

Bank lending to non-financial corporations and the real economy: a wavelet analysis*

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December 2015

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Abstract

We study the relationship between bank loans to non-financial corporations and real activity in Germany, France, Italy and Spain using wavelet analysis. Wavelet analysis allows to account for variations in the relationship between bank lending and real activity both over time and across frequencies. We find evidence of strong comovements between the growth rates in real bank loans and real GDP for all countries although with cross-country differences across the frequency spectrum. In particular, in Germany the relationship at lower frequencies is weaker compared to France but especially compared to Italy and Spain. For all countries we find that real bank lending is lagging real activity. Similar results are obtained using real business fixed investment as real activity measure. Focussing on real investment in machinery and equipment, ie excluding construction leads to a decline in the strenght of the comovements at low frequencies for France, Italy and Spain.

Keywords: bank lending to firms, real activity, business cycle, wavelet analysis

JEL classification: C30, E32, E51.

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1 Introduction

The relationship between loans and real activity has some tradition ([Bernanke and Gertler, 1989](#); [Bernanke, Gertler, and Gilchrist, 1996](#); [Kiyotaki and Moore, 1997](#)), although it played no central role in monetary policy analysis in the late 1990s and early 2000s. The financial crisis led to a renewed interest in the relationship between these variables ([Schularick and Taylor, 2012](#)).

The analyses of the link between loans and business cycles can be grouped into correlation analysis, estimation of forecasting models or into models explaining lending.

There is a series of papers analyzing the long-run (cointegrating) relationships of bank lending and real GDP growth. [Sorensen, Ibanez, and Rossi \(2009\)](#) estimate a vector error correction model in which loan growth is explained by real GDP and some other variables. In this analysis the focus is on the explanation of bank lending at low (zero) frequency, real GDP is assumed as being weakly exogenous.

[Karfakis \(2013\)](#) finds that for (quarterly) Greek data from 2000 to 2011 real credit is Granger causal for real output. Real credit is leading by about three quarters, even by accounting for external effects (trade deficit ratio). This result is robust whether first differences are used or cyclical components (at business cycle frequencies) are computed applying the HP filter.

[Antonakakis, Breitenlechner, and Scharler \(2014\)](#) analyse the dynamic interactions between credit growth and output growth applying the spillover index approach by Diebold and Yilmaz for G7 countries. They find a bidirectional causation and the link becoming more pronounced before the financial crisis.

[Zhu \(2011\)](#) applies time and frequency domain methods to analyze this relationship at business cycle frequencies, although the latter are applied only to US data. Correlation analysis on euro area data shows that there is a strong relationship between bank credit and real activity. Bank credit lags output by two quarters. This reinforces results obtained in [ECB \(2011\)](#).

[Claessens, Kose, and Terrones \(2012\)](#) argue that the duration and amplitude of financial cycles are longer and larger than those of output cycles, reflecting the build-up of instabilities. These results are based on an analysis of turning points in these series. [Drehmann, Borio, and Tsatsaronis \(2012\)](#) augment this approach by filter based methods.

Frequency methods can be applied to estimate the spectral densities of credit growth and output growth to test whether the corresponding cycles have similar characteristics. These methods assume stationarity of the time series implying a constant pattern of cyclicity over time. As these pattern may be time-varying, wavelet analysis may be more appropriate. Wavelet analysis is applied by ([Rua, 2012](#)) and ([Mandler and Scharnagl, 2014](#)) on the relationship between money and inflation in the euro area.

In this paper the relationship between bank loans to non-financial corporations and real activity is analysed for Germany, France, Italy and Spain. The advantage of this approach is that there is no need to pre-specify a specific frequency band or turning points. The estimated coherencies show whether there are shifts in the dominating periodicities or in the lead-lag relation.

2 Wavelet Analysis

Wavelet analysis is a tool for analysis in the frequency domain and thus an element of spectral analysis.¹ Spectral analysis focuses on the contribution of periodic cycles of specific frequencies to the variance of a time series or on comovements of cycles of specific frequencies in multiple time series. The standard approach to spectral analysis is based on Fourier analysis which allows to distinguish between changes across the frequency spectrum in the importance of cycles within individual time series or relationships between multiple time series. However, it has no resolution in time, i.e. it does not convey information about time variation. Furthermore, Fourier analysis is based on the assumption of stationary time series which is frequently violated in macroeconomics.

To capture time variation the short time or windowed Fourier transformation, which applies the Fourier transformation not to the full time series but to rolling subsamples has been suggested. The choice of window length is subjected to a trade-off: narrow windows provide good localization in time, i.e. result in more precision as far as the detection of changes in time is concerned while wider windows provide a better frequency resolution (e.g. Rua, 2012, p. 73). However, the short time Fourier analysis uses rolling windows with length independent of frequencies under consideration, a property which has been shown to result in a suboptimal trade-off, since the optimal window length should depend on the frequency under investigation.

Wavelet analysis provides an alternative approach to analysing changes in the periodic behaviour of time series and in the relations between multiple time series over time and across frequencies (Aguilar-Conraria, Azevedo, and Soares, 2008). Basically, wavelet analysis models periodic cycles of different frequencies in time series using flexible periodic functions with only finite length which can be stretched (scaled) to approximate lower or compressed to approximate higher frequencies. Stretching and compressing the wavelet can be thought of as corresponding to the choice of a flexible window length depending on the frequency, with wider windows being used when moving to lower frequencies. As a result, wavelet analysis implies an improved time resolution for high frequency fluctuations and improved frequency resolution for low frequencies compared to the short time Fourier transform.

