

On the composition of public spending and taxes*

George Economides

(Athens University of Economics and Business, and CESifo)

Hyun Park

(Kyung Hee University)

Apostolis Philippopoulos[†]

(Athens University of Economics and Business, and CESifo)

Stelios Sakkas

(Athens University of Economics and Business)

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Abstract

This paper studies the implications of changes in the fiscal (spending-tax) policy mix when all categories of spending and taxes are according to their *functional* breakdown. In so doing, we build a general equilibrium OLG model which naturally incorporates the main functional categories of public spending and taxes as they are in the euro area. Departing from the crisis year of 2008, the main result is that an increase in public spending on education and health would have outperformed all other changes in fiscal policy that have either happened or been debated in policy circles.

Keywords: Government expenditure, public finance, fiscal reforms.

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[†]Corresponding author: Apostolis Philippopoulos, Department of Economics, Athens University of Economics and Business, 76 Patission street, Athens 10434, Greece. Tel: +30-210-8203357. Email: aphil@aueb.gr

1 Introduction

EU policymakers have repeatedly stressed the need to reconsider the composition of public spending and taxes so as to move the economy to more efficient outcomes (see e.g. European Commission, 2008, part III). In this paper, we revisit this classic issue. We study how permanent changes in the spending-tax policy mix would affect private incentives and the macroeconomy in the medium- and long-term. The point of departure is the fiscal policy mix in the euro area when the crisis erupted in 2008.

Our work differs from most of the previous literature on the effects of fiscal policy (see section 2 below for related papers), first, because we include all main categories of public spending and taxes in a unified general equilibrium framework and, second, because the breakdown of both public spending and taxes is in terms of their *function*. A functional decomposition of public spending means that we distinguish among spending on social protection, health, education, defense and public order, etc. Similarly, we distinguish among taxes on capital, labor and consumption. Such a functional decomposition allows us to specify the behavioural channel through which changes in fiscal policy affect private incentives and in turn the macroeconomy.¹

The vehicle of analysis is a general equilibrium OLG model that includes the most important components of public spending and tax revenues in the data. In particular, we include spending on social protection (e.g. pensions), health, general public services (e.g. interest payments), education, economic affairs (e.g. public infrastructure) and defense and public-order safety, while, on the taxation side, we have capital, labor and consumption taxes. These are the most important functional categories of spending and taxes as listed in the Eurostat (2012) dataset (see Table 1 below for spending categories). Our model accommodates these categories of public spending in a natural way. In other words, public spending on health increases the probability of reaching the old age, public spending on education contributes to the accumulation of individual human capital of younger generations, public spending on economic affairs improves infrastructure and benefits the productivity of firms, public spending on social protection provides pensions to the old generation, public spending on defence and public order-safety can protect property rights, etc. To focus on the economy's growth and welfare potential in the medium- and long-term, we abstract from uncertainty so that all dynamics is driven by permanent changes in the policy mix.²

The constructed model is solved numerically using conventional parameter values and fiscal data from the euro area over 2001-2008. Then, the steady state solution of this model, called the status quo, will serve as a point of departure to study the implications of various reforms.³ By reforms, we mean ad hoc permanent

¹Usually there is an asymmetry between modeling public spending and modeling taxes. While taxes are decomposed by their function (e.g. taxes on labor income, taxes on capital income and taxes on consumption), different categories of public spending are usually modelled according to their economic transaction (e.g. transfer payments, public consumption, public investment) rather than according to their function (e.g. spending on social protection, health, education, defense and public order, etc). But, as is widely recognized, it is not clear what we mean, for instance, by public consumption or transfer payments (see e.g. European Commission, 2008, p. 139). This asymmetry leaves scope for improvement since the behavioural channel - through which changes in the mix of public spending affect private incentives and the macro economy - is not well specified.

²That is, here we do not study temporary changes in fiscal policies aiming at short-term stabilization and typically measured by multipliers. Nor we compare alternative fiscal consolidation strategies aiming at public debt reduction over time.

³Thus, we use euro area data. This means that our policy results will apply to a country in an "average" fiscal and budgetary situation. Debt crisis afflicted countries (like Greece, Cyprus, Ireland and Portugal) may require different policy measures.

changes in the spending-tax policy mix. We study three sets of policy reforms, actual and hypothetical. First, those that resemble policy measures actually taken in the euro area in the aftermath of the 2008 financial crisis (see e.g. Coenen et al., 2013). In particular, we study a rise in public spending on social protection transfers, a rise in public spending on economic affairs (as said, this is mainly spending on public infrastructure), a cut in consumption taxes and a cut in labor taxes. Second, reforms that may not have actually happened but have been debated in policy circles. For instance, an horizontal cut in all types of public spending (typically supported by conservative administrations) or a rise in capital taxes (typically supported by socialist administrations). Third, some reforms that have neither happened nor debated but deserve to be studied given the broader set of fiscal instruments available to policymakers. For instance, we experiment with changes in public spending on education, health, or defence and public order-safety. In all cases studied, we start with the assumption that there is one policy change at a time and this change is financed by public debt; this helps us to understand better how the model works. But, at the end, we also study fiscal policy mixes. The comparison of alternative policies will be in terms of (discounted lifetime) utility and output.

Our main results are as follows. An increase in public education spending, financed by a temporary rise in public debt, would have outperformed any other fiscal reforms. The beneficial effects from higher public education spending arise because this type of public spending, not only enhances the productivity of efficient labor time as one would expect, but also because it crowds in private savings. Public spending on health can become equally good as public education spending to the extent that it contributes to the accumulation of private human capital (unhealthy people cannot be efficient, irrespectively of their education level). It is worth stressing that these results are robust to allowing for private education spending to the extent that we remain within historical ranges for private and public spendings as shares of output.

By contrast, an increase in spending on public pensions (the main item of social protection) is found to be particularly bad. Such an increase distorts private incentives, by crowding out private savings, and this hurts aggregate output and welfare over time. Interestingly, a rise in spending on public pensions hurts even those who are supposed to be the main beneficiaries, namely the old households. This happens because the indirect harm (from lower savings and less aggregate output) more than outweighs any direct benefits (from the policy attempt to allocate more social resources to pensions). Actually, in our experiments, an increase in public pensions leads to outcomes worse than the status quo (SQ), which is defined as the solution with the pre-crisis fiscal policy data. It is also interesting to point out that these results hold not only under a pay-as-you-go (PAYG) public pension system, but also - although at a smaller degree - under a fully funded (FF) system. This is important because, while a PAYG system typically discourages private savings, a FF system is believed to be good for private savings.⁴ In other words, our results confirm that a FF system is superior to a PAYG system and that an increase in public pensions discourages private savings more under a PAYG system, but, the same results show that, even under a FF system, an increase in public pensions

⁴See e.g. Acemoglu (2009, chapter 9.5).

is inferior to other fiscal reforms and, especially, to an increase in public education spending.

Combining the above results, our simulations show that a fiscal policy mix, which combines a rise in public education spending with a cut in public spending on pensions, would generate substantial benefits not only to the aggregate economy but also to all generations even the old. To put it differently, a growth-enhancing education policy is the means to support the old.⁵

Other measures actually taken after the 2008 crisis - like an increase in spending on economic affairs (e.g. public infrastructure) or a reduction in consumption and labor taxes - are better than the SQ but inferior to an increase in public education spending. Finally, debated reforms supported by the political right or the political left - like an horizontal cut in all categories in public spending or a rise in capital taxes respectively - are inferior to an increase in public education spending and, in most cases, are found to be counter-productive meaning that they are inferior even to the SQ.

The rest of the paper is organized as follows. A brief review of the literature is in section 2. Section 3 reports public spending data. The benchmark model is developed in section 4. Section 5 presents the status quo solution serving as a point of departure. Fiscal reforms are studied in section 6. Richer models are studied in section 7. Section 8 summarizes.

2 Related literature and how we differ

Our work is related to two literatures. The first includes OLG models with fiscal policy. But, to the best of our knowledge, none of the previous papers has studied all main spending policy instruments at the same time. For instance, Arcalean and Schiopu (2010), Abington and Blankenau (2013) and Viaene and Zilcha (2013), among many others, have focused on public education. Cooley and Soares (1999), Krueger and Kubler (2006), Conesa and Garriga (2008), Bassetto (2008), Auerbach and Lee (2011), Kaganovich and Zilcha (2012) and Bruce and Turnovsky (2013), among many others, have focused on social security.⁶ Bhattacharya and Qiao (2007), among many others, have focused on health spending. OLG papers with a richer menu of spending policy instruments include Bouzahzah et al. (2002), Soares (2006) and Del Rey and Lopez-Garcia (2013), who have studied both public education and social security, and Dioikitopoulos (2014), who has studied both public health and education.

The second literature includes econometric studies on the growth effects of different types of public spending. See, for instance, Bleaney et al. (1999, 2001), Shelton (2007), Gemmell et al. (2012, 2014) and Acosta-Ormaechea and Morozumi (2013), while a review of the earlier empirical literature can be found in European Commission (2008, part III, chapter 3.3).

Our paper belongs to the first, OLG literature but it differs from most of the previous papers in that we incorporate all main functional categories of public spending and taxes in a unified neoclassical general

⁵For a similar policy recipe, in the context of environmental quality, see Economides and Philippopoulos (2008).

⁶Cooley and Soares (1999) also survey the earlier literature that dates back to Samuelson (1958) and Diamond (1965).

equilibrium framework and, in turn, study the effects of their changes.

3 The mix of public spending in the euro area

Table 1 presents the public spending data in the euro zone according to their functional breakdown over 2001-2012. Some data clarifications are helpful for the understanding of the model that follows. Public spending on social protection includes spending on pensions, family support, unemployment benefits, housing, sickness and disability, etc. Public spending on health includes public health, medical products and equipment, hospital services, outpatient services, etc. Public spending on general public services includes public debt payments, administrative spending, foreign economic aid, executive and legislative organs, fiscal affairs and other transfers of a general character between different levels of government. Public spending on education includes spending on pre-primary and primary education, secondary and post secondary non-higher and higher education. Public spending on economic affairs includes public infrastructure spending such as public transport, fuel and energy, mining, manufacturing and construction, communications, licenses and other related support programs. Public spending on public order-safety and defence includes military defence, civil defence, foreign military aid, police and fire protection services, law courts and prisons, etc. Other minor (quantitatively) types of spending are on environmental protection, housing and community amenities and recreation and culture.

