Why has intra-euro real exchange rate adjustment been so small? Evidence from an estimated multi-region model

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This is a very preliminary draft, or rather an annotated outline for a paper.

Abstract

This paper reviews REER adjustment inside the EA. In particular, the paper presents results from estimated multi-region models for individual EA Member States. The analysis builds on shock decompositions of the real exchange rate for individual EA Member States (the current version is limited Germany) to reveal drivers of the dynamics of the real exchange rate, i.e. factors that have fostered or hampered real appreciation or depreciation. According to the estimated model, the German REER has been driven mainly by foreign and trade-related shocks. Domestic demand shocks have played little role for the REER, which helps explaining the lack of cyclicality in the REER. There is some role for (offsetting) supply shocks, however. In particular, labour market reform in Germany has supported REER depreciation, whereas negative TFP shocks have had the opposite effect. REER and TBY appear to be driven largely by different shocks (or by similar shocks at different time), which suggests that REER dynamics did not have strong influence on the German TBY.

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1. Introduction

Real exchange rate (REER) adjustment mitigates the impact of demand rebalancing on economic activity (e.g., Krugman 1990). In particular, REER depreciation can be expected to improve net exports and, hence, limit the output and employment loss associated with a contraction of domestic demand, such as the burst of unsustainably high domestic demand in the euro area (EA) periphery in recent years.

This paper reviews REER adjustment inside the EA. In particular, the paper presents results from estimated multi-region models for individual EA Member States, following the set-up in Kollmann et al. (2016).[1] Shock decompositions of the real exchange rate between individual Memmber States and the rest of the EA (REA) reveal drivers of the dynamics of the real exchange rate, i.e. factors that have fostered or hampered real appreciation or depreciation.

REER shock decompositions can be compared to shock decompositions for the trade balance (TBY). Both variables are endogenous variables in the model. To the extent that REER adjustment has helped external rebalancing, we would expect to see TBY and REER adjustment to be driven by the same factors. If, e.g., product or labour market reforms that reduce price or wage mark-ups had been behind REER depreciation and competitiveness-driven rebalancing, we would expect mark-up shocks to figure prominently in both TBY and REER adjustment. The methodology also lends itself to counterfactual analysis, assessing the impact of changes in structural parameters. We can, e.g., discuss how, given the drivers determined in the estimation, REER and TBY adjustment would have played out with different degrees of price and wage stickiness.

The paper will start by reviewing stylised facts about REER and TBY adjustment in the EA. We then introduce the model structure, outline details of the estimation, and discuss results from shock decompositions.

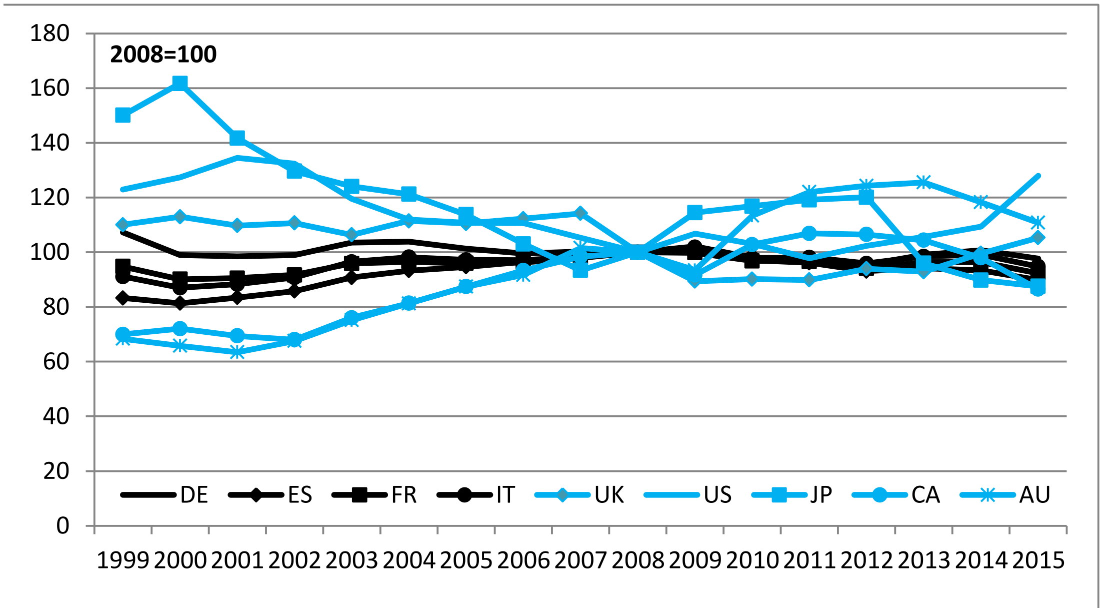
2. Stylised facts

Figure 1 shows that REER adjustment has been slow inside the EA. In particular, REER movements in the period 1999-2015 have been more pronounced in non-EA countries, which is compatible with the classical result by Mussa (1986) that REER fluctuations are dominantely driven by the volatility of nominal exchange rates, i.e. the component which is fixed for intra-EA real exchange rates. The relative stability of the REER in the economies without own nominal ex-

[1] The current draft only includes estimation results for Germany, but we envisage extension to other EA economies, notably Spain, Italy, and France.

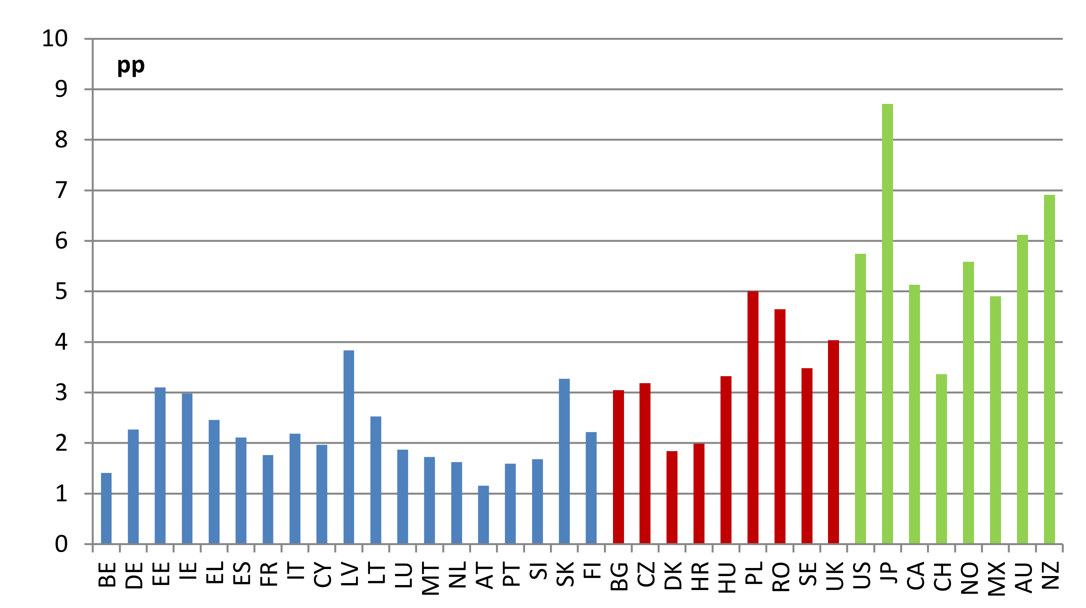
change rate is furthermore suggestive for the finding that REER dynamics has been largely non-cyclical (see, e.g., Kollmann 2016).

Figure 1: REERs in EA and non-EA countries



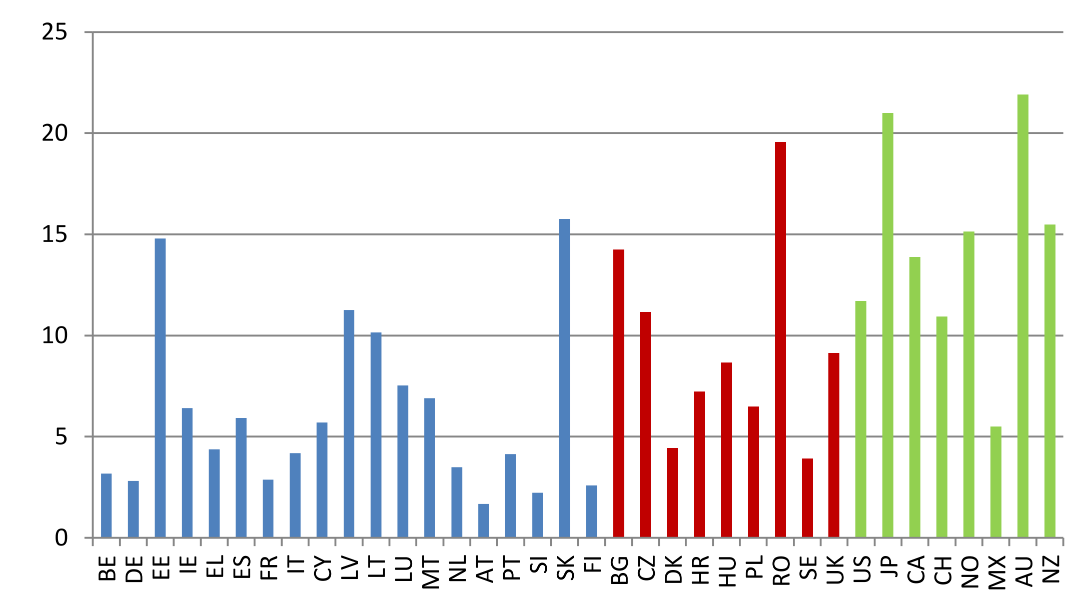
Note: REER is based on the GDP deflators. An increase in the REER indicates real effective appreciation.

