Recurrent default on public debt or policy-optimal taxation[1]

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Abstract:
This paper investigates the syndrome of "*this time is different"* with respect to Reinhart and Rogoff's (2011) interpretation of their extensive, historical data on financial default, and with particular regard to public debt in a closed-economy. Recurrent and over-generous promises to credulous investors of an *ex ante*, policy-optimal return amounts to an extra policy instrument in boosting the demand for public debt. In a numerical simulation of a version of the Diamond (1965) model, we find that the incentive for the policy-maker to pursue this strategy is trivial if taxes can be set at a policy-optimal level, but possibly over-riding if they cannot. Thus, the main result lines up with the empirical conclusion of Reinhart *et a*s (2003) that "debt intolerant countries have weak fiscal structures". The subsidiary result of the model is that defaulting countries will also have higher shares of public expenditure. This is predicts *Wagner's Law* to the extent that fiscal structure is correlated with economic development.

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1. Introduction

In an extensive, historical study, Reinhart and Rogoff (2011) find that financial default has been a recurring feature for a number of countries. Lenders, evidently, place undue and repeated trust in the ability or willingness of borrowers to repay their debts, while knowledge of their world or of history might suggest otherwise. So persistent is this phenomenon that, as an ironical description of repeatedly credulous beliefs, these authors coin the term "*this time is different*".

Chronic gullibility or failure of memory does not square with usual assumptions of rationality. It probably belongs more to a world where finite-lived agents are locked into long-term decisions, suggesting that Diamond (1965) may be an appropriate model for considering this phenomenon.[3] Each generation may start out in life blithely making the same errors as its predecessors. In a similar vein, but with regard to professional investment – where others may bear the costs – Norberg (2012) makes the claims that

> "The typical career in the financial market lasts a quarter of a century, meaning that the average person will experience only one major crisis. Lessons are thus lost, and each generation repeats the same mistakes."

If households are disposed towards optimism, policy-makers may try to exploit this by making promises that lack substance. Even without any manipulation of this kind, a problem may still remain, if the identity of the policy-maker and, hence, the nature of the policy changes periodically, so that individuals do not remain in post long enough to be able to deliver their own promises – another situation to which the OLG model is surely better suited. Moreover, it is well established that policy that is optimal *ex ante* may not be so *ex post*, following Kydland and Prescott (1977) and Barro and Gordon (1983), where rational agents discount promises that are foreseen as being not optimal *ex post*. However, in order to consider a world with beliefs that *"this time is different"*, it is necessary to relax the assumption of full rationality.

We assume that the private sector can form *rational expectations* in the standard sense of knowing the parameter values of the model, but is easily outplayed by the authorities in any

[3] Convergence to *rational expectations* in learning models may only arise asymptotically, so that if lives are finite, approximate convergence may require that the revision of beliefs is at a high frequency.

policy game. They may credulously believe promises of an *ex ante* outcome that is never delivered.[4] Thus, they are not so much unknowing as easily persuaded against their better judgements. Although this assumption is foreign to orthodox economics, it is indispensable to explaining a systematic pattern of default.

Using it as a pivotal to the analysis, while assuming that policy-makers are not necessarily predisposed towards deceit, we then proceed to investigate the economic circumstances under which they are predisposed towards misleading the private sector? Or, in other words, what kind of country might come to be characterized by serial default? We answer these questions with recourse to an OLG model of economic growth, based on Diamond (1965) and Barro (1990). The latter contribution provides a role for infrastructure expenditure, as another item that must be publically finances, while its model specification of constant returns in a broader measure of the capital stock allows a focus on steady states of economic growth.

We reach the strong conclusion that defaulting countries will be characterized by an inability to raise a policy-optimal amount of tax revenue. The manipulation of beliefs, in increasing the demand for public debt, is effectively a close substitute for a fully operational tax instrument. There will be a strong incentive to resort to this particular strategy, if the optimal amount of tax revenue is not forthcoming. Possible reasons for this are information costs, collection costs and non-compliance. As for which kind of economies that this might apply, Tanzi and Lee (2000) summarize a different discussion by saying,

> "In conclusion, in developing countries, tax policy is often the art of the possible rather than the pursuit of the optimal."

The evidence that more developed countries are less prone to serial default may be attributed, as suggested by Reinhart *at al* (2003), to the parallel development of fiscal institutions – rather than to any relative virtue.

The present model predicts that non-defaulting countries will also have higher shares of public expenditure. To the extent that these economies may also be more developed, this model also predicts the emergence of *Wagner's Law*. Thus, we can conclude that richer

[4] Reinhart and Rogoff (2011) use the term "this time is different" to describe hubristic beliefs, which because of some structural change, the normal "rules" may no longer apply. We use it in a subtly different and even more hubristic sense that each generation believes that default only belongs to the past. This is a more operational use of the concept for modelling a, typically, constant economic structure.

countries may spend proportionally more on public goods, because *they can*, while this same *ability* means they have no recurrent incentive to resort to default.

The model is presented in the following *Section 2*, while its solution is confined to an *Appendix*. *Section 3* presents the main results, and *Section 4* provides a conclusion.

2. The model

2.1 Overview of the model

The model is a version of Diamond (1965) with two overlapping generations for a closed economy. Young households save both by acquiring productive assets, leading to investment through an implicit intermediation process, and by holding the debt of their own government. These debts are issued and taxes are raised from young households in order finance public expenditure for both productive infrastructure and non-productive items and to service the outstanding debt. The stock of public debt is determined by the young households' demand, and depends on their subjective forecasts of returns. The distinctive feature of this model is that the government can manipulate these forecasts by promising unwarrantedly high returns, because of the gullibility of young households. The restraint on this policy in a two-generational setting is that the policy-maker is partly concerned with the costs of reneging in terms of the incomes of the old or previously young. As the net gains in terms of the policy function remain strictly positive, whether large or small, we assume an additional but trivial psychic cost of disappointing households' expectations, so that default is not ubiquitous, and so that we may characterize the countries that will pursue a strategy of recurrent default from those that will not.