In what follows, we will develop a model that gives a natural role to most of these spending policies.

Table 1: Average public spending 2001-2012 in the Eurozone - COFOG

Functional use	% GDP	% of total public expenditure
Social protection	18.80	39.20
Health	7.00	14.60
General public services	6.40	13.40
Education	5.30	11.00
Economic affairs	4.10	8.50
Public order and safety	1.85	3.90
Defence	1.55	3.20
Environmental protection	0.80	1.70
Housing and community amenities	1.00	1.90
Recreation and culture	1.10	3.30
Total	47.90	

4 Model

Consider a closed economy populated by overlapping generations of three-period-lived households, private firms and the government. There are N_t^y young members, N_t^m adult members and N_t^o old members. Thus, the total population is $N_t^y + N_t^m + N_t^o = N_t$ in each period. For simplicity, we assume that there is no population growth and that the numbers of young, adult and old households remain constant over time and equal to each other. There are also N_t^f private firms owned by the adult households. In order to finance its various categories of public spending, the government levies distorting taxes and issues bonds. Time is discrete and infinite.

4.1 Households

Each individual can live for 3 periods, as young, adult and old. He consumes in each period. A young individual starts with his parent's bequest and spends effort time and funds in education. When he is adult, he works and can save in the form of capital and government bonds. When he reaches the old age, which happens with probability $0 \leq q \leq 1$, he uses his own savings and social security (namely, a pension) and dies leaving an optimally chosen bequest. With probability $0 \leq 1 - q \leq 1$, he dies before reaching the old age leaving an unintended bequest. These bequests (chosen and unintended) are then inherited by the newly born young people.

4.1.1 Household's utility function

The objective of each household born at t is to maximize discounted lifetime utility defined as:

$$\begin{aligned}
 u_t = & \frac{(c_t^y)^{1-\sigma}}{1-\sigma} - \chi_n \frac{(e_t^y)^{1+\eta}}{1+\eta} + \chi_g \frac{(g_t^u)^{1-\zeta}}{1-\zeta} + \\
 & + \beta \left\{ \frac{(c_{t+1}^m)^{1-\sigma}}{1-\sigma} - \chi_n \frac{(l_{t+1}^m)^{1+\eta}}{1+\eta} + \chi_g \frac{(g_{t+1}^u)^{1-\zeta}}{1-\zeta} \right\} \\
 & + \beta^2 q \left\{ \frac{(c_{t+2}^o)^{1-\sigma}}{1-\sigma} + \chi_g \frac{(g_{t+2}^u)^{1-\zeta}}{1-\zeta} + \beta \chi_b \frac{(b_{t+2}^o)^{1-\xi}}{1-\xi} \right\}
 \end{aligned} \tag{1}$$

where c_t^y , c_{t+1}^m , c_{t+2}^o is consumption when young, adult and old respectively, e_t^y is effort time spent in education when young, l_{t+1}^m is effort time spent in work when adult, $g_t^u \equiv \frac{G_t^u}{N_t}$ denotes per capita public spending on "utility-enhancing" public goods and services (see subsection 4.3 below for the definition of utility-enhancing public goods and services) and b_{t+2}^o is a bequest chosen by the old (the way we model the bequest motive follows e.g. Acemoglu (2009, chapter 9.6) and Coeurdacier et al. (2015)). Also, the parameter $0 < \beta < 1$ is the subjective time preference rate and σ , η , ζ , χ_n , χ_g , χ_b , ξ are standard preference parameters. As said above, $0 \leq q \leq 1$ is the probability of an adult reaching the old age; the adult may die before reaching the old age with probability $0 \leq 1 - q \leq 1$.

4.1.2 Household's budget constraints and bequests

The budget constraints of the household when young, adult and old are respectively:

$$(1 + \tau_t^c) c_t^y + z_t^y = (1 - \tau_t^b) b_{t-1}^y \quad (2)$$

$$(1 + \tau_{t+1}^c) c_{t+1}^m + k_{t+1}^m + d_{t+1}^m = (1 - \tau_{t+1}^n - \phi_{t+1}) w_{t+1} h_{t+1}^m l_{t+1}^m \quad (3)$$

$$(1 + \tau_{t+2}^c) c_{t+2}^o + b_{t+2}^o = [1 - \delta^k + (1 - \tau_{t+2}^k) r_{t+2}] k_{t+1}^m + (1 - \tau_{t+2}^k) \pi_{t+2}^o + (1 + \rho_{t+2}) d_{t+1}^m + s_{t+2}^o \quad (4)$$

where z_t^y is private spending on education, h_{t+1}^m is the stock of private human capital when adult (see below for its motion), w_{t+1} is the wage earned when adult, k_{t+1}^m is savings in the form of physical capital, d_{t+1}^m is savings in the form of government bonds, r_{t+2} is the return to physical capital, ρ_{t+2} is the return to government bonds, π_{t+2}^o is dividends from firms, s_{t+2}^o is the pension given to each old person from the government, and b_{t-1}^y is an initial endowment which is taken as given by the household (nevertheless, in equilibrium, b_{t-1}^y will be proportional to what is left by the older generations - see subsection 4.4 below).⁷ Finally, $0 < \tau_t^c, \tau_t^k, \tau_t^b < 1$ are tax rates on consumption, capital income and bequests respectively, while $0 < \tau_t^n + \phi_t < 1$ denotes the total tax rate on labor income, where ϕ_t is the tax rate used to finance social security and τ_t^n is the tax rate used to finance other government spending (see subsection 4.3 below for further details).⁸

Equation (4) holds with probability $0 \leq q \leq 1$ only. With probability $0 \leq 1 - q \leq 1$, the adult dies before reaching the old age. In this case, he leaves an enforced or unintended bequest, denoted as Ω_{t+2}^o . The enforced bequest when the adult dies before reaching the old age is his whole wealth, namely:

$$\Omega_{t+2}^o \equiv [1 - \delta^k + (1 - \tau_{t+2}^k) r_{t+2}] k_{t+1}^m + (1 - \tau_{t+2}^k) \pi_{t+2}^o + (1 + \rho_{t+2}) d_{t+1}^m \quad (5)$$

Therefore, the initial endowment of the young, b_{t-1}^y , is exogenously given to the household when it solves its optimization problem but, in equilibrium, it will be a weighted average of the bequest voluntarily chosen by the old, b_{t+2}^o , and the enforced bequest, Ω_{t+2}^o , in case the adult dies suddenly before reaching the old age, where the weights are respectively the probability of reaching the old age, q , and the probability of suddenly passing away, $1 - q$.⁹ When the universe starts, b_{t-1}^y is given by an initial condition.

4.1.3 Household's human capital

Following the related literature (see e.g. Blankenau and Simpson, 2004, and Blankenau, 2005), the motion of household's human capital is defined as (see also Stokey, 1996, for a similar functional form although in

⁷As said, this is as in Acemoglu (2009, chapter 9.6) and Coeurdacier et al. (2015).

⁸This modelling is as in e.g. Bruce and Turnovsky (2013). Alternatively, we could assume that the labor income tax, τ_t^n , is imposed after we deduct social security contributions, namely, to have $(1 - \tau_{t+1}^n)(1 - \phi_{t+1})w_{t+1}h_{t+1}^m l_{t+1}^m$ in the adult's budget constraint (see e.g. Conesa and Garriga (2008)). We report that our main results do not depend on the particular way we model the social security tax (results are available upon request).

⁹See Quadrini and Rios-Rull (2014), for various assumptions about the use of the assets left by the deceased households.

a different context):

$$h_{t+1}^m = (1 - \delta^h) h_t^y + B (e_t^y)^\theta \left[\gamma (z_t^y)^\nu + (1 - \gamma) (\bar{G}_t^e + \lambda \bar{G}_t^h)^\nu \right]^{\frac{1-\theta}{\nu}} \quad (6)$$

Thus, individual human capital is augmented by effort time spent in school, e_t^y , by private spending on education, z_t^y , and by government spending per young person on education, $\bar{G}_t^e \equiv \frac{G_t^e}{N_t^y}$, as well as on health, $\bar{G}_t^h \equiv \frac{G_t^h}{N_t^m}$, where the parameter $0 \leq \lambda \leq 1$ measures how much public spending on health contributes to the quality of human capital (unhealthy people cannot be efficient, irrespectively of their education level).¹⁰ Also, $B > 0$, $0 \leq \theta \leq 1$, $0 \leq \gamma \leq 1$ and $0 \leq \nu \leq 1$ are parameters. The idea behind this functional form is that effort time in school is combined, through a Cobb-Douglas technology, with total (private and public) education spending, with weights θ and $1 - \theta$ respectively, to augment private human capital. In turn, private and public education spending, with weights γ and $1 - \gamma$ respectively, are combined into an aggregate through a CES technology with an elasticity of substitution $1/(1 - \nu)$.¹¹ Finally, $B > 0$ is a scale parameter.

4.1.4 The probability of reaching the old age

For simplicity, we assume that the probability of an adult reaching the old age, q , depends only on public spending on health and in particular on public spending on health per adult, $\bar{G}_t^h \equiv \frac{G_t^h}{N_t^m}$.¹² This is denoted as $q(\bar{G}_t^h)$, where $q(\cdot)$ is increasing and concave. For convenience, we use the functional form:

$$q(\bar{G}_t^h) = \Xi \left(1 + \frac{\bar{G}_t^h}{1 + \bar{G}_t^h} \right) \quad (7)$$

where the parameter $\Xi > 0$ will be calibrated so as the probability to be within usual ranges (see also e.g. Chakraborty, 2004, and Dioikitopoulos, 2014).