Figure 2: Year-on-year absolute REER change (1999-2015)



Note: REER is based on the GDP deflators.

Figure 3: Standard deviation of annual REER series (1999-2015)



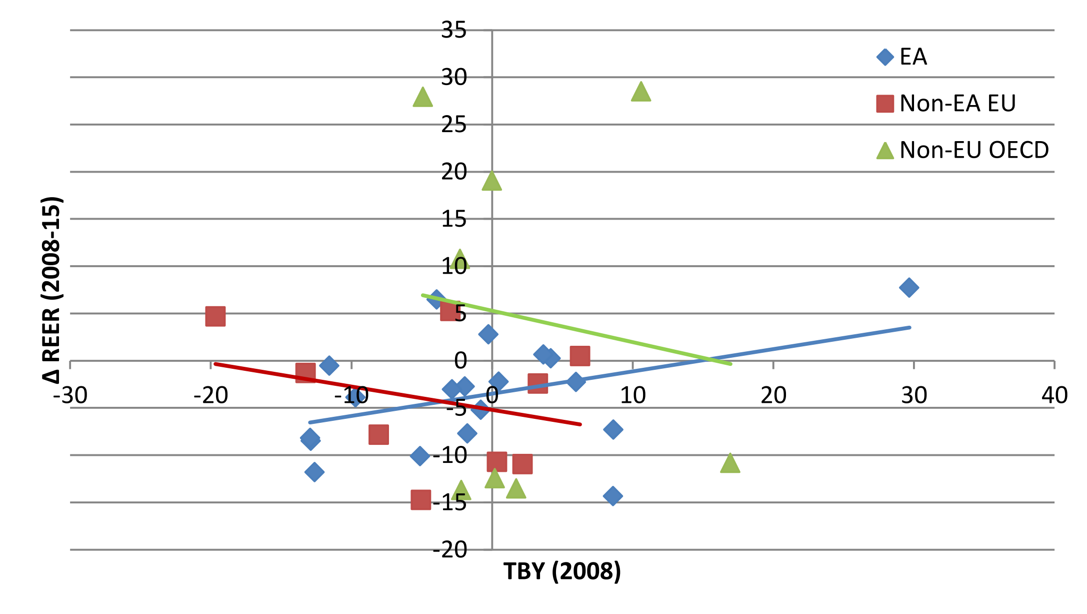
Note: REER is based on the GDP deflators.

The Figures 2 and 3 underline this observation by showing that year-on-year REER volatility has been least pronounced in EA Member States compared to non-EA EU countries and other economies with flexible nominal exchange rate. Note that those current EA Members States with relatively strong REER movement during 1999-2015 have been those that were still outside EMU during much of that time period.[2]

The link between REER and TBY adjustment appears to be weak. For EA Member States there is a tendency that a more negative TBY position in 2008 has been followed by stronger REER depreciation (Figure 4) and that stronger real depreciation has been associated overall with stronger TBY improvement (Figure 5) in a similar way that real appreciation has been associated with TBY deterioration during 1999-2008 (Figure 6).

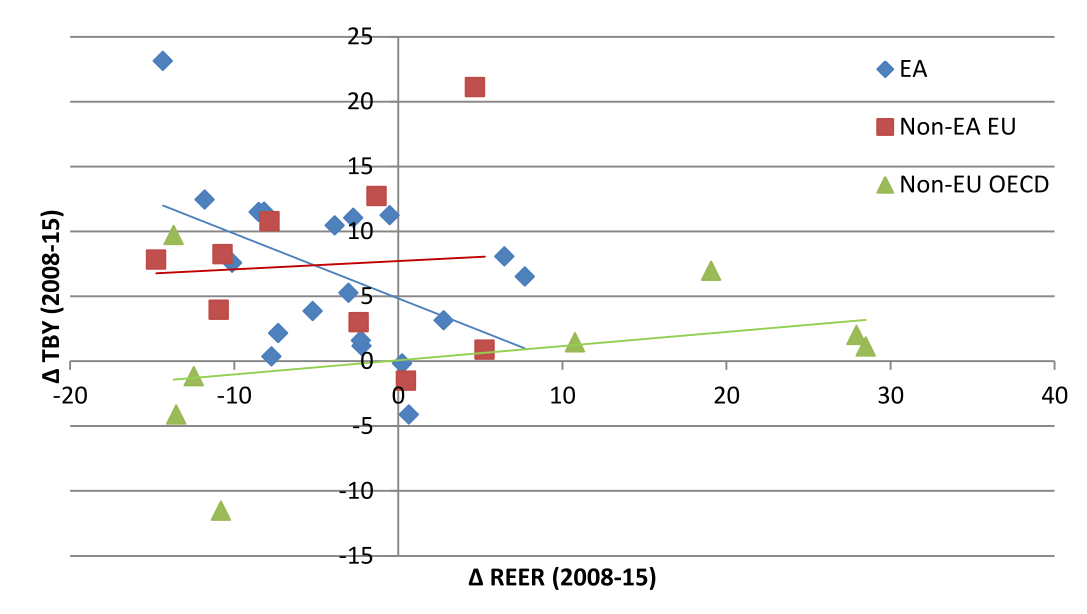
[2] While theoretically possible, the hypothesis that low REER volatility in EA Member States results from a low occurence of shocks (besides shocks to currency premia) has little plausibility given the pronounced shocks and cyclical fluctuations in a number of EA Member States over the period considered.

Figure 4: External imbalances and subsequent REER adjustment (EA-EU-OECD)



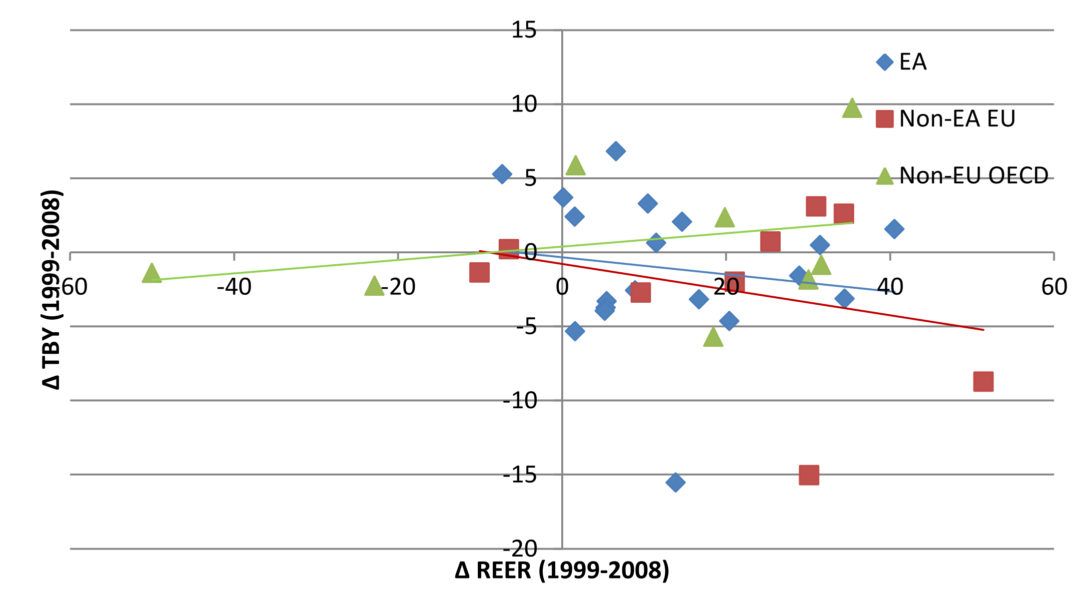
Note: REER is based on the GDP deflators. An increase in the REER indicates real effective appreciation.

Figure 5: REER and TBY adjustment 2008-15 (EA-EU-OECD)



Note: REER is based on the GDP deflators. An increase in the REER indicates real effective appreciation.

