Deep Recessions

Tatiana Kirsanova* University of Glasgow
Charles Nolan† University of Glasgow
Maryam Shafiei Deh Abad‡ University of Glasgow

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Abstract

This paper studies the conditions under which a 'modest' financial shock can trigger a deep recession with a prolonged period of recovery. We suggest that two factors can generate such a profile. The first is that the economy has accumulated a moderately high level of private debt by the time the adverse shock occurs. The second factor is when monetary policy is restricted by the zero lower bound. When present, these factors can result in a sharp contraction in output followed by a slow recovery. Perhaps surprisingly, we use a standard DSGE model with financial frictions along the lines of Jermann and Quadrini (2012) to demonstrate this result and so do not need to rely on dysfunctional interbank markets.

Key Words: financial frictions, credit boom, stagnation, ZLB

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*Address: Economics, Adam Smith Business School, Gilbert Scott Building, University of Glasgow, Glasgow G12 8QQ; e-mail tatiana.kirsanova@glasgow.ac.uk

†Address: Economics, Adam Smith Business School, Gilbert Scott Building, University of Glasgow, Glasgow G12 8QQ; e-mail charles.nolan@glasgow.ac.uk

‡Address: Economics, Adam Smith Business School, Gilbert Scott Building, University of Glasgow, Glasgow G12 8QQ; e-mail m.shafiei-deh-abad.1@research.gla.ac.uk

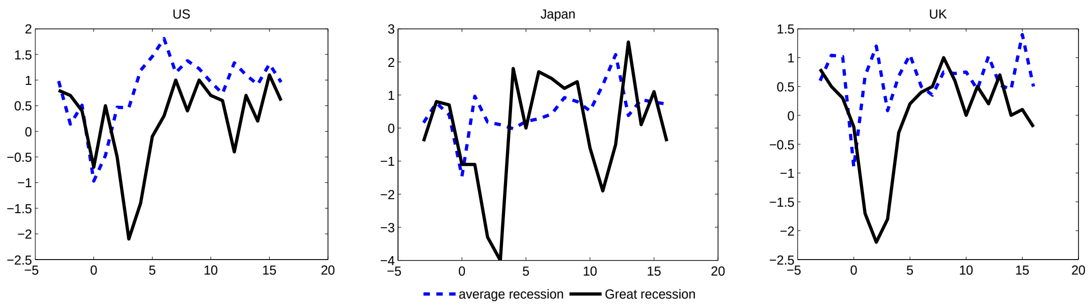
1 Introduction

Almost a decade has passed since the onset of the Great Recession in 2008, but for a number of economies recovery is slow and remains fragile. In fact, some even worry it has ushered in a new era of permanently lower trend growth. The origins of some of these concerns are reflected in Figure 1, which plots all post-war recessions for the UK, US and Japan.. The Great Recession stands out for at least two reasons. First, in each country the average recession is not as *deep* as the Great Recession. Second, in each country the average recession experiences its lowest output growth rate at the onset of the recession (period zero in the figure). Thereafter, economies generally return to a more normal growth path within one or two quarters. By way of contrast, following the onset of the Great Recession growth reached its minimum some two to four quarters later and growth remains at the lower end of the typical post-war experience; recovery has been *slow* by recent historical standards.

The role of 'financial frictions' in explaining these observed patterns has been identified as central by many researchers and policymakers and there is now a large and growing body of research which seeks to provide quantitative macroeconomic models to explain how seemingly well-functioning economies might 'unexpectedly' end up with financial crises. For example, Boissay et al. (2015) demonstrate how an 'innocuous' positive productivity shock can lead to rare but deep financial crises. The role of financial shocks in generating realistically frequent recessions is discussed in Nolan and Thoenissen (2009), Jermann and Quadrini (2012), Christiano et al. (2013) and Mumtaz and Zanetti (2016) to mention only a few. Notably, Jermann and Quadrini (2012) propose a macroeconomic model with financial frictions which is quite closely aligned with the US data since the early 1990s. The authors point out, however, that their model cannot replicate the *large* reduction in hours worked and output observed during the Great Recession. Moreover, so far researchers in this literature have generally focussed less on explaining the second aspect of the Great Recession that we highlighted above: the 'slow recovery'.[1]

In this paper we demonstrate how a simple log-linear DSGE model with financial frictions *a la* Jermann and Quadrini (2012) and with nominal rigidities *a la* Rotemberg (1983) is capable of generating the observed *deep* recessions, and *slow* recoveries that we argue are present in the data. In this model, the recession is triggered by a financial shock which reduces the proportion of output which banks will be able to recover when firms default. The implied reduction in bank

[1] This is despite the two-decades of slow growth affecting Japan since the early 1990s. An interesting recent exception is a paper by Benigno and Fornaro (2015) emphasising non-linear features in a growth model to explain 'stagnation traps'.



The period zero corresponds to the start of each of post-war recessions.[2]

Figure 1: Post-war recessions in the US, Japan and UK

credit requires substantial deleveraging in the economy and this leads to a recession. This result is interesting as some have argued that the onset of the crisis was essesntially an adverse credit event (see e.g., Taylor, 2008), whilst others attribute a large role to adverse selction and moral hazard problems in the interbank/money markets (Boissay et al., 2015). We suspect both explanations likely played a role, but our contribution in this paper is simply to argue that standard models of financial frictions explain more than perhaps is generally realised.

Two assumptions are helpful in generating a deep recession, consistent with that observed during the recent financial crisis. The first is the existence of 'overlending'. An initially high level of debt as indeed was observed prior to the Great Recession in many developed countries, see e.g. Schularick and Taylor (2012). Following the shock the reduction in debt presages a fall in the capital stock. In turn the lower capital stock requires less finance and these two effects reinforce one another and the sluggish adjustment of both stocks results in a much greater reduction of capital, output and hours worked. We show that the higher the initial debt to output ratio, the sharper the subsequent recession. For example, if the stock of lending is 10 percent above its steady state level, a financial shock nearly doubles the consequent reduction in output, compared with the case when debt is initially at steady state.

The second assumption is the existence of the zero lower bound (ZLB) on nominal interest rates. We demonstrate that the ZLB, alone, can generate sharp and deep recessions. The inability of the interest rate to help 'spread' the cost of the required deleveraging over many periods, results in an immediate and sharp reduction in the capital stock. Once the capital stock returns to the optimal level, further adjustment is slow. This is why the ZLB scenario facilitates capturing the second stylised fact: Once the initial large reduction in output is corrected, the speed of further recovery is substantially slowed down as compared to the case when monetary policy operates without constraints.

The paper is organized as follows. In the next section we present the model. We discuss the empirical evidence and the corresponding calibration of the model in section 3. In section 4 we discuss the sequence of policy experiments, which show how to generate stylized post-crisis dynamics like those with which we motivated this paper. Section 5 concludes.

2 The Model

We present a simple model with firms' borrowing constraints *a la* Jermann and Quadrini (2012) and with nominal rigidities *a la* Rotemberg (1983). The economy is populated by households and firms. Firms use labor and capital to produce differentiated goods. Firms issue equity and debt and use intra-period loans to finance working capital. Firms face credit restrictions because financial intermediaries fear they may not repay those loans. The detailed model of the economy is presented in this section.