4.1.5 Household's optimality conditions

Households act competitively taking prices, policy instruments and aggregate outcomes as given. They solve their problem at the start of their lives. Thus, they choose c_t^y , c_{t+1}^m , c_{t+2}^o , e_t^y , l_{t+1}^m , k_{t+1}^m , d_{t+1}^m , b_{t+2}^o subject to their budget constraints and the motion of their human capital.¹³ The first-order conditions include these constraints and also the optimality conditions for e_t^y , z_t^y , l_{t+1}^m , k_{t+1}^m , d_{t+1}^m and b_{t+2}^o respectively:

¹⁰Thus, individual human capital accumulation is an increasing function of both private and public spending on education, and this reflects the idea that public spending applies more to primary and secondary education, while private spending applies more to college education and on-the-job training.

¹¹Thus, the elasticity parameter, ν , is a measure of substitutability between the two types of spending, private and public. If $\nu = 1$, the two types become perfect substitutes, while if $\nu = 0$, the function turns to a Cobb-Douglas (see Stokey, 1996).

¹²Since this probability does not depend on private decisions, like private spending on health or private income, there is nothing that the household can do to affect survival (see also Quadrini and Rios-Rull (2014)). A generalization could be to assume $q(h_{t+1}^{hea}) = \Xi \left(1 + \frac{h_{t+1}^{hea}}{1 + h_{t+1}^{hea}} \right)$, where the motion of household's health capital, h_{t+1}^{hea} , is $h_{t+1}^{hea} = (1 - \delta^{hea}) h_t^{hea} + B^{hea} (z_t^{hea})^v (\bar{G}_t^h)^{1-v}$, where z_t^{hea} is private spending on health. If we set $\delta^{hea} = 1$, $B^{hea} = 1$ and $v = 0$, we go back to the simple specification used here.

¹³As said, since the household takes policy as given, it also takes q as given. Besides, it does not internalize the link between social security pensions and social security taxes (see also e.g. Conesa and Garriga (2008)).

$$\chi_n (e_t^y)^\eta = \frac{\beta (c_{t+1}^m)^{-\sigma} (1 - \tau_{t+1}^n - \phi_{t+1}) w_{t+1} l_{t+1}^m B \theta (e_t^y)^{\theta-1} [\gamma (z_t^y)^\nu + (1 - \gamma) (\bar{G}_t^e + \lambda \bar{G}_t^h)^\nu]^{\frac{1-\theta}{\nu}}}{(1 + \tau_{t+1}^c)} \quad (8)$$

$$\frac{(c_t^y)^{-\sigma}}{(1 + \tau_t^c)} = \frac{\beta (c_{t+1}^m)^{-\sigma} (1 - \tau_{t+1}^n - \phi_{t+1}) w_{t+1} l_{t+1}^m B (e_t^y)^\theta \gamma (1 - \theta) [\gamma (z_t^y)^\nu + (1 - \gamma) (\bar{G}_t^e + \lambda \bar{G}_t^h)^\nu]^{\frac{1-\theta}{\nu}-1}}{(z_t^y)^{1-\nu} (1 + \tau_{t+1}^c)} \quad (9)$$

$$\chi_n (l_{t+1}^m)^\eta = \frac{(c_{t+1}^m)^{-\sigma} (1 - \tau_{t+1}^n - \phi_{t+1}) w_{t+1} h_{t+1}^m}{(1 + \tau_{t+1}^c)} \quad (10)$$

$$\frac{(c_{t+1}^m)^{-\sigma}}{(1 + \tau_{t+1}^c)} = \frac{\beta q (c_{t+2}^o)^{-\sigma} [(1 - \delta^k) + (1 - \tau_{t+2}^k) r_{t+2}]}{(1 + \tau_{t+2}^c)} \quad (11)$$

$$\frac{(c_{t+1}^m)^{-\sigma}}{(1 + \tau_{t+1}^c)} = \frac{\beta q (c_{t+2}^o)^{-\sigma} (1 + \rho_{t+2})}{(1 + \tau_{t+2}^c)} \quad (12)$$

$$\frac{(c_{t+2}^o)^{-\sigma}}{(1 + \tau_{t+2}^c)} = \beta \chi_b (b_{t+2}^o)^{-\xi} \quad (13)$$

4.2 Private firms

There are $f = 1, 2, \dots, N_t^f$ firms. Firms act competitively taking prices, policy instruments and aggregate outcomes as given. Thus, each firm chooses its capital and labor inputs, denoted as k_t^f and l_t^f , to maximize profits given by:

$$\pi_t^f \equiv y_t^f - r_t k_t^f - w_t l_t^f \quad (14)$$

where output, y_t^f , is produced by the function (see also e.g. Lansing (1998)):

$$y_t^f = A (k_t^f)^{\alpha_1} (l_t^f)^{\alpha_2} \left(\frac{K_t^g}{N_t^f} \right)^{1-\alpha_1-\alpha_2} \quad (15)$$

where K_t^g is the total stock of public capital (see below for its motion) and $A > 0$, $0 < \alpha_1, \alpha_2 < 1$ are technology parameters.

The first-order conditions for private inputs, k_t^f and l_t^f , are:

$$r_t = \frac{\alpha_1 y_t^f}{k_t^f} \quad (16)$$

$$w_t = \frac{\alpha_2 y_t^f}{l_t^f} \quad (17)$$

so that each firm's profits are:

$$\pi_t^f = (1 - \alpha_1 - \alpha_2) y_t^f \quad (18)$$

4.3 Government

We now present government policy. Following most of the related literature, we assume that there is a consolidated general budget and a separate social security budget, where the latter is balanced (see e.g. Soares (2006), Conesa and Garriga (2008) and Bruce and Turnovsky (2013)).

4.3.1 Consolidated government budget constraint

The government uses capital taxes, τ_t^k , labor taxes, τ_t^n , consumption taxes, τ_t^c , and taxes on bequests, τ_t^b , to finance public spending on health, G_t^h , public education spending, G_t^e , public investment spending on infrastructure, G_t^i , public spending on defense and public order-safety, G_t^d , and public spending on the rest of government activities denoted as G_t^c (see below for G_t^c). As said, there is also public spending on pensions but this is financed by social security contributions (see next subsection). Also, D_t is the beginning-of-period total stock of one-period maturity government bonds. Thus, in aggregate terms, the consolidated government budget constraint is:

$$G_t^h + G_t^e + G_t^i + G_t^d + G_t^c + (1 + \rho_t) D_t = D_{t+1} + T_t \quad (19)$$

where tax revenues, T_t , are:

$$T_t \equiv \tau_t^c (N_t^y c_t^y + N_t^m c_t^m + q N_t^o c_t^o) + \tau_t^n N_t^m w_t l_t^m h_t^m + \tau_t^k N_t^o (r_t k_{t-1}^m + \pi_t^o) + \tau_t^b N_t^y b_{t-1}^y \quad (20)$$

For convenience, concerning the public spending instruments, G_t^h , G_t^e , G_t^i , G_t^d and G_t^c , we will work in terms of their GDP shares defined respectively as $s_t^{g^h} \equiv \frac{G_t^h}{N_t^f y_t^f}$, $s_t^{g^e} \equiv \frac{G_t^e}{N_t^f y_t^f}$, $s_t^{g^i} \equiv \frac{G_t^i}{N_t^f y_t^f}$, $s_t^{g^d} \equiv \frac{G_t^d}{N_t^f y_t^f}$ and $s_t^{g^c} \equiv \frac{G_t^c}{N_t^f y_t^f}$. Notice that one of the fiscal policy instruments needs to follow residually to close the above budget constraint; except otherwise said, we will assume that this is the end-of-period public debt, D_{t+1} .

4.3.2 Budget constraint of the public pension system (PAYG)

The public pension system operates separately with its own budget. We start by assuming a pay-as-you-go (PAYG) system, as is the case in most countries. Thus, each old household receives a pension s_t^o when retired (see equation (4)) and these social security benefits are financed by a dedicated tax, $0 \leq \phi_t \leq 1$, on the labor income of the currently working individuals (see equation (3)).¹⁴

¹⁴Here we assume the standard PAYG social security system: all social security benefits of retirees are financed by social security taxes paid by those currently working (see e.g. Cooley and Soares (1999), Soares (2006), Krueger and Kubler (2006), Bruce and Turnovsky (2013), etc). We could enrich this system by assuming that only a fraction is funded by the PAYG system and the rest is funded by the general government budget. See below in section 7 for a fully funded system. See e.g. Docquier and Paddison (2003), Auerbach and Lee (2011) and Kaganovich and Zilcha (2012), for different social security systems.

Thus, aggregating over the relevant households, the budget constraint of the social security pension system is:

$$G_t^s = qN_t^o s_t^o = \phi_t N_t^m w_t l_t^m h_t^m \quad (21)$$

where G_t^s is total public spending on pensions.

For convenience, as we did with the other public spending items, we will work with the output share of public spending on pensions defined as $s_t^{g^s} \equiv \frac{G_t^s}{N_t^f y_t^f}$. Notice that one of the fiscal policy instruments needs to follow residually to close the above budget constraint; except otherwise said, we will assume that this is the social security tax rate, ϕ_t .

4.3.3 The role of public spending in our model

The meaning of $s_t^{g^h}$, $s_t^{g^e}$, $s_t^{g^i}$ and $s_t^{g^d}$ is straightforward and their values will be set as in the data. Regarding G_t^c (or its output share, $s_t^{g^c}$), which includes the rest of public spending, it is defined to be as public spending on general public services net of debt payments (recall that spending on debt payments is already captured by $\rho_t D_t$ in our model), plus public spending on housing and community amenities, recreation, culture and environmental protection, plus public spending on social security net of pensions (recall that spending on pensions is already included in our model - see next subsection); these are the rest of government activities in the data (see Table 1 above).