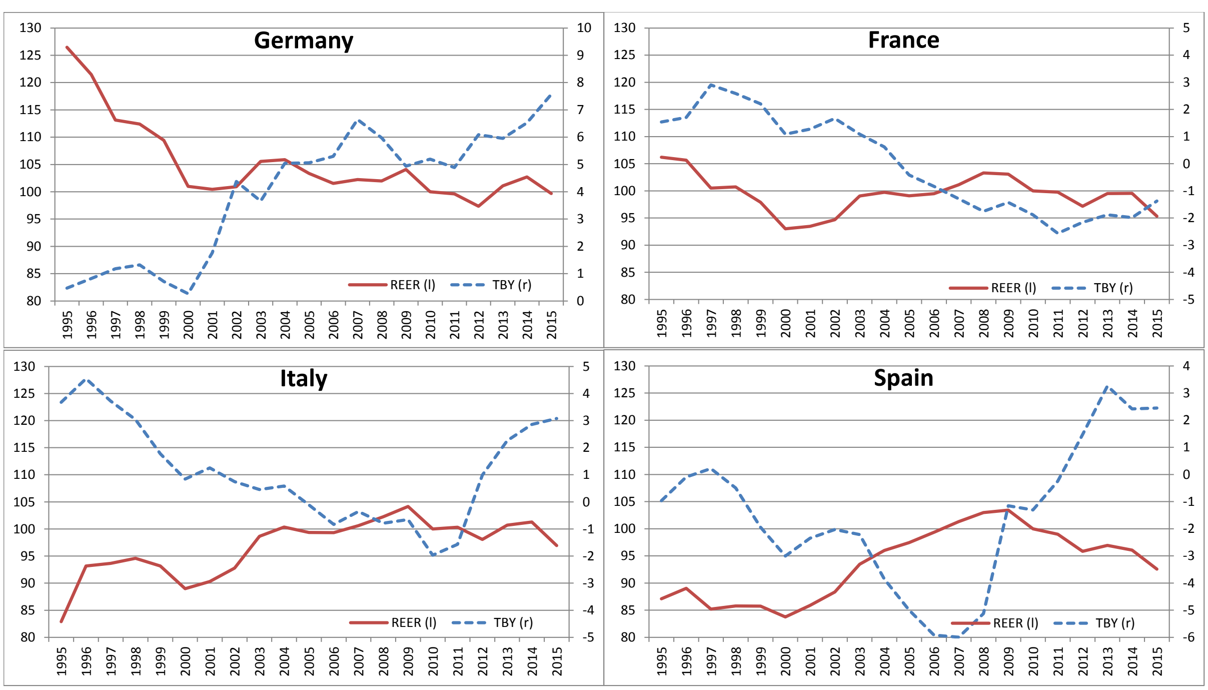
Figure 6: REER and TBY adjustment 1999-2008 (EA-EU-OECD)



Note: REER is based on the GDP deflators. An increase in the REER indicates real effective appreciation.

Country profiles for Germany, France, Italy, and Spain, however, show that the REER-TBY co-movement has not been particularly strong over time (Figure 7). Furthermore, co-movement does not imply causality, but may be due to common causes. In particular, booms (busts) in domestic demand that move the TBY through imports also move the REER through growing (declining) inflation pressure.

Figure 7: REER and TBY paths 1999-2015 (DE-FR-IT-ES)



Note: REER is based on the GDP deflators. An increase in the REER indicates real effective appreciation.

Nevertheless, Figure 7 provides evidence for differing country paths in recent TBY rebalancing. In particular, TBY adjustment in Spain has been accompanied by real depreciation and, helped by the associated gain in price competitiveness, stronger export growth, whereas Italy has witnessed little REER depreciation and TBY adjustment rather predominantly through an import decline

3. Model description

The analysis uses the multi-region open-economy framework of Kollmann et al. (2016) and adapts it to a setting with an EA Member State (MS), the REA, and the RoW. The EA Member State block of the model is rather detailed, while the REA and RoW blocks are more stylized. The EA MS block assumes two (representative) households, a number of layers of firms and a government. EA MS households provide labor services to firms. One of the two households (savers, or 'Ricardians') in each country has access to financial markets, and she owns her country's firms. The other (liquidity-constrained, or 'non-Ricardian') household has no access to financial markets, does not own financial or physical capital, and in each period only consumes the disposable wage and transfer income. The preferences of both types of household exhibit habit formation in both consumption and leisure, a feature which allows for better capturing persistence of the data.

There is a monopolistically-competitive sector producing differentiated goods in the EA MS, using domestic labor and capital and being able to. The firms in the sector maximize the present value of dividends at a discount factor that is strictly larger than the risk-free rate and varies over time. This is a short-cut for capturing financial frictions facing firms; it can, e.g., be interpreted as a 'principal agent friction' between the owner and the management of the firm. Optimization is subject to investment and labor adjustment costs and a varying capacity utilization rate, which lets the model better capture the dynamics of the current account and other macro variables. Total output in the EA MS is produced by combining the domestic differentiated goods bundle with energy input. EA MS wages are set by monopolistic trade unions. Nominal differentiated goods prices are sticky as are the wages paid to the workers. Fiscal authorities in the EA MS impose distortive taxes and issue debt.

The REA and RoW blocks are simplified compared to the EA MS block. Specifically, the REA and RoW consists of a budget constraint for the representative household, demand functions for domestic and imported goods (derived from CES consumption good aggregators), a production technology that uses labor as the sole factor input, and a New Keynesian Phillips curve. The REA and RoW blocks abstracts from capital accumulation.

The behavioral relationships and technology are subject to autocorrelated shocks denoted by ε_t^x , where *x* stands for the type of shock. ε_t^x will generally follow an AR(1) process with autocorrelation coefficient $\rho^x < 1$ and innovation u_t^x: $(\varepsilon_t^x) = \rho^x(\varepsilon_t^x) + u_t^x$. There is also a separate category of shocks, denoted A_t^x, whose logs are integrated of order 1.[3] With the exception of the TFP shocks, these shocks are modelled as ARIMA(1,1,0) shocks.[4]

We next present a detailed description of the EA MS block, followed by an overview of the REA and RoW model blocks. Throughout the derivation the following indexing convention will be preserved. Indices *i* and *j* index firms and households, respectively. These indices will usually be dropped when the equilibrium conditions are derived due to the representative household/firm assumption. Index *l* indicates sovereign states or economic regions. Finally, index *k* will always indicate the 'domestic' economy. This index will be generally dropped for parameters (even if they are country-specific), but will be usually preserved for variables.

3.1. EA Member State households

The household sector consists of a continuum of households $j \in [0;1]$. There are two types of households, savers ("Ricardians", superscript *s*) who own firms and hold government and foreign bonds and liquidity-constrained households (subscript *c*) whose only income is labor income and who do not save. The share of savers in the population is ω^s .

Both households enjoy utility from consumption C_{jkt}^r and incur disutility from labor N_{jkt}^r ($r = s, c$). On top of this, Ricardian's utility depends also on the financial assets held.

Date *t* expected life-time utility of household *r*, is defined as:

$$U_{jkt}^r = \sum_{s=t}^{\infty} \varepsilon_{kt}^c \beta^{s-t} u_{jkt}^r(\cdot)$$

where β is the (non-stochastic) discount factor (common for both types of households) and ε_{kt}^c is the saving shock.

3.1.1. Ricardian household

The Ricardian households work, consume, own firms and receive nominal transfers T_{jkt}^s from the government. Ricardians have full access to financial markets and are the only households who

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[3] These, in particular, include the TFP shock and the final demand productivity shocks.
[4] TFP is driven by 3 shocks, see below.

own financial assets $\frac{A_{jkt}}{P_{kt}^{c,vat}}$ where $P_{kt}^{c,vat}$ is consumption price, including VAT.[5] Financial wealth of household *j* consists of bonds $\frac{B_{jkt}}{P_{kt}^{c,vat}}$ and shares $\frac{P_{kt}^{S} S_{jkt}}{P_{kt}^{c,vat}}$, where P_{kt}^{S} is the nominal price of shares in *t* and S_{jkt} the number of shares held by the household:

$$\frac{A_{jkt}}{P_{kt}^{c,vat}} = \frac{B_{jkt}}{P_{kt}^{c,vat}} + \frac{P_{kt}^{S} S_{jkt}}{P_{kt}^{c,vat}}$$

It is assumed that households invest only in domestic shares. Bonds consist of government domestic $\frac{B_{jkkt}^{g}}{P_{kt}^{c,vat}}$ and foreign bonds $\frac{e_{lkt}\ B_{jlkt}^{g}}{P_{kt}^{c,vat}}$ and private risk-free bonds $\frac{B_{jkt}^{rf}}{P_{kt}^{c,vat}}$ (in zero supply):

$$\frac{B_{jkt}}{P_{kt}^{c,vat}} = \frac{B_{jkt}^{rf}}{P_{kt}^{c,vat}} + \sum_{l} e_{lkt} B_{jlkt}^{g}$$

with e_{lkt} the bilateral exchange rate and $e_{kkt} \equiv 1$.[6] The budget constraint of a saver household *j* is:

$$(1-\tau^{N})W_{kt}N_{jkt}^{s} + \sum_{l}(1+i_{lt-1}^{g})e_{lkt}B_{jlkt-1}^{g} + (1+i_{t-1}^{rf})B_{jkt-1}^{rf} + (P_{kt}^{S} + P_{kt}^{Y}d_{kt})S_{jkt-1} + div_{kt} + T_{jkt}^{s} - tax^{s}{}_{jkt} = P_{kt}^{c,vat}C_{jkt}^{s} + A_{jkt}$$

where W_{kt} is the nominal wage rate, P_{kt}^{Y}, is GDP price deflator, i_{lt-1}^{g} are interest rates on government bonds of region *l*, i_{t-1}^{rf} is interest rate on risk-free bond, T_{jkt}^{s} are government transfers to savers and $tax^{s}{}_{jkt}$ are lump-sum taxes paid by savers. Note that savers own all the firms in the economy. div_{kt} represent the profits of all firms other than differentiated goods producers (the latter producers transfer profits to savers by paying dividends d_{kt}).

