A solution method for DSGE models with regionally biased monetary policymakers: the case of ECB voting reform

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Abstract

The euro adoption in Lithuania in 2015 triggered the entry into force of a major institutional change in the ECB Governing Council, i.e. rotation of voting rights between country representatives. In the likely case of home-biased voting preferences, such a rotation scheme leads to violation of the assumption that monetary policy parameters are constant. This paper introduces an extension to the algorithm of solving DSGE models under such circumstances. We generalize the standard, Blanchard-Kahn-like methods of solving DSGE models to the case compatible with the new ECB setup (or any similar setup, such as Fed's), i.e. time-varying non-stochastic structural parameters, recurring in a finite cycle. Using a standard, New Keynesian open economy model, we apply the proposed algorithm to demonstrate that the impact of rotation on macroeconomic volatility should remain limited, at least under moderate home bias in policymakers' preferences and historical degree of asymmetry in shocks.

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Keywords: solving rational expectations models, DSGE, ECB Governing Council, monetary policy, monetary union.

1 Introduction

It may have been hardly noticeable for the euro area economy as a whole, but it eventually triggered a major institutional change: on 1st January 2015, Lithuania became the 19th member of the euro

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area. The number of euro area countries thereby exceeded 18, which had long before been defined as the point at whichthe European Central Bank Governing Council (the euro area's monetary policy council) shall introduce the new voting system. In this system, only part of the Council members (majority of them being governors of euro countries' central banks) are entitled to vote on the interest rates, and the voting rights rotate over time.

Until 2015, the Council was composed of all the national central bank governors from the euro area countries with the right of vote in every decision meeting, as well as the ECB Board of Directors. It has long been acknowledged that, in this institutional setup, further euro area enlargement would imply a growing number of the former group, which in turn would lower the effectiveness of the decision process due to coordination problems (see e.g. Gerlach-Kristen, 2005). In 2003, the Treaty of Nice initially defined the rotation framework and the European Union leaders decided to set the implementation date at the moment when the number of euro area countries exceeds 15; once that was the case in 2009, the details of the rotating scheme were issued (European Central Bank, 2009), but the implementation was further postponed until the number of countries exceeds 18.

In a perfect world, it would not be incorrect to view the composition of the Council as pure technicality, as the members fulfil the same mandate of maintaining the price stability in the euro area as a whole, without any regional perspective (see European Central Bank, 2003, for some discussion). However, in the post-crisis Europe (and elsewhere) growing national or regional centrifugal forces have been increasingly visible – *Brexit* being the most prominent example. Under such circumstances, one can expect that country representatives in supranational bodies such as the ECB are likely to remain under some kind of pressure to reveal more home-biased policy preferences. Even before the financial, economic and euro crisis, a number of authors (Baldwin et al., 2001; de Grauwe, 2003) pointed to the risk of growing (and over-proportional) impact of the EU's New Member States (Central and Eastern Europe). If we acknowledge this home bias in monetary policy modelling, even to a minor extent, then an introduction of a rotation scheme can constitute a new source of macroeconomic volatility.

In this paper, we demonstrate that a DSGE model of monetary policy that incorporates (i) a rotation scheme and (ii) some home bias of the Council members in preferred interest rate decisions leads to new methodological challenges. Standard solution techniques applicable to constant-parameter DSGE models are not applicable here (Blanchard and Kahn, 1980; Uhlig, 1999; Klein, 2000; Christiano, 2002), and the available solution techniques for time-varying parameters are designed to account for different types of time-variability patterns (Markov-Switching DSGE – Farmer et al., 2011, occasionally binding constraints –Guerrieri and Iacoviello, 2015 or solution under terminal conditions – Jung et al., 2005). We address this issue by generalizing the algorithm of Klein (2000) to the case of time-varying, nonstochastic parameters recurring in a finite cycle. We build upon a similar exercise by Torój (2009) that was applied, however, to an *ad-hoc* model rather than a micro-founded DSGE model. Our main objective is to use the proposed algorithm to simulate the impact of the rotation scheme on the macroeconomic volatility. However, the proposed method could in principle be applied to efficiently handle any parameter fluctuation problem of this type, in particular – seasonality patterns, which are normally absent from DSGE modelling.

The rest of the paper is organized as follows. Section 2 reviews previous literature on rotation schemes in multinational or federalist decision-making bodies, especially in monetary policy. Section 3 presents an illustrative, standard New Keynesian DSGE model of a monetary union and develops the extensions that challenge the standard solution procedures when a rotation scheme is introduced for home-biased policymakers. Section 4 proposes a method of solving a model with variable coefficients that recur in a finite cycle. Section 5 presents an application of the considered methods in simulations. Section 6 concludes.

2 Literature review

The new rotation system at the ECB Governing Council envisages the partitioning of the euro area countries into 2 rotation groups: 5 states with 4 votes and the rest of the states with 11 votes. Furthermore, upon exceeding the number of 22 member countries, 3 rotation groups would be created: 5 states with 4 votes, half of the states (rounded up if necessary) with 8 votes and the rest of the states with 3 votes. The distribution of states between groups is based on the ranking with respect to the following indicator:

$$V_j = \frac{5}{6} \cdot \frac{GDP_j}{GDP_{EA}} + \frac{1}{6} \cdot \frac{ABSMFI_j}{ABSMFI_{EA}} \tag{1}$$

where: GDP – gross domestic product at market prices, $ABSMFI$ – aggregated balance sheet of monetary financial institutions, EA – euro area index, j – index for state j. The update of the ranking will be performed every time the Council is extended, or every 5 years.

The main voice of criticism related to the new system focuses on the fact that it re-emphasises the national composition of the Council, thereby 're-nationalising' the euro monetary policy and hence taking a step back in the monetary unification of the euro area (cf. Belke, 2003; Friedrich, 2003). A number of empirical analyses expose the relationship between the individual interest rate decisions of monetary policymakers and the economic situation of their region of origin. This could be the case not only for the euro area (Heinemann and Huefner, 2004), but also for monetary policy councils of highly integrated, federalist states: USA (Gildea, 1992; Meade and Sheets, 2005) and Germany (Berger and de Haan, 2002).

Further reservations towards the new voting system made in the literature were related i.a. to: low transparencyBelke (2003), arbitrary construction of the indicator V_j (not least leading the inclusion of Luxembourg in the second group – Belke, 2003; Meade, 2003), inability to address the issue of inefficient collective decision making process (the number of voters remained high and the discussion rights – unrestricted), as well as the absence of EU New Member States from the reforming process (Kosior et al., 2009).

Table 1: Voting reform in the ECB Governing Council: review of the literature

Study	Tools applied	Conclusions
Aksoy et al. (2002)	Standard New Keynesian framework. Cross-country heterogeneity stems from different monetary policy preferences, transmission mechanisms and business cycle developments.	The Board of Directors can effectively lead the monetary policy even when governors of individual central banks are home biased. Pro-european focus, however, maximizes the welfare.
Bénnasy-Quéré and Turkisch (2005)	Regional bias combined with rotation system. No endogeneity of future output or inflation with respect to interest rates.	Introduction of rotation will impact the effectiveness of ECB policy to a limited extent. Low rotation frequency would be beneficial to the „old" member states.
Paczyński (2006)	Regional bias combined with rotation system. Various degrees of home bias and decision rules considered.	Substantial home bias of the Council members might lead to serious policy errors.
Belke and Styczynska (2006)	Voting power indices and the regional bias of the Board's members.	The rotation system strengthens the ECB Board of Directors and – marginally – the big euro area economies. Sudden shifts in voting power could boost output and inflation volatility.
Fahrholz and Mohl (2006)	Voting power indices.	The rotation system strengthens the ECB Board of Directors and – marginally – the big euro area economies.
Kosior et al. (2008)	Voting power indices. New Keynesian model.	Pro-european focus of the Council's members minimizes output and inflation volatility.
Berger and Knuetter (2012)	Literature review, critical discussion.	'Only a move toward more centralised decision making promises efficient, timely, and transparent decisions that avoid national biases.'

Source: Kosior et al. (2008); author.

Based on previous literature (see Table 1), and focusing on the aim and scope of this paper, a few major points can be made. Firstly, national focus of policymakers appears to exist, and the 'one-size-fits-all' problem is likely to aggravate it. Secondly, any national focus of the Council members generates welfare losses and difficulties in policy making and coordination. Thirdly, shifts of voting power between Council meetings will take place. The latter effect could be anticipated by the markets in the entire euro area, and this motivates the use of rational expectations DSGE model in simulating the effects of this reform in terms of growing volatility. Such a perspective is largely missing in the literature, and the following sections attempt to fill this gap.

3 DSGE model of the monetary union

This Section presents a standard New Keynesian DSGE model of a 2-sector, open economy. The model builds strongly upon multi-region currency union models, such as e.g. ones considered in the works by Benigno (2004), Lombardo (2006), Brissimis and Skotida (2008), Kolasa (2009) and Torój (2016). A number of nominal and real rigidities are included in the model, such as staggered price and wage setting, backward-looking indexation schemes and consumption habits. For clarity of presentation, we discuss a 2-country version of the model below (home *versus* foreign economy), but generalisation to a higher number of countries made in the following Sections is straightforward. Henceforth, parameters describing the foreign economy are denoted analogously to home economy and marked with an asterisk, e.g. σ and σ^*. Lowercase letters denote the log-deviations of their uppercase counterparts from the steady-state values.

3.1 Households

The analysed, 2-region economy is inhabited by a continuum of infinitely lived households, represented by the interval $[0;1]$, whereby the households in the first region (say, home economy) are indexed over $[0;w]$ (relative size of the region: w), and the in the second region (say, foreign economy) over $[w;1]$. A representative household in the home economy derives utility from consumption and disutility from hours worked. The constant relative returns to scale utility function takes the following form (cf. Galí, 2008; we drop index $j \in \langle 0;w \rangle$ for variables U, C, H and N using the fact that the household is representative):

$$U_t\left(C_t, N_t, H_t\right) = \epsilon_{d,t}\frac{\left(C_t - H_t\right)^{1-\sigma}}{1-\sigma} - \epsilon_{l,t}\frac{N_t^{1+\phi}}{1+\phi}, \tag{2}$$

where C_t – consumption at t, H_t – stock of consumption habits at t, N_t – hours worked at t, $\epsilon_{d,t}$ – demand shock at t, $\epsilon_{l,t}$ – labour supply shock at t, $\sigma > 0$ and $\phi > 0$. Consumption habits are assumed to be proportional to consumption at $t-1$ (see Fuhrer, 2000; Smets and Wouters, 2003):

$$H_t = hC_{t-1}, \tag{3}$$

with $h \in [0;1)$. Apart from supplying labour and consuming, all households can access complete markets of Arrow securities priced D_t at the moment of purchase. This leads to both intertemporal consumption reallocations and international risk sharing (see also Chari et al., 2002; Galí, 2008; Kolasa, 2009; Lipińska, 2014), equalising marginal utility from consumption both across space and time. Households also receive a lump-sum subsidy (which is a frequent technical assumption made to restore an efficient steady state in the presence of firm's market power in the monopolistic competition model, cf. Galí, 2008), denoted as T_t net of lump-sum taxation.

Domestic households maximize at t the discounted flow of expected future utilities:

$$E_t\sum_t^{\infty}\beta^t U\left(C_t, N_t, H_t\right) \rightarrow \underset{C_t, N_t}{max}, \tag{4}$$

where $\beta \in (0,1)$ is the discount factor. Maximization of (4) is subject to a sequence of the following period budget constraints faced by a representative household:

$$P_tC_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t \tag{5}$$

where P_t denotes the price of the consumption unit.

The representative household consumes two bundles – tradable and nontradable goods:

$$C_t \equiv \left[(1-\kappa)^{\frac{1}{\delta}} C_{T,t}^{\frac{\delta-1}{\delta}} + \kappa^{\frac{1}{\delta}} C_{N,t}^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}}, \tag{6}$$

where $\kappa \in (0;1)$ characterizes the steady-state share of nontradables in the home economy and $\delta > 0$ is the elasticity of substitution between the goods produced in both sectors.