Following usual practice (see e.g. European Central Bank, 2009), we will treat the sum of G_t^c and G_t^d as utility-enhancing public activities, or as "pure public goods", entering directly the household's utility function (1); thus, we assume $G_t^u \equiv G_t^c + G_t^d$ in equation (1). We report, however, that when we simply use $G_t^u \equiv G_t^c$ or when we also allow other categories of public spending (like education, etc) to be included into the definition for G_t^u , the main results are not affected, especially the results for output. In addition, in section 7 below, we will allow public spending on defense and public order-safety, G_t^d , to play a more explicit role by also protecting property rights.

Public investment spending on infrastructure, G_t^i , is used to augment public capital. Thus,

$$K_{t+1}^g = (1 - \delta^g) K_t^g + G_t^i \quad (22)$$

where $0 < \delta^g < 1$ is a depreciation parameter.

4.4 Market-clearing conditions

In the labor market, we have:

$$N_t^f l_t^f = N_t^m l_t^m h_t^m$$

In the capital market, we have:

$$N_t^f k_t^f = N_t^o k_{t-1}^o = N_{t-1}^m k_{t-1}^m$$

In the government bond market, we have:

$$D_{t+1} = N_t^m d_t^m$$

In the dividend market, we have:

$$N_t^f \pi_t^f = N_t^o \pi_t^o$$

In the "bequest market", we have:

$$N_t^y b_{t-1}^y = q N_{t-1}^o b_{t-1}^o + (1 - q) N_{t-1}^m \Omega_{t-1}^o$$

where

$$\Omega_{t-1}^o \equiv [1 - \delta^k + (1 - \tau_{t-1}^k) r_{t-1}] k_{t-2}^m + (1 + \rho_{t-1}) d_{t-2}^m + (1 - \tau_{t-1}^k) \pi_{t-1}^o$$

thus, the currently young inherit the bequests (chosen and unintended) of the previously old.

In a PAYG public pension system, we have:

$$q N_t^o s_t^o = N_t^m \phi_t w_t l_t^m h_t^m$$

In the goods market, we have:

$$N_t^y c_t^y + N_t^y z_t^y + N_t^m c_t^m + q N_t^o c_t^o + N_t^m k_t^m + G_t^c + G_t^h + G_t^e + G_t^i + G_t^d = N_t^m w_t l_t^m h_t^m + N_t^m (1 - \delta^k + r_t) k_{t-1}^m$$

so that (using the market-clearing conditions), we get the economy's resource constraint:

$$N_t^y c_t^y + N_t^y z_t^y + N_t^m c_t^m + q N_t^o c_t^o + N_t^m [k_t^m - (1 - \delta^k) k_{t-1}^m] + G_t^c + G_t^h + G_t^e + G_t^i + G_t^d = N_t^f y_t^f$$

or, working with population shares $n_t^y \equiv \frac{N_t^y}{N_t}$, $n_t^m \equiv \frac{N_t^m}{N_t}$, $n_t^o \equiv \frac{N_t^o}{N_t}$:

$$n_t^y c_t^y + n_t^y z_t^y + n_t^m c_t^m + q n_t^o c_t^o + n_t^m [k_t^m - (1 - \delta^k) k_{t-1}^m] + \left(s_t^h + s_t^e + s_t^i + s_t^d + s_t^o \right) n_t^f y_t^f = n_t^f y_t^f$$

where public spending items in small letters denote per capita values and also

$$n_t^f y_t^f = A (n_{t-1}^m k_{t-1}^m)^{\alpha_1} (n_t^m l_t^m h_t^m)^{\alpha_2} (n_t^m k_t^g)^{1-\alpha_1-\alpha_2} \text{ and } k_t^g \equiv \frac{K_t^g}{N_t^f}.$$

4.5 Market equilibrium system (for any feasible policy)

We now present the market equilibrium (ME) system which holds for any feasible policy. In this equilibrium, households maximize welfare, firms maximize profits, all constraints are satisfied and all markets clear. This system is comprised of the following equations ex ante:

Young:

$$\chi_n (e_t^y)^\eta = \frac{\beta (c_{t+1}^m)^{-\sigma} (1 - \tau_{t+1}^n - \phi_{t+1}) w_{t+1} l_{t+1}^m B \theta (e_t^y)^{\theta-1} [\gamma (z_t^y)^\nu + (1 - \gamma) (\bar{G}_t^e + \lambda \bar{G}_t^h)^\nu]^{\frac{1-\theta}{\nu}}}{(1 + \tau_{t+1}^c)} \quad (23)$$

$$\frac{(c_t^y)^{-\sigma}}{(1 + \tau_t^c)} = \frac{\beta (c_{t+1}^m)^{-\sigma} (1 - \tau_{t+1}^n - \phi_{t+1}) w_{t+1} l_{t+1}^m B (e_t^y)^\theta \gamma (1 - \theta) [\gamma (z_t^y)^\nu + (1 - \gamma) (\bar{G}_t^e + \lambda \bar{G}_t^h)^\nu]^{\frac{1-\theta}{\nu}-1}}{(z_t^y)^{1-\nu} (1 + \tau_{t+1}^c)} \quad (24)$$

$$(1 + \tau_t^c) c_t^y + z_t^y = (1 - \tau_t^b) b_{t-1}^y \quad (25)$$

Adult:

$$\chi_n (l_t^m)^\eta = \frac{(c_t^m)^{-\sigma} (1 - \tau_t^n - \phi_t) w_t h_t^m}{(1 + \tau_t^c)} \quad (26)$$

$$\frac{(c_t^m)^{-\sigma}}{(1 + \tau_t^c)} = \frac{\beta q (c_{t+1}^o)^{-\sigma} [(1 - \delta^k) + (1 - \tau_{t+1}^k) r_{t+1}]}{(1 + \tau_{t+1}^c)} \quad (27)$$

$$\frac{(c_t^m)^{-\sigma}}{(1 + \tau_t^c)} = \frac{\beta q (c_{t+1}^o)^{-\sigma} (1 + \rho_{t+1})}{(1 + \tau_{t+1}^c)} \quad (28)$$

$$(1 + \tau_t^c) c_t^m + k_t^m + d_t^m = (1 - \tau_t^n - \phi_t) w_t h_t^m l_t^m \quad (29)$$

$$h_{t+1}^m = (1 - \delta^h) h_t^y + B (e_t^y)^\theta [\gamma (z_t^y)^\nu + (1 - \gamma) (\bar{G}_t^e + \lambda \bar{G}_t^h)^\nu]^{\frac{1-\theta}{\nu}} \quad (30)$$

Old:

$$\frac{(c_t^o)^{-\sigma}}{(1 + \tau_t^c)} = \beta \chi_b (b_t^o)^{-\xi} \quad (31)$$

$$q (1 + \tau_t^c) c_t^o + q b_t^o + (1 - q) \Omega_t^o = [1 - \delta^k + (1 - \tau_t^k) r_t] k_{t-1}^m + (1 - \tau_t^k) \pi_t^o + (1 + \rho_t) d_{t-1}^m + q s_t^o \quad (32)$$

Government:

$$(s_t^{g^h} + s_t^{g^e} + s_t^{g^i} + s_t^{g^d} + s_t^{g^c}) n_t^f y_t^f + (1 + \rho_t) n_t^m d_{t-1}^m = \quad (33)$$

$$= n_t^m d_t^m + \tau_t^c (n_t^y c_t^y + n_t^m c_t^m + q n_t^o c_t^o) + \tau_t^n n_t^m w_t l_t^m h_t^m + \tau_t^k n_t^m r_t k_{t-1}^m + \tau_t^b n_t^y b_{t-1}^y$$

$$q n_t^o s_t^o = n_t^m \phi_t w_t l_t^m h_t^m \quad (34)$$

$$k_{t+1}^g = (1 - \delta^g) k_t^g + s_t^{g^i} n_t^f y_t^f \quad (35)$$

$$s_t^{g^s} \equiv \frac{q n_t^o s_t^o}{n_t^f y_t^f} \quad (36)$$

In the above equations, we use:

$$n_t^f y_t^f = A (n_t^m k_{t-1}^m)^{\alpha_1} (n_t^m l_t^m h_t^m)^{\alpha_2} (n_t^f k_t^g)^{1-\alpha_1-\alpha_2}$$

$$n_t^y b_{t-1}^y = q n_{t-1}^o b_{t-1}^o + (1-q) n_{t-1}^m \Omega_{t-1}^o$$

$$\Omega_{t-1}^o \equiv [1 - \delta^k + (1 - \tau_{t-1}^k) r_{t-1}] k_{t-2}^m + (1 + \rho_{t-1}) d_{t-2}^m + (1 - \tau_{t-1}^k) \pi_{t-1}^o$$

$$\pi_t^o = \frac{n_t^f}{n_t^o} \pi_t^f = \frac{(1 - \alpha_1 - \alpha_2) n_t^f y_t^f}{n_t^o}$$

$$r_t = \frac{\alpha_1 y_t^f}{k_t^f} = \frac{\alpha_1 N_t^f y_t^f}{N_t^f k_t^f} = \frac{\alpha_1 n_t^f y_t^f}{n_t^m k_{t-1}^m}$$

$$w_t = \frac{\alpha_2 y_t^f}{l_t^f} = \frac{\alpha_2 N_t^f y_t^f}{N_t^f l_t^f} = \frac{\alpha_2 n_t^f y_t^f}{n_t^m l_t^m h_t^m}$$

$$q_t = \Xi \left(1 + \frac{\bar{G}_t^h}{1 + \bar{G}_t^h} \right)$$

$$\text{where } \bar{G}_t^e \equiv \frac{G_t^e}{N_t^y} = \frac{s_t^{g^e} N_t^f y_t^f}{N_t^y} = \frac{s_t^{g^e} n_t^f y_t^f}{n^y} \text{ and } \bar{G}_t^h \equiv \frac{G_t^h}{N_t^y} = \frac{s_t^{g^h} N_t^f y_t^f}{N_t^y} = \frac{s_t^{g^h} n_t^f y_t^f}{n^y}.$$

We thus have a second-order dynamic system of 14 equations in 14 endogenous variables. The 14 endogenous variables are $\{z_t^y, c_t^y, c_t^m, c_t^o, e_t^y, l_t^m, k_t^m, \rho_t, h_t^m, s_t^o, \phi_t, d_t^m, b_t^o, k_{t+1}^g\}_{t=0}^\infty$.

