Real-Time Forecasting with a MIDAS VAR

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Abstract

This paper presents a stacked vector MIDAS type mixed frequency VAR (MFVAR) forecasting model that intends to be general and yet compact, flexible and yet parsimonious in terms of parametrization as well as easy to estimate. First, we develop a compact mixed frequency VAR framework for multiple variables and multiple frequencies using a stacked vector approach. Second, we integrate the mixed frequency VAR with a non-linear Almon MIDAS polynomial scheme which is designed to reduce the parameter space while keeping models flexible. Third, we show how to recast the resulting stacked vector MIDAS type non-linear MFVAR into a linear equation system that can be easily estimated equation by equation using standard OLS. In order to empirically evaluate the predictive accuracy of our model, we conduct a pseudo out-of-sample forecasting exercise using US real-time data. The MFVAR substantially improves accuracy upon a standard VAR for different VAR specifications: Root mean squared forecast errors for, e.g., GDP growth get reduced by 30 to 50 percent for forecast horizons up to six months and by about 20 percent for a forecast horizon of one year.

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1 Introduction

Vector autoregression (VAR) models are a standard tool for macroeconomic forecasting. While VAR models do not necessarily outperform alternative approaches in forecasting a single variable, they capture the dynamics in multiple time series and, hence, allow to generate *coherent* forecasts of multiple variables. A challenge to joining multiple macroeconomic variables in a VAR is that macroeconomic variables are usually sampled at different frequencies. For instance, GDP comes at a quarterly frequency whereas inflation is published monthly and short term interest rates are quoted at a daily or even higher frequency. The traditional solution is to simply time-aggregate all higher frequency series to the frequency of the lowest frequency series in the sample. A VAR including GDP, inflation and a interest rate will then be a quarterly frequency VAR (QFVAR). However, such time aggregation comes at costs. First, any new data releases or data revisions, which occur *within* the lowest frequency, are only taken into account after the end of each lowest frequency period. To stick to the above example, a QFVAR does not allow to consider inter-quarterly inflation and interest rate releases and inter-quarterly GDP revisions before the end of a quarter. This delayed processing of information potentially impairs (inter-quarterly) forecasts and nowcasts. Second, the time aggregation implies a peculiar constraint on the parameters attached to higher frequency variables that is potentially quite suboptimal.

To overcome the aforementioned drawbacks, we integrate a mixed frequency VAR (MFVAR) framework with MIxed DAta Sampling (MIDAS). The MIDAS approach has originally been developed by Ghysels and co-authors (Ghysels *et al.*, 2007, Andreou *et al.*, 2010, e.g.) for forecasting with single equations (cf. the applications by Clements and Galvão, 2008, 2009, Armesto *et al.*, 2010, Drechsel and Scheufele, 2012, Andreou *et al.*, 2013, Ferrara *et al.*, 2014, Mikosch and Zhang, 2014, among many others). In a first step, we propose a general – and yet compact – MFVAR forecasting framework using a stacked vector approach. The MFVAR forecasting framework is general in the sense that it allows for multiple variables and multiple frequencies. Second, we integrate the MFVAR with a non-linear Almon MIDAS polynomial scheme which is designed to reduce the parameter space while keeping models flexible. Third, we show how to recast the resulting stacked vector MIDAS type non-linear MFVAR into a linear equation system. The MIDAS parameters can then be easily estimated equation by equation using standard ordinary least squares (OLS). Ultimately, we conduct a pseudo out-of-sample forecast evaluation exercise using quarterly and higher frequency US real-time data. Our MFVAR substantially improves forecast accuracy upon a standard QFVAR for different VAR specifications. Root mean squared forecast errors for, e.g., GDP growth get reduced by 30 to 50 percent for forecast horizons up to six months and by about 20 percent for a forecast horizon of one year.

This paper relates to a small but growing literature on mixed frequency VAR models which fall in two basic classes.[1] The first class uses a state space approach. The general idea is to conceptually assume the mixed frequencies away by reformulating any lower frequency series as a partially latent high frequency series. The Kalman filter or, in a Bayesian context, the Gibbs sampler then provide the possibility to estimate the partially

[1]Following Cox (1981) the first class may be referred to as parameter-driven, while the second approach may be qualified as observation-driven (cf. Ghysels, 2012).

latent VAR process. See Kuzin *et al.* (2011) and Bai *et al.* (2013) for state space type MFVAR models using the Kalman filter, and Chiu *et al.* (2012) and Schorfheide and Song (2013) for state space type MFVARs using the Gibbs sampler. The second class – to which our paper belongs – employs a stacking approach: all variable observations, that pertain to the same lowest frequency period in the sample, are stacked into one vector. For instance, when the sample includes quarterly GDP, monthly inflation and a daily interest rate, the quarterly frequency is the lowest frequency in the sample and, thus, the stacked vector for each quarter t includes one quarterly GDP observation, three monthly inflation observations and 60 daily interest rate observations (assuming a month has 20 trading days).[2] Based on the stacked vectors a VAR process of any desired order can then be modeled. Further, in order to achieve parsimony MIDAS style polynomial structures can then be imposed on the parameter space of the stacked vector type MFVAR, resulting in a stacked vector MIDAS type MFVAR. The MIDAS approach aims to reduce the parameter space while keeping models flexible (cf. the citations in the previous paragraph). Ghysels (2012) provides a rich exposition of a stacked vector MIDAS type MFVAR model. Franses *et al.* (2011) employ a stacked vector MIDAS type MFVAR to study the effects of monetary policy shocks on macroeconomic aggregates using daily interest rate data and monthly macroeconomic indicators. Foroni *et al.* (2014) introduce Markov switching to both a state space type and a stacked vector MIDAS type bivariate MFVAR model and compare these (and several other bivariate) models in terms of predictive accuracy.

Our contribution differs from Ghysels (2012) in the following respects. First, Ghysels mainly concentrates on impulse response analysis, whereas we focus on forecasting. Second, ideally one would like to have a MFVAR framework that allows for multiple variables and multiple frequencies and is still straightforward and compact. While Ghysels' framework in principle allows for more multiple frequencies it becomes conceptually untractable as the number of frequencies grows. For this reason Ghysels limits his exposition to two frequencies (cf. Ghysels, 2012, p. 3). In contrast, our MFVAR framework is designed for multiple variables and multiple frequencies while still being relatively compact and straightforward. Third, Ghysels' and our framework differ in the way they handle the relationships between variable observations that pertain to the same lowest frequency period t and, thus, are stacked in the same vector (for instance, the relationship between the GDP observation of a quarter and the first monthly inflation observation of the same quarter, or the relationship between the second and the first monthly inflation observation of the same quarter). These within-t relationships are crucial for now- and forecasting since they allow to incorporate data releases or revisions which occur *within* period t (*within* a quarter, for instance). In order to capture the aforementioned relationships Ghysels (2012, p. 12ff) proposes a scheme which augments a reduced form MFVAR with a matrix structure which derives from the Choleski lower triangular decomposition. In contrast, we propose to capture the within-t relationships via a recursive form type MFVAR scheme. While the two schemes may ultimately be transformed into each other they fundamentally differ in terms of exposition.

