Human Capital in a Credit Cycle Model[1]

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Abstract

We augment a model of endogenous credit cycles (Matsuyama et al. (2016)) with human capital to study the impact of human capital on the stability of central economic aggregates. Human capital is modelled as pure external effect of production following a 'learning-by-producing' approach. Agents have access to two different investment projects, Good and Bad projects, which differ substantially in their next generations spillover effects. The former generate pecuniary externalities and technological spillovers through human capital formation whereas the latter fail to do so. Moreover, the latter are subject to financial frictions, the so-called borrowing constraint. Due to the heterogeneous projects and the borrowing constraint endogenous credit cycles occur and a pattern of boom and bust cycles can be observed. We explore the impact of human capital on the overall system's stability by numerical simulations with different parameter combinations representing distinct scenarios. From an economic perspective, human capital has an ambiguous effect on the evolution of the output path. Depending on the strength of the financial friction and the production share of human capital it either amplifies or mitigates output fluctuations. This analysis shows that human capital is an essential factor for economic stability and sustainable growth as a high human capital share tends to make the system's stability robust against shocks.

Keywords: Human capital, Learning-by-producing, Credit cycles, Financial instability.

JEL Codes: C61, E32, E24, J24.

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1 Introduction

Technological progress is one of the key factors in economic growth literature. As the benchmark model (see Solow (1956)) assumes decreasing returns on capital, a key feature of the growth literature deals with this assumption. Technological progress thus is assumed to be the mitigation effect on the decreasing returns of capital. On the one hand this affects the labour productivity directly (if technical progress is assumed to be labour augmenting) and the capital accumulation process indirectly. For an empirical assessment see Solow (1957), where *output growth* is broke down to growth in labour, capital and technology. Solow himself introduced technological progress exogenously with an own variable and observed the following: the output per unit of effective labour along a balanced growth path is determined, amongst others, by the technological progress.

In this article we will take a closer look to human capital as integral part of technological progress as it plays a major role in macroeconomic processes. Not only since Mankiw et al. (1992) introduced human capital in a growth context by augmenting the standard Solow model to take empirical facts into account[1]. Starting with Lucas (1988) where human capital has internal and external effects on the economy's productivity a broad branch of literature emerged and now human capital is a major part in every economic growth textbook. Recent publications focus on the role of human capital empirical studies about economic growth (see, for example Barro (2001)). The main, very brief summarized, findings are that human capital enhances productivity and thus increases growth. Little is known about the effects in a nonlinear dynamic setting, especially when it comes to models where irregular cyclicity is an issue. Therefore we suggest to include human capital as production factor in a credit cycle model. Matsuyama (2013) and Matsuyama et al. (2016) propose a credit cycle model where financial market frictions cause irregular credit cycles. The existing model uses for simulation purposes a simple Cobb-Douglas technology with physical capital and labour. Our aim is to analyse the effects of human capital on both, the economic implications and the system's long run stability. In fact, we take a production function with labour, physical and human capital as a starting point and assume that human capital is a pure external effect of production and transferred intergenerationally. In an economy where young and old generations coexist (i.e. overlapping generation structure), old agents transfer their knowledge

[1] Also Mankiw (1995) points out that 'knowledge' is the sum of all technological and scientific idiosyncrasies and 'human capital' is transmitted through studying to the agents. Therefore we hope to significantly contribute to this process.

about production processes to the young. On the one hand, this drives up the expected profits (which are assumed to be the realised profits, i.e. perfect foresight is assumed) of projects with positive pecuniary external effects (i.e. "Good" projects) and thus make it easier to compete with non-spillover projects (i.e. "Bad" projects). On the other hand it rises the wage rate of the young which is crucial as Bad projects can only be financed by a collateral. Thus, an increase of the net worth eases the borrowing constraint. Therefore it needs to be studied if one effect dominates the other or how they are interlinked. We expect significant contributions to the question of stability characteristics of human capital in an intergenerational non linear model setting with both, pecuniary and technological externalities.

The rest of this paper is organised as follows: In section 2 we will briefly sum up the original model and the key mechanisms which lead to fluctuations and instability but omit the detailed derivation, as this can be found in the aforementioned publications. Section 3 continues by describing the human capital extensions and the new dynamical law of motion. Section 4 is dedicated to a a detailed simulation exercise where we use numerical simulations to analyse various scenarios and compare it to the original publication. Section 5 concludes.

2 The Mechanism

This section gives a detailed summary of the model structure, the extensions and the core mechanism. The basic framework is close to Matsuyama et al. (2016) which uses an overlapping generations model (see Diamond (1965)) with two period lives. Time is discrete and extends from zero to infinity, $t = 1, 2, 3, ...$ In each period one final good, the numeraire, is produced which can be used for investments or for consumption. As we want to study the dynamic effects of human capital, the final goods sector uses following Cobb-Douglas technology

$$Y_t = AK_t^{\alpha} H_t^{\gamma} L_t^{1-\alpha-\gamma}, \tag{2.1}$$

where A denotes some exogenous Hicks-neutral technical progress; K_t is physical capital, H_t is human capital and L_t is labour at time t; and $(\alpha + \gamma) < 1$ are the production elasticities. Using the notation in "units of labour" and the normalisation $(1 - \alpha)A = 1$

$$\frac{Y_t}{L_t} = y_t = \frac{1}{1-\alpha} k_t^{\alpha} h_t^{\gamma}. \tag{2.2}$$

The production function is similar to the one introduced in Mankiw et al. (1992). However, our approach substantially differs, since we do not consider a resource requirement for human capital formation. We assume that human capital formation is a pure external effect of production ('learning by producing') and that human capital is transferred from the old to the young generation. Thus, a straightforward law of motion for the accumulation follows

$$h_{t+1} = \sigma_h y_t + (1 - \delta_h) h_t, \tag{2.3}$$

where σ_h is the strength of the external effect and δ_h a depreciation rate on human capital which we consider to be well below unity.