4.6 Solution methodology

We will work as follows. In the next section (section 5), we solve the above model numerically employing common parameter values and fiscal data from the euro area. The steady state solution of this model will be defined as the "status quo". In turn, the rest of the sections will study the transition dynamics when we depart from this status quo steady state and travel to new, reformed steady states (fiscal policy reforms are defined in subsection 6.1 below). To compute the transition dynamics, we will take a first-order approximation of the model around the new, reformed steady state, when the initial conditions of the predetermined state variables are those of the status quo solution. For our numerical solutions, we use the Dynare toolkit. Notice that, since the model is kept deterministic for simplicity, transition dynamics will be driven by reforms in fiscal (tax-spending) policy only.

5 Parameters, data and the status-quo solution

This section solves the above model economy numerically using fiscal policy data from the euro area. The values of parameters and policy variables are listed in Tables 2a and 2b respectively, while related details are discussed in subsections 5.1 and 5.2. In turn, the resulting steady state solution is in subsection 5.3.

5.1 Parameter values

Parameter values for preference and technology are reported in Table 2a. We use conventional values. We report at the outset that our main results are robust to changes in these parameter values (details are in subsection 6.4 and section 7 below). Thus, although our numerical simulations below are not meant to provide a rigorous quantitative study, they illustrate the qualitative dynamic features of the model in a robust way.

The time unit is meant to be a period consisting of 25 years. Regarding preference parameters, we use values used by most of the related literature. The time preference rate, β , is set at 0.985^{25} so as to be consistent with a value for the real interest rate around 5% per year. The weight given to public goods and services in the household's utility function, χ_g , is initially set at 0.1. The elasticity of intertemporal substitution, σ , the inverse of Frisch labour elasticity, η , and the elasticity of bequests in the utility are set as in related studies. The preference parameter related to effort time, χ_n , is set at 7; this implies hours of work within usual ranges.

Regarding technology parameters in the production function of goods (see equation (15)), the Cobb-Douglas exponents of physical capital and labor are set at 0.35 and 0.60 respectively, so that the exponent of public capital is 0.05 (the latter is close to public investment as share of output). The scale parameter in the same function, A , is set at 2. As is usual in the OLG literature, we set the values of the depreciation rates of physical, human and public capital equal to 1. For simplicity, we set equal population shares (however, in the data, adults have the highest share in total population). In the human capital production function (see equation (6)), the values of B and θ are close to the values used by the growth literature; these parameter values imply hours of education within usual ranges. Following Stokey (1996), the elasticity parameter, ν , is set at the neutral value of 0.5 (this implies $1/(1 - \nu) = 2$ so that private and public education spending are good substitutes). The weight given to private vis-a-vis public spending in the same function, γ , is set at 0.25; this implies that, in equilibrium, private education spending as share of GDP is around 0.5%, which is as in the European data (see section 7 below for sensitivity). As regards the parameter λ , measuring the contribution of public spending on health in the creation of private human capital, we start with a modest value of 0.1 (again, see section 7 below for sensitivity).

5.2 Fiscal policy data

We use spending and tax policy data for the Eurozone. The data are from Eurostat. To solve the model, we use data averages over the pre-crisis period, 2001-2008. Their values are listed in Table 2b. In other words, regarding the exogenously set fiscal policy instruments, we choose to use the means in the data over the pre-crisis period (2001-2008) rather than over the whole euro period (2001-2012). This is because the crisis year, 2008, triggered a number of policy changes or, at least, it led to a discussion of several policy changes. We thus find it more natural to use the 2001-2008 solution as a point of departure to study various reforms.

Nevertheless, we report that our qualitative results do not change when we use the means in the data over the whole period (2001-2012) or when we simply use the year 2008 values.

Specifically, regarding public spending, we make use of the disaggregation of the international Classification of the Functions of Government (COFOG) in the framework of the European System of National Accounts (ESA95). This is comprised by the functional categories listed in Table 1. According to our model above, we use data on education, health, defense and public-order safety, old age pensions (which are a sub-category of social protection expenditure) and economic affairs. These spending items as shares of GDP ($s_t^{g^e}, s_t^{g^h}, s_t^{g^d}, s_t^{g^s}, s_t^{g^i}$) are set at their data averages (see Table 2b). The rest of spending items are included in s^{g^c} , as defined in subsection 4.3.1 above, amounting to around 14% of GDP. Then, total government spending is around 43% of GDP, which is close to the 2001-2008 data average.

The values of the tax rates τ^c , τ^k , τ^b and τ^n are set respectively at 0.194, 0.295, 0.04 and 0.26, which are the average effective tax rates in the euro zone. Notice that the value of 0.26 for τ^n does not include the social security tax rate, ϕ . The value of the latter, as implied by the value of s^{g^s} in the data, is around 0.13. This in turn implies a total tax rate on labor income, $\tau^n + \phi$, of 0.39 which is close to the data average.

Table 2a: Parameter values

Parameters	Value	Description
β	0.985 ²⁵	time preference rate
σ	1	elasticity of intertemporal substitution
χ_n	7	preference parameter related to effort time
χ_b	1	preference parameter related to bequests
χ_g	0.100	preference parameter related to public spending
ζ	1	elasticity of public consumption in utility
η	1	inverse of Frisch labour supply elasticity
ξ	1	elasticity of bequests in utility
n^y	1/3	share of young in the population
n^m	1/3	share of adults in the population
n^o	1/3	share of old in the population
Ξ	0.600	parameter in the probability of reaching the old age
Γ	0.800	parameter in the degree of protection of property rights
A	2	scale parameter in production of goods
α_1	0.350	share of capital
α_2	0.600	share of labour
B	15	scale parameter in production of human capital
θ	0.750	productivity of education time
λ	0.100	contribution of public spending on health in human capital
ν	0.500	elasticity parameter in human capital
γ	0.250	weight to private spending in human capital
δ^k	1	physical capital depreciation rate
δ^h	1	human capital depreciation rate
δ^g	1	public capital depreciation rate

Table 2b: Fiscal policy variables (2001-8)

Policy instruments	Value	Description
τ^c	0.194	consumption tax rate
τ^n	0.260	labour tax rate (used to finance spending except social security)
ϕ	0.130	labor tax rate (used to finance social security spending)
τ^k	0.295	capital tax rate
τ^b	0.040	bequest tax rate
$s_t^{g^e}$	0.052	government spending on education as share of output
$s_t^{g^h}$	0.067	government spending on health as share of output
$s_t^{g^i}$	0.041	government spending on infrastructure as share of output
$s_t^{g^s}$	0.081	government spending on pensions as share of output
$s_t^{g^d}$	0.033	government spending on defense & public-order as share of output
$s_t^{g^c}$	0.140	government spending on the rest as share of output

5.3 Steady state solution (the status quo)

This subsection presents the steady state solution of the ME system presented in subsection 4.5 above. In this numerical solution, variables do not change over time and the parameter values and the exogenous policy instruments have been set as in Tables 2a and 2b respectively. The solution is reported in Table 3. As can be seen, the solution is meaningful. It is also empirically relevant: most of the implied ratios are not different from their values in the 2001-8 data (e.g. consumption to GDP, public debt to GDP, private education spending to GDP, time allocated to work or school, etc).¹⁵ This solution, called the status quo, will serve as a point of departure in what follows.

¹⁵Notice that the solution for the probability of an adult reaching the old age (see equation (7) above) is around $q = 0.52$. We report that, through recalibration of the parameter Ξ in that equation, the solution for q can rise and the main results do not change.

Table 3: Steady state solution (status quo)

Variable	Solution	Variable	Solution
c^y	0.1121	h^m	2.4793
c^m	0.1228	s^o	0.0874
c^o	0.1335	$Y = n^f y^f$	0.1885
e^y	0.3210	b^o	0.1093
l^m	0.4478	b^y	0.1441
k^m	0.0456	r	0.0605
k^g	0.0075	w	0.3059
q	0.5182	$\frac{c}{y}$	0.5400
z^y	0.0028	ρ	0.0293
u	-5.9918	$\frac{d^m}{Y}$	0.6035

Notes: (a) u follows from the household's utility function (1). (b) $\frac{c}{y} = \frac{(n^y c^y + n^m c^m + q n^o c^o)}{n^f y^f}$.

6 Fiscal policy reforms

We now study the implications of various policy (spending/tax) reforms. As said above, by reforms we mean permanent changes in the composition of public spending and taxes.

6.1 Discussion of fiscal policy reforms studied

We study three sets of fiscal policy changes. First, changes that, at least qualitatively, are in the spirit of policy changes actually observed in the euro area in the aftermath of the 2008 crisis. Second, changes that may not have actually happened but have been debated in policy circles during the same period. Third, some changes that have neither happened nor debated but are natural candidates to be studied given the broader set of fiscal instruments in the hands of policymakers.

In all cases studied, we depart from the status quo solution in Table 3 above. Also, we assume that any exogenous changes in policy instruments are permanent and are accommodated by adjustments in public debt; in other words, public debt is the residually determined policy variable and this happens both in the transition and in the new reformed steady state (any other public financing cases are reported below). Moreover, to make different fiscal policies comparable, we compare changes in policy instruments that represent fiscal packages of the same size of GDP. In particular, all spending-tax changes will worth 1% of GDP.¹⁶ In the case of public spending, this is straightforward; a category of spending as share of GDP is increased by 1 pp (say, from 8% to 9% of GDP). In the case of tax rates, a tax rate is cut so as the associated fall in tax revenues as share of GDP to be 1pp again (see right below for details). Finally, to understand the logic of our results, and following usual practice, we start by considering one reform at a time.