The remainder of the paper is structured as follows: Section 2 presents our stacked vector MIDAS type mixed frequency VAR framework. In particular, we show how to recast the stacked vector MIDAS type non-linear MFVAR into a linear equation system that can

[2] The order of the stacking may differ according to research interest and publication structure (cf. the discussion in Ghysels, 2012, p. 6f).

be easily estimated equation by equation using standard OLS. Subsequently, we analyze whether our stacked vector MIDAS type mixed frequency VAR is helpful for forecasting in real time. Section 3 describes the pseudo out-of-sample forecast evaluation set up and the real-time data used, and Section 3 presents the results of the empirical assessment. Finally, Section 5 provides conclusions and possible directions of further research.

2 A general MIDAS VAR framework

Let there be a set of time series variables of different frequencies. Denote each variable by $y_{i,t-1+\frac{\tau_i}{\mathcal{T}_i}}$ where i is the variable subscript with $i = 1, \ldots, I$ and $t-1+\frac{\tau_i}{\mathcal{T}_i}$ is the time period subscript. $t = 1, \ldots, T$ denotes the lowest frequency periods such that $\mathcal{T}_i$ is the frequency of variable i in each period t and $\tau_i = 1, \ldots, \mathcal{T}_i$ denotes the subperiods of variable i in each period t. For instance, when the set of variables comprises quarterly, monthly and daily variables, $t = 1, \ldots, T$ denotes the quarters, $\mathcal{T}_i = 1/3/60$ is the frequency of any quarterly/monthly/working daily variable in each quarter t and $\tau_i = 1/1, 2, 3/1, \ldots, 60$ denotes the quarter/months/working days in each quarter t.[3] Stack all observations of variable i in period $t-p$ with $p \in \mathbb{N}_0$ starting with the last observation in $t-p$ and ending with the first observation in $t-p$ to get

$$\underset{\mathcal{T}_i \times 1}{\mathbf{y}_{i,t-p}} \equiv \begin{bmatrix} y_{i,t-p-1+\frac{\mathcal{T}_i}{\mathcal{T}_i}} \\ \vdots \\ y_{i,t-p-1+\frac{1}{\mathcal{T}_i}} \end{bmatrix},$$

and let

$$\underset{\sum_{i=1}^{I} \mathcal{T}_i \times 1}{\mathbf{y}_{t-p}} \equiv \begin{bmatrix} y_{1,t-p} \\ \vdots \\ y_{I,t-p} \end{bmatrix}.$$

Further, lag each element in $y_{i,t-p}$ by $\frac{1}{\mathcal{T}_i}$ to get

$$\underset{\mathcal{T}_i \times 1}{\mathbf{y}_{i,t-p-\frac{1}{\mathcal{T}_i}}} \equiv \begin{bmatrix} y_{i,t-p-1+\frac{\mathcal{T}_i}{\mathcal{T}_i}-\frac{1}{\mathcal{T}_i}} \\ \vdots \\ y_{i,t-p-1+\frac{1}{\mathcal{T}_i}-\frac{1}{\mathcal{T}_i}} \end{bmatrix},$$

[3] We model the time period subscript as $t-1+\frac{\tau_i}{\mathcal{T}_i}$ in order to let τ_i denote the τ_i-*th* subperiod in period t. Clements and Galvão (2008, 2009), e.g., model the time period subscript as $t - \frac{\tau_i}{\mathcal{T}_i}$ so that τ_i denotes the $(\mathcal{T}_i - \tau_i)$-*th* subperiod. Both variants are equally possible. For ease of exposition we assume in this section that each variable $y_{i,t-1+\frac{\tau_i}{\mathcal{T}_i}}$ with $i \in \{1, \ldots I\}$ is released simultaneously with all other variables $y_{j,t-1+\frac{\tau_j}{\mathcal{T}_j}}$ with $j \in \{1, \ldots I\}$ and with $t-1+\frac{\tau_i}{\mathcal{T}_i} = t-1+\frac{\tau_j}{\mathcal{T}_j}$ directly after the end of period $t-1+\frac{\tau_i}{\mathcal{T}_i}$ (no ragged edge). For instance, any quarterly variable is released directly after the end of a quarter simultaneously with any other quarterly variable and with any monthly variable observation on the third month of that quarter. Of course, our empirical model application presented in Section 4 explicitly models real world ragged edges and delayed releases.

let

$$\underset{(P+1)\cdot\mathcal{T}_i\times 1}{\mathbf{x}_{i,t}} \equiv \begin{bmatrix} y_{i,t-0-\frac{1}{\mathcal{T}_i}} \\ y_{i,t-1-\frac{1}{\mathcal{T}_i}} \\ \vdots \\ y_{i,t-P-\frac{1}{\mathcal{T}_i}} \end{bmatrix},$$

and let

$$\underset{(P+1)\cdot\sum_{i=1}^{I}\mathcal{T}_i\times 1}{\mathbf{x}_t} \equiv \begin{bmatrix} x_{1,t} \\ \vdots \\ x_{I,t} \end{bmatrix}.$$[4]

Model each element of $\mathbf{y}_{t-1}$, namely each $y_{i,t-1+\frac{\tau_i}{\mathcal{T}_i}}$, as a function of $\mathbf{x}_t$,

$$\begin{aligned} \underset{1\times 1}{y_{i,t-1+\frac{\tau_i}{\mathcal{T}_i}}} &= \mathbf{a}_{i,\tau_i} \quad \mathbf{x}_t \quad + \epsilon_{i,t-1+\frac{\tau_i}{\mathcal{T}_i}} \qquad (1) \\ &= [\mathbf{a}_{i,\tau_i,1} \dots \mathbf{a}_{i,\tau_i,I}] \begin{bmatrix} \mathbf{x}_{1,t} \\ \vdots \\ \mathbf{x}_{I,t} \end{bmatrix} + \epsilon_{i,t-1+\frac{\tau_i}{\mathcal{T}_i}}, \end{aligned}$$

where $\epsilon_{i,t-1+\frac{\tau_i}{\mathcal{T}_i}}$ is an error term, where $\mathbf{a}_{i,\tau_i}$ is the parameter vector partitioned into I subvectors each of which is defined as

$$\underset{1\times(P+1)\cdot\mathcal{T}_j}{\mathbf{a}_{i,\tau_i,j}} \equiv [\mathbf{0}_{i,\tau_i,j,1}\ \boldsymbol{\alpha}_{i,\tau_i,j}\ \mathbf{0}_{i,\tau_i,j,2}] \qquad (2)$$

for $j = 1, \dots, I$. $\mathbf{0}_{i,\tau_i,j,1}$ denotes a row vector of zeros of dimension $\mathcal{T}_j - \lceil\frac{\tau_i}{\mathcal{T}_i}\mathcal{T}_j\rceil$, $\mathbf{0}_{i,\tau_i,j,2}$ denotes a row vector of zeros of dimension $\lceil\frac{\tau_i}{\mathcal{T}_i}\mathcal{T}_j\rceil - \lceil\frac{\mathcal{T}_j}{\mathcal{T}_i}\rceil$ and the row vector

