future path of interest rates, so as to influence not only the short-term rates but also longer-term rates which are largely determined by expectations of future short-term rates. Second, in order to increase support for lending, a targeted longer-term refinancing operations (TLTRO) program, with attractive associated interest rates over a period of two years, has been implemented in July 2014. The objective of TLTROs was to (i) encourage banks to lend more to non-financial corporations and to households and (ii) send a signal about future short-term rates, since loans were allotted fully and at a fixed rates (with early repayment possible without penalty). Third, public and private sector asset purchase programs have been conducted. In October 2014, the Eurosystem launched a first package of quantitative easing in the form of a dual purchase program of private sector assets aimed at promoting high-quality securitization and reducing the risk premium inflating the lending rates to NFCs. From September 2014, a target size for the balance sheet of the Eurosystem was specified, indicating that the ECB intended to return to the levels prevailing in early 2012, i.e. a balance of EUR 3,000 billion, equivalent to around 30% of euro area GDP (against EUR 2,000 billion at the end of the third quarter of 2014). In January 2015, the ECB decided to expand the previous asset purchase program to include public sector securities. The monthly purchases of public and private sector securities under this expanded asset purchase program were carried out between March 2015 and March 2016 for a total amount of EUR 60 billion per month.

In December 2015, the asset purchase program was extended until at least March 2017. In March 2016 the ECB announced a new extension of the program, including an increase in the monthly amount of purchases under the asset purchase program from EUR 60 billion to EUR 80 billion, the inclusion of investment grade bonds issued by NFCs in the scope of the asset purchase program, and a series of four targeted longer-term refinancing operations was launched: the TLTRO II.

Figure 4. Observed series and counterfactual estimate

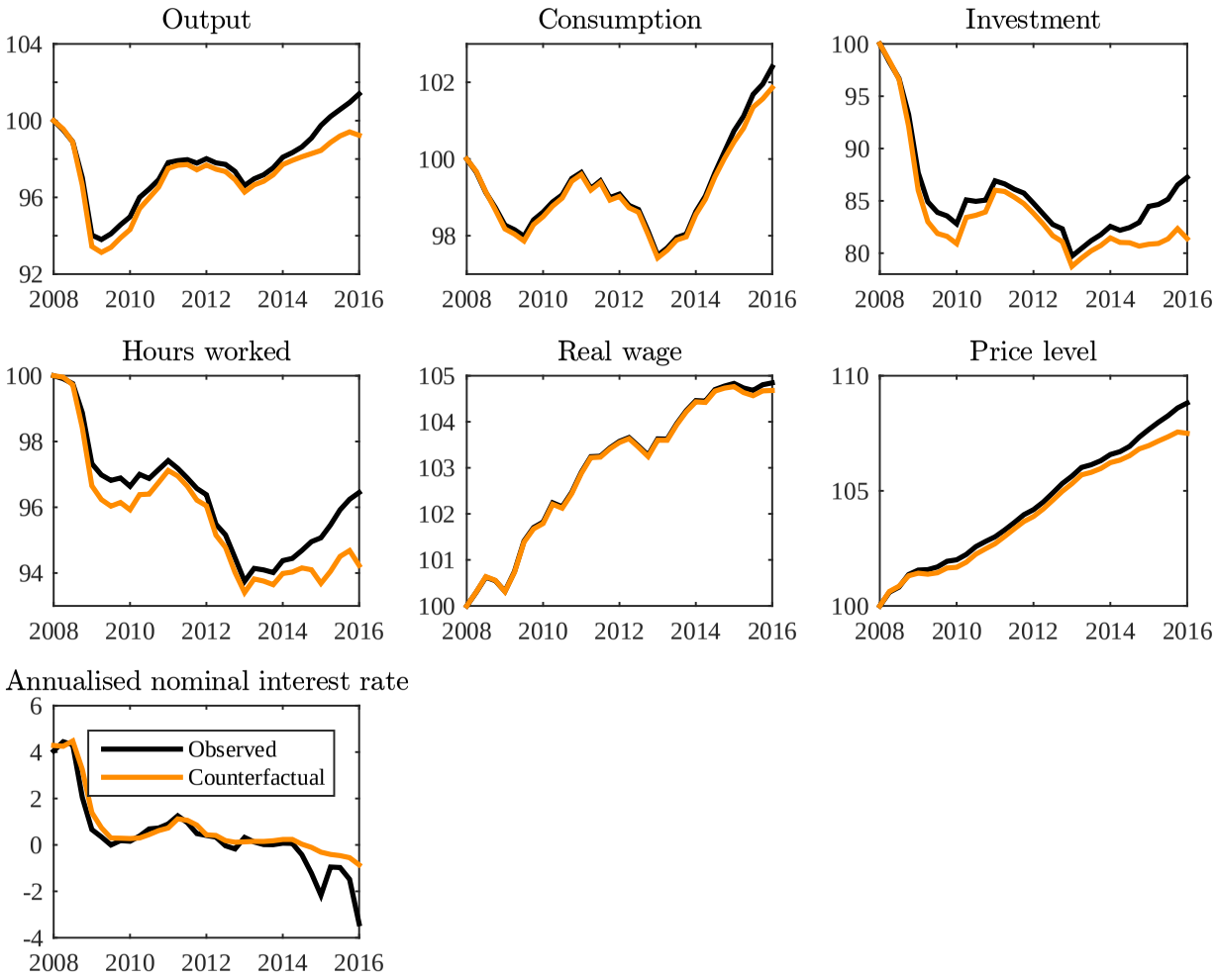
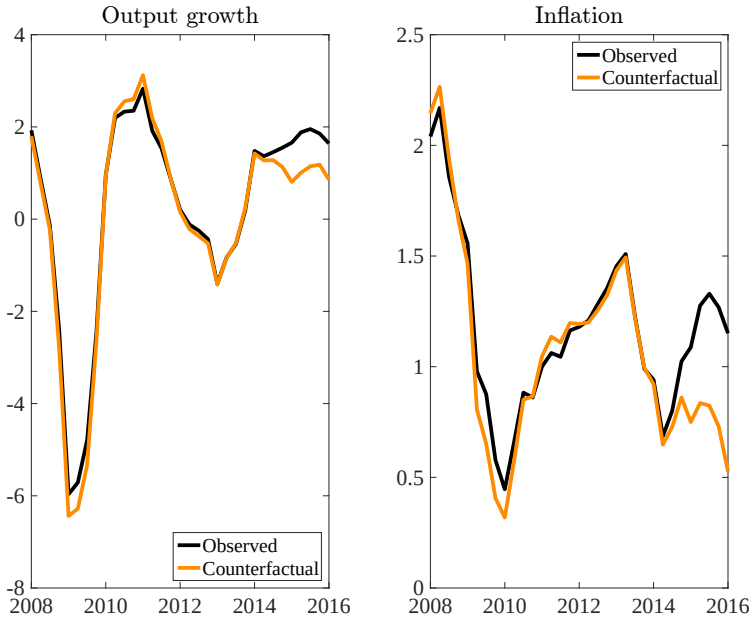


