Intertemporal tradeoffs in exchange rate management*

Nikola Tarashev and Anna Zabai†

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Abstract

Could a weaker central bank be less likely to abandon its currency peg? Traditional currency-crisis models provide a firm answer: No. We argue that the answer stems from these models' narrow focus on the short-term implications of a central bank's response to a speculative attack. The answer may reverse if we recognize that a currency peg attenuates time consistency issues, which are more pronounced for weaker central banks. By withstanding an attack and buttressing the credibility of its currency peg, a weaker monetary authority stands to gain more in the long term.

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†Bank for International Settlements. E-mail: nikola.tarashev@bis.org and anna.zabai@bis.org

1 Introduction

In this paper, we revisit the legacy of Rogoff (1985), who defines the strength of a central bank as the monetary authority's capacity to control inflation. In a small open economy with a flexible exchange rate regime, such as a managed float, this strength determines the capacity to control currency devaluation. The weaker the central bank, the greater its temptation to outsmart markets with a surprise inflation/devaluation in order to boost output. However, the central bank cannot systematically surprise rational markets. The greater is the central bank's temptation, the more forceful are private agents' preemptive actions, which results in higher devaluation without output gains. In other words, a weaker central bank faces greater time inconsistency problems à la Barro and Gordon (1983).

Prototypical second-generation models of currency crises have shown that the central bank's strength plays a key role also when the currency regime is an *adjustable peg*. Under such a regime, the central bank needs to take two decisions: first, whether to abandon the peg and, if it does, by how much to devalue. The prototypical models confine the two choices within a single period, featuring a one-shot game between the central bank and private agents (see for example Morris and Shin, 1998; Obstfeld and Rogoff, 1996; Obstfeld, 1994). In a one shot game, the impact of time inconsistency on the size of devaluation (outside the peg) goes hand in hand with that on the likelihood of abandoning the peg. Confronted with stronger pressure from the private sector, a weaker central bank is more likely to abandon the peg and then devalue by more.

Despite its intuitive appeal, this conclusion rests on an incomplete argument, which does not consider a key reason why a central bank would want to peg its currency in the first place. Namely, a credible peg helps address time inconsistency problems by tying the central bank's hands. And credibility takes time to build: to benefit from a credible peg in the long run, a central bank has to peg in the short run. Thus, given that a weaker central bank faces bigger time inconsistency issues, it stands to lose more from abandoning the peg, untying its hands and having to manage a float down the road. The question then becomes whether the one-shot game conclusions of prototypical models are robust to allowing for intertemporal considerations that boost the resolve of a weak central bank to withstand pressure on the currency peg. Even though a weaker central would devalue by more once in devaluation mode, could it be less likely to enter such a mode than a stronger central bank, all else the same?

Framework. In this paper, we contribute to the currency crisis literature by departing from the traditional one-shot game setup in order to study explicitly the intertemporal trade-offs that a central bank faces in managing its currency.

We start with a model of imperfect competition in the labour market, which provides microfoundations for the central bank's time inconsistency problem. This is a model of a small open economy, in which the domestic price level pins down the nominal exchange rate. Each period, workers set nominal wages in anticipation of the price level, targeting a real wage that maximizes the return from working. Importantly, workers exploit their market power to set their wages at a mark-up above the socially optimal wage. Acting after nominal wages have been set, the central bank picks the price level. The higher is the nominal wage mark-up, the greater is the pressure on the central bank to raise output by inflating prices, i.e. by devaluing. In each period, the central bank weighs this pressure against its own dislike for inflation. The central bank's weight on inflation-driven – relative to output-driven – losses captures its strength, i.e. the strength to resist labour-market pressure.

Our model implies that (i) a credible peg is the socially optimal equilibrium consistent with rational decision making; but (ii) the central bank cannot attain this equilibrium if – period by period – it focuses exclusively on its current output- and inflation-driven losses. A credible peg is optimal because it eliminates inflation-driven losses, while output-driven losses are independent of the exchange rate regime in equilibrium. But labour market distortions create incentives for the central bank to deviate from this equilibrium after workers have set their wages. This is the classic time inconsistency problem.

We explore two different ways of attaining partial credibility for the peg. For the first, we follow prototypical second generation models of currency crises: i.e. assume an ad hoc fixed cost of abandoning the peg, which could be interpreted as a reputational cost to the central banker. This leads to a one-shot game, with all actions and payoffs occurring within a single period: the short run. The second approach is to consider two repeated games: one in the short and one in the long run. If the central bank preserves the peg in the short run, it joins a currency union and thus credibly ties its hands for the long run. By contrast, abandoning the peg in the short run undermines the central bank's credibility, unties its hands and leads endogenously to a managed float regime in the long run. In effect, the second approach – which is an innovation of our paper – replaces the ad hoc commitment device with endogenous long-term (net) benefits of pegging in the short run.[1]

Results. To determine how the likelihood of abandoning the peg depends on the central bank's strength, it is necessary to consider the relationship between this strength and the short-term (net) costs of pegging. In line with traditional second-generation models, this relationship is negative. All else the same, workers expect greater devaluation from a weaker central bank and set higher nominal wages, thus raising the spectre of greater output losses if the peg survives. The short-term costs of withstanding workers' pressure – i.e. the short-term

[1] The long term benefits we derive are similar in spirit to those analysed in different settings by Clerc et al. (2011), Cooley and Quadrini (2003) and Alesina and Barro (2002). These articles do not consider speculative attacks but study instead the flexibility vs. credibility tradeoffs of giving up monetary independence for a currency union.

costs of pegging – are thus higher for a weaker central bank.

In the one-shot game, the short-term cost of pegging is the only channel through which the central bank's strength affects the likelihood of devaluation. Facing higher short-term costs, a weaker central bank is less likely to preserve the peg. This is the case under perfect foresight – which gives rise to multiple equilibria – and under dispersed information – which leads to a unique threshold equilibrium (Morris and Shin, 1998). As this is a standard result, our main question is whether it reverses in a repeated game setting.

In the repeated game setting, the central bank's decision about the currency regime is based both on the short-term (net) costs and the long-term (net) benefits of pegging. Like the short-term costs, the long-term benefits are now also endogenous and are also negatively related to the central bank's strength. Facing greater time-inconsistency issues on its own, a weaker central bank stands to gain more from upholding the peg and joining a currency union than does a stronger central bank.

The analysis of the repeated games setting boils down to comparing two (negative) sensitivities. These are: (i) the sensitivity of the short-term cost of pegging with respect to the central bank's strength (henceforth, the "short term sensitivity"); and (ii) the corresponding sensitivity of the long-term benefit of pegging in the short run (the "long term sensitivity"). *A weaker central bank is more likely to uphold the peg if and only if the absolute value of the long-term sensitivity is higher than that of the short-term sensitivity.*

In studying the repeated games setting, we first assume perfect foresight. Similar to the one-shot game, workers' perceptions again underpin multiple equilibria. In contrast to the one-shot game, however, workers' perceptions now determine also the sign of the relationship between the central bank's strength and the likelihood of upholding the peg. If workers perceive that the peg will be abandoned as soon as this is rational, then they act anticipating a higher devaluation rate from a weaker central bank. This feeds into nominal wages, implying that the short-term cost of pegging is extremely sensitive to the central bank's strength. As a result, the short-term sensitivity is higher than the long-term sensitivity, thus delivering the standard result that a weaker central bank is more likely to abandon the peg.

The story reverses if workers perceive a peg as long as it is rational to do so. In this case, workers perceive the central bank's hands as tied. As a result, the central bank's strength is irrelevant for workers' wages and, by extension, does not influence the short-term pressure on the peg. This brings the short-term sensitivity below the long-term one, implying that a weaker central bank is more likely to uphold the peg.

We then study repeated games under dispersed information in the private sector. Dispersed information introduces strategic uncertainty: each worker is unsure about the actions and beliefs of others. Strategic uncertainty bites only in the short run, as only then is the fate of the peg still unknown and depends on workers' collective action, i.e. the aggregate wage bill. Thus, in forming beliefs about the exchange rate, each worker needs to form beliefs about other workers' actions. In line with global games lessons, strategic uncertainty gives rise to a unique equilibrium because it impairs workers' coordination capacity (Morris and Shin, 1998).

For our analysis, it is important that – by impairing coordination – strategic uncertainty dampens the sensitivity of workers' collective action to the central bank's strength.[2] In particular, we find that the short-term costs of pegging is less sensitive to the central bank's strength under dispersed information than under the perfect-foresight scenario in which workers perceive a devaluation. At the same time, strategic uncertainty does not affect the long-run sensitivity, as the currency regime is common knowledge in the long run.

On its own, however, strategic uncertainty does not lower the short-term elasticity enough to reverse the relationship between the central bank's strength and the probability of abandoning the peg. Even when strategic uncertainty is at its highest level – i.e. even when there is only private information about the economy's exogenous fundamentals – the short-term sensitivity remains higher that the long-term sensitivity. Thus, we still obtain that a weaker central bank is more likely to abandon the peg.

In looking for a mechanism that would further dampen the short-term sensitivity, we let the wages of *some* of the workers be predetermined in the short term, e.g. the result of rigid contracts. Trivially, workers with predetermined wages cannot engage in a preemptive action even if they perceive a high likelihood of devaluation. In the spirit of Corsetti et al. (2004), this also reduces the aggressiveness of the preemptive actions by the rest of the workers, those with flexible wages. We thus find that pre-determined wages dampen not only the short-term pressure on the currency regime but also the sensitivity of the short-term costs of pegging to the central bank's strength. By contrast, the corresponding long-term sensitivity is unchanged, as wages are flexible in the long run. This opens the door to reversing the traditional result on the relationship between the central bank's strength and the likelihood of abandoning the peg. Indeed, we show that a sufficiently large fraction of predetermined wages drives the short-term sensitivity below the long-term one, implying that the probability of maintaining the peg increases as the central bank weakens.

Roadmap. The rest of the paper is organized as follows. We introduce a macro model with labour-market distortions in the next section. Then, in Section 3, we present a linear-quadratic reduced form of this model, which gives rise to micro-founded social welfare. Combining social welfare with a term capturing the central bank's dislike of inflation delivers the central bank's period loss, which is the backbone of our analysis. In the subsequent

[2] Angeletos et al. (2007) and Chamley (2003) study how strategic uncertainty and the capacity to coordinate evolve over time as private agents learn from past outcomes in a dynamic setting.

sections, we conduct the analysis in four variants of the model. In Section 4, we study a one-shot game, first under common knowledge and then under dispersed information. In Section 5, we switch to repeated games, again under common knowledge and then dispersed information. The repeated games setting allows us to also the study the implications of there being predetermined wages in the short run.

2 The model

We present our model in this section.

Agents. The economy comprises a mainland and a continuum of islands. It is populated by a household and a firm, which operates on the mainland and also runs a continuum of subsidiaries, each on a different island. Islands are indexed by $i \in [0, 1]$. There is also a monetary authority, i.e. a central bank.

The household has a continuum of workers, each represented by a point in the square $[0, 1]^2$. At the beginning of each period, a measure-one continuum of workers is matched with a specific island, and leaves the mainland to go to work. Once on the island, the workers obey a set of rules (spelled-out below) and in exchange they all receive the same amount of consumption.

There is a single consumption good and it is produced by the firm on the mainland. The production of that good requires island-specific intermediary goods, which are produced on the islands by the local subsidiaries. Imperfect substitutability of intermediate goods endows workers with market power.

Preferences and production technology. The household has standard preferences over consumption and leisure,

$$\mathbb{E}_{i,0} \sum_{t=1}^{\tau} \beta^{t-1} \left(\frac{C_t^{1-\varsigma}}{1-\varsigma} - \int \frac{N_{i,t}^{1+\varphi}}{1+\varphi} di \right), \tag{1}$$

where C_t denotes final good consumption, which is the same across labour types; $N_{i,t}$ the quantity of labour supplied by the workers on island i; and $\beta \in (0, 1)$ is the discount factor. We study below a one-shot game – where $\tau = 1$ – and a repeated-games setting – where $\tau = 2$.

The household's budget constraint is given by

$$P_t C_t \leq \int W_{i,t} N_{i,t} di + T_t, \tag{2}$$

where P_t denotes the price level; $W_{i,t}$ is the nominal wage paid to workers on island i and T_t represents nominal transfers from the firm to the household.

On the mainland, the firm produces the consumption good according to the production technology

$$Y_t = AN_t^{1-\alpha}, \tag{3}$$

where A denotes total factor productivity and $N_t \equiv \left(\int N_{i,t}^{\frac{\nu-1}{\nu}} di\right)^{\frac{\nu}{\nu-1}}$ – with $\nu > 1$ – is a CES index of intermediate goods, produced according to subsidiaries' linear technology for transforming island-specific labour input into an island-specific intermediate good. Defining the aggregate wage index as $W_t \equiv \left(\int W_{i,t}^{1-\nu} di\right)^{\frac{1}{\nu-1}}$, the firm's profits are:

$$P_t Y_t - W_t N_t. \tag{4}$$

The firm chooses the labour index N_t and the cross-island labour allocation $\{N_{i,t}\}_{i\in[0,1]}$ to maximize profits, taking the price level P_t and the wage index W_t as given.[3] Maximizing (4) accordingly yields demand for the labour index

$$N_t = A^{\frac{1}{\alpha}} \left(\frac{W_t}{P_t}\right)^{-\frac{1}{\alpha}} (1-\alpha)^{\frac{1}{\alpha}}, \tag{5}$$

as well as for island-specific labour,

$$N_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\nu} N_t. \tag{6}$$

The set of rules, that workers adhere to in order to receive the same consumption, are as follows. The workers on any particular island i have to accept the wage set by a local trade union and then stand ready to supply as much labour as the local subsidiary demands at that wage. The trade union on island i comprises the continuum of identical workers sent by the household to that island, i.e. a coalition. This union chooses the nominal wage $W_{i,t}$ at the beginning of period t to maximize the expected return from labour of its coalition:

$$\mathbb{E}_{i,t}\left[C_t^{-\varsigma}\frac{W_{i,t}}{P_t}N_{i,t} - \frac{N_{i,t}^{1+\varphi}}{1+\varphi}\right],$$

subject to demand for island-specific labour $N_{i,t}$, equation (6).[4] This gives rise to the following first order condition for the nominal wage:

$$\mathbb{E}_{i,t}\left[C_t^{-\varsigma}\frac{W_{i,t}}{P_t}N_{i,t} - \frac{\nu}{\nu-1}N_{i,t}^{1+\varphi}\right] = 0, \tag{7}$$

[3] The mainland firm's equilibrium profits, which are rebated lump sum to the household, equal $\alpha P_t Y_t = T_t$.

[4] Introducing labour market frictions through unions which optimise expected return from labour is a standard modeling device in the structural macro (i.e. DSGE) literature. For a full treatment, see Christiano et al. (2010).

We refer to this equation as the optimal wage-setting condition. The first term in the square bracket represents the marginal return from working, converted in utility terms by applying the marginal utility of consumption, $C_t^{-\varsigma}$, while the second is the marginal cost of supplying labour, augmented by the wage markup, $\frac{\nu}{\nu-1}$, which varies inversely with the elasticity of substitution across intermediate inputs ν. Intuitively, the smaller the substitutability across inputs and their corresponding labour types, the larger the market power of workers and the higher the individually optimal wage. Importantly, the individually optimal wage is higher than the socially optimal wage and converges to it as the elasticity of substitution across inputs becomes arbitrarily large, $\nu \to \infty$.

Monetary authority. We ignore the microfoundations of money and close the model by imposing an ad-hoc cash-in-advance constraint on total expenditure as in Angeletos and La'O (2011),

$$P_t Y_t = M_t \tag{8}$$

where M_t can either be interpreted as money supply or as nominal aggregate demand. By selecting M_t, the central bank pins down P_t in accordance with the currency regime it is in (see also Woodford, 2003, Chapter 3.2.1).

Currency regime. We also assume that absolute purchasing power parity holds. Thus, the real exchange rate is $\text{RER}_t = P_t/e_t P^* = 1$, where P^* denotes the foreign price level and e_t is the nominal exchange rate: the domestic currency price of the foreign currency. Assuming further that the foreign price level is fixed, $P^* = 1$, we obtain $P_t = e_t$. Domestic inflation is thus equivalent to devaluation.

The price level depends on the currency regime that the central bank operates in. If the regime is a currency union, the central bank *has to* peg the exchange rate: i.e. it sets the price at its level in the previous period. If the regime is a managed float, the central bank is free to choose the price level.

3 Reduced form of the model

We now present a linear-quadratic reduced-form of the above model. Lower-case letters denote log deviations from the deterministic steady state. Because unions' market power raises the privately optimal wage above the socially optimal one, and there are no corrective taxes, the steady state is inefficient. Since unions are made of workers, in the remainder of the paper we will use "unions" and "workers" interchangeably.

Workers. In Appendix A.1, we show that the wage-setting condition equation (7) log-linearizes to $\mathbb{E}_{i,t}\left[-\varsigma c_t + w_{i,t} - p_t - \varphi n_{i,t}\right] = 0$. Here, $-\varsigma c_t + w_{i,t} - p_t$ is the return from working (in terms of utility), and $-\varphi n_{i,t}$ is the cost of working. The optimal wage equates the two. Using the production function for the final good (3), the optimality condition for average labour (5) and the expression for island-specific labour demand (6), we can rewrite the log-linear wage-setting condition as:

$$w_{i,t} = \mathbb{E}_{i,t}\left[\delta_p p_t + (1-\delta_p)\, w_t\right], \tag{9}$$

where w_t denotes the linearized version of the wage index, $w_t = \int w_{i,t} di$, and $\delta_p = \frac{\varsigma(1-\alpha)+\varphi+\alpha}{\alpha(1+\nu\varphi)} > 0$. In the remainder of the paper, we refer to w_t as the "average wage".