$$\underset{1\times P\cdot\mathcal{T}_j+\lceil\frac{\mathcal{T}_j}{\mathcal{T}_i}\rceil}{\boldsymbol{\alpha}_{i,\tau_i,j}} \equiv [\alpha_{i,\tau_i,j,1}\ \dots\ \alpha_{i,\tau_i,j,K}],$$

where $K = P \cdot \mathcal{T}_j + \lceil\frac{\mathcal{T}_j}{\mathcal{T}_i}\rceil$.[5] We model $\boldsymbol{\alpha}_{i,\tau_i,j}$ in two alternative ways. An intuitive strategy is to regard each element of $\boldsymbol{\alpha}_{i,\tau_i,j}$ as an unrestricted parameter. Equation (1) is then a linear regression equation with $\mathcal{T}_j - \lceil\frac{\mathcal{T}_j}{\mathcal{T}_i}\rceil$ zero restrictions and with K unknown unrestricted parameters, $\alpha_{i,\tau_i,j,1}, \dots, \alpha_{i,\tau_i,j,K}$, which can be estimated via ordinary least squares (OLS). This is the unrestricted MIDAS (U-MIDAS) approach proposed by Foroni *et al.* (2012). Alternatively, following Ghysels and co-authors (Ghysels *et al.*, 2007, Andreou *et al.*, 2010, e.g.) one can regard $\boldsymbol{\alpha}_{i,\tau_i,j}$ as a vector of unknown weights each of which is a function of

[4] Stacking of lagged variables into a vector $\mathbf{x}_t$ follows Hamilton (1994, p. 292) and Hayashi (2000, p. 397).

[5] The ceiling function $\lceil z\rceil \equiv \min\{n \in \mathbb{Z} | n \geq z\}$, where z is a real number and n is an integer from the set of all integers $\mathbb{Z}$. By setting $P_i = P$ for all i and by specifying $\mathbf{a}_{i,\tau_i,j}$ as in Equation (2) we impose – for ease of exposition – that ... Our empirical model application presented in Section 3 ... The low frequency index $p = 1, \dots, P$ does neither figure in $\mathbf{a}_{i,\tau_i,j}$ nor in $\boldsymbol{\alpha}_{i,\tau_i,j}$. $\boldsymbol{\alpha}_{i,\tau_i,j}$ may run over multiple low frequency periods.

the unknown parameter vector $\boldsymbol{\theta}_{i,\tau_i,j}$ and of the lag index k with $k = 1, \ldots, K$. We model the weight function as a non-exponential Almon lag polynomial,

$$\alpha_{i,\tau_i,j,k} = \alpha_{i,\tau_i,j,k}(\boldsymbol{\theta}_{i,\tau_i,j}, k) = \alpha_{i,\tau_i,j,k}(\theta_{i,\tau_i,j,0}, \ldots, \theta_{i,\tau_i,j,Q}, k) = \sum_{q=0}^{Q} \theta_q k^q, \tag{3}$$

where $Q \in \mathbb{N}$ denotes the order of the Almon lag polynomial.[6] Let

$$\underset{\mathcal{T}_j \times \mathcal{T}_j \cdot (P+1)}{A_{i,j}} \equiv \begin{bmatrix} \mathbf{a}_{i,1,j} \\ \vdots \\ \mathbf{a}_{i,\mathcal{T}_j,j} \end{bmatrix},$$

and let

$$\underset{\sum_{i=1}^{I} \mathcal{T}_j \times (P+1) \cdot \sum_{j=1}^{I} \mathcal{T}_j}{A} \equiv \begin{bmatrix} A_{1,1} & \ldots & A_{1,I} \\ \vdots & \ddots & \vdots \\ A_{I,1} & \ldots & A_{I,I} \end{bmatrix}.$$

Further, let

$$\underset{\mathcal{T}_i \times 1}{\boldsymbol{\epsilon}_{i,t}} \equiv \begin{bmatrix} \epsilon_{i,t-1+\frac{\mathcal{T}_i}{\mathcal{T}_i}} \\ \vdots \\ \epsilon_{i,t-1+\frac{1}{\mathcal{T}_i}} \end{bmatrix},$$

and let

$$\underset{\sum_{i=1}^{I} \mathcal{T}_i \times 1}{\boldsymbol{\epsilon}_t} \equiv \begin{bmatrix} \boldsymbol{\epsilon}_{1,t} \\ \vdots \\ \boldsymbol{\epsilon}_{I,t} \end{bmatrix}.$$

The mixed frequency vector autoregressive (VAR) process then generally writes

$$\begin{bmatrix} \mathbf{y}_{1,t} \\ \vdots \\ \mathbf{y}_{I,t} \end{bmatrix} = \begin{bmatrix} A_{1,1} & \ldots & A_{1,I} \\ \vdots & \ddots & \vdots \\ A_{I,1} & \ldots & A_{I,I} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{1,t} \\ \vdots \\ \mathbf{x}_{I,t} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\epsilon}_{1,t} \\ \vdots \\ \boldsymbol{\epsilon}_{I,t} \end{bmatrix},$$

or more compactly

$$\mathbf{y}_t = A\mathbf{x}_t + \boldsymbol{\epsilon}_t. \tag{4}$$

The parameters in matrix A can be estimated row by row. In case of U-MIDAS this can be easily done via OLS. The case of non-exponential Almon lag polynomial is less straightforward. First, since each non-exponential Almon lag polynomial weight function $\alpha_{i,\tau_i,j,k}$ is highly non-linear in its parameters $\boldsymbol{\theta}_{i,\tau_i,j}$, OLS is not feasible for a direct estimation of matrix A. Second, since each row of A contains a multitude of non-exponential Almon lag

[6]The literature knows yet other weight functions (see Ghysels *et al.*, 2007, e.g.). The reason for concentrating on the non-exponential Almon lag polynomial is discussed below. For ease of exposition we assume $Q_{i,\tau_i,j} = Q$ for all $\boldsymbol{\theta}_{i,\tau_i,j}$. Our empirical model application presented in Section 3 allows the Almon lag polynomial order $Q_{i,\tau_i,j}$ to be specific to $\boldsymbol{\theta}_{i,\tau_i,j}$.

polynomial weight functions, $\alpha_{i,\tau_i,1,1}, \ldots \alpha_{i,\tau_i,1,K}, \alpha_{i,\tau_i,2,1}, \ldots \alpha_{i,\tau_i,2,K}, \ldots \ldots \alpha_{i,\tau_i,I,1}, \ldots \alpha_{i,\tau_i,I,K}$, even non-linear optimization methods reach their limits in practice.[7] Fortunately, we can recast Equation (4) in a linear form. For this, use Equation (3) to re-write vector $\boldsymbol{a}_{i,\tau_i,j}$ from Equation (2) as

$$\boldsymbol{a}_{i,\tau_i,j} = \boldsymbol{\theta}_{i,\tau_i,j} M_{i,\tau_i,j},$$

with the Almon parameter vector

$$\underset{1\times(Q+1)}{\boldsymbol{\theta}_{i,\tau_i,j}} \equiv \left[\begin{array}{ccc} \theta_{i,\tau_i,j,0} & \ldots & \theta_{i,\tau_i,j,Q} \end{array}\right]$$