We define the gross nominal return on domestic shares as:

$$1 + i_{kt}^{s} = \frac{P_{kt}^{S} + P_{kt}^{Y}d_{kt}}{P_{kt-1}^{S}}$$

The instantaneous utility functions of savers, $u^{s}(\cdot)$, is defined as:

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5 Note that $P_{kt}^{c,vat}$ is related to P_{kt}^{C}, the private consumption deflator in terms of input factors, by the formula: $P_{kt}^{c,vat} = (1+\tau_{k}^{C})P_{kt}^{C}$ where τ^{c} is the tax on consumption.

6 For simplicity, at this moment the model assumes only one type of foreign bonds, B_{RoWkt}^{g}, issued by RoW and denominated in RoW currency.

$$u^s\left(C^s_{jkt}, N^s_{jkt}, \frac{U^A_{jkt-1}}{P^{c,vat}_{kt}}\right)$$
$$= \frac{1}{1-\theta}\left(C^s_{jkt} - hC^s_{kt-1}\right)^{1-\theta} - \frac{\omega^N \varepsilon^U_{kt}}{1+\theta^N}\left(C^s_{kt}\right)^{1-\theta}\left(N^s_{jkt} - h_N N^s_{kt-1}\right)^{1+\theta^N}$$
$$- \left(C^s_{kt} - hC^s_{kt-1}\right)^{-\theta}\frac{U^A_{jkt-1}}{P^{c,vat}_{kt}}$$

where $C^s_{kt} = \int C^s_{jkt}$, $C_{kt} = \omega^s C^s_{kt} + (1-\omega^s)C^c_{kt}$; $h, h_N \in (0;1)$ measure the strength of the external habits in consumption and labor and $\varepsilon^U_{kt}$is the labor supply (or wage mark-up) shock . The disutility of holding financial assets, U^A_{jkt-1}, is defined as:

$$U^A_{jkt-1} = \sum_l \left(\left(\alpha^{bB0}_{lk} + \varepsilon^B_{lkt-1}\right) e_{lkt-1} B^g{}_{jlkt-1}\right) +$$

$$\left(\left(\alpha^{sS0}_k + \varepsilon^S_{kt-1}\right) P^s_{st-1}\, S_{jkt-1}\right)$$

The Ricardian household problem leads to the following first order conditions (FOC).[7]

The FOC w.r.t. savers' consumption produces:

$$\varepsilon^C_{kt}\left(C^s_{kt} - hC^s_{kt-1}\right)^{-\theta} = \lambda^s_{kt}$$

where λ^s_{kt} is the Lagrange multiplier on the budget constraint.

FOC w.r.t. domestic risk-free bond:

$$\beta E_t\left[\frac{\lambda^s_{kt+1}}{\lambda^s_{kt}}\frac{1+i^{rf}_{kt}}{1+\pi^{C,vat}_{kt+1}}\right] = 1$$

FOC w.r.t. domestic government bonds:

$$\beta E_t\left[\frac{\lambda^s_{kt+1}}{\lambda^s_{kt}}\frac{1+i^g_{kt} - \varepsilon^B_{kt} - \alpha^{b0}_{kk}}{1+\pi^{C,vat}_{kt+1}}\right] = 1$$

with $\pi^{C,vat}_{kt}$ the consumption deflator inflation rate and ε^B_{kt} the risk-premium on government bonds.

FOC w.r.t. RoW government bonds:

$$\beta E_t\left[\frac{\lambda^s_{kt+1}}{\lambda^s_{kt}}\frac{\left(1+i^g_{RoWkt}\right)\frac{e_{RoWkt+1}}{e_{RoWkt}} - \varepsilon^B_{RoWkt} - \left(\alpha^{b0}_{RoWk} + \alpha^{b1}_{RoWk}\frac{e_{RoWkt}B^g_{RoWkt}}{P^Y_k Y_k}\right)}{1+\pi^{C,vat}_{kt+1}}\right] = 1$$

where ε^B_{RoWkt} the risk-premium on RoW bonds.

[7] See subsection 3.1.3 for the labor supply condition.

FOC w.r.t. domestic stocks:

$$\beta E_t\left[\frac{\lambda^s_{kt+1}}{\lambda^s_{kt}}\frac{(1+i^s_{kt+1})-\varepsilon^S_{kt}-\alpha^{s0}_{kk}}{1+\pi^{C,vat}_{t+1}}\right]=1$$

where ε^S_{kt} the risk-premium on stocks. The above optimality conditions are similar to a textbook Euler equation, but incorporate asset-specific risk premia, which depend on an exogenous shock ε^A_{kt} as well as the size of the asset holdings as a share of GDP. Taking into account the Euler equation for the risk-free bond and approximating, they simplify to the familiar expressions:

$$i^g_{kt}=i^{rf}_{kt}+rprem^g_{kt}$$

$$E_t\left[\frac{e_{RoWkt+1}}{e_{RoWkt}}\right]i^g_{RoWkt}=i^{rf}_{kt}+rprem^g_{RoWkt}$$

$$i^s_{kt}=i^{rf}_{kt}+rprem^s_{kt}$$

In the equations above, $rprem^g_{kt}$ is the risk premium on domestic government bonds. Similarly, $rprem^g_{RoWkt}$ is the risk premium on domestic government bonds sold abroad (to RoW). This feature of the model, hence, helps capture international spillovers that occur via the financial market channel. Finally, $rprem^s_{kt}$ is a crucial risk premium on domestic shares. It is introduced to capture in a stylized manner financial frictions that are commonly believed to have contributed to the first phase of the financial crisis and may have contributed to its second phase, see also subsection 3.2.2, below.[8]

3.1.2. liquidity-constrained household

The liquidity-constrained household consumes her disposable after-tax wage and transfer income in each period of time ('hand-to-mouth'). The period t budget constraint of the liquidity-constrained household is:

$$(1+\tau^C_k)P^C_{kt}C^c_{jkt}=(1-\tau_k{}^N)W_{kt}N^c_{kt}+T^c_{kt}-tax^c{}_{jkt}.$$

The instantaneous utility functions for liquidity-constrained households. $u^c(\cdot)$, is defined as:

$$u^c\left(C^c_{jkt},N^c_{jkt}\right)=\frac{1}{1-\theta}\left(C^c_{jkt}-hC^c_{kt-1}\right)^{1-\theta}-\left(C^c_{kt}\right)^{1-\theta}\frac{\omega^N exp(u^U_{kt})}{1+\theta^N}\left(N^c_{jkt}-h_N N^c_{kt-1}\right)^{1+\theta^N}$$

with $C^c_{kt}=\int C^c_{jkt}$.

3.1.3. Labor supply

Trade unions are maximizing a joint utility function for each type of labor. It is assumed that types of labor are distributed equally over Ricardian and liquidity-constrained households with

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[8] Observationally, this approach is equivalent to exogenous risk premia as well as risk premia derived in the spirit of Bernanke, Gertler & Gilchrist.

their respective population weights. The wage rule is obtained by equating a weighted average of the marginal utility of leisure to a weighted average of the marginal utility of consumption times the real wage adjusted for a wage mark-up. Nominal rigidity in wage setting is introduced in the form of adjustment costs for changing wages. The wage adjustment costs are borne by the household. Real wage rigidity is also allowed, given the following optimality condition:

$$\left((1+\mu_t^w)\frac{\omega^s V_{1-l,jkt}^s+(1-\omega^s)V_{1-l,jkt}^c}{\omega^s U_{c,jkt}^s+(1-\omega^s)U_{c,jkt}^c}(1+\tau_k^C)p_{kt}^C\right)^{1-\gamma^{wr}}\left((1-\tau_k{}^N)\frac{W_{kt-1}}{P_{kt-1}^Y}\right)^{\gamma^{wr}} = (1-\tau_k{}^N)\frac{W_{kt}}{P_{kt}^Y} +$$
$$\gamma^w(\pi_t^w-(1-sf^w)\pi_{t-1}^w)(1+\pi_t^w)$$
$$-\gamma^w\frac{L_{t+1}}{L_t}\frac{1+\pi_{t+1}^y}{1+i_{t+1}^{sd}}(\pi_{t+1}^w-(1-sf^w)\pi_t^w)(1+\pi_{t+1}^w)$$

where μ_t^w is the wage mark-up, γ^{wr} is the degree of real wage rigidity, γ^w is the degree of nominal wage rigidity and sf^w is the degree of forward-lookingness in the labor supply equation.