The domestic consumption of tradables at t consists of goods produced at home, $C_{H,t}$, and abroad, $C_{F,t}$:

$$C_{T,t} \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} . \tag{7}$$

An analogous relationship holds for the foreign economy. Given this, α is an intuitive measure of degree of openness and hence $1-\alpha$ captures the home bias in consumption. $\eta > 0$ is the elasticity of substitution between home and foreign tradable goods.

The consumption of domestic tradable goods in the home economy ($C_{H,t}$) and in the foreign one ($C^*_{H,t}$) is defined respectively as:

$$C_{H,t} \equiv \left[\left(\frac{1}{w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_0^w C^j_{H,t,k} dj \right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk \right]^{\frac{\varepsilon_T}{\varepsilon_T-1}}, \qquad C^*_{H,t} \equiv \left[\left(\frac{1}{w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_0^w C^{j*}_{H,t,k} dj \right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk \right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} . \tag{8}$$

Domestic households also consume foreign tradable goods bundle ($C_{F,t}$). Foreign goods are also obviously consumed by foreign consumers ($C^*_{F,t}$):

$$C_{F,t} \equiv \left[\left(\frac{1}{1-w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_w^1 C^j_{F,t,k} dj \right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk \right]^{\frac{\varepsilon_T}{\varepsilon_T-1}}, \qquad C^*_{F,t} \equiv \left[\left(\frac{1}{1-w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_w^1 C^{j*}_{F,t,k} dj \right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk \right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} . \tag{9}$$

The parameter $\varepsilon_T > 1$ measures the elasticity of substitution between various types of goods in international trade, k indexes the varieties of goods, and j – the households (integral over j reflects the difference in both economies' size).

The nontradable consumption bundles, domestic ($C_{N,t}$) and foreign ($C_{N*,t}$), are characterized in a similar fashion as:

$$\begin{gathered} C_{N,t} \equiv \left[\left(\frac{1}{w}\right)^{\frac{1}{\varepsilon_N}} \int_0^1 \left(\int_0^w C^j_{N,t,k} dj \right)^{\frac{\varepsilon_N-1}{\varepsilon_N}} dk \right]^{\frac{\varepsilon_N}{\varepsilon_N-1}}, \\ C_{N*,t} \equiv \left[\left(\frac{1}{1-w}\right)^{\frac{1}{\varepsilon_{N*}}} \int_0^1 \left(\int_w^1 C^{j*}_{N*,t,k} dj \right)^{\frac{\varepsilon_{N*}-1}{\varepsilon_{N*}}} dk \right]^{\frac{\varepsilon_{N*}}{\varepsilon_{N*}-1}} . \end{gathered} \tag{10}$$

Consequently, ε_N and ε_{N*} are defined as elasticities of substitution between various types of nontradable goods, in the home and foreign economy. Consumption bundles defined in (8)-(10) are characterised by their respective unit prices $P_{H,t}$, $P_{F,t}$, $P_{N,t}$ and $P_{N*,t}$.

By solving the representative household problem, we establish the optimum consumption of individual varietes as a function of their price differentials versus their respective consumption baskets, as well as the volumes of these baskets. We aslo obtain the Euler equations for consumption (domestic and foreign), labour supply equation (domestic and foreign) and international risk sharing condition.

The labour supply condition, stating equality between marginal rate of substitution between consumption and leisure, mrs_t, and the real wage, $w_t - p_t$, holds only in the long run. In the short run, households cannot freely adjust nominal wages. We apply a simplified version of labour market rigidities based on the proposal of Erzeg et al. (2000), in which nominal wages are sticky and follow the Calvo (1983) scheme. Only a fraction of households, $1 - \theta^w \in (0; 1)$, can renegotiate their wages in every period. This fraction remains constant and households allowed to reoptimize are selected at random. In particular, the probability of being allowed to renegotiate the wage does not depend on the amount of time elapsed since the last change. Other households partly index their their wages to past consumer inflation. Their fraction is represented by the parameter $\omega^w \in (0; 1)$. Under monopolistic competition in the labour market, individual domestic and foreign households supply differentated types of labour services with the elasticity of substitution ε_w. These assumptions lead to wage Phillips curve, describing nominal wage dynamics π^w.

3.2 Producers

The producers of every variety k in the tradable or nontradable bundle face a single-factor Cobb-Douglas production function with constant returns to scale (see Galí, 2008). This leads to sectoral production functions, and further – to real marginal costs equations in every sector (H, F, N, N^*). Supply shocks may occur in every sector (denoted as ϵ_t^H and ϵ_t^N in the home economy, and ϵ_t^F and ϵ_t^{N*} in the foreign one).

Producers maximise the discounted flow of future expected profits under constraints resulting from the presence of nominal price rigidities in the economy. Following the usual approach in the New Keynesian literature, we model these rigidities by means of the Calvo (1983) scheme. In a given period, a fraction θ of producers are not allowed to reoptimise their prices in reaction to economic innovations and must sell at the price from the previous period. The probability of being allowed to reoptimise the price is equal across producers: $1 - \theta$ in each period, independently of the amount of time elapsed since the last price change. Fraction ω of the producers able to change the price use a backward-looking indexation scheme. This mechanism leads to a hybrid Phillips curve (see Galí and Gertler, 1999; Galí et al., 2001).

Separate θ and ω paramterers characterise each of 4 production sectors $(H, F, N, N*)$. Domestic consumer inflation rate (π_t) is a weighted average of relevant sectoral inflation rates (π_t^H, π_t^F, π_t^N). The same is true for the foreign economy (up to weights, and with π_t^{N*} instead of π_t^N).

3.3 Market clearing conditions

The markets of every product variety j, whether supplied in the home or foreign economy, clears when its output equals its worldwide consumption:

$$Y_{t,k} = \int_0^1 C_{t,k}^j dj. \tag{11}$$

Indices k can be grouped into four markets: H, F, N and N* goods, in which the resulting, aggregate conditions hold. To derive them, it is convenient to defne the following price ratios: first, bilateral terms of trade between the home and foreign economy price ratio of domestic tradable output to foreign tradable output:

$$S_t \equiv \frac{P_{H,t}}{P_{F,t}}, \tag{12}$$

and second, internal terms of trade as price ratio between tradables and nontradables (separately in the home and foreign economy):

$$X_t \equiv \frac{P_{T,t}}{P_{N,t}}, \qquad X_t^* \equiv \frac{P_{T,t}^*}{P_{N,t}^*}. \tag{13}$$

whereby $P_{T,t}$ is defined, consistently with (7), as an aggregate of domestic and foreign tradable goods. Using (11), (12) and (13), as well as household optimality conditions, we can express the output is as a function of domestic consumption, foreign consumption, terms of trade, domestic and foreign internal terms of trade, and the model parameters.

3.4 Monetary policy: baseline framework

The union-wide monetary policy is described by a Taylor (1993) rule with smoothing:

$$i_t = (1-\rho)\left[r^* + \pi^* + \gamma_\pi\left(\tilde{\pi}_t - \pi^*\right) + \gamma_y \tilde{y}_t\right] + \rho i_{t-1}, \tag{14}$$

with i_t – nominal central bank rate at time t, $\tilde{y}_t$ – output in the monetary union, $\tilde{\pi}_t$ – inflation rate in the monetary union (as deviations from the steady state), r^* – natural interest rate, π^* – inflation target of the union's central bank, $\rho \in (0;1)$ – smoothing parameter, $\gamma_\pi > 1$, $\gamma_y > 0$ – parameters for central bank reaction to deviation of inflation from the inflation target and output, respectively. The condition $\gamma_\pi > 1$ is required for the Taylor principle to be satisfied and the equilibrium to be determinate (Taylor, 1993). The inflation rate and output in the entire monetary union are calculated as weighted averages over the member countries:

$$\tilde{\pi}_t = \sum_{j=1}^{n} w_j \pi_{j,t}, \qquad \pi_{j,t} = (1-\kappa)(1-\alpha)\pi_{j,t}^H + (1-\kappa)\alpha\pi_{j,t}^F + \kappa\pi_{j,t}^N, \tag{15}$$

$$\tilde{y}_t = \sum_{j=1}^{n} w_j y_{j,t}, \qquad y_{j,t} = (1-\kappa)y_{j,t}^H + \kappa y_{j,t}^N. \tag{16}$$

In the baseline case, country weights (vector $\mathbf{w}_{n\times 1}$) reflect relative sizes of n economies ($j = 1,...,n$) participating in the monetary union. Technically, as the ECB defines the price stability target in terms of the area-wide Harmonized Index of Consumer Prices dynamics („close to, but below 2%"), the weights could be associated with country weights for the area-wide HICP formula, published by the Eurostat. These are derived from national accounts as the share of consumption spendings of households in a given country in the analogous value for the euro area.[1] They evolve sluggishly and most of their volatility was triggered by accessions to the euro area (Greece, Slovenia, Malta, Cyprus, Slovakia).

3.5 Extension: rotation scheme in the ECB Governing Council

In line with the streams of criticism reported in Section 2, *in the alternative case* we can assume that every central bank governor implicitly prefers some nominal interest rate level, conditional upon the (possibly asymmetric) cyclical position of their country of origin:

$$i_{j,t} = (1-\rho)\left[r^* + \pi^* + \gamma_\pi\left(\pi_{j,t} - \pi_j^*\right) + \gamma_y y_{j,t}\right] + \rho i_{t-1}. \tag{17}$$

If he or she wanted to reduce the cyclical stress in their country of origin (see Clarida et al., 1999; Calmfors, 2007), they would be inclined to vote in favour of interest rate changes towards $i_{j,t}$, even if these changes were at odds with (14).[2] The final preference of the national central bank governor, declared in the voting, is defined as a weighted average of the „pro-european" rate in (14) and the preferred rate for his country of origin, as in (17):

$$\tilde{i}_{j,t} = (1-hb)\, i_t + hb i_{j,t}. \tag{18}$$

The parameter $hb \in [0;1]$ measures the 'home bias' in the decision of the Council's members. In this paper, we assume equal hb across all Council members, possibly as a symmetric multi-period equilibrium. With fully 'pro-European' voters, $hb = 0$. The other limiting case of fully home-biased voters occurs when $hb = 1$.

The outcome of voting at t is approximated by the arithmetic average over preferences submitted by the governors allowed to vote at t. Let $a_{j,t}$ be a dummy equal 1 when country j representative has got the right to vote at t and 0 otherwise. With these assumptions, the final interest rate decision of the ECB can be written as:

$$\bar{i}_t = \frac{1}{\sum_j a_{j,t}} \sum_{j=1}^{n} a_{j,t} \cdot \left[(1-\alpha)\, i_t + \alpha i_{j,t}\right]. \tag{19}$$

Note that, in our numerical example, however, we do not consider the option of home-biased ECB Board members (the generalisation is straightforward).

Substituting (14)-(16) and (17) into (19), we obtain the final form of the Taylor rule for the ECB:

$$\begin{aligned} \bar{i}_t &= (1-\rho)\{r^* + \pi^* + \\ &+ \left[(1-\alpha)\,\mathbf{w}^T + \alpha \mathbf{a}_t^T\right]\gamma_\pi\left(\boldsymbol{\pi_t} - \boldsymbol{\pi^*}\right) + \left[(1-\alpha)\,\mathbf{w}^T + \alpha \mathbf{a}_t^T\right]\gamma_y \mathbf{y_t}\} + \\ &+ \rho i_{t-1}, \end{aligned} \tag{20}$$

where symbols in bold subscripted t are vectors of size $n \times 1$ containing a sequence of identically denoted variables over countries, and $\boldsymbol{\pi}^* = \pi^* \cdot \mathbf{1}_{n\times 1}$.

Note that if the home bias of the central bank governors is non-zero, the rotation scheme implies that the Taylor rule parameters for inflation rates and output gaps in individual economies vary in time. In other words, country weights in the nominal interest rate equation, as opposed to equations (15)-(16), are non-constant. In consequence, so are the parameter matrices in the log-linearised model. That is exactly what prevents us from applying standard solution methods.

3.6 Stochastic properties of the shocks

For every type of shock in the model ε_t^j (with $j \in \{D, D^*, H, F, N, N^*, W, W^*\}$ and $\varepsilon_t^j = \ln \epsilon_t^j$), we assume an autoregressive process:

$$\varepsilon_t^j = \rho_j \varepsilon_{t-1}^j + u_t^j, \tag{21}$$

with $u_t^j \sim N\left(0, \sigma_j^2\right)$. We allow the shocks of given types to be correlated between regions (i.e. D with D^*, H with F, N with N^*, W with W^*), but we assume independence between types of shocks.