The time structure of public receipts and payments is also important. The amount of primary expenditure, both productive and non-productive, and taxes are predetermined. Debt-servicing is treated as a residual to endogenously satisfy the government budget constraint. As the amount of debt to be serviced is, naturally, predetermined, it is specifically the debt-servicing interest rate which is the endogenous variable. Default on public debt is defined where – outside of *rational expectations* – this *actual* rate is lower than the *promised* rate. A persistent wedge may separate these two rates, if households are recurrently gullible. This gives the government the scope to raise the primary deficit by boosting the demand for its prospective debt, while at the same time paying less than promised on its existing debt. The government' incentive to pursue is based on both its desires to spend – on both productive and non-productive items – and to reduce the tax burden of young, saving households.

2.2 Household demands for capital and for public debt

Portfolio demands for public debt and for the second asset, capital, are first determined. The household's felicity or periodic utility function is specified as $u = u(F(E(c)), \min(c))$, where $F(E(c))$ is the forecast of the mean of consumption and $\min(c)$ is its known minimum; where $\partial u/\partial F(E(c)) > 0$, $\partial^2 u/\partial F(E(c))^2 < 0$, $\partial u/\partial \min(c) > 0$, $\partial^2 u/\partial \min(c)^2 < 0$. Closed-form solutions may be obtained by using a log-linear specification, $u = (1/(1+\alpha))\ln F(E(c)) + (\alpha/(1+\alpha))\ln \min(c)$, where α represents the degree of risk-aversion.[5] Risk-neutrality is defined where $\alpha = 0$, where the household's objective collapses in to a standard maximization of expected utility, $u = \ln F(E(c))$; while absolute risk-aversion, where $\alpha \to \infty$, leads to *maxmin* behaviour, the maximization of $u = \ln \min(c)$. Apart from providing analytical convenience, this specification also accommodates *down-side* as opposed to *symmetric* risk, which may be more relevant empirically.[6]

The model is the two-period one of Diamond (1965). Consumption is deemed to be ceratin in the first period, so that $c_t^Y = F(E(c_t^Y)) = \min(c_t^Y)$ and

$$U_t = (1-\sigma)\ln c_t^Y + \sigma\left((1/(1+\alpha))\ln F\left(E(c_{t+1}^O)\right) + (\alpha/(1+\alpha))\ln \min(c_{t+1}^O)\right). \qquad (1)$$

The parameter σ represents relative time-preference. Young households receive a gross wage, w_t, which is taxed at the rate τ_t. They may save by holding any combination of deposits, d_t, and public debt, b_t. Deposit saving provides the funds for financing the subsequent period's capital stock, k_{t+1}, $k_{t+1} = d_t$, while holding public debt is a source of crowding-out. We assume there are no other taxes.

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[5] This approach allows portfolio balance for two assets. The weightings may also be interpreted as probabilities in a two state case.

[6] See Ang, Chen and Xing (2006).

The actual returns on capital and fixed-price debt for period $t+1$ are denoted as R^K_{t+1} and R^B_{t+1}. With regard to the return on public debt, we differentiate between the *objective* mean $E(R^B_{t+1})$, the *promised* value, $P_t(R^B_{t+1})$, and the *subjective forecast*, $S_t(R^B_{t+1})$, where the promises and forecasts are made in period t and the outcomes in period $t+1$. The standard assumption of rational expectations is that $S_t(R^B_{t+1}) = E(R^B_{t+1})$ is in stark contrast with the present one of complete gullibility where $S_t(R^B_{t+1}) = P(R^B_{t+1})$. The *this time is different* syndrome is defined by adding an inequality, so that $S_t(R^B_{t+1}) = P(R^B_{t+1}) > E(R^B_{t+1})$ holds systematically. The assumption of *rational expectations* is maintained for forecasting the (unpromised) returns to capital, $S_t(R^K_{t+1}) = E(R^K_{t+1})$. Since this requires a knowledge of the model's parameters, the implication is that the other element of irrationality is based on the gullibility rather than the unknowingness of young households.

As expected returns on capital are typically higher than those on public debt, $E(R^K_{t+1}) > S(R^B_{t+1})$, the second inequality $\min(R^B_{t+1}) > \min(R^K_{t+1})$ is required for portfolio balance under the present specification. A simplifying assumption is that in the worst possible state neither the interest nor the principal is returned on capital, so that $\min(R^K_{t+1}) = 0$, implying

$$\ln U^Y_t = (1-\sigma)\ln\left((1-\tau_t)w_t - k_{t+1} - b_t\right)$$
$$+\sigma\left(\frac{1}{1+\alpha}\ln\left(E(R^K_{t+1})k_{t+1} + S_t\left(E(R^B_{t+1})\right)b_t\right) + \frac{\alpha}{1+\alpha}\ln\min(R^B_{t+1})b_t\right)$$

Utility is maximized by choices both of a total amount of saving and of a portfolio composition.[7] These give rise to the two asset demands,

$$b_t = \sigma\beta_t(1-\tau_t)w_t, \qquad k_{t+1} = \sigma(1-\beta_t)(1-\tau_t)w_t, \tag{2}$$

where, β_t, $0 \le \beta_t \le 1$, is the portfolio share of public debt, such that

[7] Specifying logarithmic preferences and omitting a second period earned income ensures that the saving and portfolio decisions are separable.