and the transformation matrix

$$\underset{(Q+1)\times(P+1)\cdot\mathcal{T}_j}{M_{i,\tau_i,j}} \equiv \left[\begin{array}{cccccccc} \mathbf{0}_{i,\tau_i,j,1} & 1 & 1 & 1 & . & . & . & 1 & \mathbf{0}_{i,\tau_i,j,2} \\ \mathbf{0}_{i,\tau_i,j,1} & 1 & 2 & 3 & . & . & . & K & \mathbf{0}_{i,\tau_i,j,2} \\ \mathbf{0}_{i,\tau_i,j,1} & 1 & 4 & 9 & . & . & . & K^2 & \mathbf{0}_{i,\tau_i,j,2} \\ \mathbf{0}_{i,\tau_i,j,1} & . & . & . & . & . & . & . & \mathbf{0}_{i,\tau_i,j,2} \\ \mathbf{0}_{i,\tau_i,j,1} & . & . & . & . & . & . & . & \mathbf{0}_{i,\tau_i,j,2} \\ \mathbf{0}_{i,\tau_i,j,1} & . & . & . & . & . & . & . & \mathbf{0}_{i,\tau_i,j,2} \\ \mathbf{0}_{i,\tau_i,j,1} & 1 & 2^Q & 3^Q & . & . & . & K^Q & \mathbf{0}_{i,\tau_i,j,2} \end{array}\right],$$

Further, re-write vector $\mathbf{a}_{i,\tau_i}$ from Equation (1) as

$$\mathbf{a}_{i,\tau_i} = \boldsymbol{\theta}_{i,\tau_i} M_{i,\tau_i}$$

with

$$\underset{1\times I\cdot(Q+1)}{\boldsymbol{\theta}_{i,\tau_i}} \equiv \left[\begin{array}{ccc} \boldsymbol{\theta}_{i,\tau_i,1} & \ldots & \boldsymbol{\theta}_{i,\tau_i,I} \end{array}\right]$$

and

$$\underset{I\cdot(Q+1)\times(P+1)\cdot\sum_{j=1}^{I}\mathcal{T}_j}{M_{i,\tau_i}} \equiv \left[\begin{array}{cccc} M_{i,\tau_i,1} & \mathbf{0}_{Q+1\times(P+1)\cdot\mathcal{T}_2} & \ldots & \mathbf{0}_{Q+1\times(P+1)\cdot\mathcal{T}_j} \\ \mathbf{0}_{Q+1\times(P+1)\cdot\mathcal{T}_1} & M_{i,\tau_i,2} & \ldots & . \\ . & \mathbf{0}_{Q+1\times(P+1)\cdot\mathcal{T}_2} & \ldots & . \\ . & . & \ldots & . \\ . & . & \ldots & \mathbf{0}_{Q+1\times(P+1)\cdot\mathcal{T}_j} \\ \mathbf{0}_{Q+1\times(P+1)\cdot\mathcal{T}_1} & \mathbf{0}_{Q+1\times(P+1)\cdot\mathcal{T}_2} & \ldots & M_{i,\tau_i,I} \end{array}\right],$$

where $\mathbf{0}_{Q+1\times(P+1)\cdot\mathcal{T}_j}$ denotes a $(Q+1\times(P+1)\cdot\mathcal{T}_j)$-dimensional matrix of zeros. Let

$$\underset{I\cdot(Q+1)\times 1}{\boldsymbol{x}^*_{i,\tau_i,t}} \equiv M_{i,\tau_i}\mathbf{x}_t, \tag{5}$$

where $\boldsymbol{x}^*_{i,\tau_i,t}$ can be interpreted as a row vector of transformed data in the sense that the row vector of data, $\mathbf{x}_{i,t}$, is transformed by premultiplying it with the number matrix M^*_{i,τ_i}. Use Equation (5) to rewrite the basic regression equation (1) as

$$y_{i,t-1+\frac{\tau_i}{\mathcal{T}_i}} = \boldsymbol{\theta}_{i,\tau_i}\boldsymbol{x}^*_{i,\tau_i,t} + \epsilon_{i,t-1+\frac{\tau_i}{\mathcal{T}_i}} \tag{6}$$

[7] Specifically, each row of A contains $I \cdot K = I \cdot (P \cdot \mathcal{T}_j + \lceil \frac{\mathcal{T}_j}{\mathcal{T}_i} \rceil)$ non-exponential Almon lag polynomial weight functions.

Let

$$\underset{\mathcal{T}_i \times 1}{\boldsymbol{\theta}_i \boldsymbol{x}^*_{i,t}} \equiv \begin{bmatrix} \boldsymbol{\theta}_{i,\mathcal{T}_i} \boldsymbol{x}^*_{i,\mathcal{T}_i,t} \\ \vdots \\ \boldsymbol{\theta}_{i,1} \boldsymbol{x}^*_{i,1,t} \end{bmatrix},$$

and let

$$\underset{\sum_{i=1}^{I} \mathcal{T}_i \times 1}{\boldsymbol{\theta} \boldsymbol{x}^*_t} \equiv \begin{bmatrix} \boldsymbol{\theta}_1 \boldsymbol{x}^*_{1,t} \\ \vdots \\ \boldsymbol{\theta}_I \boldsymbol{x}^*_{I,t} \end{bmatrix}.$$

The mixed frequency VAR then writes

$$\mathbf{y}_t = \boldsymbol{\theta} \boldsymbol{x}^*_t + \boldsymbol{\epsilon}_t. \tag{7}$$

The θ-parameters in Equation (7) can be easily estimated row by row via OLS. Thereupon, the α-weights in matrix A of the general VAR equation (4) can be calculated using Equation (3). Notably, Equation (7) does not yet conform to a standard VAR matrix notation because each element, $y_{i,t-1+\frac{\tau_i}{\mathcal{T}_i}}$, in the left-hand side variable vector, $\mathbf{y}_t$, is a function of an element-specific right-hand side data vector, $\boldsymbol{x}^*_{i,\tau_i,t}$ rather than being a function of always the same right-hand side data vector or matrix. Let

$$\underset{\mathcal{T}_i \times I \cdot (Q+1)}{\Theta_i} \equiv \begin{bmatrix} \boldsymbol{\theta}_{i,\mathcal{T}_i} \\ \vdots \\ \boldsymbol{\theta}_{i,1} \end{bmatrix},$$

and let

$$\underset{\sum_{i=1}^{I} \mathcal{T}_i \times I \cdot (Q+1)}{\Theta} \equiv \begin{bmatrix} \Theta_1 \\ \vdots \\ \Theta_I \end{bmatrix}.$$

Further let

$$\underset{\mathcal{T}_i \times I \cdot (Q+1)}{X^*_{i,t}} \equiv \begin{bmatrix} \boldsymbol{x}^{*\prime}_{i,\mathcal{T}_i,t} \\ \vdots \\ \boldsymbol{x}^{*\prime}_{i,1,t} \end{bmatrix},$$

and let

$$\underset{\sum_{i=1}^{I} \mathcal{T}_i \times I \cdot (Q+1)}{X^*_t} \equiv \begin{bmatrix} X^*_{1,t} \\ \vdots \\ X^*_{I,t} \end{bmatrix}.$$