The basic building block is a so-called mother wavelet ψ from which, by scaling and translating a variety of wavelets can be generated

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{|s|}} \psi\left(\frac{t-\tau}{s}\right), \quad (1)$$

where $s \neq 0$ is a scaling or dilation factor which controls the width of the wavelet (an increase in s stretches the wavelet in time). High frequency fluctuations imply low values of the scaling factor while low frequency fluctuations imply high values for s . The translation parameter τ controls the location of the wavelet, i.e. changes in τ shift the wavelet in time. The function ψ has to fulfil some requirements in order to have the properties of wavelets.²

¹This section draws heavily on Aguiar-Conraria and Soares (2014). An introduction to wavelet analysis is also provided by Rua (2012).

²For details see, for example Percival and Walden (2002).

The continuous wavelet transform (CWT) of a time series $x(t)$ with respect to the wavelet ψ is obtained by projecting $x(t)$ on a family of wavelets $\{\psi_{\tau,s}\}$ and is defined as

$$W_x(\tau, s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|s|}} \psi^* \left(\frac{t - \tau}{s} \right) dt, \quad (2)$$

where $*$ denotes the complex conjugate. The CWT may also be represented in the frequency domain as

$$W_x(\tau, s) = \frac{\sqrt{|s|}}{2\pi} \int_{-\infty}^{\infty} \psi^*(s\omega) X(\omega) e^{i\omega\tau} d\omega,$$

where $X(\omega)$ denotes the Fourier transform of $x(t)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt,$$

and ω is the angular frequency.

Various wavelet functions have been suggested. The one most widely used in applications in economics and which we will also use in this paper is the Morlet wavelet

$$\psi_{\omega_0}(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}, \quad (3)$$

with parameter ω_0 . The Morlet wavelet is complex valued allowing for an analysis of phase differences, i.e. lead-lag relations between time series. The most common choice is $\omega_0 = 6$ which results in some desirable properties of the Morlet wavelet (e.g. [Aguilar-Conraria and Soares, 2014](#), p. 352). In particular, this specific choice yields a simple relation between scale (s) and frequency ω , $\omega \approx \frac{1}{s}$ and implies an optimal joint time-frequency resolution (e.g. [Aguilar-Conraria and Soares, 2014](#), p. 352).

Using the CWT, the wavelet power spectrum is defined as

$$WPS_x(\tau, s) = |W_x(\tau, s)|^2. \quad (4)$$

The wavelet power spectrum can be interpreted as the local variance of the time series $x(t)$ at each point in time and frequency. In the case of a complex-valued wavelet, the corresponding wavelet transform is also complex-valued and can be decomposed into a real part, the amplitude, $|W_x(\tau, s)|$, and its imaginary part, the phase, $|W_x(\tau, s)| e^{i\phi_x(\tau, s)}$. The phase-angle $\phi_x(\tau, s)$ is

$$\phi_x(\tau, s) = \arctan \left(\frac{\Im \{W_x(\tau, s)\}}{\Re \{W_x(\tau, s)\}} \right), \quad (5)$$

where \Im denotes the imaginary part and \Re the real part of the wavelet power spectrum. Thus, the phase angle is only defined for complex-valued wavelets.

Considering the relationship between two time series $x(t)$ and $y(t)$ the cross wavelet

transform is defined as

$$W_{xy} = W_x W_y^*, \quad (6)$$

where $*$, as before, denotes the complex conjugate.

From the cross wavelet transform of $x(t)$ and $y(t)$ and the wavelet power spectra of both time series the wavelet coherency can be derived as

$$R_{xy}(s) = \frac{|S(s^{-1}W_{xy}(s))|}{\sqrt{S(s^{-1}|W_x|^2)}\sqrt{S(s^{-1}|W_y|^2)}}. \quad (7)$$

The wavelet coherency can be interpreted as local correlation between the two time series, similar to a correlation coefficient. S is a smoothing operator with respect to time and scale. Without smoothing the wavelet coherency would be equal to one across all times and scales. If the wavelet coherency is complex valued it can be decomposed into its real and imaginary parts \Re and \Im , and the wavelet phase difference can be computed as

$$\phi_{x,y}(s, \tau) = \arctan\left(\frac{\Im\{W_{xy}(\tau, s)\}}{\Re\{W_{xy}(\tau, s)\}}\right), \quad (8)$$

with $\phi_{x,y}(s, \tau) = \phi_x(s, \tau) - \phi_y(s, \tau)$. The phase difference $\phi_{x,y}(s, \tau) \in [-\pi, \pi]$ provides information about the lead-lag relationships between the two time series. If $\phi_{x,y}(s, \tau) = 0$, the series x and y move together at the given scale and time. If $\phi_{x,y}(s, \tau) \in (0, \frac{\pi}{2})$, series x leads y and if $\phi_{x,y}(s, \tau) \in (-\frac{\pi}{2}, 0)$, y leads x .³ Using the phase difference, the time lag or time difference, which gives the lead or lag of the series in the time domain can be calculated as

$$\Delta T_{x,y}(s, \tau) = \frac{\phi_{x,y}(s, \tau)}{\omega}. \quad (9)$$

3 Empirical Results

We use quarterly data for the period from 1980 until the end of 2014. For the lending series we use data on stocks of loans of MFIs (monetary financial institutions) to non-financial corporations in Germany, France, Italy and Spain. After transactions-based flow data becomes available in 1997 we use this series to construct series of notional stocks which are adjusted for non-transaction related changes such as statistical reclassifications, revaluation etc. Furthermore, after 2009 the series is also adjusted for loan sales and securitisations.⁴ The loan series are deflated using the GDP deflator for each country. For the activity measures we use for each country real GDP, real business fixed investment and real investment in machinery and equipment. For the empirical analysis all series are transformed into annual growth rates.

³However, the economic interpretation of the lead-lag pattern is not as clear cut. See Section 3, footnote 15.

⁴For details, see the technical notes to the ECB's Statistical Bulletin. Our results are robust with respect to using the stocks (outstanding amounts) over the entire sample period instead of using the notional stocks when possible.