$$\beta_t = \frac{\alpha}{1+\alpha}\left(\frac{1}{1-S_t(R_{t+1}^B)/E(R_{t+1}^K)}\right), \quad \frac{\partial\beta_t}{\partial\alpha} = \frac{1}{(1+\alpha)^2}\left(\frac{1}{1-S_t(R_{t+1}^B)/E(R_{t+1}^K)}\right) > 0 . \quad (3)$$

The demand for public debt, as the safer asset, is naturally increasing in the parameter α representing risk-aversion. A relevant point for later is that the financing identity for capital accumulation, namely, total saving less public debt acquisition, $k_{t+1} = \sigma(w_t - T_t) - b_t$, makes plain that the latter causes more crowding-out than taxation, since $\partial k_{t+1}/\partial b_t = -1$, $\partial k_{t+1}/\partial T_t = -\sigma$ and $\sigma < 1$, where young households also consume part of their incomes.

2.3 *Production*

Firms produce using labour and private capital, l_t and k_t, as inputs using under a CRS technology.

$$y_t = (1+\varepsilon_t)A\left(g_t^P l_t\right)^{1-\mu} k_t^{\mu}, \qquad E(\varepsilon_t) = 0 . \quad (4)$$

Production also depends upon the level of public infrastructure capital, g_t^P, such that there are also constant returns in a wider measure of capital as in Barro (1992). Output is also subject to a multiplicative stochastic shock, ε_t, with a zero mean,

It is assumed that the demand for labour is determined by the maximization of *expected* profit, $A\left(g_t^P L_t\right)^{1-\mu} k_t^{\mu} - k_t R_t^K - l_t w_t$, leading to a wage of $w_t = (1-\mu)Ak_t^{\mu} g_t^{P1-\mu} l_t^{-\mu}$. By contrast, private capital is paid the residual from *actual* output, giving a stochastic return of $R_t^K = (\mu+\varepsilon_t)A\left(g_t^P l_t\right)^{1-\mu} k_t^{\mu-1}$, as the source of uncertainty. A lower bound for ε_t of $\min(\varepsilon) = -\mu$ supports the earlier assumption that $\min(R_{t+1}^K) = 0$. The normalization $l_t = 1$ then fixes the wage and the expectedreturn on capital as

$$w_t = (1-\mu)Ag_t^{P1-\mu} k_t^{\mu}, \qquad E(R_t^K) = \mu A g_t^{P1-\mu} k_t^{\mu-1} \quad (5)$$

2.4 *The government budget constraint*

The government spends on both non-productive and productive items of expenditure, g_t^N, g_t^P, in addition to servicing its existing debt, $R_t^B b_{t-1}$. Total expenditure is financed by a combination of labour taxes, $\tau_t w_t$, and new issues of public debt, b_t,

$$g_t^N + g_t^P + R_t^B b_{t-1} = \tau_t w_t + b_t . \tag{6}$$

2.4 Policy

The policy objective is to maximize a geometric weighted average of the disposable income of the young, of debt-servicing expenditure, of the following period's capital stock and of non-productive government expenditure,

$$\ln Z_t = \ln\left((1-\tau_t)w_t\right) + \eta \ln\left(R_t^B b_{t-1}\right) + \varpi \ln k_{t+1} + \gamma \ln g_t^N \tag{7}$$

Debt-servicing expenditure, weighted by η, also reflects a cost of default, as well as defining the minimum and safe part of the income of the old, as a counterpart to the income of the young.[8] The policy-maker is also concerned with the accumulation of capital ($\varpi > 0$), which determines the incomes of all future generations. Choices of intergenerational redistribution, reflected in the parameters, η and ϖ, are implicit in the analysis. Defaulting on interest payments to the old allows the young to pay less in tax, while inducing them to hold more public debt means that future generations will inherit less capital. Finally, while productive public expenditure, g_t^P, is of indirect importance in determining the wage, w_t, and thence also the accumulation of capital, k_{t+1}, non-productive public expenditure, g_t^N, yields direct utility.[9]

Referring back to equation (6), the policy-maker has the potential of *three* formal policy instruments: taxation, τ_t, and productive and non-productive public expenditure, g_t^P and g_t^N. The debt term, b_{t-1}, is predetermined, while its level, b_t, is determined by current household demand on the basis of their forecasts concerning the future return factor,

[8] This term could be replaced by the *total* income of old households, but this would merely dampen the results, while making them stochastic.
[9] This could be either because public officials are also self-interested or because households derive utility from non-productive public goods (in a way that is separable from their consumption preferences to maintain the existing derivation of their functions).

$S_t(R^B_{t+1})$. The manipulation of these beliefs through the promised at time t of the return $P_t(R^B_{t+1})$, such that $S_t(R^B_{t+1}) = P_t(R^B_{t+1}) > E(R^B_{t+1})$, is tantamount to the use of a *fourth* policy instrument in raising the demand for public debt to provide additional finance.

The current, actual return, R^B_t, is then determined as the residual that satisfies the budget constraint equation (6), for a given set of current and previous policy choices and promises, $\left(\tau_t, \tau_{t-1}, g^P_t, g^N_t, P_t(R^B_{t+1}), F_{t-1}(R^B_t)\right)$, as

$$R^B_t = \frac{(1-\phi)\tau w_t + \beta_t \sigma(1-\tau_t)w_t - g^N_t - g^P_t}{\beta_{t-1}\sigma(1-\tau_{t-1})w_{t-1}},$$

$$\text{where} \quad \beta_t = \beta\left(P_t(R^B_{t+1})\right), \qquad \beta_{t-1} = \beta\left(P_{t-1}(R^B_t)\right). \tag{8}$$

The discrepancy between R^B_t and $P_{t-1}(R^B_t)$ reflects a dynamic-inconsistency issue in terms of the policy objective, of which credulous agents are unaware.

