Public Debt and the Cyclicality of Fiscal Policy*

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Abstract

We present a theory linking the cyclicality of fiscal policy to inherited public debt. When debt is low, fiscal policy is countercyclical, in the sense that the government responds to reductions in output by cutting the tax rate. Above a threshold level of debt, however, optimal fiscal policy becomes procyclical. This creates the possibility of self-fulfilling crises, in which output is low because households expect high taxes, and the government sets high taxes because output is low. Our model suggests why highly indebted governments might implement procyclical fiscal policy during recessions, even without facing high sovereign risk premia.

Keywords: Public debt, fiscal policy cyclicality, coordination failures, Laffer curve.

JEL Classification: E62, H63.

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1 Introduction

Public debt to GDP ratios in advanced economies have been rising since the mid-1970s, and have recently reached levels not seen since just after World War II (Abbas et al., 2011). The recent financial crisis and the ensuing Great Recession exacerbated this trend through bailouts, stimulus packages, rising unemployment claims, and falling tax revenues. This has led to a heated debate over the pace of fiscal consolidation, with one side emphasizing the burden on economic growth imposed by high levels of public debt, and the other warning that pursuing austerity during a recession could be very costly or even self-defeating.

In this paper we present a new theory that provides a partial reconciliation of these two views. We show that there is a threshold level of debt above which the economy is vulnerable to self-fulfilling fiscal crises. However, the mechanism that makes such crises possible is that fiscal policy becomes procyclical, in the sense that the government's optimal response to a reduction in output is to raise the tax rate.[1] Thus, our model lends qualified support to both sides of the debate over fiscal consolidation: the proximate cause of the crisis is the government's desire to raise the tax rate in a recession, but the source of this desire is the high level of public debt.

In Calvo (1988) and related papers, investors' expectations of sovereign default cause them to charge a risk premium that makes default more likely.[2] Corsetti et al. (2013) argue that this sovereign risk channel provides a motivation for fiscal consolidation. However, even countries that did not face an increase in sovereign risk premia have pursued fiscal consolidation in the years since the onset of the Great Recession. Our focus in this paper is not on self-fulfilling expectations of sovereign default, but on another type of self-fulfilling macroeconomic crisis caused by high levels of public debt. Accordingly, we focus on cases in which investors charge the lowest risk premium compatible with the economy's fundamentals. Our analysis suggests why a highly indebted government might adopt procyclical fiscal policy during a recession, even without facing a high sovereign risk premium. Indeed, in our baseline model debt is risk free because the government is committed to repaying it, and debt sustainability is ensured by future fiscal capacity.[3]

Unlike a committed Ramsey policymaker, the government in our model takes households' current labour supply decisions and output as given when setting the contemporaneous tax rate and issuing new debt. Fiscal policy is therefore a function of current output, as well as of the inherited stock of public debt. This leads to a standard time inconsistency problem of the kind identified by Kydland and Prescott

[1]Since "procyclical" can be used to describe both variables that are positively correlated with output and policies that exacerbate the business cycle, there is potential for ambiguity when describing the cyclicality of tax rates. Throughout this paper, we use "procyclical" to refer to a negative correlation of tax rates and output, that is, tax policy that could exacerbate output fluctuations.

[2]See also Cooper (2015) and Lorenzoni and Werning (2013), for instance.

[3]We show in section 5.3 that our results are robust to allowing for future government default.

(1977). Whatever the level of public debt, the government always chooses a higher contemporaneous tax rate than a Ramsey policymaker would choose, because it does not internalize the distortionary effect on current output. However, the key insight of our analysis is that the government's inability to commit to a tax rate can have even more severe consequences, because when debt is high fiscal policy becomes procyclical, thereby inducing a coordination problem among households.

When the economy suffers a fall in output, there are two countervailing effects on the government's optimal choice of the contemporaneous tax rate. The first is that, for given tax rates, current consumption falls relative to future consumption. This provides the government with a *consumption-smoothing* motive to reduce the contemporaneous tax rate relative to the future tax rate. The second effect, which we call the *tax-base* effect, is that the contemporaneous tax base shrinks, meaning that the government must raise tax rates at some point in order to remain solvent in the long run.

When the inherited stock of public debt is low, the consumption-smoothing effect dominates. This means fiscal policy is countercyclical: the government's optimal response to a fall in output is to cut the tax rate and issue more debt, postponing the necessary tax collection to the future. A household that expected aggregate labour supply to be low would therefore anticipate a low tax rate, and choose a high level of labour supply itself. Under these conditions there is no scope for coordination failure, and our economy has a unique equilibrium.

However, when the inherited level of public debt is high, the tax-base effect dominates. Optimal fiscal policy then becomes procyclical, because deferring all fiscal consolidation (tax increases) when output is low would impose an unacceptable burden on future consumption. This unleashes the possibility of multiple equilibria. In the good equilibrium, labour supply is high because households anticipate a low tax rate, and the government optimally chooses a low tax rate because output is high. In bad equilibria, which we label *fiscal policy traps*, households restrict their labour supply in anticipation of a high tax rate, and the resulting low output induces the government to fulfil their pessimistic expectations by setting a high tax rate.[4] Welfare is lower in fiscal policy trap equilibria than in the high tax-base, low tax-rate equilibrium.

We conduct our analysis in a deliberately stylized environment, so as to clearly characterize the mechanism relating the level of debt to the cyclicality of fiscal policy and the possible occurrence of fiscal crises. In our baseline model we abstract away from household borrowing and saving decisions, and we assume that government spending is exogenous and that debt policy is subject to a hard no-default constraint. In section 5 we relax these simplifying assumptions in turn and show that our core results continue to hold.

[4] Our paper is therefore related to Albanesi et al. (2003) and other papers in the literature on monetary policy expectation traps, in which a monetary authority without commitment finds it optimal to validate private-sector expectations of high inflation.

The idea that high levels of public debt can pose a threat to the economy is most famously associated with Reinhart et al. (2012). In particular, they argue that countries with sovereign debt to GDP ratios above 90 percent have significantly lower rates of economic growth on average. The burden of distortionary taxation imposed by debt service could explain why high levels of debt might reduce growth, but not why there might be a discrete drop in growth above some threshold level of debt. Our model contributes a novel explanation for why there might be such a threshold effect, based on self-fulfilling beliefs about the stance of fiscal policy. In our model, a country with a level of public debt just above the threshold is exposed to the risk of a high-tax, low-output equilibrium. If this equilibrium is selected, the country's economic performance will be significantly worse than that of a similar country with a public debt level just below the threshold.

Schmitt-Grohé and Uribe (1997) introduce similar concerns about taxpayer coordination failure into a dynamic model with income and capital taxation. In their set-up, the government runs a balanced budget each period. In contrast, we study an environment where debt issuance is a choice variable and we show how its inherited stock determines whether the economy is fragile or not.

Cole and Kehoe (2000) consider a dynamic environment in which the government is prone to self-fulfilling debt rollover crises. They assume a constant tax rate, and allow the government to adjust its debt level by varying its expenditure. In their model, there is a source of domestically initiated crisis, via capital accumulation. By reducing saving, households reduce capital next period and bring the economy into the crisis zone where market shutdown is an option, hence making the initial belief that drove the reduction in saving self-fulfilling.

Ortigueira and Pereira (2016) study the implications of retroactive taxation in an infinite-horizon economy with capital accumulation. They show that the government's lack of intra-period commitment to the income tax rate leads to a continuum of Markov-perfect equilibria, each with different steady-state tax rates and levels of public debt. Unlike in the present paper, the scope for coordination failure arises through households' consumption decisions rather than through their labour supply decisions.

The rest of the paper is organized as follows. In section 2, we present the general framework of analysis. Section 3 sets out our main analytical results. Next, in section 4, we illustrate by way of an example the mechanism by which the cyclicality of fiscal policy depends on the inherited debt position and can lead to a self-fulfilling crisis. In section 5, we build on this example to demonstrate the robustness of our results to relaxing several of our baseline assumptions. Section 6 concludes.

2 A Model of Taxpayer Coordination Failure

In this section, we first outline the mechanism by which taxpayer coordination failure can arise in a static environment, and then present the general framework of our

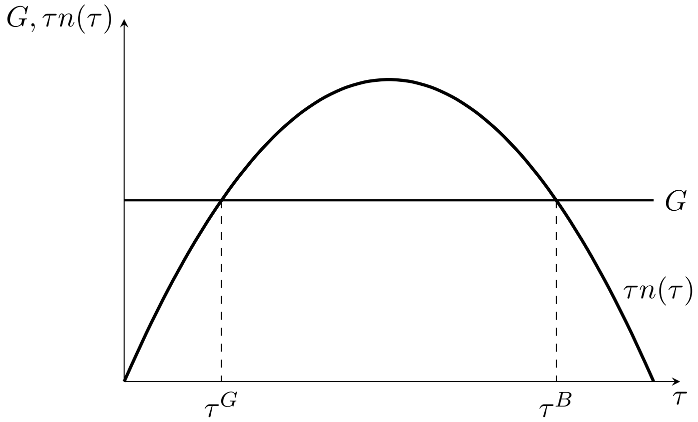


Figure 1: Equilibria on Either Side of the Laffer Curve

This figure outlines the coordination problem created by the government's inability to commit to a tax rate. For a given level of government expenditure, there are two levels of labour supply, associated with different tax rates, that satisfy the government budget constraint. The equilibrium with high labour supply and a low tax rate provides higher utility than the one with low labour supply and a high tax rate.

dynamic analysis.