3.7 Parameter values

The parameter values used in the simulation were derived from Torój (2016) as the respective parameters for the euro area. Part of them were calibrated, and the rest – estimated with Bayesian methods as posterior means (see Table 2).

Table 2: Parameters of the model

parameter	value	parameter	value	parameter	value	parameter	value
β	0.995	h	0.3627	θ_H	0.3519	ω_H	0.2106
α	0.3835	σ	1.0321	θ_N	0.7512	ω_N	0.7224
κ	0.7822	η	0.4009	θ_W	0.6136	ω_W	0.3350
δ	0.8754	ϕ	2.2064				
σ_d	0.0257	correlation D	0.3422	ρ_d	0.5881	γ_π	1.5699
σ_H	0.0158	correlation $H-F$	0.2634	ρ_H	0.3028	γ_ρ	0.6737
σ_N	0.0195	correlation $N-N^*$	0.2034	ρ_N	0.3768	γ_y	0.8670
σ_W	0.0880	correlation W	0.1713	ρ_W	0.5417		

Source: author.

4 Solving the linear rational expectations model: baseline and extended version

4.1 Baseline solution

For the sake of completeness and transparency of presentation, in this Subsection, we present the method of Klein (2000) to establish notation and lay ground for analogies. For future reference, we also indicate the points in the algorithm that will need to be changed when the assumption of parameter constancy will be relaxed.

The model composed of log-linearised equations including the Taylor rule in variant (14) can be cast into the following matrix form:

$$\mathbf{A}E_t\mathbf{x}_{t+1} = \mathbf{B}\mathbf{x}_t + \mathbf{C}\mathbf{f}_t, \tag{22}$$

whereby $\mathbf{x}_t$ contains all the log-deviations of model variables from the steady state, $\mathbf{f}_t$ – vector of shocks, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ – constant matrices of model parameters.

The solution of a dynamic linear model with rational expectations written as (22) is a transformation of (22) into a recursive law of motion (see Blanchard and Kahn, 1980; Uhlig, 1999; Klein, 2000; Sims, 2001):

$$\mathbf{x}_t = \mathbf{M}\mathbf{x}_{t-1} + \mathbf{N}\mathbf{f}_t. \tag{23}$$

Klein (2000) applies to matrices $\mathbf{A}$ and $\mathbf{B}$ in (22) a complex generalized Schur decomposition. It produces matrices $\mathbf{Q}$, $\mathbf{Z}$, $\mathbf{S}$ and $\mathbf{T}$ such that

$$\begin{array}{c}\mathbf{QAZ} = \mathbf{S} \\ \mathbf{QBZ} = \mathbf{T}\end{array}, \tag{24}$$

whereby $\mathbf{S}$ and $\mathbf{T}$ – upper triangular matrices, $\mathbf{Q}$ and $\mathbf{Z}$ – unitary matrices ($\mathbf{QQ}^H = \mathbf{Q}^H\mathbf{Q} = \mathbf{ZZ}^H = \mathbf{Z}^H\mathbf{Z} = \mathbf{I}$).[3] Without loss of generality, suppose that $\mathbf{x}_t$ is partitioned into $\mathbf{x}_{\mathbf{1},t}$ containing variables predetermined at t and $\mathbf{x}_{\mathbf{2},t}$ containing variables non-predetermined at t.

Klein (2000) defines the following substitution:

$$\tilde{\mathbf{x}}_t = \mathbf{Z}^H \mathbf{x}_t \ . \tag{25}$$

Given (25) and after conformable partitioning of the matrices, we can express (22) as

$$\begin{bmatrix} \mathbf{S_{11}} & \mathbf{S_{12}} \\ 0 & \mathbf{S_{22}} \end{bmatrix} E_t \begin{bmatrix} \tilde{\mathbf{x}}_{\mathbf{1},t+1} \\ \tilde{\mathbf{x}}_{\mathbf{2},t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{T_{11}} & \mathbf{T_{12}} \\ 0 & \mathbf{T_{22}} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}_{\mathbf{1},t} \\ \tilde{\mathbf{x}}_{\mathbf{2},t} \end{bmatrix} + \begin{bmatrix} \mathbf{Q_1} \\ \mathbf{Q_2} \end{bmatrix} \mathbf{C}\mathbf{f}_t. \tag{26}$$

Upper-triangularity of $\mathbf{S}$ and $\mathbf{T}$ has decoupled the lower portion of the redefined vector $\tilde{\mathbf{x}}_t$, which can be solved out of (26) as follows:

$$\tilde{\mathbf{x}}_{\mathbf{2},t} = \mathbf{T_{22}^{-1}} \mathbf{S_{22}} E_t \tilde{\mathbf{x}}_{2,t+1} - \mathbf{T_{22}^{-1}} \mathbf{Q_2} \mathbf{C} \mathbf{f}_t. \tag{27}$$

Iterating (27) forward and using the law of iterated expectations (see Ljungqvist and Sargent, 2004), we express $\tilde{\mathbf{x}}_{\mathbf{2},t}$ as the following infinite sum:

$$\tilde{\mathbf{x}}_{\mathbf{2},t} = \lim_{k\to\infty} \left(\mathbf{T_{22}^{-1}} \mathbf{S_{22}}\right)^k E_t \tilde{\mathbf{x}}_{2,t+1} - \mathbf{T_{22}^{-1}} \left[\sum_{k=0}^{\infty} \left(\mathbf{S_{22}} \mathbf{T_{22}^{-1}}\right)^k \mathbf{Q_2} \mathbf{C} E_t \mathbf{f}_{t+k}\right]. \tag{28}$$

Note that the forward iteration of (27) to (28) exploits the assumption parameter constancy.

According to Proposition 1 by Blanchard and Kahn (1980), there exists a unique solution if the number of explosive eigenvalues (i.e. lying outside the unit circle) equals the number of non-predetermined variables. If this applied to the generalized eigenvalues of $\mathbf{A}$ and $\mathbf{B}$, all of the eigenvalues concentrated in the block (2,2) would be explosive and hence the infinite sum would exist and the limit would converge to zero.

Another condition formulated by Blanchard and Kahn (1980) is that the exogenous variables in $\mathbf{f}_t$ do not „explode too fast" in expectations:

$$\forall t \quad \exists \bar{\mathbf{f}}_t \in R^k, \theta_t \in R \quad \forall i \geq 0 \quad -(1+i)^{\theta_t} \bar{\mathbf{f}}_t \leq E\left(f_{t+i}\right) \leq (1+i)^{\theta_t} \bar{\mathbf{f}}_t. \tag{29}$$

In rational expectations models, autoregressive shocks are commonly assumed (as we do in (21)), so let us define a VAR representation:

$$\mathbf{f}_t = \mathbf{\Phi} \mathbf{f}_{t-1} + \varepsilon_t. \tag{30}$$

with $E_t\boldsymbol{\varepsilon}_{t+k}=0$, $k=1,2,....$ Stationarity of this process, i.e. nonexplosive eigenvalues of $\boldsymbol{\Phi}$, allow us to calculate (31). As $E_t\mathbf{f}_{t+k}=\boldsymbol{\Phi}^k f_t$ given (30), we can rewrite (31) as

$$\tilde{\mathbf{x}}_{\mathbf{2},t}=-\mathbf{T}_{\mathbf{22}}^{-1}\left[\sum_{k=0}^{\infty}\Big(\underbrace{\mathbf{S}_{\mathbf{22}}\mathbf{T}_{\mathbf{22}}^{-1}}_{\mathbf{F}}\Big)^k\underbrace{\mathbf{Q}_{\mathbf{2}}\mathbf{C}}_{\mathbf{G}}\underbrace{\boldsymbol{\Phi}}_{\mathbf{H}}{}^k\right]f_t=-\mathbf{T}_{\mathbf{22}}^{-1}\mathbf{L}\mathbf{f}_t. \tag{31}$$

Following Klein (2000), we calculate the elements of $\mathbf{L}$ using the vectorization operator:[4]

$$vec\left(\mathbf{L}\right)=\left[\mathbf{I}-\mathbf{H}^T\otimes\mathbf{F}\right]^{-1}vec\left(\mathbf{G}\right). \tag{32}$$

We use the solution for the unstable component, (31), in the upper portion of (26):

$$\mathbf{S}_{\mathbf{11}}E_t\tilde{\mathbf{x}}_{\mathbf{1},t+1}-\mathbf{S}_{\mathbf{12}}\mathbf{T}_{\mathbf{22}}^{-1}\mathbf{L}\boldsymbol{\Phi}\mathbf{f}_t=\mathbf{T}_{\mathbf{11}}\tilde{\mathbf{x}}_{\mathbf{1},t}-\mathbf{T}_{\mathbf{12}}\mathbf{T}_{\mathbf{22}}^{-1}\mathbf{L}\mathbf{f}_t+\mathbf{Q}_{\mathbf{1}}\mathbf{C}\mathbf{f}_t. \tag{33}$$

Rewriting (25) with the standard partitioning and using (31), we obtain::

$$\begin{bmatrix}\mathbf{x}_{\mathbf{1},t}\\ \mathbf{x}_{\mathbf{2},t}\end{bmatrix}=\begin{bmatrix}\mathbf{Z}_{\mathbf{11}} & \mathbf{Z}_{\mathbf{12}}\\ \mathbf{Z}_{\mathbf{21}} & \mathbf{Z}_{\mathbf{22}}\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{x}}_{\mathbf{1},t}\\ -\mathbf{T}_{\mathbf{22}}^{-1}\mathbf{L}\mathbf{f}_t\end{bmatrix}. \tag{34}$$

This allows us to express $\tilde{\mathbf{x}}_{\mathbf{1},t}$ in terms of $\mathbf{x}_{\mathbf{1},t}$:

$$\tilde{\mathbf{x}}_{\mathbf{1},t}=\mathbf{Z}_{\mathbf{11}}^{-1}\mathbf{x}_{\mathbf{1},t}+\mathbf{Z}_{\mathbf{11}}^{-1}\mathbf{Z}_{\mathbf{12}}\mathbf{T}_{\mathbf{22}}^{-1}\mathbf{L}\mathbf{f}_t. \tag{35}$$

Using (35) in (36) and the predeterminacy of $\mathbf{x}_{\mathbf{1},t+1}$ at t, i.e. $\mathbf{x}_{\mathbf{1},t+1}=E_t\mathbf{x}_{\mathbf{1},t+1}$, we obtain:

$$\mathbf{x}_{\mathbf{1},t+1}=\mathbf{Z}_{\mathbf{11}}\mathbf{S}_{\mathbf{11}}^{-1}\mathbf{T}_{\mathbf{11}}\mathbf{Z}_{\mathbf{11}}^{-1}\mathbf{x}_{\mathbf{1},t}+\left[\mathbf{Z}_{\mathbf{11}}\mathbf{S}_{\mathbf{11}}^{-1}\left(\mathbf{T}_{\mathbf{11}}\mathbf{Z}_{\mathbf{11}}^{-1}\mathbf{Z}_{\mathbf{12}}-\mathbf{T}_{\mathbf{12}}\right)\mathbf{T}_{\mathbf{22}}^{-1}\mathbf{L}-\left(\mathbf{Z}_{\mathbf{12}}-\mathbf{Z}_{\mathbf{11}}\mathbf{S}_{\mathbf{11}}^{-1}\mathbf{S}_{\mathbf{12}}\right)\mathbf{T}_{\mathbf{22}}^{-1}\mathbf{L}\boldsymbol{\Phi}+\mathbf{Z}_{\mathbf{11}}\mathbf{S}_{\mathbf{11}}^{-1}\mathbf{Q}_{\mathbf{1}}\mathbf{C}\right]\mathbf{f}_t. \tag{36}$$

Turning to $\mathbf{x}_{\mathbf{2},t}$, it can be expressed in terms of $\mathbf{x}_{\mathbf{1},t}$ and $\mathbf{f}_t$ using (34) and (35):

$$\mathbf{x}_{\mathbf{2},t}=\mathbf{Z}_{\mathbf{21}}\mathbf{Z}_{\mathbf{11}}^{-1}\mathbf{x}_{\mathbf{1},t}+\left(\mathbf{Z}_{\mathbf{21}}\mathbf{Z}_{\mathbf{11}}^{-1}\mathbf{Z}_{\mathbf{12}}-\mathbf{Z}_{\mathbf{22}}\right)\mathbf{T}_{\mathbf{22}}^{-1}\mathbf{L}\mathbf{f}_t. \tag{37}$$

Equations (36) and (37) are the solution of the model (22).