An analytically convenient feature of the model is that that while the previous period's beliefs $S_{t-1}(R^B_t)$ determine the inherited debt, b_{t-1}, they have no effect on the residually and subsequently amount of debt-servicing, $b_{t-1}R^B_t$, since a higher value for $S_{t-1}(R^B_t)$ is exactly offset by a proportionally lower R^B_t. Thus, greater optimism with regard to the *believed* value, $S_{t-1}(R^B_t)$ only leads to a bigger disappointment in the *actual* one, R^B_t.[10]

Substituting equations (2), (3) – where $S_t(R^B_{t+1}) = P_t(R^B_{t+1})$ – and (6) into (7) gives

$$\begin{aligned} \ln Z_t &= \ln\left((1-\tau_t)w_t\right) + \eta \ln\left((1-\phi)\tau_t w_t + \beta\left(P_t(R^B_{t+1})\right)\sigma(1-\tau_t)w_t - g^P_t - g^N_t\right) \\ &+ \varpi \ln\left(\left(1-\beta\left(P_t(R^B_{t+1})\right)\right)\sigma(1-\tau_t)w_t\right) + \gamma \ln g^N_t \end{aligned}. \tag{9}$$

There is evidently a value of β_t, based on a credulously believed promise, $P_t(R^B_{t+1})$, which maximizes the value of the policy objective. The previous promise, $P_{t-1}(R^B_t)$, which led to β_{t-1}, is dynamically-inconsistent, since its actual value is given by the separate equation (8). The governing assumption of this analysis is that promises made by the policy-maker of a

[10] Symmetrically, unwarranted pessimism leads to pleasant surprises.

future bond returns, whether made in bad or naïve, good faith, are always believed.[11] Thus, it is not concerned with the potential for the private sector to be deceived, but with the incentive for the public sector to mislead.

If promised exceed actual returns, $P_t(R^B_{t+1}) > R^B_{t+1}$, the demand for public debt is inflated; and the degree of default may is reflected in the ratio, Δ , $\Delta \equiv P_t\,(R^B_{t+1})\big/R^B_{t+1}$. For a closed economy, it might be more appropriate to assume that default comes through unanticipated inflation rather than the more blatant form of the non-payment of nominal interest. If the nominal return on debt is fixed, then the default ratio, Δ, is interpreted as the unanticipated factor inflation, $\Pi_{t+1}\big/P_t(\Pi_{t+1})$. If, then, gullible households believe promises that prices will be stable, $P_t(\Pi_{t+1}) = 1$, then the default ratio has the further interpretation of being the actual the inflation factor, Π_{t+1}. Technically, the model allows the possibility too of perpetually falling prices, causing unanticipated real bonuses on the holding of public debt and a distribution *from* tax payers to the holders of the debt. [12] However, since this scenario is not one that seems to be prevalent,[13] and because the present concern is with default, the model is parameterised to reflect the normal experience of rising prices.

The analysis is concerned with measuring the gains from default in terms to the policy-maker, where a second, but small and exogenous cost of disappointing households' expectations is also assumed. Thus, if the gains from default are negligible, a policy of

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[11]If households believe they will receive the *ex post* outcome, $S_t\,(R^B_{t+1}) = R^B_{t+1}$, they must also believe there is a *rational expectations* steady state. Leading equation (8) one period shows that for current expectations to be strictly rational, there must be a *foreknowledge* of the following generation's beliefs, $S_{t+1}(R^B_{t+2})$, while at best there can only be *beliefs* about future beliefs. A steady state requires that each generation believes their successors will form the same beliefs as themselves. Designating current beliefs on future beliefs as $S_t\big(S_{t+1}(R^B_{t+2})\big)$, a stationary *REE* requires that $S_t\big(S_{t+1}(R^B_{t+2})\big) = S_t\,(R^B_{t+1}) = R^B_{t+1},\ \forall t$. This is the benchmark for measuring the effects of persuading households to believe otherwise, $S_t\,(R^B_{t+1}) > R^B_{t+1}$.

[12] Ferguson (2002) claims that the holders of the public debt in nineteenth century Britain, a politically well-connected minority, gained from an extended period of deflation at the expense of an unenfranchised, tax-paying majority.

[13] One reason why a real redistribution of this kind may no longer apply may be that many of the owners of the public debt may gain instead from the *ex post* redistribution furnished by a pay-as-you-go social security scheme.

manipulating beliefs would not be pursued, and households would resort to a base case of forming rational and dynamically-consistent expectations. Alternatively, if the gains are non-negligible, governments will willingly entice households into holding unduly large amounts of public debt. On this basis of the measured gains from default, we can then distinguish two types of country and identify the characteristics of those which are prone to claiming that *this time is different*.

3. Results

3.1 The default ratio or the unanticipated inflation factor

The details are consigned to the *Appendix*, while the results are presented here. The *default ratio* or the unanticipated inflation factor for the case of false promises is solved as

$$\Delta \equiv \frac{\Pi_t}{P_{t-1}(\Pi_t)} = \frac{P_{t-1}(R_t^B)}{R_t^B} = \frac{\Psi - \alpha\omega - \left(\Psi + \varpi\left((1+\alpha)\sigma^{-1} - \alpha\right)\right)\tau_{t-1}}{\varpi\Gamma(1+\alpha)\left(1+\left((1/\sigma)-1\right)\tau_{t-1}\right)\left(\tau_t + \beta_t\sigma(1-\tau_t)\right)},$$

$$\Gamma \equiv (1-\mu)\eta/\mu\Psi, \qquad \Psi \equiv \left(\eta + \gamma + (1-\mu)(1+\varpi)\right)/\mu. \tag{10}$$

First, it is apparent that a sufficiently large degree of risk-aversion in α, implies that households may happily hold large levels of (safe) public debt without needing an inducement of inflated real returns or an unduly low inflation, $\partial\Delta/\partial\alpha < 0$. The default ratio or unanticipated inflation factor is decreasing in η (through Γ), the weighting on the incomes of the old as the holders of public debt. A higher weighting placed on the accumulation of capital also encourages default in order to build up the disposable income of the young as the base for saving.