2.1 One-Period Economy with a Balanced Budget

Consider a static environment, as proposed by Cooper (1999, 131–132), in which the government must finance a fixed level of expenditure G through a proportional ex post tax on labour income.[5] The economy is populated by a mass-one continuum of ex ante identical households, indexed by $i \in [0,1]$, who derive utility from consumption, c_i, and disutility from labour supply, n_i. Production is linear, so with a proportional tax rate τ, household i's consumption is $c_i = (1-\tau)n_i$. Since the pre-tax real wage is fixed at unity, households' optimal labour supply will be a function of the tax rate: $n_i = n(\tau)$. We assume that the substitution effect dominates the income effect in the utility function, so that labour supply is decreasing in the tax rate: $dn(\tau)/d\tau < 0$.

The government's budget balance constraint is $\tau n = G$, where $n = \int_i n_i di$ is aggregate labour supply. The government must pay for its fixed expenditure, so the tax rate will depend negatively on the tax base: $\tau = G/n$. This creates strategic complementarities among households: the higher is aggregate labour supply, the lower will be the tax rate, and so the higher is household i's optimal labour supply.

The equilibrium condition is $\tau n(\tau) = G$. As Figure 1 shows, there are two Pareto-ranked equilibria: an equilibrium with a low tax rate τ^G and high labour supply $n(\tau^G)$, and a Pareto-dominated equilibrium with a high tax rate τ^B and low labour supply $n(\tau^B)$. Given the presence of strategic uncertainty over the tax rate, households may coordinate on the inefficient Nash equilibrium, which lies on the downward-sloping part of the Laffer curve.

[5]We can think of G as pre-contracted expenses that do not enter into household utility directly.

If the government could credibly commit to a tax rate, this strategic uncertainty among households would disappear. However, in a static environment with fixed expenditure, the government has no choice but to respond to a revenue shortfall by raising the tax rate. The combination of an inability to commit to a tax rate and an absolute requirement to balance the budget leads to the possibility of coordination failure. The first of these assumptions is reasonable: sovereign governments cannot in fact commit to keep tax rates constant regardless of the state of the economy.[6] However, the balanced-budget view of fiscal policy is less realistic because governments routinely borrow to cover revenue shortfalls when output is lower than expected (and even balanced-budget constitutional amendments can be overturned).

The focus of our paper is therefore to analyse what happens when the government can issue new debt rather than increase taxes in the event of a revenue shortfall. Does this allow the government to eliminate the source of taxpayer coordination failure and steer the economy to the more efficient outcome with a low tax rate and high labour supply? Our answer will be that this depends on the inherited debt level. If the outstanding debt burden is sufficiently low, then the government's ability to adjust its debt position in the event of a revenue shortfall will ensure that there is a unique, low-tax equilibrium. However, if the inherited stock of debt is large enough then the government will optimally respond to lower output with higher taxes, unleashing the possibility of a fiscal policy trap.

2.2 Two-Period Economy with Taxes and Debt Issuance

We consider a two-period economy: $t = 1, 2$. The government inherits a level of debt B_1, owed to foreign investors. In period 1, households choose labour supply and produce accordingly. The government then sets its fiscal policy, choosing the tax rate on labour income τ_1 and the new debt B_2 to be issued to foreign investors. This debt is backed by future primary fiscal surpluses and is always repaid in period 2, so the government can borrow at the risk-free rate R between periods 1 and 2. We interpret the terminal period 2 as the long run.

The focus of our analysis is on the determinants of labour supply and fiscal policy in period 1. We next describe these choices.

2.2.1 Households' Preferences and Choices

There is a unit mass of households in the economy, indexed by $i \in [0, 1]$, who live over the two periods. To simplify the exposition, we assume for the moment that

[6] Income tax policy can change relatively quickly, particularly during crises, and even retroactive tax increases are not unheard of. On 6 November 2012, voters in California passed Proposition 30, which included increases in top marginal tax rates that applied retroactively to income earned since 1 January 2012. The Minnesota omnibus tax bill (HF 677), signed into law on 23 May 2013, included a new top income tax bracket and an increase in the alternative minimum tax rate, both of which applied retroactively to the beginning of 2013.

none of the households borrow or save between periods 1 and 2.[7] Since households are atomistic, they do not internalize the impact of their labour supply choices on the government's choices of tax rate and debt issuance. In period 1, household i forms a belief about the tax rate τ_1 and solves:

$$\max_{n_{1,i}} \; u(c_{1,i}) - g(n_{1,i}) \tag{1}$$

$$\text{subject to}$$

$$c_{1,i} = (1-\tau_1) z_1 f(n_{1,i}). \tag{2}$$

Consumption utility is increasing and concave: $u'(\cdot) > 0$ and $u''(\cdot) < 0$; and labour disutility is increasing and convex: $g'(\cdot) > 0$ and $g''(\cdot) < 0$.

The individual production function is $y_{1,i} = z_1 f(n_{1,i})$, where $z_1 > 0$ is an aggregate productivity parameter and $f(\cdot)$ is an increasing function that exhibits weakly decreasing returns to scale and is unbounded above: $f'(\cdot) > 0$, $f''(\cdot) \leq 0$ and $\lim_{n \to +\infty} f(\cdot) = +\infty$.

The labour supply decision $n(\tau_1)$ is implicitly defined by the following first-order condition:

$$(1-\tau_1) z_1 f'(n_{1,i}) u'\big((1-\tau_1) z_1 f(n_{1,i})\big) = g'(n_{1,i}). \tag{3}$$

We assume that the curvature of the utility function is such that substitution effects dominate income effects:

$$u'(c) + c u''(c) > 0 \quad \forall c \geq 0.$$

This ensures that labour supply is a decreasing function of the tax rate:

$$\frac{dn(\tau_1)}{d\tau_1} = \frac{z_1 f'(\cdot)\big(u'(\cdot) + c_1 u''(\cdot)\big)}{(1-\tau_1) z_1 f''(\cdot) u'(\cdot) + \big((1-\tau_1) z_1 f'(\cdot)\big)^2 u''(\cdot) - g''(\cdot)} < 0. \tag{4}$$

2.2.2 Government's Preferences and Choices

The government faces an intertemporal tax-smoothing problem. It has an inherited stock of debt owed to foreign investors, B_1, which it is committed to repaying. In each period, the government also has to finance an exogenous amount of expenses $G_t \geq 0$, which do not enter into household utility directly.[8] Given inherited debt B_1 and aggregate labour supply n_1, it optimally sets the tax rate τ_1 and issues new debt B_2 to risk-neutral foreign investors. For now, we assume the government is committed to repaying its debt obligations.[9] Future fiscal capacity determines the

[7] We demonstrate in section 5.1 below that our analysis goes through whenever there is a strictly positive fraction $\lambda > 0$ of hand-to-mouth households who can neither borrow nor save.

[8] In section 5.2, we endogenize short-run public expenditure G_1.

[9] This assumption is introduced to highlight the fact that the mechanism at play in our analysis, namely the link between inherited debt, the cyclicality of fiscal policy and the possibility of taxpayer coordination failure, is not driven by self-fulfilling increases in sovereign risk premia. In section 5.3,

maximum amount of debt $\bar{B}_2$ that can be issued in period 1.

The government's maximization problem is as follows:

$$\max_{\tau_1, B_2} \quad u(c_1) - g(n_1) + \beta V(B_2) \tag{5}$$

subject to

$$B_1 + G_1 \leq \tau_1 z_1 f(n_1) + \frac{B_2}{R} \tag{6}$$

$$B_2 \leq \bar{B}_2. \tag{7}$$

The function $V(\cdot)$ captures the continuation utility of the economy when in period 1 the government issues bonds with face value B_2 to be repaid in period 2. The government budget constraint (6) states that debt service and government expenditure in period 1 must be financed by proportional taxes on output and new debt issuance. Expression (7) states that, because of the long-run solvency requirement, the government also faces a borrowing limit $\bar{B}_2$.

The continuation utility function $V(\cdot)$ satisfies the following concavity assumptions:

$$V'(\cdot) < 0, \quad V''(\cdot) < 0. \tag{8}$$

In addition, we assume for now that

$$\lim_{B_2 \to \bar{B}_2} V'(\cdot) = -\infty, \tag{9}$$

which states that the marginal utility of a reduction in the future debt burden approaches infinity as the government approaches its debt limit. We verify below that this condition is satisfied for natural specifications of $V(\cdot)$ in which the cost of issuing additional debt in period 1 is higher taxes and lower consumption in period 2.