For tractability we consider the limits in which the household is risk neutral, $\varsigma \to 0$ and preferences are linear in labour, $\varphi \to 0$. The framework in Hansen (1985), which features indivisible labour with lotteries, helps rationalize the latter assumption. Indeed, for any preferences over labour in that framework, workers behave as if these preferences were linear. In this special case, the wage-setting condition is given by

$$w_{i,t} = \mathbb{E}_{i,t}\left[p_t\right], \tag{10}$$

which allows us to focus on workers' incentives to stabilize their real wage.

Central bank. In each period the central bank has two options: (i) peg the currency, i.e. set $p_t = p_{t-1}$; or (ii) abandon the peg and manage a float. If it abandons the peg, the central bank manages the float by picking p_t to minimize the following period loss,

$$L_t^A = \underbrace{\frac{1}{2}\frac{\alpha}{1-\alpha}\left(\hat{y}_t - \hat{y}^*\right)^2}_{\text{align w/ steady state}} \quad \underbrace{-\,\theta_t\left(\hat{y}_t - \hat{y}^*\right)}_{\text{correct steady state inefficiency}} \quad \underbrace{+\,\frac{\chi}{2}p_t^2}_{\text{minimise inflation}}. \tag{11}$$

where the difference $\hat{y}_t - \hat{y}^*$ represents the deviation of the welfare-relevant output gap $\hat{y}_t$ from its steady state value $\hat{y}^*$. Each gap is relative to the level of output that would prevail in the absence of the labour market distortions discussed in Section 2. According to the first term in the period loss, the central bank seeks to align the output gap with its steady-state value. However, the steady-state output is inefficiently low because of the sub-optimally high wages stemming from the labour market distortions: i.e. $\hat{y}^* < 0$. This underpins the second term in the period loss, with $\theta_t > 0$ capturing the central bank's drive to raise the welfare-relevant output above its steady-state value. Finally, after normalizing p_{t-1} to 0, the third term captures the central bank's dislike for inflation.[5]

We can relate the loss function (11) to the microfoundations presented in Section 2. If θ_t were constant and equal to the inverse of the elasticity of substitution across intermediate inputs $1/\nu$, the first two terms in (11) would be equal to a second-order approximation of the aggregate welfare losses accruing to households (see Appendix A.2). Hence, the central

[5] The assumption $p_{t-1} = 0$ is without loss of generality. Relaxing it amounts to treating all endogenous variables as *deviations* from p_{t-1}, which does not affect any of the derivations.

bank objective (11) departs from social welfare in two ways. First, the central bank places a different, time-varying, weight on negative output gaps – and thus on labour market distortions – than households. Second, it has a stronger aversion to inflation than households: hence $\chi > 0$.[6] We will see below that, in order for equilibrium inflation to be bounded, it is necessary that the central bank's differ from the household's.

Using the production function for the final good (3), the optimality condition for the labour index (5) we can write the "aggregate supply" curve of this economy as $y_t = -\frac{1-\alpha}{\alpha}(w_t - p_t)$, where y_t is the log-deviation of output from steady state. Observing that $y_t = \hat{y}_t - \hat{y}^*$ and substituting in (11) we finally obtain a version of the period loss in which the only endogenous magnitudes stem from the households' and the central bank's choice variables, w_t and p_t, respectively:

$$L_t^A = \frac{1}{2}(w_t - p_t)^2 + \theta_t (w_t - p_t) + \frac{\chi}{2} p_t^2. \tag{12}$$

If the central bank abandons the peg, it chooses the price that minimizes these losses, taking the average wage as given:

$$p_t = \frac{1}{1+\chi}(w_t + \theta_t). \tag{13}$$

This price reflects the trade-offs faced by the central bank. First, the monetary authority seeks to stabilize the welfare-relevant output gap, that is, to minimize $\frac{1}{2}(w_t - p_t)^2$. On its own, this takes the price level p_t towards the average wage w_t. Second, in order to mitigate the negative output gap, the authority targets a negative real wage: $\theta_t (w_t - p_t) < 0$. Added to the first consideration, this increases the price level to $w_t + \theta_t$. In this sense, $\theta_t > 0$ leads to an inflation bias in price setting under a managed float. Third, the central bank dislikes inflation. This lowers the optimal price level to $\frac{1}{1+\chi}(w_t + \theta_t)$, where $\frac{1}{1+\chi} < 1$.

[6] Our model does not generate a tradeoff between inflation and output stabilisation. Woodford (2003, Chapter 6) shows that in New Classical models with imperfect competition and price/wage rigidities, social welfare can be approximated as a linear combination of the (square of the) output gap and a price/wage dispersion term. The exact way in which changes in the price level affect price dispersion depends on the details of price/wagesetting. In our model, the approximation yields a wage dispersion term that is positive only under dispersed information. Wage dispersion arises because of an information friction that is not affected by changes in the price level. As an alternative modelling strategy, we could have set up the reduced form model as a game between the monetary authority and *pricesetting firms* who choose prices given their expectations about the nominal value of aggregate demand. However, such a model would not have delivered a microfounded output/inflation tradeoff, because the output gap is proportional to inflation surprises, which are the source of price dispersion. Woodford also shows that a sufficient assumption to microfound an quadratic *price* inflation term in the welfare criterion is Calvo pricesetting. We did experiment with Calvo pricing for final good producers. There is a tradeoff, however, between generating an endogenous inflation term in the central bank's objective and obtaining a tractable expression for the linearised wagesetting condition. Under monopolistic competition amongst final producers, we could not have conveniently exploited the optimality condition for final producers to write aggregate labour N_t as an explicit function of the real wage, $W_t - P_t$ (see Appendix A.1). With Calvo pricing, the optimality condition for final producers would have been a complicated forward-looking expression involving expectations of future marginal costs, and hence of future real wages.

The loss-minimizing price level given by (13) increases with the inflation bias θ_t and increases with the aggregate nominal wage w_t. Thus, the higher are nominal wages, the wider is the wedge between (i) the price level minimizing the central bank's period loss and (ii) the price level consistent with a peg, $p_t = 0$. In this sense, the level of nominal wages represents workers' pressure on the currency peg. However, the greater is the central bank's aversion to inflation, the stronger is its resistance to this pressure: the wedge between p_t in equation (13) and $p_t = 0$ decreases in χ for a given w_t. We thus refer to χ as the central bank's strength.

Time inconsistency of monetary policy. In choosing whether to peg or set the price level according to (13), the central bank compares the associated losses. As regards the period loss in (12), we have:

$$L_t^A\Big| \text{ abandon peg} = \frac{1}{2}\frac{\chi}{1+\chi}\left(w_t+\theta_t\right)^2 - \frac{1}{2}\theta_t^2$$
$$L_t^A\Big| \text{ peg} = \frac{1}{2}\left(w_t+\theta_t\right)^2 - \frac{1}{2}\theta_t^2$$

Accordingly, the *net* cost of pegging is given by

$$c\left(w_t,\theta,\chi\right) \equiv L_t^A\Big| \text{ peg} - L_t^A\Big| \text{ abandon peg} = \frac{1}{2}\frac{\left(w_t+\theta_t\right)^2}{1+\chi} > 0. \tag{14}$$

Thus, if the central bank considers period losses only, devaluing is a dominant strategy for *any* given wage bill. Intuitively, devaluation reduces sufficiently the output gap to more than compensate for the higher inflation in the central bank's period loss. *Thus, pegging is time inconsistent.*

This implies that, for the peg to be partially credible, abandoning it must generate losses to the central bank over and above the period loss in (12). We consider two alternative ways of introducing such losses.

1. We suppose that workers and the central bank are engaged in a one-shot game (i.e. $\tau = 1$ in equation (1)). In this case, the central bank faces a fixed ad hoc cost $\bar{v}$ if and only if it abandons the peg. This is the textbook setup of e.g. Obstfeld and Rogoff (1996).

2. A key modelling innovation of this paper is to introduce an *endogenous* alternative to the above ad hoc cost of abandoning the peg. Namely, we consider sequential games over two periods: the short run and the long run ($\tau = 2$).

   (a) If the central bank pegs in the short run, it has the option to join a currency union. If it does join, it pegs unconditionally in the long run as well.

(b) If the central bank abandons the peg in the short run, then it cannot join the currency union and faces only its period loss (12) in the long run. Thus, pegging becomes time inconsistent in the long run and the central bank is sure to float then.

Fundamentals. The only source of uncertainty in this model is the dislike of the central bank for negative output gaps, θ_t. We refer to θ_t as the "fundamentals" of the economy. A higher value of θ_t leads to a higher inflationary bias and thus corresponds to worse fundamentals. At the beginning of each period, nature draws θ_t from a uniform distribution with support on $\left[\theta_L, \theta^H\right]$, where $\theta_L > 0$. The fundamentals are i.i.d. over time.

Workers' information. In setting the wage rate, the workers on each island make use of the information they have at the beginning of the period. We consider two different information structures. The first is perfect foresight, whereby workers observe the fundamentals θ_t before choosing the optimal nominal wage. The second structure features uncertainty about θ. All workers start with a common prior that is different from the true distribution of θ_t. Under the prior, θ_t is a Gaussian random variable, $\theta_t \sim N(\theta_0, \sigma_0^2)$, and i.i.d. across periods. Then the workers on each island i observe an island-specific private signal:

$$x_{i,t} = \theta_t + \sigma_x \xi_{i,t}, \tag{15}$$

where the noise $\xi_{i,t} \sim N(0,1)$ is i.i.d. across periods and in the cross-section and is independent of θ. The private signal implies that the subjective posterior distributions of the fundamentals differ across islands,

$$\theta_t \sim N\left(\psi x_{i,t} + (1-\psi)\,\theta_0, \sigma^2_{\theta|x}\right), \tag{16}$$

where $\psi \equiv \frac{\sigma_0^2}{\sigma_0^2+\sigma_x^2}$ and $\sigma^2_{\theta|x} \equiv \frac{\sigma_0^2\sigma_x^2}{\sigma_0^2+\sigma_x^2}$. In analyzing the dispersed information setting, we focus on the limit $\sigma_x \to 0$.

4 One-shot game

We now study the reduced form model introduced in Section 3 when workers and the central bank are engaged in a one-shot game. In this game, the central bank abandons the peg for a managed float, if and only if the net payoff to defending the peg is negative:

$$\bar{v} \leq c\,(w, \theta, \chi)\,, \tag{17}$$

where the right-hand side is as defined in equation (14) and we have suppressed time subscripts.

4.1 Perfect foresight

Consider first the case in which workers possess perfect foresight. In this case, all wages are identical and equal to the aggregate wage, $w_i = w$. By the wage-setting equation (10) we then obtain simply

$$w = p. \tag{18}$$

Then, if a peg is sustained in equilibrium, $p(\theta, \chi) = 0$, equations (10) and (12) deliver the average wage and the period loss:

$$p(\theta, \chi) = 0,\ w(\theta, \chi) = 0,\ L^A(\theta, \chi) = 0. \tag{19}$$

Alternatively, for a managed float to be an equilibrium outcome, the same two equations together with (13) imply the following price, wage and loss:

$$p(\theta, \chi) = \frac{\theta}{\chi},\ w(\theta, \chi) = \frac{\theta}{\chi},\ L^A(\theta, \chi) = \frac{1}{2}\frac{\theta^2}{\chi}. \tag{20}$$

As anticipated above, a difference between the central bank's and the household's dislike for inflation, i.e. $\chi \neq 0$, is necessary for a bounded equilibrium.

The second-generation literature on currency crises has established that, in settings like the one in the current subsection, workers acting before the central bank gives rise to multiple equilibria for intermediate values of the fundamentals. This is indeed the case.[7]

Proposition 1 (equilibrium with common knowledge)**.** *There are two threshold values of the fundamentals. The first one, denoted by $\underline{\theta}$, is the positive solution to $\bar{v} - c\left(\frac{\theta}{\chi}, \theta, \chi\right) = 0$:*

$$\underline{\theta} = \sqrt{2\bar{v}\frac{\chi^2}{1+\chi}}. \tag{21}$$

The second one, denoted by $\overline{\theta}$, is the positive solution to $\bar{v} - c(0, \theta, \chi) = 0$:

$$\overline{\theta} = \sqrt{2\bar{v}(1+\chi)} \tag{22}$$

For low values of the fundamentals, $\theta \leq \underline{\theta}$, there exists a unique equilibrium such that the rule in (17) leads the central bank to sustain the peg and deliver the outcome summarized in equation (19)*. For high values of the fundamentals, $\theta > \overline{\theta}$, there exists a unique equilibrium such that the rule in* (17) *leads the central bank to abandon the peg and deliver the outcome summarized in equation* (20)*. For intermediate values of the fundamentals, $\theta \in \left(\underline{\theta}, \overline{\theta}\right]$, there*

[7] Throughout the paper, we consider only negative critical values of the fundamentas, which are associated with a devaluation of the currency under a managed float. We abstract from positive critical values, which are associated with revaluations. See also Appendix C.2.

are multiple equilibria: the central bank pegs or devalues, depending on whether workers anticipate a peg or a devaluation.

There are two important aspects of the equilibrium under perfect foresight in the one-shot game. First, under a peg, the central bank sits tight and its strength does not matter: as shown in (19) neither the price, nor wages, nor the period loss depends on χ. Second, these variables *do* depend on χ under a managed float, as shown by (20). In particular, since a stronger central bank is better at managing inflation/devaluation, the equilibrium price, wage and period loss all decrease in χ.

The multiplicity region of the state space, $\left(\underline{\theta}, \overline{\theta}\right]$, shares important properties with the multiplicity region in the reduced form model of Morris and Shin (1998, henceforth MS). In the common knowledge benchmark of the MS model, the worst fundamentals supporting a peg are such that the benefit of maintaining the peg equal the cost of pegging in the absence of pressure from the private sector. In our case, the corresponding benefit is on the left-hand side of expression (17): $\bar{v}$. And the corresponding cost is on the right-hand side for $w = p = 0$. In the MS framework, a downward shift of the cost-of-pegging schedule strengthens the peg. Corollary 1 below shows that this is the case in our model, too. An increase in χ – corresponding to a stronger central bank – results in such a shift and also strengthens the peg.

The parallels between the MS and our model extend also to the best fundamentals that can witness a devaluation. In the MS model, these fundamentals worsen (i.e. the peg strengthens) as the size of the potential devaluation declines. Similarly, equation (20) shows that, under a managed float, the devaluation size declines in χ while the upper threshold, $\overline{\theta}$, increases in χ.

Corollary 1. *Both the lower threshold, $\underline{\theta}$, and the upper threshold, $\overline{\theta}$, of the multiplicity region are increasing in central bank conservatism, χ.*

In anticipation of our analysis below, it is useful to step back and consider carefully the drivers of the relationship between the central bank's strength and a threshold value of the fundamentals. Denoting such a value with the generic θ^t, we know that it solves the indifference condition for the monetary authority, $\bar{v} = c\left(w, \theta^t, \chi\right)$. Totally differentiating this condition with respect to χ, we obtain the derivative we are after:

$$\frac{d\theta^t}{d\chi} = \frac{\frac{d}{d\chi}\bar{v} - \left(\frac{\partial}{\partial\chi}c\left(w, \theta^t, \chi\right) + \frac{\partial}{\partial w}c\left(w, \theta^t, \chi\right)\frac{\partial}{\partial\chi}w\left(\theta^t, \chi\right)\right)}{\frac{\partial}{\partial w}c\left(w, \theta^t, \chi\right)\frac{\partial}{\partial\theta}w\left(\theta^t, \chi\right) + \frac{\partial}{\partial\theta}c\left(w, \theta^t, \chi\right)}, \tag{23}$$

Quite intuitively – and as confirmed by (14), (19) and (20) – the denominator of this expression is positive. The three underlying drivers operate in the same direction. First, the cost of pegging increases with the pressure from the private sector: $\frac{\partial}{\partial w}c\left(w,\theta^t,\chi\right) > 0$. Second, this pressure builds up as the fundamentals worsen: $\frac{\partial}{\partial \theta}w\left(\theta^t,\chi\right) > 0$ (recall that a higher fundamental θ means higher distortions). Third, a deterioration in the fundamentals also increases the cost of pegging directly: $\frac{\partial}{\partial \theta}c\left(w,\theta^t,\chi\right) > 0$. These effects will remain qualitatively the same in all the specifications we study below.

Thus, turning to the numerator in expression (23), it is the relative size of two sensitivities that determines whether the central bank's strength raises or lowers the threshold value of the fundamentals. The first is the sensitivity of the benefit of pegging to the central bank's strength: $\frac{d\bar{v}}{d\chi}$. In the present setting, this is zero. The second is the sensitivity of the cost of pegging to the central bank's strength: $\frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi} + \frac{\partial c}{\partial \chi}$. Intuitively – and as confirmed by (14), (19) and (20) – this sensitivity is negative: a stronger central bank (i) perceives a lower cost of pegging, $\frac{\partial c}{\partial \chi} < 0$, and (ii) is better at managing the devaluation pressure from the private sector: $\frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi} < 0$.

In the current setting, a stronger central bank faces higher fundamental thresholds because (in absolute terms) the benefit of pegging is less sensitive than the cost of pegging to the central bank's strength. Recall Proposition 1, which states that the peg is abandoned when the fundamentals rise above the relevant threshold. Thus, facing a higher threshold, a stronger central bank is *less* likely to abandon the peg.

4.2 Dispersed information

We remain in the one-shot game but now suppose that each worker does not observe θ directly but observes a private signal x, as defined in equation (15). We consider the limit in which the noise in private signals vanishes: $\sigma_x \to 0$.

This setting delivers two main takeaways. First, in line with lessons from the global games literature, the dispersed information leads to a unique equilibrium. Second, similar to the multiple equilibria in the perfect-foresight setting above, the unique equilibrium implies that a stronger central bank is less likely to abandon the peg.