Factor markets are competitive and the factors are rewarded with

$$\rho_t = \frac{\partial f(k_t, h_t)}{\partial k_t} = \frac{\alpha}{1-\alpha} k_t^{\alpha-1} h_t^{\gamma} \quad \text{and} \quad w_t = f(k_t, h_t) - k_t \frac{\partial f(k_t, h_t)}{\partial k_t} = k_t^{\alpha} h_t^{\gamma}. \tag{2.4}$$

This formulation implies that workers do not only receive the marginal product of labour, but also the marginal product of human capital (which is similar to Mankiw et al. (1992)).

Agents are born at the beginning of each period and stay active for two periods. Young agents are endowed with one unit of labour and the human capital that they inherited from the previous generation; they work in their first period and thereby they accumulate human capital. At the end of their first period, i.e. in point of time $t + 1$, they earn the factor reward of labour, save everything (savings rate equals unity) and thus accumulate wealth. At the same time, the young generation becomes the old generation (and a new young generation is born to which the human capital is transferred) and they have to decide how to use their wealth (accumulated in form of the numeraire) in order to maximise their consumption at the end of their second period. They have three possibilities to allocate wealth: They can (1) invest in a "Good" investment project or (2) they can start a "Bad" investment project; in addition, (3) they can lend funds to other same cohort agents. Good projects are investments in the final goods production sector that uses the final good as physical capital input, k_{t+1}. Assuming perfect foresight, the expected return of this investment project type is equal to the marginal product of capital, $\rho^e_{t+1} = \rho_{t+1}$. Thus, Good projects fuel production processes that generate labour income for the next generation and induce human capital formation. Instead, Bad projects do not involve production processes. They can be seen as simple trading or storing activities, and essentially fail to create any positive externalities for the next generations. Those type of projects are assumed to require an indivisible amount of $m > 1$ units of the final good and to transform it into mB units of the final good in period $t+1$. The known and constant parameter $B > 0$ indicates the profitability of the Bad projects. In addition, it is assumed that agents who want to run those projects need to borrow $m - w_t > 0$ at an interest rate r_{t+1}; this interest rate is agreed upon in $t+1$ and has to be paid at the end of period $t+1$. Since Bad projects require credit financing lending is a third option for wealth allocation. Given the possibility of investing in Good projects, the interest rate on credit has to be equal to the expected marginal product of capital $r_{t+1} = \rho^e_{t+1}$.

Now two constraints enter the game: The *profitability* constraint and the *borrowing* constraint. The profitability constraint follows from the consideration that agents only intend to start Bad projects if their return is greater or equal to the return of simple lending or investing in Good projects. Thus,

$$B \geq r_{t+1} = \rho^e_{t+1}\Big(= \frac{\partial f(k_{t+1}, h_{t+1})}{\partial k_{t+1}}\Big) = \rho_{t+1}. \tag{2.5}$$

The borrowing constraint takes capital market imperfections into account: Due to financial frictions agents can borrow only against a collateral. In addition, only a fraction of the expected project revenue can be pledged for the repayment; this fraction reflects the credit market imperfection and is denoted by $0 < \mu < 1$. As information is complete the borrowing constraint requires that:

$$\mu m B \geq r_{t+1}(m - k_t^\alpha h_t^\gamma) \qquad \text{or} \tag{2.6}$$

$$\frac{\mu m B}{m - k_t^\alpha h_t^\gamma} \geq r_{t+1}. \tag{2.7}$$

The lender will only lend up to $\frac{\mu m B}{r_{t+1}}$ which implicitly sets a minimum net worth requirement[2] for agents interested in starting a Bad project. If the financial friction is severe (i.e. $\mu = 0$) the left hand side of equation 2 equals zero implying that the net worth of the agents is always too low to start a Bad project. The other extreme case is the absence of a friction (i.e. $\mu = 1$) where agents can fully pledge their revenues as a collateral to their lenders. The borrowing constraint (7) set a tighter limit for r_{t+1} than the profitability constraint (5) if:

$$\frac{\mu m B}{m - k_t^\alpha h_t^\gamma} < B \tag{2.8}$$

[2] Recall, that $w_t = k_t^\alpha h_t^\gamma$.

$$k_t < k_\mu = \left(m(1-\mu)h_t^{-\gamma}\right)^{1/\alpha} \tag{2.9}$$

The critical value, k_μ which is a function of human capital, separates two regions in the $[k, h]$-phase space, where either BC or PC is binding. It is strictly decreasing with higher k and lower h.