2.1 Households

There is a continuum of homogeneous households of measure one. Households are indexed by j. The typical household seeks to maximize the following utility function:

$$\max_{C_t^j, n_t, b_{t+1}, s_{t+1}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U\left(C_t^j, n_t^j\right)$$

where $\mathbb{E}_t$ indicates expectations conditional on information available at time t; $0 < \beta < 1$ is the discount factor; C_t and n_t are a consumption aggregate and labor supply in period t, respectively. The period utility is:

$$U\left(C_t^j, n_t^j\right) = \frac{C_t^{j1-\sigma}}{1-\sigma} - \alpha \frac{n_t^{j1+\psi}}{1+\psi}$$

where σ is elasticity of relative risk aversion, ψ is the elasticity of labour supply and α is a 'preference' parameter. The households are assumed to hold equity, shares and corporate bonds. The household's budget constraint in nominal term can be written as:

$$W_t n_t^j + b_t P_t + s_t\left(D_t + \bar{p}_t\right) + P_t \Phi_t = P_t q_t b_{t+1} + s_{t+1}\bar{p}_t + P_t C_t^j + P_t T_t \quad (1)$$

where W_t is the nominal wage rate, P_t is the price of goods, $\bar{p}_t$ is the market price of shares, D_t is the dividend, s_t is equity holdings. $b_t P_t$ is the market value of one-period nominal bonds held by the households, T_t is a government transfer and the nominal return on bonds is

$$1 + i_t = \frac{\Pi_{t+1}}{q_t}.$$

Finally, $P_t\Phi_t$ is nominal profit from the ownership of final good firms.

The standard optimization of utility with respect to C_t^j, n_t^j , b_{t+1} , and s_{t+1}, subject to the budget constraint yields the following system of first order conditions:

$$\begin{aligned} 0 &= U_{n,t} + U_{C,t}w_t; && (2) \\ q_t &= \beta \mathbb{E}_t \frac{U_{C,t+1}}{U_{C,t}}; && (3) \\ \frac{\bar{p}_t}{P_t} &= \beta \mathbb{E}_t \frac{U_{C,t+1}}{U_{C,t}} \frac{(D_{t+1} + \bar{p}_{t+1})}{P_{t+1}}; && (4) \\ b_t &= q_t b_{t+1} + s_{t+1}\frac{\bar{p}_t}{P_t} + C_t^j + T_t - s_t\frac{(D_t + \bar{p}_t)}{P_t} - \frac{W_t}{P_t}n_t^j - \Phi_t. && (5) \end{aligned}$$

Here U_C and U_n indicate derivatives of $U\left(C_t^j, n_t^j\right)$ with respect to C_t^j and n_t^j respectively. These conditions are familiar, reflecting optimal labour supply decisions, bond purchases, equity purchases and intertemporal resource allocation.

2.2 Firms

There are two types of firms in this economy. There are flexible price intermediate goods producers and monopolistically competitive retailers. We discuss each in turn.

2.2.1 Intermediate goods producers

Intermediate goods producers have access to a standard production technology

$$y_t = F(e^{z_t}, k_t, n_t) = Ae^{z_t}k_t^{\theta}n_t^{1-\theta} \quad (6)$$

where A is a constant productivity shifter, e^{z_t} is stochastic productivity common to all firms, n_t is the labor input which can be flexibly changed at time t, k_t is the capital stock determined at time $t-1$ and θ is the capital share. Capital accumulates according to:

$$k_{t+1} = (1-\delta)\,k_t + I_t \quad (7)$$

where I_t is investment and δ is the depreciation rate. Firms use equity and debt to finance their operations. They prefer debt, b_t, to equity because of debt's tax advantage: see, Jermann and Quadrini (2012).

The budget constraint can be written as:

$$\frac{P_{mt}}{P_t}F(e^{z_t}, k_t, n_t) + \frac{b_{t+1}}{R_t} = b_t + \frac{W_t}{P_t}n_t + I_t + \frac{\Psi\left(D_t, D_{t-1}\right)}{P_t} \quad (8)$$

where P_{mt} is the nominal price of produced intermediate goods, $R_t = 1 + r_t(1 - \tau_t)$ is the after tax return on bonds, and $1 + r_t = \frac{1+i_t}{\Pi_{t+1}}$. $\Psi(D_t, D_{t-1})$ is the nominal payout to shareholders.

We assume that firms raise funds via both intertemporal debt, b_t, and an intraperiod loan, L_t , to finance working capital. They pay back the interest-free intraperiod loan at the end of the period. Firms start the period with intertemporal debt b_t and they choose labour, investment in capital, the dividend, D_t , and new intertemporal debt, b_{t+1}, *before* producing. Therefore the payments to workers $W_t n_t$, suppliers of investment goods $P_t I_t$, shareholders $\Psi(D_t)$ and bondholders $P_t b_t$ are made ahead of the realization of revenues. The intraperiod loan contracted by the firm will cover these costs as follows:

$$L_t = P_t I_t + W_t n_t + \Psi(D_t) + P_t b_t - P_t \frac{b_{t+1}}{R_t}.$$

From here and the budget constraint $L_t = P_{mt} F(e^{z_t}, k_t, n_t)$ is repaid at the end of the period and is free of interest.

The ability of firms to borrow is bounded because they may choose to default on their debt. Default arises after the realization of revenues but before repaying the intraperiod loan. The total liabilities of the firm at that time are $L_t +$ $P_t q_t b_{t+1}$, as it will need to pay back the loan and buy back all the bonds. The total liquid resources of the firm are $L_t = P_{mt} F(e^{z_t}, k_t, n_t)$. These can be 'diverted' by the firm and so cannot be recovered by the lender after a default. Then, the only asset available to the lender is capital $P_t k_{t+1}$. Following Jermann and Quadrini (2012), we assume that the liquidation value of capital is unknown at the moment of contracting the loan. With probability Ξe^{ξ_t} the full value $P_t k_{t+1}$ will be recovered, but with probability $1 - \Xi e^{\xi_t}$ the liquidation value is zero. Therefore the enforcement constraint will be as follows:

$$\Xi e^{\xi_t} (P_t k_{t+1} - P_t q_t b_{t+1}) \geq P_{mt} F(e^{z_t}, k_t, n_t). \tag{9}$$

Whilst Ξe^{ξ_t} is stochastic and depends on (uncertain) markets conditions, it is the same for all firms. This variable is what is identified as a 'financial shock'.

The firm's nominal payout to shareholders is assumed to be subject to a quadratic adjustment cost:

$$\Psi(D_t, D_{t-1}) = D_t + \kappa \left(\frac{D_t}{D_{t-1}} - 1 \right)^2 D_t$$

where the nominal equity payout D_t is given and $\kappa \geq 0$ is a parameter.

Each firm maximizes profit subject to budget constraint (8) and enforcement constraint (9), so the Lagrangian is:

$$
\begin{aligned}
L &= \sum_{t=0}^{\infty} m_{0,t}\left(\frac{D_t}{P_t}+\mu_t\left(\Xi e^{\xi_t}\left(Q_t k_{t+1}-q_t b_{t+1}\right)-\frac{P_{mt}}{P_t}F(e^{z_t},k_t,n_t)\right)\right. \\
&\quad +\lambda_t\left(\left(1-\delta\right)k_t+\frac{P_{mt}}{P_t}F(e^{z_t},k_t,n_t)+\frac{b_{t+1}}{R_t}-b_t-\frac{W_t}{P_t}n_t\right. \\
&\quad \left.\left.-\frac{k_{t+1}}{\psi_{t+1}}-\left(\frac{D_t}{P_t}+\kappa\left(\frac{D_t}{D_{t-1}}-1\right)^2\frac{D_t}{P_t}\right)\right)\right)
\end{aligned}
$$

where $m_{t,t+1}$ is the stochastic discount factor, μ_t and λ_t are Lagrange multipliers.

The first order conditions are (8), (9) and derivatives with respect to $n_t, k_{t+1}, b_{t+1}, D_t$:

$$0 = (\lambda_t-\mu_t)\,X_t F_n(e^{z_t},k_t,n_t)-\lambda_t w_t \tag{10}$$

$$0 = \mu_t\Xi e^{\xi_t}-\lambda_t+\mathbb{E}_t m_{t+1}\left(\left(\lambda_{t+1}-\mu_{t+1}\right)X_{t+1}F_k(e^{z_{t+1}},k_{t+1},n_{t+1})+\lambda_{t+1}\left(1-\delta\right)\right) \tag{11}$$

$$0 = \frac{\lambda_t}{R_t}-\mu_t\Xi e^{\xi_t}\frac{1}{1+r_t}-\mathbb{E}_t m_{t+1}\lambda_{t+1} \tag{12}$$

$$
\begin{aligned}
0 &= 1+\mathbb{E}_t m_{t,t+1}\lambda_{t+1}2\kappa\left(\frac{D_{t+1}}{D_t}-1\right)\frac{D_{t+1}^2}{D_t^2}\frac{1}{\Pi_{t+1}} \\
&\quad -\lambda_t\left(1+2\kappa\left(\frac{D_t}{D_{t-1}}-1\right)\frac{D_t}{D_{t-1}}+\kappa\left(\frac{D_t}{D_{t-1}}-1\right)^2\right)
\end{aligned} \tag{13}
$$

where $X_t=\frac{P_{mt}}{P_t}$, $w_t=\frac{W_t}{P_t}$ and $\Pi_t=\frac{P_t}{P_{t-1}}$ is gross inflation, F_n and F_k are derivatives of $F(e^{z_t},k_t,n_t)$ respect to n and k.