We focus on an equilibrium in which the central bank follows a threshold devaluation strategy. We conjecture and verify that there exists a $\theta^* > 0$ such that p is given by equation (13) if $\theta \geq \theta^*$, and $p = 0$ otherwise. The wage-setting condition (10) delivers an expression for the individually optimal wage as a function of private information x, the threshold θ^* and the strength of the authority χ,

$$w_x\left(x,\theta^*,\chi\right) = \mathbb{E}\left[p\left(\theta,\theta^*,\chi\right)|x\right], \tag{24}$$

where the expectation is taken with respect to the posterior distribution of θ given the private signal x. Equation (24) highlights the conjecture that the individually optimal wage and the equilibrium price both depend on the threshold θ^* and the central bank's strength χ. However, the individual wage reflects the worker's private signal x, while the price reflects the fundamentals θ. Integrating across households we obtain an expression characterizing the average wage bill

$$w(\theta,\theta^*,\chi) = \mathbb{E}\left[\mathbb{E}\left[p(\theta,\theta^*,\chi)\,|x\right]|\theta\right], \tag{25}$$

where the outermost expectation is now taken with respect to the distribution of the private signal x conditional on the fundamental θ. In Appendix C we show that the average wage bill $w(\theta,\theta^*,\chi)$ is the fixed point of an operator,

$$w(\theta,\theta^*,\chi) = T[w](\theta,\theta^*,\chi). \tag{26}$$

The operator T is defined as

$$T[w](\theta,\theta^*,\chi) \equiv \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(w(\zeta,\theta^*,\chi)-\zeta\right)\phi\left(\zeta,\bar{\theta}(\theta),\sigma^2\right)d\zeta, \tag{27}$$

with $\sigma^2 \equiv \sigma_x^2\frac{\psi^2\left(\sigma_0^2+\sigma_x^2\right)+\sigma_0^2}{\sigma_0^2+\sigma_x^2}$ and $\bar{\theta}(\theta) \equiv \psi\theta + (1-\psi)\theta_0$. The function $\phi(\cdot,\mu,s)$ denotes the density of a Gaussian random variable with mean μ and variance s. We show in Appendix C.1 that the operator T has a unique fixed point, so the candidate equilibrium aggregate wage function is well-defined for any $\theta > 0$. We also show that this function is differentiable in all its three arguments.

Lemma 2 (Existence and uniqueness of the wage function)**.** *If workers believe the central bank follows a threshold strategy such that* $p = \frac{1}{1+\chi}(w+\theta)$ *if* $\theta \geq \theta^* < 0$ *and* $p = 0$ *otherwise, the aggregate nominal wage* $w(\theta,\theta^*,\chi)$ *is the unique fixed point of the operator* T *defined by equation* (27)*.*

Sketch of the proof. Although the operator T is not a contraction, the term $\frac{1}{1+\chi} \in (0,1)$. As a result, we can show that, starting from an appropriately selected function w_0, the sequence $\{T^n[w_0]\}$ is a Cauchy sequence in $\mathbb{R}$, and so for $n \to +\infty$ it converges to some w^* which is a fixed point of T by the continuity of T. We show uniqueness by contradiction.

We next verify that it is indeed optimal for the central bank to abandon the peg when $\theta \geq \theta^*$. By equation (17), the central bank switches to a managed float if and only if the cost of pegging is sufficiently high relative to the benefit:

$$\bar{v} \leq c\left(w(\theta,\theta^*,\chi),\theta^*,\chi\right), \tag{28}$$

with θ^* implicitly defined by $\bar{v} = c\left(w(\theta^*,\theta^*,\chi),\theta^*,\chi\right)$. Hence, to verify the conjecture that a threshold equilibrium exists, it suffices to show that the right-hand side of (28) is increasing in θ. We show this in Appendix C.4 and thus obtain the following proposition:

Proposition 3. *There exists a threshold equilibrium such that the central bank abandons the peg if fundamentals are sufficiently bad, $\theta \geq \theta^*$. The threshold θ^* is sandwiched between its common knowledge counterparts, $\underline{\theta} < \theta^* < \overline{\theta}$.*

In order to solve for the equilibrium average wage, illustrated in Figure 1, we need to resort to numerical methods. In a related setting, Guimaraes and Morris (2007) solve for the equilibrium in closed form. We cannot use the techniques in that paper because the private sector payoffs in our model are affected both by the likelihood of a devaluation and by its size.This implies a global game with continuous actions for *both* the private sector and the monetary authority.

That said, we prove analytically that the stronger is the central bank, i.e. the higher is χ, the higher is the threshold θ^*. We start with the analogue of (23) under dispersed information :

$$\frac{d\theta^*}{d\chi} = \left. \frac{\frac{d}{d\chi}\bar{v} - \left(\frac{\partial}{\partial w}c\left(w,\theta,\chi\right)\frac{\partial}{\partial \chi}w\left(\theta,\theta^*,\chi\right) + \frac{\partial}{\partial \chi}c\left(w,\theta,\chi\right)\right)}{\frac{\partial}{\partial w}c\left(w,\theta,\chi\right)\left(\frac{\partial}{\partial \theta}w\left(\theta,\theta^*,\chi\right) + \frac{\partial}{\partial \theta^*}w\left(\theta,\theta^*,\chi\right)\right) + \frac{\partial}{\partial \theta}c\left(w,\theta,\chi\right)}\right|_{\theta=\theta^*}. \tag{29}$$

Now, we need to consider separately the dependence of the wage bill on (i) the fundamentals and (ii) the threshold value of the fundamentals. Nevertheless, we show in Appendix C.4 that the denominator is still negative and for the same reasons as in the perfect-foresight setting: the cost of pegging declines as the fundamentals improve.

So, it again boils down to comparing the sensitivity of the benefit of pegging to the central bank's strength, $\frac{d\bar{v}}{d\chi}$, with the corresponding sensitivity of the cost of pegging, $\frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi} + \frac{\partial c}{\partial \chi}$. Since the benefits of pegging in the one-shot game stem from an ad hoc constant, the former sensitivity is zero. And, as in the perfect-foresight case, the cost of pegging is decreasing in χ (see (14) and results in Appendix C.3). Thus, the cost of pegging is more sensitive to χ than the benefit of pegging, implying that the critical threshold increases in χ: $\frac{d\theta^*}{d\chi} > 0$. In other words, a stronger central bank is less likely to abandon the peg.

Figure 2 illustrates this result as well as the statement in Proposition 3 that the critical threshold under dispersed information is sandwiched between the common knowledge thresholds.

5 Repeated games

In this section, the mechanism for attaining partial credibility of the peg stems from allowing the central bank and workers to play two subsequent games: one in the short run (SR) and one in the long run (LR). Each game features the same (net) cost of pegging, conditional on the aggregate wage: as given by expression (14).

5.1 Long run

The central bank's options in the long run depend on its actions in the short run.

If the central bank pegs in the short run, then it can – if it wishes – join a currency union that enforces the peg in the long run. Once in the currency union, there is no uncertainty about the exchange rate regime: i.e. the peg is perfectly credible. This means that equation (19) applies and the central bank's losses are zero.

Alternatively, the central bank may not enter the currency union in the long run. This could happen for one of two reasons. First, the central may choose to not exercise its option after pegging in the short run. Second, it would not have the option to join the currency union if it devalues in the short run. In either case, the central bank would face only the period loss (12) in the long run, which, as we showed above, implies that pegging is time inconsistent.

Time inconsistency of the peg implies that there is again no uncertainty about the exchange rate regime: if the central bank devalues in the short run, there is common knowledge that the regime is a managed float in the long run. In turn, common knowledge about the regime implies that the aggregate wage bill is the same irrespective of whether workers (i) observe θ_{LR} directly, as in Section 4.1; or (ii) observe infinitely precise private signals about θ_{LR}, as in Section 4.2. In Appendix E, we show that, under either information structure, the long-term wage rate is $\frac{1}{2}\frac{1}{\chi}\theta_{LR}$ under a managed float. Then, as in equation (20), the central bank's costs long-run costs under a managed float are equal to $\frac{1}{2}\frac{\theta_{LR}^2}{\chi}$.

The central bank's long-run costs under a (perfectly credible) peg are unambiguously lower than those under a (perfectly credible) float: $0 < \frac{1}{2}\frac{\theta_{LR}^2}{\chi}$. It follows that, if the central bank has the option of joining the currency union in the long run, it always exercises it. In comparison to a managed float, a currency union delivers a net benefit: $\frac{1}{2}\frac{\theta_{LR}^2}{\chi} > 0$. This is the long-run benefit of pegging in the short run.

5.2 Short run

The long-term benefit of joining the currency union gives rise to partial credibility of the peg in the short run. In particular, the central bank abandons the peg in the short run if and only if:

$$\underbrace{\beta\frac{1}{2}\frac{1}{\chi}\mathbb{E}\left[\theta_{LR}^2\right]}_{\text{long-term benefit}} \equiv v(\chi) \leq c(w,\theta_{SR},\chi) \equiv \underbrace{\frac{1}{2}\frac{1}{1+\chi}(w(\theta_{SR},\chi)+\theta_{SR})^2}_{\text{short-term cost}} \tag{30}$$

where $\beta \in (0,1)$ is the discount factor and the expectation operator incorporates the unconditional distribution of θ_{LR}. Inequality (30) is the repeated-games analogue of inequality (17) from the one shot-game setting. On the left-hand side, the endogenous long-term benefit of pegging, $v(\chi)$, replaces the ad hoc fixed benefit in the one-shot game. On the right-hand side, the short-term cost of pegging is as given by expression (14).

We will now study how the central bank's strength, χ, affects the likelihood of a devaluation, i.e. the value of fundamental threshold(s). As in the one-shot game, this will boil down to comparing the sensitivity of the short-term cost of pegging to χ with the corresponding sensitivity of the long-term benefit. The short term cost has not changed and, as we showed and explained above, it decreases with χ. Importantly, the long-term benefit has changed and – as expression (30) reveals – it also decreases in χ. To see the intuition, recall that the long-term benefit of pegging in the short term is the benefit of solving the time inconsistency problem by tying the central bank's hands within a currency union. Since a stronger central bank is better at managing inflation/devaluation on its own, it faces a smaller time inconsistency problem and thus stands to benefit less from joining the currency union.

In the remaining three subsections, we dissect the conditions under which the "short-term" sensitivity is weaker than the "long-term" sensitivity. When this is the case, a stronger central bank is more likely to abandon the peg in the short run.

5.2.1 Perfect foresight

First, let agents set their wages under perfect knowledge of the current fundamentals. As in the one-shot game, there are again multiple equilibria characterized by the respective short-run thresholds, $\underline{\theta}$ and $\overline{\theta}$ i.e. Proposition 1 still applies, with the caveat that the thresholds need to be replaced by

$$\underline{\theta} = \sqrt{v(\chi)\frac{\chi^2}{1+\chi}}, \tag{31}$$

and

$$\overline{\theta} = \sqrt{v(\chi)(1+\chi)}, \tag{32}$$

respectively.[8]

The impact of the central bank's strength, χ, on either threshold can again be summarized in the expression

$$\frac{d\theta^t}{d\chi} = \frac{\frac{dv(\chi)}{d\chi} - \left(\frac{\partial}{\partial w}c(w,\theta^t,\chi)\frac{\partial}{\partial \chi}w(\theta^t,\chi) + \frac{\partial}{\partial \chi}c(w,\theta^t,\chi)\right)}{\frac{\partial}{\partial w}c(w,\theta^t,\chi)\frac{\partial}{\partial \theta}w(\theta^t,\chi) + \frac{\partial}{\partial \theta}c(w,\theta^t,\chi)} \tag{33}$$

where θ^t stands for either $\underline{\theta}$ or $\overline{\theta}$, and w denotes the short-run average wage bill. Similar to

[8] To lighten notation, we do not explicitly index short-run objects by the subscript SR. This should not be a source of confusion as the long-run value of the fundamentals θ_{LR} only enters the equilibrium analysis indirectly through its second moment $\mathbb{E}\left[\theta_{LR}^2\right]$, which is subsumed in the long-term benefit $v(\chi)$.

the one-shot game, the short-term cost of pegging decreases as the fundamentals improve (i.e. the denominator is still positive, $\frac{\partial c}{\partial w}\frac{\partial w}{\partial \theta} + \frac{\partial c}{\partial \theta} > 0$) and as the strength of the central bank increases (i.e. the term in parentheses in the numerator is still negative, $\frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi} + \frac{\partial c}{\partial \chi} < 0$).

In contrast to the corresponding one-shot-game-expression (23), however, the long-term benefit of pegging, $v(\chi)$, now depends on the strength of the central bank χ. This property of the repeated-games setup leads to the result that, in contrast to the one-shot game, χ has opposite effects on the two thresholds.

Corollary 2. *The lower threshold of the multiplicity region, $\underline{\theta}$, is increasing in central bank's strength, χ. The upper threshold, $\overline{\theta}$, is decreasing in χ.*

Proof. See Appendix F.

Corollaries 1 and 2 have similar implications for the lower threshold $\underline{\theta}$. This threshold stems from workers perceiving a devaluation. Since devaluation is sensitive to the central bank's strength, χ, so is the average wage bill, i.e. so is workers' pressure on the authority to abandon the peg. This makes the short-term cost of pegging more sensitive to χ than the long-term benefit of pegging, both in the one-shot and the repeated-games settings: $\left|\frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi} + \frac{\partial c}{\partial \chi}\right| > \left|\frac{dv}{d\chi}\right|$. The upshot is that – if workers anticipate a devaluation as long as it is rational to do so – a stronger central bank abandons the peg at weaker fundamentals: $\bar{\theta}$ decreases with χ.

By contrast, Corollaries 1 and 2 differ in terms of the upper threshold $\overline{\theta}$. This threshold stems from workers perceiving a peg. Under a peg, the central bank's strength, χ, does not matter and the wage bill is insensitive to it: $\frac{\partial w}{\partial \chi} = 0$. The sensitivity of the short-term cost of pegging to χ, $\left|\frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi} + \frac{\partial c}{\partial \chi}\right|$, thus weakens but remains negative: this is a flip side of the fact that – irrespective of the wage bill – a stronger central bank dislikes inflation/devaluation more: $\frac{\partial c}{\partial \chi} < 0$. Since the ad hoc long-term benefit of pegging in the one-shot game is insensitive to χ, however, the "short term" sensitivity still dominates. The story is reversed in the repeated games setting, as then the endogenous long-term benefits of pegging are sensitive to χ: $\frac{dv}{d\chi} < 0$ and $\left|\frac{\partial c}{\partial \chi} + \frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi}\right| = \left|\frac{\partial c}{\partial \chi}\right| < \left|\frac{dv}{d\chi}\right|$, implying that a *weaker* central bank abandons the peg for weaker fundamentals: $\underline{\theta}$ is increasing in χ. Thus, if there are repeated games and workers anticipate a peg, a weaker central bank is less likely to abandon the peg.

5.2.2 Dispersed information

Suppose that workers observe noisy private signals about the fundamentals, as defined in equation (15). As in the one-shot game, there is a threshold equilibrium in the repeated games setting. Proposition 3 still applies, with the threshold value of the fundamentals, θ^*

being equal to the positive root of $v(\chi) = c(w(\theta^*, \theta^*, \chi), \theta^*, \chi)$. Implicitly differentiating the indifference condition for the monetary authority with respect to χ, we obtain:

$$\frac{d\theta^*}{d\chi} = \left. \frac{\frac{d}{d\chi}v(\chi) - \left(\frac{\partial}{\partial w}c(w,\theta,\chi)\frac{\partial}{\partial \chi}w(\theta,\theta^*,\chi,\lambda) + \frac{\partial}{\partial \chi}c(w,\theta,\chi)\right)}{\frac{\partial}{\partial w}c(w,\theta,\chi)\left(\frac{\partial}{\partial \theta}w(\theta,\theta^*,\chi,\lambda) + \frac{\partial}{\partial \theta^*}w(\theta,\theta^*,\chi,\lambda)\right) + \frac{\partial}{\partial \theta}c(w,\theta,\chi)} \right|_{\theta=\theta^*}. \tag{34}$$

We obtain again that the short-term cost of pegging declines as the fundamentals improve (implying that $\frac{\partial c}{\partial w}\left(\frac{\partial w}{\partial \theta} + \frac{\partial w}{\partial \theta^*}\right) + \frac{\partial c}{\partial \theta} > 0$) and as the strength of the central bank increases ($\frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi} + \frac{\partial c}{\partial \chi} < 0$). In contrast to the one-shot setting, the long-term benefits of a devaluation decrease in the central bank's strength, $\frac{dv}{d\chi} < 0$.

Thus, to determine the direction of the impact of χ on the threshold of the fundamentals, θ^*, we again need to compare the "short term" sensitivity $\left|\frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi} + \frac{\partial c}{\partial \chi}\right|$ to the "long-term" sensitivity $\left|\frac{dv}{d\chi}\right|$. There being dispersed information does not affect the latter sensitivity because the currency regime is common knowledge in the long term. We thus turn to the former, short-term sensitivity. Since dispersed information creates strategic uncertainty among workers in the short run, it impairs their coordination capacity. As a result, the short-term sensitivity under dispersed information is *below* the corresponding short-term sensitivity when workers have perfect foresight and coordinate their wages on the perception of a *devaluation*.[9] But it is also *above* the corresponding short-term sensitivity when workers have perfect foresight and coordinate their wages on the perception of a *peg*.

This leaves open the question as to whether the "short term" sensitivity is larger than the "long term" sensitivity under dispersed information. We answer this question by resorting to numerical simulations, which reveal that, in qualitative terms, the short-term cost of pegging is still more sensitive to changes in χ than is the long-term benefit. Thus, as portrayed in Figure 3 and Figure 4, θ^* is an increasing function of χ. In other words, a stronger central bank is less likely to abandon the peg.

5.2.3 Predetermined wages

In this subsection, we explore a mechanism that can reverse the relationship between the central bank's strength and the likelihood of a devaluation. This mechanism affects only the sensitivity of the short-term cost of pegging to the central bank's strength and brings it below the corresponding long-term sensitivity. As it hinges on the endogenous intertemporal trade-off of the repeated-games setting, the mechanism does not work in the one-shot game.