4.2 Extended solution: model with finite-cycle time-varying parameters

The time-varing Taylor rule (20) requires solving a model like (22), but with time-varying parameters[5]:

$$\mathbf{A}_{(t)}E_t\left(\mathbf{x}_{t+1}\right)=\mathbf{B}_{(t)}\mathbf{x}_t+\mathbf{C}_{(t)}\mathbf{f}_t. \tag{38}$$

It is useless to start with a single generalized Schur decomposition because the factor matrices we would obtain inherit the nonconstancy and parameter matrices for $\mathbf{x}_t$ and $E_t\left(\mathbf{x}_{t+1}\right)$ would not be upper triangular as we need.[6] Instead, we exploit the assumption that $\mathbf{A}_{(t)}$ and $\mathbf{B}_{(t)}$ vary in time, but the values recur after m periods, i.e. $\mathbf{A}_{(t+j)}=\mathbf{A}_{(t+j+i\cdot m)}$ and $\mathbf{B}_{(t+j)}=\mathbf{B}_{(t+j+i\cdot m)}$ for each $j=0,1,...,m-1$ and each $i\in\mathbb{N}$. Let us first factorize the matrices $\mathbf{A}_{(t)}$ and $\mathbf{B}_{(t)}$ for each period in the cycle using a sequence of generalized complex Schur decompositions:

$$\begin{array}{c}\mathbf{Q}_{(t)}\mathbf{A}_{(t)}\mathbf{Z}_{(t)}=\mathbf{S}_{(t)}\\ \mathbf{Q}_{(t)}\mathbf{B}_{(t)}\mathbf{Z}_{(t)}=\mathbf{T}_{(t)}\end{array}, \tag{39}$$

with $\mathbf{S}$,$\mathbf{T}$,$\mathbf{Q}$ and $\mathbf{Z}$ bearing the same properties as their counterparts in the standard case (Subsection 4.1). For the decomposition to be unique, we impose a restriction that diagonal elements of $\mathbf{S}$ and $\mathbf{T}$ are ordered in such a way that generalized eigenvalues of $\mathbf{A}$ and $\mathbf{B}$ (equal $\frac{S_{i,i}}{T_{i,i}}$) ascend with rising index i.

Using (24) we can rewrite (38) for each t as:

$$\mathbf{S}_{(t)}\mathbf{Z}_{(t)}^{H}E_t\mathbf{x}_{t+1}=\mathbf{T}_{(t)}\mathbf{Z}_{(t)}^{H}\mathbf{x}_t+\mathbf{Q}_{(t)}\mathbf{C}_{(t)}\mathbf{f}_t. \tag{40}$$

Let us write the equation for t, $t+1$, ..., $t+m-1$ and solve each of them for $\mathbf{x}$:

$$\begin{array}{ll}\mathbf{x}_t= & \mathbf{Z}_{(t)}\mathbf{T}_{(t)}^{-1}\mathbf{S}_{(t)}\mathbf{Z}_{(t)}^{H}E_t\left(\mathbf{x}_{t+1}\right)-\mathbf{Z}_{(t)}\mathbf{T}_{(t)}^{-1}\mathbf{Q}_{(t)}\mathbf{C}_{(t)}\mathbf{f}_t\\ \mathbf{x}_{t+1}= & \mathbf{Z}_{(t+1)}\mathbf{T}_{(t+1)}^{-1}\mathbf{S}_{(t+1)}\mathbf{Z}_{(t+1)}^{H}E_{t+1}\left(\mathbf{x}_{t+2}\right)-\mathbf{Z}_{(t+1)}\mathbf{T}_{(t+1)}^{-1}\mathbf{Q}_{(t+1)}\mathbf{C}_{(t+1)}\mathbf{f}_{t+1}\\ \vdots & \\ \mathbf{x}_{t+m-1}= & \mathbf{Z}_{(t+m-1)}\mathbf{T}_{(t+m-1)}^{-1}\mathbf{S}_{(t+m-1)}\mathbf{Z}_{(t+m-1)}^{H}E_{t+m-1}\left(\mathbf{x}_{t+m}\right)+\\ & -\mathbf{Z}_{(t+m-1)}\mathbf{T}_{(t+m-1)}^{-1}\mathbf{Q}_{(t+m-1)}\mathbf{C}_{(t+m-1)}\mathbf{f}_{t+m-1}\end{array}. \tag{41}$$

A bottom-up sequence of substitutions and the law of iterated expectations allow us to write an equation for $\mathbf{x}_t$:[7]

$$\mathbf{x}_t = \underbrace{\left[\prod_{i=0}^{m-1} \mathbf{Z}_{(t+i)} \mathbf{T}_{(t+i)}^{-1} \mathbf{S}_{(t+i)} \mathbf{Z}_{(t+i)}^{H}\right]}_{\mathbf{D}_{(t)}} E_t\left(\mathbf{x}_{t+m}\right) + \\ -\underbrace{\left\{\left[\sum_{k=1}^{m-1}\left(\prod_{l=1}^{k} \mathbf{Z}_{(t+l-1)} \mathbf{T}_{(t+l-1)}^{-1} \mathbf{S}_{(t+l-1)} \mathbf{Z}_{(t+l-1)}^{H}\right) \mathbf{Z}_{(t+k)} \mathbf{T}_{(t+k)}^{-1} \mathbf{Q}_{(t+k)} \mathbf{C}_{(t+k)} E_t \mathbf{f}_{t+k}\right] + \mathbf{Z}_{(t)} \mathbf{T}_{(t)}^{-1} \mathbf{Q}_{(t)} \mathbf{C}_{(t)} \mathbf{f}_t\right\}}_{\Sigma_{k=0}^{m-1} \mathbf{R}_{\mathbf{k}(t)} E_t \mathbf{f}_{t+k}}. \tag{42}$$

Note that at this point we assume that economic agents trust in the new system's sustainability and know the sequence of countries' rotation.

Once again, we perform a complex generalized Schur decomposition of $\mathbf{D}_{(t)}$ and $\mathbf{I}$ (as the parameter matrix for $\mathbf{x}_t$):

$$\begin{array}{c} \mathbf{Q}_{(t)} \mathbf{D}_{(t)} \mathbf{Z}_{(t)} = \mathbf{S}_{(t)} \\ \mathbf{Q}_{(t)} \mathbf{I} \mathbf{Z}_{(t)} = \mathbf{T}_{(t)} \end{array}, \tag{43}$$

with the usual restriction on ordering generalized eigenvalues. Let us define an auxiliary variable:

$$\tilde{\mathbf{x}}_t = \mathbf{Z}_{(t)}^{H} \mathbf{x}_t \ . \tag{44}$$

In line with the conventional treatment in the literature, let $\mathbf{x}_t$ be ordered in such a way that the first partition ($\mathbf{x}_{\mathbf{1},t}$) contains variables predetermined at t. Analogous partitioning of $\tilde{\mathbf{x}}_t$, substitution of (43) and (25) into (42), premultiplication by $\mathbf{Q}_{(t)}$ and conformable partitioning of $\mathbf{S}_{(t)}$, $\mathbf{T}_{(t)}$ and $\mathbf{Q}_{(t)}$ yield:

$$\begin{bmatrix} \mathbf{S}_{\mathbf{11}(t)} & \mathbf{S}_{\mathbf{12}(t)} \\ 0 & \mathbf{S}_{\mathbf{22}(t)} \end{bmatrix} E_t \begin{bmatrix} \tilde{\mathbf{x}}_{\mathbf{1},t+m} \\ \tilde{\mathbf{x}}_{\mathbf{2},t+m} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{\mathbf{11}(t)} & \mathbf{T}_{\mathbf{12}(t)} \\ 0 & \mathbf{T}_{\mathbf{22}(t)} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}_{\mathbf{1},t} \\ \tilde{\mathbf{x}}_{\mathbf{2},t} \end{bmatrix} + \begin{bmatrix} \mathbf{Q}_{\mathbf{1}(t)} \\ \mathbf{Q}_{\mathbf{2}(t)} \end{bmatrix} \left(\Sigma_{k=0}^{m-1} \mathbf{R}_{\mathbf{k}(t)} E_t \mathbf{f}_{t+k}\right). \tag{45}$$

Following Klein (2000), we solve the lower, decoupled row of (26) for $\tilde{\mathbf{x}}_{2,t}$:

$$\tilde{\mathbf{x}}_{\mathbf{2},t} = \mathbf{T}_{\mathbf{22}(t)}^{-1} \mathbf{S}_{\mathbf{22}(t)} E_t \tilde{\mathbf{x}}_{2(t),t+m} - \mathbf{T}_{\mathbf{22}(t)}^{-1} \mathbf{Q}_{\mathbf{2}(t)} \left(\Sigma_{k=0}^{m-1} \mathbf{R}_{\mathbf{k}(t)} E_t \mathbf{f}_{t+k}\right). \tag{46}$$

The finite cycle of length m, in which the parameters of $\mathbf{A}_{(t)}$ and $\mathbf{B}_{(t)}$ recur, implies $\mathbf{D}_{(t)} = \mathbf{D}_{(t+m)}$

and $\mathbf{R}_{\mathbf{k}(t)} = \mathbf{R}_{\mathbf{k}(t+m)}$ for each k. We can therefore shift (42) m periods forward without changing the parameters:

$$\mathbf{x}_{t+m} = \mathbf{D}_{(t)} E_{t+m} \left(\mathbf{x}_{t+2m} \right) + \Sigma_{k=0}^{m-1} \mathbf{R}_{\mathbf{k}(t)} E_{t+m} \mathbf{f}_{t+m+k}. \tag{47}$$

Matrices $\mathbf{Q}$, $\mathbf{Z}$, $\mathbf{S}$ and $\mathbf{T}$, resulting from the Schur decomposition of both matrices of interest in the above system, will equal those obtained in (43). Then, we can shift (27) by any multiple of m without changing the parameters:

$$\begin{aligned} &\tilde{\mathbf{x}}_{2,t+m} = \mathbf{T}^{-1}_{\mathbf{22}(t)} \mathbf{S}_{\mathbf{22}(t)} E_{t+m} \tilde{\mathbf{x}}_{2,t+2m} - \mathbf{T}^{-1}_{\mathbf{22}(t)} \mathbf{Q}_{\mathbf{2}(t)} \left(\Sigma_{k=0}^{m-1} \mathbf{R}_{\mathbf{k}(t)} E_{t+m} \mathbf{f}_{t+m+k} \right) \\ &\tilde{\mathbf{x}}_{2,t+2m} = \mathbf{T}^{-1}_{\mathbf{22}(t)} \mathbf{S}_{\mathbf{22}(t)} E_{t+2m} \tilde{\mathbf{x}}_{2,t+4m} - \mathbf{T}^{-1}_{\mathbf{22}(t)} \mathbf{Q}_{\mathbf{2}(t)} \left(\Sigma_{k=0}^{m-1} \mathbf{R}_{\mathbf{k}(t)} E_{t+2m} \mathbf{f}_{t+2m+k} \right) \; . \\ &\vdots \end{aligned} \tag{48}$$

As in (**??**), a sequence of substitutions in (27) and (48) and iterating expectations allow us to express $\tilde{\mathbf{x}}_{\mathbf{2},t}$ as an infinite sum:

$$\tilde{\mathbf{x}}_{\mathbf{2},t} = -\sum_{i=0}^{+\infty} \left\{ \left(\mathbf{T}^{-1}_{\mathbf{22}(t)} \mathbf{S}_{\mathbf{22}(t)} \right)^i \mathbf{T}^{-1}_{\mathbf{22}(t)} \mathbf{Q}_{\mathbf{2}(t)} \left(\Sigma_{k=0}^{m-1} \mathbf{R}_{\mathbf{k}(t)} E_t \mathbf{f}_{t+i\cdot m+k} \right) \right\}. \tag{49}$$

At this point, we need to know the expected path of future random disturbances, conditional on the information that agents have at t.[8] Like in Subsection (4.1), we proceed with an autoregressive error term (30).