Since $V(\cdot)$ is decreasing in B_2, the government budget constraint (6) will be satisfied with equality. Substituting this into the government's objective function (5) and differentiating with respect to the short-run tax rate τ_1 yields the following first-order condition:

$$u'\big((1-\tau_1) z_1 f(n_1)\big) = -\beta R V'\big(R(B_1 + G_1 - \tau_1 z_1 f(n_1))\big). \tag{10}$$

Equation (10) implicitly defines the tax policy function $\tau(n_1, B_1)$. We demonstrate below that the optimal short-run tax rate is unambiguously increasing in the inherited debt level B_1, but that the sign of its derivative with respect to short-run labour supply n_1 is ambiguous. When $d\tau(\cdot)/dn_1 > 0$, we say that fiscal policy is *countercyclical*, meaning a drop in output induces the government to lower the tax rate; when $d\tau(\cdot)/dn_1 < 0$, we say that fiscal policy is *procyclical*, meaning a drop in output induces the government to raise the tax rate. We also show that the cyclicality of fiscal policy and the number of equilibria in this economy depend on the

we relax this assumption and show that our results still hold.

inherited level of debt.

2.2.3 Equilibrium Definition

The relevant choices of households and the government are both made in period 1. The government inherits an amount of debt B_1. Households form expectations about fiscal policy, supply labour and produce accordingly. Given its outstanding debt and the economy's tax base, the government sets fiscal policy to maximize the lifetime utility of the population.

The relevant state variables for the government's decisions are aggregate labour supply, n_1, and the inherited amount of debt B_1. Given (n_1, B_1), the government sets the tax rate τ_1 and issues new bonds B_2. We denote the policy functions $\tau(n_1, B_1)$ and $B(n_1, B_1)$. In the long run, i.e. in period 2, debt is fully repaid.

Accordingly, an equilibrium in this environment is defined as follows:

Definition. *A rational expectations equilibrium is a labour supply decision n_1, a tax rate τ_1 and debt issuance B_2 such that:*

- *Given outstanding debt B_1, households form rational expectations about fiscal policy, and supply labour n_1 to maximize their intratemporal utility* (1).

- *Given (n_1, B_1), the government sets the tax rate τ_1 and issues debt B_2 to maximize aggregate lifetime utility* (5) *subject to its budget constraint* (6) *and borrowing limit* (7).

Some comments are in order. First, we spell out the game and equilibrium definition as sequential actions, where households supply labour and then the government sets taxes. Similar economic interactions would prevail if moves were simultaneous. On the other hand, it is essential that the government does not move first. Indeed, if the government had a way to act as a Stackelberg leader and commit to its policy, it would naturally solve the coordination problem by choosing a tax rate on the left-hand side of the Laffer curve.[10]

Second, although the government takes labour supply as given and therefore does not face a Laffer curve, Nash equilibrium requires consistency between the tax rate the private sector expects and the tax rate the government chooses. All equilibria must therefore be on the labour income Laffer curve, but not all points on the Laffer curve will be equilibria.

Third, our analysis below will yield conditions under which the equilibrium is unique or not. If the policy functions of households and the fiscal authority exhibit substitutability, which we interpret as fiscal policy being *countercyclical*, then there

[10] For comparison, a government with the ability to commit within period 1 to a tax rate would solve maximization problem (5) subject to constraints (6) and (7) and the additional constraint $n_1 = n(\tau_1)$, implicitly defined by equation (3).

will be a unique equilibrium. If instead they exhibit complementarity, i.e. if fiscal policy is *procyclical*, then there may be multiple equilibria.[11]

The next section is dedicated to deriving conditions on the inherited level of debt that give rise to complementarities and create the possibility of fiscal policy traps.

3 Analysis

This section establishes the key result of the paper, namely that the level of debt is critical to the cyclicality of fiscal policy and can induce complementarities that give rise to fiscal policy traps. The argument is built on a geometric interpretation of the model in (n_1, τ_1) space.[12] Equilibria in this environment can be represented by intersections of the labour supply function $n(\tau_1)$ and the tax policy function $\tau(n_1, B_1)$. We show that there are three threshold levels of inherited debt, $B_1^* \leq \hat{B}_1 < \bar{B}_1$, such that when $B_1 < B_1^*$ a unique equilibrium is guaranteed, when $\hat{B}_1 < B_1 < \bar{B}_1$ there will be multiple equilibria, and when $B_1 > \bar{B}_1$ there will not be any equilibria. This result supports our key idea that the level of debt is critical in creating the potential for self-fulfilling fiscal crises.

The analysis is structured as follows. We begin by characterising the labour supply function, which is everywhere downward sloping and invariant to the inherited debt stock B_1. We then characterize the government's tax policy function, starting with the limits imposed by the government's budget constraint and borrowing limit. Unlike the labour supply function, the tax policy function's position and slope does depend on the inherited debt stock B_1.

We then show that when inherited debt is sufficiently low ($B_1 < B_1^*$), the tax policy function is upward sloping (countercyclical) at least until it crosses the labour supply function, thereby ensuring a unique equilibrium. Alternatively, when the inherited amount of debt is high enough ($B_1 > \hat{B}_1$), the tax policy function crosses the labour supply function at least twice. This situation gives rise to multiple equilibria.

We conclude this section with an economic interpretation of why the slope of the tax policy function is ambiguous and depends on the inherited debt stock B_1. We decompose the government's optimal response to a change in labour supply into two countervailing effects: a tax-base effect and a consumption-smoothing effect.

3.1 Properties of the Labour Supply Function

From (4) we know that labour supply is a monotonically decreasing function of the tax rate, so the labour supply function $n(\tau_1)$ is downward sloping in (n_1, τ_1) space. Optimal labour supply is zero when the tax rate is 100 percent, and $n(0) > 0$

[11]Formally, since $dn(\tau_1)/d\tau_1 < 0$, the policy functions exhibit complementarities if and only if $d\tau(n_1, B_1)/dn_1 \leq 0$.

[12]The geometric approach is very convenient, both for preserving generality of the results and for conveying the main intuitions underlying our analysis.

when the tax rate is zero. The labour supply function starts at $(0, 1)$ and cuts the horizontal axis at $(n(0), 0)$. It continues below the horizontal axis, because greater effort can be induced by negative tax rates (i.e. labour income subsidies).

Optimal labour supply depends only on the tax rate τ_1, so the labour supply function is unaffected by changes in the inherited debt stock B_1 or in the government's debt issuance B_2. Figure 2 summarizes the properties of $n(\tau_1)$, the reaction function of households.

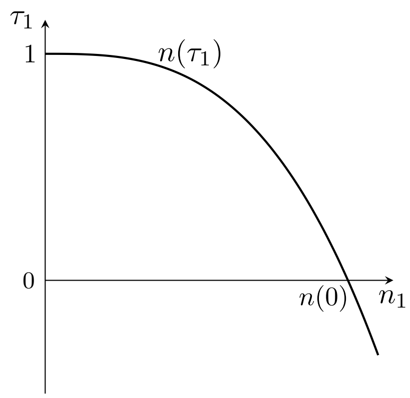


Figure 2: Labour Supply Function

3.2 Properties of the Tax Policy Function

The number of intersections (and hence the number of equilibria) therefore depends on the shape of the tax policy function, which, as we show in this section, *does* depend on the debt stock B_1 as well as on the quantity of labour supplied, n_1. We show that changes in B_1 both shift the tax policy function and alter its slope, thereby affecting the number of equilibria.

3.2.1 Constraints on the Government's Choice of Tax Rate

Let us first consider the constraints the government faces. The *borrowing limit* $\bar{B}_2$ in (7) is the highest level of debt that the government can feasibly repay in period 2 (often referred to in the literature as the "natural" borrowing limit). This of course depends on the government's fiscal capacity in period 2. Let the *maximum rollover threshold* debt level,

$$\hat{B}_1 = \bar{B}_2/R - G_1, \tag{11}$$

be the inherited debt level at which the government is exactly solvent in period 2 if it collects zero revenue in period 1. For debt levels strictly above this threshold, the government cannot repay its debts in period 2 without collecting some tax revenue in period 1. For debt levels strictly below this threshold, on the other hand, the government can in fact afford to *subsidize* labour supply in period 1 by setting a negative income tax rate $\tau_1 < 0$ and still be solvent in period 2.[13]

For now, we only consider equilibria in which the government repays its debts in period 2. Accordingly, we define the *lower bound on short-run labour supply* $\underline{n}(B_1)$ as the level of short-run labour supply at or below which the government's fiscal policy is not well defined because repayment of the debt is not feasible. Formally, we have:

$$\underline{n}(B_1) = \begin{cases} f^{-1}\left(\frac{B_1 - \hat{B}_1}{z_1}\right) & \text{if } B_1 > \hat{B}_1, \\ 0 & \text{if } B_1 \leq \hat{B}_1. \end{cases}$$

Since it is the short-run tax rate that matters for labour supply decisions, it is convenient to rewrite the government's constraints in terms of this tax rate. We define the *minimum short-run tax rate* $\underline{\tau}(n_1, B_1)$ as the tax rate in period 1 that, given the inherited debt level B_1, the economy's tax base $y_1 = z_1 f(n_1)$ and the government's budget constraint (6), requires the government to issue debt up to its borrowing limit $\bar{B}_2$. The tax rate $\underline{\tau}(\cdot)$ is therefore the lowest tax rate in period 1 such that full repayment of the public debt is feasible in period 2. As the borrowing limit depends on the government's long-run fiscal capacity, so does the minimum short-run tax rate. Formally, using the government budget constraint, $\underline{\tau}(\cdot)$ is given by:

$$\underline{\tau}(n_1, B_1) = \frac{B_1 - \hat{B}_1}{z_1 f(n_1)}, \quad n_1 > 0, \, n_1 \geq \underline{n}(B_1).$$

Figure 3 illustrates the characterisation of the minimum short-run tax rate $\underline{\tau}(\cdot)$. As the inherited debt level B_1 increases, for a given labour supply n_1, the tax rate must rise to ensure long-run solvency, so the curve shifts up. If $B_1 > \hat{B}_1$, positive short-run tax revenue is needed to ensure long-run solvency, but the higher is the short-run labour supply n_1, the lower is the minimum tax rate. If $B_1 < \hat{B}_1$, the government can afford to set negative rates $\tau_1 < 0$ (i.e. to subsidize labour), but the higher is the short-run labour supply, the smaller this subsidy has to be. For $B_1 = \hat{B}_1$, no short-run revenue is needed to ensure long-run solvency, but the government cannot afford subsidies, either.