Let a share $\lambda \in [0, 1)$ of the workers have predetermined wages in *the short run* but let

[9] This stems from the smaller sensitivity of the wage bill to χ under dispersed information than under perfect foresight and a commonly perceived devaluation. Concretely, on the basis of results in Appendix C.2 and Appendix C.4), we obtain that $-\frac{\partial}{\partial \chi}w(\theta, \theta^*, \chi) < \frac{\chi}{\kappa+\chi}\left(-\frac{\kappa}{\chi^2}\theta\right) < -\frac{\kappa}{\chi^2}\theta$.

all workers have fully flexible wages in the long run. A higher λ means that more workers cannot respond to the perceived likelihood or magnitude of devaluations. Thus, the higher is λ, the less sensitive is the short-term wage bill to the strength of the central bank, χ, and to the fundamentals, θ. In comparison to the specifications studied above, this lowers the sensitivity of the short-term cost of pegging to changes in χ. And because of the flexibility of long-run wages, the value of λ plays no role in the long run.

Perfect foresight. Under perfect foresight, Proposition 1 continues to apply and there are multiple equilibria. The upper dominance threshold $\bar{\theta}$ is unaffected by the existence of passive workers because it stems from active workers perceiving a peg and thus behaving as if they were passive themselves.[10] Thus, we again obtain $\frac{d\bar{\theta}}{d\chi} < 0$.

In Appendix G, we derive the lower threshold, $\underline{\theta}$, which reflects the share of predetermined wages:

$$\underline{\theta} = \sqrt{\frac{v(\chi)}{\frac{1+\chi}{(\lambda+\chi)^2}}}, \tag{35}$$

where $v(\chi)$ is as defined in expression (30). We thus obtain the following result.

Corollary 3. *The lower threshold of the multiplicity region decreases in the central bank's strength, χ, for a sufficiently high fraction of predetermined wages, i.e. for $\lambda \in \left(\frac{\chi}{1+2\chi}, 1\right]$, and increases otherwise. The upper threshold, $\bar{\theta}$, is unaffected by λ and decreases in χ.*

To see the parallel with the corresponding result in Corollary 2 of Section 5.2.1, note that the impact of the central bank's strength on the lower threshold is now given by:

$$\frac{d\underline{\theta}}{d\chi} = \frac{\frac{dv(\chi)}{d\chi} - \left(\frac{\partial}{\partial w}c(w,\underline{\theta},\chi,\lambda)\frac{\partial}{\partial\chi}w(\underline{\theta},\chi,\lambda) + \frac{\partial}{\partial\chi}c(w,\underline{\theta},\chi,\lambda)\right)}{\frac{\partial}{\partial w}c(w,\underline{\theta},\chi,\lambda)\frac{\partial}{\partial\theta}w(\underline{\theta},\chi,\lambda) + \frac{\partial}{\partial\theta}c(w,\underline{\theta},\chi,\lambda)}. \tag{36}$$

In comparison to equation (33), the long-term benefit of pegging, $v(\chi)$, is the same. In addition, the short-term cost of pegging still decreases in the central bank's strength, i.e. $\frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi} + \frac{\partial c}{\partial \chi} < 0$, and as the fundamentals improve, i.e. $\frac{\partial c}{\partial w}\frac{\partial w}{\partial \theta} + \frac{\partial c}{\partial \theta} > 0$. That said, as the share of predetermined wages rises – i.e. as λ approaches 1 – the equilibrium converges to the one in which all workers expect the peg to be maintained, i.e. the equilibrium in which the short-term sensitivity is lower than the long-term sensitivity (recall Section 5.2.1): $\left|\frac{dv(\chi)}{d\chi}\right| > \left|\frac{\partial c}{\partial w}\frac{\partial w}{\partial \theta} + \frac{\partial c}{\partial \theta}\right|$. By continuity, the last inequality holds for $\lambda < 1$ and we thus obtain Corollary 3.

Dispersed information. Suppose now that there is dispersed information about θ. Then Proposition 3, which established the existence of a threshold equilibrium under dis-

[10] This is without loss of generality. Even if we had allowed the predetermined wage to be poisitive, the sensitivity of the lower threshold $\underline{\theta}$ with respect to χ would be independent of both the preset wage level and λ.

persed information, still applies, with the caveat that the aggregate wage now satisfies

$$w\left(\theta,\theta^{*},\chi,\lambda\right)=\left(1-\lambda\right)\mathbb{E}_{x}\left[w_{1}\left(x,\theta^{*},\chi,\lambda\right)|\theta\right], \tag{37}$$

where $w_1\left(x,\theta^{*},\chi,\lambda\right)$ denotes the wage set by active workers, and is as given in equation (10). The devaluation condition of the authority is still given by (17), with the average wage w now given by (37). Implicitly differentiating this condition, we generalize equation (34) to obtain the impact of the central bank's strength on the threshold value of the fundamentals in the presence of passive workers:

$$\frac{d\theta^{*}}{d\chi}=\left.\frac{\frac{d}{d\chi}v\left(\chi\right)-\left(\frac{\partial}{\partial w}c\left(w,\theta,\chi,\lambda\right)\frac{\partial}{\partial\chi}w\left(\theta,\theta^{*},\chi,\lambda\right)+\frac{\partial}{\partial\chi}c\left(w,\theta,\chi\right)\right)}{\frac{\partial}{\partial w}c\left(w,\theta,\chi\right)\left(\frac{\partial}{\partial\theta}w\left(\theta,\theta^{*},\chi,\lambda\right)+\frac{\partial}{\partial\theta^{*}}w\left(\theta,\theta^{*},\chi,\lambda\right)\right)+\frac{\partial}{\partial\theta}c\left(w,\theta,\chi\right)}\right|_{\theta=\theta^{*}(\lambda)} \tag{38}$$

The long-term benefit of pegging, $v\left(\chi\right)$, is the same as in equation (34). In addition, the short-term cost of pegging still decreases in the central bank's strength, i.e. $\frac{\partial c}{\partial w}\frac{\partial w}{\partial\chi}+\frac{\partial c}{\partial\chi}<0$, and as the fundamentals improve, i.e. $\frac{\partial c}{\partial w}\frac{\partial w}{\partial\theta}+\frac{\partial c}{\partial\theta}>0$. However, a higher λ dampens the sensitivity of the average wage bill with respect to χ, $\left|\frac{\partial w}{\partial\chi}\right|$, and, with it, dampens the short-term sensitivity $\left|\frac{\partial c}{\partial w}\frac{\partial w}{\partial\chi}+\frac{\partial c}{\partial\chi}\right|$.

We resort to numerical simulations to explore whether dampening the elasticity of the average wage bill is sufficient to ensure that the long-term sensitivity dominates the short-term sensitivity, $\left|\frac{dv}{d\chi}\right|>\left|\frac{\partial c}{\partial w}\frac{\partial w}{\partial\chi}+\frac{\partial c}{\partial\chi}\right|$. We find that this can indeed be the case even for very small λ, as illustrated in Figure 5 and Figure 6 where we obtain that the dispersed information threshold is decreasing in χ.

Thus, provided that a sufficiently large fraction of the wages are predetermined, there is a reversal of the conventional wisdom: a *weaker* central bank is *less* likely to abandon the peg. This is the case when active workers have perfect foresight and are inclined to coordinate their actions on the perception of a peg. And it is also the case under dispersed information.

6 Conclusion

In this paper, we built on microfoundations and developed a repeated games version of the textbook second-generation currency crises model, in which the central bank faces time inconsistency issues. In our model, both the short-term cost and the long-term benefit of pegging decrease with the strength of the central bank, i.e. the central bank's dislike of inflation. This implies that a weaker central bank is more likely to preserve the peg in order to solve its time inconsistency issues, provided that one condition is satisfied: namely, the long term benefit is more sensitive than the short term cost to changes in the central bank's strength. This condition would indeed hold for a sufficiently passive private sector, e.g. a sector in which a sufficiently high fraction of wages are predetermined.

Appendices

A The linearized model

In this section we present a log linear approximation of the wage-setting equation and of social welfare. Together, these two blocks constitute the "reduced form" of our setup. Throughout this appendix we use lowercase to denote log of variables, and hats to denote deviations of log variables from their log values in the approximating equilibrium. So for any variable X_t, $x_t = \ln(X_t)$ and $\hat{x}_t \equiv \ln(X_t) - \ln(X^*)$.

We linearize around a symmetric two-period $t = 1, 2$ equilibrium of the non-linear model with no uncertainty. Given (a^*, p^*), this equilibrium is a vector (c^*, n^*, w^*). We consider the same process for productivity and in the approximated and approximating equilibrium, so $\hat{a}_t = a^* - a^* = 0$. We also assume that in the approximating equilibrium the wage markup is the same as in the equilibrium to be approximated, so $\hat{\mu}_t = \mu_t - \mu = 0$. We assume that the approximating equilibrium is not efficient. In particular, labour market distortions drive a wedge between the marginal rate of substitution between consumption and leisure MRS^* and the marginal product of labour MPN^*. From the wage-setting equation, (7), we obtain that $\frac{W^*}{P^*} = \mathcal{M}^* MRS^*$, where $\mathcal{M}^*_t \equiv \frac{\nu}{\nu-1}$ and $MRS^*_t \equiv (C^*)^{\varsigma} (N^*)^{\varphi}$. From the optimality condition for the final good producer, (5), we obtain that $\frac{W^*}{P^*} = MPN^*$, where $MPN^* \equiv (1-\alpha)(N^*_t)^{-\alpha}$. The equilibrium is inefficient since $\frac{MPN^*}{MRS^*} = \mathcal{M}^* > 1$. Since $MPN^* > MRS^*$ and MPN^* is decreasing in N^* while MRS^* is increasing, we have that employment is too low, and output is accordingly also too low.

A.1 Approximating the wage-setting condition

We next log-linearize the wage-setting condition (7),

$$\mathbb{E}\left[C_t^{-\varsigma}\frac{W_{i,t}}{P_t}N_{i,t}\right] = \mathbb{E}\left[\mu_t N_{i,t}^{1+\varphi}\right],$$

with $\mu \equiv \ln\{\mathcal{M}\} = \ln\left(\frac{\nu}{\nu-1}\right)$. Consider first the term within the expectation on the left-hand side. Taking a first order Taylor approximation around (c^*, w^*, p^*, n^*) returns

$$C_t^{-\varsigma}\frac{W_{i,t}}{P_t}N_{i,t} = \exp\left\{-\varsigma c^* - p^* + w^* + n^*\right\}\left[1 - \varsigma(c_t - c^*) - (p_t - p^*) + (w_{i,t} - w^*) + (n_{i,t} - n^*)\right]. \tag{A.1}$$

A first order Taylor approximation of the term within the expectation on the right-hand side around n^* instead returns

$$\mu N_{i,t}^{1+\varphi} = \exp\{\mu + (1+\varphi)\,n^*\}\,[1 + (1+\varphi)\,(n_{i,t} - n^*)]\,. \tag{A.2}$$

The wage-setting condition evaluated in the approximation point returns $w^* - p^* = \mu + \varsigma c^* + \varphi n^*$. As a result, we can write (A.1) as

$$C_t^{-\varsigma}\frac{W_{i,t}}{P_t}N_{i,t} = \exp\{\mu + (1+\varphi)\,n^*\}\,[1 - \varsigma\,(c_t - c^*) - (p_t - p^*) + (w_{i,t} - w^*) + (n_{i,t} - n^*)]\,.$$

Taking expectations and using again the result that $w_t^* - p^* = \mu + \varsigma c^* + \varphi n_t^*$, we finally obtain

$$w_{i,t} = \mathbb{E}[\mu + p_t + \varsigma c_t + \varphi n_{i,t}]. \tag{A.3}$$

Log-linearizing the labour demand equation, (6), we obtain

$$n_{i,t} = -\nu\,(w_{i,t} - w_t) + n_t, \tag{A.4}$$

so then substituting back into expression (A.5) and collecting terms, we end up with

$$(1+\varphi\nu)\,w_{i,t} = \mathbb{E}[\mu + p_t + \varsigma c_t + \varphi\nu w_t + \varphi n_t] \tag{A.5}$$

We now re-write equation (A.5) so the right hand side depends only on TFP a_t, the log aggregate nominal wage w_t and the log price level, p_t. Using the goods market clearing condition $c_t = y_t$, the (log) production function $y_t = a_t + (1-\alpha)\,n_t$ (see (3)), and the (log) optimality condition for firms, $\ln(1-\alpha) + a_t - \alpha n_t = w_t - p_t$ (see (5)) we obtain

$$\begin{aligned} w_{i,t} \;=\; & \mathbb{E}\left[\frac{\varphi+\varsigma}{\alpha(1+\nu\varphi)}a_t + \frac{\varphi+\varsigma(1-\alpha)}{\alpha(1+\nu\varphi)}\ln(1-\alpha)\right. \\ & \left. + \frac{\varsigma(1-\alpha)+\alpha+\varphi}{\alpha(1+\nu\varphi)}p_t - \frac{\varsigma(1-\alpha)+\varphi(1-\alpha\nu)}{\alpha(1+\nu\varphi)}w_t\right]. \end{aligned}$$

And now in deviations from the approximating equilibrium we write the wage-setting equation more compactly as

$$\hat{w}_{i,t} = \mathbb{E}\left[\delta_a\hat{a}_t + \delta_{p,t}\hat{p}_t + \delta_{w,t}\hat{w}_t\right], \tag{A.6}$$

where hats denote deviations from the approximating equilibrium and the coefficients are given as

$$\delta_{a,t} \equiv \tfrac{\varphi+\varsigma}{\alpha(1+\nu\varphi)}, \quad \delta_p \equiv \tfrac{\varsigma(1-\alpha)+\alpha+\varphi}{\alpha(1+\nu\varphi)}, \quad \delta_w \equiv -\tfrac{\varsigma(1-\alpha)+\varphi(1-\alpha\nu)}{\alpha(1+\nu\varphi)},$$

Under the assumption that $(\varsigma, \varphi) = (0, 0)$, we finally end up with

$$\hat{w}_{i,t} = \mathbb{E}\left[\hat{p}_t\right].$$

A.2 Approximating social welfare

Taking a second order Taylor approximation of social welfare, we obtain

$$\int_0^1 (U_{i,t} - U^*)\, di = (C^*)^{-\varsigma} C^* \left(\hat{y}_t + \frac{1-\varsigma}{2} \hat{y}_t^2 \right) - (N^*)^{\varphi} N^* \left(\int_0^1 \hat{n}_{i,t} di + \frac{1+\varphi}{2} \int_0^1 \hat{n}_{i,t}^2 di \right), \tag{A.7}$$

Our objective is to write the second term in the sum $\int_0^1 \hat{n}_{i,t} di + \frac{1+\varphi}{2} \int_0^1 \hat{n}_{i,t}^2 di$ so it depends on $\hat{y}_t$ and $\hat{y}_t^2$ only.

To that end, recall the definition of the intermediate good index, $N_t = \left(\int (N_{i,t})^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}}$. It implies that

$$1 \equiv \int \left(\frac{N_{i,t}}{N_t} \right)^{\frac{\nu-1}{\nu}} di.$$

Taking a second order Taylor approximation of $\left(\frac{N_{i,t}}{N_t} \right)^{\frac{\nu-1}{\nu}}$ returns

$$\left(\frac{N_{i,t}}{N_t} \right)^{\frac{\nu-1}{\nu}} = 1 + \frac{\nu-1}{\nu} (\hat{n}_{i,t} - \hat{n}_t) + \frac{1}{2} \left(\frac{\nu-1}{\nu} \right)^2 (\hat{n}_{i,t} - \hat{n}_t)^2.$$

Hence, we obtain

$$1 \equiv \int \left(\frac{N_{i,t}}{N_t} \right)^{\frac{\nu-1}{\nu}} di = 1 + \frac{\nu-1}{\nu} \int (\hat{n}_{i,t} - \hat{n}_t)\, di + \frac{1}{2} \left(\frac{\nu-1}{\nu} \right)^2 \int (\hat{n}_{i,t} - \hat{n}_t)^2 di.$$

Re-arranging we get

$$\begin{aligned} \int \hat{n}_{i,t} di &= \hat{n}_t - \frac{1}{2} \frac{\left(\frac{\nu-1}{\nu}\right)^2}{\frac{\nu-1}{\nu}} \int (\hat{n}_{i,t} - \hat{n}_t)^2 di \\ &= \hat{n}_t - \frac{1}{2} (\nu - 1)\, \nu \int (\hat{w}_{i,t} - \hat{w}_t)^2 di \end{aligned} \tag{A.8}$$

where the second line uses the expression for labour demand, $\hat{n}_{i,t} = -\nu (\hat{w}_{i,t} - \hat{w}_t) + \hat{n}_t$. This takes care of the first integral in $\int_0^1 \hat{n}_{i,t} di + \frac{1+\varphi}{2} \int_0^1 \hat{n}_{i,t}^2 di$. We still need to get rid of the second integral,

$$\begin{aligned} \int_0^1 \hat{n}_{i,t}^2 di &= \int (-\nu (\hat{w}_{i,t} - \hat{w}_t) + \hat{n}_t)^2 di \\ &= \hat{n}_t^2 + \nu^2 \int (\hat{w}_{i,t} - \hat{w}_t)^2 di - 2\nu \hat{n}_t \int (\hat{w}_{i,t} - \hat{w}_t)\, di. \end{aligned}$$

Consider the third term, $\int (\hat{w}_{i,t} - \hat{w}_t)\, di$. Taking a first order Taylor approximation of the average wage $\left(\frac{W_{i,t}}{W_t}\right)^{1-\nu}$ returns

$$1 \equiv \int \left(\frac{W_{i,t}}{W_t}\right)^{1-\nu} di = 1 + (1-\nu) \int (\hat{w}_{i,t} - \hat{w}_t)\, di + \frac{1}{2}(1-\nu)^2 \int (\hat{w}_{i,t} - \hat{w}_t)^2\, di,$$

that is,

$$\int (\hat{w}_{i,t} - \hat{w}_t)\, di = \frac{1}{2}(\nu - 1) \int (\hat{w}_{i,t} - \hat{w}_t)^2\, di.$$