In order to bring the Almon type mixed frequency vector autoregressive process into a standard VAR matrix notation, rewrite Equation (7) as

$$\mathbf{y}_t = S \operatorname{vec}(\Theta X^{*\prime}_t) + \boldsymbol{\epsilon}_t. \tag{8}$$

The selection matrix

$$\underset{L\times L\cdot L}{S} \quad \equiv \quad (\boldsymbol{I}_L * \boldsymbol{I}_L)'$$

where $\boldsymbol{I}_L$ denotes the identity matrix of size $L \equiv \sum_{i=1}^{I} \mathcal{T}_i$ and where the (column-wise) Khatri-Rao product

$$\boldsymbol{I} * \boldsymbol{I} \quad \equiv \quad [\boldsymbol{I}_l \otimes \boldsymbol{I}_l]_l \,,$$

where l denotes the l-*th* column of $\boldsymbol{I}$ with $l = 1, \ldots, L$.[8] Use the fact that for any two matrices B with size $R \times T$ and C with size $U \times V$

$$\text{vec}(BC) = (C' \otimes I_R)\, \text{vec}(B)$$

to rewrite Equation (8) as

$$\mathbf{y}_t \quad = \quad S(X_t^* \otimes I_L)\, \text{vec}(\Theta) + \boldsymbol{\epsilon}_t, \tag{9}$$

where $S(X_t^* \otimes I_L)$ with size $L \times I \cdot (Q+1) \cdot L$ builds the right-hand side data matrix and $\text{vec}(\Theta)$ with size $I \cdot (Q+1) \cdot L \times 1$ is the parameter vector. Equation (9) conforms to the standard VAR matrix notation in the sense that each element, $y_{i,t-1+\frac{\tau_i}{\mathcal{T}_i}}$, in the left-hand side variable vector, $\mathbf{y}_t$, is a function of always the same right-hand side data matrix, namely of $S(X_t^* \otimes I_L)$. On the other hand, Equation (9) deviates from the standard VAR matrix notation in that the data come in matrix form (but not in vector form) while the parameters come in vector form (but not in matrix form). Importantly, the θ-parameters in Equation (9) can be easily estimated row by row via OLS. As each row of $S(X_t^* \otimes I_L)$ contains only $I \cdot (Q+1)$ non-zero elements, a single row can only be used to estimate $I \cdot (Q+1)$ θ-parameters in $\text{vec}(\Theta)$. Hence, all rows in Equation (9) are needed to fully estimate the $I \cdot (Q+1) \cdot L$ parameters in $\text{vec}(\Theta)$. Upon estimation of $\text{vec}(\Theta)$, the α-weights in matrix A of the general VAR equation (4) can be calculated using Equation (3).

3 Data and forecast evaluation set up

Can our stacked vector MIDAS type mixed frequency VAR help improve forecasts upon a standard single frequency VAR? In order to answer this question we conduct a pseudo-out-of sample forecast evaluation with real time data.

The real-time data used in our analysis comprise quarterly GDP and the monthly consumer price index, unemployment rate, industrial production and housing starts. Data sources are the ArchivaL Federal Reserve Economic Data (ALFRED) published by the Federal Reserve Bank of St. Louis and the Federal Reserve Bank of Philadelphia Real-Time Data Set for Macroeconomists. In addition, we employ a set of time series that are not subject to data revisions: the ISM indices for manufacturing, supplier delivery times and orders, S&P 500 stock market index, 10 year treasury bond yield, 3 month treasury bill yield, new unemployment claims and average weekly hours worked by production and

[8]See Khatri and Rao (1968).

supervisory workers. Following common practice in the literature higher than monthly frequency series are time-aggregated to monthly frequency (cf. Carriero *et al.*, 2012 and Schorfheide and Song, 2013, e.g.).

Our real-time dataset covers vintages from January 1985 to August 2012, which allows us to use 344 samples for the forecast evaluation. The data of each vintage starts in January 1970 and is extended each vintage, i.e. the sample for the first vintage of January 1985 covers data from January 1970 to January 1985. In order to reproduce the available information a forecaster would have had at any vintage we also take the publication lag into consideration. Almost all of the used variables are not available directly but are published with a time lag. Only the S&P 500, 10 year treasury bond yield and the 3 month treasury bill yield, which are available daily, are directly available at the end of a month (lead variables). The other variables (lag variables) are available latest at the end of the next month. All monthly variables are available before the first publication of the GDP.

Our benchmark model is a standard quarterly frequency VAR (QF-VAR) where all higher than quarterly frequency series are time-aggregated to quarterly frequency. Our forecast performance measure is the difference between the root mean squared forecast error (RMSFE) of the MIDAS VAR and the QF-VAR in percent of the RMSFE of the QF-VAR,

$$\Delta RMSFE = 100 * \left(\frac{RMSFE^{MIDAS\ VAR} - RMSFE^{QF\text{-}VAR}}{RMSFE^{QF\text{-}VAR}} \right)$$

We refer to $\Delta RMSFE$ as the relative change in RMSFE. The *more negative* $\Delta RMSFE$ is, the *better* performs the MIDAS VAR relative to the QF-VAR in terms of predicitive power. The MIDAS VAR and the QF-VAR always have the same amount of lagged information available. Thus, differences in forecast performance between competing models can solely accrue from two sources: how flexible – or parsimonious – the models are in terms of parametrization, and whether they can incorporate higher frequency data updates (new releases or revisions).

The evaluation of real-time forecasts depends also on the choice of the actual data with which the forecast is compared. There have been several benchmark revisions since 1985; the latest one occurred in mid-2014 and included a substantial redefinition of gross fixed capital formation A forecaster in 1985 could not have predicted such a definition change. Thus we follow the approach of Carriero *et al.* (2012) and use the second estimate of quarterly GDP for the forecast evaluation.As a robustness test we also check the forecast performance with the first and third estimate of GDP.

4 Empirical results

The following sections present the results of the forecast evaluation outlined in Section 3. We compare MIDAS VARs and QF-VARs for four alternative specification setups: few variables and few lags, few variables and many lags, higher number of variables and few lags, and higher number of variables and many lags. Competing MIDAS VARs and QF-VARs always have the same amount of lagged information available. Thus, differences in forecast performance between competing models can solely accrue from two sources: how flexible – or parsimonious – the models are in terms of parametrization, and whether they can incorporate higher frequency data updates (new releases or revisions).