With $E_t \boldsymbol{\varepsilon}_{t+k} = 0$, $k = 1, 2, ...$, we can write the infinite sum (49) as

$$\begin{aligned} \tilde{\mathbf{x}}_{\mathbf{2},t} &= -\sum_{i=0}^{+\infty} \left[\left(\mathbf{T}^{-1}_{\mathbf{22}(t)} \mathbf{S}_{\mathbf{22}(t)} \right)^i \mathbf{T}^{-1}_{\mathbf{22}(t)} \mathbf{Q}_{\mathbf{2}(t)} \left(\Sigma_{k=0}^{m-1} \mathbf{R}_{\mathbf{k}(t)} \boldsymbol{\Phi}^{i\cdot m+k} \mathbf{f}_t \right) \right] = \\ &= -\sum_{i=0}^{+\infty} \left[\left(\underbrace{\mathbf{T}^{-1}_{\mathbf{22}(t)} \mathbf{S}_{\mathbf{22}(t)}}_{\mathbf{F}_{(t)}} \right)^i \underbrace{\mathbf{T}^{-1}_{\mathbf{22}(t)} \mathbf{Q}_{\mathbf{2}(t)} \left(\Sigma_{k=0}^{m-1} \mathbf{R}_{\mathbf{k}(t)} \boldsymbol{\Phi}^k \right)}_{\mathbf{G}_{(t)}} \cdot \left(\underbrace{\boldsymbol{\Phi}^m}_{\mathbf{H}_{(t)}} \right)^i \right] \mathbf{f}_t = \\ &= -\mathbf{L}_{(t)} f_t. \end{aligned} \tag{50}$$

Using (51) again, we calculate the elements of $\mathbf{L}_{(t)}$ by means of the vectorization operator:

$$vec \left(\mathbf{L}_{(t)} \right) = \left[\mathbf{I} - \mathbf{H}^T_{(t)} \otimes \mathbf{F}_{(t)} \right]^{-1} vec \left(\mathbf{G}_{(t)} \right). \tag{51}$$

The existence of the infinite sum stems from (i) fulfilled assumptions of the Blanchard-Kahn theorem (exactly all unstable generalized eigenvalues of $\mathbf{A}$ and $\mathbf{B}$ concentrated in the partition (2,2) of matrices $\mathbf{S}$ and $\mathbf{T}$) as well as (ii) stability of the process (30) (eigenvalues of $\mathbf{\Phi}$ lower than 1 in absolute terms). Substitute (50) into (25) after premultiplication by $\mathbf{Z}_{(t)}$ and conformable partitioning:

$$\begin{bmatrix} \mathbf{x}_{\mathbf{1},t} \\ \mathbf{x}_{\mathbf{2},t} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{\mathbf{11}(t)} & \mathbf{Z}_{\mathbf{12}(t)} \\ \mathbf{Z}_{\mathbf{21}(t)} & \mathbf{Z}_{\mathbf{22}(t)} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}_{\mathbf{1},t} \\ -\mathbf{L}_{(t)}\mathbf{f}_t \end{bmatrix}. \tag{52}$$

After solving out $\tilde{\mathbf{x}}_{\mathbf{1},t}$ from (34), we obtain a linear relationship linking $\mathbf{x}_{\mathbf{1},t}$, $\mathbf{x}_{\mathbf{2},t}$ and $\mathbf{f}_t$:

$$\mathbf{x}_{\mathbf{2},t} = \mathbf{Z}_{\mathbf{21}(t)}\mathbf{Z}_{\mathbf{11}(t)}^{-1}\mathbf{x}_{\mathbf{1},t} + \left(\mathbf{Z}_{\mathbf{21}(\mathbf{t})}\mathbf{Z}_{\mathbf{11}(t)}^{-1}\mathbf{Z}_{\mathbf{12}(t)} - \mathbf{Z}_{\mathbf{22}(t)}\right)\mathbf{L}_{(t)}\mathbf{f}_t. \tag{53}$$

We exploit the predeterminacy of $\mathbf{x}_{\mathbf{1},t}$ to get:

$$\begin{aligned} E_t\left(\mathbf{x}_{t+1}\right) &= E_t\begin{pmatrix} \mathbf{x}_{\mathbf{1},t+1} \\ \mathbf{x}_{\mathbf{2},t+1} \end{pmatrix} = \\ &= \begin{bmatrix} \mathbf{x}_{\mathbf{1},t+1} \\ \mathbf{Z}_{\mathbf{21}(t+1)}\mathbf{Z}_{\mathbf{11}(t+1)}^{-1}\mathbf{x}_{\mathbf{1},t+1} + \left(\mathbf{Z}_{\mathbf{21}(t+1)}\mathbf{Z}_{\mathbf{11}(t+1)}^{-1}\mathbf{Z}_{\mathbf{12}(t+1)} - \mathbf{Z}_{\mathbf{22}(t+1)}\right)\mathbf{L}_{(t+1)}\underbrace{E_t\mathbf{f}_{t+1}}_{\mathbf{\Phi}f_t} \end{bmatrix} = \\ &= \begin{bmatrix} \mathbf{I} \\ \mathbf{Z}_{\mathbf{21}(t+1)}\mathbf{Z}_{\mathbf{11}(t+1)}^{-1} \end{bmatrix}\mathbf{x}_{\mathbf{1},t+1} + \begin{bmatrix} 0 \\ \left(\mathbf{Z}_{\mathbf{21}(t+1)}\mathbf{Z}_{\mathbf{11}(t+1)}^{-1}\mathbf{Z}_{\mathbf{12}(t+1)} - \mathbf{Z}_{\mathbf{22}(t+1)}\right)\mathbf{L}_{(t+1)}\mathbf{\Phi} \end{bmatrix}\mathbf{f}_t. \end{aligned} \tag{54}$$

Using (53), we can also replace $\mathbf{x}_{\mathbf{2},t}$ in $\mathbf{x}_t$:

$$\begin{aligned} \mathbf{x}_t &= \begin{bmatrix} \mathbf{x}_{\mathbf{1},t} \\ \mathbf{x}_{\mathbf{2},t} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{\mathbf{1},t} \\ \mathbf{Z}_{\mathbf{21}(t)}\mathbf{Z}_{\mathbf{11}(t)}^{-1}\mathbf{x}_{\mathbf{1},t} + \left(\mathbf{Z}_{\mathbf{21}(t)}\mathbf{Z}_{\mathbf{11}(t)}^{-1}\mathbf{Z}_{\mathbf{12}(t)} - \mathbf{Z}_{\mathbf{22}(t)}\right)\mathbf{L}_{(t)}\mathbf{f}_t \end{bmatrix} = \\ &= \begin{bmatrix} \mathbf{I} \\ \mathbf{Z}_{\mathbf{21}(t)}\mathbf{Z}_{\mathbf{11}(t)}^{-1} \end{bmatrix}\mathbf{x}_{\mathbf{1},t} + \begin{bmatrix} 0 \\ \left(\mathbf{Z}_{\mathbf{21}(t)}\mathbf{Z}_{\mathbf{11}(t)}^{-1}\mathbf{Z}_{\mathbf{12}(t)} - \mathbf{Z}_{\mathbf{22}(t)}\right)\mathbf{L}_{(t)} \end{bmatrix}\mathbf{f}_t. \end{aligned} \tag{55}$$

In the example considered here, the vector of predetermined variables $\mathbf{x}_{\mathbf{1},t}$ contains lags of all elements in $\mathbf{x}_{\mathbf{2},t}$. Accordingly, some rows in $\mathbf{A}_{(t)}$, $\mathbf{B}_{(t)}$ and $\mathbf{C}_{(t)}$ were trivial identities defining the equivalence between some elements of $\mathbf{x}_{\mathbf{1},t+1}$ and $\mathbf{x}_{\mathbf{2},t}$. With the relation between $\mathbf{x}_{\mathbf{1},t}$ and $\mathbf{x}_{\mathbf{2},t}$ in hand, we can drop these rows and denote the remaining matrices as $\overline{\mathbf{A}}_{(t)}$, $\overline{\mathbf{B}}_{(t)}$ and $\overline{\mathbf{C}}_{(t)}$. Rewrite (38) without these rows, using (54) and (55):

$$\overline{\mathbf{A}}_{(t)} \begin{bmatrix} \mathbf{I} \\ \mathbf{Z_{21}}_{(t+1)}\mathbf{Z_{11}}^{-1}_{(t+1)} \end{bmatrix} \mathbf{x}_{\mathbf{1},t+1} + \overline{\mathbf{A}}_{(t)} \begin{bmatrix} 0 \\ \left(\mathbf{Z_{21}}_{(t+1)}\mathbf{Z_{11}}^{-1}_{(t+1)}\mathbf{Z_{12}}_{(t+1)} - \mathbf{Z_{22}}_{(t+1)}\right)\mathbf{L}_{(t+1)}\mathbf{\Phi} \end{bmatrix} \mathbf{f}_t =$$
$$= \overline{\mathbf{B}}_{(t)} \begin{bmatrix} \mathbf{I} \\ \mathbf{Z_{21}}_{(t)}\mathbf{Z_{11}}^{-1}_{(t)} \end{bmatrix} \mathbf{x}_{\mathbf{1},t} + \overline{\mathbf{B}}_{(t)} \begin{bmatrix} 0 \\ \left(\mathbf{Z_{21}}_{(\mathbf{t})}\mathbf{Z_{11}}^{-1}_{(\mathbf{t})}\mathbf{Z_{12}}_{(\mathbf{t})} - \mathbf{Z_{22}}_{(\mathbf{t})}\right)\mathbf{L}_{(\mathbf{t})} \end{bmatrix} \mathbf{f}_t + \overline{\mathbf{C}}\mathbf{f}_t. \tag{56}$$

The solution of (56) with respect to $\mathbf{x}_{1,t+1}$ is the searched law of motion of the form (23):

$$\mathbf{x}_{\mathbf{1},t+1} = \left(\overline{\mathbf{A}}_{(t)} \begin{bmatrix} \mathbf{I} \\ \mathbf{Z_{21}}_{(t+1)}\mathbf{Z_{11}}^{-1}_{(t+1)} \end{bmatrix}\right)^{-1} \overline{\mathbf{B}}_{(t)} \begin{bmatrix} \mathbf{I} \\ \mathbf{Z_{21}}_{(t)}\mathbf{Z_{11}}^{-1}_{(t)} \end{bmatrix} \mathbf{x}_{\mathbf{1},t} +$$
$$+ \left(\overline{\mathbf{A}}_{(t)} \begin{bmatrix} \mathbf{I} \\ \mathbf{Z_{21}}_{(t+1)}\mathbf{Z_{11}}^{-1}_{(t+1)} \end{bmatrix}\right)^{-1} \cdot$$
$$\cdot \left(\overline{\mathbf{C}} + \overline{\mathbf{B}}_{(t)} \begin{bmatrix} 0 \\ \left(\mathbf{Z_{21}}_{(t)}\mathbf{Z_{11}}^{-1}_{(t)}\mathbf{Z_{12}}_{(t)} - \mathbf{Z_{22}}_{(t)}\right)\mathbf{L}_{(t)} \end{bmatrix} - \overline{\mathbf{A}}_{(t)} \begin{bmatrix} 0 \\ \left(\mathbf{Z_{21}}_{(t+1)}\mathbf{Z_{11}}^{-1}_{(t+1)}\mathbf{Z_{12}}_{(t+1)} - \mathbf{Z_{22}}_{(t+1)}\right)\mathbf{L}_{(t+1)}\mathbf{\Phi} \end{bmatrix}\right) \mathbf{f}_t. \tag{57}$$

5 Simulation results

The model developed in Section 3 and solved with method described in Subsection 4.2 was subsequently used to simulate the impact of rotation in the ECB Governing Council on macroeconomic volatility. To focus the attention, homogeneity of monetary union members with respect to parameter values from Table 2 was initially assumed (and subsequently relaxed in the sensitivity analysis).