Substituting back in the expression for $\int_0^1 \hat{n}_{i,t}^2 di$ we obtain

$$\int_0^1 \hat{n}_{i,t}^2 di = \hat{n}_t^2 + \nu^2 \int (\hat{w}_{i,t} - \hat{w}_t)^2\, di,$$

since $-\nu \hat{n}_t (\nu - 1) \int (\hat{w}_{i,t} - \hat{w}_t)^2$ is a term of order higher than 2. Letting $var_i \{w_{i,t}\} \equiv \int (\hat{w}_{i,t} - \hat{w}_t)^2\, di$, we can finally write $\int_0^1 \hat{n}_{i,t} di + \frac{1+\varphi}{2} \int_0^1 \hat{n}_{i,t}^2 di$ as

$$\int \hat{n}_{i,t} di + \frac{1+\varphi}{2} \int_0^1 \hat{n}_{i,t}^2 di = \hat{n}_t + \frac{1+\varphi}{2} \hat{n}_t^2 + \frac{1}{2} \nu (\nu\varphi + 1)\, var_i \{w_{i,t}\}.$$

Substituting back into equation equation (A.7), we are left with

$$\int_0^1 \frac{(U_{i,t} - U^*)}{(C^*)^{-\varsigma} C^*} di = \hat{y}_t + \frac{1-\varsigma}{2} \hat{y}_t^2 - \frac{(N^*)^{\varphi} N^*}{(C^*)^{-\varsigma} C^*} \left(\hat{n}_t + \frac{1+\varphi}{2} \hat{n}_t^2 + \frac{1}{2} \nu (\nu\varphi + 1)\, var_i \{w_{i,t}\} \right).$$

Now using the production function $\hat{y}_t = \hat{a}_t + (1-\alpha)\hat{n}_t$, substituting in and ignoring terms independent of policy we obtain

$$\begin{aligned} \int_0^1 \frac{U_{i,t} - U^*}{(C^*)^{-\varsigma} C^*} di =& \hat{y}_t + \frac{1-\varsigma}{2} \hat{y}_t^2 \\ & - \frac{(N^*)^{\varphi} N^*}{(C_t^*)^{-\varsigma} C^*} \left(\frac{\hat{y}_t - \hat{a}_t}{1-\alpha} + \frac{1+\varphi}{2} \frac{(\hat{y}_t - \hat{a}_t)^2}{(1-\alpha)^2} + \frac{\nu(1+\varphi\nu)}{2} var_i \{w_{i,t}\} \right). \end{aligned}$$

Now consider now the term $\frac{(N^*)^{\varphi} N^*}{(C^*)^{-\varsigma} C^*}$,

$$\begin{aligned} \frac{(N^*)^{\varphi} N^*}{(C_t^*)^{-\varsigma} C^*} &= MRS^* \frac{N^*}{C^*} = \frac{MRS^*}{MPN^*}(1-\alpha) \text{ since } MPN^* = (1-\alpha) C_t^*/N^* \\ &= \left(1 - \frac{1}{\nu}\right)(1-\alpha) \end{aligned}$$

where the second line follows since $MRS^* = \frac{\frac{W_t^*}{P^*}}{\mathcal{M}}$ and $MPN^* = \frac{W^*}{P^*}$, and recalling that $\mathcal{M} = \frac{\nu}{\nu - 1}$. The term $\frac{1}{\nu}$ can thus be interpreted as steady state distortions. We assume that

$\frac{1}{\nu}$ is small enough that its product with a second order term can be ignored. As a result,

$$\int_0^1 \frac{U_{i,t}-U^*}{\left(C^*\right)^{-\varsigma} C^*} di \approx \hat{y}_t + \frac{1-\varsigma}{2}\hat{y}_t^2 - \left(1-\frac{1}{\nu}\right)(1-\alpha)\left(\frac{\hat{y}_t-\hat{a}_t}{1-\alpha} + \frac{1+\varphi}{2}\frac{\left(\hat{y}_t-\hat{a}_t\right)^2}{\left(1-\alpha\right)^2}\right) - \frac{\Upsilon}{2} var_i\left\{w_{i,t}\right\}$$
$$= -\frac{1}{2}\left(\varsigma + \frac{\alpha+\varphi}{1-\alpha}\right)\left(\hat{y}_t - \frac{\varphi+1}{\varsigma+\alpha+\varphi-\varsigma\alpha}\hat{a}_t\right)^2 + \frac{1}{\nu}\hat{y}_t - \frac{\Upsilon}{2} var_i\left\{w_{i,t}\right\},$$

where $\Upsilon \equiv (1-\alpha)\,\nu\,(1+\varphi\nu)$. Note that

$$\begin{aligned} \hat{y}_t - \frac{\varphi+1}{\varsigma+\alpha+\varphi-\varsigma\alpha}\hat{a}_t &= y_t - y^* \text{ since } \hat{a}_t = 0 \\ &= y_t - y_t^e + y_t^e - y^* \\ &= y_t - y_t^e - \left(y^* - y_t^e\right) \end{aligned}$$

Hence we can write the losses as

$$L = \frac{1}{2}\left(\varsigma + \frac{\alpha+\varphi}{1-\alpha}\right)\left(\left(y_t - y_t^e\right) - \left(y^* - y_t^e\right)\right)^2 - \frac{1}{\nu}\left(\left(y_t - y_t^e\right) - \left(y^* - y_t^e\right)\right) + \frac{1}{2}\Upsilon var_i\left\{w_{i,t}\right\}.$$

This is useful for interpretation. Moreover, observe that since $\hat{y}_t = \hat{a}_t + (1-\alpha)\left(\hat{a}_t - (\hat{w}-\hat{p})\right)$ and $\hat{a}_t = 0$, we can write this as

$$\begin{aligned} L &= \frac{1}{2}\left(\varsigma + \frac{\alpha+\varphi}{1-\alpha}\right)\hat{y}_t^2 - \frac{1}{\nu}\hat{y}_t + \frac{\Upsilon}{2} var_i\left\{w_{i,t}\right\}, \\ &= \frac{1}{2}\left(\varsigma + \frac{\alpha+\varphi}{1-\alpha}\right)\left(\frac{1-\alpha}{\alpha}\right)^2 (\hat{w}-\hat{p})^2 + \frac{1}{\nu}\left(\frac{1-\alpha}{\alpha}\right)(\hat{w}-\hat{p}) + \frac{\Upsilon}{2} var_i\left\{w_{i,t}\right\}. \end{aligned}$$

Now consider the term $\frac{\Upsilon}{2} var_i\left\{w_{i,t}\right\}$. Under common knowledge about θ, all workers set the same wage, so this term is zero because $var_i\left\{w_{i,t}\right\} = 0$. Under dispersed information about θ, on the other hand, $var_i\left\{w_{i,t}\right\} > 0$. The variance of wages, however, arises purely because of heterogenous beliefs, not because of fluctuations in the price level. Hence, in the case of heterogenous information, we can treat $\frac{\Upsilon}{2} var_i\left\{w_{i,t}\right\}$ as a term independent of monetary policy in the approximation of social welfare. Hence, we finally obtain

$$L = \frac{1}{2}(\hat{w}-\hat{p})^2 + \frac{1}{\nu}\frac{1}{\left(\varsigma + \frac{\alpha+\varphi}{1-\alpha}\right)\left(\frac{1-\alpha}{\alpha}\right)}(\hat{w}-\hat{p}).$$

B Aggregate wage with dispersed info

ior about θ with mean θ_0 and variance σ_0^2. Recall that the distribution of signals is a Gaussian with mean θ and variance σ_x^2. The posterior belief about θ conditional on the private signal x is therefore also a Gaussian with mean $\psi x + (1-\psi)\,\theta_0$ and variance $\sigma_{\theta|x}^2 = \frac{\sigma_0^2\sigma_x^2}{\sigma_0^2+\sigma_x^2}$, where

$\psi \equiv \frac{\frac{1}{\sigma_x^2}}{\frac{1}{\sigma_0^2}+\frac{1}{\sigma_x^2}}$.

Dropping hats and time subscripts, the optimal wage equation for a worker who has observed signal x becomes

$$w_i(x,\theta^*,\chi) = \int_{-\infty}^{+\infty} p(\zeta,\theta^*,\chi)\phi\left(\zeta,\psi x + (1-\psi)\,\theta_0,\sigma_{\theta|x}\right) d\zeta,$$

Next, aggregate over x to obtain

$$w\left(\theta,\theta^*,\chi\right) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p\left(\zeta,\theta^*,\chi\right)\phi\left(\zeta,\psi x + (1-\psi)\,\theta_0,\sigma_{\theta|x}\right)\phi\left(x,\theta,\sigma_x\right) d\zeta dx,$$

We next show that we can integrate out x, since

$$\phi\left(\zeta,\psi x + (1-\psi)\,\theta_0,\sigma_{\theta|x}\right)\phi\left(x,\theta,\sigma_x\right) = \phi\left(\zeta,\bar{\theta}\left(\theta\right),\sigma\right)\phi\left(x,\mu_1,\sigma_1\right)$$

with

$$\bar{\theta}\left(\theta\right) \equiv \psi\theta + (1-\psi)\,\theta_0, \quad \sigma^2 \equiv \sigma_x^2\frac{\psi^2\left(\sigma_0^2+\sigma_x^2\right)+\sigma_0^2}{\sigma_0^2+\sigma_x^2}$$

and

$$\mu_1 \equiv \frac{\sigma_0^2\theta+\psi\left(\sigma_0^2+\sigma_x^2\right)\zeta+\psi(\psi-1)\left(\sigma_0^2+\sigma_x^2\right)\theta_0}{\sigma_0^2+\psi\left(\sigma_0^2+\sigma_x^2\right)+\psi(\psi-1)\left(\sigma_0^2+\sigma_x^2\right)}, \quad \sigma_1^2 \equiv \frac{\sigma_0^2\sigma_x^2}{\psi^2\left(\sigma_0^2+\sigma_x^2\right)+\sigma_0^2}.$$

Consider then the product of the two Gaussian densities,

$$\frac{1}{\sqrt{2\pi\sigma_x^2}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{\frac{\sigma_0^2\sigma_x^2}{\sigma_0^2+\sigma_x^2}}}\exp\left\{-\frac{1}{2\sigma_x^2}\left(x-\theta\right)^2 - \frac{1}{2}\frac{1}{\frac{\sigma_0^2\sigma_x^2}{\sigma_0^2+\sigma_x^2}}\left(\zeta-\psi x-(1-\psi)\,\theta_0\right)^2\right\}.$$

After some manipulations of the term in curly braces we obtain:

$$\begin{aligned}&-\frac{1}{2}\frac{1}{\frac{\sigma_0^2\sigma_x^2}{\psi^2\left(\sigma_0^2+\sigma_x^2\right)+\sigma_0^2}}\left(x-\frac{\sigma_0^2\theta+\psi\left(\sigma_0^2+\sigma_x^2\right)z+\psi\left(\psi-1\right)\left(\sigma_0^2+\sigma_x^2\right)w}{\sigma_0^2+\psi\left(\sigma_0^2+\sigma_x^2\right)+\psi\left(\psi-1\right)\left(\sigma_0^2+\sigma_x^2\right)}\right)^2\\ &-\frac{1}{2}\frac{1}{\sigma_x^2\frac{\psi^2\left(\sigma_0^2+\sigma_x^2\right)+\sigma_0^2}{\sigma_0^2+\sigma_x^2}}\left(z-\psi\theta-(1-\psi)\,w\right)^2\end{aligned}$$

Since $\frac{\sigma_0^2\sigma_x^2}{\psi^2\left(\sigma_0^2+\sigma_x^2\right)+\sigma_0^2}\sigma_x^2\frac{\psi^2\left(\sigma_0^2+\sigma_x^2\right)+\sigma_0^2}{\sigma_0^2+\sigma_x^2} = \sigma_x^2\frac{\sigma_x^2\sigma_0^2}{\sigma_0^2+\sigma_x^2} = \sigma_x^2\sigma_{\theta|x}^2$, we get the result above.

Integrating out x, the aggregate wage equation can be written as

$$w\left(\theta,\theta^*,\chi\right) = \int_{-\infty}^{+\infty} p(\zeta,\theta^*,\chi)\phi\left(\zeta,\bar{\theta}\left(\theta\right),\sigma\right) d\zeta, \qquad \text{(B.9)}$$

where $\bar{\theta}\left(\theta\right)$ and σ^2 have been defined above.

C Results about the wage equation

We now prove existence, uniqueness and monotonicity of the aggregate wage. Suppose workers believe that the central bank follows a threshold strategy such that

$$p(\zeta, \theta^*, \chi) = \begin{cases} \frac{1}{1+\chi}(w+\zeta) & \text{if } \zeta \leq \theta^* \\ 0 & \zeta > \theta^* \end{cases},$$

for some $\theta^* > 0$. Then we can write the wage equation as

$$w(\theta, \theta^*, \chi) = \int_{-\infty}^{+\infty} a(\zeta, \theta^*, \chi)(w(\zeta, \theta^*, \chi) + \zeta)\, \phi\left(\zeta, \bar{\theta}(\theta), \sigma\right) d\zeta, \tag{C.10}$$

where the function $a(\zeta, \theta^*, \chi)$ is defined as

$$a(\zeta, \theta^*, \chi) = \begin{cases} 0 & \text{if } \zeta \geq \theta^* \\ \frac{1}{1+\chi} & \zeta \geq \theta^* \end{cases}. \tag{C.11}$$

Note that since $\chi > 0$, $0 < a \leq \frac{1}{1+\chi} < 1$.

Consider the operator associated with the right hand side of equation (C.10),

$$T[x](\theta, \theta^*, \chi) \equiv \int_{-\infty}^{+\infty} a(\zeta, \theta^*, \chi)(w(\zeta, \theta^*, \chi) + \zeta)\, \phi\left(\zeta, \bar{\theta}(\theta), \sigma\right) d\zeta. \tag{C.12}$$

In principle, the operator T maps the space of continuous functions on $\mathbb{R} \times \mathbb{R}_+^2$ such that the integral $\int a(\zeta, \theta^*, \chi)(w(\zeta, \theta^*, \chi) + \zeta)\, \phi\left(\zeta, \bar{\theta}(\theta), \sigma\right) d\zeta$ exists onto itself. We consider however only positive values of θ^*, $\theta^* > 0$, and only values of $\chi > 0$. We also make enough assumptions on the information structure to ensure that negative values of θ can be safely ignored.

C.1 Proof of Lemma 2

Existence part. We next show that there exists a fixed point of T. The proof is by induction. We start from some x_0 and we iterate. We show that the iteration procedure converges.

Fix (θ, θ^*, χ). Consider a particular starting point, $x_0(\theta, \theta^*, \chi) = 0$. Letting $x_1 = T[x_0]$, we can write

$$\begin{aligned} |x_1 - x_0| &= \frac{1}{1+\chi}\left|\int_{\theta^*}^{+\infty} \zeta\phi\left(\zeta, \bar{\theta}(\theta), \sigma\right) d\zeta\right| \\ &\leq \frac{1}{1+\chi}\left|\bar{\theta}(\theta)\, \Phi\left(\frac{\bar{\theta}(\theta) - \theta^*}{\sigma}\right) + \sigma\phi\left(\frac{\theta^* - \bar{\theta}(\theta)}{\sigma}\right)\right| \end{aligned}$$

$$\leq \frac{1}{1+\chi} Q(\theta)$$

with $Q(\theta) \equiv \left| \bar{\theta}(\theta) \Phi\left(\frac{\bar{\theta}(\theta)-\theta^*}{\sigma}\right) + \sigma\phi\left(\frac{\theta^*-\bar{\theta}(\theta)}{\sigma}\right) \right|$. Iterating again, $x_2 = T^2[x_0]$, we have that

$$\begin{aligned} |x_2 - x_1| &= \left| \int_{\theta^*}^{+\infty} \frac{1}{1+\chi} \left(x_1(\zeta, \theta^*, \chi) - x_0(\zeta, \theta^*, \chi)\right) \phi\left(\zeta, \bar{\theta}(\theta), \sigma\right) d\zeta \right| \\ &\leq \frac{1}{1+\chi} \left| \int_{\theta^*}^{+\infty} \left(x_1(\zeta, \theta^*, \chi) - x_0(\zeta, \theta^*, \chi)\right) \phi\left(\zeta, \bar{\theta}(\theta), \sigma\right) d\zeta \right| \\ &\leq \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} \left| x_1(\zeta, \theta^*, \chi) - x_0(\zeta, \theta^*, \chi) \right| \phi\left(\zeta, \bar{\theta}(\theta), \sigma\right) d\zeta \\ &\leq \left(\frac{1}{1+\chi}\right)^2 Q(\theta). \end{aligned}$$

By induction, then,

$$|x_n - x_{n-1}| \leq \left(\frac{1}{1+\chi}\right)^n Q(\theta).$$

Observe that for fixed θ, $\lim_{n\to+\infty} \left(\frac{1}{1+\chi}\right)^n Q(\theta) = 0$ since $\left|\frac{1}{1+\chi}\right| < 1$. Given that the right hand side converges to zero as $n \to \infty$, for every $\varepsilon > 0$ there exists an N such that for all $n, m \geq N$ we have that

$$|x_n - x_m| \leq \varepsilon,$$

which means that $\{x_n\}$ is a sequence on a complete metric space ($\mathbb{R}$ with the absolute value norm). By completeness of $\mathbb{R}$, there exists some x^* such that $x_n \to x^*$. By continuity of the operator T, we get that x^* is a fixed point, since

$$T[x^*] = T\left[\lim_{n\to+\infty} x_n\right] = \lim_{n\to+\infty} T(x_n) = \lim_{n\to+\infty} x_{n+1} = x^*.$$

This established convergence to a fixed point (existence), but there may be more than one.
Uniqueness part. For uniqueness, consider first x^a and x^b. Define $x_n^a \equiv T^n[x^a]$ and $x_n^b \equiv T^n\left[x^b\right]$. Observe that $\left|x_n^a - x_n^b\right|$ is bounded by

$$\left|x_n^a - x_n^b\right| \leq \left(\frac{1}{1+\chi}\right)^n M,$$

for some $M > 0$. It follows that $\lim_{n\to+\infty} \sum_{j=n}^{+\infty} \left|x_j^a - x_j^b\right| = 0$, so

$$\sum_{j=0}^{+\infty} \left|x_j^a - x_j^b\right| < +\infty.$$

Suppose that there are two fixed points, x^* and x^{**}. Then we have that

$$\sum_{j=0}^{+\infty} \left| T^j \left[x^*\right] - T^j \left[x^{**}\right] \right| = \sum_{j=0}^{+\infty} \left| x^* - x^{**} \right| < +\infty,$$

which implies that $x^* = x^{**}$.