Of course, if the inherited level of debt B_1 is too high, the government will be unable to raise enough revenue to remain solvent, and there will be no equilibrium. Clearly, if the required revenue in period 1 exceeds that which would be raised at the peak of the Laffer curve, repayment will not be feasible. However, the maximum inherited debt level that can be sustained in equilibrium is less than this level. The government's lack of commitment reduces the amount of tax revenue it can raise in equilibrium.[14]

[13]Note that for large enough G_1, the threshold $\hat{B}_1$ is negative, meaning that the government must inherit net *claims* on foreign wealth in order to be able to afford not to collect any revenue in period 1.

[14]If households were to supply the amount of labour consistent with the peak of the Laffer curve, the government would optimally choose to raise the tax rate.

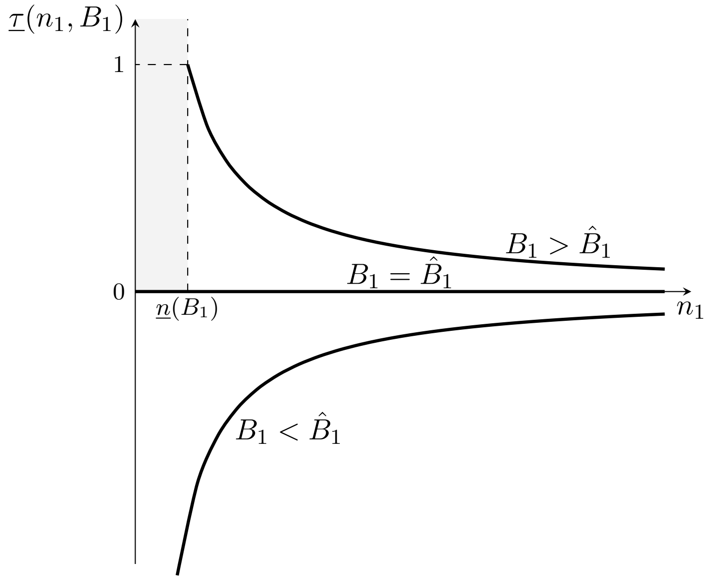


Figure 3: Minimum Short-Run Tax Rate $\underline{\tau}(n_1, B_1)$

This figure summarizes the constraints on the government's optimization problem. It displays the minimum short-run tax rate induced by inherited public debt, labour supply and future fiscal capacity.

Accordingly, we define $\bar{B}_1$ as the upper limit on the amount of inherited debt B_1 that the government can sustain in equilibrium. It is derived as follows. In equilibrium, households' expectations of the tax rate in period 1 must be correct, and labour supply must be optimal: $n_1 = n(\tau_1)$. Equilibrium also requires that the tax rate is set optimally given the level of output and the inherited debt level, that is, $\tau_1 = \tau(n_1, B_1)$. Equilibrium tax revenue in period 1 will therefore be given by the Laffer curve $\tau_1 z_1 f\big(n(\tau_1)\big)$. Therefore, the maximum inherited debt level $\bar{B}_1$ is such that, by raising the maximum tax revenue and issuing the maximum amount of debt $\bar{B}_2$, the government has just enough resources to finance its spending G_1 in period 1. It is the highest level of inherited debt B_1 that satisfies the following two equations:

$$R\left(\bar{B}_1 + G_1 - \tau(n_1, \bar{B}_1) z_1 f\left(n\left(\tau(n_1, \bar{B}_1)\right)\right)\right) = \bar{B}_2,$$
$$\tau_1 = \tau(n_1, \bar{B}_1).$$

3.2.2 Borrowing Limit Does Not Bind in Equilibrium

We have now defined all the ingredients necessary to prove that the borrowing limit (7) does not bind in equilibrium. This justifies restricting our attention to interior solutions of the government's maximization problem. This point is formalized in Lemma 1.

Lemma 1. *For all $B_1 < \bar{B}_1$ and for all $n_1 > \underline{n}(B_1)$, we have $B(n_1, B_1) < \bar{B}_2$ and $\tau(n_1, B_1) > \underline{\tau}(n_1, B_1)$. That is, the borrowing limit* (7) *does not bind, and* *the optimal short-run tax rate is strictly greater than what is required for long-run solvency.*

Proof. Suppose, on the way to a contradiction, that there exist $B_1 < \bar{B}_1$ and $n_1 > \underline{n}(B_1)$ such that the optimal debt issuance is $B(n_1, B_1) = \bar{B}_2$ and the optimal short-run tax rate is $\tau(n_1, B_1) = \underline{\tau}(n_1, B_1)$. From (9), we have $V'(\bar{B}_2) = -\infty$. Given $n_1 > \underline{n}(B_1)$ and our curvature assumptions on the utility function, for all $\tau_1 < 1$ we have $u'\big((1-\tau_1)z_1 f(n_1)\big) < +\infty$. The combination of $V'(\cdot) = -\infty$ and $u'(\cdot) < +\infty$ violates the government's first-order condition (10). Given $B_1 < \bar{B}_1$ and $n_1 > \underline{n}(B_1)$, it is feasible for the government to raise the short-run tax rate to $\tilde{\tau} \in \big(\underline{\tau}(n_1, B_1), 1\big)$ and reduce debt issuance to $\tilde{B}_2 < \bar{B}_2$. Relative to the candidate policy, this alternative policy produces an arbitrarily large long-run marginal benefit at a strictly finite short-run marginal cost, and so the candidate policy $B(n_1, B_1) = \bar{B}_2$ and $\tau(n_1, B_1) = \underline{\tau}(n_1, B_1)$ cannot be optimal. ■

This Lemma tells us that the optimal short-run tax rate $\tau(n_1, B_1)$ will be the interior solution implicitly defined by the first-order condition (10). Since the government's budget constraint (6) will be satisfied with equality, the debt issuance decision $B(n_1, B_1)$ will be given by:

$$B_2 = R\big(B_1 + G_1 - \tau(n_1, B_1) z_1 f(n_1)\big).$$

Since households' decisions depend only on the tax rate τ_1, we are interested mainly in the properties of the tax policy function $\tau(n_1, B_1)$.

3.2.3 Optimal Tax Rate Is Increasing in Inherited Debt

We first show that an increase in B_1 induces an increase in the tax rate τ_1 for any level of labour supply n_1.[15]

Lemma 2.

$$\frac{d\tau(n_1, B_1)}{dB_1} = \frac{\beta R^2 V''(\cdot)}{z_1 f(n_1)\big(u''(\cdot) + \beta R^2 V''(\cdot)\big)} > 0. \tag{12}$$

Proof. The expression is derived by totally differentiating the government's first-order condition (10) with respect to B_1 and rearranging. Standard assumptions on the curvature of the utility functions, $u''(\cdot) < 0$ and $V''(\cdot) < 0$, guarantee that the expression is positive. ■

The economic intuition behind this result is straightforward. An increase in the inherited debt stock B_1 means the government is poorer overall. In order to remain solvent, it must raise taxes in period 1, period 2, or both. Given that the marginal utility of consumption is decreasing in both periods, optimality requires

[15] In (n_1, τ_1) space, this feature is represented by an upward shift of the tax policy function as the inherited debt stock B_1 increases.

the government to spread the pain of an increase in B_1 over both periods, meaning the short-run tax rate τ_1 must rise.

3.2.4 Tax Policy Function Is Upward Sloping Whenever Negative

We are mainly interested in the slope of the tax policy function, that is, how the optimal tax rate responds to changes in labour supply. Taking the total derivative of (10) with respect to n_1, we get:

$$\frac{d\tau(n_1, B_1)}{dn_1} = \frac{f'(n_1)}{f(n_1)} \left(\frac{(1-\tau_1)u''(\cdot) - \tau_1 \beta R^2 V''(\cdot)}{u''(\cdot) + \beta R^2 V''(\cdot)} \right). \tag{13}$$

In general the sign of this expression is ambiguous, and depends on the inherited debt stock B_1.[16] However, the expression is unambiguously positive (and therefore the tax policy function is upward sloping, i.e. countercyclical) whenever the short-run tax rate τ_1 is negative:

Lemma 3.

$$\frac{d\tau(n_1, B_1)}{dn_1} > 0 \quad \forall \tau(n_1, B_1) < 0.$$

Proof. Totally differentiating the government's first-order condition (10) with respect to n_1 and rearranging yields:

$$\frac{d\tau(n_1, B_1)}{dn_1} = \frac{f'(n_1)}{f(n_1)} \left(\frac{u''(c_1)}{u''(c_1) + \beta R^2 V''(B_2)} - \tau(n_1, B_1) \right).$$

Since $\beta \geq 0$, $u''(\cdot) < 0$ and $V''(\cdot) < 0$, we have

$$\frac{u''(c_1)}{u''(c_1) + \beta R^2 V''(B_2)} \in [0, 1].$$

Since $f(\cdot) > 0$ and $f'(\cdot) > 0$, the whole expression must be positive whenever $\tau(n_1, B_1) < 0$. ■

3.3 Equilibria

Combining the analysis of households' labour supply function and the government's tax policy function, we are now ready to derive conditions under which the equilibrium of the economy is unique or not, i.e. conditions under which fiscal policy traps can arise.

