Extracting the relevant trends for applied portfolio management

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Abstract

As financial return series comprise relevant information about risk and dependence, historical return series describe the underlying information for applied portfolio management. Although market quotes are measured periodically, the data contains information on short-run as well as long-run trends of the underlying return series. A simulation study and an analysis of daily market prices reveal the relevance of short-run information for applied portfolio management.

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1   Introduction

Historical market prices describe the underlying information of financial risk management. Although, financial data is tracked periodically i.e. daily, weekly or monthly, historical market prices comprise information on both short- and long-run seasonalities of the underlying return series. This paper decomposes financial return series into different seasonalities and provides an assessment of the relevance of particular seasonalities for daily portfolio management.

In order to decompose financial return series into its underlying trends, Percival and Walden (2000) and Gençay et al. (2001) introduced wavelet decomposition to financial return series and triggered a growing field of literature which deals with decomposition of financial return series into short-run and long-run seasonalities.

In this context, Gençay et al. (2003) and Gençay et al. (2005) apply the Capital Asset pricing model (CAPM) to decomposed return series of international stock indices and illustrate that systematic risk increases for long-run seasonalities. Rua and Nunes (2012) confirm this finding for emerging markets whereas Gallegati (2012) decomposes return series of stock market indices of G7 countries, Brazil and Hong Kong to assess changing correlation regimes between decomposed return series. As a result, dependence schemes between decomposed return series are described by different patterns, especially dependence between long-run seasonalities appears to be stronger than suggested by the original data. Reboredo and Rivera-Castro (2014) assess dependence between decomposed European and US stock and oil prices and find evidence of increased dependence between long-run seasonalities after 2008. Dewandaru et al. (2015) analyze dependence between Asian stocks, Andries et al. (2014) between intersest rates, stock prices and exchange rates and Berger and Salah Uddin (2016) dependence between commodities, and Tan et al. (2014) analyze dependence between US and Asian equity markets.

As dependence between assets presents a crucial information in the context of periodical portfolio allocations, financial portfolios are impacted by misspecified

dependence parameters (see Fantazzini 2009). Moreover, according to Ane and Kharoubi (2003), misspecified dependence structure accounts for (up to) 20% of the overall portfolio risk.

In this vein, the wavelet decomposition of financial return series implies a decomposition of variance and covariance of the underlying return series, namely the relevant information for applied portfolio management. Consequently, the decomposition of return series leads to a decomposition of both the risk of an asset (conditional variance) and diversification effects between assets (covariance) whereas information on different seasonalities (i.e. short run, middle run and long run information) directly impact portfolio allocations. Furthermore, wavelet analysis does not only allow for a decomposition but also for a reconstruction of a decomposed financial return series and as presented by Berger and Gençay (2016), return series can be reconstructed by excluding particular seasonalities.

Triggered by the ongoing discussion on dependence structure between decomposed return series, we introduce decomposed information on long-run and short-run seasonalities to periodical portfolio management and evaluate the relevance of different seasonalities within an out-of-sample analysis. Doing so allows us to evaluate the relevance of changing dependence schemes between decomposed financial time series for applied portfolio management.

Therefore, we draw on Berger and Gençcay (2016) and apply wavelet filter to decompose return series into different components and reconstruct the return series, which enables us to exclusively take particular trends into account. That is, we reconstruct filtered return series by taking either short-run, middle-run or long-run seasonalities into account. Furthermore, we apply the reconstructed versions of the original return series to build portfolio allocations and assess their out-of-sample performance. The assessment will be twofold:

First we set up a simulation analysis, and simulate return series which are described by different patterns of long-memory effects to assess the relevance of short

and long-memory of a return series on portfolio allocations. By investigating reconstructed return series which take either short-run or long-run memory into account, we shed light on the relevant information for applied portfolio management in the absence of incomplete information on the underlying market conditions.

Second we assess the out-of-sample performance of global mean variance efficient portfolios which are based on reconstructed return series and compare the performance with the mean-variance efficient portfolio allocations based on un-decomposed daily data. To take account for different market sizes and regimes, we assess stocks that are listed at leading indices of both developed and emerging stock markets. The results indicate that middle-run and long-run information can be excluded from the original time series without impacting the out-of-sample performance of daily portfolio management.

The remainder is structured as follows. Section 2 describes the relevant methodology and the simulation study is presented in Section 3. Results regarding the empirical portfolio analysis are given in section 4 and section 5 concludes.

2 Methodology

In this section, we present methodological approach of wavelet filtering that enables us to decompose and reconstruct the underlying return series. As well, we introduce the portfolio allocation algorithm which will be applied to the reconstructed return series and relevant quality criteria of our analysis.

2.1 Maximal overlap discrete wavelet transform

In this section, we introduce the maximal overlap discrete wavelet transform (MODWT) as described by Gençay et al. (2001) and Percival and Walden (2000)[1]. MODWT approach describes an expansion of the classical approach of discrete wavelet transformation (DWT) (see Zhu et al. 2014). As the amount of observations remains constant at each level of decomposition[2] and is characterized by shift invariance, MODWT approach is predestined for a rolling window out-of-sample analysis.

As presented by Gençay et al. (2001), the choice of wavelet filter is directly linked to a scaling filter and describes the core of wavelet decomposition.

Let $h_{j,l}$ be the DWT wavelet filter with $l = 1, ..., L$ describing the length of the filter and $j = 1, ..., J$ the level of decomposition, then the corresponding scaling filter is determined by $g_{j,l}$.[3]

Further, as the MODWT filter describes an expansion of the DWT concept, the MODWT wavelet and scaling filter are directly obtained from DWT filters by:

$$\tilde{h}_{j,l} = h_{j,l}/2^{j/2}, \tag{1}$$

and

$$\tilde{g}_{j,l} = g_{j,l}/2^{j/2}. \tag{2}$$

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[1]In contradiction to Fourier Analysis, the decomposition of a return series via wavelet approach events can still be localized throughout the decompositions.

[2]Due to boundary conditions, only the observations at the beginning of each series get reduced.

[3]The respective scaling (low pass) filter $g_{j,l}$, depends on $h_{j,l}$ by Quadratic mirror filtering and is given by $g_l = -1^l h_l$.