2.2.2 Retailers

We introduce nominal rigidities *a la* Rotemberg (1983). Each final good producer i buys goods at price P_{mt}, and repackages them, producing the final good, which may also be costlessly transformed into capital. It sets its optimal price p_t^i and produces quantity $y_t\,(i)$. The firm faces a familiar demand for its good

$$y_t\,(i)=\left(\frac{p_t^i}{P_t}\right)^{-\varepsilon}Y_t.$$

Here the elasticity of substitution between any pair of goods is given by $\varepsilon>1$. Firm chooses price p_t^i which solves the following optimization problem:

$$
\begin{aligned}
&\mathbb{E}_t\sum_{\tau=t}^{\infty}m_{t,\tau}\left(\frac{p_t^i}{P_t}y_t^i-\frac{P_{mt}}{P_t}y_t^i-\frac{\omega}{2}\left(\frac{p_t^i}{p_{t-1}^i}-1\right)^2Y_t\right) \\
=\ &\mathbb{E}_t\sum_{\tau=t}^{\infty}m_{t,\tau}\left(\left(\frac{p_t^i}{P_t}\right)^{1-\varepsilon}Y_t-\frac{P_{mt}}{P_t}\left(\frac{p_t^i}{P_t}\right)^{-\varepsilon}Y_t-\frac{\omega}{2}\left(\frac{p_t^i}{p_{t-1}^i}-1\right)^2Y_t\right)
\end{aligned}
$$

where $\frac{\omega}{2}\left(\frac{p_t^i}{p_{t-1}^i}-1\right)^2 Y_t$ represents the cost of adjusting prices.

The aggregated first order condition is:

$$\omega\left(\Pi_t-1\right)\Pi_t=(1-\varepsilon)+\varepsilon X_t+\omega m_{t,t+1}\left(\Pi_{t+1}-1\right)\frac{Y_{t+1}}{Y_t}\Pi_{t+1} \tag{14}$$

Finally, the profit Φ_t in the household budget constraint can be found from the aggregation of firms' budget constraints:

$$P_t\Phi_t=P_tY_t-P_{mt}Y_t-\frac{\omega}{2}\left(\Pi_t-1\right)^2 Y_tP_t \tag{15}$$

2.3 Private Sector Equilibrium and Market Clearing

Private Sector Equilibrium is determined by the system (2)-(5), (8), (9), (10)-(13), (14). We substitute out equations which determine share prices, and arrive to the following system

$$0 = \left(\lambda_t-\mu_t\right)X_tF_n(e^{z_t},k_t,n_t)-\lambda_t w_t \tag{16}$$

$$0 = -\lambda_t+\mu_t\Xi e^{\xi_t}+\beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\left(\left(\lambda_{t+1}-\mu_{t+1}\right)X_{t+1}\theta\frac{Y_{t+1}}{k_{t+1}}+\lambda_{t+1}\left(1-\delta\right)\right) \tag{17}$$

$$0 = \frac{\lambda_t}{R_t}-\mu_t\Xi e^{\xi_t}\frac{\Pi_{t+1}}{1+i_t}-\beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\lambda_{t+1} \tag{18}$$

$$\begin{aligned} 0 &= 1+2\kappa\beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\frac{\lambda_{t+1}}{\Pi_{t+1}}\left(\frac{d_{t+1}}{d_t}\Pi_{t+1}-1\right)\left(\frac{d_{t+1}}{d_t}\Pi_{t+1}\right)^2 \\ &\quad -\lambda_t\left(1+2\kappa\left(\frac{d_t}{d_{t-1}}\Pi_t-1\right)\frac{d_t}{d_{t-1}}\Pi_t+\kappa\left(\frac{d_t}{d_{t-1}}\Pi_t-1\right)^2\right) \end{aligned} \tag{19}$$

$$X_tY_t = \Xi e^{\xi_t}\left(k_{t+1}-b_{t+1}\frac{\Pi_{t+1}}{1+i_t}\right) \tag{20}$$

$$\frac{b_{t+1}}{R_t} = b_t+w_tn_t+k_{t+1}-\left(1-\delta\right)k_t+d_t\left(1+\kappa\left(\frac{d_t}{d_{t-1}}\Pi_t-1\right)^2\right)-X_tY_t \tag{21}$$

$$\omega\left(\Pi_t-1\right)\Pi_t = (1-\varepsilon)+\varepsilon X_t+\omega\beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\left(\Pi_{t+1}-1\right)\frac{Y_{t+1}}{Y_t}\Pi_{t+1} \tag{22}$$

$$\frac{\Pi_{t+1}}{1+i_t} = \beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}} \tag{23}$$

$$\alpha n_t^{\psi} = w_tC_t^{-\sigma} \tag{24}$$

where $Y_t=Ae^{z_t}k_t^{\theta}n_t^{1-\theta}$, $R_t=1+r_t(1-\tau_t)$ and $1+r_t=\frac{\Pi_{t+1}}{1+i_t}$.

Finally, the resource constraint yields

$$Y_t=C_t+k_{t+1}-\left(1-\delta\right)k_t+\kappa\left(\frac{d_t}{d_{t-1}}\Pi_t-1\right)^2 d_t+\frac{\omega}{2}\left(\Pi_t-1\right)^2 Y_t \tag{25}$$

and system (16)-(25) is used to determine equilibrium variables for $\lambda_t, \mu_t, X_t, C_t, k_t, \Pi_t, n_t, d_t, w_t, b_t$ given the policy instruments i_t and τ_t.

2.4 Linearization

We linearize the system around the efficient steady state, which is the flexile price equilibrium without credit shocks. Specifically, for every variable Z_t we define $\hat{z}_t = \log \frac{Z_t}{Z}$ where Z is the steady state level. We then define $\tilde{z}_t = \hat{z}_t - \hat{z}_t^n$ where $\hat{z}_t^n$ is the log-deviation from the efficient steady state. We arrive to the following system:

$$
\begin{aligned}
\tilde{w}_t &= \frac{\mu}{1-\mu}\left(\tilde{\lambda}_t - \tilde{\mu}_t\right) + \tilde{x}_t + \theta\tilde{k}_t - \theta\tilde{n}_t \\
\tilde{\lambda}_t &= \beta\left(X\theta\frac{Y}{k}\left(\tilde{\lambda}_{t+1} - \mu\tilde{\mu}_{t+1}\right) + X\theta\frac{Y}{k}(1-\mu)\left(\theta\tilde{k}_{t+1} + (1-\theta)\tilde{n}_{t+1} - \tilde{k}_{t+1} + \tilde{x}_{t+1}\right)\right. \\
&\quad \left. - (1-\mu\Xi)\sigma\tilde{c}_{t+1} + (1-\delta)\tilde{\lambda}_{t+1}\right) + \mu\Xi\left(\tilde{\mu}_t + \tilde{\xi}_t\right) + \beta(1-\mu\Xi)\sigma\tilde{c}_t \\
0 &= \beta\left(\sigma\tilde{c}_t - \sigma\tilde{c}_{t+1} + \tilde{\lambda}_{t+1}\right) - \frac{1}{R}\left(\hat{\lambda}_t - \frac{(1-\tau)}{\beta R}(\tilde{\imath}_t - \tilde{\pi}_{t+1}) + \frac{r\tau}{R}\tilde{\tau}_t\right) \\
&\quad + \frac{\Xi\mu}{1+r}\left(\hat{\mu}_t + \tilde{\xi}_t - \tilde{\imath}_t + \tilde{\pi}_{t+1}\right) \\
\tilde{\lambda}_t &= 2\kappa\beta\left(\tilde{\pi}_{t+1} + \tilde{d}_{t+1} - \tilde{d}_t\right) - 2\kappa\left(\tilde{\pi}_t + \tilde{d}_t - \tilde{d}_{t-1}\right) \\
\theta\tilde{k}_t &= \frac{\Xi}{XY}\left(k\left(\tilde{k}_{t+1} + \tilde{\xi}_t\right) - \frac{b}{1+r}\left(\tilde{b}_{t+1} - \tilde{\imath}_t + \tilde{\pi}_{t+1} + \tilde{\xi}_t\right)\right) - (1-\theta)\tilde{n}_t - \tilde{x}_t \\
K\tilde{k}_{t+1} &= XY\left(\tilde{x}_t + \theta\tilde{k}_t + (1-\theta)\tilde{n}_t\right) + \frac{b}{R}\left(\tilde{b}_{t+1} - \frac{(1-\tau)}{\beta R}(\tilde{\imath}_t - \tilde{\pi}_{t+1}) + \frac{r\tau}{R}\tilde{\tau}_t\right) \\
&\quad - wn(\tilde{w}_t + \tilde{n}_t) - b\tilde{b}_t - d\tilde{d}_t + (1-\delta)K\tilde{k}_t \\
\tilde{\pi}_t &= \frac{\varepsilon X}{\omega}\tilde{x}_t + \beta\mathbb{E}_t\tilde{\pi}_{t+1} \\
\tilde{c}_t &= \tilde{c}_{t+1} - \frac{1}{\sigma}(\tilde{\imath}_t - \tilde{\pi}_{t+1}) \\
\tilde{w}_t &= \psi\tilde{n}_t + \sigma\tilde{c}_t \\
Y\theta\tilde{k}_t &= C\tilde{c}_t + K\tilde{k}_{t+1} - K(1-\delta)\tilde{k}_t - Y(1-\theta)\tilde{n}_t
\end{aligned}
$$

2.5 Policy

Monetary policy is assumed to behave optimally, minimizing an *ad hoc* welfare loss, given by the objective

$$L = \mathbb{E}_0 \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\tilde{\pi}_t^2 + \varkappa_y\tilde{y}_t^2 + \varkappa_\iota(\tilde{\imath}_t - \tilde{\imath}_{t-1})^2\right).$$

Thus we assume that the policymaker has objectives over inflation and output, $\tilde{y}_t = \theta\tilde{k}_t + (1-\theta)\,\tilde{n}_t$, as well as over interest rate smoothing. This policy objective has significan empirical support, one recent discussion can be found in Chen, Kirsanova, and Leith (2013).[34]

Note that the interest rate may be constrained by the ZLB. We compute numerically the implications of such a restriction using the approach developed in Laseen and Svensson (2011) and extended to the case of discretion in Chen et al. (2013).

3 Calibration

The model is calibrated to a quarterly frequency. We fix $\beta = 0.9825$. The capital depreciation rate is set to $\delta = 0.025$. The capital ratio in production function is set to $\theta = 0.36$, and the mean value of A is normalized to 1. The tax wedge which corresponds to the advantage of debt over equity is determined to be $\tau = 0.35$, and the dividend adjustment cost parameter set to $\kappa = 0.146$ as in Jermann and Quadrini (2012).

We calibrate the steady state debt to output ratio to match the data. The quarterly ratio of debt to output for the non-financial business sector is 3.25 over the sample period 1984:I-2010:II, see the top panel in Figure 2. In order to match that, we set the steady state value of the financial variable, Ξ , to 0.1634.[5]

Parameters of the household utility function are determined as follows. The calibration of the Frisch intertemporal elasticity of substitution in labor supply, ψ, is assumed to be equal to 1 and the risk aversion parameter is: $\sigma = 2$.The relative weight on the disutility of labour, $\alpha = 1.8834$, is chosen so as to set steady state hours worked equal to 0.3.

We calibrate the measure of price stickiness, $\omega = 80$, in a way that corresponds to a probability of firms changing prices every 3 quarters (in a corresponding Calvo model). The elasticity of substitution between any pair of goods ε is equal to 11 in steady state which gives a 10% mark up.

Parameters of the policy objective function are chosen to be $\varkappa_y = 0.5$ and $\varkappa_\iota = 0.6$, see Chen, Kirsanova, and Leith (2013).[6]

It remains to calibrate the shock and the initial states to simulate the scenarios of interest. The second panel in Figure 2 plots the historical data of corporate debt to output ratio (quarterly).

[3] See also estimation of policy objectives in e.g. Dennis (2006), Ilbas (2010) and Givens (2012).

[4] We can derive microfounded objectives for some other classes of policies using approach developed by Schmitt-Grohe and Uribe (2004). The comparison of those policies for different specifications of objectives allows demonstrates great robustness of results to specifications of policy oobjectives.

[5] Data sources: NIPA and FoF tables. The calculations follow Jermann and Quadrini (2012).

[6] The results are very robust to wide range of parameters $\varkappa_y$ and $\varkappa_\iota$ between zero and one.

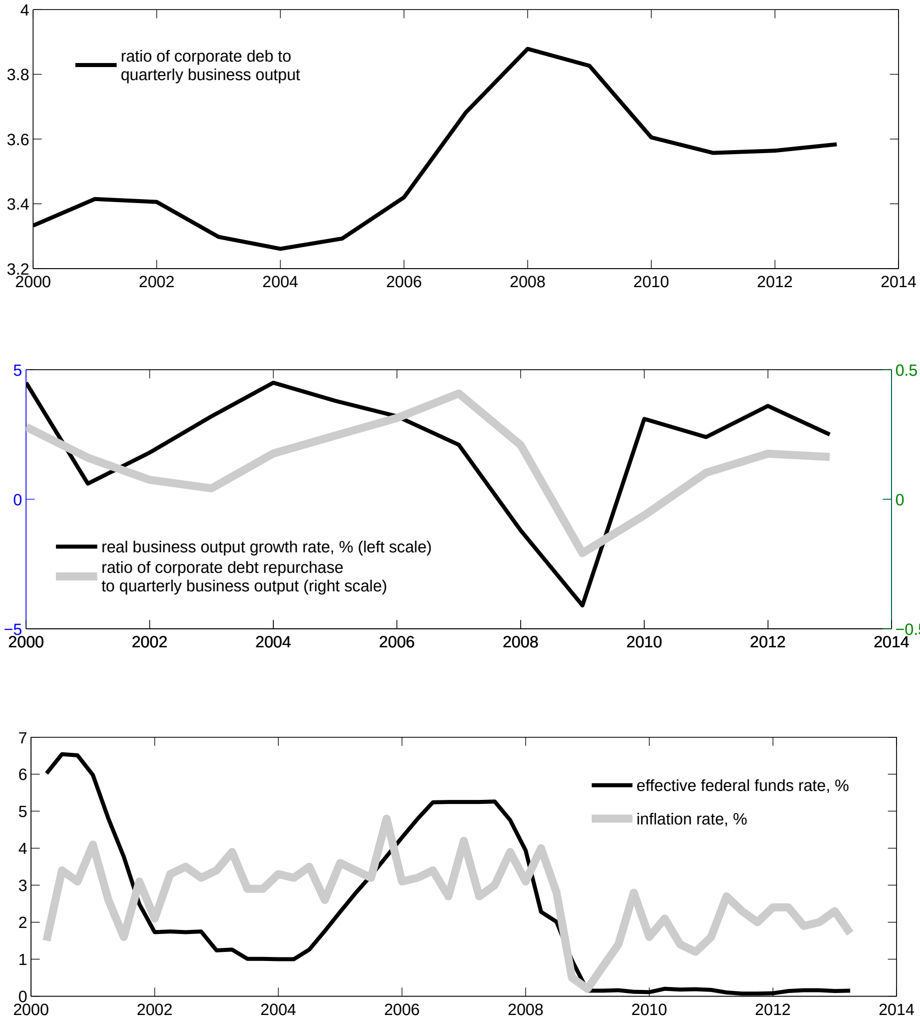
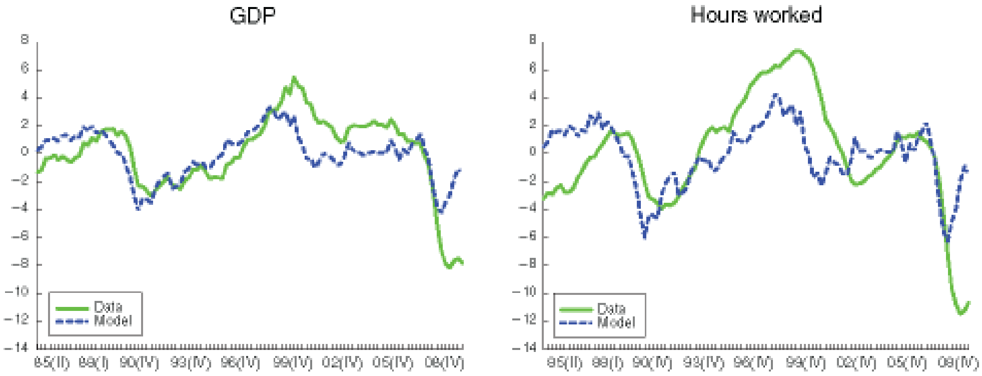