Differentiability. Observe that while the function $x_0(\theta, \theta^*, \chi)$ is not differentiable with respect to θ , the function $T[x_0](\theta, \theta^*, \chi)$ and all its higher-order iterates $\{T^n[x_0]\}_{n\geq 2}$ instead are differentiable with respect to all three arguments θ, θ^* and χ. It follows that the fixed point of the T operator x^* is also differentiable.

C.2 Wage sandwiching

To verify that it is indeed optimal for the monetary authority to abandon the peg when $\theta \geq \theta^*$, we need to establish that the candidate equilibrium wage function is sandwiched between its common knowledge counterparts. For this, we need to be able to abstract from negative values of the fundamentals, which we formalize with the following assumption.

Assumption 1. *Assume that θ_0, σ_0 and σ_x are such that $\left| \int_{-\infty}^{0} \zeta \phi \left(\zeta, \bar{\theta}\left(\theta\right), \sigma\right) d\zeta \right| < \varepsilon$, for ε arbitrarily small and $\theta > 0$.*

Lemma 4. *If $\theta > 0$ and $\theta^* > 0$ the candidate equilibrium aggregate nominal wage $w\left(\theta, \theta^*, \chi\right)$ is sandwiched between (i) the aggregate nominal wage under common knowledge when workers expect the central bank to uphold the peg and (ii) the aggregate nominal wage under common knowledge when they expect the bank to abandon the peg, $w(\theta, \theta^*, \chi) \in \left(0, \frac{\theta}{\chi}\right)$.*

The proof follows. Recall that

$$T\left[x\right] = \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} \left(x\left(\zeta, \theta^*, \chi\right) + \zeta\right) \phi\left(\zeta, \bar{\theta}\left(\theta\right), \sigma\right) d\zeta.$$

Now define $\underline{T}\left[x\right]$ as the operator associated with the right hand side of the wage equation when workers expect the central bank to never abandon the peg,

$$\underline{T}\left[x\right] \equiv \frac{1}{1+\chi} \int_{-\infty}^{+\infty} 0 \phi\left(\zeta, \bar{\theta}\left(\theta\right), \sigma\right) d\zeta. \tag{C.13}$$

This operator has a unique fixed point,

$$\underline{x}\left(\theta, \theta^*, \chi\right) = 0.$$

Observe that for $\theta \to -\infty$, applying the T operator is equivalent to applying the $\underline{T}$ operator. Hence, for θ arbitrarily large and negative the wage rate under dispersed information converges to the wage rate under common knowledge when workers expect the peg to be upheld.

Otherwise, we obtain that

$$T\left[\underline{x}\right]-\underline{T}\left[\underline{x}\right]=T\left[\underline{x}\right]=\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(\underline{x}\left(\theta,\theta^*,\chi\right)+\zeta\right)\phi\left(\zeta,\bar{\theta}\left(\theta\right),\sigma\right)d\zeta>0$$

since $\theta^*>0$ and $\bar{\theta}\left(\theta\right)>0$. Moreover,

$$T^2\left[\underline{x}\right]-\underline{T}^2\left[\underline{x}\right]=\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(T\left[\underline{x}\right]\left(\zeta,\theta^*,\chi\right)+\zeta\right)\phi\left(\zeta,\bar{\theta}\left(\theta\right),\sigma\right)d\zeta>0,$$

since $T\left[\underline{x}\right]>0$ and $\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\zeta\phi\left(\zeta,\bar{\theta}\left(\theta\right),\sigma\right)d\zeta>0$.
By induction, we obtain that $\lim_{n\to\infty}\left(T^n\left[\underline{x}\right]-\underline{T}^n\left[\underline{x}\right]\right)>0$. Since $\lim_{n\to\infty}T^n\left[\underline{x}\right]=x^*$, and $\lim_{n\to\infty}T^n\left[\underline{x}\right]=\underline{x}$, it follows that $x^*-\underline{x}>0$.

Now, define $\overline{T}\left[x\right]$ as the operator associated with the right hand side of the wage equation when workers expect the central bank to always abandon the peg,

$$\overline{T}\left[x\right]\equiv\frac{1}{1+\chi}\int_{-\infty}^{+\infty}\left(x\left(\zeta,\theta^*,\chi\right)+\zeta\right)\phi\left(\zeta,\bar{\theta}\left(\theta\right),\sigma\right)d\zeta. \tag{C.14}$$

For $\theta\to+\infty$, applying the T operator is equivalent to applying the $\overline{T}$ operator. For θ arbitrarily large and positive the wage rate under dispersed information converges to the wage rate under common knowledge when workers expect a devaluation. Like $\underline{T}\left[x\right]$, the operator $\overline{T}\left[x\right]$ also has a unique fixed point,

$$\overline{x}\left(\theta,\chi\right)=\frac{1}{\chi}\bar{\theta}\left(\theta\right).$$

Assume that $\sigma_x\to0$ so $\psi\to1$ and $\bar{\theta}\left(\theta\right)\to\theta$. Now consider

$$\begin{aligned}T\left[\overline{x}\right]-\bar{T}\left[\overline{x}\right] &= \frac{1}{\left(1+\chi\right)^2}\int_{\theta^*}^{+\infty}\left(\frac{1}{\chi}\zeta+\zeta\right)\phi\left(\zeta,\theta,\sigma\right)d\zeta-\frac{1}{\left(1+\chi\right)^2}\int_{-\infty}^{\infty}\zeta\phi\left(\zeta,\theta,\sigma\right)d\zeta\\ &= -\frac{1}{\left(1+\chi\right)^2}\int_{-\infty}^{\theta^*}\zeta\phi\left(\zeta,\theta,\sigma\right)d\zeta<0\end{aligned}$$

since $\theta^*>0$ and by Assumption 1 $\int_{-\infty}^{\theta^*}\zeta\phi\phi\left(\zeta,\theta,\sigma\right)d\zeta=\int_0^{\theta^*}\zeta\phi\left(\zeta,\theta,\sigma\right)d\zeta>0$. As for the second iteration round,

$$\begin{aligned}T^2\left[\overline{x}\right]-\bar{T}^2\left[\overline{x}\right] &= \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(T\left[\bar{x}\right]\left(\zeta,\theta^*,\chi\right)+\zeta\right)\phi\left(\zeta,\theta,\sigma\right)d\zeta\\ &\quad -\frac{1}{1+\chi}\int_{-\infty}^{\infty}\left(\bar{T}\left[\bar{x}\right]\left(\zeta,\theta^*,\chi\right)+\zeta\right)\phi\left(\zeta,\theta,\sigma\right)d\zeta\end{aligned}$$

$$\begin{aligned} &= \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(T\left[\overline{x}\right](\zeta,\theta^*,\chi)-\bar{T}\left[\overline{x}\right](\zeta,\theta^*,\chi)\right)\phi(\zeta,\theta,\sigma)\,d\zeta \\ &\quad -\frac{1}{1+\chi}\int_{-\infty}^{\theta^*}\left(\bar{T}\left[\overline{x}\right](\zeta,\theta^*,\chi)+\zeta\right)\phi(\zeta,\theta,\sigma)\,d\zeta, \end{aligned}$$

that is,

$$\begin{aligned} T^2\left[\overline{x}\right]-\bar{T}^2\left[\overline{x}\right] &= \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(T\left[\overline{x}\right](\zeta,\theta^*,\chi)-\bar{T}\left[\overline{x}\right](\zeta,\theta^*,\chi)\right)\phi(\zeta,\theta,\sigma)\,d\zeta \\ &\quad -\frac{1}{(1+\chi)^2}\int_{-\infty}^{\theta^*}\zeta\phi(\zeta,\theta,\sigma)\,d\zeta, \\ &< 0 \end{aligned}$$

where the last inequality follows from the fact that $T\left[\overline{x}\right]-\bar{T}\left[\overline{x}\right]<0$ and that $\int_{-\infty}^{\theta^*}\zeta\phi(\zeta,\theta,\sigma)\,d\zeta>0$. Again by induction, then, $x^*-\overline{x}<0$.

C.3 Wage monotonicity

Lemma 5. *The candidate wage function $w(\theta,\theta^*,\chi)$ is (i) increasing in θ; (ii) increasing in θ^* and (iii) decreasing in χ.*

The proof follows. **Part (i).** We now show that the fixed point of T is increasing in θ. Using the change of variable $z=\frac{\zeta-\bar{\theta}(\theta)}{\sigma}$we write the T operator as

$$T[x]=\frac{1}{1+\chi}\int_{\frac{\theta^*-\bar{\theta}}{\sigma}}^{\infty}\left(x\left(\sigma z+\bar{\theta}(\theta),\theta^*,\chi\right)+\sigma z+\bar{\theta}(\theta)\right)\phi(z,0,1)\,dz.$$

Now suppose that x is differentiable with respect to its first argument. Further assume that $x(\theta^*,\theta^*,\chi)>0$ and $\frac{\partial}{\partial\theta}x(\theta,\theta^*,\chi)>0$, where the notation $\frac{\partial}{\partial\theta}x$ denotes the partial derivative of x with respect to its first argument. Then we obtain

$$\begin{aligned} \frac{\partial}{\partial\theta}T[x] =& \frac{1}{1+\chi}\psi\left(x(\theta^*,\theta^*,\chi)+\theta^*\right)\phi\left(\theta^*,\bar{\theta}(\theta),\sigma\right) \\ &+\frac{1}{1+\chi}\int_{\frac{\theta^*-\bar{\theta}}{\sigma}}^{\infty}\left(\frac{\partial}{\partial\theta}x\left(\sigma z+\bar{\theta}(\theta),\theta^*,\chi\right)+\psi\right)\phi(z,0,1)\,dz>0. \end{aligned}$$

Consider now $x_0(\theta,\theta^*,\chi)=0$. Then we have that

$$T[x_0]=\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\zeta\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right)d\zeta>0,$$

since $\theta^*>0$. Now using the change of variable $z=\frac{\zeta-\bar{\theta}(\theta)}{\sigma}$, we write

$$T[x_0]=\frac{1}{1+\chi}\int_{\frac{\theta^*-\bar{\theta}}{\sigma}}^{+\infty}\left(\sigma z+\bar{\theta}(\theta)\right)\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right)\sigma dz.$$

Differentiating with respect to θ we obtain

$$\frac{\partial}{\partial\theta}T[x_0] = \frac{1}{1+\chi}\psi\theta^*\phi\left(\theta^*,\bar{\theta}(\theta),\sigma\right) + \frac{1}{1+\chi}\psi\int_{-\infty}^{\frac{\theta^*-\bar{\theta}}{\sigma}}\phi(z,0,1)\,dz > 0,$$

since $\theta^* > 0$ and $\phi(z,0,1)$ is a density. This establishes that $x_1 = T[x_0]$ satisfies $x_1(\theta^*,\theta^*,\chi) > 0$ and $\frac{\partial}{\partial\theta}x_1(\theta,\theta^*,\chi) > 0$. Now consider the second iterate. We have that

$$T[x_1] = \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(x_1(\zeta,\theta^*,\chi)+\zeta\right)\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right)d\zeta > 0,$$

since $x_1(\zeta,\theta^*,\chi) > 0$ for $\theta^* > 0$ and $\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\zeta\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right)d\zeta > 0$ for $\theta^* > 0$. Again differentiating with respect to θ we obtain

$$\begin{aligned}\frac{\partial}{\partial\theta}T^2[x_0] =& \frac{1}{1+\chi}\psi\left(x_1(\theta^*,\theta^*,\chi)+\theta^*\right)\phi\left(\theta^*,\bar{\theta}(\theta),\sigma\right)\\ &+\frac{1}{1+\chi}\int_{\frac{\theta^*-\bar{\theta}}{\sigma}}^{\infty}\left(\frac{\partial}{\partial\theta}x_1\left(\sigma z+\bar{\theta}(\theta),\theta^*,\chi\right)+\psi\right)\phi(z,0,1)\,dz > 0,\end{aligned}$$

where the inequality follows from $\theta^* > 0$, $x_1(\theta^*,\theta^*,\chi) > 0$, $\frac{\partial}{\partial\theta}x_1 > 0$ and the fact that $\phi(z,0,1)$ is a density. We have thus shown that $x_2 = T[x_1]$ satisfies $x_2(\theta^*,\theta^*,\chi) > 0$ and $\frac{\partial}{\partial\theta}x_2(\theta,\theta^*,\chi) > 0$. By induction, we obtain that $\lim_{n\to+\infty} x_n = x^* = T[x^*]$ is also increasing. Since x^* is the unique fixed point of T, the wage function is monotonically increasing.

Part (ii). We now show that the fixed point of T is decreasing in θ^*. Again, fix $x_0(\theta,\theta^*,\chi) = 0$. The first iterate $T[x_0] = \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\zeta\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right)d\zeta$ is decreasing in θ^*,

$$\frac{\partial T[x_0]}{\partial\theta^*} = -\frac{1}{1+\chi}\theta^*\phi\left(\theta^*,\bar{\theta}(\theta),\sigma\right) < 0,$$

since $\theta^* > 0$ and $\phi\left(\theta^*,\bar{\theta}(\theta),\sigma\right) > 0$. Then consider the second iterate,

$$T^2[x_0] = \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(x_1(\zeta,\theta^*,\chi)+\zeta\right)\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right)d\zeta,$$

where $x_1 = T[x_0]$. Differentiating with respect to θ^* we obtain

$$\begin{aligned}\frac{\partial T^2[x_0]}{\partial\theta^*} =& -\frac{1}{1+\chi}\left(x_1(\theta^*,\theta^*,\chi)+\theta^*\right)\phi\left(\theta^*,\bar{\theta}(\theta),\sigma\right)\\ &+\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\frac{\partial}{\partial\theta^*}x_1(\zeta,\theta^*,\chi)\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right)d\zeta\\ &< 0,\end{aligned}$$

where the inequality follows from $x_1(\theta^*,\theta^*,\chi) > 0$ (see part (i) of the proof), $\theta^* > 0$ and $\frac{\partial}{\partial\theta^*}x_1 < 0$. We have thus established that $x_2(\theta,\theta^*,\chi) = T^2[x_0](\theta,\theta^*,\chi)$ is decreasing in θ^*.

By induction, we obtain that $\lim_{n\to+\infty} x_n(\theta,\theta^*,\chi) = x^*(\theta,\theta^*,\chi) = T[x^*](\theta,\theta^*,\chi)$ is also decreasing in θ^*. Since x^* is the unique fixed point of T, x^* is monotonically decreasing in θ^*.

Part (iii). We now show that the fixed point of T is decreasing in χ. Fix $x_0(\theta,\theta^*,\chi) = 0$. Differentiating $T[x_0] = \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} \zeta\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right) d\zeta$ with respect to χ we obtain

$$\frac{\partial T[x_0]}{\partial\chi} = -\frac{1}{(1+\chi)^2}\int_{\theta^*}^{+\infty} \zeta\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right) d\zeta < 0,$$

since $\int_{\theta^*}^{+\infty} \zeta\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right) d\zeta > 0$ for $\theta^* > 0$. Differentiating the second iterate with respect to χ we obtain

$$\begin{aligned}\frac{\partial T^2[x_0]}{\partial\chi} &= -\frac{1}{(1+\chi)^2}\int_{\theta^*}^{+\infty} \left(x_1(\zeta,\theta^*,\chi)+\zeta\right)\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right) d\zeta \\ &\quad + \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} \frac{\partial}{\partial\chi}x_1(\zeta,\theta^*,\chi)\,\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right) d\zeta.\end{aligned}$$

Since $x_1 = \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} \zeta\phi\left(\zeta,\bar{\theta}(\theta),\sigma\right) d\zeta > 0$ and $\frac{\partial}{\partial\chi}x_1 < 0$, it follows that $x_2(\theta,\theta^*,\chi) = T^2[x_0](\theta,\theta^*,\chi)$ is decreasing in χ. By induction, we obtain that $\lim_{n\to+\infty} x_n(\theta,\theta^*,\chi) = x^*(\theta,\theta^*,\chi) = T[x^*](\theta,\theta^*,\chi)$ is also decreasing in χ. Since x^* is the unique fixed point of T, x^* is monotonically decreasing in χ.

C.4 Proof of Proposition 3

First, we verify the conjecture. The central bank switches to a managed float if and only if the cost of pegging $c(w,\theta,\chi)$ is sufficiently high relative to the benefit v,

$$v \leq c\left(w(\theta,\theta^*,\chi),\theta^*,\chi\right).$$

for all $\theta \geq \theta^* > 0$, with θ^* implicitly defined as a positive root of

$$v = \frac{1}{2}\frac{1}{1+\chi}\left(w(\theta,\theta,\chi)+\theta\right)^2. \tag{C.15}$$

Fix χ and let $\Omega(\theta)$ denote the right hand side of equation (C.15). By Lemma 4, $\theta < w(\theta,\theta,\chi)+\theta < \left(1+\frac{1}{\chi}\right)\theta$ for all $\theta > 0$. Moreover, we have argued that as $\theta\to 0$, $w(\theta,\theta,\chi)+\theta\to\theta$. On the other hand, $w(\theta,\theta,\chi)+\theta\to\frac{\theta}{\chi}$ as $\theta\to+\infty$. Hence it must be the case that for $\theta\to 0$, $\Omega(\theta)\to 0$. For $\theta\to+\infty$, on the other hand, $\Omega(\theta)\to+\infty$. Since $\Omega(\theta)$ is a continuous function on $(0,+\infty)$, to verify the conjecture that a threshold equilibrium exists, it suffices to show that that $\Omega(\theta)$ is increasing in θ for $\theta > 0$. We have that

$$\frac{d\Omega(\theta)}{d\theta} = \frac{(w(\theta,\theta^*,\chi)+\theta)}{1+\chi}\left(\frac{\partial}{\partial\theta}w(\theta,\theta^*,\chi)+\frac{\partial}{\partial\theta^*}w(\theta,\theta^*,\chi)+1\right)\Bigg|_{\theta^*=\theta>0}.$$

First, observe that by Lemma 4 $w(\theta,\theta,\chi)+\theta > \theta$. We now worry about the second term, which is potentially a problem because $\frac{\partial}{\partial\theta}w > 0$ but $\frac{\partial}{\partial\theta^*}w < 0$. However, the first derivative dominates for any $\theta > 0$ and $\theta^* > 0$.