Figure 4 plots the levels of the observed variables and their counterfactual estimates. In order to facilitate reading, all paths have been normalized to 100 in 2008Q1, with the exception of the interest rate which starts at its historical value. We clearly show that, without unconventional monetary policy measures, all macroeconomic variables would have been lower. Our results, over the 2008Q1-2016Q1 period, imply a cumulative loss of output of around 19% of its pre-crisis level.

The bulk of this effect stems from the large decline in investment (whose cumulated loss reaches 58%). The difference in the price level is more modest (around 9%). The muted effect of QE on inflation, relative to GDP, is corroborated by Andrade, Breckenfelder, De Fiore, Karadi and Tristani (2016) and Sahuc (2016). More importantly, we note that unconventional measures have helped avoid (i) deflation episodes in 2009Q2 and 2016Q1 and (ii) a slowdown in price increases in 2015.

Figure 5. Year-on-year output growth and inflation rates



This translates into year-on-year (y-o-y) inflation and GDP growth differentials of 0.1% and 0.2% on average over the period 2008Q1-2016Q1, respectively. Drawing attention on the period 2014Q1-2016Q1, when public and private sector asset purchase programs have been announced and conducted, y-o-y inflation and GDP growth would have been lower by 0.25% and 0.56%, respectively (Figure 5). Gauging the impact of unconventional monetary policies depends of which shocks are driving fluctuations.

The historical contribution of the different types of shocks to GDP growth and inflation show that, although the dominant source of secular shifts in inflation is driven by price and wage markup shocks, monetary policy plays a significant positive role over the 2008-2016 period (see Figure D1 of Appendix D).

Using an estimated two-region DSGE model, that combines data on government debt stocks and yields across maturities, Hohberger, Priftis and Vogel (2017) find similar effects. Specifically, between 2015Q1 and 2016Q2, their shock decompositions suggest a positive average contribution of the ECB asset purchases to output growth and inflation ranging between 0.3% and 0.7% and 0.3% and 0.4%, respectively, depending on the presence of a temporarily binding zero-bound constraint. On the same period, we obtain a gain of 0.86% for output growth and 0.4% for inflation. These differences may be due to other parallel measures, such as TLTROs or forward guidance.

4.3. **Robustness.** Naturally, there is uncertainty underlying any estimate of the efficacy of the ECB's unconventional measures. Some of this uncertainty is associated with the measure of the shadow rate itself, as it is deduced from a statistical model and is not directly observed. Therefore, we compare our results to those obtained using four alternative shadow rates that are available for the euro area, proposed by Kortela (2016), Krippner (2016), Lemke and Vladu (2016), and Wu and Xia (2016). Their specifications typically vary in terms of the number of factors explaining the yield curve and in terms of the lower bound imposed on short-term rates. Kortela (2016) incorporates a time-varying lower bound for nominal interest rates in the shadow rate model, in order to take account for the recent changes of the deposit facility rate into negative territory. Krippner (2016) uses a two-factor shadow-rate model with a fixed 12.5 basis-point lower bound.[6] Lemke and Vladu (2016) use a shadow-rate model that allows for several shifts in the lower bound (they ultimately retain two deterministic sub-periods). Wu and Xia (2016) propose an approximation which renders non-linear term structure models highly tractable. In their euro-area analysis, they set the lower bound equal to the deposit facility rate when the latter goes into negative territory.

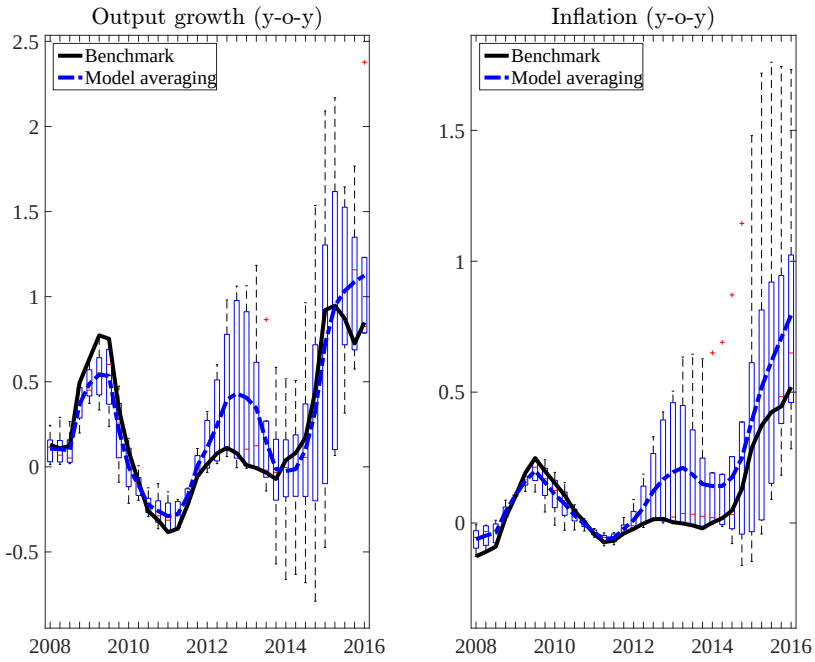
Although all shadow rates show similar trends, there may be differences in level.[7] The reason is the following. By construction, a shadow rate absorbs the extent to which the yield

[6] See also Halberstadt and Krippner (2016) for an application of this indicator to study its relationships with prices and output developments across conventional and unconventional environments.