Analogously to Matsuyama et al. (2016) we define the maximal pledgeable rate of return, $R(k_t, h_t)$, that an agent with the net worth $w_t = k_t^\alpha h_t^\gamma$ can pledge to the lender without violating a constraint:

$$R(k_t, h_t) \equiv B\min\left\{\frac{\mu}{1-\frac{k_t^\alpha h_t^\gamma}{m}}, 1\right\} = \begin{cases} \dfrac{\mu B}{1-\frac{k_t^\alpha h_t^\gamma}{m}} & \text{if } k_t \leq k_\mu \quad \text{i.e. if BC is tighter} \\ B & \text{if } k_t \geq k_\mu \quad \text{i.e. if PC is tighter} \end{cases} \tag{2.10}$$

3 The dynamic equations and the phase space

In equilibrium following equation must hold with equality:

$$\rho_{t+1}^e = \rho_{t+1} = r_{t+1} = \frac{\partial f(k_{t+1}, h_{t+1})}{\partial k_{t+1}} = \frac{\alpha}{1-\alpha} k_{t+1}^{\alpha-1} h_{t+1}^\gamma \geq R(k_t, h_t) \tag{3.1}$$

If $\rho_{t+1} = r_{t+1} < R(k_t, h_t)$ would hold with strict inequality, agents always want to start Bad projects (higher returns) but nobody would provide the required credit as the rate of return of lending is to low, which is a contradiction and therefore not possible. In the case of $\rho_{t+1} > R(k_t, h_t)$ agents would never run Bad projects due to a violation of the profitability or the borrowing constraint.

Following Matsuyama et al. (2016), we differentiate a *non-distortionary* and a *distortionary* case (see figure 1 that represents the phase space):

I The *non-distortionary case* in which the borrowing constraint is never binding and aggregate credit is thus allocated efficiently, occurs if $\left.\frac{\partial f(k_{t+1}, h_{t+1})}{\partial k_{t+1}}\right|_{k_t = k_\mu} > B$. In that case, for low k_t and thus a high return on Good projects, $\frac{\partial f(k_{t+1}, h_{t+1})}{\partial k_{t+1}} > B$ and Bad projects will not be started as they are less profitable than the Good. All available credit flows into the Good projects and the corresponding dynamic equation is

$$k_{t+1} = \Psi_L = k_t^\alpha h_t^\gamma. \tag{3.2}$$

Increasing k_t reduces the return on Good projects, until reaching a threshold $k_B > k_\mu$, defined by $k_{t+1} = k_t^\alpha h_t^\gamma \, (= w_t)$ and $\frac{\partial f(k_{t+1}, h_{t+1})}{\partial k_{t+1}} = B$; the profitability of Good and Bad projects is equal. Beyond that point any additional credit flows in Bad projects and investment in Good projects k_{t+1} is determined by the profitability constraint, i.e. by $\frac{\partial f(k_{t+1}, h_{t+1})}{\partial k_{t+1}} = B$. The corresponding dynamic equation is

$$k_{t+1} = \Psi_R = \left(\frac{1}{B} \frac{\alpha}{1-\alpha} \right)^{\frac{1}{1-\alpha}} (h_{t+1})^{\frac{\gamma}{1-\alpha}} . \tag{3.3}$$

Note that in this case the financial frictions parameter μ does not occur in the dynamics; thus, this case is indeed *non- distortionary*. The boundary condition for this case is $\left. \frac{\partial f(k_{t+1}, h_{t+1})}{\partial k_t+1} \right|_{k_t = k_\mu} = B$. Observe that for $k_t = k_\mu$ the net worth is given as $w_t = (1-\mu)\, m$ and the output as $y_t = \frac{1}{1-\alpha} (1-\mu)\, m$. Using $k_{t+1} = w_t$ and $h_{t+1} = \sigma_h y_t + (1-\delta_h) h_t$ allows to determine the threshold explicitly as

$$\overline{h} = \frac{1}{1-\delta_h} \left[\left(\frac{1}{B} \frac{\alpha}{1-\alpha} \right)^{-\frac{1}{\gamma}} ((1-\mu)\, m)^{\frac{1-\alpha}{\gamma}} - \frac{\sigma_h}{1-\alpha} (1-\mu)\, m \right] . \tag{3.4}$$

The *non-distortionary case* occurs for $h_t > \overline{h}$ (above the dashed line in figure 1).

II The *distortionary case*, in which the borrowing constraint impinges upon the dynamics and financial frictions play a role, occurs if $\frac{\partial f(k_{t+1}, h_{t+1})}{\partial k_{t+1}} |_{k_t = k_\mu} < B$, or if $h_t < \overline{h}$ (see below the dashed line in figure 1). Again, for low k_t agents do not intend to start Bad projects because of the high profitability of Good projects. Increasing k_t reduces the profitability of Good projects and at some $k_B < k_\mu$ the profitability of both investment types will be equal and agents start to prefer Bad projects. However, since the wage rate and thus the net worth is still low, the maximum pledgeable rate of return for credit is lower than the return on Good investments and agents cannot obtain the required credit – the borrowing constraint is still binding and agents continue to invest only in the Good projects. In that region of the phase space, the law of motion is given by:

$$k_{t+1} = \Psi_L = k_t^\alpha h_t^\gamma . \tag{3.5}$$

Further increasing k_t raises the agents' wage rate and thus their net worth, which increases the maximum pledgeable rate of return on credit and eases the borrowing constraint. At a threshold k_c, the borrowing constraint is satisfied with equality (while the entire net worth is still invested in Good projects) and the maximum pledgeable rate of return on credit is equal to the profitability of Good projects. k_c is thus implicitly defined by $\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}} = \frac{\alpha}{1-\alpha} k_{t+1}^{\alpha-1} h_{t+1}^{\gamma} = R(k_t, h_t) = \dfrac{\mu m B}{m - k_t^{\alpha} h_t^{\gamma}}$, in which $k_{t+1} = k_t^{\alpha} h_t^{\gamma}\,(= w_t)$. Beyond this threshold, for $k_t > k_c$ (but $k_t < k_\mu$) credit starts to flow into Bad projects. Investment in Good projects is determined by $\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}} = \frac{\alpha}{1-\alpha} k_{t+1}^{\alpha-1} h_{t+1}^{\gamma} = R(k_t, h_t) = \dfrac{\mu m B}{m - k_t^{\alpha} h_t^{\gamma}}$. Solving for k_{t+1}, the law of motion for that region of the phase space is determined by

$$k_{t+1} = \Psi_M = \left(\frac{1}{\mu B} \frac{\alpha}{1-\alpha} \left(1 - \frac{k_t^{\alpha} h_t^{\gamma}}{m} \right) \right)^{\frac{1}{1-\alpha}} (h_{t+1})^{\frac{\gamma}{1-\alpha}} . \tag{3.6}$$