Figure 2: Historical data in the US.



Source: Figure 4 in Jermann and Quadrini (2012)

Figure 3: Response of Jermann and Quadrini (2012) model to Financial Shocks

The average value of this ratio during 1984-2009 is 3.25. The peak of 3.87 in 2008 was somewhat above the average value, and the consequent reduction to 3.55 in 2011 constitutes a reduction of about 10% relative to its peak. We use these numbers as a guide to our simulations.

Note that the model suggests the following steady state relationship

$$\frac{b}{Y}=\frac{\theta\left(\varepsilon-1\right)}{\varepsilon\left(2-\frac{1}{\beta R}-\beta\left(1-\delta\right)\right)}-\left(\frac{\theta\left(\varepsilon-1\right)\left(\frac{1}{\beta R}-1\right)}{\varepsilon\left(2-\frac{1}{\beta R}-\beta\left(1-\delta\right)\right)}+\frac{\left(\varepsilon-1\right)}{\beta\varepsilon}\right)\frac{1}{\Xi}$$

so that a reduction in the debt to output ratio can be explained by a reduction in the recovered share of output, Ξ, as all other parameters are structural. Rough calculations indicate that a shock of about 10% is not unreasonable.

Based on this evidence, we consider an AR(1) credit shock $\hat{\xi}_t=\rho\hat{\xi}_{t-1}+\varepsilon_t$ with persistence $\rho=0.95$, and we examine the dynamic implications of a negative 10% innovation in ε_t.

4 Discussion

Figure 3 is taken from Jermann and Quadrini (2012) and it demonstrates the ability of an RBC-type model with financial frictions to explain the three episodes of recessions. As the authors note, it is evident that the model fails to account for the depth of the Great Recession.

In this paper we claim that deep recessions can still be generated by a very similar model. To support this claim we run two numerical experiments. In the first experiment we demonstrate how an initial state with 'excess' lending results in deeper recessions following a financial shock. In the

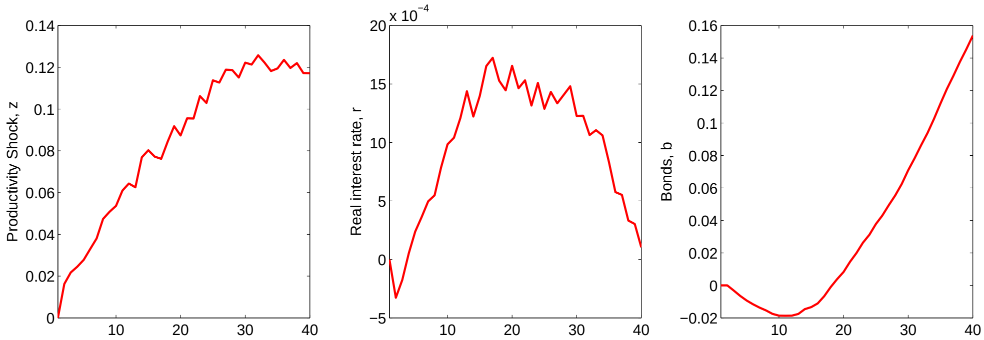


Figure 4: Ten years of positive productivity shock

second experiment we demonstrate how ZLB on interest rates may results in a large reduction in output.

4.1 The effect of over-lending

We start with the baseline scenario of a negative credit shock, impacting the economy which is initially at steady state. Such a shock reduces the proportion of output which banks will be able to recover in the case of default. Banks lend to firms at the beginning of the period, so that firms are able to pay wages. As the enforcement constraint is always binding, the difference between bonds and capital is covered by a loan. As the financial shock reduces the probability of recovery, the amount of bank lending falls. Firms which are not able to obtain funds up front have to deleverage or reduce production. Firms reduce their labour demand, produce less output and also pay lower wages, see Figure 5. The equilibrium prices of intermediate goods and final goods fall as a result of lower income and lower demand. Firms reduce the amount of borrowing. Both constraints for firms are tightened, as the values of the Lagrange multipliers indicate.

In response to lower inflation and output the central bank reduces the nominal interest rate sufficiently to guarantee a reduction in the real rate. Low real interest rates result in falling consumption profile over time.

Lower interest rates also make it easier for firms to pay out the existing stock of corporate debt, so it helps to reduce the debt stock quickly. In addition, output falls by less than wages, profits of firms fall and so dividends fall.

The reduction of output doubles if the financial shock requires greater deleveraging. We can illustrate this in the following scenario. Suppose the economy suffers from an one-off but