$V_{N,jkt}^x$, for x=s,c, is the marginal disutility of labor, defined as:

$$V_{N,jkt}^x = \omega^N exp(u_{kt}^U)C_{kt}{}^{1-\theta}(N_{jkt}^x-h_N N_{kt-1}^x)^{\theta^N}$$

3.2. EA Member State production sector

3.2.1. Total output demand

Total output O_{kt} is produced by perfectly competitive firms by combining value added, Y_{kt}, with energy input, Oil_{kt}, using the following CES production function:

$$O_{kt} = \left[(1-s^{Oil})^{\frac{1}{\sigma^O}}(Y_{kt})^{\frac{\sigma^O-1}{\sigma^O}}+(s^{Oil})^{\frac{1}{\sigma^O}}(OIL_{kt})^{\frac{\sigma^O-1}{\sigma^O}}\right]^{\frac{\sigma^O}{\sigma^O-1}}$$

where s^{Oil} is the energy input share in total output and elasticity σ^o is inversely related to the steady state output price gross mark-up. It follows that the demand for Y_{kt} and OIL_{kt} by total output producers is, respectively:

$$Y_{kt} = (1-s^{Oil})\left(\frac{P_{kt}^Y}{P_{kt}^O}\right)^{-\sigma^O} O_{kt}$$

$$OIL_{kt} = s^{Oil}\left(\frac{P_{kt}^{Oil}}{P_{kt}^O}\right)^{-\sigma^O} O_{kt}$$

where P_{kt}^Y and P_{kt}^{Oil} are price deflators associated with Y_{kt} and Oil_{kt}, respectively, and the total output deflator P_{kt}^O is such that:

$$P_{kt}^O = \left[(1-s^{Oil})(P_{kt}^Y)^{1-\sigma^O}+s^{Oil}(P_{kt}^{Oil})^{1-\sigma^O}\right]^{\frac{1}{1-\sigma^O}}$$

3.2.2. Differentiated goods supply

Each firm $i \in [0;1]$ produces a variety of the domestic good which is an imperfect substitute for varieties produced by other firms. Given imperfect substitutability, firms are monopolistically competitive in the goods market and face a downward-sloping demand function for goods. Domestic final good producers then combine the different varieties into a homogenous good and sell them to domestic final demand goods producers and exporters.

Differentiated goods are produced using total capital K_{ikt-1}^{tot} and labour N_{ikt} which are combined in a Cobb-Douglas production function:

$$Y_{ikt} = (A_{kt}^{Y} N_{ikt})^{\alpha} (cu_{ikt} K_{ikt-1}^{tot})^{1-\alpha}$$

where A_{kt}^{Y} is labour-augmenting productivity shock common to all firms in the differentiated goods sector and cu_{ikt} is firm-specific level of capital utilization. Total Factor Productivity, TFP_{kt}, can therefore be defined as:

$$TFP_{kt} = (A_{kt}^{Y})^{\alpha}.$$

We allow for three types of shocks related to the technology: a temporary shock ε_{kt}^{AY} which accounts for temporary deviations of A_{kt}^{Y} from its trend, $\bar{A}_{kt}^{Y}$, and two shocks related to the trend components itself:

$$\log(A_{kt}^{Y}) - \log(\bar{A}_{kt}^{Y}) = \varepsilon_{kt}^{AY}$$

$$\log(\bar{A}_{kt}^{Y}) - \log(\bar{A}_{kt-1}^{Y}) = g_{kt}^{\overline{AY}} + \varepsilon_{kt}^{L\overline{AY}}$$

$$g_{kt}^{\overline{AY}} = \rho^{\overline{AY}} g_{kt-1}^{\overline{AY}} + \varepsilon_{kt}^{G\overline{AY}} + \left(1 - \rho^{\overline{AY}}\right) g^{\overline{AY}}$$

with $g^{\overline{AY}}$ being the long-run technology growth.

Total capital is a sum of private installed capital, K_{ikt}, and public capital, K_{ikt}^{g}:

$$K_{ikt}^{tot} = K_{ikt} + K_{ikt}^{g}$$

The producers maximize the value of the firm, V_{kt}, equal to a discounted stream of future dividends, $V_{kt} = d_{kt} + E_t[sdf_{kt+1} V_{kt+1}]$, with the stochastic discount factor

$$sdf_{kt} = \left(1 + i_{kt}^{sd}\right) / \left(1 + \pi_{kt}^{C,vat}\right) \approx \left(1 + i_{kt-1}^{rf} + rprem_{kt-1}^{s}\right) / \left(1 + \pi_{kt}^{C,vat}\right)$$

which depends directly on the investment risk premium, $rprem_{kt-1}^{s}$. The dividends are defined as:

$$d_{ikt} = (1 - \tau_k^K) \left(\frac{P_{ikt}^{Y}}{P_{kt}^{Y}} Y_{ikt} - \frac{W_{kt}}{P_{kt}^{Y}} N_{ikt} \right) + \tau_k^K \delta \frac{P_{kt}^{I}}{P_{kt}^{Y}} K_{ikt-1} - \frac{P_{kt}^{I}}{P_{kt}^{Y}} I_{ikt} - adj_{ikt}$$

where I_{ikt} is physical investment, P^I_{kt} is investment price, τ^K_k is the profit tax, δ is capital depreciation rate and adj_{ikt} are adjustment costs associated with price P^Y_{ikt} and labour input N_{ikt} adjustment or moving capacity utilization cu_{ikt} and investment I_{ikt} away from their optimal level:

$$adj_{ikt} = adj(P^Y_{ikt}) + adj(N_{ikt}) + adj(cu_{ikt}) + adj(I_{ikt}) \text{ where}$$
$$adj(P^Y_{ikt}) = \frac{\gamma^p}{2} Y_{kt}\left(\frac{P^Y_{ikt}}{P^Y_{ikt-1}} - 1\right)^2$$
$$adj(N_{ikt}) = \frac{\gamma^n}{2} Y_{kt}\left(\frac{N_{ikt}}{N_{ikt-1}} - 1\right)^2$$
$$adj(cu_{ikt}) = \frac{P^I_{kt}}{P^Y_{kt}} K_{ikt-1}\left(\gamma^{u,1}(cu_{ikt} - 1) + \frac{\gamma^{u,2}}{2}(cu_{ikt} - 1)^2\right)$$
$$adj(I_{ikt}) = \frac{P^I_{kt}}{P^Y_{kt}}\left(\frac{\gamma^{I,1}}{2} K_{kt-1}\left(\frac{I_{ikt}}{K_{kt-1}} - \delta\right)^2 + \frac{\gamma^{I,2}}{2}\frac{(I_{ikt} - I_{ikt-1})^2}{K_{kt-1}}\right)$$

The maximization is subject to production function, standard capital accumulation equation:

$$K_{ikt} = (1-\delta)K_{ikt-1} + I_{ikt}$$

and the usual demand condition which inversely links demand for variety *i* goods and the price of the variety:

$$Y_{ikt} = \left(\frac{P^Y_{ikt}}{P^Y_{kt}}\right)^{-\sigma^y} Y_{kt}$$

Let $adj_{X,ikt}$ for $X = P^Y, N, cu, I$ denote additional dynamic terms due to the existence of adjustment costs. Let also define $g^X_{kt} := \frac{X_{kt} - X_{kt-1}}{X_{kt-1}}$ the net growth rate of variable $X = N, Y, I, C, \ldots$ and $\pi^X_{kt} := \frac{\Delta P^X_{kt}}{P^X_{kt-1}}$ the inflation rate of a price deflator associated with variable $X = N, Y, I, C, \ldots$ The main optimality conditions of the differentiated goods producers are as follows.

The usual equality between the marginal product of labor and labor cost holds, with a wedge driven by the labor adjustment costs:

$$\mu^y_{kt}\alpha\frac{Y_{kt}}{N_{kt}} - adj_{N,ikt} = (1-\tau^k)\frac{W_{kt}}{P^Y_{kt}}$$

with μ^y_{kt} being inversely related to the price mark-up. The capital optimality condition reflects the usual dynamic trade-off faced by the firm:

$$\frac{1+\pi^y_{kt+1}}{1+i^{sd}_{kt+1}}\frac{P^I_{kt+1}/P^Y_{kt+1}}{P^I_{kt}/P^Y_{kt}}\left(\mu^y_{kt+1}(1-\alpha)\frac{P^Y_{kt+1}Y_{ikt+1}}{P^I_{kt+1}K^{tot}_{ikt}} + \tau^k\delta - adj^{cu}_{kt}/K_{ikt} + (1-\delta)Q_{kt+1}\right) = Q_{kt}$$

where Q_{kt} has the usual Tobin's interpretation.