Figure 1: Wavelet coherency of growth in real loans to non-financial corporations and real GDP

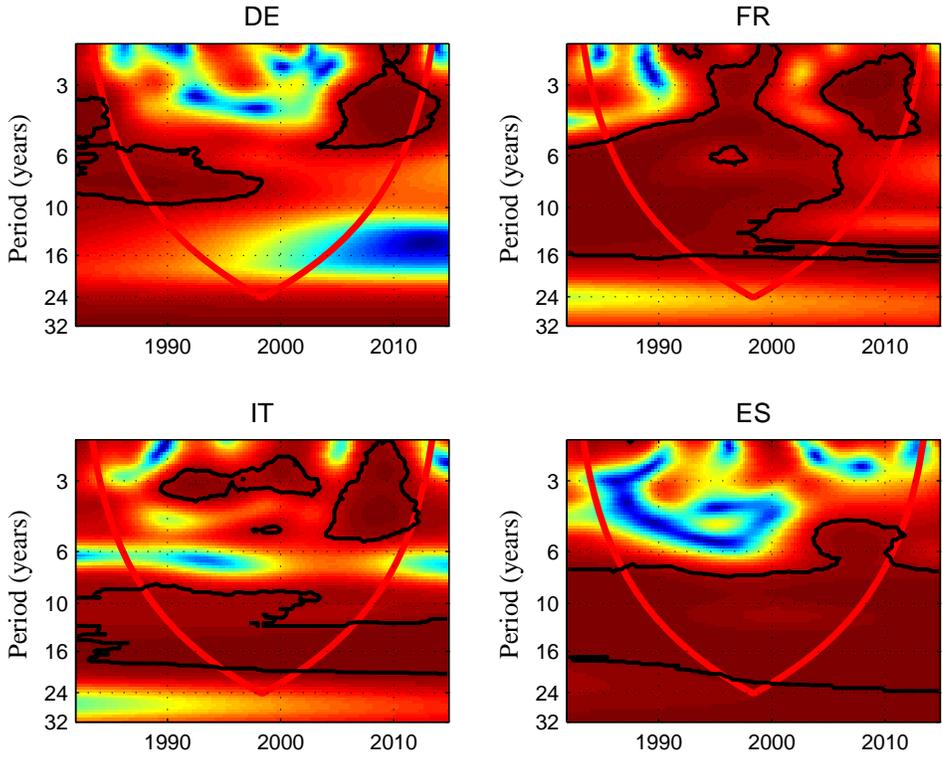


Figure 1 displays for each country the estimated wavelet coherency across time (horizontal axis) and across frequencies (vertical axis).⁵ The coherency is colour coded and increasing from blue to red. The black lines indicate coherencies that are significantly different from zero at the 5% level. (??? correct).⁶ The curved red lines indicate the “cone of influence” which is the boundary of the region for which the coherency can be interpreted.⁷ For Germany the results indicate strong comovements between loan growth and real GDP for fluctuations with a length of about six to ten years until the late 1990s while the relationship at lower frequencies turns out to be less pronounced. After the mid-2000s we estimate strong coherencies at somewhat higher frequencies than before. In contrast, for France until about 2000 the significant coherencies extend over a much broader frequency spectrum and for fluctuations with a length of about 16 years we estimate significant coherencies for all time periods within the cone of influence. This stable relationship at low frequencies is also estimated for Italy and Spain where it extends over a broader range (for fluctuations with periods about 10 to 20 years in Italy and about 7 to 20 years in Spain). At higher frequencies the results indicate strong comovements also for Italy from the early 1990s to the mid 2000s. For Germany, France and to some extent also for Italy there are strong comovements at frequencies associated with business cycle movements that is for fluctuations with period of up to about ten years.⁸ In contrast, for Italy, but even more so for Spain the main comovements are at the upper bound of this frequency range and extend to fluctuations that can be interpreted as trend movements.⁹ The higher coherencies that are estimated for Germany, France, and Italy at frequencies with period length of up to six years in the late 2000s are likely to be related to the financial crisis which led to both a collapse in bank lending and a strong contraction in output.

As another form of visualisation Figure 2 displays the average of the wavelet coherency within three different frequency ranges (periods of 2-6, 6-10, and 10-16 years) for each country and shows the evolution of these average coherencies over time. In all four

⁵All estimations were performed using the AST-toolbox for MATLAB by Aguiar-Conraria and Soares. <https://sites.google.com/site/aguiarconraria/joanasoares-wavelets/>

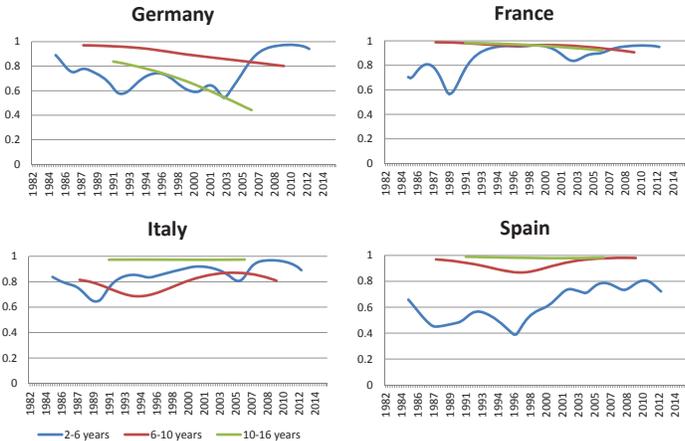
⁶Significance is tested using a simulation procedure in which under the null hypothesis of a zero coherency two uncorrelated AR processes are fitted to both time series. From these estimates artificial data is simulated. By estimating the wavelet coherency for each simulated pair of time series the distribution of wavelet coherencies under the null hypothesis can be obtained. For details, see [Aguiar-Conraria and Soares \(2014\)](#).