Again, let $\sigma_x \to 0$ so $\psi = 1$. Let $x_0(\theta,\theta^*,\chi) = 0$. By previous arguments, we have that

$$\frac{\partial}{\partial\theta}T[x_0]+\frac{\partial}{\partial\theta^*}T[x_0] = \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\phi(\zeta,\theta,\sigma)\,d\zeta > 0.$$

Consider now the second iterate. Then we have that

$$\begin{aligned}\frac{\partial}{\partial\theta}T^2[x_0]+\frac{\partial}{\partial\theta^*}T^2[x_0] &= \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(\frac{\partial}{\partial\theta}x_1(\theta,\theta^*,\chi)+1\right)\phi(\zeta,\theta,\sigma)\,dz\\ &+\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\frac{\partial}{\partial\theta^*}x_1(\zeta,\theta^*,\chi)\,\phi(\zeta,\theta,\sigma)\,d\zeta\\ &> 0\end{aligned}$$

since $x_1 = T[x_0]$ and we have shown that $\frac{\partial}{\partial\theta}T[x_0]+\frac{\partial}{\partial\theta^*}T[x_0] > 0$. By induction, then, $\lim_{n\to+\infty}\left(\frac{\partial}{\partial\theta}T^n[x_0]+\frac{\partial}{\partial\theta^*}T^n[x_0]\right) = \frac{\partial}{\partial\theta}\lim_{n\to+\infty}T^n[x_0]+\frac{\partial}{\partial\theta^*}\lim_{n\to+\infty}T^n[x_0] = \frac{\partial}{\partial\theta}x^*+\frac{\partial}{\partial\theta^*}x^* > 0$. Since x^* is the unique fixed point of T, it follows that the aggregate wage equation satisfies the condition that $\frac{\partial}{\partial\theta}w+\frac{\partial}{\partial\theta^*}w > 0$.

There remains to check that $\theta^* \in \left(\underline{\theta},\overline{\theta}\right)$. This follows immediately from the fact that $\theta < w(\theta,\theta,\chi) < \frac{\theta}{\chi}$ and the definitions of $\underline{\theta}$ and $\overline{\theta}$.

D Proof of Corollary 1

Recall the expressions for the thresholds in the main body, (21) and (22). Differentiating with respect to χ we obtain that both thresholds are increasing in χ,

$$\begin{aligned}sign\left(\frac{\partial\underline{\theta}}{\partial\chi}\right) &= sign\left(\frac{d}{d\chi}\frac{\chi^2}{1+\chi}\right) = sign\left(\frac{\chi}{(\chi+1)^2}(\chi+2)\right) = +,\\ sign\left(\frac{\partial\overline{\theta}}{\partial\chi}\right) &= sign\left(\frac{d}{d\chi}\chi\right) = sign(1) = +.\end{aligned}$$

E The long-run average wage bill

Consider perfect foresight. Then if the central bank has maintained the peg in the short-run, workers expect the peg to the maintained in the long-run as well, and by equation (10) the average wage bill is $w_{LR}(\theta_{LR},\chi)=0$. If, on the other hand, the central bank has not maintained the peg, workers expect a devaluation, in which case the average long run bill is $w_{LR}(\theta_{LR},\chi)=\theta_{LR}/\chi$.

Consider dispersed information. If workers expect the peg to be maintained, the average wage bill solves

$$w_{LR}(\theta_{LR},\chi)=\int_{-\infty}^{+\infty}0\phi\left(\zeta,\bar{\theta}(\theta_{LR}),\sigma\right)d\zeta,$$

and so again, $_{LR}(\theta_{LR},\chi)=0$. If workers expect a devaluation, the average wage bill solves

$$w_{LR}(\theta_{LR},\chi)=\overline{T}[w_{LR}](\theta_{LR},\chi),$$

where the operator $\overline{T}[x]$ is defined as $\overline{T}[x](\theta_{LR},\chi)=\frac{1}{1+\chi}\int_{-\infty}^{+\infty}(x(\zeta,\chi)+\zeta)\,\phi\left(\zeta,\overline{\theta}(\theta_{LR}),\sigma\right)d\zeta$. It is easy to verify that $w_{LR}(\theta_{LR},\chi)=\theta_{LR}/\chi$ is the fixed point of $\overline{T}$ when $\sigma_x\to 0$.

F Proof of Corollary 2

By implicit differentiation, we obtain

$$\begin{aligned}
sign\left(\frac{\partial\underline{\theta}}{\partial\chi}\right) &= sign\left(\frac{d}{d\chi}\frac{\chi^2}{1+\chi}v(\chi)\right)=sign\left(\frac{\chi}{(\chi+1)^2}(\chi+2)\,v(\chi)+\frac{\chi^2}{1+\chi}\frac{dv}{d\chi}\right)\\
&= sign\left(\frac{\chi}{(\chi+1)^2}(\chi+2)\,v-\frac{\chi^2}{1+\chi}\frac{1}{\chi}v\right)=sign\left(v\frac{\chi}{(\chi+1)^2}\right)=+,
\end{aligned}$$

where the second line uses the result that $\frac{dv}{d\chi}=-\frac{v}{\chi}$. As for the upper threshold, we have that

$$sign\left(\frac{\partial\overline{\theta}}{\partial\chi}\right)=sign\left(\frac{d}{d\chi}(1+\chi)\,v(\chi)\right)=sign\left(v-(1+\chi)\frac{1}{\chi}v\right)=sign\left(-\frac{v}{\chi}\right)=-.$$

Hence, the lower threshold is still increasing but there is a reversal for the upper threshold.

G Proof of Corollary 3

Suppose that active agents expect a devaluation. By equation (10), recalling that the devaluation price is given by (13) and using the fact that $w=(1-\lambda)\,w_1+\lambda w_2$, we obtain that for $w_2\geq 0$ active wages are given by

$$w_1=\frac{\theta+\lambda w_2}{\lambda+\chi}$$

with the corresponding average wage given by

$$w = \frac{1-\lambda}{\lambda+\chi}\theta + \lambda\frac{\chi+1}{\lambda+\chi}w_2.$$

If the authority were to indeed devalue, it would set a price equal to

$$p = \frac{1}{\lambda+\chi}\theta + \frac{\lambda}{\lambda+\chi}w_2.$$

The central bank then devalues if and only if

$$\frac{\beta}{\chi}\mathbb{E}\left[\theta_{LR}^2\right] \leq \frac{1+\chi}{(\lambda+\chi)^2}\left(\theta_{SR}+\lambda w_2\right)^2$$

which enables us to characterize the lower dominance threshold as

$$\underline{\theta} = \sqrt{v(\chi)\frac{(\lambda+\chi)^2}{1+\chi}} - \lambda w_2.$$

Now consider $\underline{\theta}$. Differentiating with respect to χ, we obtain

$$\begin{aligned} sign\left(\frac{\partial\underline{\theta}}{\partial\chi}\right) &= sign\left(\frac{\partial}{\partial\chi}\frac{(\lambda+\chi)^2}{\chi}\frac{1}{1+\chi}\right) \\ &= sign\left(P(\lambda)\right) \end{aligned}$$

The sign of this expression depends on the term $P(\lambda) = -(2\chi+1)\lambda + \chi$. Observe that the function P is continuous, with

$$\begin{aligned} \lim_{\lambda\to 0} P(\lambda) &= \chi > 0, \\ \lim_{\lambda\to 1} P(\lambda) &= -(1+\chi) < 0. \end{aligned}$$

Moreover, P is decreasing in λ,

$$\frac{dP}{d\lambda} = -(2\chi+1) < 0.$$

So there exists a range of values of λ, $\left(\frac{\chi}{1+2\chi}, 1\right]$ such that $P(\lambda) < 0$. As a result, we have that

$$\frac{\partial\underline{\theta}}{\partial\chi} < 0 \text{ for } \lambda \in \left(\frac{\chi}{1+2\chi}, 1\right],$$

and $\frac{\partial\underline{\theta}}{\partial\chi} > 0$ otherwise. In addition, we have that as λ increases, the sensitivity of $\underline{\theta}$ to χ also decreases,

$$sign\left(\frac{\partial}{\partial\lambda}\left(\frac{\partial\overline{\theta}}{\partial\chi}\right)\right) = sign\left(\frac{\partial}{\partial\lambda}\left(-(\lambda-\chi+2\lambda\chi)\frac{\lambda+\chi}{\chi^2(\chi+1)^2}\right)\right).$$

$$
\begin{aligned}
&= -sign(\frac{\partial}{\partial \lambda}(\lambda - \chi + 2\lambda\chi)(\lambda + \chi)) \\
&= -sign(2\left(\lambda + \chi^2 + 2\lambda\chi\right)) \\
&= -
\end{aligned}
$$

Now suppose active workers expect a peg. Since active wages are given by $w_1 = 0$, the corresponding average wage is given by

$$
w = \lambda w_2.
$$

The central bank then devalues if and only if

$$
v(\chi) \leq \frac{1}{1+\chi}(\lambda w_2 + \theta_{SR})^2
$$

which enables us to characterize the upper dominance threshold as

$$
\overline{\theta} = \sqrt{v(\chi)(1+\chi)} - \lambda w_2.
$$

Since $\overline{\theta}$ is computed under the assumption that active workers expect the peg to be upheld, the sensitivity of $\overline{\theta}$ to λ is unaffected by λ, $\frac{\partial}{\partial \lambda}\left(\frac{\partial \overline{\theta}}{\partial \chi}\right) = 0$.

To lighten the notation in the body of the paper we set $w_2 = 0$. This assumption is without loss of generality since w_2 is just a constant and it does not affect the sensitivity of the critical thresholds with respect to the strength of the monetary authority, χ.

H Results about $w(\theta, \theta^*, \chi)$ when $\lambda > 0$

Consider the variant of the model in which a fraction λ of workers cannot readjust their wages. The individually optimal wage for workers who can optimize is $w_{1,i} = \mathbb{E}_\theta[p|x]$. Averaging across these workers, we obtain

$$
\begin{aligned}
w_1(x, \theta^*, \chi, \lambda) &= \mathbb{E}_x\left[\mathbb{E}_\theta[p|x]|\theta\right] \\
&= \int_{-\infty}^{\infty} p(\zeta, \theta^*, \chi)\,\phi(\zeta, \theta, \sigma)\,d\zeta \\
&= \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}(w(\zeta, \theta^*, \chi, \lambda) + \zeta)\,\phi(\zeta, \theta, \sigma)\,d\zeta \\
&= \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}((1-\lambda)\,w_1(\zeta, \theta^*, \chi, \lambda) + \lambda w_2 + \zeta)\,\phi(\zeta, \theta, \sigma)\,d\zeta.
\end{aligned}
$$

Setting $w_2 = 0$ and observing that the average wage $w = (1-\lambda) w_1$, we can write the equation for the average wage as

$$w_\lambda (\theta, \theta^*, \chi, \lambda) = (1-\lambda) \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} (w_\lambda (\zeta, \theta^*, \chi, \lambda) + \zeta) \phi (\zeta, \theta, \sigma) d\zeta.$$

where we have used the notation w_λ to denote the average wage bill function in the presence of predetermined wages to avoid any confusion with the benchmark average bill function w that applies when $\lambda = 0$. Let T_λ denote the operator associated with the right hand side of this equation, so $T_\lambda = (1-\lambda) T$. Since $1-\lambda < 1$, all the results that apply to the average wage defined using T also apply to the average wage defined using T_λ.

Next, we show that the average wage is less sensitive to both χ and θ. We start by showing that a positive λ reduces the aggregate wage. Fix $x_0 = 0$. Then $T[x_0] > T_\lambda [x_0]$ since $\frac{\lambda}{1+\chi} \int_{\theta^*}^{+\infty} (w_\lambda (\zeta, \theta^*, \chi, \lambda) + \zeta) \phi (\zeta, \theta, \sigma) d\zeta > 0$ for $\theta^* > 0$. Moreover,

$$\begin{aligned} T^2 [x_0] - T_\lambda^2 [x_0] =& \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} (T[x_0] (\zeta, \theta^*, \chi) + \zeta) \phi (\zeta, \theta, \sigma) d\zeta \\ &- \frac{1-\lambda}{1+\chi} \int_{\theta^*}^{+\infty} (T_\lambda [x_0] (\zeta, \theta^*, \chi, \lambda) + \zeta \phi) (\zeta, \theta, \sigma) d\zeta \\ =& \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} (T[x_0] (\zeta, \theta^*, \chi) - T_\lambda [x_0] (\zeta, \theta^*, \chi, \lambda)) \phi (\zeta, \theta, \sigma) d\zeta \\ &+ \frac{\lambda}{1+\chi} \int_{\theta^*}^{+\infty} \zeta \phi (\zeta, \theta, \sigma) d\zeta > 0. \end{aligned}$$

By induction and using previous results about existence and uniqueness of a fixed point of the two operators, introducing passive workers reduces the average wage, $w(\theta, \theta^*, \chi) > w_\lambda (\theta, \theta^*, \chi, \lambda)$. Observe that as a result, it must be the case that $\theta_\lambda^* > \theta^*$. This is because the critical threshold solves $v = c(w, \theta, \chi)$, and the cost of pegging c is increasing in w.

We now show that the wage is less sensitive to χ when $\lambda > 0$. The argument relies on the proof that the fixed point of T is decreasing in χ. Again, fix $x_0 (\theta, \theta^*, \chi) = 0$. Differentiating with respect to χ we obtain

$$\left| \frac{\partial T[x_0]}{\partial \chi} \right| - \left| \frac{\partial T_\lambda [x_0]}{\partial \chi} \right| = \frac{\lambda}{(1+\chi)^2} \int_{\theta^*}^{+\infty} \zeta \phi \left(\zeta, \bar{\theta}(\theta), \sigma \right) d\zeta > 0,$$

since $\int_{\theta^*}^{+\infty} \zeta \phi \left(\zeta, \bar{\theta}(\theta), \sigma \right) d\zeta > 0$ for $\theta^* > 0$. Differentiating the second iterate with respect to χ we obtain

$$\begin{aligned} & \left| \frac{\partial T^2 [x_0]}{\partial \chi} \right| - \left| \frac{\partial T_\lambda^2 [x_0]}{\partial \chi} \right| \\ &= \frac{1}{(1+\chi)^2} \int_{\theta^*}^{+\infty} ((T[x_0] (\zeta, \theta^*, \chi) - T_\lambda [x_0] (\zeta, \theta^*, \chi, \lambda))) \phi \left(\zeta, \bar{\theta}(\theta), \sigma \right) d\zeta \end{aligned}$$

$$+\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(\left|\frac{\partial}{\partial\chi}T\left[x_0\right]\left(\zeta,\theta^*,\chi\right)\right|-\left|\frac{\partial}{\partial\chi}T_\lambda\left[x_0\right]\left(\zeta,\theta^*,\chi,\lambda\right)\right|\right)\phi\left(\zeta,\bar{\theta}\left(\theta\right),\sigma\right)d\zeta>0$$

where the last inequality follows from the fact that $T\left[x_0\right]\left(\zeta,\theta^*,\chi\right)>T_\lambda\left[x_0\right]\left(\zeta,\theta^*,\chi\right)>0$ for $\lambda>0$ and that $\left|\frac{\partial T[x_0]}{\partial\chi}\right|-\left|\frac{\partial T_\lambda[x_0]}{\partial\chi}\right|>0$. Again by induction and using previous results about existence and uniqueness of a fixed point of the two operators, introducing passive workers makes the average wage less sensitive to χ, $0>\frac{\partial w_\lambda}{\partial\chi}>\frac{\partial w}{\partial\chi}$.

We now show that the wage is less sensitive to θ and θ^* when $\lambda>0$. Let $x_0\left(\theta,\theta^*,\chi\right)=0$. By previous arguments, we have that

$$\frac{\partial}{\partial\theta}T\left[x_0\right]+\frac{\partial}{\partial\theta^*}T\left[x_0\right]-\frac{\partial}{\partial\theta}T_\lambda\left[x_0\right]-\frac{\partial}{\partial\theta^*}T_\lambda\left[x_0\right]=\frac{\lambda}{1+\chi}\int_{\theta^*}^{+\infty}\phi\left(\zeta,\theta,\sigma\right)d\zeta>0.$$

Consider now the second iterate. Then we have that

$$\begin{aligned}
&\frac{\partial}{\partial\theta}T\left[x_0\right]+\frac{\partial}{\partial\theta^*}T\left[x_0\right]-\frac{\partial}{\partial\theta}T_\lambda\left[x_0\right]-\frac{\partial}{\partial\theta^*}T_\lambda\left[x_0\right]\\
&=\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left[\frac{\partial}{\partial\theta}T\left[x_0\right]\left(\theta,\theta^*,\chi\right)+\frac{\partial}{\partial\theta^*}T\left[x_0\right]\left(\zeta,\theta^*,\chi\right)\right]\phi\left(\zeta,\theta,\sigma\right)dz\\
&-\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left[\frac{\partial}{\partial\theta}T_\lambda\left[x_0\right]\left(\theta,\theta^*,\chi,\lambda\right)+\frac{\partial}{\partial\theta^*}T_\lambda\left[x_0\right]\left(\theta,\theta^*,\chi,\lambda\right)\right]\phi\left(\zeta,\theta,\sigma\right)dz\\
&+\frac{\lambda}{1+\chi}\int_{\theta^*}^{+\infty}\phi\left(\zeta,\theta,\sigma\right)dz\\
&+\frac{\lambda}{1+\chi}\int_{\theta^*}^{+\infty}\frac{\partial}{\partial\theta}T_\lambda\left[x_0\right]\left(\theta,\theta^*,\chi,\lambda\right)\phi\left(\zeta,\theta,\sigma\right)dz\\
&+\frac{\lambda}{1+\chi}\int_{\theta^*}^{+\infty}\frac{\partial}{\partial\theta^*}T_\lambda\left[x_0\right]\left(\theta,\theta^*,\chi,\lambda\right)\phi\left(\zeta,\theta,\sigma\right)d\zeta>0
\end{aligned}$$

where the last inequality follows because $\frac{\partial T[x_0]}{\partial\theta}+\frac{\partial T[x_0]}{\partial\theta^*}-\frac{\partial T_\lambda[x_0]}{\partial\theta}-\frac{\partial T_\lambda[x_0]}{\partial\theta^*}>0$ and $\frac{\partial T_\lambda[x_0]}{\partial\theta}+\frac{\partial T_\lambda[x_0]}{\partial\theta^*}>0$. Again by induction and using previous results about existence and uniqueness of a fixed point of the two operators, introducing passive workers results in $\frac{\partial w}{\partial\theta}+\frac{\partial w}{\partial\theta^*}>\frac{\partial w_\lambda}{\partial\theta}+\frac{\partial w_\lambda}{\partial\theta^*}>0$.