After crossing the next threshold, $k_t > k_\mu$, the borrowing constraint is not binding any more, and investment in Good projects k_{t+1} is determined by the profitability constraint, i.e. by $\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}} = B$. All additional credit flows into Bad projects and the dynamics follow again Ψ_R,

$$k_{t+1} = \Psi_R = \left(\frac{1}{B} \frac{\alpha}{1-\alpha} \right)^{\frac{1}{1-\alpha}} (h_{t+1})^{\frac{\gamma}{1-\alpha}} . \tag{3.7}$$

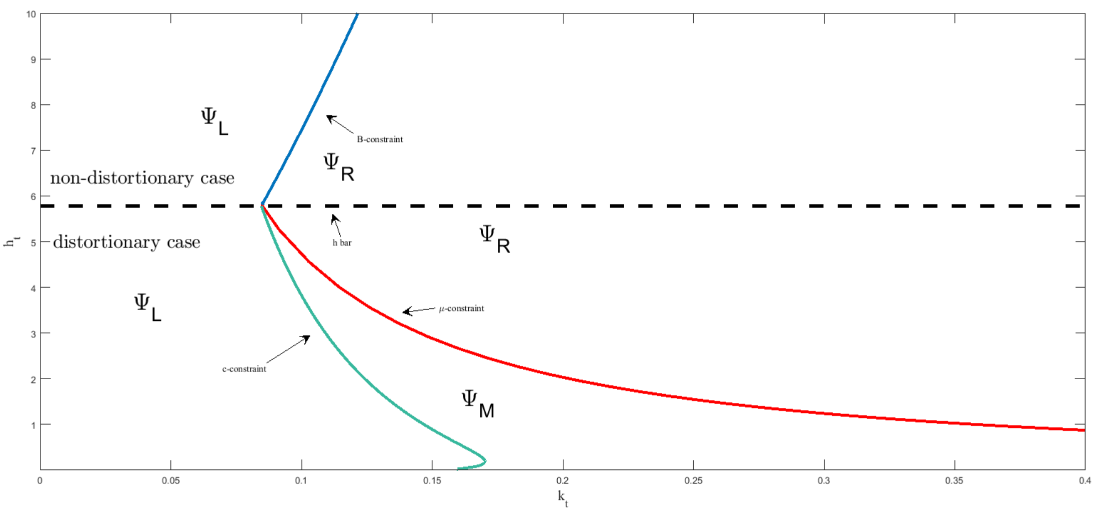

Figure 1: Phase space

Following tables provide a summary of the relevant thresholds and their specifications. Figure 1 indicates the phase space formed by those thresholds.

Threshold	Meaning
k_t	physical capital in time t
k_B	Good is as profitable as Bad
k_c	BC is binding
k_μ	BC is not binding anymore

Table 1: Threshold values

Threshold	Theoretical specification	CD specification
k_B	$\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}} = B$ $k_{t+1} = w_t$	$k_B = \left[\frac{1}{B}\frac{\alpha}{1-\alpha}h_{t+1}^{\gamma}\right]^{\frac{1}{\alpha(1-\alpha)}} h_t^{-\frac{\gamma}{\alpha}}$
k_c	$\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}} = R(k_t, h_t)$ $k_{t+1} = w_t$	$k_c = \left[\frac{1}{\mu B}\frac{\alpha}{1-\alpha}\left(1 - \frac{k_c^{\alpha}h_t^{\gamma}}{m}\right)h_{t+1}^{\gamma}\right]^{\frac{1}{\alpha(1-\alpha)}} h_t^{-\frac{\gamma}{\alpha}}$
k_μ	PC = BC	$k_\mu = \left[m(1-\mu)h_t^{-\gamma}\right]^{1/\alpha}$

Table 2: Threshold values cont'd

Putting above mentioned equations together, we now can construct the dynamical system taking human capital as an external effect into account, for the *non distortionary* case, if $h_t > \overline{h}$:

$$\Psi : \begin{pmatrix} k_{t+1} \\ h_{t+1} \end{pmatrix} \mapsto \begin{pmatrix} k_{t+1} = \begin{cases} \Psi_L = k_t^\alpha h_t^\gamma & \text{if } k_t \leq k_B \\ \Psi_R = \left(\dfrac{\alpha}{B(1-\alpha)}\right)^{\frac{1}{1-\alpha}} \left[h_{t+1}\right]^{\frac{\gamma}{1-\alpha}} & \text{if } k_t \geq k_B \end{cases} \\ h_{t+1} = \left\{ \frac{\sigma_h}{1-\alpha} k_t^\alpha h_t^\gamma + (1-\delta_h) h_t \right. \qquad \forall k_t \end{pmatrix} \tag{3.8}$$

and for the *distortionary* case, if $h_t < \overline{h}$:

$$\Psi : \begin{pmatrix} k_{t+1} \\ h_{t+1} \end{pmatrix} \mapsto \begin{pmatrix} k_{t+1} = \begin{cases} \Psi_L = k_t^\alpha h_t^\gamma & \text{if } k_t \leq k_c \\ \Psi_M = \left[\dfrac{1}{\mu B} \dfrac{\alpha}{1-\alpha} (1 - \frac{k_t^\alpha h_t^\gamma}{m})\right]^{\frac{1}{1-\alpha}} \left[h_{t+1}\right]^{\frac{\gamma}{1-\alpha}} & \text{if } k_c \leq k_t \leq k_\mu \\ \Psi_R = \left(\dfrac{\alpha}{B(1-\alpha)}\right)^{\frac{1}{1-\alpha}} \left[h_{t+1}\right]^{\frac{\gamma}{1-\alpha}} & \text{if } k_t \geq k_\mu \end{cases} \\ h_{t+1} = \left\{ \frac{\sigma_h}{1-\alpha} k_t^\alpha h_t^\gamma + (1-\delta_h) h_t \right. \qquad \forall k_t \end{pmatrix}. \tag{3.9}$$

Thus, the system is continuous piecewise smooth, two-dimensional in k and h, with seven parameters, $\alpha, \gamma, \mu, m, B, \sigma_h, \delta_h$. Following restrictions apply: $\alpha + \gamma < 1$, $0 < \mu, \sigma_h, \delta_h < 1$, $B > 0$ and $m > 1$. Similar to the original model, the law of motion for k_t is crucial for the dynamics. But as human capital has an additional positive effect on the next period net worth, it also affects the magnitude of k each period. For the learning-by-producing external effect of human capital, the parameter σ_h indicates the strength of this effect. This knowledge is assumed to be highly persistent therefore the depreciation rate (or rate of forgetfulness) is set far below unity.

4 Dynamic analysis

In this version of the model, human capital enhances the profitability of the next generation's Good projects (higher h_t increases $\rho^e_{t+1} = \frac{\partial f(k_{t+1}, h_t)}{\partial k_{t+1}} = \frac{\alpha}{1-\alpha} k_{t+1}^{\alpha-1} h_{t+1}^\gamma$, the expected reward of starting a Good project in period t). Thus, investing into Bad projects, which neither generate pecuniary nor technological externalities for the next generation, becomes more unattractive. In terms of the model, the profitability constraint tightens up. Through this mechanism human capital is expected to serve as a stabiliser as it creates more incentives (i.e. profit) to start Good projects. On the other hand, it also might boost the general output as the consumption of the old generation (which allocated their net worth at the end of period t) at the end of period $t+1$ is now higher, as human capital is also included in the final good, the numeraire.

Before starting the dynamical analysis, we shall point to the fact, that by setting $\gamma = 0$ and $h_0 = 0$, the model collapses in the original Matsuyama et al. (2016) model if the dynamics are expressed in k_t. Thus, we cannot directly compare the results to the original publication as there the analysed variable is w_t, the agent's net worth. We will present the dynamics of k_t, the physical capital stock per unit of labour. As the two-dimensional equation which governs the dynamics are too complicated to analyse analytically we stick to the method of simulation to determine stability features. However, omitting a detailed analytical treatment will help not to blur the core mechanism. The critical parameters in the original paper were the strength of the credit frictions (μ), the gross return (B) and the fixed investment size (m) of the Bad projects. Our set-up provide new parameters concerning human capital, like production elasticity (γ), depreciation rate (δ_h) and the strength of the external effect σ_h. Thus, and comprehensive study of the dynamics might go beyond the scope of this paper and therefore we concentrate on some empirical backed parametrization. For a first analysis, we fix $\delta_h = 0.05$ and $\sigma_h = 0.5$ and check the dynamical system in the (μ, B) parameter space. For the capital share we follow the standard macroeconomic literature and set $\alpha = 0.4$ (see parametrization, for example, in Bernanke & Gertler (1989)). Although we point out that this model is highly stylized therefore an exact parametrization seem not achievable. Following figures show the effect of an increase in γ on the (μ, B) parameter space.

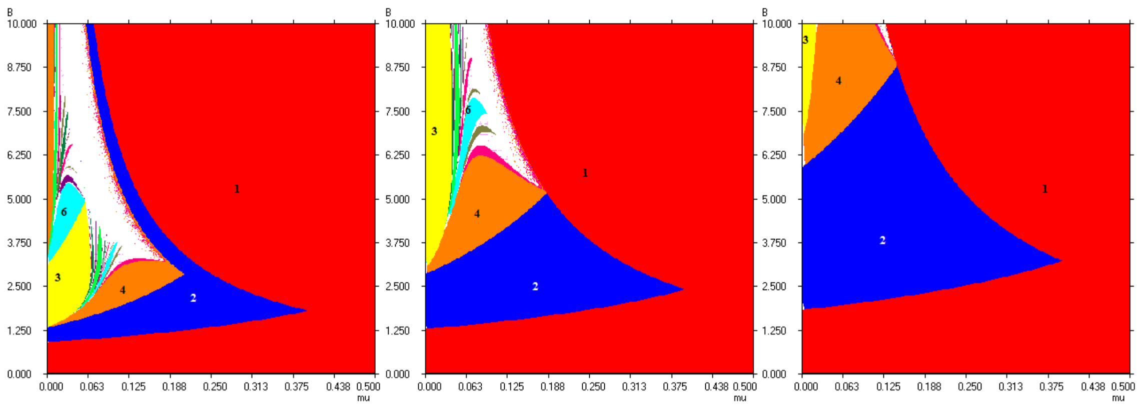

Figure 2: (μ, B) plane with $\gamma = 0.05$ (left), $\gamma = 0.10$ (center) and $\gamma = 0.15$ (right)