⁷The flexible window length that is used for the wavelet transform increases when moving to lower frequencies. Thus the estimation of the coherency requires more and more observations on both sides of the point in time of interest. Thus, the time period for which a meaningful coherency can be computed shrinks for lower frequencies because of the limited length of the sample period. Technically, if there is an insufficient number of past or future observations to estimate the the wavelet power spectra and cross spectrum at a given point in time the algorithm extends the sample forward or backward by “reflecting” the first or last observations, respectively. The time-frequency combinations outside of the cone of influence are subject to this procedure.

⁸For an upper bound on business cycle fluctuations in the euro area of about ten to twelve years, see [Musso \(2004\)](#).

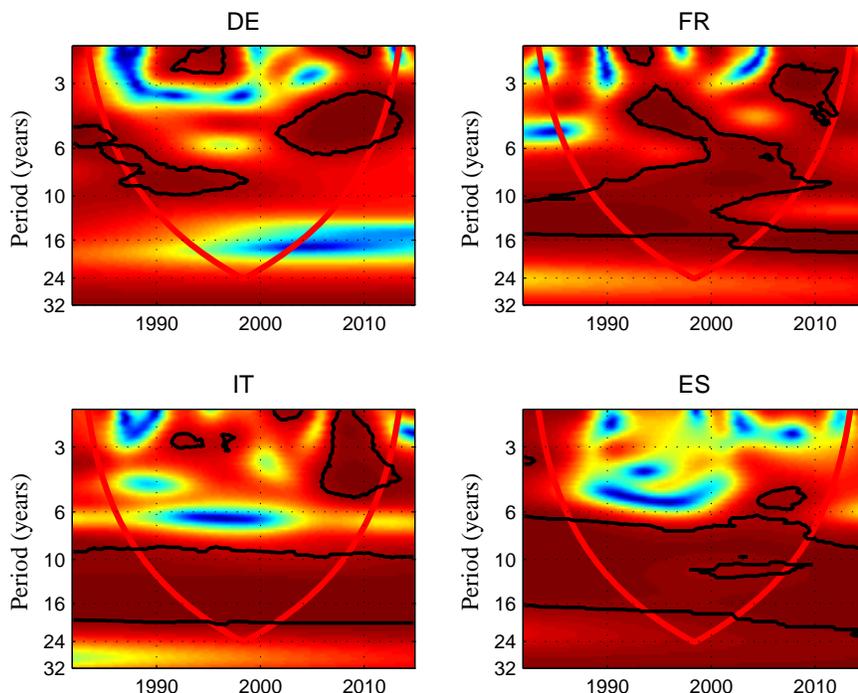
⁹The range of frequencies we actually can analyze is restricted by the available data as the cone of influence narrows down when moving to longer periods of fluctuations and finally collapses to range zero. Thus, we can analyse comovements in the trend components only to a limited extent and will consider results for the lower frequency range in the graphs, i.e. 10 years and longer, as giving some indication about the correlation of the trends.

Figure 2: Average wavelet coherency of growth in real loans to non-financial corporations and real GDP for selected frequency bands



Note: average coherency across different frequency bands. Different estimation periods reflect the contraction of the cone of influence when moving to lower frequencies.

Figure 3: Wavelet coherency of growth in real loans to non-financial corporations and real business fixed investment

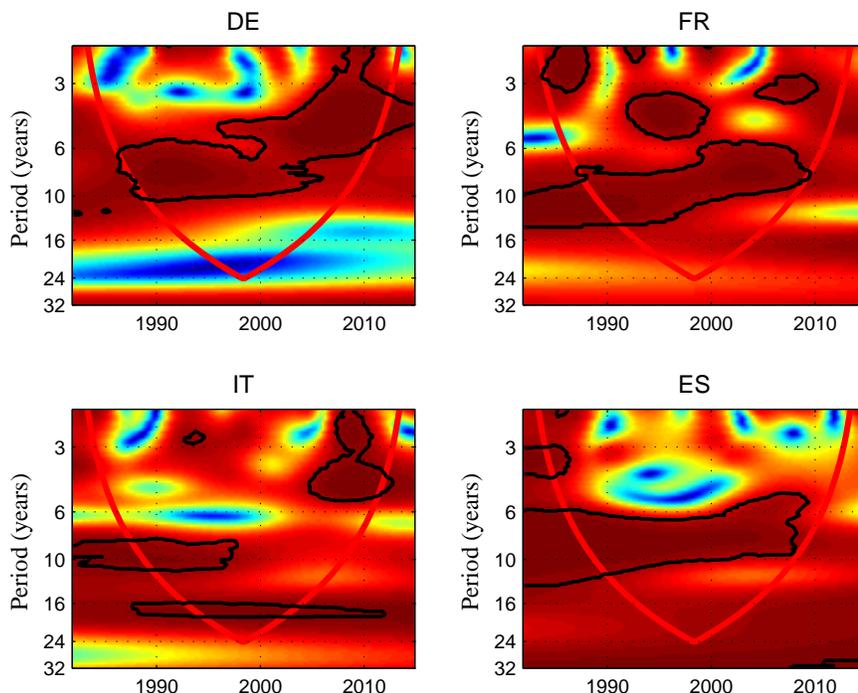


countries coherency in the higher frequency band displays more time variation compared to the middle and lower frequency bands. For Germany, the coherency in the low frequency band is relatively low compared to the other countries (it is close to one in Italy and Spain) and has experienced a marked decline over time. Thus, in Germany the average coherency in the middle frequency range (6-10 years) is higher than that for the low frequency range throughout the sample period while in France both coherencies are very close to each other and in Italy and Spain the order is reversed.