[7] The series are displayed on Figure E1 of Appendix E.

curve is constrained by the lower bound. Therefore, the higher the lower bound, the more binding the constraint is and thus the lower the shadow rate. Vice versa, when the lower bound is set to a smaller value (and enters into negative territory), this loosens the constraint and the shadow rate responds by increasing.

Figure 6. Unconventional policy gains from alternative shadow rates *(percent)*



Note: The black line represents the percentage gain when using our benchmark shadow rate. The dotted line represents the percentage gain computed in averaging the results obtained with the five shadow rates. On each box, the central mark indicates the median, and the bottom and top edges of the box indicate the 25th and 75th percentiles, respectively. The outliers are plotted individually using the '+' symbol.

We reproduce the same exercise as in the previous section for each shadow-rate measure. This allows us to have a distribution of the gains made by unconventional policies across the five indicators. A standardized way of displaying such a distribution is to use a non-parametric box plot that graphically depicts the gains through their quartiles. Figure 6 shows box plots for y-o-y output growth and y-o-y inflation rates and provides three interesting results. First, the distribution can be quite large when shadow rates fall into negative territory, with deviations up to 1% for output growth and 0.5% for inflation. Second, we note that the shadow rate proposed in this paper ("benchmark") is in the low range of evaluations, in other words our results are rather conservative. Third, to account for the uncertainty surrounding

the shadow rate, an econometrician could prefer using an average of the results obtained with each of the available indicators ("model averaging"). In this case, we see that the gains are slightly larger than those obtained using our indicator. In particular, over the period 2014Q1-2016Q1, y-o-y inflation and GDP growth would have been lower by 0.4% and 0.6%, respectively.

Table 6. Cumulative loss (2008Q1-2016Q1)

Variable	Measure				
	Benchmark	Kortela	Krippner	Lemke-Vladu	Wu-Xia
Output	19.44	18.74	37.57	7.36	21.46
Consumption	2.62	2.60	7.73	1.11	7.12
Investment	58.76	56.64	108.88	22.11	57.01
Hours worked	20.35	19.25	36.16	7.39	18.25
Real wage	2.57	2.86	7.64	1.23	5.82
Price level	8.84	9.53	23.57	5.50	17.82

Note: The cumulative loss associated with the variable x_t is $\sum \left(\frac{x_t^c}{x_t^o} - 1 \right)$, where x_t^o is the observed level and x_t^c is the counterfactual.

In addition, Table 6 displays the quantification of unconventional monetary policies for all macroeconomic variables and all alternative shadow rates as measured by the cumulative losses over the crisis period. It confirms that our indicator is in the low end of the estimates. It also shows that our findings are close to those obtained with Kortela (2016)'s measure (the correlation between the two measures is 0.97), while Wu and Xia (2016)'s output is the one that least correlates with our measure (0.90).[8]

5. Concluding remarks

In this paper, we estimate a medium-scale DSGE model in which the policy rate is replaced by a shadow rate, and perform counterfactual analyses. This allows us to quantify the macroeconomic effects of the European Central Bank's unconventional monetary policies. Overall, our results suggest that, without unconventional measures, the euro area would have suffered *(i)* a cumulative loss of output of around 19% of its pre-crisis level since the

[8]This can possibly be explained by the fact that shadow rates have been reported to be sensitive to the model specification and data used (see Christensen and Rudebusch, 2015; Bauer and Rudebusch, 2016).

Great Recession, *(ii)* deflation episodes in 2009Q1 and 2016Q1 and *(iii)* a slowdown in price increases in 2015 and 2016. This translates into year-on-year inflation and GDP growth differentials of 0.25% and 0.56%, respectively, over the period 2014Q1-2016Q1.

Our analysis also highlights that we can still use standard DSGE models in low interest rate environments when using an unconstrained proxy of the monetary policy stance such as the shadow rate. Indeed, the introduction of the shadow rate allows us to bypass problems associated with the ELB, especially when the latter varies over time as in the euro area. It makes the approach appealing from a policy point of view because evaluations can be easily updated with traditional tools.

However, there are many extensions to this current work, both from a modeling and an econometric perspective. In particular, endogenously deriving the shadow rate within the structural model, by attempting to directly introduce a term structure into the model (along the lines of Ang and Piazzesi, 2003; Hordahl et al., 2006) would be a promising step. The complexity is that the shadow rate block remains nonlinear and therefore solving and estimating the model as a whole is not trivial.

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APPENDIX A: EVIDENCE ON THE SHADOW RATE