We observe a shrinkage of the area with high-order periodicity (white) but also an expansion of parameter combinations which eventually lead to period two and four cycles. On the one hand we find a destabilisation as a stable fixed point is harder to achieve with intermediate values of γ but also irregular cycles (i.e. cycles with period higher than 12) are less likely to occur. But, in fact, a much higher gross rate of return is necessary to enter the regions of high periodicity compared to the original model. Thus, we conclude that human capital -to be even more precise- the output elasticity of human capital (γ) serves as stabiliser in the following way: It pushed the required profitability, such that the profitability constraint is fully met, up. Therefore with lower gross returns, required credit will simply not flow into Bad projects which avoids the propagated mechanism of boom and bust cycles. On the other hand, for intermediated high values of B we indeed observe rich and complex dynamic behaviour of k, the physical capital which needs further investigation.

We stick now to the case, where we enter the region of high periodicity to check for interesting dynamic phenomena. For instance, Matsuyama et al. (2016) reports the so-called *corridor stability* for the parameter α which exhibits not only some nice dynamic properties but also a strong economic rational. As we use an augmented production function, we concentrate again on γ. We find the same phenomenon for certain γ-values, depending on the magnitude of B and μ.

Figure 3 reports the bifurcation scenario of parameter γ. The left panel shows the bifurcation structure for an intermediate range of γ-values. We observe a flip bifurcation (indicated by the green point) as the eigenvalue of the system equals minus one. The bifurcation at the red point is a *border collision bifurcation (BCB)* as the trajectory crosses the k_{mu} border, thus it moves from the second to the third segment. For $\gamma \gtrsim 0.257$ it stays at the third regime. Out of a period two cycle a fixed point is born. By enlarging the interval around $\gamma = 0.14$ (see boxed region) and varying the computing directions[3] we observe following phenomena: There is a coexisting stable period two cycle and a stable fixed point and, moreover, also a period two saddle point (indicated by the dotted line). This is exactly the phenomenon which Matsuyama et al. (2016) reported for parameter combination μB. The corridor is spanned by the unstable period two cycle (see dotted line).

To confirm those presumptions we perform some numerical simulations which leads to following results, reported in table 3. There is a triple cycle coexistence, a fixed point, a stable and a saddle period two.

[3]We initialise on the previous value.

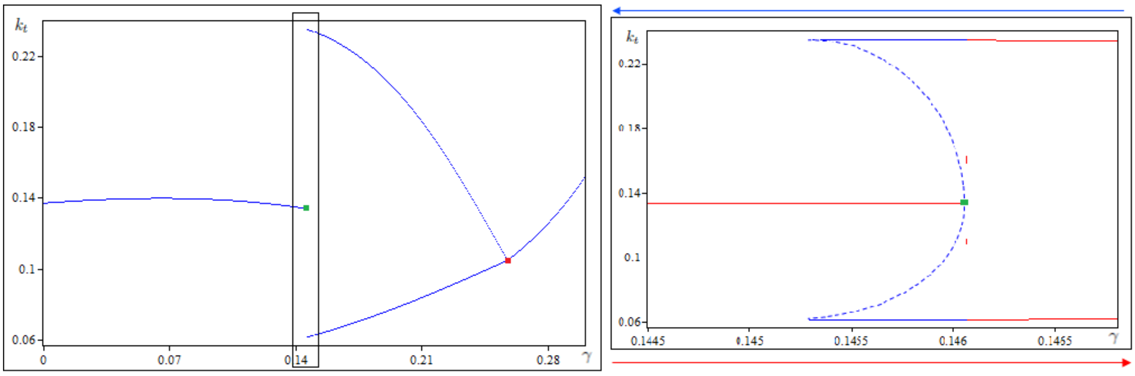

Figure 3: Bifurcation scenario of γ (with $B = 5, \mu = 0.25$); arrows indicate the direction of computation

Period	Classification	$L_x = \{(k^*, h^*)\}$
[1]	stable	$\{(0.1367, 10.5120)\}$
[2]	stable	$\{(0.0615, 10.4802), (0.2356, 10.3407)\}$
[2]	saddle	$\{(0.0771, 10.5087), (0.2074, 10.4044)\}$

Table 3: Numerical simulation results, $\gamma = 0.1455$, where L_x indicates the periodical fixed points.

Most interestingly, the basin of attraction[4], displayed in figure 4 shows some structures which we need to discuss in detail. The red area indicates a fixed point basin, the yellow and blue area a period two basin, depending where the cycle starts (0.06 or 0.23). The boundary between the (yellow) period two and the fixed point basin is the k_{mu} threshold. That is where the borrowing constraint is not binding any more. The basin boundary is spanned by the unstable period two cycle in a triangular shape forming a *corridor*.

Why is *corridor stability* an important issue? The parameter indicates the output elasticity with respect to human capital. In general, a larger γ means a higher persistence in human capital accumulation which indicated the importance of this parameter. If a shock hits this parameter the magnitude of the shock is crucial for the system. This corridor stabilty implies that is robust and self-correcting against small shocks but unstable against shocks with higher magnitude (see Leijonhufvud (1973) for a qualitative treatment of this issue). In this situation even a small positive shock in γ becomes *catastrophic* and *irreversible* if the flip bifurcation point is crossed. The latter means by reverting to the original parameter value, we will not come back to

[4]In terms of dynamical system theory, a basin of attraction of an attractor is the set of all initial conditions converging, after sufficient transient iterations, to that attractor.