The results in Figure 3 consider real business fixed investment in place of real GDP as real activity measure and show that the results shown before are broadly robust with respect to this modification. However, when using real investment in machinery and equipment (Figure 4) the estimated coherency increases somewhat for Germany at business cycle frequencies while the comovements for the other countries tend to become weaker at all frequency ranges but particularly so at the lower frequencies. Since the difference in the investment series is investment in construction this result suggests that construction is likely to be an important determinant of the low-frequency relationship between lending and investment in France, Italy and Spain. In contrast, for Germany removing construction enhances the comovements between loan growth and investment.

The lead-lag structure between real loans and real activity can be analyzed using the time difference (9). Figure 5 to Figure 7 show the average time difference between the cycles in the growth rates in real loans and real GDP for frequencies with periods of 2-6, 6-10 and 10-16 years. Time differences are measured on the vertical axis in years and should only be interpreted for the time periods within the cone of influence in Figure 1.

Figure 4: Wavelet coherency of growth in real loans to non-financial corporations and real investment in machinery and equipment

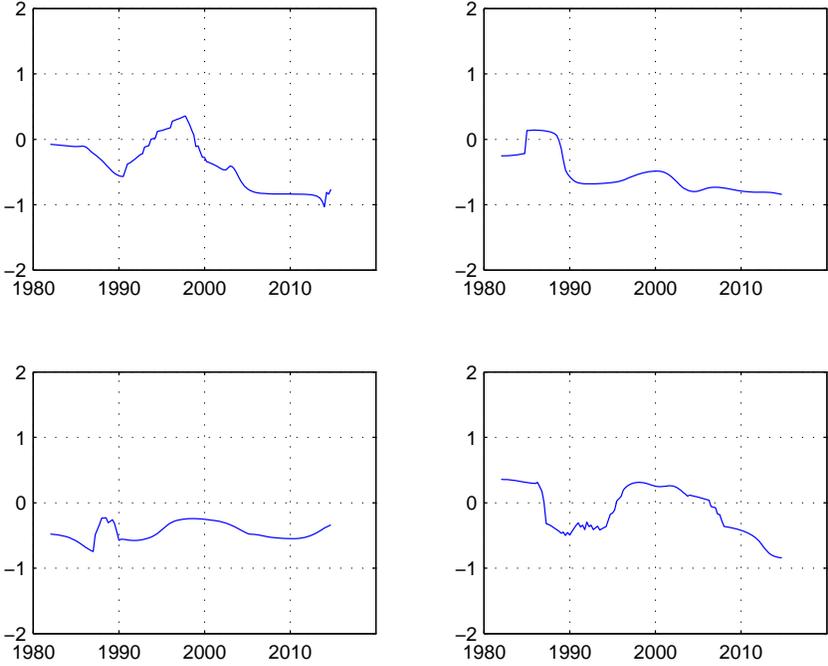


A negative time difference implies that real GDP growth leads real loan growth and vice versa. Again, the estimates for the higher frequency band turn out to be more volatile than those for the lower bands. For fluctuations with periods between 6 and 10 years we estimate a lag of half a year up to about two years between real lending behind real activity in all countries. While this lag does not change much for France, Italy and Spain it has been markedly increasing in Germany over the 2000s. For the low frequency components the estimates show lags of lending relative to real GDP of 2-3 years (France and Italy) or 1.5 to 2 years (Spain) over the time period for which results can be interpreted (1990Q3 to 2007Q3). For Germany the results indicate contemporaneous correlation, however since the coherency over this frequency band is relatively low for Germany, the results should not be overemphasised. Over the higher frequency range the time differences indicate a time lag of lending relative to economic activity, as well, with a possible exception of Spain where for about half of the sample we estimate a short lead of real lending.¹⁰

Figure 8 to Figure 10 show that the results obtained from using real GDP as activity measure are robust to using real business fixed investment instead. The results in Figure 11 to Figure 13 apply to the case of using real investment in machinery and equipment and again turn out to be similar to those obtained for real GDP except for the case of

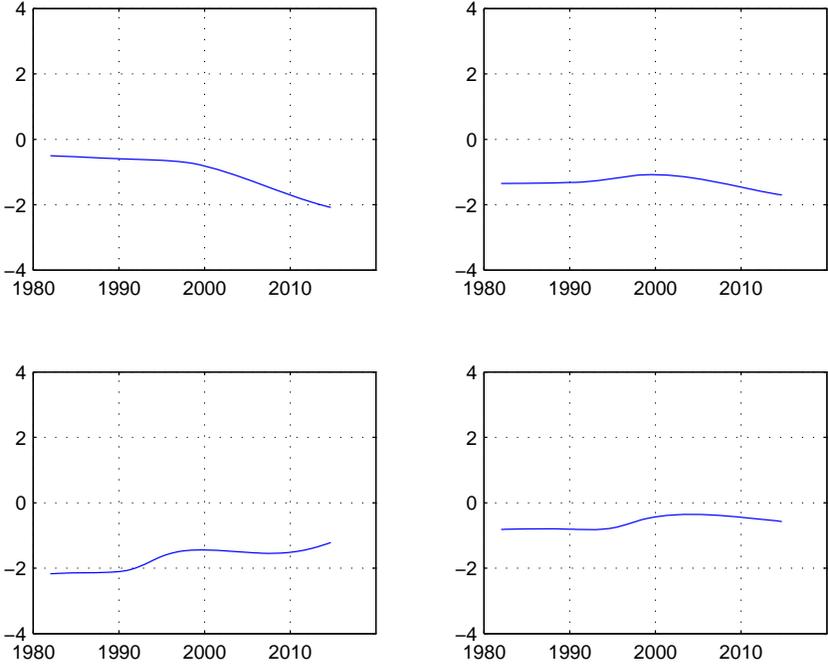
¹⁰As explained before the phase differences from which the time differences are derived refer to the relative position of the two time series' cycles on the unit circle and the distinction between lead and lag is not entirely clear, as a short lead could also be interpreted as a long lag and vice versa. Given the average length of the cycles analysed here compared to the estimated time differences the interpretation given in the text seems to be the more plausible one.