Figure A1. Monetary policy measures in the euro area and the shadow rate

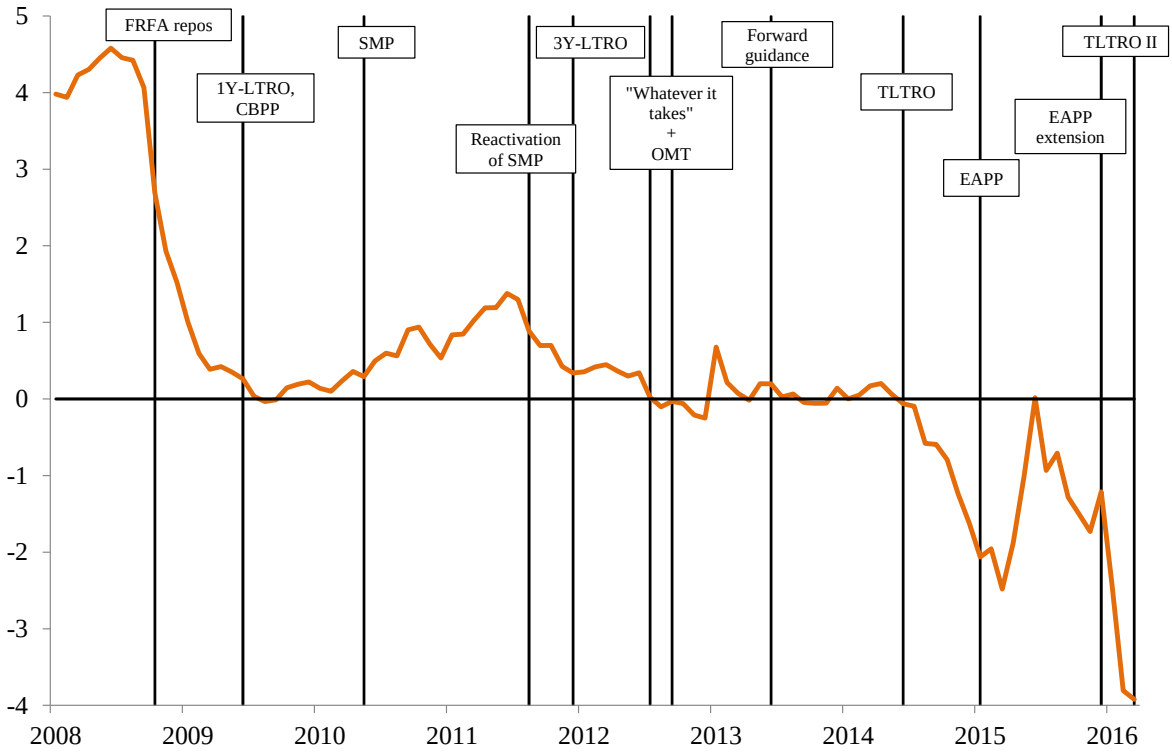
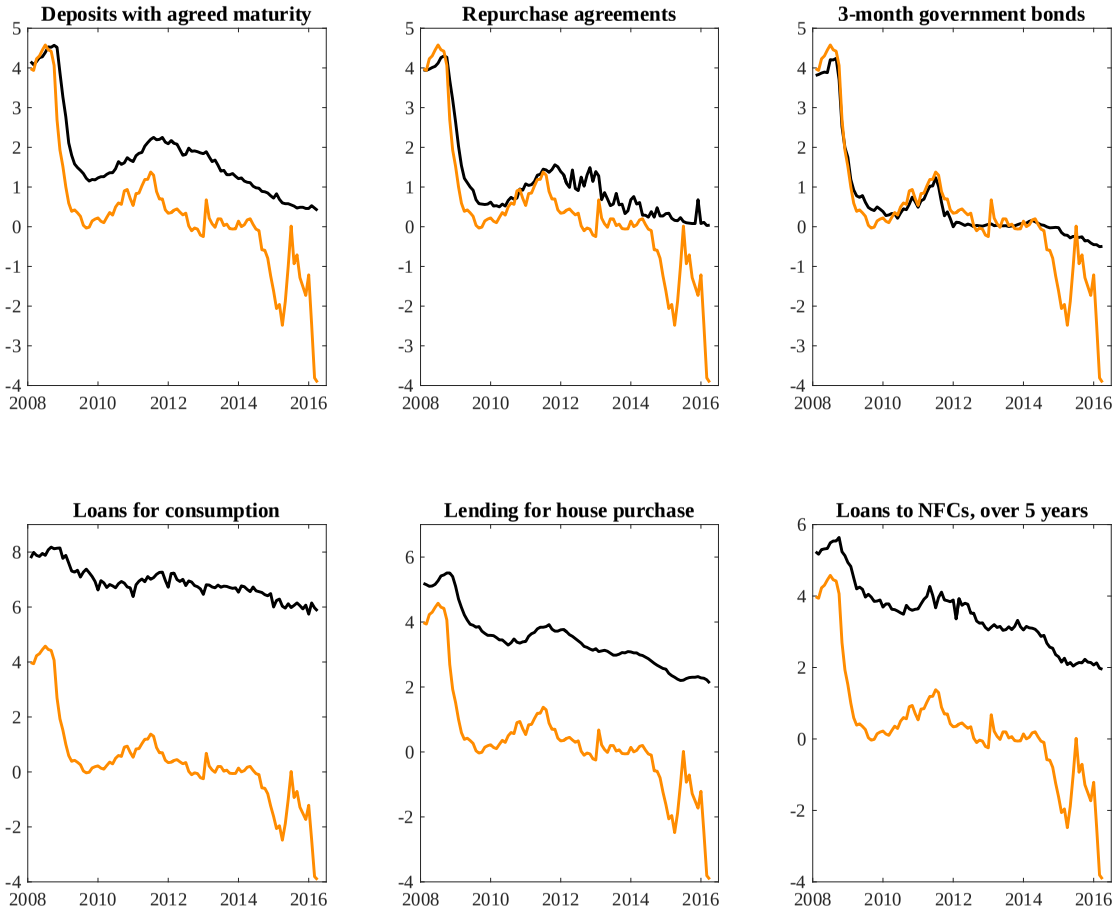


Figure A2. Private and public interest rates and the shadow rate



Note: The orange line is the shadow rate and the black line represents the interest rate indicated in the title of each panel. Deposits and loans are new business. Deposits with agreed maturity are mainly time deposits with a given maturity that may be subject to the payment of a penalty in the event of early withdrawal. Deposits with an agreed maturity of up to two years are included in M2. Repurchase agreement (or "repo") refers to the sale of a security on one date with an agreement for the seller to buy it back on a later date. All rates come from the Statistical Data Warehouse of the ECB.

APPENDIX B: MACROECONOMIC MODEL: EQUILIBRIUM CONDITIONS, STEADY-STATE AND LOG-LINEARIZATION

B.1. Equilibrium conditions

This section reports the first-order conditions for the agents' optimizing problems and the other relationships that define the equilibrium of the model.