In this vein, as the underlying data of this study is described by daily return series, $r = \{r_t, \ t = 0, 1, 2, ..., N-1\}$, to decompose the series into J frequencies, wavelet coefficients of level j are achieved by the convolution of r and the MODWT filters (see Percival and Walden 2000) :

$$\widetilde{W}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} r_{t-l \ mod \ N}, \tag{3}$$

and

$$\widetilde{V}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} r_{t-l \ mod \ N}. \tag{4}$$

with $L_j = (2^j - 1)(L - 1) + 1$. According to the presented MODWT, wavelet coefficients at all scales are characterized by the same amount of observations as the original return series r and can be expressed in matrix notation :

$$\widetilde{W}_{\mathbf{j}} = \tilde{\omega}_j r \tag{5}$$

and

$$\widetilde{V}_{\mathbf{j}} = \tilde{v}_j r. \tag{6}$$

As we aim to utilize MODWT for an out-of-sample portfolio application, we are interested in the maximum number of the boundary-free coefficients. Therefore, we use the Haar filter which has the smallest number of coefficients leading to $\tilde{h}_{1,0} = \frac{1}{2}, \ \tilde{h}_{1,1} = -\frac{1}{2}$ and $\tilde{g}_{1,0} = \frac{1}{2}, \ \tilde{g}_{1,1} = \frac{1}{2}$ for $j = 1$.

Due to the fact, that our study aims at the exclusion of particular seasonalities from the original return series, we make use of the properties of wavelet analysis that allow for a reconstruction of the decomposed series. Thus, based on the DWT specific concept of multi-resolution analysis (MRA), the underlying original return series can be reconstructed by simply summing up all coefficients and the smoothed

version of decomposition step J:

$$r = \sum_{j=1}^{J} \tilde{\omega}_j^T \widetilde{W}_j + \tilde{v}_J^T \tilde{V}_J = \sum_{j=1}^{J} \widetilde{D}_j + \tilde{S}_J. \tag{7}$$

In this setup $\widetilde{D}_j = \tilde{\omega}_j^T \widetilde{W}_j$ describes the detail coefficients and $\tilde{S}_J = \tilde{v}_J^T \tilde{V}_J$ the corresponding smoothed version of the return series. Further, $\widetilde{D}_j$ deals as the local details of the trend at level j and captures the short term dynamics (low levels) of the original return series whereas long-term fluctuations are described by high levels. Consequently, $\tilde{S}_J$ is defined as the smoothed version of the time series.

Based on the introduced setup, which allows for a decomposition and reconstruction of the underlying return series, we follow Berger and Gençay (2016) and reconstruct decomposed return series by excluding particular levels of decomposition.

Hence, after a return series is decomposed into J scales, we reconstruct the series by excluding the highest scales that comprise the long-run information of the underlying return series. More concretely, in our analysis, we decompose every series eight times and discuss three different reconstructed versions of the original return series[4]:

$$r_{SR} = \sum_{j=1}^{3} \widetilde{D}_j, \tag{8}$$

$$r_{MR} = \sum_{j=4}^{5} \widetilde{D}_j, \tag{9}$$

$$r_{LR} = \sum_{j=6}^{8} \widetilde{D}_j. \tag{10}$$

Consequently, based on eight decomposition levels of the original return series, we achieve the reconstructed return series that exclusively comprises short-run information (r_{SR}), by summing up the relevant detail coefficients $\widetilde{D}_1, \widetilde{D}_2, \widetilde{D}_3$. r_{MR} the middle-run trend and r_{LR} the long-run trend are constructed in the same way. For a thorough introduction to MODWT in the context of financial data we refer to

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[4]We refer to Table 1 for an economic discussion of the applied setup.

Gençay et al. (2001) and for an intuitive economic introduction to wavelet wavelet analysis we refer to Crowley (2007).

2.2 Portfolio allocation

Based on historical financial return series, as well as its filtered versions (see equations (8)-(9)), we introduce the competing versions of the original return series (comprising short run, middle run and long run respectively), to applied portfolio management.

Due to the focus on the return series, we apply the covariance matrices of the reconstructed return series to the widely accepted Markowitz portfolio optimization setup (see Markowitz 1952) and assess the global minimum-variance allocation, whereas we restrict our analysis to the absence of short sellings:

$$\min_{w_t}\ w_t^T H_t w_t \quad s.t.\ 1_N^T w_t = 1. \tag{11}$$

In this setup, only the estimate of the covariance matrix (H_t) of the underlying series (either original or reconstructed) impacts portfolio allocations. As this strategy ignores expected returns, differences between portfolio allocations are directly linked to differences in the underlying covariance matrices[5].

In order to discuss the relevance of the underlying covariance matrices which contain information on different memories, we assess the out-of-sample performance of the global mean variance efficient portfolio allocation by several backtesting criteria.

Analogous to the study of De Miguel et al. (2009), we also evaluate the out-of-sample returns by different performance metrics.

In order to compare portfolio allocations which aim at minimizing particular trends, portfolio allocations we define allocations based on un-decomposed data as the benchmark and assess the information ratio of strategy k against a benchmark b

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[5]Berger (2016) initiated the application of decomposed return series to applied portfolio management.

strategy. As presented by Grinold and Kahn (2000) an adequate information ratio is given as follows:

$$IR_k = \frac{\frac{1}{n}\sum(r_k - r_b)}{\hat{\sigma}_{TE}} = \frac{\hat{\mu}}{\hat{\sigma}_{TE}}. \tag{12}$$

Here, r_k and r_b describe the vector of portfolio returns for strategy k and b respectively and $\hat{\sigma}_{TE}$ describes the standard deviation of the tracking error (TE), i.e. portfolio return relative to benchmark returns (see .

To assess the risk adjusted out-of-sample returns, we build the out-of-sample Sharpe ratio of strategy k

$$SR_k = \frac{\hat{\mu}_k}{\hat{\sigma}_k}. \tag{13}$$

Here, $\hat{\mu}_k$ describes the out-of-sample returns generated by strategy k divided by their sample standard deviation $\hat{\sigma}_k$.[6] To add to the Sharpe ratio performance, especially in case of negative average returns, we assess two alternative measures which add to the information provided the Sharpe ratio. As introduced by Sortino (1991), we assess the Sortino ratio as a natural extension of the Sharpe ratio:

$$SoR_k = \frac{\hat{\mu}_k}{\sqrt{\frac{1}{n}\sum_{t=1}^{T}(min(r_k,0))^2}}. \tag{14}$$

Additionally, as described by Shadwick and Keating (2002), we take account for the Ω-ratio, to capture the information in the higher moments of return distribution:

$$OR_k = \frac{\int_0^\infty (1 - F(r_k))dr_k}{\int_\infty^0 F(r_k)dr_k} \tag{15}$$

As we deal with daily prices, we set the threshold to 0 which leads us to distinguish between upside and downside potential.

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[6]Please note, that the information ratio and the Sharpe ratio of an Asset versus a riskless benchmark are equivalent.

3 Simulation analysis

To assess the impact of long-run seasonalities on portfolio allocations, we set up a simulation analysis that allows us to control for the existence of long-memory effects of the underlying return series. Therefore, we simulate daily return series which are characterized by different memory regimes (short run, middle run and long run) to assess the performance of portfolio allocations that take particular seasonalities into account.