Figure 5: Average time difference between growth in real loans to non-financial corporations and real GDP - fluctuations with period of 2-6 years



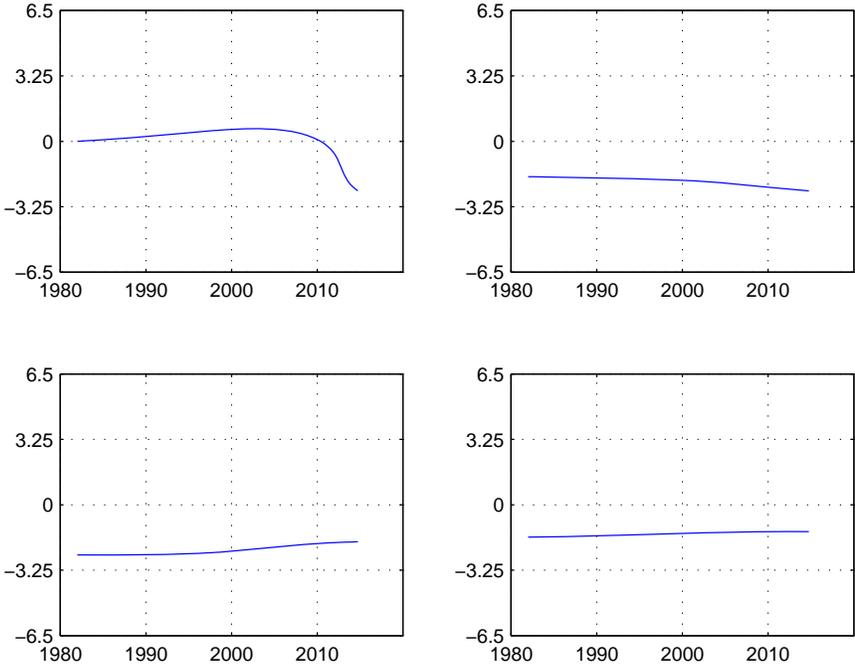
Note: top row from left to right: Germany and France; bottom row from left to right: Italy and Spain.

Figure 6: Average time difference between growth in real loans to non-financial corporations and real GDP - fluctuations with period of 6-10 years



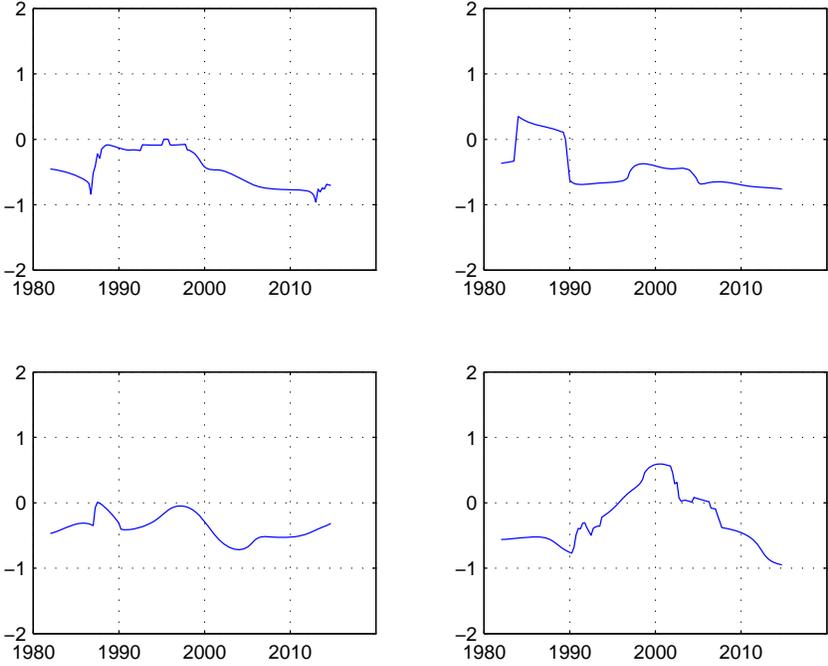
Note: top row from left to right: Germany and France; bottom row from left to right: Italy and Spain.

Figure 7: Average time difference between growth in real loans to non-financial corporations and real GDP - fluctuations with period of 10-16 years



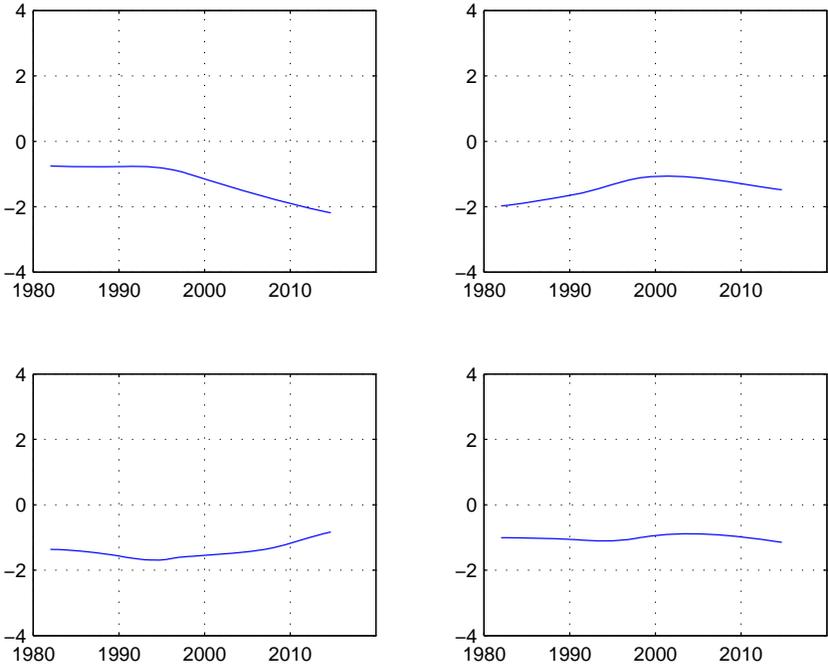
Note: top row from left to right: Germany and France; bottom row from left to right: Italy and Spain.