3-variable MIDAS VAR with short memory

In a first step it might be interesting to see whether MIDAS VAR models improve forecasts upon a standard VAR when model specifications are kept minimal. Following common practice in macroeconomics our minimum VAR specification includes three variables only: GDP growth, consumer price inflation and a (3-months) short-term interest rate (treasury bill rate). Figure 1 presents results for an Almon MIDAS VAR of order 1 and a U-MIDAS VAR both with one quarterly GDP growth lag, three monthly inflation lags and three monthly interest rate lags. The benchmark model is a quarterly frequency VAR (QF-VAR) with one quarterly lag of GDP growth, inflation and interest rate, respectively. As regards forecasting GDP growth, the two MIDAS VAR specifications reduce the RMSFE by more than 30 percent compared to the QF-VAR for forecast horizons of one to six months. The RMSFE improvement gradually declines as the forecast horizon increases, but is still substantial for longer horizons: One year before publication of GDP the MIDAS VAR forecasts are still more than 20 percent better than the QF-VAR forecasts. For forecast horizons of 22 months or longer the MIDAS VARs yield no improvement over the QF-VAR anymore. For inflation, the Almon MIDAS VAR and the U-MIDAS VAR yield very high RMSFE reductions for short forecast horizons (more than 65 percent for months 1 to 3). On the other hand, RMSFE improvements quickly vanish as the forecast horizon grows. When it comes to forecasting the interest rate, the Almon MIDAS VAR and the U-MIDAS VAR improve forecasts substantially throughout all considered forecast horizons as compared to the QF-VAR. The RMSFE reductions reach more the 75 percent for horizons of one to three months. Forecast improvements gradually decline with an increase in the forecast horizon, but even two years before GDP publication the MIDAS VARs reduce the RMSFE by 5 percent or more compared to the QF-VAR. All previous findings remain robust when we iterate the forecast evaluation for the first or third GDP estimate instead of the second estimate (see Section 3).

– Figure 1 about here –

3-variable MIDAS VAR with long memory

VAR models of higher order than the ones presented in the previous section arguably achieve better forecasting performance for longer forecast horizons. Thus, it is important do know whether MIDAS VARs still improve forecasts upon a standard VAR for specifications with longer time series memory. Figure 2 shows the results for an Almon MIDAS VAR of order 2 and a U-MIDAS VAR both with two quarterly GDP growth lags, six monthly inflation lags and six monthly interest rate lags. The benchmark model here is a QF-VAR with two quarterly lags of GDP growth, inflation and interest rate, respectively, where two lags are chosen in order to provide each competing model with the same amount of lagged information (see Section 3). To allow for even longer memory, Figure 3 depicts results for an Almon MIDAS VAR of order 2 and a U-MIDAS VAR both with four quarterly GDP growth lags, twelve monthly inflation lags and twelve monthly interest rate lags. In accordance with the above reasoning about the appropriate information set, the benchmark model now is a QF-VAR with four quarterly lags of GDP growth, inflation and interest rate, respectively. Generally, the RMSFE patterns in Figures 2 and 3 strongly resemble the patterns in Figure 1: the two MIDAS VAR specifications yield substantial improvements in forecast accuracy upon the QF-VAR for GDP growth, inflation as well as the interest rate. That said, for the long memory specification the U-MIDAS VAR performs clearly worse than the Almon VAR when it comes to forecasting GDP growth. When the forecast horizon exceeds 13 months the U-MIDAS VAR forecasts are even less accurate than the QF-VAR forecasts. This finding is not surprising: U-MIDAS specifications easily become overparametrized when the lag order grows. As a consequence, U-MIDAS is less suitable for – and is actually not meant for – forecasting with longer lags.

– Figures 2 and 3 about here –

6-variable MIDAS VAR with short memory

The previous minimum scale VAR specifications including only GDP growth, inflation and an short-term interest rate provide us with first evidence on the potential of MIDAS VAR models for forecasting. However, in order to improve predictive accuracy forecasters usually employ VAR models with more than just the aforementioned variables. So, do MIDAS VARs still improve forecasts upon standard VARs when more variables are included? To answer this question we include three additional variables which are commonly considered to be helpful for now- or forecasting GDP growth: industrial production, housing starts and the Standard & Poor's 500 stock market index (S&P 500).[9] Figure 4 depicts results for an Almon MIDAS VAR of order 1 and a U-MIDAS VAR both with one quarterly GDP growth lag and three monthly lags of inflation, the interest rate, industrial production, housing starts and the S&P 500. The benchmark model here is a QF-VAR with one quarterly lag of each of the aforementioned six variables. Thus, as before each competing model has the same amount of lagged information such that differences in forecast performance cannot accrue from differences in the (lagged) information set. The RMSFE patterns in Figure 4 strongly resemble the patterns presented in the previous subsections: the two MIDAS VAR specifications substantially improve predictive accuracy upon the QF-VAR for GDP growth, inflation and the interest rate.

– Figure 4 about here –

6-variable MIDAS VAR with long memory

The previous section has shown that small scale MIDAS VARs substantially improve predictive accuracy upon a standard small scale VAR when only short time series memory is taken into account (one quarterly lag or three monthly lags). Is this gain is robust to allowing for longer time series memory (which increases the risk of overparametrization)?

[9]Indeed, we find that a small scale MIDAS VAR (with industrial production, housing starts, S&P 500, GDP growth, inflation and the interest rate) improves forecast accuracy upon a minimum MIDAS VAR (with only the latter three variables). Equally, the corresponding small scale benchmark QF-VAR performs better than the corresponding minimum scale benchmark QF-VAR. A rigorous variable selection procedure might deliver small scale specifications with still greater forecasting ability. That said, it is not the goal of this paper to find the best model specification. Rather, we compare MIDAS VARs and standard VARs for several sensible, yet alternative model specifications in order to see whether MIDAS VARs robustly outperform standard VARs.

Figure 5 (or Figure 6) shows the results for an Almon MIDAS VAR of order 2 and a U-MIDAS VAR both with two (or four) quarterly GDP growth lags and six (or twelve) monthly lags of inflation, the interest rate, industrial production, housing starts and the S&P 500. The benchmark model is a QF-VAR with two (or four) quarterly lags for each of the aforementioned six variables such that each competing model has again the same amount of lagged information (see Section 3). The Almon MIDAS VAR still largely outperforms the benchmark QF-VAR in terms of forecast accuracy. A comparison between the RMSFE improvements resulting from the longer memory Almon MIDAS VARs in Figures 5 and 6 with the RMSFE improvements from the short memory Almon MIDAS VAR in Figure 4 yields only few differences: Regarding GDP growth, the forecast improvement is slightly smaller for longer horizons. For inflation and the interest rate, the longer horizon forecast improvement is substantially larger.

– Figures 5 and 6 about here –

5 Conclusion

A challenge to joining multiple macroeconomic variables in a VAR is that macroeconomic variables are usually sampled at different frequencies. For instance, GDP comes at a quarterly frequency whereas inflation is published monthly and short term interest rates are quoted at a daily or even higher frequency. The traditional solution is to simply time-aggregate all higher frequency series to the frequency of the lowest frequency series in the sample. A VAR including GDP, inflation and a interest rate will then be a quarterly frequency VAR (QFVAR). However, such time aggregation comes at costs. First, any new data releases or data revisions, which occur *within* the lowest frequency, are only taken into account after the end of each lowest frequency period. To stick to the above example, a QFVAR does not allow to consider inter-quarterly inflation and interest rate releases and inter-quarterly GDP revisions before the end of a quarter. This delayed processing of information potentially impairs (inter-quarterly) forecasts and nowcasts. Second, the time aggregation implies a peculiar constraint on the parameters attached to higher frequency variables that is potentially quite suboptimal.