The most important results, however, are that the default ratio is inversely related to taxes, both present and previous, $\partial\Delta/\partial\tau_t < 0$ and $\partial\Delta/\partial\tau_{t-1} < 0$. Previously lower taxes would have spurred promises of exaggerated prospective returns, while currently lower taxes would require lower actual ones because of less revenue to service the debt. A kind of recursive or *Ponzi*–type behaviour is implied by this, since default at any time relates to an earlier promise of returns, which may have been made to alleviate the scale of default, which in turn would relate to an even earlier promise.

3.2 *The gains from believed false promises of the ex ante optimal return*

After solving the two generational incomes and the two policy choices of productive and non-productive public expenditure, the policy objective in equation (9) becomes

$$\ln Z_t''' = \ln(1-\tau_t) + \Psi \ln\left(\tau_t + \beta_t \sigma (1-\tau_t)\right) + \varpi \ln\left((1-\beta_t)\sigma(1-\tau_t)\right) + .. \quad ,$$

$$\Psi \equiv \left(\eta + \gamma + (1-\mu)(1+\varpi)\right)/\mu \ . \qquad (11)$$

This leaves the possibility of a further two policy choices: a formal one, namely, the level of taxation, τ_t, and an informal one in promised returns, $P_t(R_{t+1}^B)$.

Two hypothetical economies, *A* and *B*, are considered, which are identical in all respects, except that the taxes may be set at a policy-optimal level in *Economy A*, but are constrained to be at a lower figure of 15% of GDP in *Economy B*.[14] For a common set of parameter values, the policy-gains from offering credulous households an *ex ante*, policy-optimal return $P_t(R_{t+1}^B)$ in excess of the ex post outcome, $S_t(R_{t+1}^B) = P_t(R_{t+1}^B) > R_{t+1}^B$, compared with the fully rational outcome strategy, $S_t(R_{t+1}^B) = R_{t+1}^B$ are evaluated for each of the two economies. To establish a benchmark, the common productivity parameter is set at value to ensure that the lowest outcome for the economic growth rate (for economy *B* where $F_t(R_{t+1}^B) = P_t(R_{t+1}^B) > R_{t+1}^B$) is benchmarked at zero. All the other rates of growth and of return are expressed in annual terms. The numerical results are presented in the following *Table*.

Table: Values of the solutions[15] **where** $\alpha = 1/4,\ \eta,\varpi,\sigma,\gamma = 1/3,\quad \mu = 1/2$					
	Economy A: taxes are policy-optimal			**Economy B: taxes are constrained to be 15% of GDP (30% of wage)**	
Beliefs	***RE***	***TTID***		***RE***	***TTID***
β	0.26154	0.33333		0.23442	0.74603

[14] As we assume only labour income is taxed and as we assign a labour income share of 50%, the calculations are based on a labour income tax of 30%.

[15] We have set chosen the value of A, so that the lowest growth rate is benchmarked at zero.

Taxation/income	31.74%	31.25%		15%	15%
Expected interest rate on capital p.a.	7.23%	7.23%		5.41%	6.28%
Expected interest rate on public debt p.a.	2.98%	4.55%		-0.26%	5.34%
Actual interest rate on public debt p.a.	2.98%	1.97%		-0.26%	-5.19%
Unanticipated inflation rate p.a.	0%	2.53%		0%	10.81%
Growth rate p.a.	2.19%	1.97%		2.35%	0%
Total expend/income	26.39%	26.39%		14.04%	18.77%
Utility gain from TTID	0.13%			40.58%	

For *Economy A*, where tax revenue can be determined optimally, the deceit strategy causes the portfolio share of public debt to rise modestly from 26% to 33%, allowing taxes to fall by a meagre one-half of a percentage point. This is based on promised real rates of turn on public debt of 4.55% instead of 2.98%, leading to the even lower figure of 1.97%. With fixed nominal interest rates, the implication is of small unexpected inflation of 2.53%. The effect is to reduce economic growth falls by one fifth of a percentage point, since public debt has a greater crowding-out effect than taxes. However, the outstanding result is that the welfare gain in terms of the policy function is of the same magnitude, being measured at a negligible one-eighth of a percentage point. Thus, if a policy-optimal amount of tax revenue can be raised, only the most unconscionable policy-maker would attempt to disappoint households' expectations in this way for such a small gain.

For *Economy B*, where taxes are constrained to be 15% of GDP, the promise of exaggerated returns on public causes a more than three-fold increase in the portfolio share of public debt from 23% to 74%. This is based on a promised of 5.34% instead of -0.26%, leads to an actual return of -5.19%. Thus, if the nominal interest rate on public debt is fixed, default comes through an anticipated inflation of 10.81%, while the reduction in the economic growth is 2.35%.[16] Although these two responses are sizeable, the most dramatic result is in the scale of the policy welfare gain of 40%. One might conclude that for economies where there is a problem in raising taxes only the most virtuous policy-maker would refrain from making promises of public debt interest rates that cannot be fulfilled.

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[16] Naïve expectations, leading to an unduly high demand for public debt, thus imply a negative generational externality through an excessive crowding-out of the capital stock.