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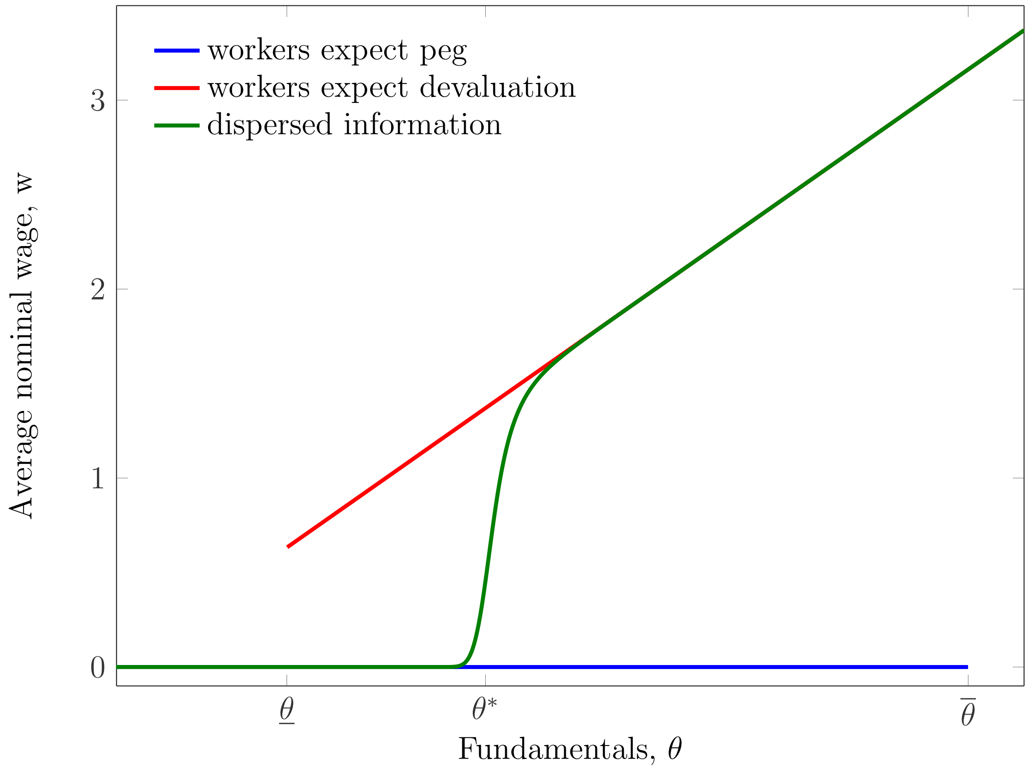

Figure 1: Equilibria under perfect foresight (red and blue lines) and dispersed information (green line). Figure drawn for $\chi = .25$, $\sigma = 0.01$.

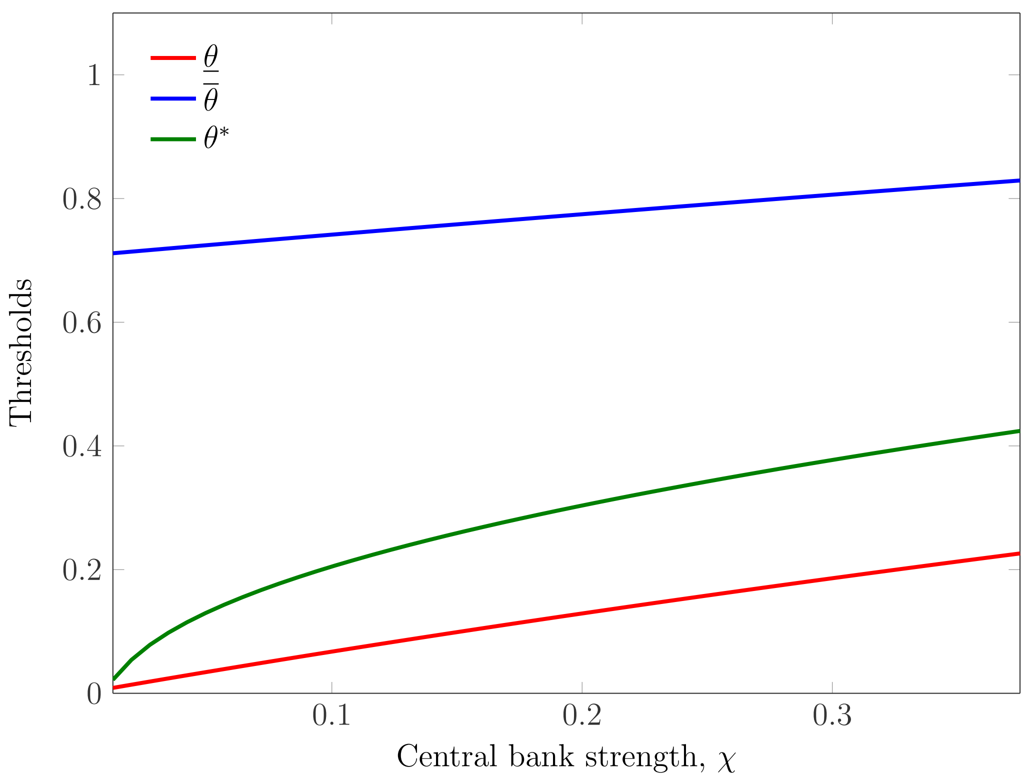

Figure 2: One-shot game. All thresholds rise with χ. Figure drawn for $\sigma = 0.01$.

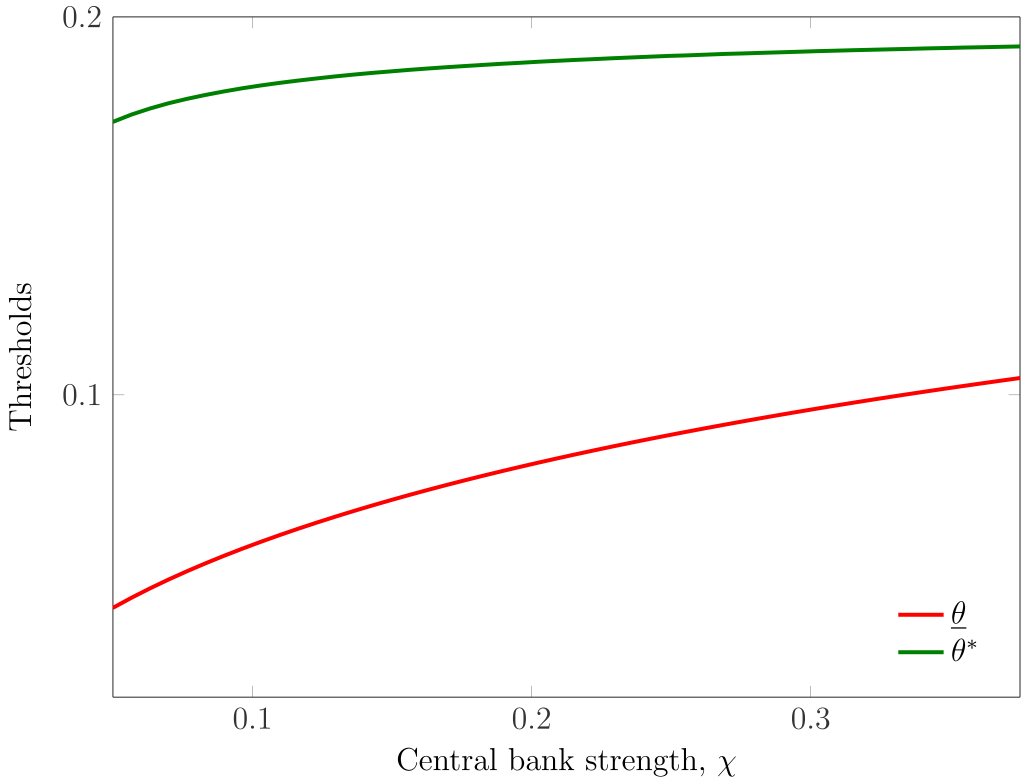

Figure 3: Repeated games. Strategic uncertainty makes θ^* less sensitive to χ than $\underline{\theta}$. Figure drawn for $\sigma = 0.01$, $\beta\mathbb{E}[\theta_{LR}^2] = 0.04$.

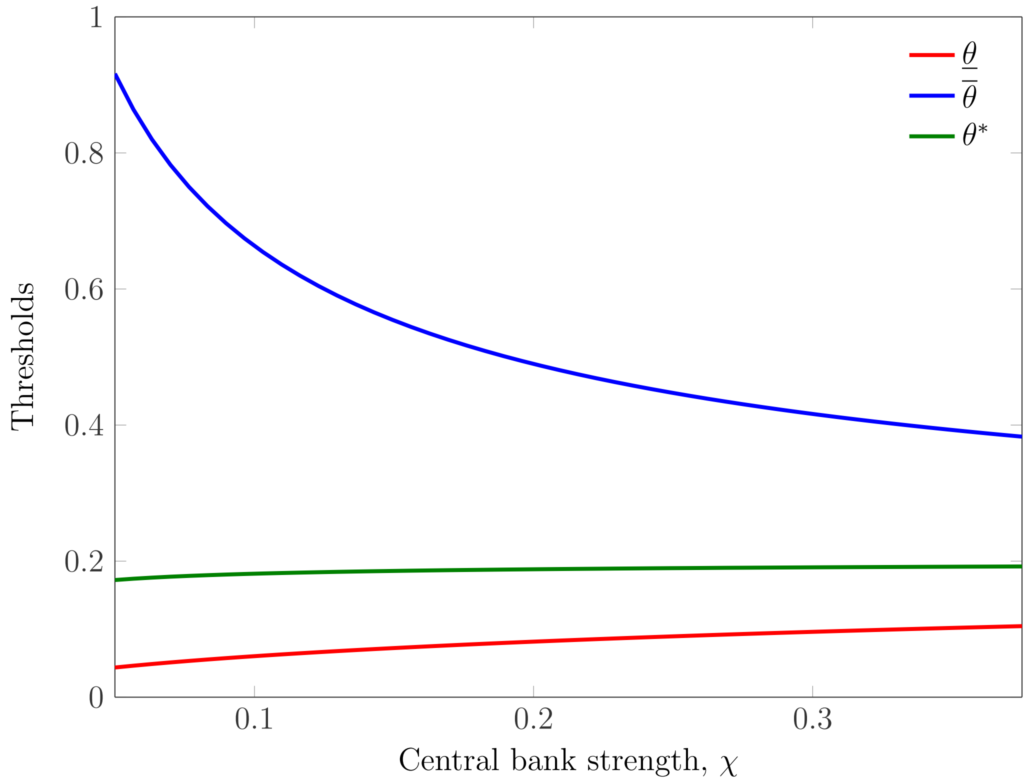

Figure 4: Repeated games. The upper threshold decreases in χ. Figure drawn for $\sigma = 0.01$, $\beta\mathbb{E}[\theta_{LR}^2] = 0.04$.

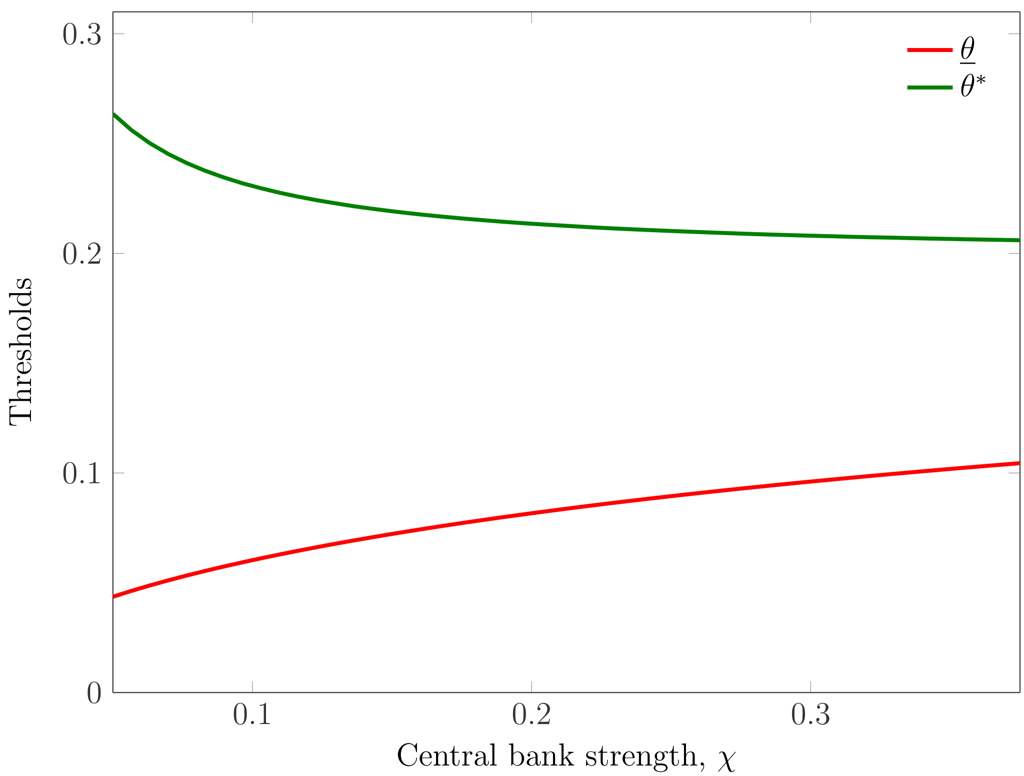

Figure 5: : Repeated games and pre-determined wages. The dispersed-information threshold decreases in χ. Figure drawn for $\sigma = 0.01$, $\beta\mathbb{E}[\theta_{LR}^2] = 0.04$, $\lambda = 0.05$.

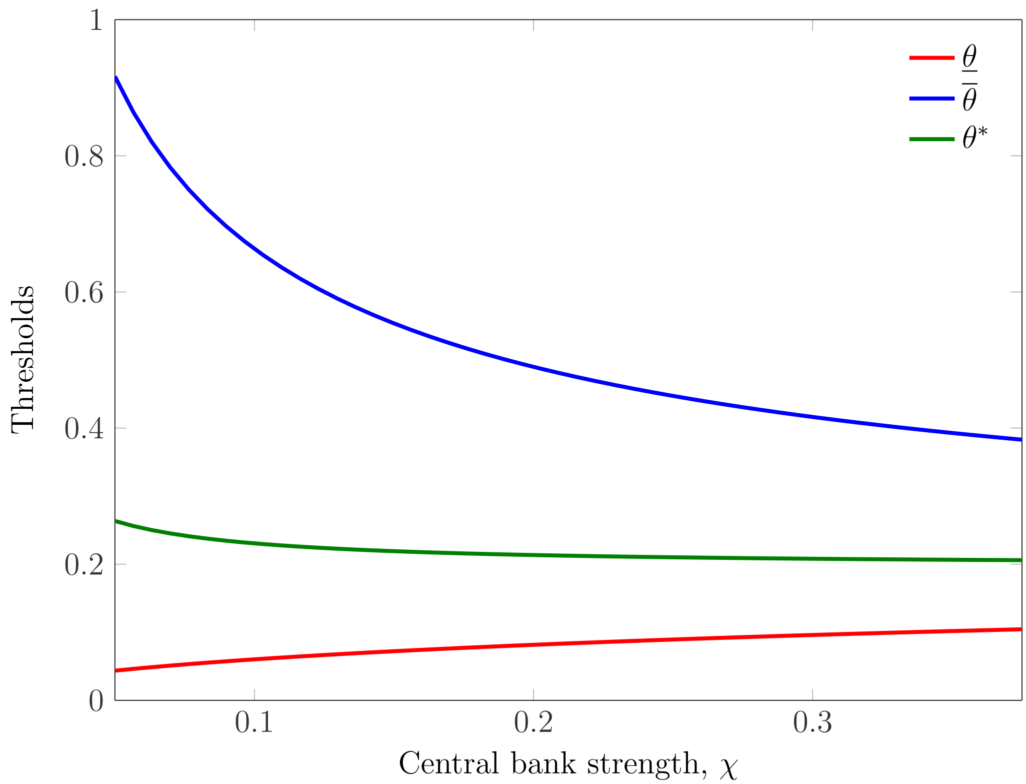

Figure 6: Repeated games and pre-determined wages. The upper perfect-foresight threshold and the dispersed-information threshold both decrease in χ. Figure drawn for $\sigma = 0.01$, $\beta\mathbb{E}[\theta_{LR}^2] = 0.04$, $\lambda = 0.05$.