We consider various simulation scenarios that differ in the following dimensions:

1. **Rotation frequency**: quarterly (rotation in every period), semi-annual (rotation every two periods) and annual (rotation every four periods).

2. **Home bias**: $hb = 0$, 0.1, 0.2, 0.3, 0.4 and 0.5. The parameter hb is additionally downward-scaled by a factor of $\frac{19}{25}$ corresponding to the presence of the ECB Executive Board in the ECB Governing Council. The Board is, by assumption, not home-biased.

Table 3: Simulation scenarios for rotation in ECB Governing Council

Scenario	Baseline				Alternative I				Alternative II					
country #	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**5**	**6**
1	1	0	0	1	0	1	0	1	1	0	1	1	0	0
2	1	1	0	0	1	0	1	0	1	1	0	0	1	0
3	0	1	1	0	1	0	1	0	0	1	1	0	0	1
4	0	0	1	1	1	1	0	0						
5					1	1	0	0						
% voting	50	50	50	50	80	60	40	20	66,(6)	66,(6)	66,(6)	33,(3)	33,(3)	33,(3)

Source: author.

3. **Asymmetries in size**: in the baseline scenario, the monetary union is composed of 4 equally-sized economies, in the first alternative scenario – the sizes are: $w_1 = 0.4$, $w_2 = 0.3$, $w_3 = 0.2$ and $w_4 = 0.1$.[9]

4. **Potential mismatch between voting frequency and size of the economy**. In the baseline scenario, there is no mismatch, as two (out of four) representatives of equally-sized countries are entitled to vote in 50% of meetings. The first alternative scenario also envisages no mismatch: the country with weight 0.4 participates in 80% of votes, the following ones: in 60%, 40% and 20% respectively (see Table 3). However, in the second alternative scenario, the monetary union is composed of 6 countries weighted $\frac{7}{27}$, $\frac{6}{27}\left(=\frac{2}{9}\right)$, $\frac{5}{27}$, $\frac{4}{27}\left(=\frac{1}{9}\right)$ and $\frac{3}{27}$. These countries were segmented into 2 rotation groups: first three (represented by 2 voters) and last three (represented by 1 voter). The first group members hence participate in $\frac{2}{3}$ of votes, the second in $\frac{1}{3}$ of votes. When country sizes are taken into account, it is easy to notice that country 1 and 4 are under-represented, and countries 3 and 6 – over-represented in terms of voting frequency.

For each variant of simulations, a path of 10000 observations was generated. To test the statistical significance of variance differentials, the exercise was repeated 100 times. To isolate the effect of rotation in the Council, tables 4-8 contain the variance of individual model variables rescaled in such a way that the case of $hb = 0$ equals 100.

The simulations in the baseline scenario of equally sized economies (see Table 4 and Figure 1) confirm the intuition that introducing the rotation into the Council increases the volatility of macroeconomic variables when coupled with home-biased preferences of policymakers. With $hb = 0.5$ and semi-annual rotation, the variance of output grows by 0.08% in the tradable sector and much more, by 0.65%, in the nontradable sector. The analogous growth for inflation rates is 0.02% and 0.015%. Volatility of consumption grows by 0.5%, while volatility of the real wages – by 0.1%.

Table 4: Rotation in ECB Governing Council: variance of individual variables (the case of $hb = 0$ equals 100)

Frequency	Variable (Home bias)	π^H w=0,25	π^N w=0,25	c w=0,25	rw w=0,25	y^H w=0,25	y^N w=0,25
Annual	**0.0**	**100.0000**	**100.0000**	**100.0000**	**100.0000**	**100.0000**	**100.0000**
	0.1	100.0005	100.0009	100.0254***	100.0049*	100.0035	100.0349***
	0.2	100.0052***	100.0065	100.0982***	100.0238***	100.0181***	100.1406***
	0.3	100.0141***	100.0167**	100.2182***	100.0567***	100.0438***	100.3173***
	0.4	100.0271***	100.0314***	100.3855***	100.1036***	100.0805***	100.5648***
	0.5	100.0443***	100.0505***	100.6***	100.1645***	100.1284***	100.8834***
Semi-annual	**0.0**	**100.0000**	**100.0000**	**100.0000**	**100.0000**	**100.0000**	**100.0000**
	0.1	100.0013**	100.0015*	100.024***	100.0065***	100.0041***	100.0305***
	0.2	100.004***	100.0037**	100.0859***	100.0193***	100.0144***	100.1104***
	0.3	100.0082***	100.0067**	100.1858***	100.0386***	100.0311***	100.2395***
	0.4	100.0137***	100.0104***	100.3236***	100.0643***	100.0541***	100.418***
	0.5	100.0206***	100.0149***	100.4993***	100.0964***	100.0834***	100.6459***
Quarterly	**0.0**	**100.0000**	**100.0000**	**100.0000**	**100.0000**	**100.0000**	**100.0000**
	0.1	100.0002	99.9998	100.0058***	100.0003	100.0008	100.0066***
	0.2	100.0005*	99.9998	100.023***	100.0012	100.0038***	100.0262***
	0.3	100.0009**	99.9998	100.0518***	100.0029**	100.0092***	100.0589***
	0.4	100.0014**	99.9998	100.092***	100.0054***	100.0167***	100.1047***
	0.5	100.0021***	99.9999	100.1437***	100.0085***	100.0266***	100.1636***

rw– real wages ($w_t - p_t$). Differences between a given variant and the case of $hb = 0$ may be significant at 1% (***), 5% (**) or 10% (*) level.
Source: author.

The variance grows monotonically with increasing hb. This dependence is nonlinear: incrementing hb by 0.1 from 0 leads to higher increments of variance in subsequent steps, for all variables and rotation frequencies (see Figure 1). In some cases, under low hb, the difference in variance as compared to $hb = 0$ turned out to be statistically insignificant.

Under less frequent rotation (e.g. annual), the variance of all variables grows more than under less frequent rotation. This is related to interest rates smoothing in the Taylor rule which prevents home-biased policymakers from pushing the nominal interest rate towards the level preferred by their country of origin because they do not have sufficient time if they rotate more often. This effect has heterogeneous impact on individual variables: less frequent rotation boosts the volatility of inflation rates and real wages to a higher extent, but the volatility of consumption and output – to a lower extent.

While qualitatively unsurprising, these results may be viewed as very limited on the quantitative level. For $hb = 0.5$, the growth in variance did not exceed 1% for any variable or simulation scenario.

Under diversified sizes of monetary union members (first alternative scenario), the above results were in fact replicated (see Table 5). The differences in increase of variances between $hb = 0.5$ and $hb = 0$ mostly did not exceed 0.1 percentage point.

Table 5: Rotation in ECB Governing Council: variance for different home bias (hb) as a function of economy size (the case of $hb = 0$ equals 100)

Frequency	Variable	π^H				π^N				c			
		4 countries				4 countries				4 countries			
	Weight	w=0.4	w=0.3	w=0.2	w=0.1	w=0.4	w=0.3	w=0.2	w=0.1	w=0.4	w=0.3	w=0.2	w=0.1
Annual	0.0	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**
	0.1	100.002**	100.004***	100.005***	99.995***	100.019***	99.999	99.966***	100.022***	99.998	100.018***	100.006*	100.01**
	0.2	100.005***	100.004**	100.011***	100.002	100.018***	100.003	99.972***	100.013***	100.068***	100.091***	100.051***	100.057***
	0.3	100.018***	100.016***	100.021***	100.007**	100.025***	100.024***	99.995	100.039***	100.173***	100.193***	100.133***	100.133***
	0.4	100.026***	100.028***	100.03***	100.02***	100.049***	100.026***	100.003	100.039***	100.314***	100.335***	100.258***	100.251***
	0.5	100.042***	100.042***	100.046***	100.035***	100.073***	100.053***	100.02*	100.066***	100.508***	100.516***	100.412***	100.397***
Semi-annual	0.0	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**
	0.1	100.000	100.000	100.000	100.000	99.999	100.000	100.000	99.999	100.023***	100.024***	100.014***	100.022***
	0.2	100.001	100.002	100.001	100.001	100.000	100.000	100.000	100.000	100.081***	100.079***	100.055***	100.069***
	0.3	100.004**	100.005***	100.004**	100.004**	100.001	100.002	100.002	100.002	100.173***	100.165***	100.125***	100.143***
	0.4	100.008***	100.01***	100.009***	100.009***	100.004	100.005	100.005	100.004	100.299***	100.282***	100.222***	100.243***
	0.5	100.014***	100.017***	100.015***	100.015***	100.008	100.009*	100.009	100.008	100.459***	100.43***	100.348***	100.369***
Quarterly	0.0	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**
	0.1	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.011***	100.005**	100.008***	100.003*
	0.2	100.000	100.000	100.000	100.000	99.999	99.999	99.999	99.999	100.037***	100.025***	100.028***	100.018***
	0.3	100.000	100.000	100.000	100.000	99.999	99.999	99.999	99.999	100.079***	100.058***	100.061***	100.045***
	0.4	100.000	100.001	100.000	100.000	99.999	99.999	99.999	99.999	100.136***	100.104***	100.107***	100.083***
	0.5	100.001	100.001	100.001	100.001	99.999	99.999	99.999	99.999	100.209***	100.165***	100.164***	100.132***

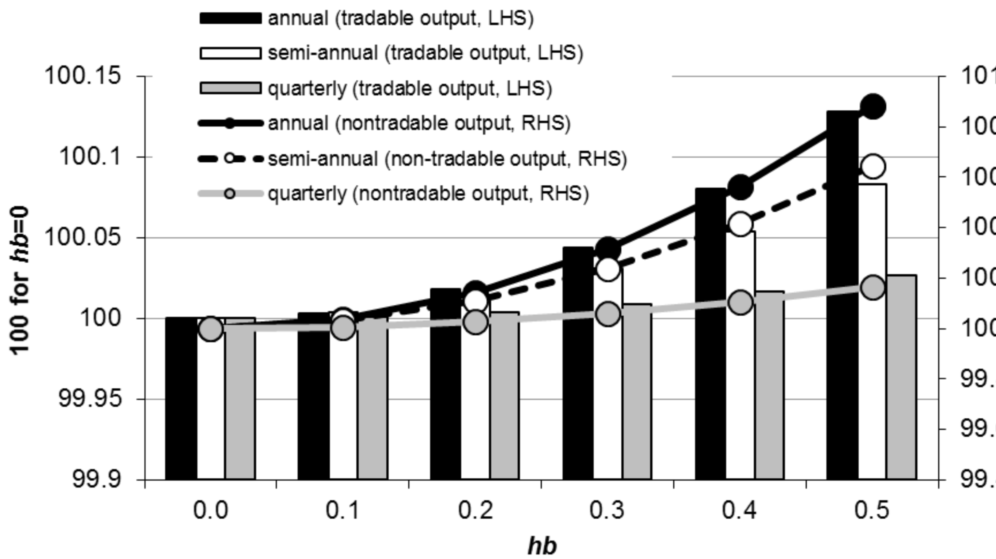
Home bias (rows 0.0–0.5)

Frequency	Variable	rw				y^H				y^N			
		4 countries				4 countries				4 countries			
	Weight	w=0.4	w=0.3	w=0.2	w=0.1	w=0.4	w=0.3	w=0.2	w=0.1	w=0.4	w=0.3	w=0.2	w=0.1
Annual	0.0	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**
	0.1	100.01***	99.989***	100.011***	99.988***	100.007***	100.000	100.011***	99.991***	99.99*	99.988*	100.043***	99.986**
	0.2	100.042***	100.000	100.026***	100.009**	100.025***	100.006*	100.027***	100.004	100.122***	100.083***	100.106***	100.068***
	0.3	100.07***	100.033***	100.056***	100.03***	100.044***	100.027***	100.045***	100.02***	100.278***	100.236***	100.229***	100.169***
	0.4	100.104***	100.075***	100.094***	100.074***	100.067***	100.061***	100.071***	100.051***	100.486***	100.457***	100.407***	100.353***
	0.5	100.156***	100.125***	100.142***	100.12***	100.106***	100.097***	100.108***	100.085***	100.784***	100.728***	100.623***	100.552***
Semi-annual	0.0	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**
	0.1	100.000	100.004*	100.002	100.000	100.004***	100.004***	100.004***	100.004***	100.03***	100.029***	100.016***	100.025***
	0.2	100.007**	100.014***	100.01***	100.006*	100.012***	100.014***	100.013***	100.013***	100.11***	100.101***	100.07***	100.085***
	0.3	100.021***	100.03***	100.024***	100.019***	100.027***	100.028***	100.027***	100.027***	100.238***	100.216***	100.162***	100.18***
	0.4	100.041***	100.054***	100.046***	100.038***	100.046***	100.048***	100.047***	100.047***	100.415***	100.374***	100.292***	100.309***
	0.5	100.068***	100.084***	100.074***	100.064***	100.071***	100.074***	100.072***	100.071***	100.64***	100.575***	100.46***	100.472***
Quarterly	0.0	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**	**100.000**
	0.1	99.999	99.999	100.000	99.999	100.002**	100.001**	100.002**	100.001	100.011***	100.005*	100.007***	100.002
	0.2	100.000	100.000	100.001	100.000	100.006***	100.005***	100.006***	100.004***	100.04***	100.026***	100.028***	100.017***
	0.3	100.001	100.001	100.003	100.001	100.012***	100.012***	100.012***	100.01***	100.087***	100.063***	100.064***	100.045***
	0.4	100.003	100.003	100.006**	100.003	100.021***	100.021***	100.022***	100.018***	100.153***	100.116***	100.113***	100.086***
	0.5	100.007**	100.007**	100.01***	100.007**	100.033***	100.032***	100.033***	100.029***	100.237***	100.185***	100.177***	100.14***

Home bias (rows 0.0–0.5)

Differences between a given variant and the case of $hb = 0$ may be significant at 1% (***), 5% (**) or 10% (*) level.
Source: author.