3.1 Setup of the analysis

In order to mimic the conditional volatility patterns of daily return series, we assume a process that is described by time varying conditional volatility. For that reason, we apply an extension of the widely accepted GARCH approach (Bollerslev (1986)) to simulate daily return series of realistic length. Specifically, we control for memory effects by applying the FIGARCH(1,d,1) approach as presented by (Baille et al. (1996)) to generate conditional volatility processes wich are given as follows:

$$\sigma_t^2 = \Omega + (1 - \beta(L) - \phi(L)(1 - L)^d)r_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{16}$$

Here, Ω describes a constant, r_{t-1} the return from the previous period. The parameter d controls for the memory of the process and allows the autocorrelation of the process to decay at a hyperbolic rate wheras $\phi(L)$ and $\beta(L)$ describe the lag polynomials. According to Baillie and Morana (2009).

As presented by Gençay et al. (2001), FIGARCH$(1, d, 1)$ processes with $d \in [0, 0.5]$ are predestined to mimic conditional financial return variance with different memory schemes, whereas the memory of the conditional volatility increases with d. Because of that, we apply different parameterizations of $d = 0.05, 0.15, 0.35$ and 0.45 to discuss four different memory schemes, namely the transition from short memory (d=0.05) to long-term memory (d=0.45). Additionally, to control for the memory

of the simulated return series, and assume a constant mean return.[7]

As we focus on portfolio allocations based on simulated return series, we assess multiple simulated return series simultaneously and introduce realistic dependence schemes between the simulated univariate return series. In line with stylized facts about the dependence of financial return series, i.e. dependence varies over time, we introduce time varying dependence via dynamic conditional correlation approach, as introduced by Engle (2002).

For this reason, as presented by Engle (2009), based on multiple simulated FI-GARCH(1,d,1) series, we implement the assigned dynamic correlation (DCC) structure between series i and series j following iterative multi-period process. Let $\bar{R}$ be the sample correlation and α_{DCC} and β_{DCC} the DCC parameters. Based on this step, we proceed iteratively, so to say, conditionally on period t, we model the dependence structure between asset i and j for $t + 1$:

$$Q_{t+1} = (1 - \alpha_{DCC} - \beta_{DCC}) * \bar{R} + \alpha_{DCC} * (\epsilon_{i,t}^{sim\flat}\epsilon_{j,t}^{sim}) + \beta_{DCC}(Q_t) \qquad (17)$$

$$R_{t+1} = diag\left\{Q_{t+1}^{-1/2}\right\} Q_{t+1} diag\left\{Q_{t+1}^{-1/2}\right\}, \qquad (18)$$

$$(\epsilon_{i,t+1}^{sim}\ \epsilon_{j,t+1}^{sim}) = (\epsilon_{i,t+1}\ \epsilon_{j,t+1}) * \sqrt{R_{t+1}}. \qquad (19)$$

In this setup β_{DCC} represents the persistency of the process.

Here parameters α_{DCC} and β_{DCC} control for the news impact and persistence of the process. As described in Engle (2009), financial return series are typically described by a parameterization of $\alpha = 0.05$ and $\beta = 0.90$.[8] As a target correlation, we mimic stocks which are listed at the same index and assume slightly positive correlated assets and assess a portfolio that consists of five simulated stocks which are characterized by a correlation matrix as follows:

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[7]It is to note, that the application of an AR process impacts the memory of the underlying return series and makes it difficult to control for the memory of the underlying return series. Moreover, we discuss both positive and negative average returns.

[8]A detailed description on the simulation of time varying conditional correlation is given by Berger (2016).

$$\bar{R} = \begin{bmatrix} 1 & 0.8 & 0.4 & 0.3 & 0.5 \\ & 1 & 0.2 & 0.1 & 0.3 \\ & & 1 & 0.4 & 0.1 \\ & & & 1 & 0.7 \\ & & & & 1 \end{bmatrix} \tag{20}$$

Based on the simulated return series, we decompose each series into three different trends, as described by equations (7) - (10) in section 2.1 . Then, we estimate the mean-var efficient portfolio allocations (equation (11)) based on the decomposed and the original return series and analyze the out-of-sample performance of each allocation strategy via rolling window analysis.

The setup of the simulation study can be summarized as follows:

1) We generate five return series comprising 1500 observations via FIGARCH(1,d,1) approach.

2) We introduce time varying conditional correlation via DCC approach.

3) We decompose each return series via MODWT approach.

4) We reconstruct the decomposed return series and achieve r_{SR}^{sim}, r_{MR}^{sim} and r_{LR}^{sim} and the simulated return series r^{sim} and assess 500 -1500.

5) We apply rolling window approach: 1+t:500+t.

6) We apply Markowitz approach.

7) We assess the out-of-sample performance.

For each parameterization of d we repeat the simulation 1.000 times to assess the robustness of the results.

3.2 Results of the analysis

As the presented results are valid for each simulation-run, in the remainder of this section, we present the analysis of one simulation-run for each memory scenario.[9][10]

[Insert Table 2 about here.]

Table 2 presents the results of the conducted simulation analysis. That is, for four different scenarios of time series memory ($d = 0.05$, $d = 0.15$, $d = 0.35$ and $d = 0.45$), we assess the performance of portfolio allocations that minimize the covariance matrix of reconstructed return series which exclusively take short-run information (SR), middle-run information (MR) or long-run information (LR) into account. As a benchmark, we assess portfolio allocations based on the original (undecomposed) simulated return series (Original). The presented results, describe the out-of-sample performance for 1000 days.

Obviously, absolute average returns increase when the simulated return series are characterized by longer memory ($d = 0.05$: -0.0364% and -0.2313% for $d = 0.45$).

Although different memory scenarios lead to different average returns, our results provide evidence for the importance of short-run information. Comparing the out-of-sample performance of portfolios which minimize the covariance matrix of the original simulated return series against the portfolios that minimize the reconstructed return series indicates that the SR out-of-sample returns are closer to the original return than MR and LR.

For instance, if the simulated return series is characterized by less memory (d=0.05), the average out-of-sample portfolio returns based on the original return series are -0.0364% and SR -0.0363% whereas MR and LR lead to -0.0367% and -0.0353% respectively. This tendency remains stable for different memory-

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[9]For sake of page constraints, average statistics for all 1.000 simulations for each scenario are available upon request.

[10]As the presented results are robust against simulated up and downward-trends, we present the results for simulated return series which are characterized by negative average returns.

scenarios, indicating that information on the short-run memory of a return series provide the relevant information for portfolio allocations.

The applied information ratio (IR_{mv}) allows for a comparison of the portfolio strategies against the applied benchmark (Original). In this particular simulation setup, lower info ratios are preferred against higher values, indicating that the assessed strategy does not deviate from the out-of-sample returns based on un-decomposed return series. The results suggest, that SR leads to out-of-sample returns which are closest to the benchmark, whereas MR and LR result in larger deviations from the benchmark and to more volatile out-of-sample returns. This finding indicates that the relevant information for daily portfolio management can be described by decomposed short-run trends. Moreover, the results of the presented simulation study demonstrate, that the long-run information can be excluded from the underlying return series without impacting portfolio allocations.