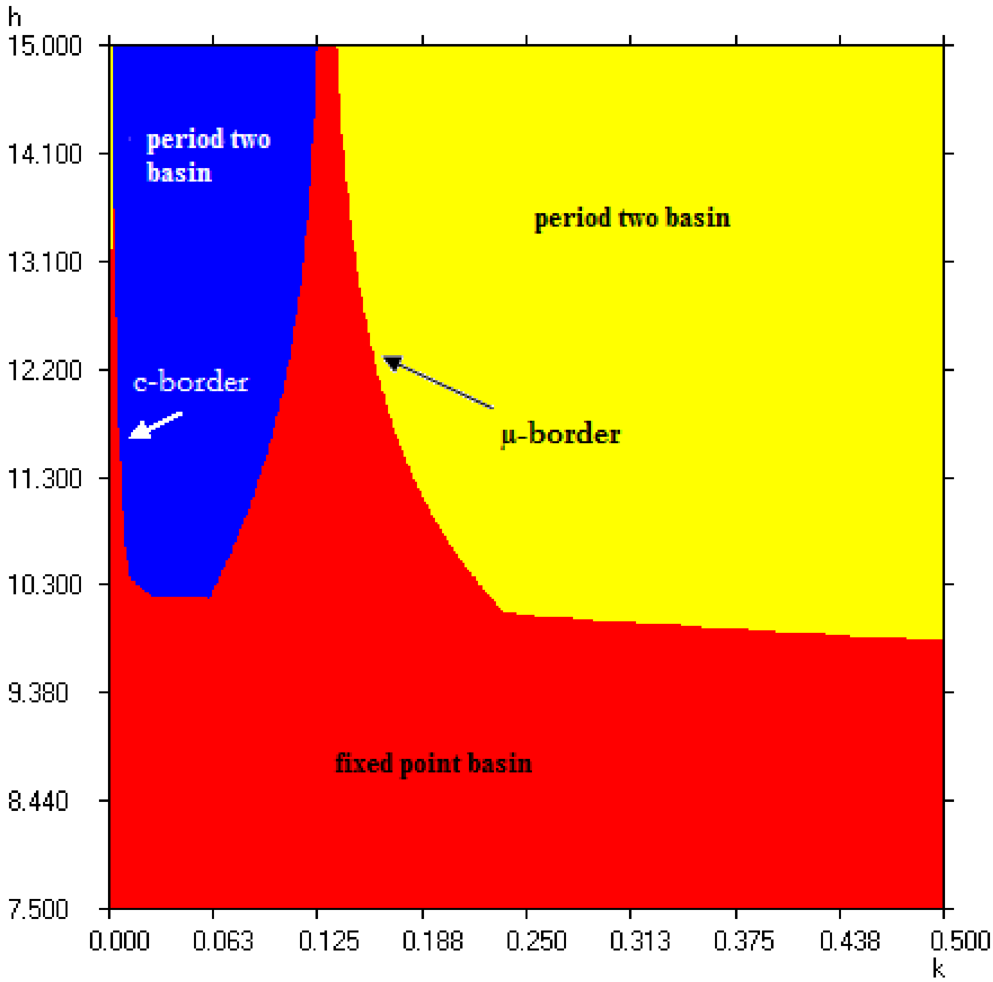

Figure 4: Basin of attraction

the stable fixed point but remain at the (stable) period two cycle (transition from the red to the blue bifurcation path in figure 3). The former characteristic is crucial for our model: Human capital brings another possible component into the model as it produces instability for a wide parameter range of γ. Even though the credit market friction parameter is set such that a convergence towards a stable steady state could be achieved without human capital. For an intermediate high human capital output elasticity (starting around $\gamma = 0.257$) again a stable steady state is born from a BCB. At least at this parameter configuration, human capital produces instability. Also in the augmented model, the corridor stability remains present for the friction parameter μ. We refer for a detailed study to the original publication.

The next situation analyses the interaction effects between the human capital parameter and the strength of the credit market friction. There exists a $[\mu, \gamma]$-parameter continuum where periods higher than order 11 occur. The left panel of figure 5 shows this situation and the right panel shows an enlargement of the boxed area on the left panel.