Effective capital:

$$K_t = u_t \bar{K}_{t-1}$$

Capital accumulation:

$$\bar{K}_t = (1-\delta)\,\bar{K}_{t-1} + \varepsilon_{i,t}\left(1 - \Psi\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$$

Marginal utility of consumption:

$$\Lambda_t = \frac{\varepsilon_{b,t}}{C_t - hC_{t-1}} - \beta h \mathrm{E}_t\left\{\frac{\varepsilon_{b,t+1}}{C_{t+1} - hC_t}\right\}$$

Consumption Euler equation:

$$\Lambda_t = \beta S_t \mathrm{E}_t\left\{\Lambda_{t+1}\frac{P_t}{P_{t+1}}\right\}$$

Investment equation:

$$1 = Q_t\varepsilon_{i,t}\left[1 - \Psi\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}}\Psi'\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta \mathrm{E}_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1}\varepsilon_{i,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 \Psi'\left(\frac{I_{t+1}}{I_t}\right)\right\}$$

Tobin's Q:

$$Q_t = \beta \mathrm{E}_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\left[\frac{R^k_{t+1}}{P_{t+1}}u_{t+1} - \vartheta\left(u_{t+1}\right) + (1-\delta)\,Q_{t+1}\right]\right\}$$

Capital utilization:

$$R^k_t = P_t \vartheta'\left(u_t\right)$$

Production function:

$$Y_{i,t} = {K_{i,t}}^{\alpha}\left[Z_t N_{i,t}\right]^{1-\alpha} - Z_t\Omega$$

Labor demand:

$$W_t = (1-\alpha)\,Z_t\left(\frac{K_t}{Z_t N_t}\right)^{\alpha} MC_t$$

where MC_t is the nominal marginal cost.
Capital renting:

$$R^k_t = \alpha\left(\frac{K_t}{Z_t N_t}\right)^{\alpha-1} MC_t$$

Price setting:

$$\mathrm{E}_t \sum_{s=0}^{\infty}\left(\beta\theta_p\right)^s \frac{\Lambda_{t+s}}{\Lambda_t} Y^{\star}_{t,t+s}\left[P^{\star}_t \Pi^p_{t,t+s} - \varepsilon_{p,t+s} MC_{t+s}\right] = 0$$

Aggregate price index:

$$P_t = \left[\left(1-\theta_p\right) \left(P_t^\star\right)^{1/\left(\varepsilon_{p,t}-1\right)} + \theta_p \left(\pi^{1-\gamma_p} \pi_{t-1}^{\gamma_p} P_{t-1} \right)^{1/\left(\varepsilon_{p,t}-1\right)} \right]^{\left(\varepsilon_{p,t}-1\right)}$$

Wage setting:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \left(\beta\theta_w\right)^s \Lambda_{t+s} N_{t,t+s}^\star \left[\frac{W_t^\star}{P_{t+s}} \Pi_{t,t+s}^w - \varepsilon_{b,t+s} \varepsilon_{w,t+s} \frac{\left(N_{t,t+s}^\star\right)^\nu}{\Lambda_{t+s}} \right] = 0$$

Aggregate wage index:

$$W_t = \left[\left(1-\theta_w\right) \left(W_t^\star\right)^{1/\left(\varepsilon_{w,t}-1\right)} + \theta_w \left(\gamma_z \pi^{1-\gamma_w} \pi_{t-1}^{\gamma_w} W_{t-1} \right)^{1/\left(\varepsilon_{w,t}-1\right)} \right]^{\left(\varepsilon_{w,t}-1\right)}$$

Government spending:

$$G_t = \left(1 - \frac{1}{\varepsilon_{g,t}}\right) Y_t$$

Monetary policy rule:

$$\frac{S_t}{S} = \left(\frac{S_{t-1}}{S}\right)^{\varphi_s} \left[\left(\frac{\pi_t}{\pi}\right)^{\varphi_\pi} \left(\frac{Y_t}{\gamma_z Y_{t-1}} \right)^{\varphi_y} \right]^{(1-\varphi_s)} \varepsilon_{s,t}$$

Resource constraint:

$$Y_t = C_t + I_t + G_t + \vartheta\left(u_t\right) \bar{K}_{t-1}$$
$$\Delta_{p,t} Y_t = \left(u_t \bar{K}_{t-1}\right)^\alpha \left[Z_t N_t\right]^{1-\alpha} - Z_t \Omega$$

B.2. Stationary equilibrium

To find the steady state, we express the model in stationary form. Thus, for the non-stationary variables, let lower-case notations denote their value relative to the technology process Z_t:

$$\begin{array}{llll} y_t \equiv Y_t / Z_t & k_t \equiv K_t / Z_t & \bar{k}_t \equiv \bar{K}_t / Z_t & i_t \equiv I_t / Z_t \qquad c_t \equiv C_t / Z_t \\ g_t \equiv G_t / Z_t & \lambda_t \equiv \Lambda_t Z_t & w_t \equiv W_t / \left(Z_t P_t\right) & w_t^\star \equiv W_t^\star / \left(Z_t P_t\right) \end{array}$$

where we note that the marginal utility of consumption Λ_t will shrink as the economy grows, and we express the wage in real terms. Also, we denote the real rental rate of capital and real marginal cost by

$$r_t^k \equiv R_t^k / P_t \text{ and } mc_t \equiv MC_t / P_t,$$

and the optimal relative price as

$$p_t^\star \equiv P_t^\star / P_t.$$

Then we can rewrite the model in terms of stationary variables as follows.
Effective capital:

$$k_t = \frac{u_t \bar{k}_{t-1}}{\varepsilon_{z,t}}$$

Capital accumulation:

$$\bar{k}_t = \left(1-\delta\right) \frac{\bar{k}_{t-1}}{\varepsilon_{z,t}} + \varepsilon_{i,t} \left(1 - \Psi\left(\frac{i_t}{i_{t-1}} \varepsilon_{z,t} \right) \right) i_t$$

Marginal utility of consumption:

$$\lambda_t = \frac{\varepsilon_{b,t}}{c_t - h\frac{c_{t-1}}{\varepsilon_{z,t}}} - \beta h \mathrm{E}_t \left\{ \frac{\varepsilon_{b,t+1}}{\varepsilon_{z,t+1}\left(c_{t+1} - h\frac{c_t}{\varepsilon_{z,t+1}}\right)} \right\}$$

Consumption Euler equation:

$$\lambda_t = \beta R_t \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\varepsilon_{z,t+1}\pi_{t+1}} \right\}$$

Investment equation:

$$1 = q_t \varepsilon_{i,t} \left[1 - \Psi\left(\frac{i_t}{i_{t-1}}\varepsilon_{z,t}\right) - \frac{i_t}{i_{t-1}}\varepsilon_{z,t}\Psi'\left(\frac{i_t}{i_{t-1}}\varepsilon_{z,t}\right) \right] + \beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t \varepsilon_{z,t+1}} q_{t+1}\varepsilon_{i,t+1} \left(\frac{i_{t+1}}{i_t}\varepsilon_{z,t+1}\right)^2 \Psi'\left(\frac{i_{t+1}}{i_t}\varepsilon_{z,t+1}\right) \right\}$$