3.3.1 Unique Equilibrium When Debt Is Low

Combining Lemmas 2 and 3, we can show that there will be a unique equilibrium whenever the inherited debt stock B_1 is sufficiently low.

[16] In section 3.4 below, we provide some economic analysis of this ambiguity by decomposing the government's response to a change in labour supply into a tax-base effect and a consumption-smoothing effect.

Proposition 1. *Let B_1^* be such that $\tau(n(0), B_1^*) = 0$. Then for all $B_1 < B_1^*$, there will be a unique equilibrium.*

Proof. From Lemma 2 we know that $\tau(n(0), B_1) < \tau(n(0), B_1^*) = 0$ for all $B_1 < B_1^*$, that is, the optimal tax rate will be negative whenever labour supply is $n(0)$ and inherited debt is less than the threshold value B_1^*. Then from Lemma 3 we know that whenever $B_1 < B_1^*$ the tax policy function will be negative valued and upward sloping for all values of labour supply $n_1 \leq n(0)$, and indeed will continue to slope upwards at least until it cuts the horizontal axis. Before it does so, it will cut the (downward-sloping) labour supply function exactly once. ■

3.3.2 Multiple Equilibria When Debt Is High

The government's budget constraint (6) means that if it inherits a sufficiently large stock of debt B_1, it will be forced to collect tax revenue in period 1 in order to stay within its borrowing limit (7). Whenever the inherited debt level B_1 is high enough that the government must collect taxes in period 1 (but not so high that repayment becomes infeasible), the economy will exhibit multiple equilibria.

Proposition 2. *Let $\hat{B}_1 = \bar{B}_2/R - G_1$, where $\bar{B}_2$ is the natural borrowing limit, and let $\bar{B}_1$ be the highest inherited debt level for which an equilibrium exists. Then for all $B_1 \in (\hat{B}_1, \bar{B}_1)$, the economy exhibits multiple equilibria.*

Proof. For all $B_1 \in (\hat{B}_1, \bar{B}_1)$, we have $\tau(\underline{n}(B_1), B_1) = \underline{\tau}(\underline{n}(B_1), B_1) = 1 > n^{-1}(\underline{n}(B_1))$. That is, when inherited debt is above the maximum rollover threshold $\hat{B}_1$ and short-run labour supply is at its minimum value $\underline{n}(\cdot)$, the government's optimal short-run tax rate is 100 percent, because this is the only feasible choice. We know that 100 percent is higher than the tax rate that would induce labour supply of $\underline{n}(\cdot) > 0$, because labour supply is decreasing in the tax rate and it is optimal not to work when the tax rate is 100 percent.

For all $B_1 \in (\hat{B}_1, \bar{B}_1)$, we have $\tau(n(0), B_1) > \underline{\tau}(n(0), B_1) > n^{-1}(n(0)) = 0$. This says that, when inherited debt is above the maximum rollover threshold $\hat{B}_1$ and labour supply is at the value that would optimally be chosen if the tax rate were zero, the optimal tax rate is in fact strictly positive.

These two pieces tell us that the optimal tax curve lies above the labour supply curve at two points: when n_1 is at the minimum level consistent with solvency, $\underline{n}(B_1)$, and when n_1 is at the point consistent with zero taxes, $n(0)$. There cannot be an equilibrium to the left of (i.e. with a lower labour supply than) $\underline{n}(B_1)$, because solvency would be violated whatever fiscal policy the government chose. We also know that there cannot be an equilibrium to the right of (i.e. with a higher labour supply than) $n(0)$, because the labour supply curve is negative valued after that point, and $\underline{\tau}(n_1, B_1)$ is strictly positive for all n_1 whenever $B_1 > \hat{B}_1$. So if an equilibrium exists, it must be between $\underline{n}(B_1)$ and $n(0)$. Apart from the special case of tangency (with $B_1 = \bar{B}_1$), if the optimal tax curve crosses below the labour supply

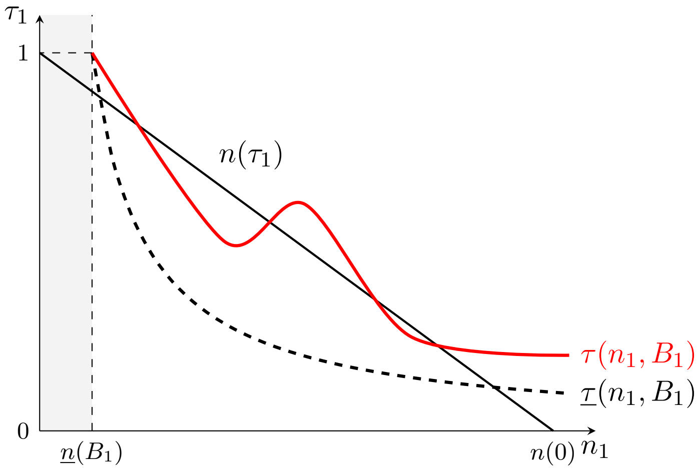


Figure 4: Existence of Fiscal Policy Traps

curve somewhere to the right of $\underline{n}(B_1)$, it must cross it again in order to be above it at $n(0)$. ■

3.3.3 Welfare Ordering of Equilibria

Proposition 3. *The equilibria in Proposition 2 with higher labour supply* n_1 *Pareto dominate those with lower labour supply.*

Proof. Since all households are ex ante identical and all equilibria are symmetric, the welfare ordering of equilibria depends on the utility of the representative household.

All equilibria must lie on the labour supply curve $n(\tau_1)$, which is downward sloping, so equilibria featuring higher short-run labour supply n_1 must also feature a lower short-run tax rate τ_1. The short-run tax rate τ_1 enters into the household budget constraint (2), and since labour supply cannot be negative, a reduction in τ_1 expands the household's choice set, meaning the household is (weakly) better off in period 1.

All that remains to be shown is that in equilibria with higher short-run labour supply, the representative household is also better off in period 2. Since $V'(B_2) < 0$, we need to show that the government's optimal debt issuance B_2 is lower in equilibria featuring higher short-run labour supply n_1. To see this, note that optimal fiscal policy must satisfy the first-order condition (10):

$$u'\big((1-\tau_1)z_1 f(n_1)\big) = -\beta V'(B_2).$$

Consider two equilibria, one "good" and one "bad", with $n_1^G > n_1^B$ and $\tau_1^G < \tau_1^B$. Now suppose (on the way to a contradiction) that the good equilibrium features higher debt issuance: $B_2^G > B_2^B$. Then from $V''(B_2) < 0$ we have $V'(B_2^G) < V'(B_2^B)$, meaning $-\beta V'(B_2^G) > -\beta V'(B_2^B)$. In order for the government's first-order condition to be satisfied in both equilibria, we would therefore need $u'\big((1-\tau_1^G)z_1 f(n_1^G)\big) >$

$u'\big((1-\tau_1^B)z_1 f(n_1^B)\big)$. However, given that output is increasing in labour supply and $u''(\cdot) < 0$, this would require $\tau_1^G > \tau_1^B$, which cannot be the case because by hypothesis the good equilibrium features a lower tax rate. ■

A lower tax rate in period 1 means households are wealthier in period 1. Since substitution effects dominate income effects, their response is to increase their labour supply, which increases the government's tax base. This induces a reduction in the government's optimal debt issuance, so households are wealthier in period 2 as well.

3.4 Tax-Base and Consumption-Smoothing Effects

We now provide some economic intuition for our main result that optimal fiscal policy is procyclical when the burden of inherited debt is large. We do so by providing a decomposition of the effect of a change in labour supply on the optimal tax rate. We identify two countervailing effects at play, which we label *tax-base* and *consumption-smoothing* effects.

Consider a reduction in period 1 labour supply n_1. Ceteris paribus, this reduces period 1 consumption relative to period 2 consumption, thereby providing the government with a consumption-smoothing motive to reduce the period 1 tax rate relative to the period 2 tax rate. On the other hand, when the period 1 tax rate is positive, a reduction in period 1 labour supply shrinks the overall tax base. In order for the government to remain solvent, therefore, the average tax rate across periods 1 and 2 must rise.