Although simulated return series are characterized by long-memory schemes ($d = 0.45$), minimizing the covariance matrix of the reconstructed return series which exclusively comprises information on the short-run trends of the original series leads to similar out-of-sample performance, according to its benchmark.

4 Empirical Study

4.1 Data

The data set of the empirical study comprises different stocks which are listed at leading indices of nine different countries. In order to indicate robustness of the empirical results, we analyze different currency denominations and focus on both developed and emerging markets. We discuss assets which are listed at the leading North American, German and British stock markets as representatives of developed markets. Additionally, we assess Canadian and Australian stocks as representatives for smaller indexes. To analyze emerging markets, we stick to the definition of O'Neil (2001) and assess stocks which are listed at the leading indices of the so called BRIC states. That is, we assess shares which are listed at the Brazilian, Russian, Indian and Chinese stock exchanges. For all countries we assess daily market quotes and analyze more than 11 years of data ranging from 2.1.2006-20.5.2016.[11] By splitting our sample into sub-samples, we are able to take account for the market turmoil beginning at 2007 .

[Insert Table 3 about here.]

Table 3 presents an overview of the applied data. For all countries, we analyze all stocks which are listed at the leading stock index of the respective country. Moreover, we exclude all stocks which were listed or de-listed after 2006 from our analysis to ensure a consistent sample size for the assessment of different sub-samples.

[Insert Table 4 about here.]

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[11]This period refers to the limits of the assessed out-of-sample period and comprises up to 2675 daily market quotes. Due to data intensive rolling window and wavelet analysis, the assessed market prices range from 6.8.2003 up to 20.5.2016 (up to 3305 observations). Due to country specific bank holidays, the amount of observations differs marginally by each country.

Table 4 presents the averaged descriptive statistics of the analyzed assets for each stock index. Obviously, the assessed stocks which are listed at indices of developed markets are characterized by lower risk (σ ranges between 0.007 and 0.008) and extreme negative losses in comparison to positive gains (skew ranges between -0.039 and -0.796) in comparison to stocks which are listed at indices of emerging markets which are characterized by higher risk and a longer right tail (σ ranges between 0.010 and 0.016; skew ranges between 0.029 and 1.687). According to the presented averaged descriptive statistics, the investigated stocks of small markets are characterized by similar risk like developed markets (σ ranges between 0.007 and 0.009) but by positive skewness like emerging markets (skew ranges between 0.212 and 0.365) and describe an interesting compromise between stocks of developed and emerging markets.[12].

4.2 Empirical Results

Analogous to the presented simulation analysis, we assess reconstructed return series and we start the empirical analysis by assessing the out-of-sample performance of the portfolios that comprise stocks of developed markets. The results of the out-of-sample performance of the global minimum variance portfolios in the time from 2006 until 2010 are presented in Table 5.

[Insert Table 5 about here.]

Table 5 provides the descriptive statistics and the performance metrics of the daily out-of-sample portfolio returns from 2006 until 2010. The results for the period between 2010 until 2016 and the crisis-period from 2007 until 2009 are presented in Table 6 and Table 7 respectively. The out-of-sample performance of the global mean-var-efficient minimum variance portfolios based on particular trends indicate,

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[12]As our analysis comprises 265 stocks, a detailed list of descriptive statistics for each individual asset is available upon request to the authors

that the decomposition of return series into different trends impacts the portfolio performance. Both the exclusive focus on middle-run and long-run trends does not lead to an improved portfolio performance in comparison to portfolios that take the complete information, which is provided by the original return series, into account. For instance, mean-var efficient portfolio allocations which comprise stocks that are listed at the Dow Jones index (DJI 30) lead to an average return of 0.0148% and a Sharpe ratio of 0.0136 whereas middle-run and long-run trends lead to lower average returns (middle run: 0.0021%; long run: 0.0013%) and lower Sharpe ratios (middle run: 0.0018; long run: 0.0011). In contradiction to that, an exclusive focus on short-run trends leads to marginally improved performance metrics (average out-of-sample return: 0.0167%; Sharpe ratio 0.0153). Generally, the presented figures suggest that daily rebalanced portfolio allocations that aim at minimizing middle-and long-run trends of the assessed series, do not improve the out-of-sample performance in terms of the applied portfolio metrics. Turning to the assessment of short-run trends, the results indicate that the application of decomposed short-run trends lead to an improvement in terms of the applied quality criteria. In comparison to the out-of-sample performance of portfolios that minimize the conditional covariance matrix of daily data, the particular focus on short-run trends leads to higher average returns, larger Sharpe and sortino ratios for the assessed stocks of all 9 countries.

[Insert Table 6 about here.]

Table 6 provides the results for the time from 2010 until 2016. Although both market times are characterized by different market regimes, i.e. the time from 2006 until 2010 includes the outbreak and the recovery of financial crisis whereas the period between 2010 until 2016 is described by market upturns as a result of historically low interest rates, the findings for the period between 2006 until 2010 hold for the period

2010 until 2016. Again, by explicitly focusing on the short-run information of the underlying daily return series, the assessed performance metrics can be improved. Long-run information do not lead to an improvement of the applied quality criteria. For both subsamples, extracting the short-run information from daily return series leads to an improvement of the applied metrics, indicating that the relevant information for applied portfolio management is adequately described by daily short-run fluctuations.

[Insert Table 7 about here.]

With particular focus on the turmoil market times between June 2007 and June 2009, see Table 7, portfolios that exclusively take the covariance matrix of short-run trends into account lead to smaller negative returns (e.g. DJI 30: -0.0066% daily data and -0.0039% short run; TSX 60: -0.0206% daily data and -0.0196% short run; RTS: -0.0572% daily data and -0.0501% short run) than the ordinary portfolios. Again, the results indicate that the short-run trends incorporate the relevant information for daily portfolio management, when markets are characterized by larger volatility and collective market downturns. In this vein, the maximum drawdown in turmoil market times gets marginally reduced when decomposed return series that exclusively describe the short-run trends are applied.

However, the presented results indicate, that the relevant information for daily portfolio management is adequately described by the extracted short-run information of the respective daily return series. Therefore, exclusively focusing on the short-run information of the assessed financial return series leads to slightly improved results with respect to the assessed quality criteria. Although recent studies mainly point at stronger dependence regimes between the long-run seasonalities of stock returns (see Gallegati 2012, Rua and Nunes 2014, Tan et al. 2014 amongst others), our results indicate that the information on different long-run seasonalities

is of limited use for applied portfolio management.