¹⁶We report that our qualitative results are robust to fiscal changes of bigger, or smaller, magnitude. See also below.

The reforms studied are listed in Table 4. The first set of reforms (namely, reforms that are in the spirit of actual policy measures) includes reforms R1-R4. In particular, R1 assumes 1 pp permanent increase on public pensions as share of GDP, s^{g^s} ; ¹⁷ R2 assumes 1 pp permanent increase in public spending on economic affairs, s^{g^i} (which, as said, is basically infrastructure spending); R3 assumes a cut in the consumption tax rate, τ^c , associated with 1 pp permanent decrease in consumption tax revenues; ¹⁸ and R4 assumes a cut in the labor tax rate, τ^n , associated with 1 pp permanent decrease in labor income tax revenues. As said above, these changes, at least qualitatively, are in accordance with fiscal policy measures taken in the euro area after the 2008 crisis (see e.g. Coenen et al. (2013)).

The second set of reforms includes R5-R6. In particular, R5 assumes an horizontal permanent cut in all public spending items by 0.165 percentage points which amounts to a reduction in overall public spending by 1 pp; R6 assumes a cut in the capital tax rate, τ^k , associated with 1 pp permanent decrease in capital income tax revenues. Notice that we study an horizontal cut in all public spending categories because a smaller public sector is ideologically supported by several right-wing governments in Europe these days ¹⁹ and, in the same spirit, we study a rise in capital taxes because this is ideologically supported by left-wing governments.

The third set includes reforms R7-R9. In particular, R7 assumes 1 pp permanent increase in public education spending, s^{g^e} ; R8 assumes 1 pp permanent increase in public spending on health, s^{g^h} ; finally, R9 assumes 1 pp permanent increase in public spending on defence and public order-safety, s^{g^d} .

We will also report what would have happened if the policy variables had remained at their 2008 values for ever (this SQ solution serves as a natural benchmark).

Table 4: Fiscal (spending-tax) policy reforms

Reforms	Description (as shares of GDP)
R1	1 pp increase in spending on pensions (s^{g^s})
R2	1 pp increase in spending on economic affairs (s^{g^i})
R3	1 pp decrease in consumption tax revenues (via cuts in τ^c)
R4	1 pp decrease in labor income tax revenues (via cuts in τ^n)
R5	1 pp decrease in overall public spending
R6	1 pp increase in capital income tax revenues (via increases in τ^k)
R7	1 pp increase in spending on education (s^{g^e})
R8	1 pp increase in spending on health (s^{g^h})
R9	1 pp increase in spending on defence and public-order safety (s^{g^d})

¹⁷That is, it rises from 8% of GDP to 9%.

¹⁸This is modeled as follows. Consumption tax revenues as share of GDP in the SQ solution in Tables 2b and 3 are $\tau^c \frac{n^y c^y + n^m c^m + q n^o c^o}{n^f y^f} = 0.194 \cdot 0.54 = 0.105$. In the reformed economy this is reduced permanently by 1pp, i.e. to 0.095. Then, along the transition of the reformed economy, the consumption tax rate follows from the equation $\tau_t^c \frac{n_t^y c_t^y + n_t^m c_t^m + q n_t^o c_t^o}{n_t^f y_t^f} = 0.095$.

We work similarly in the case of labor income taxes, τ_t^n , in R4 and capital income taxes, τ_t^k , in R6.

¹⁹Our choice of an horizontal cut in all public spending categories can also be motivated by the fact that in several policy reports (see e.g. European Commission, 2014, part IV), it is recommended that debt consolidation is successful when we cut spending on public wages. Here, since we do not have separate data on public wages for each functional category of public spending, we ad hoc cut all public spending categories by 0.165 percentage points.

6.2 Results for discounted lifetime output and welfare

Tables 5 and 6 report results for discounted lifetime output (defined as $\sum_{t=0}^{\infty} \beta^t y_t$) and discounted lifetime utility (defined as $\sum_{t=0}^{\infty} \beta^t u_t$) respectively for each regime listed in Table 4. As said, the adjusting public financing instrument is the end-of-period public debt in each period. In turn, the implied utility gains, or losses, for each generation separately (young, adult and old) vis-a-vis the SQ case, measured in terms of consumption equivalence units, are reported in Table 7.

Table 5: Discounted lifetime output

SQ	R1	R2	R3	R4	R5	R6	R7	R8	R9
0.5992	0.5825	0.6147	0.6052	0.6119	0.5925	0.5891	0.6195	0.6101	0.6067

Table 6: Discounted lifetime utility

SQ	R1	R2	R3	R4	R5	R6	R7	R8	R9
-19.0423	-19.2268	-18.9774	-18.99557	-18.9353	-19.0450	-19.1523	-18.8624	-19.0737	-19.0603

Table 7: Welfare gains/losses (from Table 6)

	R1	R2	R3	R4	R5	R6	R7	R8	R9
ξ^y	-0.0144	-0.0171	-0.0092	-0.0147	0.0176	-0.0037	-0.0036	-0.0264	-0.0240
ξ^m	-0.0563	0.0544	0.0366	0.0689	-0.0287	-0.0351	0.0772	0.0385	0.0373
ξ^o	-0.0192	0.0094	-0.0081	0.0071	0.0021	-0.0350	0.0434	-0.0144	-0.0226

Note: A positive value for the consumption equivalence, ξ , means a welfare gain vis-a-vis SQ.

The results in Tables 5 and 6 imply that the best reform, both in terms of discounted lifetime output and utility, would be an increase in public education spending, namely R7. At the other end, the worst outcome is given by R1, namely an increase in spending on public pensions; actually, R1 is inferior even to the status quo (SQ). R2 (an increase in spending on economic affairs), R3 (a cut in consumption taxes) and R4 (a cut in labor taxes) are better than SQ, but worse than R7. R5 (an horizontal cut in all categories of public spending) and R6 (an increase in capital taxes) are worse than SQ both in terms of lifetime output and utility. R8 (higher spending on health) and R9 (higher spending on defence and public order-safety) are better than SQ in terms of output but not in terms of utility.

The simulated series, and the associated impulse response functions (see Figure 1), reveal that the social benefits from higher public education spending arise, not only because private human capital increases as one would naturally expect, but also because this type of public spending leads to relative high private savings, as measured by $k_{t+1}^m + d_{t+1}^m$ in the adults' budget constraint (see equation (3)). Higher private savings are, in turn, good for aggregate output and hence for social welfare in equilibrium. The opposite happens in the case of public spending on pensions funded by a PAYG system: now an increase in the latter is particularly bad for private savings and eventually for aggregate output (see Figure 2).²⁰

²⁰This is a well recognized result: social security, via a PAYG system, decreases private saving and hurts capital formation. See e.g. Cooley and Soares (1999), Bouzahzah et al. (2002), Docquier and Paddison (2003), Krueger and Kubler (2006) and Soares (2006). Krueger and Kubler (2006) provide a quantitative evaluation of the pluses and minuses of a PAYG social security system, where the tradeoff is between intergenerational risk-sharing and a lower capital stock. Here, as said, we abstract from

Figure 1: From SQ to R7

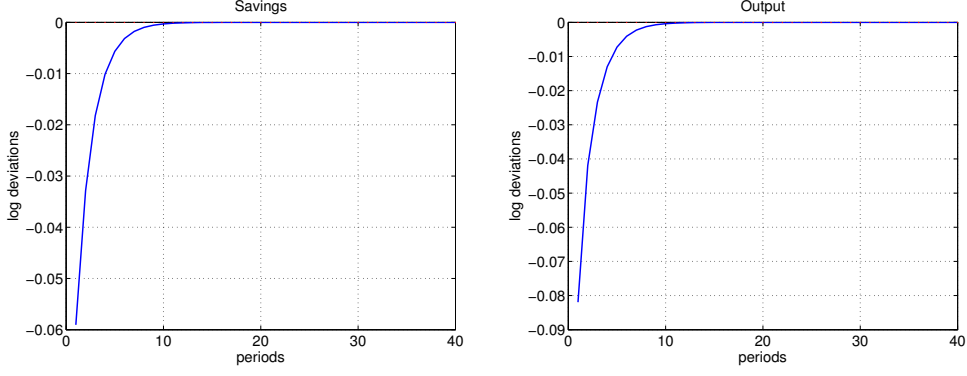
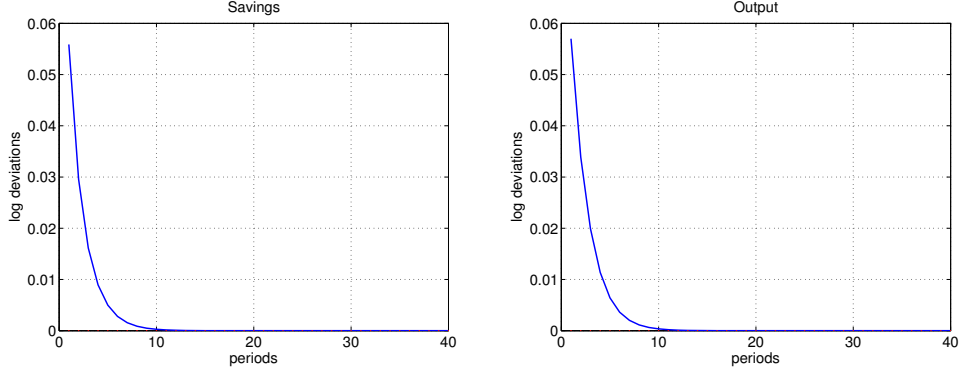


Figure 2: From SQ to R1



Consider finally the results in Table 7. In terms of lifetime welfare gains/losses for each generation separately, an increase in public education spending (R7) produces a welfare gain of 7.72 and 4.34 percent of consumption for adults and the old respectively (i.e. 7.72 and 4.34 of a percentage point of consumption). By contrast, an increase in social protection spending (R1) proves to be counter-productive for everybody, even for the old generation. In other words, even for the old households, the indirect harm from lower savings and aggregate output more than outweighs any direct benefits from the policy attempt to allocate more social resources to their pensions. Notice that, in most cases, the young are affected by less than the adults or the old; this is because the income of the young comes from bequests only which appear to be less sensitive to exogenous changes (see also Quadrini and Rios-Rull (2014)).