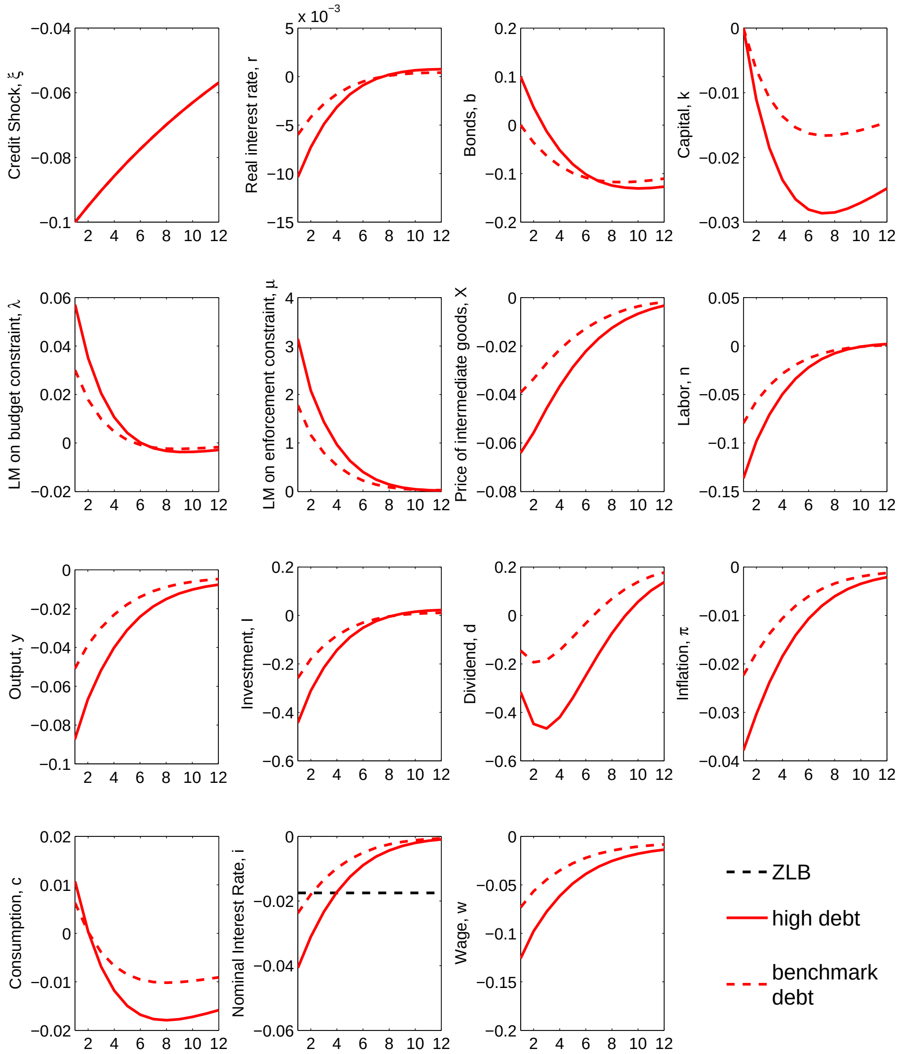


Figure 5: The effect of high corporate debt

permanent 'capital quality' shock where agents suddenly realise that the level of accumulated debt is above the current level of the underlying capital stock. The resulting deleveraging brings the level of debt down towards the steady state level of new, quality-adjusted capital. In figure 3 we assume that the over-lending is 10 percent, i.e., as a result of the permanent 'shock' the initial debt to output ratio is exactly 10 percent higher than its steady state level. Such a degree of over-lending may be plausible, see Section 3. Moreover, this excess can easily be achieved in this model, see Figure 4, which demonstrates that the stock of outstanding corporate debt accumulates more than the required 10 percent above the steady state level in the course of 10 years.[7] This high initial level of debt requires greater deleveraging than in the default scenario. Following the shock the reduction in debt reduces the capital stock and the lower capital stock requires less financing. These two effects reinforce each other and the sluggish adjustment of both stocks results in much overshooting of debt below the steady state in the course of adjustment. As a result, there is greater reduction in labour demand and wages, and much lower inflation. The interest rate falls by more, not only because of inflation, but because it also helps to stabilize debt – and thus all policy-relevant variables – faster. The reduction in output and labour doubles, which is consistent with the evidence presented in Figure 3

However, in the process of this adjustment, the optimal interest rate violates the ZLB. Therefore, next we discuss the implications of the ZLB on the dynamics of the economy.

4.2 The effect of ZLB

The effect of ZLB is illustrated in Figure 6. We compare two scenarios: the first one is the default case of 'unconstrained discretion', discussed in the section above, where the interest rate can move below the ZLB, and the second scenario, where such movements are prohibited. When the optimal interest rate is constrained by the ZLB, the recession is deeper. Inflation does not fall immediately but adjusts with a delay and converges back to the steady state, and is higher than in the previous scenario without the ZLB. When the financial shock occurs both constraints are tightened and the interest is reduced but not as much as the policymaker would like. The enforcement constraint is tightened much more than in the 'no ZLB' scenario, see Figure 6. As the interest rate on debt remains 'too high', greater deleveraging is required. Consumption drastically falls, and so does output and demand. Both bond and capital stocks fall quickly and by large

[7] Boissay, Collard, and Smets (2015) discuss that the 'average' development before the deep recession is a period of positive productivity shocks, their model suggest at least 10 years of AR(1) productivity shocks with $\rho_z = 0.9$ and standard error of 0.013. We stick to Jermann and Quadrini (2012) calibration of the model and we hit the economy with positive AR(1) productivity shocks with $\rho_z = 0.95$ and standard error of 0.008.

amounts. Wages and labor fall instantaneously. The absence of monetary intervention results in a deep recession.

As capital and labour fall, the production of intermediate goods fall too. The supply of intermediate goods is greatly reduced, much lower than the demand for final goods. As a result, the initial-periods price of intermediate goods rises, and so does inflation. However, expected inflation remains negative. Together with relatively high interest rate this results in only a small reduction of the real interest rate and so consumption falls only slow over time. As a result, we observe reduction in consumption and investment reflecting the reduction in output in the first few periods following the shock.

Once the initial-periods capital and debt de-accumulation is done, the constraint is weakened substantially. There is no further need to reduce bonds and capital quickly, and no need to restrain investment as much. Investment remains negative, but somewhat higher than in the first several periods. Intermediate goods firms increase output, and the price of intermediate goods falls to equalize demand and supply. Therefore, inflation falls as costs fall. Inflation is negative and it is optimal to keep the interest rate below the steady state level in order to stabilize the economy, but the interest rate does not need to be below the ZLB. We show that it is optimal to slightly raise the interest rate above the ZLB. The higher interest rate increases the real interest rate, but it still remains below the steady state level. As such, consumption continues falling to match the desired path for capital and supply. At some point the optimal deleveraging is achieved, the real rate rises back to the steady state level and above so that expected future consumption is higher than current consumption. Finally, consumption and demand start rising, prices and inflation rise and the economy converges back to the steady state. The adjustments is however slow and the growth rate of the economy, measured by $\tilde{y}_t - \tilde{y}_{t-1}$, is noticeably slower than in scenarios with no ZLB.

Note that this model requires convergence to the well-defined steady state. That means that the economy grows faster when recovering from a negative shock than if it were not hit by a shock at all. However, we compare the speed of recovery in different scenarios after a negative financial shock. We have shown that an initial stock of overlending results in higher output growth rate and the ZLB results in lower growth rate along the most of recovery path, excluding several initial periods. The effect of the ZLB dominates, see Figure 7. In the Figure the economy is hit by a financial shock when there is 10% overlending, and there is a restriction on interest rate movements below the ZLB. This simple superposition of two scenarios generates a very substantial reduction in output, and slow recovery. The slow recovery is the 'cost' of rapid deleveraging, due