Tobin's Q:

$$q_t = \beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t \varepsilon_{z,t+1}} \left[r^k_{t+1} u_{t+1} - \vartheta\left(u_{t+1}\right) + \left(1-\delta\right) q_{t+1} \right] \right\}$$

Capital utilization:

$$r^k_t = \vartheta'\left(u_t\right)$$

Production function:

$$y_{i,t} = k^{\alpha}_{i,t} N^{1-\alpha}_{i,t} - \Omega$$

Labor demand:

$$w_t = \left(1-\alpha\right)\left(\frac{k_t}{N_t}\right)^{\alpha} mc_t$$

Capital renting:

$$r^k_t = \alpha\left(\frac{k_t}{N_t}\right)^{\alpha-1} mc_t$$

Price setting:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \left(\beta\theta_p\right)^s \frac{\lambda_{t+s}}{\lambda_t} y^{\star}_{t,t+s} \left[p^{\star}_t \frac{P_t}{P_{t+s}} \Pi^p_{t,t+s} - \varepsilon_{p,t+s} mc_{t+s} \right] = 0$$

Aggregate price index:

$$1 = \left[\left(1-\theta_p\right)\left(p^{\star}_t\right)^{1/\left(\varepsilon_{p,t}-1\right)} + \theta_p \left(\pi^{1-\gamma_p} \pi^{\gamma_p}_{t-1} \frac{1}{\pi_t} \right)^{1/\left(\varepsilon_{p,t}-1\right)} \right]^{\left(\varepsilon_{p,t}-1\right)}$$

Wage setting:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \left(\beta\theta_w\right)^s \lambda_{t+s} N^{\star}_{t,t+s} \left[w^{\star}_t \frac{P_t}{P_{t+s}} \frac{Z_t}{Z_{t+s}} \Pi^w_{t,t+s} - \varepsilon_{b,t+s}\varepsilon_{w,t+s} \frac{N^{\nu}_{t,t+s}}{\lambda_{t+s}} \right] = 0$$

Aggregate wage index:

$$w_t = \left[\left(1-\theta_w\right)\left(w^{\star}_t\right)^{1/\left(\varepsilon_{w,t}-1\right)} + \theta_w \left(\gamma_z \pi^{1-\gamma_w} \pi^{\gamma_w}_{t-1} \frac{w_{t-1}}{\pi_t \varepsilon_{z,t}} \right)^{1/\left(\varepsilon_{w,t}-1\right)} \right]^{\left(\varepsilon_{w,t}-1\right)}$$

Government spending:

$$g_t = \left(1 - \frac{1}{\varepsilon_{g,t}}\right) y_t$$

Monetary policy rule:

$$\frac{S_t}{S} = \left(\frac{S_{t-1}}{S}\right)^{\varphi_s} \left[\left(\frac{\pi_t}{\pi}\right)^{\varphi_\pi} \left(\frac{\varepsilon_{z,t} y_t}{\gamma_z y_{t-1}}\right)^{\varphi_y}\right]^{(1-\varphi_r)} \varepsilon_{s,t}$$

Resource constraint:

$$y_t = c_t + i_t + g_t + \vartheta\left(u_t\right) \bar{k}_{t-1} / \varepsilon_{z,t}$$
$$\Delta_{p,t} y_t = \left(u_t \bar{k}_{t-1}\right)^{\alpha} N_t^{1-\alpha} - \Omega$$

B.3. Steady state

We use the stationary version of the model to find the steady state, and we let variables without a time subscript denote steady-state values. First, we have that $S = \left(\gamma_z \pi\right) / \beta$ and the expression for Tobin's Q implies that the rental rate of capital is

$$r^k = \frac{\gamma_z}{\beta} - (1 - \delta)$$

and the price-setting equation gives marginal cost as

$$mc = \frac{1}{\varepsilon_p}.$$

The capital/labor ratio can then be retrieved using the capital renting equation:

$$\frac{k}{N} = \left(\alpha \frac{mc}{r^k}\right)^{1/(1-\alpha)},$$

and the wage is given by the labor demand equation as

$$w = (1 - \alpha)\, mc \left(\frac{k}{N}\right)^{\alpha}.$$

The production function gives the output/labor ratio as

$$\frac{y}{N} = \left(\frac{k}{N}\right)^{\alpha} - \frac{\Omega}{N},$$

and the fixed cost Ω is set to obtain zero profits at the steady state, implying

$$\frac{\Omega}{N} = \left(\frac{k}{N}\right)^{\alpha} - w - r^k \frac{k}{N}.$$

The output/labor ratio is then given by

$$\frac{y}{N} = w + r^k \frac{k}{N} = \frac{r^k}{\alpha} \frac{k}{N}.$$

Finally, to determine the investment/output ratio, we use the expressions for effective capital and physical capital accumulation to get

$$\frac{i}{k} = \left(1 - \frac{1 - \delta}{\gamma_z}\right) \gamma_z \Longrightarrow \frac{i}{y} = \frac{i}{k} \frac{k}{N} \frac{N}{y} = \left(1 - \frac{1 - \delta}{\gamma_z}\right) \frac{\alpha \gamma_z}{r^k}.$$

Given the government spending/output ratio g/y, the consumption/output ratio is then given by the resource constraint as

$$\frac{c}{y} = 1 - \frac{i}{y} - \frac{g}{y}.$$

B.4. Log-linearized version

We log-linearize the stationary model around the steady state. Let $\hat{\chi}_t$ denote the log deviation of the variable χ_t from its steady-state level χ: $\hat{\chi}_t \equiv \log(\chi_t/\chi)$. The log-linearized model is then given by the following system of equations for the endogenous variables.
Effective capital:

$$\hat{k}_t + \hat{\varepsilon}_{z,t} = \hat{u}_t + \widehat{\bar{k}}_{t-1}$$

Capital accumulation:

$$\widehat{\bar{k}}_t = \frac{1-\delta}{\gamma_z}\left(\widehat{\bar{k}}_{t-1} - \hat{\varepsilon}_{z,t}\right) + \left(1 - \frac{1-\delta}{\gamma_z}\right)(\hat{\imath}_t + \hat{\varepsilon}_{i,t})$$