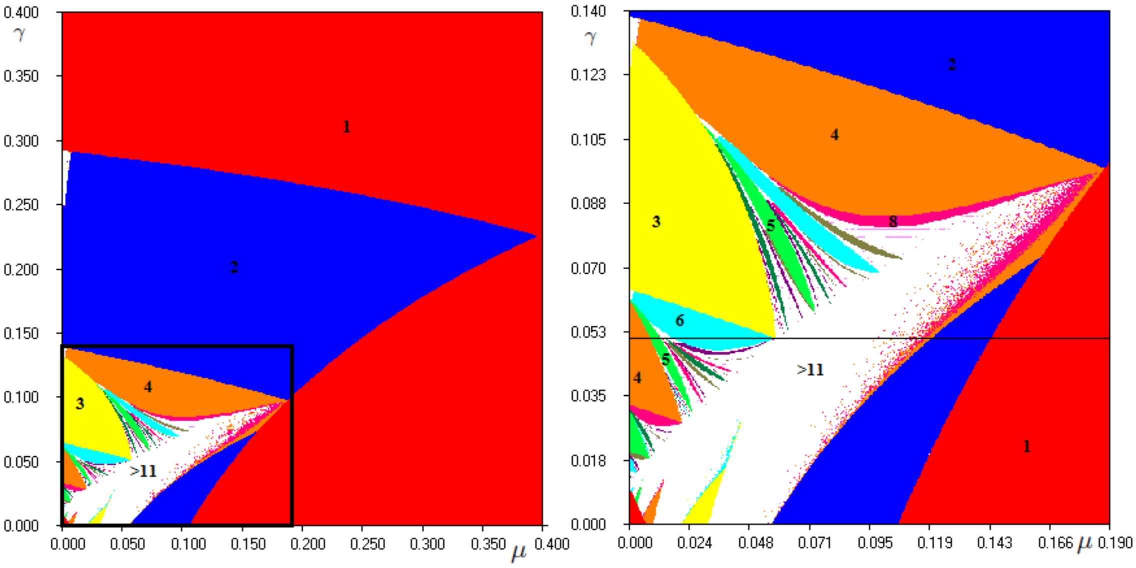

Figure 5: (μ, γ) plane with $\alpha = 0.4$ (left) and enlarged box (right). Numbers indicate length of stable cycle.

The boundary between the fixed point and the period two cycle parameter range has a clear structure. The fixed point looses its stability through a flip bifurcation, i.e. the eigenvalues of the Jacobian become minus one. Due to the complicated structure of the Jacobians (recall, that each of the three branches has its own Jacobian) an analytical treatment is omitted as it only will blur the economic meaning. We therefore stick to numerical simulation results. A closer look shows the rich internal bifurcation structure of the parameter μ.

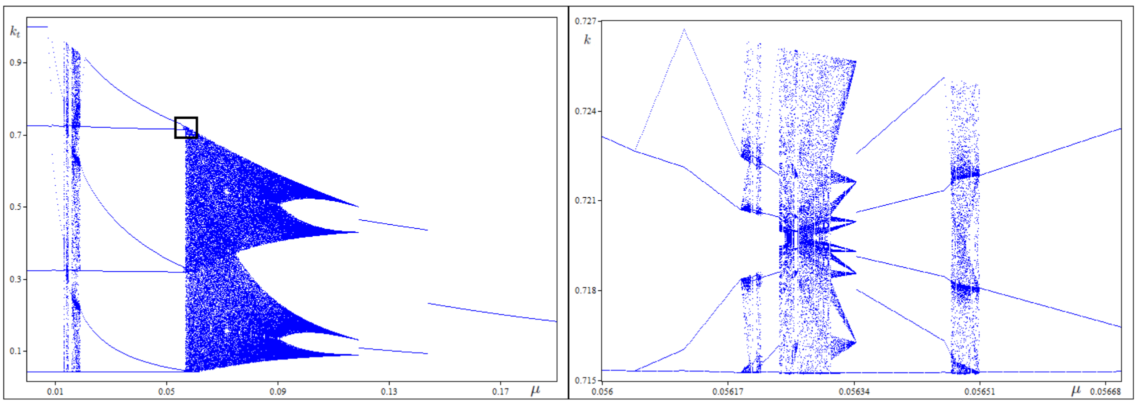

Figure 6: Bifurcations scenario of μ (left) and enlarged box (right).

By fixing $\gamma = 0.088$ we trace the bifurcation of μ through the horizontal line and observe the structures which is shown in the left panel of figure 6. The boxed area is enlarged in the right panel. We observe a –almost fractal– bifurcation structure, which can be seen on the right panel. A detailed (mathematical) bifurcation analysis is omitted as it will go far beyond the scope of this paper.

Not only the dynamics, but also the economic implications are important. Especially in economies with a very low human capital share, each change (or shock) in the financial sphere due to a negative variation in μ, the credit friction parameter, might lead to instability. This highly depends on the value of the capital share. These findings serve as a clear indicator for the robustness providing character of human capital. With high human capital shares, the economy becomes robust for changes of financial frictions, as the steady state (at least for our assumptions) is invariant under changes in μ.

5 Concluding remarks

In this paper we examined the impact of human capital on an economy with irregular output fluctuations which occurred due to credit flows into different investment projects. Human capital serves as technological component by reducing the effect of diminishing returns on physical capital. We observed new dynamical features which have an immediate impact on the system stability. It appears that the human capital share of production has a significant ambiguous impact on the stability. In general, a higher share tends to stabilise output measured in per capita physical capital. But, especially when the credit market friction is sufficiently high (i.e. μ is sufficiently low) low human capital shares introduce some instability by amplifying the cyclicity. This leads us to a first message regarding technological shocks. Matsuyama et al. (2016) reported the features of *corridor stability* for the credit market frictions parameter. We identified the same feature for the human capital share. As human capital serves as component of technological progress, we highlight the importance of technological shocks. A vast branch of literature in macroeconomic business cycle modelling deals with exogenous shocks (for example, shocks in total factor productivity, demand or supply) and their impact on stability. The shocks are usually assumed to be stochastic and following a mean reverting process. After the shock the system eventually returns to the equilibrium (the duration depends on the shock's persistence). Applied to our situation (i.e. the parameter change affects the corridor stability region), the system is robust to small shocks but suffers strongly from intermediate high shocks as it permanently looses its stability. On the other hand, a high human capital share tends to make the system's stability resistant to shocks from the credit market, as our simulation exercise clearly showed. With a high human capital share, irregular fluctuations eventually vanish. So the message is clear: Conventional linearised DSGE models might neglect the fact of nonlinearities due to linearisation around a stable steady state. By linearising the nonlinearities out, essential features of the (usually highly nonlinear) models might obscure essential interactions. In a worst case this might lead to simply wrong policy recommendations. We want to stress the fact that drawing policy recommendation out of a linearised model might induce contrary effects like destabilising an economy and should therefore carefully be considered.

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