To reiterate, if a policy-optimal level of tax revenue, τ_t^*, can be raised, there are negligible gains from raising extra finance by manipulating the private sector's demand for public debt, but if the amount of tax revenue is constrained at $\bar{\tau}_t$, $\bar{\tau}_t < \tau_t^*$, these gains may be overwhelming and are naturally commensurate with the size of the shortfall, $\tau_t^* - \bar{\tau}_t$. A strategy of default thus acts as a close substitute for an effective tax instrument. Finally, there is the subsidiary result that countries of the first type will also have a higher amount of primary public expenditure in relation to GDP.

4. Further and concluding points

The inclusion of a small, exogenous cost to the policy-maker from disappointing expectations implies that the syndrome of "*this time is different"* would pertain only to those economies with difficulties with raising taxes. If this is due to a weak fiscal structure, it may be viewed as a LDC phenomenon. This particular interpretation concurs with Reinhart, Rogoff and Savastano's (2003) finding that "debt intolerant countries have weak fiscal structures." Reinhart and Rogoff (2011) also find that developed countries tend not to default.[17]

If *Wagner's Law* also holds, namely, that richer countries tend to spend proportionally more on public expenditure, our subsidiary result provides an exemplification of this insofar as richer countries also tend to have more developed fiscal structures. A scope for raising taxes means that countries can both spend more and be less prone to financial default.

Another implication is that the practice of defaulting on public debt is likely to be due to insufficient taxes rather than to excessive expenditure. For most cases the expenditure side of the public deficit is unlikely to be behind the *"this time is different"* phenomenon, since governments, at least, in the long-run, are able to control their own expenditures. The counter-example of the German hyperinflation of 1923, caused by excessive expenditure for post-war reparations, appears to be an exception but one that actually "proves the rule" for the following three reasons. First, there was a lack of domestic control, since the reparations were imposed by external parties, namely, by the British and the French trying to recoup their

[17] A commitment to inflation targeting in these same economies may thus be seen in the light of an ability to raise taxes.

war debts; second, impositions of this kind are likely to be temporary[18]; thirdly, large expansions in public debt associated are associated with major wars, which are also, hopefully, neither permanent nor recurrent features. Persistent problems with the control of public expenditure might, of course, occur under poor governance, for example, where a military establishment has a strong political influence but without a fiscal concern with balancing the books.

If fiscal infrastructure underlies the problem and if an efficient one may be furnished above a threshold level of GDP, then countries may be able to grow out of the syndrome of "this time is different". Less optimistically, however, the existence of this syndrome may curtail the process of economic growth, extending the time it takes for countries to "graduate" from being defaulters. The numerical results above present a case where a defaulting country never grows, so, consequently, would never rise above any hypothetical threshold. In this sense, the *"this time is different"* syndrome may be a development trap, where high public debt chokes off economic growth, and where the concomitant lack of fiscal development prolongs a dependence on high public debt. This is a different from the type of trap already discussed, where there is an incentive to make promises of unwarranted returns in order to alleviate the burden of a large current debt, which only exists due to previously extravagant promises. An element of *Ponzi* behaviour in attempting to push the current burden of debt into the future would surely be compounded by such a growth trap.

This analysis has been simplified in order to a number of basic points. It could be extended in various other ways, not least by weakening the assumption of irrational beliefs. There could be heterogeneous degrees of credulity both across individuals and across time, allowing learning and forgetfulness and leading to more empirically plausible, intermittent and irregular patterns of default. [19] Reality undoubtedly lies somewhere between the standard assumption of *rational expectations* and the equally strong alternative of *extreme credulity* presented here. Default may be regarded as probabilistic but where the assigned probabilities are unknowable, but subject to over-optimism rather than the absolute gullibility of the present deterministic case.

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[18] An exception is the more historical case of fiefdom where one jurisdiction may be contracted to pay *tribute* to another one on a permanent basis.

[19] Or, *"You can fool some of people some of the time, but you can't fool all of the people all of the time."* [attributed to Abraham Lincoln].

The role that macroeconomic shocks play in process of default is another factor not to be discounted. It would also be interest to extend the model to an open-economy setting, where sovereign debt might be held both at home and abroad and where domestic and foreign investors hold globally diversified and, perhaps, home-biased portfolios. This would require a consideration of some wider political economy issues, which have been kept at bay in the present analysis, but which would certainly be relevant for any further discussion of public debt.

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Appendix: Solution

A.1 Factor prices, public expenditure and economic growth

Returning to equation (9),

$$\ln Z_t = \ln\big((1-\tau_t)w_t\big) + \eta \ln\Big((1-\phi)\tau_t w_t + \beta_t \sigma (1-\tau_t) w_t - g_t^P - g_t^N\Big) + \varpi \ln\big((1-\beta_t)\sigma(1-\tau_t)w_t\big) + \gamma \ln g_t^N ,$$

the policy-optimal level of non-productive public expenditure is

$$g_t^N = \frac{\gamma}{\gamma+\eta}\Big(\tau_t w_t + \beta_t \sigma(1-\tau_t)w_t - g_t^P\Big). \tag{A1}$$

Substituting this back into (9) gives

$$\ln Z_t' = \ln\big((1-\tau_t)w_t\big) + (\eta+\gamma)\ln\Big(\tau_t w_t + \beta_t\sigma(1-\tau_t)w_t - g_t^P\Big) + \varpi\ln\big((1-\beta_t)\sigma(1-\tau_t)w_t\big) + .. , \quad \text{or}$$

$$\ln Z_t' = \ln(1-\tau_t) + (\eta+\gamma)\ln\Big(\tau_t + \beta_t\sigma(1-\tau_t) - g_t^P \big/ w_t\Big) + \varpi\ln\big((1-\beta_t)\sigma(1-\tau_t)\big) + (1+\eta+\gamma+\varpi)\ln w_t \tag{A2}$$