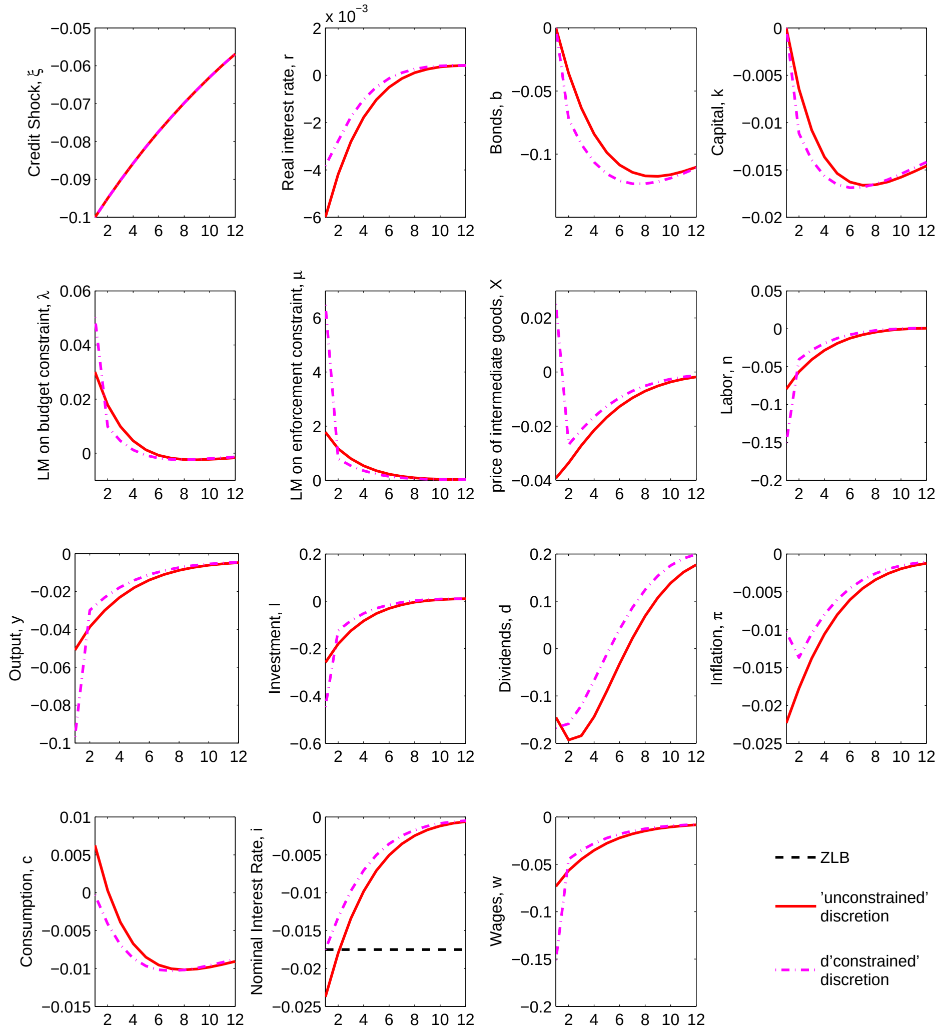


Figure 6: The effect of ZLB

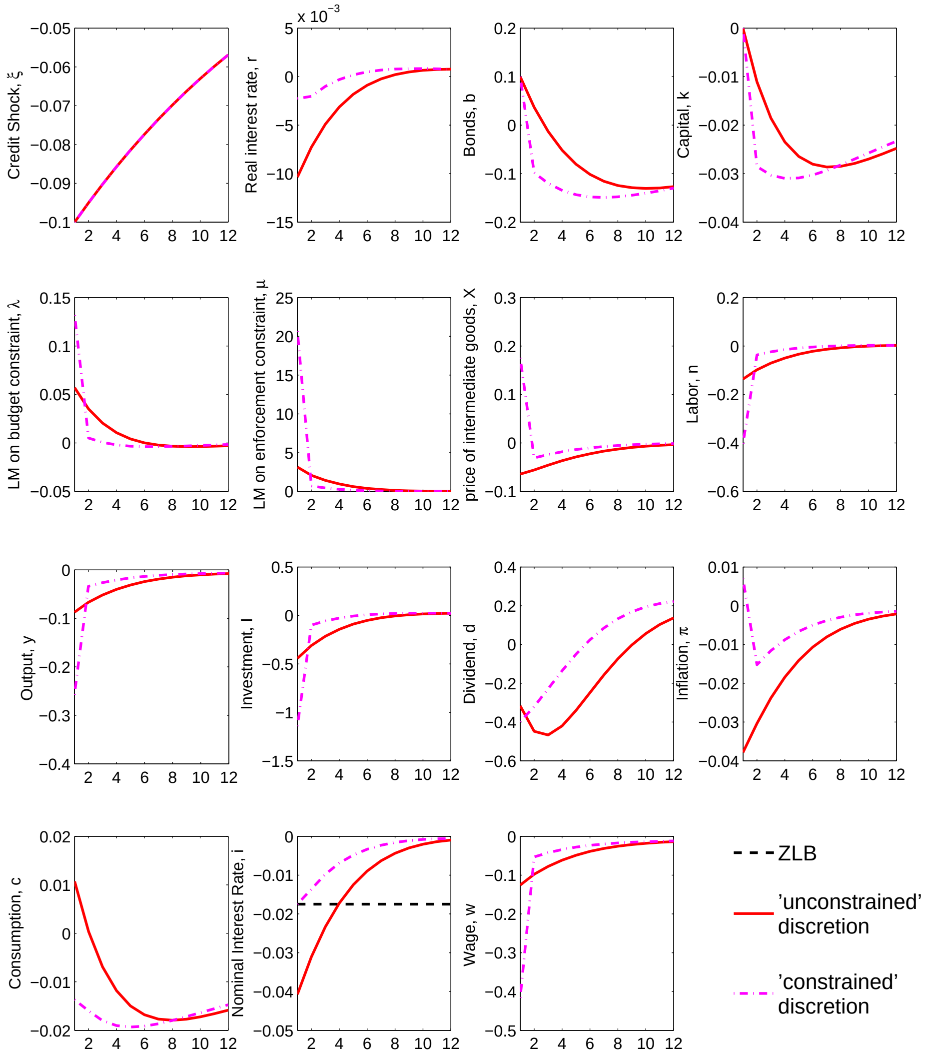


Figure 7: The effect of ZLB and over-lending.

to the presence of ZLB.

5 Conclusion

In this paper we demonstrate how a simple model with borrowing constrained firms is able to replicate two empirical facts, observed during the recent financial crisis. We demonstrate that, in response to a financial shock, the economy may generate a very deep recession following a moderate financial shock. We demonstrate that if the interest rate is bound by the zero lower bound then the dynamics of the economy also involve a 'stagnation' period, when the recovery is very slow.

References

Benigno, G. and L. Fornaro (2015). Stagnation Traps. mimeo.

Boissay, F., F. Collard, and F. Smets (2015). Booms and Banking Crises. *Journal of Political Economy*. Forthcoming.

Chen, X., T. Kirsanova, and C. Leith (2013). An Empirical Assessment of Optimal Monetary Policy Delegation in the Euro Area? Paper presented at the Bundesbank/CEPR conference 'Inflation Developments after the Great Recession', Eltville 6-7 December 2013.

Christiano, L., R. Motto, and M. Rostagno (2013). Risk Shocks. NBER Working Paper 18682.

Dennis, R. (2006). The Policy Preferences of the US Federal Reserve. *Journal of Applied Econometrics 21*, 55–77.

Givens, G. E. (2012). Estimating Central Bank Preferences under Commitment and Discretion. *Journal of Money, Credit and Banking 44*(6), 1033–1061.

Ilbas, P. (2010). Estimation of monetary policy preferences in a forward-looking model: a Bayesian approach. *International Journal of Central Banking 6*(3), 169–209.

Jermann, U. and V. Quadrini (2012). Macroeconomic Effects of Financial Shocks. *American Economic Review 102*, 238–271.

Laseen, S. and L. E. Svensson (2011). Anticipated alternative instrument-rate paths in policy simulations. *International Journal of Central Banking 7(3)*, 1–35.

Mumtaz, H. and F. Zanetti (2016). The Effect of Labor and Financial Frictions on Aggregate Fluctuations. *Macroeconomic Dynamics 20*, 313–341.

Nolan, C. and C. Thoenissen (2009). Financial shocks and the US business cycle. *Journal of Monetary Economics 56*, 596–604.

Rotemberg, J. J. (1983). Aggregate Consequences of Fixed Costs of Price Adjustment. *American Economic Review 73*, 433–36.

Schmitt-Grohe, S. and M. Uribe (2004). Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function. *Journal of Economic Dynamics and Control 28*, 755–775.

Schularick, M. and A. M. Taylor (2012). Credit Booms Gone Bust: Monetary Policy, Leverage Cycles and Financial Crises, 1870-2008. *American Economic Review 102*, 1029–61.

Taylor, J. (2008). The Financial Crisis and the Policy Responses: An Empirical Analysis of What Went Wrong. Mimeo, University of Stanford.

A ZLB under Discression: application of the Laseen-Svensson (2011) approach

Consider the standard LQ RE model. The policy objective is quadratic

$$L_t = \frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}g_s'Qg_s = \frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left(y_s'\mathcal{Q}z_s + 2y_s'\mathcal{P}u_s + u_s'\mathcal{R}u_s\right). \tag{26}$$

subject to the system of linear constarints

$$\begin{bmatrix} x_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} x_t \\ X_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}[u_t] + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}[\epsilon_t], \tag{27}$$

where $y_s = [x_s', X_s']'$. We assume A_{22} is invertible. We consider standard discretionary policy.

A.1 No binding constraints

Thi section presents the standard discretionary solution.