FOC w.r.t. investment implies that Tobin's Q varies due to the existence of investment adjustment costs:

$$Q_{kt} = 1 + adj_{I,ikt}$$

Firms adjust their capacity utilization depending on the conditions on the market via the optimality condition:

$$\frac{\mu^{y}_{kt}}{P^{I}_{kt}/P^{Y}_{kt}}(1-\alpha)\frac{Y_{kt}}{cu_{kt}} = adj_{cu,ikt}$$

Finally, the FOC w.r.t. differentiated output price pins down the price mark-up:

$$\frac{\sigma^{y}}{(\sigma^{y}-1)}\mu^{y}_{kt} = (1-\tau^{k}) + \frac{adj_{P^{Y},ikt}}{(\sigma^{y}-1)} + \varepsilon^{\mu}_{kt}$$

with ε^{μ}_{kt} being the markup shock. The latter equation, combined with the FOC w.r.t. labor implies the Phillips curve of the familiar form.

3.3. Trade

3.3.1. Import sector

Aggregate demand components

The final aggregate demand component goods C_{kt} (private consumption good), I_{kt}, (private investment good) G_{kt} (government consumption good) and I^{G}_{kt} (government investment good) are produced by perfectly competitive firms by combining domestic output, O^{Z}_{kt} with imported goods M^{Z}_{kt}, $Z = C, I, G, I^{G}$, using the following CES production function:

$$Z_{kt} = A^{p^{z}}_{kt}\left[(1-\varepsilon^{M}_{kt}s^{M,Z})^{\frac{1}{\sigma^{z}}}(O^{Z}_{kt})^{\frac{\sigma^{z}-1}{\sigma^{z}}} + (\varepsilon^{M}_{kt}s^{M,Z})^{\frac{1}{\sigma^{z}}}(M^{Z}_{kt})^{\frac{\sigma^{z}-1}{\sigma^{z}}}\right]^{\frac{\sigma^{z}}{\sigma^{z}-1}}$$

with $A^{p^{z}}_{kt}$ a shock to productivity in the sector producing goods Z and ε^{M}_{kt} is a shock to the share $s^{M,Z}$ of imports in domestic demand components. We assume that the log difference of the specific productivities, $A^{p^{z}}_{kt}$ is an AR(1), $\varepsilon^{p^{z}}_{kt}$ with mean $g^{p^{z}}$. It follows that the demand for the domestic and foreign part of demand aggregates is:

$$O^{Z}_{kt} = \left(A^{p^{z}}_{kt}\right)^{\sigma^{z}-1}(1-\varepsilon^{M}_{kt}s^{M,Z})\left(\frac{P^{O}_{kt}}{P^{Z}_{kt}}\right)^{-\sigma^{z}} Z_{kt}$$

$$M^{Z}_{kt} = \left(A^{p^{z}}_{kt}\right)^{\sigma^{z}-1}\varepsilon^{M}_{kt}s^{M,Z}\left(\frac{P^{M}_{kt}}{P^{Z}_{kt}}\right)^{-\sigma^{z}} Z_{kt}$$

where P^{Z}_{kt} are price deflators associated with Z_{kt}; they satisfy:

$$P_{kt}^{Z} = \left(A_{kt}^{p^z}\right)^{-1}\left[(1-\varepsilon_{kt}^{M}s^{M,Z})(P_{kt}^{O})^{1-\sigma^z} + \varepsilon_{kt}^{M}s^{M,Z}(P_{kt}^{M})^{1-\sigma^z}\right]^{\frac{1}{1-\sigma^z}}$$

Economy-specific final imports demand

Final imported goods are produced by perfectly competitive firms combining economy-specific homogenous imports goods, M_{lkt}, using CES production function:

$$M_{kt} = \left(\sum_{l}(s_{lkt}^{M})^{\frac{1}{\sigma^{FM}}}(M_{lkt})^{\frac{\sigma^{FM}-1}{\sigma^{FM}}}\right)^{\frac{\sigma^{FM}}{\sigma^{FM}-1}}$$

where σ^{FM} is the price elasticity of demand for country *l*'s goods and $\sum_l s_{lkt}^{M} = 1$ are import shares. The demand for goods from country *l* is then:

$$M_{lkt} = s_{lkt}^{M}\left(\frac{P_{lkt}^{M}}{P_{kt}^{M}}\right)^{-\sigma^{FM}} M_{kt}$$

while the imports price:

$$P_{kt}^{M} = \left(\sum_{l} s_{lkt}^{M}(P_{lkt}^{M})^{1-\sigma^{FM}}\right)^{\frac{1}{1-\sigma^{FM}}}$$

with P_{lkt}^{M} being the country-specific imports good prices.

Supply of economy- and sector-specific imports

The homogenous goods from country *l* are assembled by monopolistically competitive firms from economy- and sector- specific goods using a linear production function and subject to adjustment costs. All products from country *l* are initially purchased at export price P_{lt}^{X} of this country. Firms then maximize a discounted stream of profits, div_{kt}^{IM} , such that :

$$div_{ilkt}^{IM} = \frac{P_{ilkt}^{M}}{P_{kt}^{Y}}M_{ilkt} - e_{lkt}\frac{P_{lt}^{X}}{P_{kt}^{Y}}M_{ilkt} - adj_{ilkt}^{PM}$$

where adj_{ilkt}^{PM} are the adjustment costs that producers face when choosing the bilateral import price.[9] The maximization is subject to the usual inversely-sloping demand equation. These assumptions result in a simple expression for price P_{lkt}^{M} of homogenous goods from country *l*:

$$P_{lkt}^{M} = e_{lkt}P_{lt}^{X} - adj_{M,ilkt}^{PM}$$

where $adj_{M,ilkt}^{PM}$ are additional dynamic terms due to costs of adjustment.

3.3.2. Export sector

9 $adj_{ilkt}^{PM} = \frac{\gamma^{pM}}{2}\frac{P_{lkt}^{M}}{P_{kt}^{Y}}M_{lkt-1}\left(\frac{P_{ilkt}^{M}}{P_{ilkt-1}^{M}} - 1\right)^{2}$

The exporting firms are supposed to be competitive and set their prices equal to the output price, up to a shock, ε_{kt}^{X}:

$$P_{kt}^{X} = \varepsilon_{kt}^{X} P_{kt}^{O}$$

3.4. EA Member State policy

3.4.1. Monetary policy

Monetary policy is modelled by a Taylor rule where the ECB sets the policy rate i_{kt} in response to EA-wide inflation and real GDP growth. The policy rate adjusts sluggishly to deviations of inflation and GDP growth from their respective target levels; it is also subject to random shocks:

$$i_{kt} - \bar{\iota} = \rho^{i}(i_{kt-1} - \bar{\iota}) + (1-\rho^{i})\left(\eta^{i\pi}\left(0.25\left(\sum_{r=0}^{3}\pi_{kt-r}^{c+g}\right) - \bar{\pi}^{C+G}\right) + \eta^{iy}(\tilde{y}_{kt})\right) + u_{kt}^{inom}$$

where $\bar{\iota} = r + \pi^{C+G}$ is the steady state nominal interest rate, equal to the sum of the steady state real interest rate and CPI inflation and output gap $\tilde{y}_{kt} = log(Y_{kt}) - \bar{y}_{kt}$ where $\bar{y}_{t}$ is (log) potential output. The Taylor rule may be extended to deal with economies with managed exchange rates and other exchange rate regimes. It is assumed that the risk-free rate is equal to the policy rate: $i_{kt}^{sd} \equiv i_{kt}$.

3.4.2. Fiscal policy

Government expenditure and receipts can deviate temporarily from their long-run levels in systematic response to budgetary or business-cycle conditions and in response to idiosyncratic shocks. Concerning government consumption and government investment, we specify the following autoregressive equations:

$$\frac{G_{kt}}{\bar{Y}_{kt}A_{kt}^{PG}} - \bar{G} = \rho^{G}\left(\frac{G_{kt-1}}{\bar{Y}_{kt}A_{kt}^{PG}} - \bar{G}\right) + u_{kt}^{G}$$

$$\frac{I_{kt}^{G}}{\bar{Y}_{kt}A_{kt}^{PI}} - \bar{I}^{G} = \rho^{IG}\left(\frac{I_{kt-1}^{G}}{\bar{Y}_{kt}A_{kt}^{PI}} - \bar{I}^{G}\right) + u_{kt}^{IG}$$

$$\frac{T_{kt}}{\overline{P_{kt}^{Y}Y_{kt}}} - \bar{T} = \rho^{T}\left(\frac{T_{kt-1}}{\overline{P_{kt}^{Y}Y_{kt}}} - \bar{T}\right) + \eta^{DEF,T}\left(\frac{\Delta B_{kt}^{gtot}}{P_{kt}^{Y}Y_{kt}} - def^{T}\right) + \eta^{B,T}\left(\frac{B_{kt}^{gtot}}{P_{kt}^{Y}Y_{kt}} - \bar{B}_{k}^{G}\right) + u_{kt}^{T}$$

with B_{kt}^{gtot} total nominal government debt. Government transfers react to the level of government debt and the government deficit relative to the associated debt and deficit targets $\bar{B}_{k}^{G}$ and def^{T}. The government budget constraint is

$$B_{kt}^{g} = (1 + i_{kt-1}^{g})B_{kt-1}^{g} - R_{kt}^{G} + P_{kt}^{G}G_{kt} + P_{kt}^{IG}I_{kt}^{G} + T_{kt}$$

where government (nominal) revenue:

$R^G_{kt} = \tau^K_k(P^Y_{kt}Y_{kt} - W_{kt}N_{kt} - P^I_{kt}\delta_k K_{kt-1}) + \tau^N W_{kt}N_{kt} + \tau^C P^C_{kt}C_{kt} + tax_{kt}$
consists of taxes on consumption, labor and corporate income as well as lump-sum tax.