Substituting (5) for the wage into (A2) gives

$$\ln Z_t'' = \ln(1-\tau_t) + (\eta+\gamma)\ln\left(\tau_t + \beta_t\sigma(1-\tau_t) - \frac{{g_t^P}^{\mu}}{(1-\mu)A{k_t}^{\mu}}\right) + \varpi\ln\big((1-\beta_t)\sigma(1-\tau_t)\big) + (1+\eta+\gamma+\varpi)(1-\mu)\ln g_t^P + .. \tag{A3}$$

The policy-optimal level of productive public expenditure is

$$g_t^P = \left(\frac{(1-\mu)^2(1+\eta+\gamma+\varpi)\big(\tau_t + \beta_t\sigma(1-\tau_t)\big)A}{\mu(\eta+\gamma)+(1-\mu)(1+\eta+\gamma+\varpi)}\right)^{\frac{1}{\mu}} k_t , \tag{A4}$$

which, according to (5) implies a wage and an expected value return on capital of

$$w_t = (1-\mu)A\left(\frac{(1-\mu)^2(1+\eta+\gamma+\varpi)\left(\tau_t+\beta_t\sigma(1-\tau_t)\right)A}{\mu(\eta+\gamma)+(1-\mu)(1+\eta+\gamma+\varpi)}\right)^{\frac{1}{\mu}-1} k_t\,, \tag{A5}$$

$$E_{t-1}(R_t^K) = \mu\left(\frac{(1-\mu)^2(1+\eta+\gamma+\varpi)\left(\tau_t+\beta_t\sigma(1-\tau_t)\right)}{\mu(\eta+\gamma)+(1-\mu)(1+\eta+\gamma+\varpi)}\right)^{\frac{1}{\mu}-1} A^{1/\mu} \tag{A6}$$

The ratio of the productive public expenditure to the wage is

$$\frac{g_t^P}{w_t} = \left(\frac{(1-\mu)(1+\eta+\gamma+\varpi)\left(\tau_t+\beta_t\sigma(1-\tau_t)\right)}{\mu(\eta+\gamma)+(1-\mu)(1+\eta+\gamma+\varpi)}\right) \tag{A7}$$

Equations (A1) and (A7) imply the equivalent ratio for total public expenditure,

$$\frac{g_t}{w_t} = \frac{g_t^P+g_t^N}{w_t} = \left(\frac{\gamma}{\gamma+\eta}+\eta\left(\frac{\gamma\mu+(1-\mu)(1+\eta+\gamma+\varpi)}{\mu(\eta+\gamma)+(1-\mu)(1+\eta+\gamma+\varpi)}\right)\right)\left(\tau_t+\beta_t\sigma(1-\tau_t)\right)$$

Equations (A3) and (A4) give

$$\begin{aligned}\ln Z_t''' = \ln(1-\tau_t) + \left((\eta+\gamma)+\frac{1-\mu}{\mu}(1+\eta+\gamma+\varpi)\right)\ln\left(\tau_t+\beta_t\sigma(1-\tau_t)\right) \\ +\varpi\ln\left((1-\beta_t)\sigma(1-\tau_t)\right)+\,..\end{aligned} \tag{A8}$$

Equations (2) and(A5) give the growth factor for the capital stock as

$$\frac{k_t}{k_{t-1}} = (1-\mu)(1-\beta_{t-1})\sigma(1-\tau_{t-1})A^{1/\mu}\left(\frac{(1-\mu)^2\left(\tau_{t-1}+\beta_{t-1}\sigma(1-\tau_{t-1})\right)}{\frac{\mu(\eta+\gamma)}{1+\eta+\gamma+\varpi}+1-\mu}\right)^{1/\mu-1} \tag{A9}$$

A2. Actual ex post returns on public debt

Equations (8), (A1) and (A7) give

$$R_t^B = \frac{\eta\mu}{\mu(\eta+\gamma)+(1-\mu)(1+\eta+\gamma+\varpi)}\left(\frac{\tau_t+\beta_t\sigma(1-\tau_t)}{\beta_{t-1}\sigma(1-\tau_{t-1})}\right)\frac{w_t}{w_{t-1}} \tag{A10}$$

Equation (A5) implies a factor of wage growth of

$$\frac{w_t}{w_{t-1}} = \left(\frac{\tau_t+\beta_t\sigma(1-\tau_t)}{\tau_{t-1}+\beta_{t-1}\sigma(1-\tau_{t-1})}\right)^{\frac{1}{\mu}-1}\frac{k_t}{k_{t-1}} \tag{A11}$$

The last three equations give for R_t^B

$$R_t^B = \frac{\eta\mu(1-\mu)\left((1+\eta+\gamma+\varpi)(1-\mu)^2\right)^{1/\mu-1}}{\left(\mu(\eta+\gamma)+(1-\mu)(1+\eta+\gamma+\varpi)\right)^{1/\mu}}\left(\frac{1-\beta_{t-1}}{\beta_{t-1}}\right)\left(\tau_t+\beta_t\sigma(1-\tau_t)\right)^{1/\mu}A^{1/\mu}$$

(A12)

This can be expressed in relation to $E_{t-1}(R_t^K)$ in equation (A6) as

$$\frac{R_t^B}{E_{t-1}(R_t^K)} = \Gamma\left(\frac{1-\beta_{t-1}}{\beta_{t-1}}\right)\left(\tau_t+\beta_t\sigma(1-\tau_t)\right), \quad \Gamma \equiv \frac{\eta(1-\mu)}{\mu(\eta+\gamma)+(1-\mu)(1+\eta+\gamma+\varpi)}$$

(A13)