Figure 8: Average time difference between growth in real loans to non-financial corporations and real business fixed investment - fluctuations with period of 2-6 years



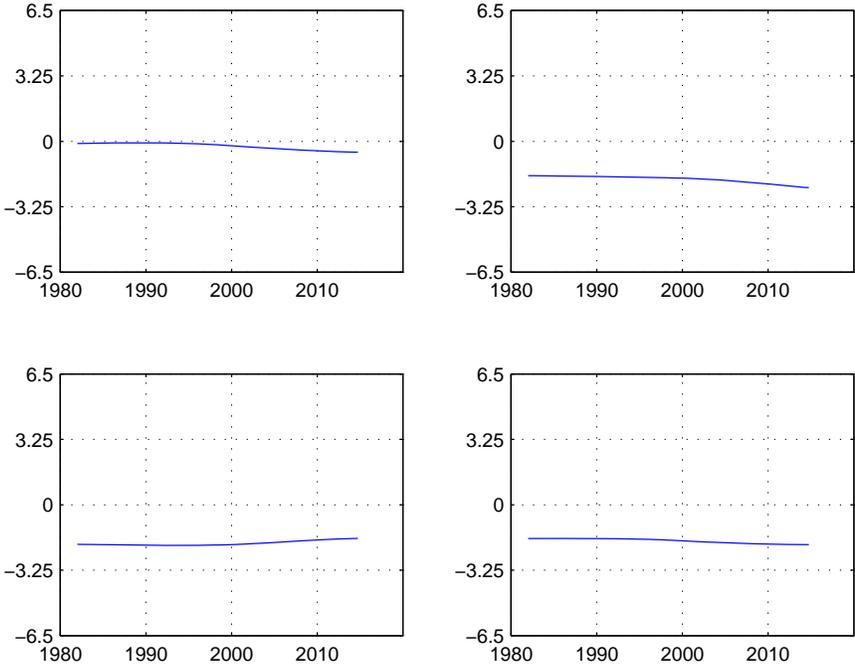
Note: top row from left to right: Germany and France; bottom row from left to right: Italy and Spain.

Figure 9: Average time difference between growth in real loans to non-financial corporations and real business fixed investment - fluctuations with period of 6-10 years



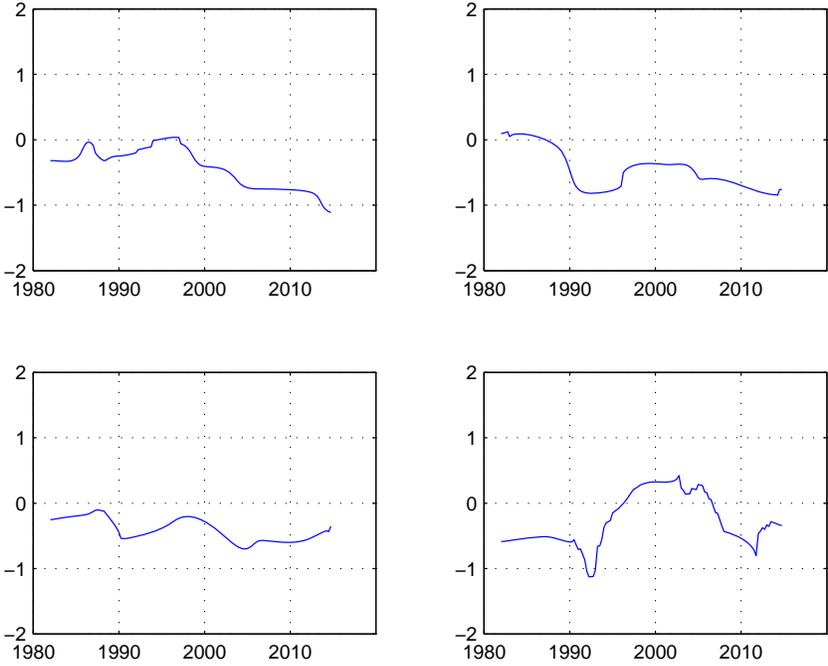
Note: top row from left to right: Germany and France; bottom row from left to right: Italy and Spain.

Figure 10: Average time difference between growth in real loans to non-financial corporations and real business fixed investment - fluctuations with period of 10-16 years



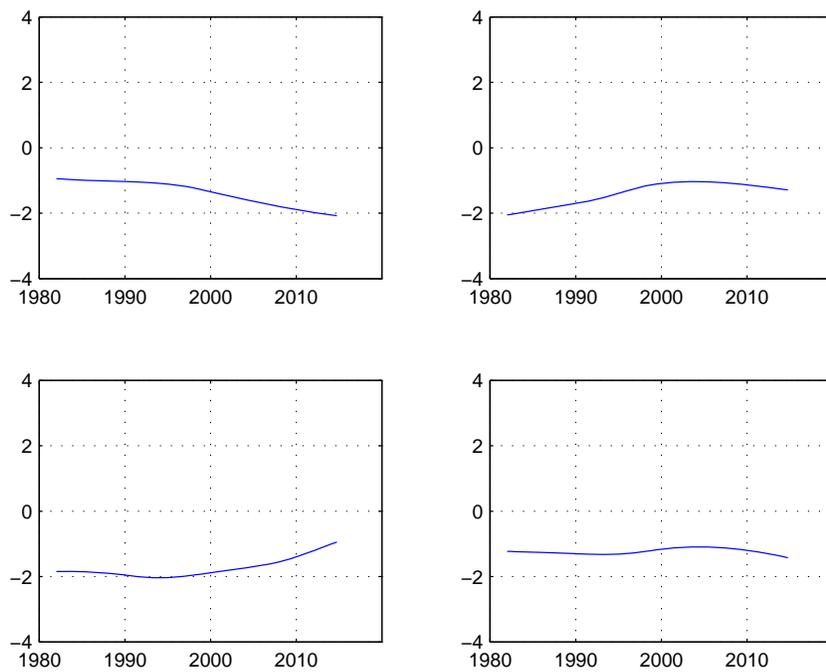
Note: top row from left to right: Germany and France; bottom row from left to right: Italy and Spain.