Finally, the accumulation equation for government capital is:

$$K^G_{kt} = \left(1 - \delta^G\right)K^G_{kt-1} + I^G_{kt}$$

3.5. The REA and RoW blocks

The model of the REA and RoW blocks (subscript k=REA,RoW) is a simplified structure with fewer shocks. Specifically, the REA and RoW consist of a budget constraint for the representative household, demand functions for domestic and imported goods (derived from CES consumption good aggregators), a production technology that uses labor as the sole factor input, and a New Keynesian Phillips curve. The REA and RoW blocks abstracts from capital accumulation. There are shocks to labor productivity, price mark-ups, the subjective discount rate, the relative preference for domestic vs. imported goods, as well as monetary policy shocks in the REA and RoW.

More specifically the budget constraint for the RoW representative household is:

$$P^Y_{RoWt}Y_{RoWt} + P^{Oil}_{RoWt}OIL_{RoWt} = P^C_{RoWt}C_{RoWt} + P^X_{RoWt}X_{RoWt} - \sum_l \frac{size_l}{size_{RoW}} e_{lRoWt}P^X_{lt}M_{lRoWt}$$

where X_{RoWt} are non-oil exports by the RoW, and the intertemporal equation for aggregate demand derived from the FOC for consumption:

$$\beta_t \frac{\lambda_{RoWt+1}}{\lambda_{RoWt}} \frac{1 + i_{RoWt}}{1 + \pi^C_{RoWt+1}} = 1$$

with $\beta_t = \beta\exp(\varepsilon^C_{RoWt})$, $\left(C_{RoWt} - hC_{RoWt-1}\right)^{-\theta} = \lambda_{RoWt}$ and ε^C_{RoWt} as the RoW demand shock. Note that

$$i_{RoWt} \equiv i^g_{RoWkt}$$

The same structure holds for the REA, with the exception that REA is not an oil exporter, but oil importer.

As for the EA MS, final aggregate demand C_{RoWt} (in the absence of investment and government spending in the REA and RoW blocks) is a combination of domestic output, Y_{RoWt} and imported goods, M_{RoWt}, using the following CES function:

$$C_{RoWt} = A^p_{RoWt}\left[\left(1 - \varepsilon^M_{RoWt}s^M\right)^{\frac{1}{\sigma}}\left(Y^C_{RoWt}\right)^{\frac{\sigma-1}{\sigma}} + \left(\varepsilon^M_{RoWt}s^M\right)^{\frac{1}{\sigma}}\left(M^C_{RoWt}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

which gives the demand for the domestic and foreign goods in RoW demand:

$$Y^C_{RoWt} = \left(A^p_{RoWt}\right)^{\sigma-1}(1-\varepsilon^M_{RoWt}s^M)\left(\frac{P^Y_{RoWt}}{P^C_{RoWt}}\right)^{-\sigma} C_{RoWt}$$

$$M^C_{RoWt} = \left(A^p_{RoWt}\right)^{\sigma-1}\varepsilon^M_{RoWt}s^M\left(\frac{P^M_{RoWt}}{P^C_{RoWt}}\right)^{-\sigma} C_{RoWt}$$

where the consumer price deflator P^C_{RoWt} satisfies:

$$P^C_{RoWt} = \left(A^p_{RoWt}\right)^{-1}[(1-\varepsilon^M_{RoWt}s^M)(P^Y_{RoWt})^{1-\sigma} + \varepsilon^M_{RoWt}s^M(P^M_{RoWt})^{1-\sigma}]^{\frac{1}{1-\sigma}}$$

The RoW non-oil output is produced with the technology:

$$Y_{RoWt} = A^Y_{RoWt}N_{RoWt}$$

Price setting for RoW non-oil output follows a New Keynesian Phillips curve:

$$\pi^Y_{RoWt} - \bar{\pi}^Y_{RoW} = \beta\frac{\lambda_{RoWt+1}}{\lambda_{RoWt}}(sfp(E_t\pi^Y_{RoWt+1} - \bar{\pi}^Y_{RoW}) + (1-sfp)(\pi^Y_{RoWt-1} - \bar{\pi}^Y_{RoW})) + \varphi^Y_{RoW}\ln(Y_{RoWt} - \bar{Y}_{RoW}) + \varepsilon^Y_{RoWt}$$

Monetary policy in the RoW follows the Taylor rule:

$$i_{RoWt} - \bar{\imath} = \rho^i(i_{RoWt-1} - \bar{\imath}) + (1-\rho^i)\left(\eta^{i\pi}(\pi^Y_{RoWt} - \bar{\pi}^Y_{RoW}) + \eta^{iy}\tilde{y}_{RoWt}\right) + \varepsilon^{inom}_{RoWt}$$

where $\tilde{y}_{RoWt}$ is the deviation of actual output from trend output.

The RoW net foreign asset (NFA) position equals minus the sum of the EA MS and REA NFA positions.

Finally, oil is assumed to be fully imported from the RoW and the oil price is assumed as follows:

$$P^{Oil}_{RoWt} = \frac{\bar{P}^Y}{A^{p^{oil}}_{ROWt}e_{RoW,US}}$$

where $A^{p^{oil}}_{ROWt}$ is oil-specific productivity and oil is priced in USD.

Total nominal exports are defined as:

$$P^X_{RoWt}X_{RoWt} = \sum_l P^X_{lRoWt}M_{RoWlt}$$

with the bilateral export price being defined as the domestic price subject to a bilateral price shock:

$$P^X_{lRoWt} = \exp(\varepsilon^{PX}_{lRoWt})\, P^Y_{RoWt}$$

3.6 Closing the economy

Market clearing requires that:

$$Y_{kt}P^{Y}_{kt} + div^{M}_{kt}P^{Y}_{kt} = P^{C}_{kt}C_{kt} + P^{I}_{kt}I_{kt} + P^{IG}_{kt}IG_{kt} + TB_{kt}$$

Export is a sum of imports from the domestic economy by other countries:

$$X_{kt} = \sum_{l} M_{klt}$$

where M_{klt} stands for imports from the domestic economy to economy *l*. The total imports are defined as:

$$P^{Mtot}_{kt}M^{tot}_{kt} = P^{M}_{kt}M_{kt} + P^{oil}_{kt}OIL_{kt}$$

where non-oil imports

$$P^{M}_{kt}M_{kt} = P^{M}_{kt}(M^{C}_{kt} + M^{I}_{kt} + M^{G}_{kt} + M^{IG}_{kt})$$

$$e_{RoWk,t}B^{w}_{k,t} = +\left(1 + i^{bw}_{t-1}\right)e_{RoWk,t}B^{w}_{k,t-1} + P^{X}_{kt}X_{kt} - \sum_{l}\frac{size_{l}}{size_{k}}e_{lkt}P^{X}_{lt}M_{lkt} - P^{Oil}_{kt}OIL_{kt} + ITR_{k}\overline{P^{Y}_{kt}Y_{kt}}$$

where $P^{X}_{kt}X_{kt} - \sum_{l}\frac{size_{l}}{size_{k}}e_{lkt}P^{X}_{lt}M_{lkt} - P^{Oil}_{kt}OIL_{kt} = TB_{kt}$ defines the trade balance, with domestic importers buying the imported good at the price $\mathrm{P}^{\mathrm{X}}_{\mathrm{lt}}$ .We allow non-zero trade balance and include an international transfer, ITR_{k}, calibrated in order to satisfy zero NFA in equilibrium.

Finally, net foreign assets of each country sum to zero:

$$\sum_{l} NFA_{lt}size_{l} = 0.$$

$size_{l}$ is the relative size of economy *l*.

4. Model solution and econometric approach

We compute an approximate model solution by linearizing the model around its deterministic steady state. Following the recent literature that estimates DSGE models, we calibrate a subset of parameters to match long-run data properties, and we estimate the remaining parameters using Bayesian methods.[10] The observables are not demeaned or detrended prior to estimation. The model is estimated on first differences of real GDP, real demand components and price indices,

[10] We use the DYNARE software (Adjemian *et al*. 2011) to solve the linearized model and to perform the estimation.

and on nominal ratios of aggregate demand components to GDP. The estimation uses quarterly data for the period 1999q1-2016q2.

5. Estimation results

5.1. Posterior parameter estimates

The posterior estimates of key model parameters for the EA are reported in Table 1. The model properties discussed in what follows are evaluated at the posterior mode of the model parameters.

[INSERT TABLE 1 AND COMPLETE DESCRIPTION]

5.2. Decomposing the REER for Germany

To quantify the role of different shocks as drivers of the REER we plot the estimated contribution of the different shocks to the (log of) REER in Figure 8. Figure 9 plots a decomposition for TBY.