Similarly to how the effect of a price change on demand can be decomposed into a substitution and an income effect, we can decompose the effect of a change in labour supply on the optimal tax rate by rewriting the slope of the tax policy function (13) as follows:

$$\frac{d\tau(n_1, B_1)}{dn_1} = (1-\tau_1)\frac{f'(n_1)u''(\cdot)}{f(n_1)\big(u''(\cdot) + \beta R^2 V''(\cdot)\big)} - \tau_1 z_1 f'(n_1)\frac{d\tau(n_1, B_1)}{dB_1}.$$

The first term captures the consumption-smoothing effect, which is unambiguously positive (meaning a reduction in labour supply prompts a reduction in the tax rate i.e. that fiscal policy is countercyclical). The second term captures the tax-base effect, which operates through the impact of a change in labour supply on the total fiscal resources available to the government. It is therefore no mere coincidence that the size of the tax-base effect is linked to the effect of a change in the inherited debt stock on the optimal tax rate, $d\tau(n_1, B_1)/dB_1$.

The relative strength of the tax-base and consumption-smoothing effects will determine the cyclicality of the government's optimal fiscal policy. When the consumption-smoothing effect dominates, fiscal policy will be countercyclical and the tax policy function will be upward sloping. Noting that the size of both effects depends on the short-run tax rate τ_1, which itself depends positively on the inherited debt level B_1

as per Lemma 2, we can see that the cyclicality of fiscal policy will depend on the inherited debt level.

However, the effect of inherited debt on the cyclicality of fiscal policy is not guaranteed to be monotonic in all cases. This potential non-monotonicity means that there may not necessarily be a cut-off level of debt above which fiscal policy switches from being countercyclical to procyclical. Nevertheless, Proposition 1 guarantees that fiscal policy will always be countercyclical over the relevant range of labour supply when inherited debt is below the threshold B_1^*, ensuring a unique equilibrium. Similarly, Proposition 2 guarantees that there will be multiple equilibria (which requires that fiscal policy is at least locally procyclical) whenever inherited debt exceeds the maximum rollover threshold $\hat{B}_1$.

Note that the sign of the tax-base effect depends on whether the period 1 tax rate is positive or negative. This provides the intuition behind the result in Lemma 3 that the tax policy function is upward sloping whenever the tax rate is negative. With a negative tax rate (i.e. a labour subsidy), a reduction in labour supply actually *reduces* the fiscal burden on the government. This reverses the usual sign of the tax-base effect, meaning it reinforces rather than counteracts the consumption-smoothing effect. With both effects acting in the same countercyclical direction, the government's fiscal policy will be unambiguously countercyclical whenever the short-run tax rate is negative. We emphasize that a negative tax rate is a *sufficient* condition for fiscal policy to be countercyclical, but not a *necessary* condition. As the example in section 4 demonstrates, fiscal policy can be countercyclical for positive tax rates, too.

4 Example with an Explicit Policy Function

In this section we present an analytical example of the class of economies described previously, and clearly highlight the general result of section 3.

We adopt the following specification. In period 1, self-employed households convert labour effort into output using the following production function:

$$y_1 = z_1 n_1^{\alpha}, \quad \alpha > 0,$$

where α captures returns to scale. The government inherits a stock of debt B_1 owed to foreigners, chooses a proportional income tax rate τ_1 and issues bonds with a period-2 face value of B_2 (again to foreigners) at the risk-free interest rate R.

Period 2, the long run, is an endowment economy in which the government levies lump-sum taxes.[17] The per-capita endowment of output is y_2, and to economize on notation we normalize period 2 government expenditure, G_2, to zero.[18]

[17]Our results do not depend on this particular specification of period 2, which we adopt because it is a particularly tractable example in which conditions (8) and (9) on the continuation utility function are satisfied.

[18]This normalization is innocuous because with lump-sum taxes in period 2, an increase in G_2

The representative household's lifetime utility is given by:

$$U(c_1, n_1, c_2) = u(c_1) - g(n_1) + \beta u(c_2),$$

where instantaneous consumption utility is given by

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad \sigma \in (0,1)$$

in both periods, and the disutility from labour effort in period 1 is given by

$$g(n_1) = \frac{n_1^{\gamma}}{\gamma}, \quad \gamma > 0.$$

Substituting the budget constraint $c_1 = (1-\tau_1)y_1$ and the production function into the objective function and solving the household's first-order condition yields the following expression for optimal labour supply:

$$n(\tau_1) = \left(\alpha\left((1-\tau_1)z_1\right)^{1-\sigma}\right)^{\frac{1}{\gamma-\alpha(1-\sigma)}}.$$

The government faces the usual budget constraint (6). With lump-sum taxation in period 2, the natural borrowing limit $\bar{B}_2$ in (7) is given by the long-run endowment y_2, since long-run consumption $c_2 = y_2 - B_2$ cannot be negative. The government's continuation utility $V(B_2)$ from issuing an amount of debt B_2 is simply households' utility $u(y_2 - B_2)$ of consuming the amount left over after lump-sum taxes are levied on the endowment to pay off the debt. It follows immediately that conditions (8) and (9) on the continuation utility function are satisfied.[19]

The maximum rollover threshold level of inherited debt, above which the government must collect revenue in period 1 in order to remain solvent, is given by:

$$\hat{B}_1 = \bar{B}_2/R - G_1 = y_2/R - G_1. \tag{14}$$

4.1 Inherited Debt and the Cyclicality of Fiscal Policy

Solving the government's optimization problem yields the following tax policy function:

$$\tau(n_1, B_1) = \frac{(\beta R)^{1/\sigma}}{R + (\beta R)^{1/\sigma}} - \frac{R(\hat{B}_1 - B_1)}{\left(R + (\beta R)^{1/\sigma}\right)z_1 n_1^{\alpha}}. \tag{15}$$

This solution allows us to characterize precisely how the cyclicality of fiscal policy depends on the inherited level of debt.

Proposition 4. *The cyclicality of fiscal policy depends on the inherited debt level*

is equivalent to a decrease in y_2.

[19] Formally, $V'(B_2) = -u'(y_2 - B_2) = -(y_2 - B_2)^{-\sigma} < 0$, $V''(B_2) = u''(y_2 - B_2) = -\sigma(y_2 - B_2)^{-1-\sigma} < 0$ and $\lim_{B_2 \to \bar{B}_2} V'(B_2) = \lim_{B_2 \to y_2} u'(y_2 - B_2) = \lim_{c_2 \to 0} c_2^{-\sigma} = +\infty$.

B_1 as follows:

$$\frac{d\tau(n_1, B_1)}{dn_1} = \frac{R(\hat{B}_1 - B_1)\alpha}{\left(R + (\beta R)^{1/\sigma}\right) z_1 n_1^{1+\alpha}} \begin{cases} > 0 & \textit{(countercyclical) if } B_1 < \hat{B}_1 \\ = 0 & \textit{(acyclical) if } B_1 = \hat{B}_1 \\ < 0 & \textit{(procyclical) if } B_1 > \hat{B}_1. \end{cases}$$

Accordingly, the equilibrium of the economy is unique if and only if $B_1 < \hat{B}_1$, and fiscal policy traps may emerge for high levels of inherited debt $B_1 > \hat{B}_1$.

Proof. Differentiation of (15) and application of Propositions 1 and 2. ■

In the proof of Proposition 2, we saw the general result that when public debt is above the maximum rollover threshold level $\hat{B}_1$, the government's tax policy function must be at least *locally* procyclical, leading to multiple equilibria. Proposition 4 shows that there is a starker relationship between the level of public debt and the cyclicality of fiscal policy in this particular case. For levels of debt above $\hat{B}_1$, fiscal policy is procyclical for *all* values of labour supply.

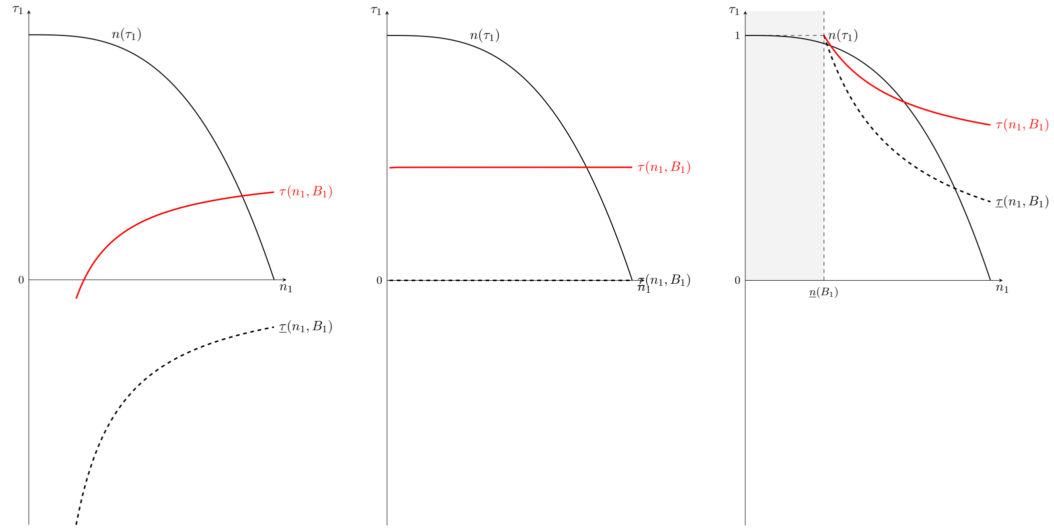
Since for any given value of inherited debt B_1 the government's tax policy function is monotonic, we can guarantee that there is a unique cutoff value of B_1, below which there will be a unique equilibrium and above which there will be two equilibria.[20] The three cases are illustrated in Figure 5. In panel (a), debt is below the threshold $\hat{B}_1$ and so the tax policy function is upward sloping for all values of labour supply. It therefore crosses the labour supply function just once, ensuring a unique equilibrium. Panel (b) shows that the equilibrium is also unique when inherited debt is equal to the threshold $\hat{B}_1$ and the tax policy function is horizontal. Whenever inherited debt exceeds this threshold, as in panel (c), the tax policy function is downward sloping for all values of labour supply and there are two equilibria.