Consequently, as both middle- and long-run trends do not contribute an improved portfolio performance, our results identify the extracted information on short-run fluctuations of the underlying return series as the relevant information for daily portfolio management. As portfolio allocations which aim at minimizing short-run seasonalities and classical portfolio allocations lead to comparable results, the empirical findings are in line with the results of the simulation analysis and provide evidence for the relevance of the short-run trends in the context of applied portfolio management. Due to the applied different stock indices and sample periods, our results are robust against different volatility and market regimes.

5   Conclusion

Triggered by the growing literature on changing dependence schemes between decomposed return series, a simulation study and a thorough empirical assessment unveal the relevance of short-run trends for applied portfolio management.

The presented simulation study indicates, that particular information of long-run seasonalities are of minor relevance for daily portfolio management. Although the underlying data is characterized by long-memory processes, excluding the information of the underlying long-run seasonalities does not impact the presented portfolio allocations. Moreover, the empirical assessment of different portfolios that comprise stocks which are listed at both developed and emerging markets provides evidence that extracting short-run trends from daily return series describes the crucial information for portfolio management.

Therefore, our results provide novel insights into the relevant information for applied portfolio management and further research should focus on the assessment of stylized properties of the short-run trends. Moreover, alternative investment strategies which take structural breaks and jumps within the short-run scales into account or the assessment portfolio Value-at-Risk as a direct function of decomposed covariance matrices present promising expansions of the presented analysis and should be investigated.

6 Tables

Definition	Horizon	Detail
Short Run	2-4 days	D1
Short Run	4-8 days	D2
Short Run	8-16 days	D3
Middle Run	16-32 days	D4
Middle Run	32-64 days	D5
Long Run	64-128 days	D6
Long Run	128-256 days	D7
Long Run	256-512 days	D8

Table 1: Economic interpretation of decomposed daily return series.

	Original	SR	MR	LR
$d = 0.05$				
$\bar{r}_{sim}$	-0,0364%	-0,0363%	-0,0367%	-0,0353%
min	-0,5354%	-0,5346%	-0,5346%	-0,5502%
max	0,3342%	0,3336%	0,3272%	0,3381%
$\bar{\sigma}_{sim}$	0,0009	0,0009	0,0009	0,0009
IR_{mv}		0,0436	-0,0349	0,0261
TE_{mv}		0,0000	0,0001	0,0004
$d = 0.15$				
$\bar{r}_{sim}$	-0,0661%	-0,0661%	-0,0661%	-0,0700%
min	-0,9148%	-0,9198%	-0,9062%	-1,0400%
max	0,7920%	0,7895%	0,7892%	0,9321%
$\bar{\sigma}_{sim}$	0,0022	0,0022	0,0022	0,0025
IR_{mv}		0,0024	0,0040	-0,0354
TE_{mv}		0,0001	0,0002	0,0011
$d = 0.35$				
$\bar{r}_{sim}$	-0,1498%	-0,1503%	-0,1489%	-0,1447%
min	-1,4204%	-1,3997%	-1,4859%	-1,6412%
max	1,0535%	1,0598%	1,0926%	1,4492%
$\bar{\sigma}_{sim}$	0,0028	0,0028	0,0029	0,0034
IR_{mv}		-0,0527	0,0317	0,0330
TE_{mv}		0,0001	0,0003	0,0015
$d = 0.45$				
$\bar{r}_{sim}$	-0,2313%	-0,2316%	-0,2379%	-0,2368%
min	-10,0287%	-10,0287%	-10,0287%	-10,7048%
max	10,3812%	10,3815%	10,4105%	11,6836%
$\bar{\sigma}_{sim}$	0,0202	0,0203	0,0203	0,0213
IR_{mv}		-0,0073	-0,0851	-0,0222
TE_{mv}		0,0004	0,0008	0,0025

Table 2: This table describes the out-of-sample results of the applied simulation study. $\bar{r}_{sim}$ describes the average out-of-sample return of the mean-var efficient portfolio, min and max describe the respective minimum and maximum out-of-sample returns and $\bar{\sigma}_{sim}$ gives the standard deviation. IR_{mv} describes the sum of deviations from the applied benchmark strategy and TE_{mv} describes the ratio of overage deviation from the benchmark devided by $\bar{\sigma}_{sim}$. Here, the benchmark is defined as the portfolio based on un-decomposed data (Original).

Data Developed Markets	Index	Period	Frequency	# Assets
United States of America	DJI 30	2006-2016	daily	28
Great Britain	FTSE 30	2006-2016	daily	27
Germany	DAX 30	2006-2016	daily	29
Small Markets				
Canada	TSX 60	2006-2016	daily	54
Australia	ASX	2006-2016	daily	19
Emerging Markets (BRIC)				
Brazil	Bovespa	2006-2016	daily	32
Russia	RTS Index	2006-2016	daily	15
India	BSE Sensex	2006-2016	daily	25
China	SSE Index	2006-2016	daily	36

Table 3: This table presents an overview of the underlying assets which are assessed in the empirical analysis. Due to the fact, that we exclude assets which were listed or de-listed in the investigated period from 2006-2016, not all assets from each index are analyzed.

Descriptive Statistics Developed Markets	μ	σ	min	max	skew	kurt
DJI 30	0.000	0.007	-0.059	0.062	-0.039	11.369
FTSE 30	0.000	0.008	-0.099	0.067	-0.796	22.917
DAX 30	0.000	0.008	-0.072	0.066	-0.119	8.702
Small Markets						
TSX 60	0.000	0.009	-0.083	0.088	0.212	21.176
ASX	0.000	0.007	-0.057	0.072	0.365	11.250
Emerging Markets						
Bovespa	0.000	0.011	-0.072	0.109	1.687	49.187
RTS Index	0.000	0.011	-0.119	0.123	0.029	21.494
BSE Sensex	0.000	0.010	-0.076	0.077	0.018	6.684
SSE Index	0.000	0.016	-0.079	0.071	0.776	55.936

Table 4: This table presents the average descriptive statistics of the assessed assets. μ and σ describe the sample average log returns and the average standard deviation for asset which is listed in the respective market. min and max describe the average minimum and maximum for each asset and skew and kurt the average skewness and kurtosis of each asset.