A3. Public debt under rational expectations

Taking expectations of the future value of the ration (A13), while assuming $E_t(\tau_{t+1}) = \tau_t$ and $E_t(\beta_{t+1}) = \beta_t$ gives

$$\frac{E_t(R_{t+1}^B)}{E_t(R_{t+1}^K)} = \Gamma\left(\frac{1-\beta_t}{\beta_t}\right)\left(\tau_t+\beta_t\sigma(1-\tau_t)\right) \quad \Gamma \equiv \frac{\eta(1-\mu)}{\mu(\eta+\gamma)+(1-\mu)(1+\eta+\gamma+\varpi)}$$

This is solved simultaneously with (3) gives a quadratic equation for the portfolio share of public debt, β_t,

$$\Gamma\sigma(1-\tau_t)\beta_t^{\,2} + \left(1+\Gamma\left(\tau_t-\sigma(1-\tau_t)\right)\right)\beta_t - \frac{\alpha}{1+\alpha} - \Gamma\tau_t = 0 \qquad \text{(A14)}$$

A4. Public debt under manipulation of beliefs

As $\tilde{\beta}_t$ is a positive monotonic function of ${}_t\tilde{R}_{t+1}^B$, we consider its value that maximizes (A8),

$$\tilde{\beta}_t = \frac{\Psi+(\varpi/\sigma)\left(1-(1-\tau_t)^{-1}\right)}{\Psi+\varpi} \quad \Psi \equiv \left((\eta+\gamma)+\frac{1-\mu}{\mu}\left(1+\eta+\gamma+\varpi\right)\right), \qquad \text{(A15)}$$

which according to (3) requires a promised return factor of

$${}_t\tilde{R}_{t+1}^B = \left(1-\left(\frac{\alpha}{1+\alpha}\right)\left(\frac{\Psi+\omega}{\Psi-(\varpi/\sigma)\,\tau_t/(1-\tau_t)}\right)\right)E_t(R_{t+1}^K) \qquad \text{(A16)}$$

This is lagged one period then divided by the actual return in (A13) to give the default ratio

$$\frac{{}_{t-1}\tilde{R}_t^B}{R_t^B} = \frac{\Psi-\alpha\omega-\left(\Psi+\varpi\left((1+\alpha)\sigma^{-1}-\alpha\right)\right)\tau_{t-1}}{\varpi\Gamma(1+\alpha)\left(1+(\sigma^{-1}-1)\tau_{t-1}\right)\left(\tau_t+\beta_t\sigma(1-\tau_t)\right)} \qquad \text{(A17)}$$

$$\tilde{\beta}_{t-1} = \frac{\Psi - \left(\Psi + (\varpi/\sigma)\right)\tau_{t-1}}{(\Psi + \varpi)(1-\tau_{t-1})} \qquad 1 - \tilde{\beta}_{t-1} = \left(\frac{\varpi}{\Psi + \varpi}\right)\left(\frac{1 + (\sigma^{-1} - 1)\tau_{t-1}}{(1-\tau_{t-1})}\right)$$

$$\frac{{}_{t-1}\tilde{R}_t^B}{R_t^B} = \frac{\Psi - \alpha\omega - \left(\Psi + \varpi\left((1+\alpha)\sigma^{-1} - \alpha\right)\right)\tau_{t-1}}{\varpi\Gamma(1+\alpha)\left(1 + (\sigma^{-1} - 1)\tau_{t-1}\right)\left(\tau_t + \beta_t\sigma(1-\tau_t)\right)}$$

The manipulation of beliefs at time $t-1$ implies $\tilde{\beta}_{t-1} = \beta_{t-1}$. Equations (A15) and (A17) give

$$\frac{{}_{t-1}\tilde{R}_t^B}{R_t^B} = \Gamma^{-1}\left(1 - \left(\frac{\alpha}{1+\alpha}\right)\left(\frac{\Psi + \omega}{\Psi - (\varpi/\sigma)\,\tau_{t-1}/(1-\tau_{t-1})}\right)\right)\frac{\sigma\Psi - (\varpi + \sigma\Psi)\tau_{t-1}}{\varpi\left(\sigma - (1+\sigma)\tau_{t-1}\right)\left(\tau_t + \beta_t\sigma(1-\tau_t)\right)}$$

$$\frac{{}_{t-1}\tilde{R}_t^B}{R_t^B} = \frac{\left(-\alpha\sigma\left(\Psi/\varpi + \alpha\right) + \left(\sigma\alpha(\Psi/\varpi + 1) - (1+\alpha)\right)\tau_{t-1}\right)}{(1+\alpha)\Gamma\left(\sigma - (1+\sigma)\tau_{t-1}\right)\left(\tau_t + \beta_t\sigma(1-\tau_t)\right)} \quad \text{check!!!!} \tag{A18}$$

A5. Policy-optimal taxes

Returning to equation (A7), the policy-maker's preference for taxation is the rate

$$1 - \tau_t = \frac{\Pi}{1 - \sigma\beta_t}, \qquad \text{where } \Pi \equiv \frac{\mu(1+\varpi)}{1 + \eta + \gamma + \varpi} \tag{A19}$$

Under policy-optimal taxes, equation (A15) for rational beliefs becomes,

$$\beta = \frac{\alpha/(1+\alpha) + (1-\Pi)\Gamma}{1 + (1-\Pi)\Gamma} \tag{A20}$$

while equation (A16) for manipulated beliefs becomes,

$$\tilde{\beta}_t = 1 + \varpi(1 - 1/\sigma), \tag{A21}$$

implying optimal taxes in this case of

$$\tilde{\tau}_t = 1 - \frac{1}{(2-\mu)(1 + \varpi + \eta + \gamma)(1-\sigma)}. \tag{A22}$$