6.3 Fiscal policy mixes

We now study fiscal policy mixes. To save on space, and building upon the above findings, we report results only for the case in which we increase public spending on education permanently (as in R7), but now we assume that, in the new steady state, this is financed by a cut in public spending on social protection (which is the opposite of R1), while the steady state public debt-to-GDP ratio is exogenously set at its data average value. In the transition, as it has been the case so far, public debt remains the residual public financing instrument. The idea behind this mix is that we increase the most productive type of public spending and uncertainty and risk. Instead, we focus on the medium and long term.

this is financed by a cut in the least productive one. Inspection of the new results in Tables 8, 9 and 10, which are like Tables 5, 6 and 7 above, implies that the welfare gains are now much more substantial and these clear gains are enjoyed by all generations.

Table 8: Steady state output and utility

	SQ	R7-R1 mix
y	0.1885	0.2620
u	-5.9918	-5.4809

Note: $s^{\bar{g}}$ falls from 0.080 to 0.042 at the new steady state.

Table 9: Discounted lifetime output and utility

	SQ	R7-R1 mix
y	0.5992	0.7189
u	-19.0423	-17.9897

Table 10: Welfare gains/losses (from Table 9)

ξ^y	0.0933
ξ^m	0.3722
ξ^o	0.1381

Note: A positive value for ξ means a welfare gain vis-a-vis SQ.

6.4 Sensitivity analysis of the above results

We report that all main results are robust to changes in the parameter values in Table 2a as well as to changes in the exogenous policy instruments of different magnitude from what has been assumed so far (i.e. larger or smaller than 1 pp of GDP). Results are available upon request. In what follows, we provide some details regarding two key parameters related to private spending on education and public spending on health.

6.4.1 The role of private spending on education

In the solutions so far, the value of the parameter $0 \leq \gamma \leq 1$, which is the weight given to private education spending vis-a-vis public education spending in the production function for new human capital (see equation (6)), has been set at 0.25. As said, this parameter value resulted in a solution for private spending on education close to the European data (0.5% of GDP). Our results remain qualitatively unchanged for γ in the range between 0 and around 0.4. Above 0.4, as expected, R7 ceases to be the best policy change. It is important to point out, however, that a value of γ above 0.4 results in solutions for the output share of private education spending outside the range of historical values. For instance, when we set $\gamma = 0.5$, this share rises to 2.5% of GDP.

We also report that our results do not change when the related elasticity parameter, ν , rises, so that the elasticity of substitution between private and public education spending, $1/(1 - \nu)$, also rises. In particular,

our results do not change when ν rises from 0.5 (which was its value in the baseline parameterization) to around 0.8.

6.4.2 The role of public spending on health

We now study higher values of the parameter λ in equation (6). Recall that λ measures the extent to which public spending on health contributes to the creation of new human capital alongside with public spending on education (as argued above, not only education, skills and knowledge, but also health and longevity can be crucial to human capital). Also recall that the results so far have been with $\lambda = 0.1$, which can be thought as a low value. We now go to the other extreme and set, say, $\lambda = 0.9$. The new numerical results are reported in Tables 11, 12 and 13. As expected, now R7 and R8 become almost identical. Namely, an increase in public spending on health becomes as good as an increase in public spending on education to the extent that the former also augments private human capital.

Table 11: Discounted lifetime output

SQ	R7	R8
0.7293	0.7472	0.7491

Table 12: Discounted lifetime utility

SQ	R7	R8
-17.6238	-17.5527	-17.5749

Table 13: Welfare gains/losses (from Table 12)

	R7	R8
ξ^y	-0.0169	-0.0172
ξ^m	0.0560	0.0561
ξ^o	0.0120	0.0161

Note: A positive value for ξ means a welfare gain vis-a-vis SQ.

7 Richer models and further sensitivity analysis

In this section, we check the robustness of our results to changes in the model. We will focus on what happens when we have a fully funded (FF) public pension system instead of a PAYG system (see subsection 7.1) and on the possible role of public spending on defense and public order-safety in the protection of property rights (see subsection 7.2). In each case, we change one thing at a time so as results to be comparable to the baseline case studied so far.

7.1 A fully funded (FF) public pension system

One could claim that, by assuming a PAYG public pension system, we have not treated public pensions fairly because, as it typically presumed, this system hurts private savings. Hence, we now resolve the model

by replacing the PAYG system with a fully funded (FF) system. The latter is widely recognized to be a more efficient system (see e.g. Acemoglu (2009, chapter 9.5)). In particular, we now assume that the government collects social security taxes from the adults (as above) but now these funds are invested in physical capital with a return $(1 - \delta^k + r_{t+1})$ and then the resulting amount is distributed to the same individuals when they reach the old age. Thus, we now consider the other polar case where all social security funds are invested in capital (see again Acemoglu (2009, chapter 9.5)).

The budget constraint of the fully funded social security system is now:

$$G_t^s = qN_t^o s_t^o = (1 - \delta^k + r_t)\phi_{t-1}N_{t-1}^m w_{t-1}l_{t-1}^m h_{t-1}^m$$

Model details are in Appendix A. Here, working as above, we just present the final numerical solutions in Tables 14, 15 and 16. These tables should be compared to Tables 5, 6 and 7 above with the PAYG system. As can be seen, a FF system improves lifetime output and lifetime utility under all fiscal regimes. It thus confirms the wide perception that a FF system is more efficient than a PAYG system. Nevertheless, the main results regarding the desirability of fiscal policy changes remain as above. Namely, R7 is again the best policy change both in terms of discounted lifetime output and discounted lifetime utility. At the other extreme, now R6 is the worst in terms of lifetime output and R8 the worst in terms of lifetime utility. Notice that again, even under a FF system, R1 scores poorly both in terms of output and utility (although it is not the worst policy change, as it was the case under a PAYG system).

Table 14: Discounted lifetime output

SQ	R1	R2	R3	R4	R5	R6	R7	R8	R9
0.7851	0.7726	0.8152	0.7977	0.8089	0.7706	0.7643	0.8221	0.8101	0.8041

Table 15: Discounted lifetime utility

SQ	R1	R2	R3	R4	R5	R6	R7	R8	R9
-17.9780	-18.0742	-18.0390	-18.0022	-17.9910	-17.9287	-18.0709	-17.9027	-18.1252	-18.1098

Table 16: Welfare gains/losses (from Table 15)

	R1	R2	R3	R4	R5	R6	R7	R8	R9
ξ^y	-0.0250	-0.0612	-0.0363	-0.0550	0.0454	0.0137	-0.0475	-0.0708	-0.0684
ξ^m	-0.0030	0.0850	0.0554	0.0925	-0.0511	-0.0501	0.1164	0.0716	0.0674
ξ^o	-0.0177	-0.0321	-0.0322	-0.0338	0.0244	-0.0294	0.0034	-0.0553	-0.0629

Note: A positive value for ξ means a welfare gain vis-a-vis SQ.

Figures 3 and 4 are like Figures 1 and 2 above showing private savings in R7 and R1 respectively. As can be seen, as it was also the case under the PAYG system, R1 (namely, a rise in public spending on pensions) leads to a fall in private savings under a FF system. On the other hand, as expected, the detrimental effect of R1 on private savings is much stronger under a PAYG system (see Figure 3) than under a FF system (see Figure 4). It should be reported here that the wide belief that an increase in public pensions stimulates (or,

at least, does not hurt) private savings and growth under a FF system can be obtained only when we assume that effort time (allocated to work and education) are exogenously set so that labor supply is inelastically supplied. In other words, the result that higher public pensions, under a FF system, are good for savings and growth is not general. And, in any case, even with exogenous labor supply, R7 remains the best fiscal policy change. These results are available upon request.

Figure 3: From SQ to R7

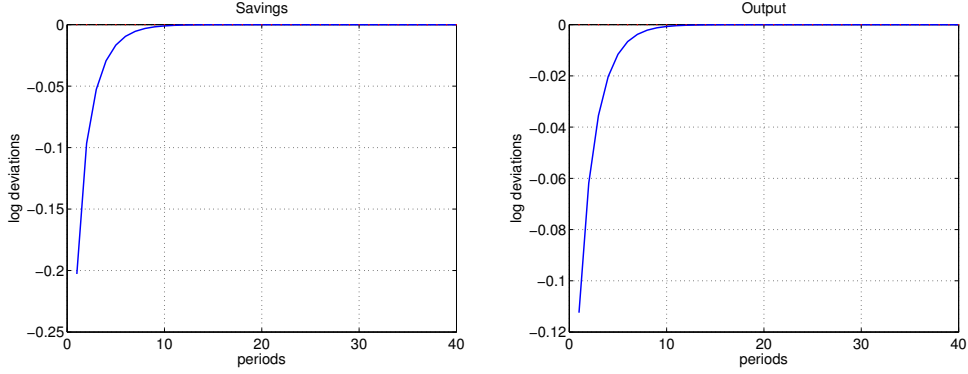
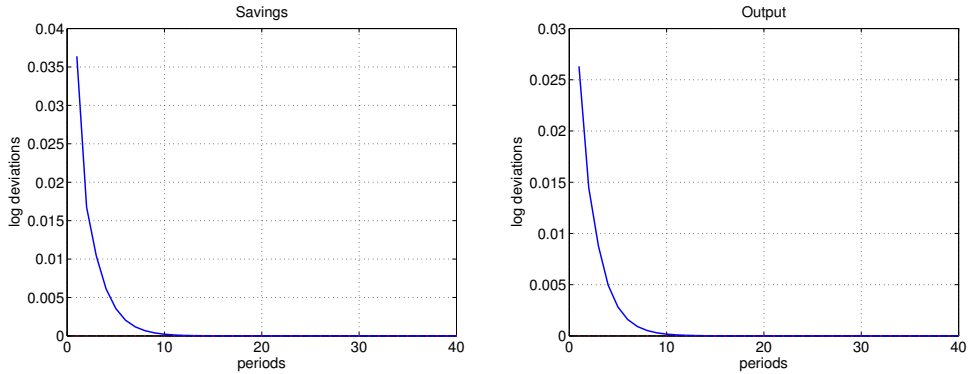


Figure 4: From SQ to R1



7.2 The role of public spending on defense and public order-safety

In the analysis so far, we have treated public spending on defense and public-order safety as a utility-enhancing policy activity only. One could claim, however, that this type of policy also provides productivity-enhancing services. For instance, it is widely recognized that public spending on defense and public-order safety (national security, police, courts, prisons, etc) is necessary for the protection of property rights. We therefore augment the model to allow for weak property rights and thus give a richer, and potentially more useful, role to public spending on defense and public order-safety. Thus, now, this type of policy not only provides utility-enhancing services as assumed so far, but it also increases the endogenously determined degree of property right protection.