2006-2010	DJI 30				FTSE 30				DAX 30			
	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run
$\bar{r}$	0,0148%	0,0167%	0,0021%	0,0013%	0,0098%	0,0122%	0,0088%	-0,0178%	-0,0230%	-0,0221%	-0,0344%	-0,0095%
min	-7,7854%	-7,6324%	-7,6949%	-7,8221%	-8,1488%	-8,0408%	-8,7883%	-10,3711%	-7,2481%	-7,5901%	-8,4755%	-7,6912%
max	9,7203%	9,8513%	10,0642%	10,3124%	9,6387%	9,7875%	8,9639%	8,6938%	7,9862%	8,2566%	7,9934%	9,3030%
$\bar{\sigma}^2$	0,0001	0,0001	0,0001	0,0002	0,0001	0,0001	0,0001	0,0002	0,0001	0,0001	0,0001	0,0001
SR_{MV}	0,0136	0,0153	0,0018	0,0011	0,0084	0,0104	0,0073	X	X	X	X	X
OR_{MV}	1,0452	1,0509	1,0057	1,0035	1,0259	1,0322	1,0221	0,9583	0,9448	0,9475	0,9200	0,9775
SoR_{MV}	0,0193	0,0217	0,0025	0,0015	0,0118	0,0146	0,0101	-0,0187	-0,0255	-0,0241	-0,0371	-0,0108
max dd	0,3839	0,3755	0,4524	0,4279	0,4083	0,4007	0,4073	0,5358	0,6145	0,6156	0,6254	0,5259
$IR_{1/N}$	0,0050	0,0072	-0,0112	-0,0123	0,0382	0,0410	0,0389	0,0101	-0,0221	-0,0211	-0,0326	-0,0071
$tracking_{1/N}$	0,0085	0,0085	0,0076	0,0075	0,0097	0,0096	0,0093	0,0094	0,0091	0,0091	0,0097	0,0094
IR_{mv}		0,0553	-0,0469	-0,0318		0,0516	-0,0027	-0,0466		0,0152	-0,0350	0,0271
TE_{mv}		0,0003	0,0027	0,0043		0,0005	0,0037	0,0059		0,0006	0,0032	0,0050

2006-2010	TSX 60				ASX 30				Bovespa			
	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run
$\bar{r}$	0,0004%	0,0019%	0,0086%	0,0036%	-0,0172%	-0,0149%	-0,0251%	-0,0289%	0,0460%	0,0486%	0,0462%	0,0470%
min	-5,1438%	-5,1241%	-6,6166%	-6,3838%	-7,5756%	-7,6570%	-7,5265%	-6,4131%	-7,0026%	-6,8893%	-8,1648%	-9,2738%
max	4,9433%	5,0497%	5,6951%	5,9811%	4,4451%	4,3195%	5,8802%	5,4767%	73,4432%	73,4432%	73,4432%	70,1435%
$\bar{\sigma}^2$	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0002	0,0002	0,0006	0,0006	0,0007	0,0007
SR_{MV}	0,0004	0,0022	0,0091	0,0035	X	X	X	X	0,0183	0,0193	0,0179	0,0179
OR_{MV}	1,0013	1,0066	1,0279	1,0108	0,9586	0,9641	0,9436	0,9384	1,1592	1,1678	1,1413	1,1139
SoR_{MV}	0,0006	0,0029	0,0122	0,0047	-0,0200	-0,0173	-0,0276	-0,0304	0,0578	0,0612	0,0524	0,0437
max dd	0,3925	0,3902	0,3269	0,4059	0,4928	0,4830	0,5271	0,6088	0,4957	0,4930	0,5370	0,6355
$IR_{1/N}$	-0,0169	-0,0155	-0,0094	-0,0157	-0,0218	-0,0189	-0,0349	-0,0360	-0,0072	-0,0062	-0,0072	-0,0072
$tracking_{1/N}$	0,0100	0,0100	0,0093	0,0088	0,0086	0,0086	0,0076	0,0084	0,0254	0,0254	0,0251	0,0239
IR_{mv}	NaN	0,0340	0,0219	0,0058		0,0599	-0,0242	-0,0248		0,0439	0,0007	0,0014
$tracking_{mv}$	0,0000	0,0004	0,0038	0,0055		0,0004	0,0033	0,0047		0,0006	0,0035	0,0074

2006-2010	RTS				BSE				SSE			
	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run
$\bar{r}$	0,0009%	0,0053%	0,0216%	0,0080%	0,0943%	0,0958%	0,0914%	0,0633%	0,0907%	0,0924%	0,0937%	0,1361%
min	-16,4032%	-15,6767%	-13,2803%	-12,9818%	-7,0145%	-7,0576%	-7,1491%	-6,4699%	-7,1185%	-7,1913%	-7,8707%	-9,8429%
max	9,6029%	10,0595%	9,3845%	12,4053%	10,2740%	10,3894%	11,7132%	7,5746%	4,0467%	4,1162%	4,4252%	5,4538%
$\bar{\sigma}^2$	0,0003	0,0003	0,0003	0,0004	0,0002	0,0002	0,0002	0,0002	0,0001	0,0001	0,0001	0,0002
SR_{MV}	0,0005	0,0030	0,0127	0,0041	0,0700	0,0709	0,0647	0,0449	0,0921	0,0925	0,0828	0,1022
OR_{MV}	1,0017	1,0103	1,0405	1,0126	1,2210	1,2239	1,2043	1,1347	1,4392	1,4389	1,3854	1,4828
SoR_{MV}	0,0007	0,0041	0,0174	0,0056	0,1005	0,1018	0,0940	0,0643	0,1339	0,1349	0,1154	0,1541
max dd	0,7337	0,7342	0,6933	0,7291	0,3167	0,3192	0,3437	0,3145	0,1828	0,1887	0,1757	0,1690
$IR_{1/N}$	-0,0128	-0,0105	-0,0017	-0,0094	0,0176	0,0191	0,0148	-0,0114	-0,0166	-0,0158	-0,0153	0,0058
$tracking_{1/N}$	0,0185	0,0182	0,0165	0,0175	0,0101	0,0102	0,0101	0,0114	0,0203	0,0203	0,0201	0,0200
IR_{mv}		0,0452	0,0414	0,0061		0,0261	-0,0075	-0,0489		0,0298	0,0080	0,0664
$tracking_{mv}$		0,0010	0,0050	0,0117		0,0006	0,0038	0,0063		0,0006	0,0037	0,0068

Table 5: Empirical Results: 2006 - 2010. $\bar{r}$ and $\bar{\sigma}^2$ describe the mean and variance and min/max the respective minimum/maximum of the out-of-sample returns. SR_{MV} describes the Sharpe Ratio of the minimum variance strategy, SoR_{MV} the Sortino ratio and OR_{MV} the Omega ratio. The maximum drawdown is given by max dd, and IR_k and $tracking_k$ describe the information on the deviation from the assessed strategy against benchmark strategy k and the respective information ratio. Here, k is given by equally weighted strategy $(1/N)$ and mean-var efficient portfolios based on un-decomposed return series (mv).