Looking at equation (14) we can see that the threshold value of debt does not depend on contemporaneous parameters, such as productivity z_1, but only on future variables, such as fiscal capacity y_2. Although an increase in productivity reduces the optimal tax rate for a given level of labour supply, it cannot eliminate the possibility of fiscal policy traps. No matter how high is productivity, if debt is above the maximum rollover threshold then fiscal policy will be procyclical. This supports the idea that future fiscal capacity is essential in steering the economy away from fiscal policy traps.

5 Robustness

Thus far we have made a number of simplifying assumptions, namely that no households can borrow or save, that government spending is exogenous, and that the government is fully committed to repaying its debts in the long run. In this section

[20] This is true whenever the tax policy function is monotonic for all values of B_1, not just for the particular example we consider here.



(a) $B_1 < \hat{B}_1$ (countercyclical) (b) $B_1 = \hat{B}_1$ (acyclical) (c) $B_1 > \hat{B}_1$ (procyclical)

Figure 5: Inherited Debt and the Cyclicality of Fiscal Policy

we present extensions of our baseline model that relax each of these assumptions in turn, and show that our results generalize.

5.1 Limited Asset Market Participation

Suppose now that some fraction $\lambda \in (0, 1]$ of households cannot borrow or save, whereas the remainder $1 - \lambda$ can borrow and save on international financial markets.[21] We denote the former group, the hand-to-mouth households, with the letter H. In period 1 they supply n_1^H labour and consume $c_1^H = (1 - \tau_1) z_1 f\left(n_1^H\right)$. Households with access to international financial markets are denoted with the letter F. In period 1 they supply n_1^F labour, borrow an amount a from foreign investors at the risk-free rate R, and consume $c_1^F = (1 - \tau_1) z_1 f\left(n_1^F\right) + a$.[22] Total period 1 output is given by $Y_1 \equiv \lambda z_1 f\left(n_1^H\right) + (1 - \lambda) z_1 f\left(n_1^F\right)$.

As in section 4 above, period 2 is an endowment economy in which the government levies lump-sum taxes to repay its debt $B_2 = R(B_1 - \tau_1 Y_1)$. Type H households consume their endowment net of lump-sum taxes, $c_2^H = y_2 - R(B_1 - \tau_1 Y_1)$, and type F households consume their endowment net of lump-sum taxes and repayment of their own debt, $c_2^F = y_2 - R(B_1 - \tau_1 Y_1 + a)$. The benevolent government's objective function is given by a weighted average of the discounted lifetime utilities of the two types of households:

$$\lambda\left(u\left(c_1^H\right) - g\left(n_1^H\right)\right) + (1 - \lambda)\left(u\left(c_1^F\right) - g\left(n_1^F\right)\right) + \beta\left(\lambda u\left(c_2^H\right) + (1 - \lambda) u\left(c_2^F\right)\right).$$

The timing within period 1 is as before, with households of both types choosing

[21] Our baseline model is nested as the special case with $\lambda = 1$.

[22] Negative values of a denote lending by type F households to foreign creditors.

their labour supply simultaneously. Type H households behave in exactly the same way as households in our baseline model above: they choose their labour supply in period 1 to satisfy their intratemporal first-order condition:

$$(1-\tau_1)z_1 f'\left(n_1^H\right) u'\big((1-\tau_1)z_1 f\left(n_1^H\right)\big) = g'(n_1^H).^{23} \tag{16}$$

Type F households choose their labour supply to satisfy their own intratemporal first-order condition:

$$(1-\tau_1)z_1 f'\left(n_1^F\right) u'\big((1-\tau_1)z_1 f\left(n_1^F\right)+a\big) = g'(n_1^F). \tag{17}$$

They also choose their borrowing a to satisfy their intertemporal first-order condition:

$$u'\big((1-\tau_1)z_1 f\left(n_1^F\right)+a\big) = \beta R u'\big(y_2 - R(B_1 - \tau_1 Y_1 + a)\big). \tag{18}$$

The government, which takes household labour supply decisions n_1^H and n_1^F and type F household borrowing a as given, sets the period-1 tax rate τ_1 to satisfy its own intertemporal first-order condition:

$$z_1\big(\lambda f\left(n_1^H\right) u'\left(c_1^H\right)+(1-\lambda) f\left(n_1^F\right) u'\left(c_1^F\right)\big) = \beta R Y_1\big(\lambda u'\left(c_2^H\right)+(1-\lambda)u'\left(c_2^F\right)\big). \tag{19}$$

An equilibrium in the extended model with limited asset market participation will consist of a quadruple $\left(\tau_1, n_1^H, n_1^F, a\right)$ such that equations (16), (17), (18) and (19) are satisfied. A striking property common to all equilibria in this environment is that, no matter how small the fraction λ of hand-to-mouth households, the remaining $1-\lambda$ households behave as if they were themselves unable to borrow or save.

Proposition 5. *When a fraction $\lambda \in (0, 1]$ of households cannot borrow or save, in equilibrium the remaining $1-\lambda$ households do not borrow or save either ($a = 0$). The set of equilibria in which $a = 0$ is the same as the set of equilibria in the baseline model, where no households have the ability to borrow or save ($\lambda = 1$).*

Proof. We first demonstrate that equilibria exist in which type F households optimally choose not to borrow or save ($a = 0$), and that these are the same equilibria that exist in our baseline model with only type H (hand-to-mouth) households. Observe from the intratemporal first-order conditions (16) and (17) that type H and type F households will optimally supply the same amount of labour if and only if $a = 0$. With $a = 0$ and $n_1^F = n_1^H$, the government's intertemporal first-order condition (19) simplifies to that of type F households, (18). This means that, given their correct anticipation of the tax rate τ_1, type H households' labour supply n_1^H, and the labour supply of their fellow type F households $n_1^F = n_1^H$, type F households will in fact optimally choose $a = 0$ in order to satisfy (18). Equilibrium requires that

[23] A corner solution with $n_1^H = 0$ is ruled out whenever $\tau_1 < 100\%$ by standard assumptions on the consumption utility and labour disutility functions.

type F households' intertemporal first-order condition (18) and the government's intertemporal first-order condition (19) are both satisfied. With $a = 0$ and $n_1^F = n_1^H$, the two different types of agents are behaving identically. It follows from this that the set of equilibria will be the same as in our baseline model.

We now demonstrate that equilibria with $a = 0$ are the only equilibria that can exist. From (16) and (17) it follows that for $a \neq 0$ we must have $n_1^F \neq n_1^H$. Total differentiation of (17) with respect to n_1^F confirms that $dn_1^F/da < 0$, and so for $a > 0$ we have $n_1^F < n_1^H$ and for $a < 0$ we have $n_1^F > n_1^H$. Equilibrium requires that first-order conditions (16), (17), (18) and (19) are all satisfied. Substituting first-order conditions (16), (17), (18) into the government's intertemporal first-order condition (19) yields a contradiction for $a \neq 0$. ∎

The intuition for this result is as follows. The government's ability to choose the tax rate after labour supply and borrowing or saving decisions have already been made allows it to determine the allocation of households' consumption across the two periods. Whenever the marginal utility of hand-to-mouth households' consumption is not equalized across time, the government has an incentive to change the tax rate in order to better smooth hand-to-mouth households' consumption. In equilibrium, households with access to international financial markets perfectly anticipate the government's incentive to use the tax rate to smooth hand-to-mouth households' consumption, and adjust their own borrowing or saving accordingly. In this way, the government's optimal reaction to an arbitrarily small degree of financial market incompleteness completely crowds out private-sector consumption smoothing.

5.2 Endogenous Government Spending

Consider a government with a high inherited level of debt. Facing a low value of labour supply, would the government prefer to increase its tax rate or to reduce government expenditure?

To endogenize the choice of public expenditure, we assume that households derive instantaneous utility $v(G_1)$ from public expenditure G_1 by the government. The next Proposition shows that the key result of the baseline model still holds, even if the possibility of adjusting government expenditure provides the government with some "breathing room": the threshold level of debt is higher, but above this threshold, fiscal policy is procyclical and there is still the risk of fiscal policy traps. We adapt the analytical specification introduced in section 4 above and assume that $v(G_1) = \frac{G_1^{1-\sigma}}{1-\sigma}$. Formally, given (n_1, B_1), the government solves:

$$\max_{\tau_1, G_1, B_2} \; u\big((1-\tau_1)z_1 f(n_1)\big) - g(n_1) + v(G_1) + \beta u(y_2 - B_2)$$

subject to the usual government budget constraint (6) and borrowing constraint (7).