Marginal utility of consumption:

$$\begin{aligned}\hat{\lambda}_t = {} & \frac{h\gamma_z}{(\gamma_z - h\beta)(\gamma_z - h)}\hat{c}_{t-1} - \frac{\gamma_z^2 + h^2\beta}{(\gamma_z - h\beta)(\gamma_z - h)}\hat{c}_t + \frac{h\beta\gamma_z}{(\gamma_z - h\beta)(\gamma_z - h)}\mathrm{E}_t\hat{c}_{t+1} \\ & - \frac{h\gamma_z}{(\gamma_z - h\beta)(\gamma_z - h)}\hat{\varepsilon}_{z,t} + \frac{h\beta\gamma_z}{(\gamma_z - h\beta)(\gamma_z - h)}\mathrm{E}_t\hat{\varepsilon}_{z,t+1} \\ & + \frac{\gamma_z}{\gamma_z - h\beta}\hat{\varepsilon}_{b,t} - \frac{h\beta}{\gamma_z - h\beta}\mathrm{E}_t\hat{\varepsilon}_{b,t+1}\end{aligned}$$

Consumption Euler equation:

$$\hat{\lambda}_t = \mathrm{E}_t\hat{\lambda}_{t+1} + \left(\hat{S}_t - \mathrm{E}_t\hat{\pi}_{t+1}\right) - \mathrm{E}_t\hat{\varepsilon}_{z,t+1}$$

Investment equation:

$$\hat{\imath}_t = \frac{1}{1+\beta}(\hat{\imath}_{t-1} - \hat{\varepsilon}_{z,t}) + \frac{\beta}{1+\beta}\mathrm{E}_t(\hat{\imath}_{t+1} + \hat{\varepsilon}_{z,t+1}) + \frac{1}{\eta_k\gamma_z^2(1+\beta)}(\hat{q}_t + \hat{\varepsilon}_{i,t})$$

Tobin's Q:

$$\hat{q}_t = \frac{\beta(1-\delta)}{\gamma_z}\mathrm{E}_t\hat{q}_{t+1} + \left(1 - \frac{\beta(1-\delta)}{\gamma_z}\right)\mathrm{E}_t\hat{r}^k_{t+1} - \left(\hat{S}_t - \mathrm{E}_t\hat{\pi}_{t+1}\right)$$

Capital utilization:

$$\hat{u}_t = \frac{1-\eta_u}{\eta_u}\hat{r}^k_t$$

Production function:

$$\hat{y}_t = \frac{y+\Omega}{y}\left(\alpha\hat{k}_t + (1-\alpha)\hat{n}_t\right)$$

Labor demand:

$$\hat{w}_t = \widehat{mc}_t + \alpha\hat{k}_t - \alpha\hat{n}_t$$

Capital renting:

$$\hat{r}^k_t = \widehat{mc}_t - (1-\alpha)\hat{k}_t + (1-\alpha)\hat{n}_t$$

Phillips curve:

$$\hat{\pi}_t = \frac{\gamma_p}{1+\beta\gamma_p}\pi_{t-1} + \frac{\beta}{1+\beta\gamma_p}\mathrm{E}_t\pi_{t+1} + \frac{\left(1-\beta\theta_p\right)\left(1-\theta_p\right)}{\theta_p\left(1+\beta\gamma_p\right)}\left(\widehat{mc}_t + \hat{\varepsilon}_{p,t}\right)$$

Wage curve:

$$\hat{w}_t = \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}\mathrm{E}_t\hat{w}_{t+1} + \frac{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)}{\theta_w\left(1+\beta\right)\left(1+\nu\frac{\varepsilon_w}{\varepsilon_w-1}\right)}\left(\widehat{mrs}_t - \hat{w}_t + \hat{\varepsilon}_{w,t}\right)$$
$$+ \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}\mathrm{E}_t\hat{\pi}_{t+1} - \frac{1}{1+\beta}\hat{\varepsilon}_{z,t} + \frac{\beta}{1+\beta}\mathrm{E}_t\hat{\varepsilon}_{z,t+1}$$

Marginal rate of substitution:

$$\widehat{mrs}_t = \nu\hat{n}_t - \hat{\lambda}_t + \hat{\varepsilon}_{b,t}$$

Government spending:

$$\hat{g}_t = \hat{y}_t + \frac{1-g/y}{g/y}\hat{\varepsilon}_{g,t}$$

Monetary policy rule:

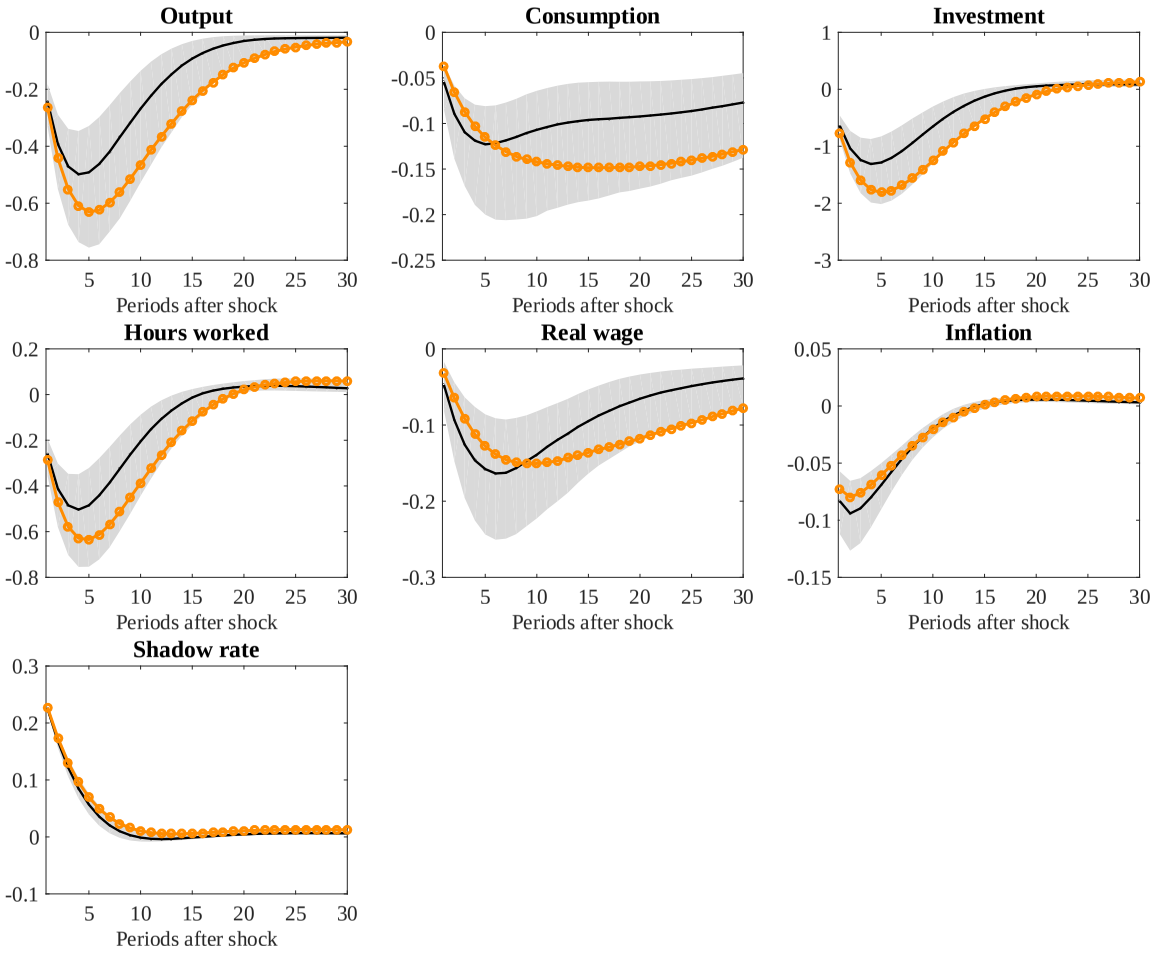
$$\hat{S}_t = \varphi_s\hat{S}_{t-1} + \left(1-\varphi_s\right)\left[\varphi_\pi\hat{\pi}_t + \varphi_y\left(\hat{y}_t - \hat{y}_{t-1} + \hat{\varepsilon}_{z,t}\right)\right] + \hat{\varepsilon}_{s,t}$$

Resource constraint:

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + \frac{g}{y}\hat{g}_t + \frac{r^k k}{y}\hat{u}_t$$

APPENDIX C: IMPULSE RESPONSE ANALYSIS

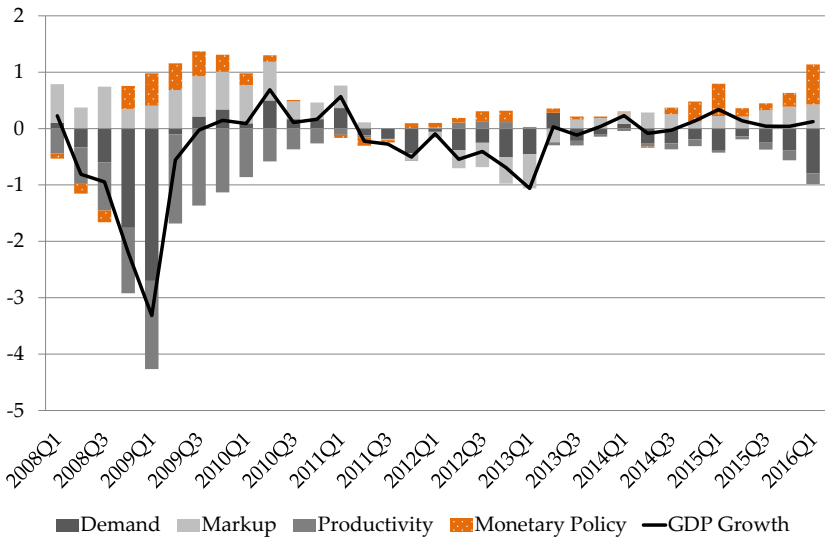
Figure C1. The impulse responses to a monetary policy shock

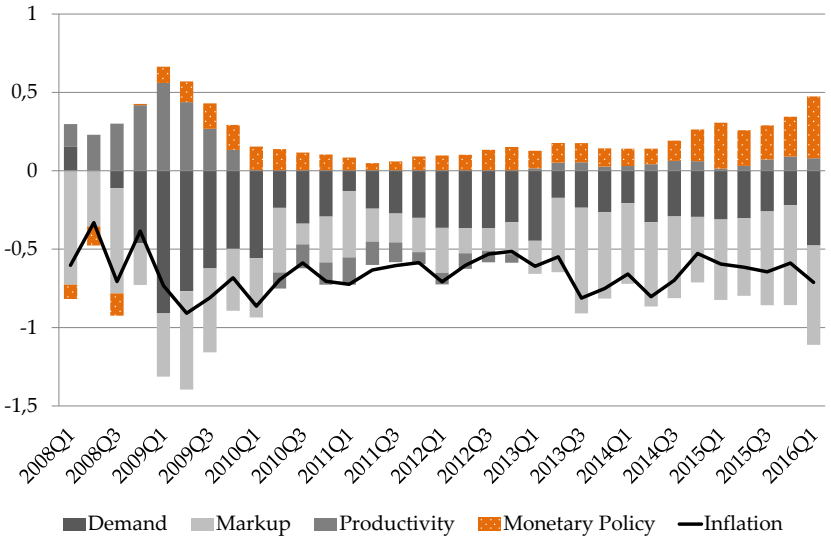


Note: The black line is the mean impulse response associated with the model estimated over the period 1980Q1-2007Q4 and the gray area is its 90 percent confidence region. The orange line is the mean impulse response associated with the model estimated over the period 1980Q1-2016Q1.

APPENDIX D: HISTORICAL DECOMPOSITION OF GDP GROWTH AND INFLATION

Figure D1. Historical decomposition of GDP growth and inflation





Note: The demand shocks include the preference, investment and government spending shocks; the markup shocks include the price and wage markup shocks. Mean inflation is estimated at 0.84 percent

APPENDIX E: ALTERNATIVE SHADOW-RATE MEASURES FOR THE EURO AREA

Figure E1. Alternative shadow rates