Figure 11: Average time difference between growth in real loans to non-financial corporations and real investment in machinery and equipment - fluctuations with period of 2-6 years



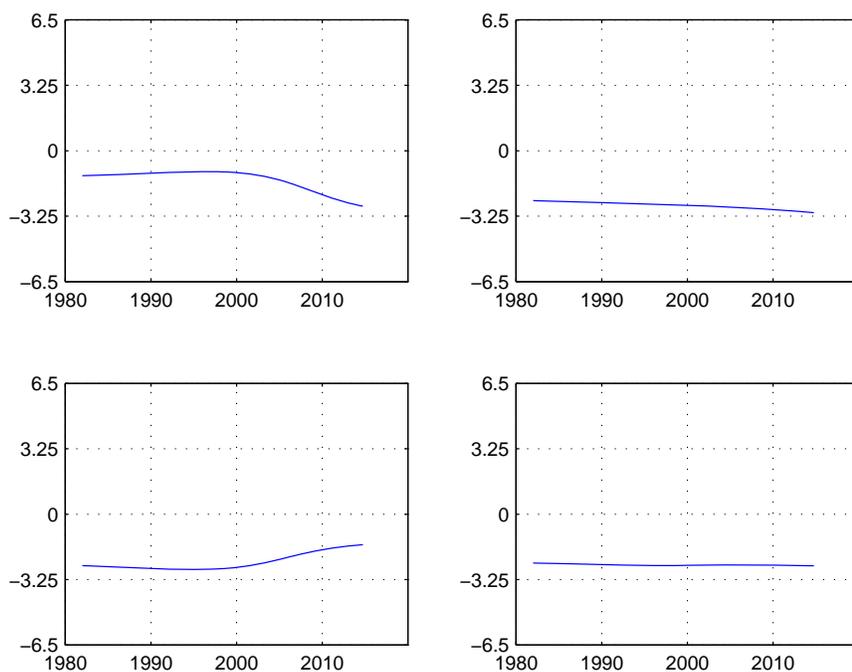
Note: top row from left to right: Germany and France; bottom row from left to right: Italy and Spain.

Figure 12: Average time difference between growth in real loans to non-financial corporations and real investment in machinery and equipment - fluctuations with period of 6-10 years



Note: top row from left to right: Germany and France; bottom row from left to right: Italy and Spain.

Figure 13: Average time difference between growth in real loans to non-financial corporations and real investment in machinery and equipment - fluctuations with period of 10-16 years



Note: top row from left to right: Germany and France; bottom row from left to right: Italy and Spain.

Germany, where the results for the 6-10 year period cycles now show a lag of bank lending relative to investment of about 1-1.5 years. Note, however that this results again applies to a frequency band where the coherencies were estimated to be not very pronounced.

4 Conclusions

The empirical literature on modeling the relationship between bank lending and economic activity assumes a stable relationship between the lending and output, production or other activity measures. We provide additional evidence on this relationship using wavelet analysis which allows us to account for the possibility of time-variation and changes across the frequency spectrum, i.e. changes depending on the length of the cycles in question. We present evidence for strong comovements between the growth rates of real bank lending to non-financial corporations and real activity in the four large euro area countries where real activity is measured by growth rates of real GDP or real investment. While there is evidence of pronounced comovements between the growth rates in real bank loans and real GDP in all countries we also find important cross-country differences. In particular, in Germany the relationship at lower frequencies is weaker compared to France but especially compared to Italy and Spain. At business cycle frequencies the comovements are more pronounced for France than for Italy and Spain where the relationship can be found more at the lower end of the frequency range. For all countries we find that real bank lending is lagging real activity at the relevant frequencies. Similar results are obtained using real business fixed investment as real activity measure. Focussing on real investment in machinery and equipment, i.e. excluding construction leads to a decline in the strength of the comovements at low frequencies for France, Italy and Spain while it enhances the strength of the comovements in the middle and upper frequency ranges in Germany.

Overall, the results suggest for all countries except for Germany stable comovements at middle to lower frequency ranges. The disruption for France is probably linked to the financial crisis. The relationship on lower frequencies appears in part to be driven by developments in business construction as moving to an investment measure that excludes construction reduces the correlations, while the move from real GDP to an investment measure that includes investment in construction does not - which does argue against housing as being behind this result.

These results also suggest that models that are based on establishing long-run equilibrium relationships between bank lending to firms and real activity capture important characteristics of the data in France, Italy and Spain. In contrast, the results can be interpreted as evidence of a comparatively weaker link between bank lending to non-financial firms and real activity measured by real GDP or investment in Germany. Investment in machinery and equipment seems to be a more useful variable to explain lending to firms in Germany but even there the comovements at low frequencies are much weaker than in the other countries.

The difference between Germany and the other countries requires further investigation. Possible explanations might be that German firms have increased their reliance on internal funding over time thus weakening the link between output or investment and bank lending or that they have been substituting bank lending by other external financing.

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