Suppose that the reaction of the private sector is given by a linear rule

$$X_t = -Nx_t. \tag{28}$$

Representation (28) can be rewritten in an equivalent form in terms of predetermined variables and controls (as did **?**). We one-period lead (28) and substitute for x_{t+1} from the first equation (27):

$$X_{t+1} = -Nx_{t+1} = -N(A_{11}x_t + A_{12}X_t + B_1u_t).$$

Combining this with the second equation in (27) we obtain:

$$\begin{aligned} X_t &= -(A_{22} + NA_{12})^{-1}[(A_{21} + NA_{11})y_t + (B_2 + NB_1)u_t] \\ &= -Jx_t - Ku_t, \end{aligned} \tag{29}$$

where

$$J = (A_{22} + NA_{12})^{-1}(A_{21} + NA_{11}), \tag{30}$$

$$K = (A_{22} + NA_{12})^{-1}(B_2 + NB_1). \tag{31}$$

The policymaker is maximising its objective function with respect to u_t, taking time-consistent reaction X_t as given and recognising dependence of X_t on policy u_s. We define Lagrangian with period term

$$H_s = \frac{1}{2}\beta^{s-t}(y_s'\mathcal{Q}y_s + 2y_s'\mathcal{P}u_s + u_s'\mathcal{R}u_s) + \lambda_{s+1}'(A_{11}x_s + A_{12}X_s + B_1u_s - y_{s+1}) + \mu_s'\left(X_s + Jx_s + Ku_s\right),$$

with λ_s and μ_s are Lagrange multipliers.

First order conditions can be written as

$$\begin{aligned} 0 &= \left(\mathcal{P}_1' - K'\mathcal{Q}_{12}'\right)x_s + \left(\mathcal{P}_2' - K'\mathcal{Q}_{22}\right)X_s + \left(\mathcal{R} - K'\mathcal{P}_2\right)u_s + \left(B_1' - K'A_{12}'\right)\beta\xi_{s+1}, \\ 0 &= \left(\mathcal{Q}_{11} - J'\mathcal{Q}_{12}'\right)x_s + \left(\mathcal{Q}_{12} - J'\mathcal{Q}_{22}\right)X_s + \left(\mathcal{P}_1 - J'\mathcal{P}_2\right)u_s - \lambda_s + \left(A_{11}' - J'A_{12}'\right)\beta\xi_{s+1}, \\ \eta_s &= -\mathcal{Q}_{12}'x_s - \mathcal{Q}_{22}X_s - \mathcal{P}_2u_s - A_{12}'\beta\xi_{s+1} \end{aligned}$$

and equations (29) and the first equation of (27). Here $\xi_s = \beta^{s-t}\lambda_s$, $\eta_s = \beta^{s-t}\mu_s$ and matrices $\mathcal{Q}$, $\mathcal{P}$ and $\mathcal{R}$ are partitioned conformally with $y_s = [x_s', X_s']'$ and u_s.

Substitue out X_s and η_s, and relabelling the matrices we arrive to the follwing linear system

$$\begin{aligned} 0 &= P^{*\prime}x_s + R^*u_s + B^{*\prime}\beta\xi_{s+1}, \\ 0 &= Q^*x_s + P^*u_s - \lambda_s + A^{*\prime}\beta\xi_{s+1}, \\ 0 &= A^*x_s + B^*u_s - x_{s+1} \end{aligned}$$

which can be written in a matrix form

$$\begin{bmatrix} I & 0 \\ 0 & \Phi_{22} \end{bmatrix} \begin{bmatrix} x_{t+1} \\ \tilde{u}_{t+1} \end{bmatrix} = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} \begin{bmatrix} x_t \\ \tilde{u}_t \end{bmatrix}, \tag{32}$$

where $\tilde{u}_t = \left[u_t', \xi_t'\right]'$and

$$\Phi_{22} = \begin{bmatrix} 0 & \beta B^{*\prime} \\ 0 & \beta A^{*\prime} \end{bmatrix}, \quad \Psi_{21} = \begin{bmatrix} -P^{*\prime} \\ -Q^{*} \end{bmatrix}, \quad \Psi_{22} = \begin{bmatrix} -R^{*} & 0 \\ -P^{*} & I \end{bmatrix},$$
$$\Psi_{11} = A^{*}, \quad \Psi_{12} = \begin{bmatrix} B^{*} & 0 \end{bmatrix}.$$

A solution to system (32) will necessarily have a linear form of

$$\tilde{u}_t = \begin{bmatrix} u_t \\ \lambda_t \end{bmatrix} = \begin{bmatrix} -F \\ S \end{bmatrix} y_t \tag{33}$$

It is straightforward to show that system matrices in (33) satisfy the following well-known Riccati equations describing solution to a discretionary problem

$$S = Q^{*} + \beta A^{*\prime} S A^{*} - \left(P^{*\prime} + \beta B^{*\prime} S A^{*}\right)\left(R^{*} + \beta B^{*\prime} S B^{*}\right)^{-1}\left(P^{*\prime} + \beta B^{*\prime} S A^{*}\right) \tag{34}$$
$$F = \left(R^{*} + \beta B^{*\prime} S B^{*}\right)^{-1}\left(P^{*\prime} + \beta B^{*\prime} S A^{*}\right) \tag{35}$$

Practically, the solution can be found with generalised Schur decomposition of (32).

A.2 Binding constraint on instrument

Following Laseen and Svensson (2011) we augment the original system by the vector of predetermined state variables $\boldsymbol{z}^t \equiv (z_{t,t}, z_{t+1,t} \ldots z_{t+T,t})'$ in order to account for the sequence of anticipated policy shocks. Vector $\boldsymbol{z}^t$ denotes a projection in period t of future realizations of shocks, $z_{t+\tau,t}, \tau = 0, 1, ..., T$. $z_{t,t}$ follows a moving average process

$$z_{t,t} = \eta_{t,t} + \sum_{s=1}^{T} \eta_{t,t-s},$$

where $\eta_{t,t-s}, s = 0, 1, ..T$, are zero-mean *i.i.d.* shocks. For $T = 0$, $z_{t,t} = \eta_{t,t}$. For $T > 0$, the stochastic shocks following a moving average process:

$$\begin{aligned} z_{t+\tau,t+1} &= z_{t+\tau,t} + \eta_{t+\tau,t+1}, \ \tau = 1, ..., T \\ z_{t+T+1,t+1} &= \eta_{t+T+1,t+1}. \end{aligned}$$

The above stochastic shocks process can be rewritten in the following matrix form

$$\boldsymbol{z}^{t+1} = \boldsymbol{A}_z \boldsymbol{z}^t + \boldsymbol{\eta}^{t+1},$$

where $\boldsymbol{\eta}^{t+1} \equiv \left(\eta_{t+1,t+1}, \eta_{t+2,t+1} \ldots \eta_{t+T+1,t+1}\right)'$ is a $(T+1)$ vector of *i.i.d.*shocks and A_z is $(n_1+1) \times (n_1+1)$ matrix

$$A_z = \begin{bmatrix} \mathbf{0}_{T\times 1} & \boldsymbol{I}_T \\ 0 & \mathbf{0}_{1\times T} \end{bmatrix}$$

We denote the vector of predetermined state variables $x_t^c = [z_t', x_t']'$, where superscript c stands for 'constrained', and vector z_t consists of anticipated shocks.

Matrices in equation (27) can be written as

$$A_{11}^c = \begin{bmatrix} A_z & 0 \\ 0 & A_{11} \end{bmatrix}, B_1^c = \begin{bmatrix} 0 \\ B_1 \end{bmatrix}$$

and matrices $\mathcal{Q}$, $\mathcal{P}$ and $\mathcal{R}$ can be redefined to account for additional state variables (we keep the same notation).

Finally, equation for policy instrument has to be augmented to account for reaction to shocks z_t. This is achieved by replacing the top left square submatrix of new Ψ_{21} with R^*. As before, the solution can be found by solving the augmented system (32) with Schur decomposition.