Figure 1: Rotation in ECB Governing Council: variance of y^H and y^N for various rotation frequencies and home bias (hb)



Source: author.

This result, at first glance, could be surprising: increasing hb leads to increasing the fit of common monetary policy to the needs of the smallest economies in an over-proportionate manner. However, a different effect has turned out to dominate: growing hb increases the macroeconomic volatility in the other, larger economies. This volatility is automatically imported into smaller economies *via* real exchange rate volatility and trade linkages. It can be concluded that in an integrated group of open economies, the monetary stability of the entire area should be rationally preferred to home-biased attitude by policymakers from small economies. Hence, the argument about the 're-nationalisation' of ECB policy may not be appropriate.

The simulations in the second alternative scenario, in turn, demonstrate that a mismatch between voting frequency and economy size has a substantial impact on macroeconomic volatilities (see Tables 6-8 and Figure 2). In the group of over-proportionately frequent voters (such as countries 3 and 6), growing hb decreases the variance of output, tradable inflation, consumption and real wages. The opposite is the case in countries 1 and 4, i.e. under-proportionately frequent voters. The only exception is the decrease in non-tradable inflation rates (very limited in countries 1 and 4, and stronger in countries 3 and 6), though at the cost of a higher volatility in output. It is noteworthy that under *one country – one vote* principle without rotation (i.e. prior to 2015), the degree of mismatch was even greater, and hence from this perspective the reform can be viewed as a step in the right direction.

More efficient stabilisation of economies 3 and 6 spills over to economies 2 and 5 that, under moderate home bias, exhibit lower volatility (although their policymakers vote at an adequate frequency). However, under strong home bias, this positive effect is dominated by negative effect of destabilising economies 1 and 4. This nonlinearity is best visible for annual rotation scheme in the case of tradable inflation, consumption, real wages and nontradable output in countries 2 and 5.

The results discussed above have been subject to sensitivity test with respect to various dimensions of country heterogeneity. In a monetary union of 4 equally sized countries and semi-annual rotation frequency in monetary policy council, the following aspects affected the results:

- **share of the nontradable sector**. In countries with higher κ, the growth of hb leads to a higher increase in variance. The higher shares of nontradables, the less efficient market-based adjustment mechanisms in the monetary union after asymmetric shocks. This is why growing inefficiency in monetary policy appears to be more detrimental to economic stability.

- **market rigidities**. Higher Calvo probabilities lead to stronger growth in volatilities under growing hb, especially in the case of output. Efficient monetary policy is more useful in the presence of nominal rigidities, and consequently the noise component introduced into the Taylor rule may be more disturbing in such a case.

- **inertia of shocks**. The patterns depend on the type of shock. Serial correlation of demand shocks remains neutral for the effects. For most of the supply shocks (and variables), high persistence implies more predictable policy actions that can be smoothed out becasue rotations are anticipated. Consequently, with higher serial correlations, the impact of growing hb on macroeconomic volatility decreases.

- **households' discount factor**. In the economies characterised by higher β, growing hb boosts the macroeconomic volatility to a highest extent.

Intuitively, the results are also sensitive to the parameters of the Taylor rule. Stronger smoothing, γ_ρ, dampens the effects of growing hb. In turn, growing values of γ_π and γ_y (increasing monetary policy activity) clearly increase the macroeconomic volatility (especially in terms of output).

Two disclaimers should be provided at this point. Firstly, the simulations were focused on cyclical rather than structural divergencies. If, for example, a group of countries characterised by a higher natural interest rate (say, East) was interested in setting systematically higher nominal nominal interest

Table 6: Variable volatilities under mismatch between voting frequency and economy size (1)

Frequency	Home bias	π^H					
	Country	**1**	**2**	**3**	**4**	**5**	**6**
	Group	1	1	1	2	2	2
	Weight 1	0.26	0.22	0.19	0.15	0.11	0.07
	Weight 2	0.22	0.22	0.22	0.11	0.11	0.11
Annual	0	100	100	100	100	100	100
Annual	0.1	100.002***	99.9973***	99.9931***	100.0025***	99.9975***	99.9932***
Annual	0.2	100.0055***	99.996***	99.9875***	100.0064***	99.9965***	99.9879***
Annual	0.3	100.0103***	99.9962**	99.9835***	100.0117***	99.9969*	99.984***
Annual	0.4	100.0167***	99.9978	99.9809***	100.0185***	99.9988	99.9816***
Annual	0.5	100.0244***	100.0009	99.9797***	100.0268***	100.0021	99.9806***
Semi-annual	0	100	100	100	100	100	100
Semi-annual	0.1	100.0013***	99.9966***	99.9918***	100.0014***	99.9968***	99.9924***
Semi-annual	0.2	100.003***	99.9934***	99.984***	100.003***	99.9938***	99.9851***
Semi-annual	0.3	100.0049***	99.9906***	99.9764***	100.0049***	99.9911***	99.978***
Semi-annual	0.4	100.0071***	99.988***	99.9691***	100.0071***	99.9887***	99.9713***
Semi-annual	0.5	100.0095***	99.9857***	99.962***	100.0096***	99.9866***	99.9648***
Quarterly	0	100	100	100	100	100	100
Quarterly	0.1	100.0009***	99.9963***	99.9915***	100.001***	99.9961***	99.9919***
Quarterly	0.2	100.002***	99.9927***	99.9831***	100.0021***	99.9924***	99.984***
Quarterly	0.3	100.0032***	99.9892***	99.9749***	100.0034***	99.9888***	99.9762***
Quarterly	0.4	100.0045***	99.9859***	99.9669***	100.0048***	99.9854***	99.9685***
Quarterly	0.5	100.006***	99.9828***	99.9589***	100.0064***	99.9821***	99.961***

Frequency	Home bias	π^N					
	Country	**1**	**2**	**3**	**4**	**5**	**6**
	Group	1	1	1	2	2	2
	Weight 1	0.26	0.22	0.19	0.15	0.11	0.07
	Weight 2	0.22	0.22	0.22	0.11	0.11	0.11
Annual	0	100	100	100	100	100	100
Annual	0.1	99.9931***	99.9921***	99.9915***	99.9933***	99.9923***	99.9915***
Annual	0.2	99.9877***	99.9858***	99.9845***	99.9881***	99.9861***	99.9846***
Annual	0.3	99.9838***	99.9809***	99.979***	99.9844***	99.9814***	99.9791***
Annual	0.4	99.9815***	99.9776***	99.975***	99.9822***	99.9782***	99.9752***
Annual	0.5	99.9807***	99.9758***	99.9726***	99.9815***	99.9766***	99.9728***
Semi-annual	0	100	100	100	100	100	100
Semi-annual	0.1	99.9918***	99.9908***	99.9899***	99.9918***	99.9909***	99.9901***
Semi-annual	0.2	99.9838***	99.9819***	99.9801***	99.9838***	99.982***	99.9804***
Semi-annual	0.3	99.976***	99.9732***	99.9704***	99.976***	99.9733***	99.971***
Semi-annual	0.4	99.9685***	99.9647***	99.961***	99.9684***	99.9648***	99.9617***
Semi-annual	0.5	99.9612***	99.9564***	99.9518***	99.9611***	99.9566***	99.9527***
Quarterly	0	100	100	100	100	100	100
Quarterly	0.1	99.9914***	99.9905***	99.9896***	99.9915***	99.9905***	99.9897***
Quarterly	0.2	99.983***	99.9811***	99.9793***	99.9831***	99.9811***	99.9796***
Quarterly	0.3	99.9747***	99.9719***	99.9692***	99.9748***	99.9719***	99.9696***
Quarterly	0.4	99.9665***	99.9628***	99.9592***	99.9667***	99.9628***	99.9597***
Quarterly	0.5	99.9584***	99.9539***	99.9493***	99.9587***	99.9538***	99.9499***

Differences between a given variant and the case of $hb = 0$ may be significant at 1% (***), 5% (**) or 10% (*) level.
Source: author.

Table 7: Variable volatilities under mismatch between voting frequency and economy size (2)

Frequency	Home bias	Variable: *c*					
	Country	**1**	**2**	**3**	**4**	**5**	**6**
	Group	1	1	1	2	2	2
	Weight 1	0.26	0.22	0.19	0.15	0.11	0.07
	Weight 2	0.22	0.22	0.22	0.11	0.11	0.11
Annual	0	100	100	100	100	100	100
Annual	0.1	100.1719***	99.9897***	99.8217***	100.1593***	99.9945*	99.8268***
Annual	0.2	100.3669***	100.0019	99.6652***	100.3398***	100.0095	99.6737***
Annual	0.3	100.5848***	100.0365***	99.5305***	100.5414***	100.045***	99.5407***
Annual	0.4	100.8258***	100.0936***	99.4176***	100.7642***	100.1011***	99.4278***
Annual	0.5	101.0897***	100.1732***	99.3265***	101.0081***	100.1776***	99.335***
Semi-annual	0	100	100	100	100	100	100
Semi-annual	0.1	100.1705***	99.987***	99.8142***	100.1591***	99.9905***	99.829***
Semi-annual	0.2	100.3544***	99.987***	99.641***	100.3304***	99.993*	99.6698***
Semi-annual	0.3	100.5518***	100.0001	99.4805***	100.5141***	100.0075	99.5222***
Semi-annual	0.4	100.7625***	100.0261***	99.3326***	100.71***	100.0339***	99.3864***
Semi-annual	0.5	100.9867***	100.0651***	99.1974***	100.9183***	100.0723***	99.2622***
Quarterly	0	100	100	100	100	100	100
Quarterly	0.1	100.1688***	99.9892***	99.814***	100.1558***	99.9872***	99.828***
Quarterly	0.2	100.3432***	99.9839***	99.6333***	100.3166***	99.9794***	99.6609***
Quarterly	0.3	100.5231***	99.9839***	99.4578***	100.4826***	99.9765***	99.4986***
Quarterly	0.4	100.7085***	99.9892**	99.2875***	100.6536***	99.9786***	99.3412***
Quarterly	0.5	100.8994***	100	99.1225***	100.8297***	99.9856**	99.1886***

Frequency	Home bias	Variable: *rw*					
	Country	**1**	**2**	**3**	**4**	**5**	**6**
	Group	1	1	1	2	2	2
	Weight 1	0.26	0.22	0.19	0.15	0.11	0.07
	Weight 2	0.22	0.22	0.22	0.11	0.11	0.11
Annual	0	100	100	100	100	100	100
Annual	0.1	100.0198***	99.9972*	99.9777***	100.0209***	99.9991	99.9784***
Annual	0.2	100.0447***	99.9997	99.9606***	100.0469***	100.0032	99.9619***
Annual	0.3	100.0748***	100.0072*	99.9486***	100.0781***	100.0125**	99.9505***
Annual	0.4	100.11***	100.02***	99.9417***	100.1143***	100.0269***	99.9442***
Annual	0.5	100.1503***	100.0378***	99.9399***	100.1556***	100.0463***	99.943***
Semi-annual	0	100	100	100	100	100	100
Semi-annual	0.1	100.0148***	99.993***	99.971***	100.0152***	99.9938***	99.9734***
Semi-annual	0.2	100.031***	99.9873***	99.9434***	100.0317***	99.9888***	99.9481***
Semi-annual	0.3	100.0485***	99.983***	99.917***	100.0495***	99.9852***	99.9242***
Semi-annual	0.4	100.0673***	99.98***	99.892***	100.0687***	99.9829***	99.9015***
Semi-annual	0.5	100.0875***	99.9783***	99.8683***	100.0891***	99.9819***	99.8802***
Quarterly	0	100	100	100	100	100	100
Quarterly	0.1	100.0143***	99.9927***	99.9711***	100.0148***	99.9924***	99.9728***
Quarterly	0.2	100.0292***	99.986***	99.9427***	100.0301***	99.9854***	99.9462***
Quarterly	0.3	100.0447***	99.9799***	99.915***	100.0461***	99.979***	99.9202***
Quarterly	0.4	100.0607***	99.9743***	99.8878***	100.0626***	99.9732***	99.8948***
Quarterly	0.5	100.0774***	99.9694***	99.8612***	100.0797***	99.9679***	99.87***

Differences between a given variant and the case of $hb = 0$ may be significant at 1% (***), 5% (**) or 10% (*) level.
Source: author.