Main results [TO BE BROADENED AND COMPLETED]:

German REER driven mainly by foreign and trade-related shocks. Domestic demand shocks have played little role for REER, which helps explaining the lack of cyclicality. There is some role for (offsetting) supply shocks, however. In particular, labour market reform in Germany has supported REER depreciation, whereas negative TFP shocks have had the opposite effect.

REER and TBY appear to be driven largely by different shocks (or by similar shocks at different time), which suggests that REER dynamics did not have strong influence on the German TBY.

Figure 8: Decomposition of REER for Germany

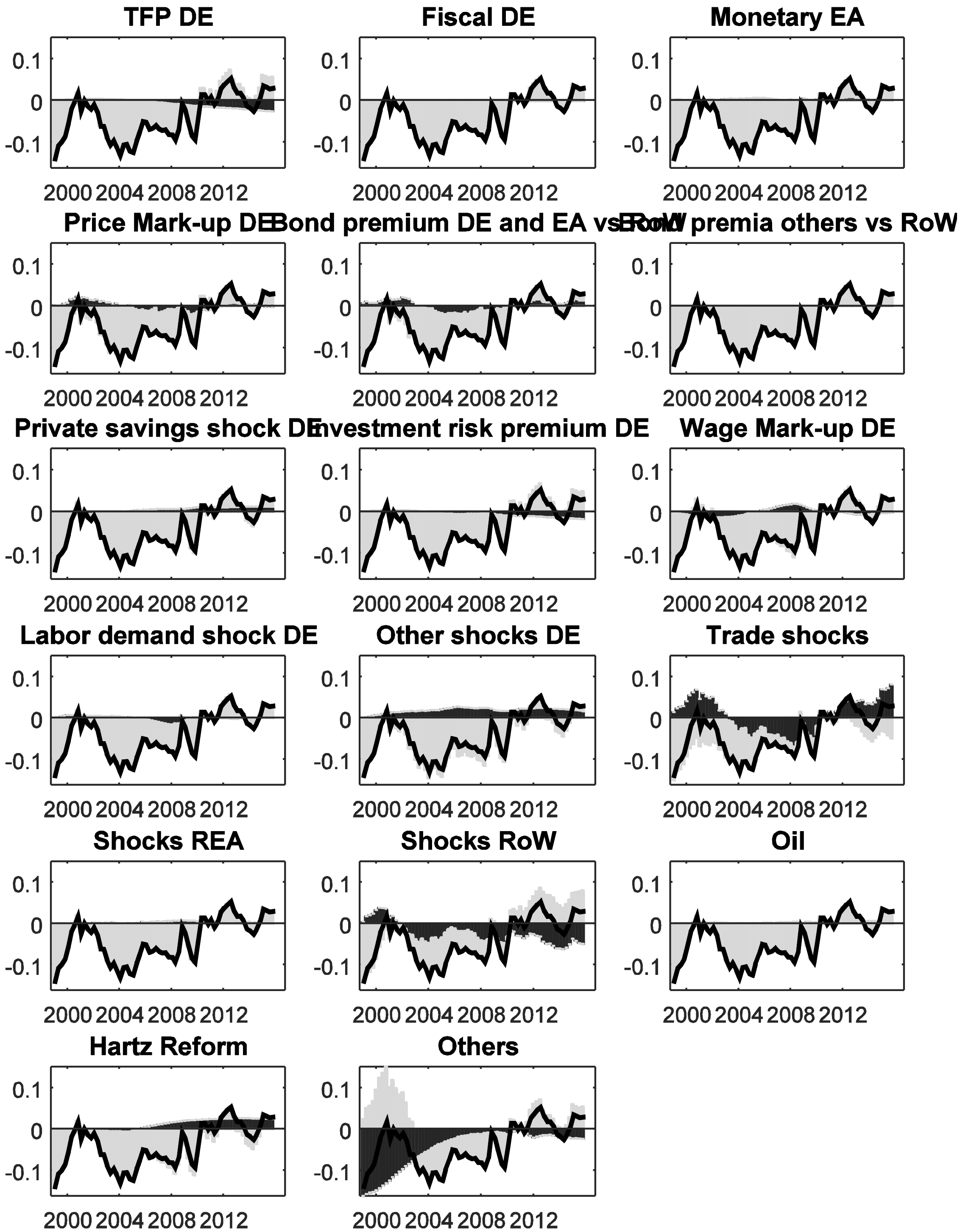
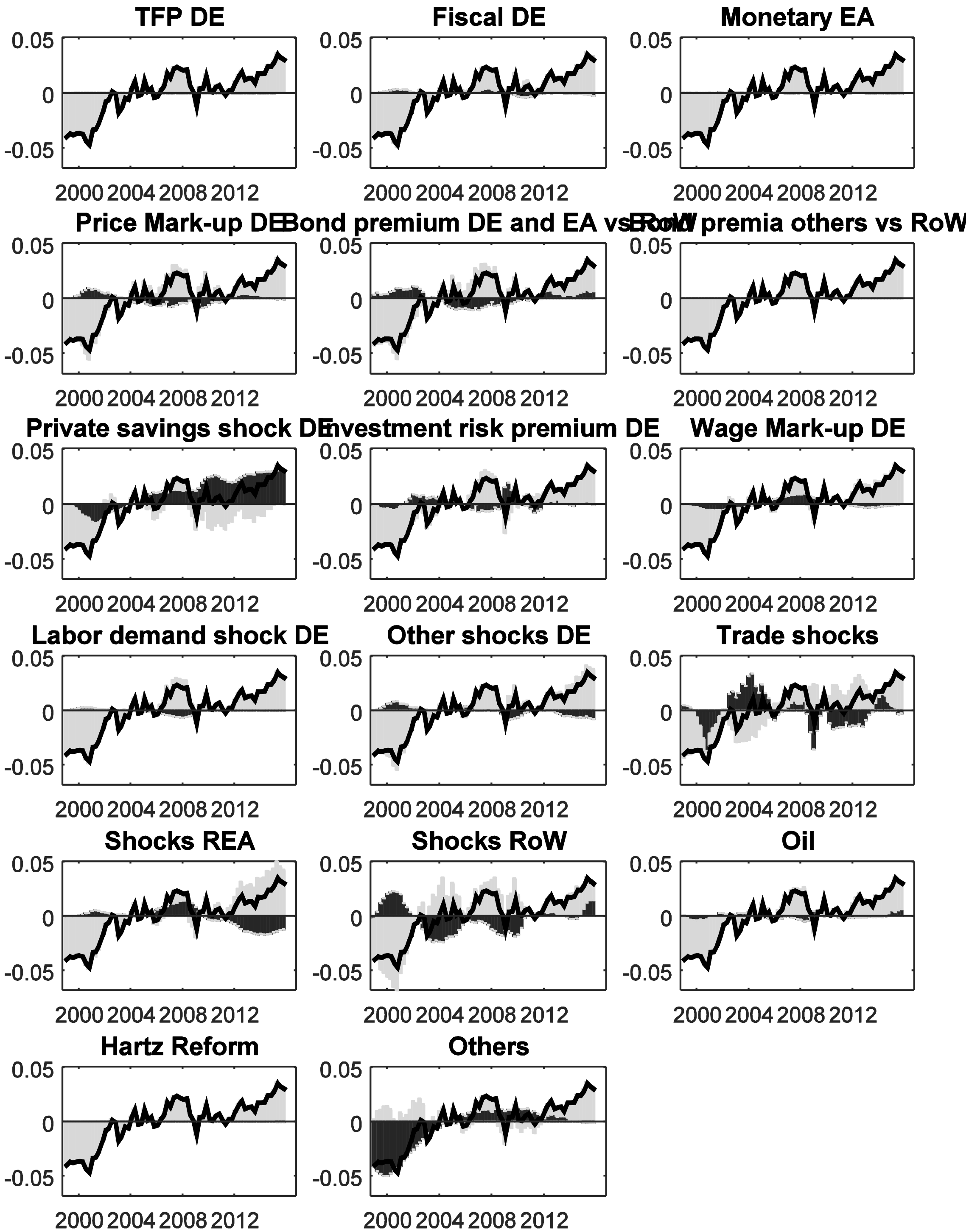


Figure 9: Decomposition of TBY for Germany



6. Conclusion

This paper reviews REER adjustment inside the EA. In particular, the paper presents results from estimated multi-region models for individual EA Member States. The analysis builds on shock decompositions of the real exchange rate for individual EA Member States (the current version is limited Germany at this point) to reveal drivers of the dynamics of the real exchange rate, i.e. factors that have fostered or hampered real appreciation or depreciation.

According to the estimated model, the German REER has beendriven mainly by foreign and trade-related shocks. Domestic demand shocks have played little role for the REER, which helps explaining the lack of cyclicality in the REER. There is some role for (offsetting) supply shocks, however. In particular, labour market reform in Germany has supported REER depreciation, whereas negative TFP shocks have had the opposite effect. REER and TBY appear to be driven largely by different shocks (or by similar shocks at different time), which suggests that REER dynamics did not have strong influence on the German TBY.

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