To overcome the aforementioned drawbacks, we integrate a mixed frequency VAR (MFVAR) framework with a MIxed DAta Sampling (MIDAS) approach which has been originally been developed for forecasting with single equations (cf. Ghysels *et al.*, 2007, Andreou *et al.*, 2010, e.g.). In a first step, we propose a general – and yet compact – MFVAR forecasting framework using a stacked vector approach. The MFVAR forecasting framework is general in the sense that it allows for multiple variables and multiple frequencies. Second, we integrate the MFVAR with a non-linear Almon MIDAS polynomial scheme which is designed to reduce the parameter space while keeping models flexible. Third, we show how to recast the resulting stacked vector MIDAS type non-linear MFVAR into a linear equation system. The MIDAS parameters can then be easily estimated equation by equation using standard OLS.

Can our stacked vector MIDAS type MFVAR help improve forecasts upon a standard single frequency VAR? In order to answer this question we conduct a pseudo-out-of sample forecast evaluation using quarterly and higher frequency US real-time data. Our MFVAR substantially improves forecast accuracy upon a standard QFVAR for different VAR specifications. Root mean squared forecast errors for, e.g., GDP growth get reduced by 30 to 50 percent for forecast horizons up to six months and by about 20 percent for a forecast horizon of one year.

The VAR specifications in our empirical application are intentionally kept simple in the sense that each variable's own lags include the same memory than its other variables lags. For instance, when the quarterly GDP growth equation has two own quarterly lags, it has six monthly inflation lags in case of the MFVAR (or two quarterly inflation lags in case of the QFVAR). The linear transformation of the stacked vector MIDAS type MFVAR could be estimated using Bayesian methods instead of OLS. This would allow to employ Bayesian MFVAR specifications with prior types, like the Minnesota prior, that are known to deliver a better forecast performance than the non-Bayesian MFVAR specifications in our empirical application. However, it is an open (and ultimately empirical) question whether such priors will improve the forecast performance of the Bayesian MFVAR *relative to the Bayesian QFVAR*. We leave this question for future research.

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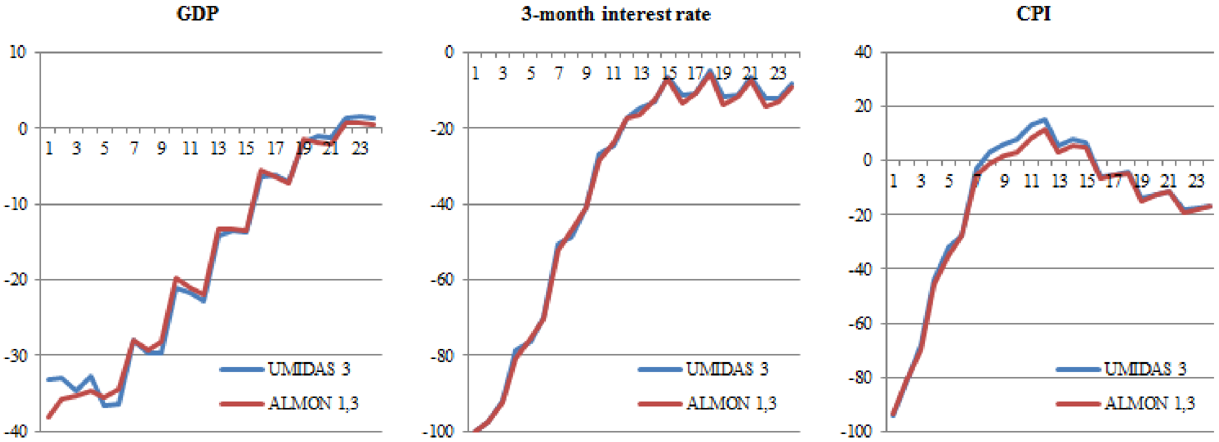
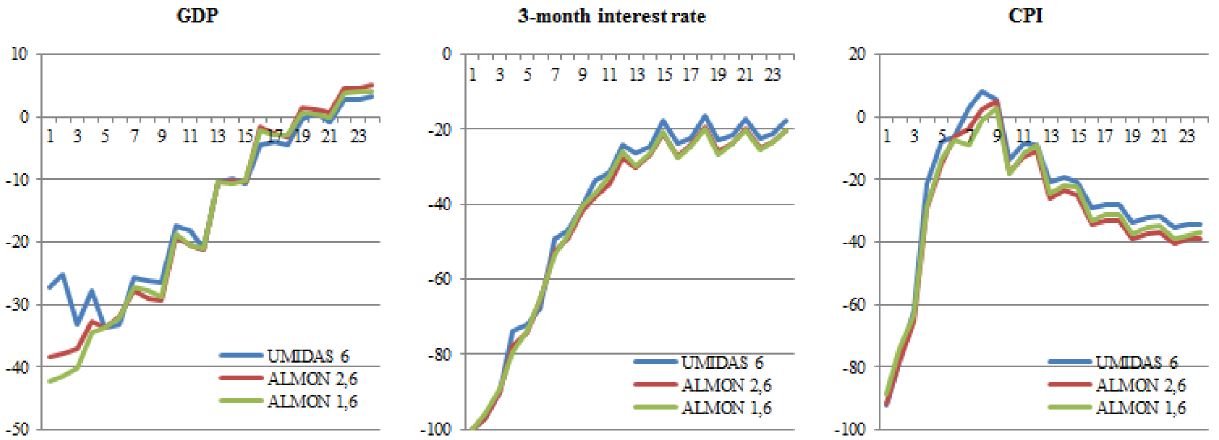
Figure 1: MIDAS VAR specifications with 3 variables and short memory



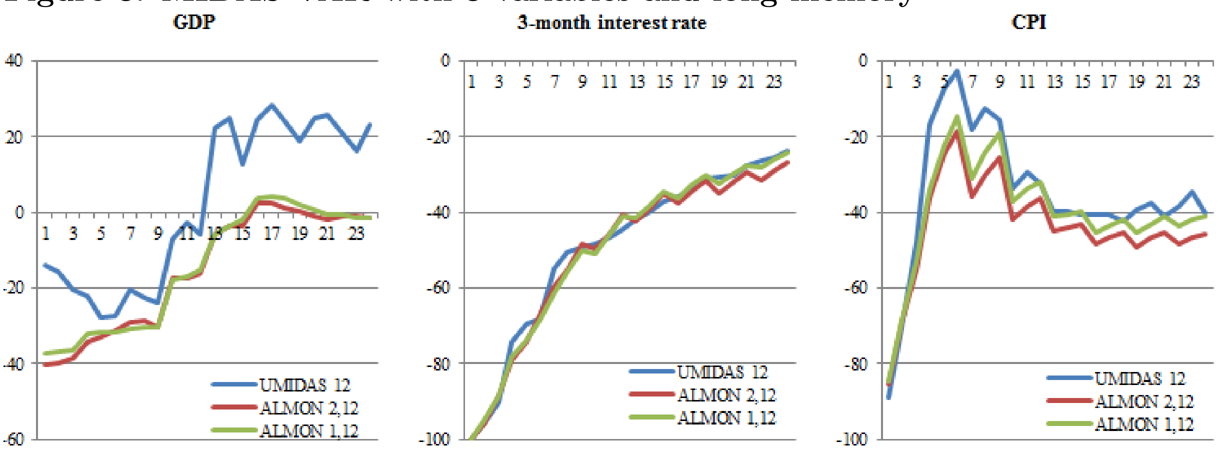
x-axis: Forecast horizon, i.e. months prior to GDP growth/interest rate/CPI inflation release. y-axis: Relative change in RMSFE, $\Delta RMSFE$, i.e. difference between RMSFE of MIDAS VAR and QF-VAR in percent of RMSFE of QF-VAR.