2010-2016	DJI 30				FTSE 30				DAX 30			
	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run
$\bar{r}$	0,0269%	0,0270%	0,0293%	0,0297%	0,0156%	0,0151%	0,0116%	0,0071%	0,0445%	0,0446%	0,0429%	0,0485%
min	-3,7954%	-3,8201%	-3,5992%	-4,0641%	-3,6648%	-3,6529%	-4,4917%	-4,2918%	-5,6470%	-5,6082%	-5,4475%	-7,6292%
max	3,6866%	3,7204%	3,9758%	4,4496%	3,9654%	3,9503%	3,9596%	3,5162%	3,3996%	3,3670%	3,6337%	3,6229%
$\bar{\sigma}^2$	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001
SR_{MV}	0,0380	0,0381	0,0406	0,0379	0,0189	0,0183	0,0135	0,0082	0,0497	0,0496	0,0474	0,0508
OR_{MV}	1,1113	1,1118	1,1194	1,1113	1,0530	1,0514	1,0384	1,0224	1,1451	1,1449	1,1374	1,1495
SoR_{MV}	0,0540	0,0542	0,0576	0,0534	0,0264	0,0256	0,0188	0,0114	0,0697	0,0696	0,0672	0,0712
max dd	0,1667	0,1645	0,1721	0,1583	0,2032	0,2019	0,2071	0,2641	0,1970	0,1970	0,2199	0,2190
$IR_{1/N}$	-0,0133	-0,0130	-0,0085	-0,0078	-0,0190	-0,0199	-0,0264	-0,0339	0,0156	0,0159	0,0131	0,0200
$tracking_{1/N}$	0,0048	0,0048	0,0047	0,0047	0,0055	0,0054	0,0054	0,0056	0,0070	0,0070	0,0072	0,0075
IR_{mv}		0,0051	0,0139	0,0084		-0,0173	-0,0163	-0,0269		0,0035	-0,0075	0,0122
$tracking_{mv}$		0,0002	0,0018	0,0033		0,0003	0,0024	0,0031		0,0003	0,0021	0,0032

2010-2016	TSX 60				ASX 30				Bovespa			
	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run
$\bar{r}$	0,0422%	0,0425%	0,0353%	0,0269%	0,0214%	0,0205%	0,0206%	0,0221%	0,0407%	0,0411%	0,0420%	0,0283%
min	-3,3768%	-3,4492%	-3,5134%	-3,4805%	-3,7120%	-3,7566%	-3,9817%	-3,9112%	-10,5745%	-10,5700%	-10,3868%	-10,0028%
max	3,0810%	3,1970%	2,8131%	2,9108%	2,4104%	2,4064%	2,6741%	2,6586%	33,5379%	33,6140%	31,6236%	28,8147%
$\bar{\sigma}^2$	0,0000	0,0000	0,0000	0,0000	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001
SR_{MV}	0,0701	0,0705	0,0544	0,0382	0,0284	0,0271	0,0261	0,0271	0,0335	0,0337	0,0348	0,0236
OR_{MV}	1,2146	1,2164	1,1600	1,1102	1,0776	1,0740	1,0721	1,0744	1,1505	1,1514	1,1488	1,0904
SoR_{MV}	0,0998	0,1005	0,0770	0,0535	0,0396	0,0377	0,0366	0,0383	0,0632	0,0636	0,0634	0,0414
max dd	0,1321	0,1265	0,1479	0,1651	0,2033	0,2076	0,1991	0,1911	0,2468	0,2504	0,2893	0,2194
$IR_{1/N}$	0,0550	0,0552	0,0443	0,0313	0,0320	0,0303	0,0303	0,0306	0,0413	0,0415	0,0438	0,0334
$tracking_{1/N}$	0,0057	0,0057	0,0055	0,0051	0,0049	0,0049	0,0049	0,0054	0,0122	0,0123	0,0119	0,0114
IR_{mv}		0,0090	-0,0265	-0,0419		-0,0330	-0,0032	0,0023		0,0110	0,0044	-0,0266
$tracking_{mv}$		0,0003	0,0026	0,0037		0,0003	0,0024	0,0031		0,0004	0,0031	0,0047

2010-2016	RTS				BSE				SSE			
	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run
$\bar{r}$	0,0557%	0,0581%	0,0514%	0,0276%	0,0577%	0,0585%	0,0480%	0,0347%	0,1984%	0,1973%	0,1884%	0,2279%
min	-11,2726%	-11,1690%	-10,8548%	-10,4222%	-5,3720%	-5,3388%	-5,2222%	-6,0037%	-10,0796%	-10,0505%	-9,9857%	-8,7335%
max	5,2000%	5,0285%	5,1904%	4,9126%	3,6149%	3,6696%	4,1139%	4,6121%	24,5473%	24,5473%	24,5473%	24,5473%
$\bar{\sigma}^2$	0,0001	0,0001	0,0002	0,0002	0,0001	0,0001	0,0001	0,0001	0,0038	0,0038	0,0038	0,0038
SR_{MV}	0,0462	0,0486	0,0395	0,0207	0,0722	0,0731	0,0580	0,0374	0,0321	0,0320	0,0305	0,0368
OR_{MV}	1,1444	1,1522	1,1206	1,0603	1,2142	1,2168	1,1675	1,1061	1,7040	1,6920	1,6244	1,7745
SoR_{MV}	0,0629	0,0661	0,0540	0,0283	0,1066	0,1081	0,0840	0,0531	0,2203	0,2153	0,1979	0,2509
max dd	0,2848	0,2814	0,3248	0,3402	0,1805	0,1780	0,1961	0,2837	0,4255	0,4360	0,4944	0,4662
$IR_{1/N}$	0,0285	0,0329	0,0190	-0,0170	0,0462	0,0477	0,0269	0,0043	0,0293	0,0291	0,0276	0,0341
$tracking_{1/N}$	0,0059	0,0058	0,0066	0,0066	0,0056	0,0056	0,0060	0,0065	0,0603	0,0602	0,0603	0,0604
IR_{mv}		0,0387	-0,0115	-0,0501		0,0211	-0,0357	-0,0495		-0,0133	-0,0293	0,0532
$tracking_{mv}$		0,0006	0,0038	0,0056		0,0004	0,0027	0,0046		0,0008	0,0034	0,0055

Table 6: Empirical Results: 2010 - 2016. $\bar{r}$ and $\bar{\sigma}^2$ describe the mean and variance and min/max the respective minimum/maximum of the out-of-sample returns. SR_{MV} describes the Sharpe Ratio of the minimum variance strategy, SoR_{MV} the Sortino ratio and OR_{MV} the Omega ratio. The maximum drawdown is given by max dd, and IR_k and $tracking_k$ describe the information on the deviation from the assessed strategy against benchmark strategy k and the respective information ratio. Here, k is given by equally weighted strategy $(1/N)$ and mean-var efficient portfolios based on un-decomposed return series (mv).