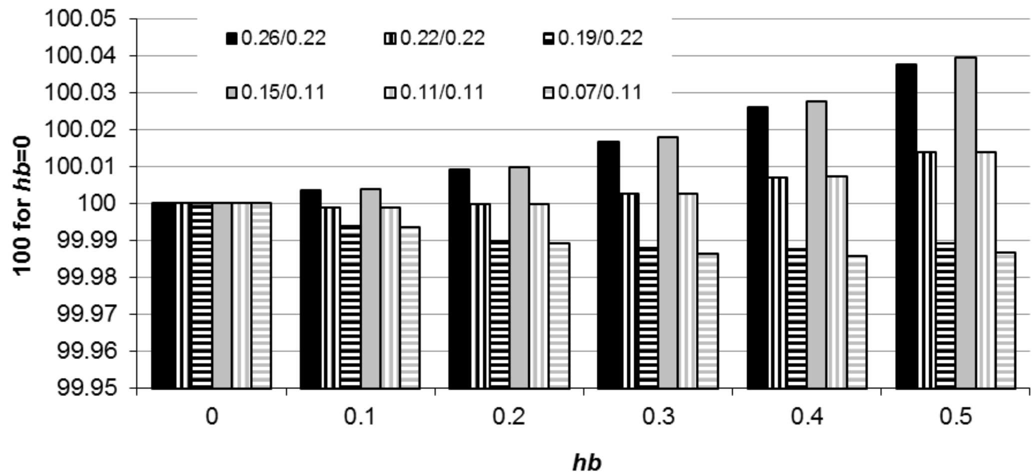
Table 8: Variable volatilities under mismatch between voting frequency and economy size (3)

Frequency	Home bias					
Variable	y^H					
Country	**1**	**2**	**3**	**4**	**5**	**6**
Group	1	1	1	2	2	2
Weight 1	0.26	0.22	0.19	0.15	0.11	0.07
Weight 2	0.22	0.22	0.22	0.11	0.11	0.11
Annual 0	100	100	100	100	100	100
Annual 0.1	100.0062***	100.0014	99.997**	100.0065***	100.0014	99.9953***
Annual 0.2	100.0161***	100.0065***	99.9977	100.0167***	100.0064**	99.9942**
Annual 0.3	100.0296***	100.0152***	100.002	100.0306***	100.0151***	99.9966
Annual 0.4	100.0467***	100.0276***	100.01**	100.0481***	100.0275***	100.0026
Annual 0.5	100.0675***	100.0436***	100.0216***	100.0693***	100.0435***	100.0122**
Semi-annual 0	100	100	100	100	100	100
Semi-annual 0.1	100.0037***	99.9989*	99.994***	100.004***	99.9989*	99.9936***
Semi-annual 0.2	100.0093***	99.9997	99.99***	100.01***	99.9998	99.9891***
Semi-annual 0.3	100.0167***	100.0025	99.9878***	100.0178***	100.0025	99.9865***
Semi-annual 0.4	100.0261***	100.0071**	99.9876***	100.0276***	100.0072**	99.9857***
Semi-annual 0.5	100.0375***	100.0137***	99.9893***	100.0393***	100.0138***	99.9868***
Quarterly 0	100	100	100	100	100	100
Quarterly 0.1	100.0043***	99.9998	99.9949***	100.0047***	99.9996	99.9944***
Quarterly 0.2	100.0094***	100.0004	99.9906***	100.0102***	100	99.9896***
Quarterly 0.3	100.0154***	100.0018	99.9871***	100.0165***	100.0013	99.9856***
Quarterly 0.4	100.0222***	100.0041**	99.9845***	100.0237***	100.0034*	99.9825***
Quarterly 0.5	100.0298***	100.0073***	99.9828***	100.0318***	100.0064***	99.9802***
Variable	y^N					
Country	**1**	**2**	**3**	**4**	**5**	**6**
Group	1	1	1	2	2	2
Weight 1	0.26	0.22	0.19	0.15	0.11	0.07
Weight 2	0.22	0.22	0.22	0.11	0.11	0.11
Annual 0	100	100	100	100	100	100
Annual 0.1	100.2368***	99.9804***	99.7475***	100.2146***	99.989**	99.7646***
Annual 0.2	100.5104***	99.9962	99.5291***	100.462***	100.0095	99.5599***
Annual 0.3	100.8207***	100.0474***	99.3449***	100.7423***	100.0614***	99.3857***
Annual 0.4	101.1678***	100.1342***	99.1948***	101.0553***	100.1449***	99.242***
Annual 0.5	101.5515***	100.2563***	99.0788***	101.401***	100.2598***	99.129***
Semi-annual 0	100	100	100	100	100	100
Semi-annual 0.1	100.2354***	99.976***	99.7357***	100.2149***	99.9818***	99.7645***
Semi-annual 0.2	100.4887***	99.9694***	99.4881***	100.4459***	99.9792***	99.544***
Semi-annual 0.3	100.7602***	99.9801**	99.2571***	100.693***	99.9921	99.3386***
Semi-annual 0.4	101.0497***	100.0081	99.0428***	100.9563***	100.0205*	99.1482***
Semi-annual 0.5	101.3572***	100.0534***	98.8452***	101.2357***	100.0645***	98.9727***
Quarterly 0	100	100	100	100	100	100
Quarterly 0.1	100.2299***	99.9759***	99.733***	100.2074***	99.9746***	99.7605***
Quarterly 0.2	100.4673***	99.9591***	99.4729***	100.4215***	99.9557***	99.5274***
Quarterly 0.3	100.7122***	99.9496***	99.2198***	100.6423***	99.9433***	99.3005***
Quarterly 0.4	100.9647***	99.9472***	98.9738***	100.8698***	99.9374***	99.0798***
Quarterly 0.5	101.2247***	99.9522***	98.7347***	101.104***	99.938***	98.8655***

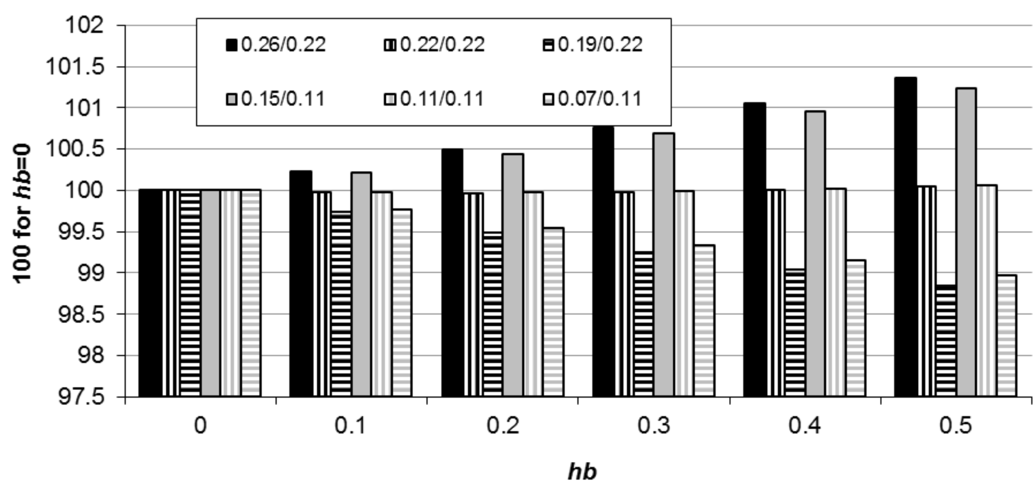
Differences between a given variant and the case of $hb = 0$ may be significant at 1% (***), 5% (**) or 10% (*) level.
Source: author.

Figure 2: Variable volatilities under mismatch between voting frequency and economy size (at biannual rotation frequency)

(a) variance of tradable output



(b) variance of nontradable output



The two numbers in legend denote: economy size and the fraction of time with the voting right.
Source: author.

rates than the others (say, West), then the impact of growing hb on volatilities would grow substantially. Secondly, it might be argued whether 18 is the optimum way to describe the aggregate preferences (in particular, policy preferences of individual countries could vary in terms of γ_ρ, γ_π and γ_y). Both questions could potentially be addressed in an optimum policy framework, which we leave for future research.

6 Conclusions

This paper generalizes the analytical methods of solving linear rational expectations models to the case of time-varying, nonstochastic parameters recurring in a finite, predefined cycle. Such a specification emerged from the inclusion of rotational voting system in a monetary policy council combined with regional bias in policymakers' preferences. A solution algorithm for DSGE model with rational expectations is proposed and exemplified with the simulated impact of ECB voting reform (in force since 2015). The conditions for existence of a unique solution correspond in a straightforward way to the standard Blanchard-Kahn conditions.

These simulations confirm the previous findings from the literature without DSGE modelling: the introduction rotation in the ECB Governing Council coupled with home bias in interest rate decisions taken by the members of the Council, increases the macroeconomic volatility in the monetary union. The magnitude of this effect, however, remains limited: the increase in variance did not exceed 1% for any variable or simulation scenario. A number of factors can, however, intensify this effect, i.a. low frequency of rotation, mismatch between the size of economies and the frequency of voting, product market rigidities and low inertia of supply shocks.

This paper contributes to the discussions on both policy and methodological levels. On the policy level, our results call for searching the rotation schemes that provide the optimum mapping between economic sizes and voting frequencies of individual countries, as well as more frequent rotation. We also demonstrated that the fulfilment of optimum currency area criteria (such as high degree of openness and integration) naturally eradicates the problem of home-biased, rotating policymakers. By assuming rationality of agents in our analysis, however, we cannot be conclusive about such issues as transparency and legitimacy of the new voting system.

On the methodological level, our contribution is – to the best of our knowledge – the first analysis of the new voting system at the ECB performed with a DSGE model. Importantly, the proposed solution algorithm is not confined to the class of problems related to collective decision-making in monetary unions or federalist currency areas. It can be applied to any DSGE analysis with predetermined, time-varying cycle of parameters. The most prominent example that springs to mind is the relatively simple, analytically elegant, but also numerically efficient inclusion of seasonality patterns into DSGE-based forecasts.

Worthwhile avenues of related future research include the extensions to (i) optimum policy framework rather than the Taylor rule (to obtain a potentially more adequate description of home-biased individual preferences and conduct a full welfare analysis), (ii) currency unions with heterogeneous natural interest rates (e.g. due to ongoing convergence process), (iii) applications to more sophisitcated DSGE models and (iv) numerical simulations for 19 euro area countries based on bilateral shock correlations and adapted to all dimensions of cross-country heterogeneity. Building game-theoretical fundamentals (see e.g. Sosnowska, 2013) is also an interesting direction.

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