Figure 2: MIDAS VAR with 3 variables and medium-long memory



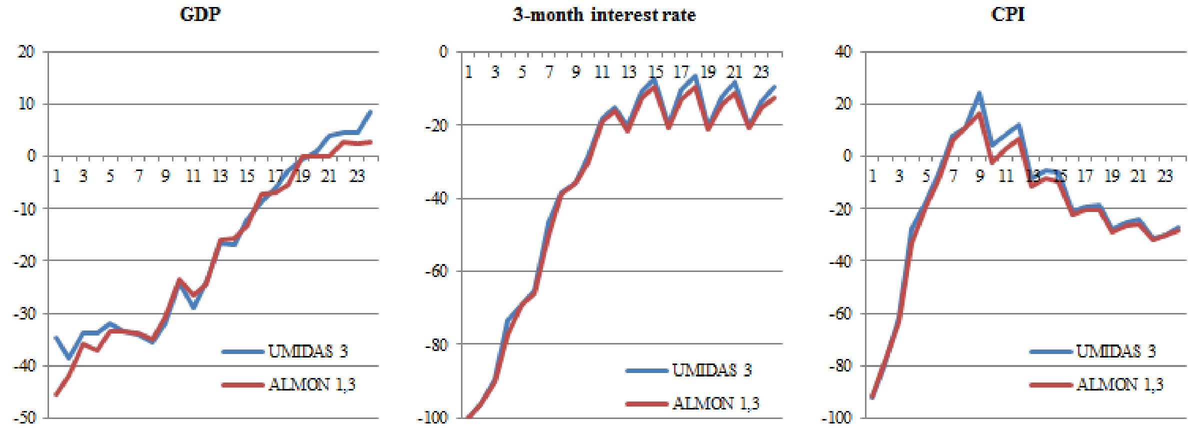
x-axis: Forecast horizon, i.e. months prior to GDP growth/interest rate/CPI inflation release. y-axis: Relative change in RMSFE, $\Delta RMSFE$, i.e. difference between RMSFE of MIDAS VAR and QF-VAR in percent of RMSFE of QF-VAR.

Figure 3: MIDAS VAR with 3 variables and long memory



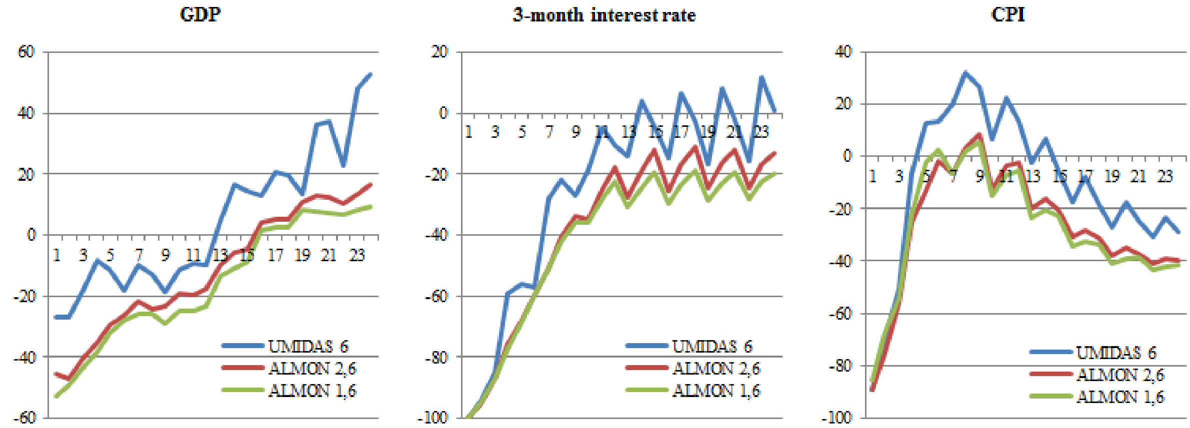
x-axis: Forecast horizon, i.e. months prior to GDP growth/interest rate/CPI inflation release. y-axis: Relative change in RMSFE, $\Delta RMSFE$, i.e. difference between RMSFE of MIDAS VAR and QF-VAR in percent of RMSFE of QF-VAR.

Figure 4: MIDAS VAR with 6 variables and short memory



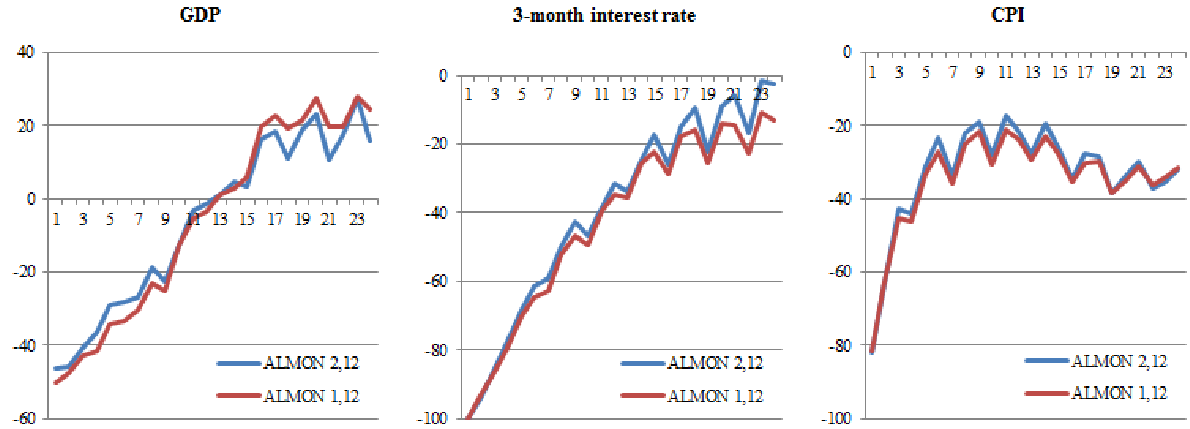
x-axis: Forecast horizon, i.e. months prior to GDP growth/interest rate/CPI inflation release. y-axis: Relative change in RMSFE, $\Delta RMSFE$, i.e. difference between RMSFE of MIDAS VAR and QF-VAR in percent of RMSFE of QF-VAR.

Figure 5: MIDAS VAR with 6 variables and medium-long memory



x-axis: Forecast horizon, i.e. months prior to GDP growth/interest rate/CPI inflation release. y-axis: Relative change in RMSFE, $\Delta RMSFE$, i.e. difference between RMSFE of MIDAS VAR and QF-VAR in percent of RMSFE of QF-VAR.

Figure 6: MIDAS VAR with 6 variables and long memory



x-axis: Forecast horizon, i.e. months prior to GDP growth/interest rate/CPI inflation release. y-axis: Relative change in RMSFE, $\Delta RMSFE$, i.e. difference between RMSFE of MIDAS VAR and QF-VAR in percent of RMSFE of QF-VAR.