In the richer model, we assume that each firm can appropriate only a fraction $0 < p \leq 1$ of its output produced, Y_t^f , because the rest, $(1 - p) Y_t^f$, can be taken away by other agents (say, adult households).²¹

²¹In a general equilibrium model, nothing is lost. Of course, ill-defined property rights distort individual incentives and this

Thus, p denotes the degree of protection of property rights in the economy. For simplicity, as we did above with the probability of reaching the old age, we assume that p depends on public expenditure on defense and public order-safety per firm, $\bar{G}_t^d \equiv \frac{G_t^d}{N_t^f}$.²² This is denoted as $p(\bar{G}_t^d)$, where $p(\cdot)$ is increasing and concave. For convenience, as we did above with the probability of reaching the old age in equation (7), we use the form:

$$p(\bar{G}_t^d) = \Gamma \left(1 + \frac{\bar{G}_t^d}{1 + \bar{G}_t^d} \right) \quad (37)$$

where the parameter $\Gamma > 0$ is calibrated so as to capture the degree of protection of property rights in the data (see also e.g. Angelopoulos et al., 2009).

Details of the new model are in Appendix B. Here, working as above, we just report the final numerical solutions. As can be seen in Tables 17, 18 and 19, the main results do not change. That is, even when we allow public spending on defense and public order-safety to play a new richer role, R9 (namely, an increase in public spending on defense and public order-safety) continues to be inferior to R7 (namely, an increase in public education spending) both in terms of output and utility. It is also important to report that our solution for the degree of property rights, p , is around 0.85, which is close to the international property rights index for the EU (see e.g. the World Bank database).

Table 17: Discounted lifetime output

SQ	R7	R9
0.4107	0.4292	0.4210

Table 18: Discounted lifetime utility

SQ	R7	R9
-21.0862	-20.9418	-21.1019

Table 19: Welfare gains/losses (from Table 18)

	R7	R9
ξ^y	-0.0255	-0.0362
ξ^m	0.0970	0.0534
ξ^o	0.0382	-0.0124

Note: A positive value for ξ means a welfare gain vis-a-vis SQ.

8 Concluding remarks

In this paper, we built a general equilibrium OLG model that included the most important functional categories of spending and taxes in the data. We then carried out a number of (actual and hypothetical) policy experiments to see what happens when spending and tax instruments change vis-a-vis their data

hurts the aggregate economy. This is also the case in our new model. See e.g. Economides et al. (2007) and Angelopoulos et al. (2009) for details and the related literature.

²²That is, we assume that the economy's degree of property rights does not depend on private decisions, like expropriation activities, rent seeking effort, etc. For such a generalization, see e.g. Economides et al. (2007) and Angelopoulos et al. (2009).

average values. The main result is that higher public spending on education and health, namely policies that enhance the accumulation of private human capital, would be the best way out of the recent crisis.

Regarding social protection spending, however, it is worth keeping in mind that here we assumed that capital markets are perfect (for instance, there are no borrowing constraints) so that agents can smooth consumption and insure themselves via private saving. It would be interesting though to see what happens when capital markets are not perfect in which case social security is not a substitute for private saving and borrowing (see e.g. Krueger and Perri (2011) and the reviews of inequality and policy by Heathcote et al. (2009) and Quadrini and Rios-Rull (2014)). Also, recall that, in the analysis above, we assumed that differences in age are the only source of heterogeneity across individuals. It would be interesting to add other sources. For instance, we could assume that households are also heterogeneous at birth, namely, there are rich-born households and poor-born households so that there are differences among households within any given cohort. In this case, public education spending may have an even stronger role to play (see e.g. Cunha and Heckman (2009) for this literature). We leave these extensions for future work.

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9 Appendix A: Fully funded (FF) public pension system

In this appendix, we model the case of a fully funded public pension system. The government collects social security taxes from adults at time $t + 1$ (as above), invests them in physical capital with a return $(1 - \delta^k + r_{t+1})$ and then distributes the resulting amount to the same individuals when they reach the old age. Thus, here we consider the best possible case where all related funds are invested in capital (see also Acemoglu (2009, chapter 9.5)). In what follows, we model what changes only.

The budget constraint of the fully funded social security system is now:

$$G_t^s = qN_t^o s_t^o = (1 - \delta^k + r_t)\phi_{t-1}N_{t-1}^m w_{t-1}l_{t-1}^m h_{t-1}^m$$

or equivalently:

$$G_t^s = qn_t^o s_t^o = (1 - \delta^k + r_t)\phi_{t-1}n_{t-1}^m w_{t-1}l_{t-1}^m h_{t-1}^m$$

In the capital market (since all social security funds are invested in capital), the market-clearing condition is now:

$$N_t^f k_t^f = N_{t-1}^m k_{t-1}^m + \phi_{t-1}N_{t-1}^m w_{t-1}l_{t-1}^m h_{t-1}^m$$

or equivalently:

$$n_t^f k_t^f = n_{t-1}^m k_{t-1}^m + \phi_{t-1} n_{t-1}^m w_{t-1} l_{t-1}^m h_{t-1}^m$$

In equilibrium, production and factor returns are now:

$$n_t^f y_t^f = A \left(n_t^f k_t^f \right)^{\alpha_1} \left(n_t^m l_t^m h_t^m \right)^{\alpha_2} \left(n_t^f k_t^g \right)^{1-\alpha_1-\alpha_2}$$

$$r_t = \frac{\alpha_1 y_t^f}{k_t^f} = \frac{\alpha_1 n_t^f y_t^f}{n_t^f k_t^f}$$

$$w_t = \frac{\alpha_2 y_t^f}{l_t^f} = \frac{\alpha_2 n_t^f y_t^f}{n_t^f l_t^f} = \frac{\alpha_2 n_t^f y_t^f}{n_t^m l_t^m h_t^m}$$

10 Appendix B: Adding weak property rights

In this appendix, we augment the baseline model to allow for weak property rights. We also allow public spending on defense and public order-safety to contribute to the protection of these rights.

As said in the text, each firm can appropriate only a fraction $0 < p \leq 1$ of its output produced, Y_t^f . The rest, $(1-p)Y_t^f$, can be taken away by adult households. Thus, p is the degree of protection of property rights in the economy. We assume that p depends on public expenditure on defence and public order-safety per firm, $\bar{G}_t^d \equiv \frac{G_t^d}{N_t^f}$. In particular, the degree of property rights is denoted as $p(\bar{G}_t^d)$, where $p(\cdot)$ is increasing and concave. For convenience, we use the functional form:

$$p(\bar{G}_t^d) = \Gamma \left(1 + \frac{\bar{G}_t^d}{1 + \bar{G}_t^d} \right) \quad (38)$$

where the parameter $\Gamma > 0$ is calibrated so as to capture the degree of protection of property rights in the data.

Then the profit function of each firm is:

$$\pi_t^f = p_t Y_t^f - r_t K_t^f - w_t L_t^f \quad (39)$$

The variable p_t is taken as given by the firm when it solves its optimization problem. The first-order conditions are now:

$$r_t = p_t \frac{\alpha_1 Y_t^f}{K_t^f} \quad (40)$$

$$w_t = p_t \frac{\alpha_2 Y_t^f}{L_t^f} \quad (41)$$

so that the firm's profits are:

$$\pi_t^f = p_t (1 - \alpha_1 - \alpha_2) Y_t^f \quad (42)$$

Regarding households, the budget constraint of the adult is now:

$$(1 + \tau_{t+1}^c) c_{t+1}^m + k_{t+1}^m + d_{t+1}^m = (1 - \tau_{t+1}^n - \phi_{t+1}) w_{t+1} h_{t+1}^m l_{t+1}^m + \frac{(1-p)N_t^f Y_t^f}{N_t^m} \quad (43)$$

where $\frac{(1-p)N_t^f Y_t^f}{N_t^m}$ is the extra income extracted by each adult in the case of weak property rights. This extra term is taken as given by the household when it solves its optimization problem.

Therefore, in the final equilibrium system, we now have:

$$\begin{aligned} \pi_t^o &= p_t \frac{n_t^f}{n_t^o} \pi_t^f = p_t \frac{(1 - \alpha_1 - \alpha_2) n_t^f Y_t^f}{n_t^o} \\ r_t &= p_t \frac{\alpha_1 Y_t^f}{K_t^f} = p_t \frac{\alpha_1 N_t^f Y_t^f}{N_t^f K_t^f} = p_t \frac{\alpha_1 n_t^f Y_t^f}{n_t^m k_{t-1}^m} \\ w_t &= p_t \frac{\alpha_2 Y_t^f}{L_t^f} = p_t \frac{\alpha_2 N_t^f Y_t^f}{N_t^f L_t^f} = p_t \frac{\alpha_2 n_t^f Y_t^f}{n_t^m l_t^m h_t^m} \end{aligned}$$

where $p_t = \Gamma \left(1 + \frac{\bar{G}_t^d}{1 + \bar{G}_t^d} \right)$.