Note that with endogenous G_1, the maximum rollover threshold level of debt becomes:

$$\hat{B}_1 = \bar{B}_2/R = y_2/R,$$

because the government has the option of setting $G_1 = 0$.

The solution to the government's maximization problem gives the following tax policy function:

$$\tau(n_1, B_1) = \frac{R + (\beta R)^{1/\sigma}}{2R + (\beta R)^{1/\sigma}} - \frac{R(\hat{B}_1 - B_1)}{\left(2R + (\beta R)^{1/\sigma}\right) z_1 n_1^{\alpha}}. \tag{20}$$

Proposition 6. *The cyclicality of fiscal policy depends on the inherited debt level B_1 as follows:*

$$\frac{d\tau(n_1, B_1)}{dn_1} = \frac{R(\hat{B}_1 - B_1)\alpha}{\left(2R + (\beta R)^{1/\sigma}\right) z_1 n_1^{1+\alpha}} \begin{cases} > 0 & \textit{(countercyclical) if } B_1 < \hat{B}_1 \\ = 0 & \textit{(acyclical) if } B_1 = \hat{B}_1 \\ < 0 & \textit{(procyclical) if } B_1 > \hat{B}_1. \end{cases}$$

Accordingly, the equilibrium of the economy is unique if and only if $B_1 < \hat{B}_1$, and fiscal policy traps may emerge for high levels of inherited debt B_1.

Proof. Differentiation of (20) and application of Propositions 1 and 2. ■

Intuitively, when G_1 and c_1 are complements, the government will optimally choose to reduce G_1 in proportion with c_1 when labour supply n_1 decreases and the country is poorer. This allows the government to raise the tax rate by less than in the case with exogenous government expenditure. Nevertheless, once the government is above its short-run borrowing limit, it will have to raise the tax rate, preserving the risk of fiscal policy traps.

5.3 Allowing for Default on Newly Issued Debt

In our baseline model, we assume that the government is committed to repaying its debts in full in period 2. This commitment implies the limit $\bar{B}_2$ to the amount of debt the government can issue in period 1 (see equation (7)). This debt issuance limit, together with the tax-base effect that becomes stronger as the government approaches it, causes optimal fiscal policy to be procyclical when the inherited debt level is high.

We now relax the hard solvency constraint and allow the government to choose strategically in period 2 whether or not to repay its debts. We show that this does not eliminate the possibility of self-fulfilling fiscal crises. In fact, the lack of commitment to debt repayement, i.e. the prospect of default in period 2, tightens the borrowing constraint in period 1. This in turn decreases the threshold level of debt $\hat{B}_1$ above which the economy is susceptible to fiscal policy traps.

5.3.1 Stochastic Long-Run Output and Strategic Default

To develop this idea, we amend the model as follows. Let long-run output y_2 be stochastic, distributed uniformly on $\left[\underline{y}_2, \bar{y}_2\right]$. We denote by $F(\cdot)$ the cumulative distribution function of y_2. As in section 4 above, the government can use lump-sum taxes in period 2 to repay its debt B_2, in which case period 2 consumption will be $c_2 = y_2 - B_2$. If instead the government chooses to default in period 2, the economy suffers the loss of a proportion δ_2 of output, so period 2 consumption becomes $c_2 = (1 - \delta_2)y_2$.

The proportional output loss δ_2 determines the government's degree of commitment to repaying its debts in period 2. The extreme case of $\delta_2 = 1$ induces a strong commitment to repay and captures the hard solvency constraint assumed up to now. At the opposite extreme of $\delta_2 = 0$, default is costless, so the government would always default. Given outstanding bonds B_2, it is optimal for the government to repay its debts in period 2 whenever output y_2 satisfies:

$$y_2 - B_2 \geq (1 - \delta_2)y_2.$$

This relation gives the threshold $\hat{y}_2(B_2)$, realizations of y_2 below which the government defaults on its bonds B_2:

$$\hat{y}_2(B_2) = B_2/\delta_2. \tag{21}$$

Risk-neutral foreign investors anticipate the strategic default decision of the government. Accordingly, the price schedule $q(B_2)$ satisfies the following no-arbitrage condition:

$$q(B_2) = \frac{1 - F\left(\hat{y}_2(B_2)\right)}{R}, \tag{22}$$

where R is the risk-free interest rate. In this expression, the credit risk associated with the issuance of bonds B_2 is captured by $F\big(\hat{y}_2(B_2)\big)$, the probability that long-run output will be below the default threshold $\hat{y}_2(B_2)$. The possibility of strategic default can lead to indeterminacy in the price schedule (22), as studied in Calvo (1988) and Cooper (2015). As our focus is on the occurrence of fiscal policy traps rather than self-fulfilling increases in sovereign risk premia, whenever several prices satisfy the price schedule (22) we assume that investors select the "fundamental" outcome with the lowest risk premium. In this case, the price of debt $q(B_2)$ is decreasing in the amount of bonds B_2 issued, reflecting the increased probability of default.

5.3.2 Lack of Commitment to Repay Reduces Borrowing Limit

We are now ready to prove that despite its capacity to default on debt in period 2, the government may still be susceptible to fiscal policy traps. As in our baseline model, government borrowing between period 1 and 2 is constrained. This is turn induces a maximum level of inherited debt $\hat{B}_1$ such that the government can roll over its obligations without having to collect tax revenue in period 1. As in the baseline model, the economy is under the threat of fiscal policy traps whenever inherited debt B_1 is above this threshold. Interestingly, this threshold is increasing in the commitment parameter δ_2. In other words, the less committed a country is to repaying its debt, the lower is the debt threshold at which it becomes vulnerable to fiscal policy traps.

Proposition 7. *Whenever the government can default on its debt in period 2, there is a debt rollover threshold $\hat{B}_1$ above which the country is subject to fiscal policy traps. The threshold is increasing in the output loss parameter δ_2 (i.e. an increase in commitment increases debt capacity).*

Proof. We first demonstrate that there is a maximum amount of revenue that the government can raise in period 1, and that this amount is decreasing in δ_2. The revenue raised in period 1 by issuing B_2 bonds is $q(B_2)B_2$, where the price schedule $q(B_2)$ satisfies (22). Using the default threshold (21), resources from debt issuance are:

$$q(B_2)B_2 = \frac{1 - F(B_2/\delta_2)}{R} B_2.$$

The right-hand side is equal to 0 for $B_2 = 0$ and for $B_2 = \delta_2 \bar{y}_2$, and is strictly positive for any value of B_2 in between. Hence the right-hand side reaches a maximum for $B_2 = \bar{B}_2 \in (0, \delta_2 \bar{y}_2)$. The maximum period 1 revenue from debt issuance is therefore $q(\bar{B}_2)\bar{B}_2$. Since the price of debt is strictly increasing in δ_2, the revenue collected $q(B_2)B_2$ is also increasing in δ_2, and so is the maximum amount that can be collected.

As in (11) above, there is a maximum amount of debt $\hat{B}_1$ that can be rolled over without raising any tax revenue in period 1. This threshold is increasing in the maximum amount of revenue that can be raised by issuing new debt, and therefore in δ_2:

$$\hat{B}_1 = q(\bar{B}_2)\bar{B}_2 - G_1.$$

If the stock of inherited debt B_1 exceeds the maximum rollover threshold $\hat{B}_1$, then as in our baseline model the government will have to gather revenue in period 1. Indeed, to remain within this limit the government must set a short-run tax rate at least equal to

$$\underline{\tau}(n_1, B_1) = \frac{B_1 - \hat{B}_1}{z_1 f(n_1)}.$$

It follows that, as before, when $B_1 > \hat{B}_1$ the optimal tax policy function is at least locally procyclical, and Proposition 2 applies.

■

The intuition behind this result is as follows. As the proportional default cost δ_2 falls, investors know that the government will default in more states of the world in period 2 because it faces a lower penalty for doing so. This causes them to charge a higher risk premium, thereby reducing the amount of revenue the government can raise in period 1 by issuing new debt. Allowing the government to default on its debts in period 2 does not, therefore, eliminate the possibility of self-fulfilling fiscal crises.[24]

6 Conclusion

The recent rise (and subsequent fall) of sovereign debt spreads in the euro area periphery has prompted renewed interest in multiple equilibria and self-fulfilling crises. Yet it was not only countries facing increased borrowing costs that pursued contractionary fiscal policies during the Great Recession.

In this paper we have proposed a potential explanation for why governments might pursue procyclical fiscal policies despite not facing increased sovereign risk premia. When the inherited stock of public debt is sufficiently high, concerns about the burden of future taxes may overwhelm concerns about preserving consumption in the face of a decline in output, making even optimal fiscal policy procyclical. This procyclicality unleashes the possibility of a different kind of crisis, fuelled not by self-fulfilling fears of higher sovereign spreads but by self-fulfilling fears of a decline in output.

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[24]One could also consider the possibility of the government defaulting on its inherited debt. However, this creates the potential for another kind of self-fulfilling crisis, in which households restrict their labour supply in anticipation of the proportional output loss associated with default, thereby increasing the tax rate necessary to repay inherited debt and inducing the government to default.

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