Table 7 — Empirical Results: 2007 – 2009

2007-2009	DJI 30				FTSE 30				DAX 30			
	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run
$\bar{r}$	-0,0066%	-0,0039%	-0,0214%	-0,0211%	-0,0192%	-0,0160%	-0,0168%	-0,0642%	-0,0843%	-0,0846%	-0,0856%	-0,0580%
min	-7,7854%	-7,6324%	-7,6949%	-7,8221%	-8,1488%	-8,0408%	-8,7883%	-10,3711%	-7,2481%	-7,5901%	-8,4755%	-7,6912%
max	9,7203%	9,8513%	10,0642%	10,3124%	9,6387%	9,7875%	8,9639%	8,6938%	7,9862%	8,2566%	7,9934%	9,3030%
$\bar{\sigma}^2$	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002
SR_{MV}	X	X	X	X	X	X	X	X	X	X	X	X
OR_{MV}	0,9848	0,9910	0,9560	0,9582	0,9607	0,9671	0,9676	0,8856	0,8339	0,8352	0,8346	0,8853
SoR_{MV}	-0,0070	-0,0041	-0,0206	-0,0194	-0,0188	-0,0157	-0,0158	-0,0545	-0,0799	-0,0789	-0,0791	-0,0578
max dd	0,3839	0,3755	0,4524	0,4279	0,4024	0,3957	0,4027	0,5301	0,6145	0,6156	0,6254	0,5225
$IR_{1/N}$	0,0135	0,0161	-0,0004	-0,0001	0,0463	0,0493	0,0510	0,0098	-0,0142	-0,0145	-0,0145	0,0087
$tracking_{1/N}$	0,0106	0,0106	0,0094	0,0092	0,0122	0,0121	0,0115	0,0116	0,0113	0,0113	0,0120	0,0116
IR_{mv}		0,0656	-0,0456	-0,0279		0,0569	0,0053	-0,0607		-0,0043	-0,0034	0,0445
$tracking_{mv}$		0,0004	0,0033	0,0052		0,0006	0,0046	0,0074		0,0007	0,0038	0,0059

2007-2009	TSX 60				ASX 30				Bovespa			
	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run
$\bar{r}$	-0,0206%	-0,0196%	-0,0103%	-0,0153%	-0,0865%	-0,0841%	-0,0841%	-0,0979%	-0,0656%	-0,0725%	-0,0725%	-0,0801%
min	-5,1438%	-5,1241%	-6,6166%	-6,3838%	-7,5756%	-7,6570%	-7,5265%	-6,4131%	-7,0026%	-6,8893%	-8,1648%	-9,2738%
max	4,9433%	5,0497%	5,6951%	5,9811%	4,4451%	4,3195%	5,8802%	5,4767%	7,2918%	7,4988%	8,0535%	12,9811%
$\bar{\sigma}^2$	0,0001	0,0001	0,0001	0,0002	0,0002	0,0002	0,0002	0,0003	0,0002	0,0002	0,0002	0,0003
SR_{MV}	X	X	X	X	X	X	X	X	X	X	X	X
OR_{MV}	0,9435	0,9468	0,9734	0,9640	0,8437	0,8507	0,8611	0,8515	0,8504	0,8570	0,8539	0,8721
SoR_{MV}	-0,0267	-0,0253	-0,0121	-0,0167	-0,0770	-0,0734	-0,0687	-0,0765	-0,0669	-0,0622	-0,0648	-0,0586
max dd	0,3925	0,3902	0,3269	0,4059	0,3098	0,3028	0,3384	0,3941	0,4957	0,4930	0,5370	0,6355
$IR_{1/N}$	-0,0077	-0,0069	0,0010	-0,0038	-0,0379	-0,0347	-0,0375	-0,0449	-0,0530	-0,0511	-0,0628	-0,0778
$tracking_{1/N}$	0,0119	0,0118	0,0108	0,0100	0,0143	0,0145	0,0128	0,0137	0,0133	0,0133	0,0124	0,0109
IR_{mv}		0,0192	0,0228	0,0080		0,0857	0,0147	-0,0113		0,0407	-0,0170	-0,0174
$tracking_{mv}$		0,0005	0,0045	0,0066		0,0005	0,0043	0,0065		0,0007	0,0041	0,0084

2007-2009	RTS				BSE				SSE			
	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run	Daily Data	Short Run	Middle Run	Long Run
$\bar{r}$	-0,0572%	-0,0501%	-0,0358%	-0,1117%	-0,0090%	-0,0075%	-0,0234%	-0,0112%	0,0115%	0,0116%	0,0136%	0,0091%
min	-16,4032%	-15,6767%	-13,2803%	-12,9818%	-6,6434%	-6,5910%	-7,1072%	-6,4699%	-2,2669%	-2,3230%	-2,3643%	-2,9543%
max	9,6029%	10,0595%	9,3845%	12,4053%	5,4387%	5,4012%	5,1287%	5,3015%	2,3724%	2,4163%	2,3460%	3,3240%
$\bar{\sigma}^2$	0,0004	0,0004	0,0003	0,0004	0,0002	0,0002	0,0002	0,0002	0,0000	0,0000	0,0000	0,0000
SR_{MV}	X	X	X	X	X	X	X	X	0,0334	0,0325	0,0316	0,0167
OR_{MV}	0,9005	0,9124	0,9392	0,8421	0,9823	0,9854	0,9581	0,9798	1,1962	1,1886	1,1683	1,0840
SoR_{MV}	-0,0383	-0,0335	-0,0260	-0,0731	-0,0086	-0,0071	-0,0209	-0,0102	0,0486	0,0472	0,0469	0,0257
max dd	0,7337	0,7342	0,6933	0,7291	0,2959	0,2963	0,3356	0,3100	0,0559	0,0564	0,0770	0,0998
$IR_{1/N}$	-0,0144	-0,0115	-0,0056	-0,0416	0,0586	0,0598	0,0482	0,0560	-0,0022	-0,0022	-0,0014	-0,0033
$tracking_{1/N}$	0,0225	0,0221	0,0197	0,0209	0,0126	0,0126	0,0124	0,0128	0,0248	0,0248	0,0246	0,0242
IR_{mv}		0,0613	0,0361	-0,0442		0,0248	-0,0349	-0,0035		0,0024	0,0110	-0,0088
$tracking_{mv}$		0,0012	0,0059	0,0123		0,0006	0,0041	0,0062		0,0002	0,0019	0,0028

Table 7: Empirical Results: 2007 - 2009. $\bar{r}$ and $\bar{\sigma}^2$ describe the mean and variance and min/max the respective minimum/maximum of the out-of-sample returns. SR_{MV} describes the Sharpe Ratio of the minimum variance strategy, SoR_{MV} the Sortino ratio and OR_{MV} the Omega ratio. The maximum drawdown is given by max dd, and IR_k and $tracking_k$ describe the information on the deviation from the assessed strategy against benchmark strategy k and the respective information ratio. Here, k is given by equally weighted strategy $(1/N)$ and mean-var efficient portfolios based on un-decomposed return series (mv).

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