The role of money in DSGE models: a forecasting perspective

This paper studies the importance of money in a New Keynesian model by considering the forecasting performance of DSGE models both without and with money in various specifications. While the estimation results are in line with previous studies, favoring the inclusion of money mostly in the form of portfolio adjustment and policy effects, a few interesting results emerge with respect to the accuracy of forecasts. For both point and density forecasts, the best results for output are obtained when the portfolio adjustment channel is eliminated, while the other two channels are kept. For inflation and the interest rate, especially during the Great Recession, models featuring money in various specifications lead to better forecasts. The forecasts with DSGE models featuring money nevertheless appear generally as inefficient and with badly calibrated density forecasts. The results in this paper parallel recent results that have shown that including financial frictions in DSGE models leads to better forecasts, at least for the Great Recession sample.

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1. Introduction

How important is money in a New Keynesian model? Leading monetary scholar Woodford's seminal papers and works, see for example Woodford (2003) or Woodford (2008), suggest that money does not matter for the conduct of monetary policy.

Recent research, however, has contradicted this thesis. Sims and Zha (2006) and Favara and Giordani (2009) used SVAR models to show that LM shocks matter for output and prices.

Structural models used to study the impact of money can be traced back to the seminal work by Nelson (2002) and Ireland (2004), with the latter relaxing the usual assumed nonseparability between consumption and real money balances. Ireland (2004) suggests that there exists evidence favoring the assumption of separability, although the role of money is not significant. Further work has been done by Andres, Lopez-Salido and Valles (2006) using a model with habit formation and price indexation, which confirms the results by Ireland on a dataset from the Euro zone. Furthermore, Andres, Lopez-Salido and Nelson (2009) extend the earlier work by Nelson (2002), Ireland (2004) and Andres, Lopez-Sallido and Valles (2006) by using a model that embeds the previous specifications as particular cases. Their most important finding is that their model favors the portfolio adjustment costs channel, at least for the US and the Euro zone.

With the help of a DSGE model, Arestis et al. (2010) found that money can significantly influence the estimates of potential output. Within a SVAR framework, according to them, money also influences monetary shocks.

Work looking for international evidence on the role of money in New Keynesian models has been carried out by Canova and Menz (2011), who estimated a simple New Keynesian model for four developed economies, the US, the Euro zone, the UK and Japan. They found strong evidence in favor of a positive role for money in explaining the business cycles of these economies (in the form of nonseparability).

In contrast to previous work, Castelnuovo (2012) contributed to the literature by considering the dynamic role of money in time through recursive estimations along a sliding window. Through Bayesian estimation, he was also able to discriminate between the models with the help of Bayes factors. According to him, the role of money was important during the 1970s but has diminished since.

The present paper contributes to the previous body of work in several ways. First of all, it is based on the state-of-art model by Andres, Lopez-Salido and Nelson (2009). This model is estimated with Bayesian techniques as in Castelnuovo (2012). In contrast to previous studies, however, the role of money is evaluated not only based on Bayes factors and estimated values of the key parameters related to money, but also on the forecasting performance of models with and without money. Furthermore, as Castelnuovo (2012) argued, it is important to have a grasp on the time-varying role of money, which, in our case, is done by recursively estimating and forecasting the models. Second, in evaluating the role of money in the forecasting accuracy for key variables like output, inflation and interest rate, we also contribute to the forecasting literature by studying how certain micro-foundations (money, in this case) influence forecasting accuracy, using both point and density forecasts. Unfortunately, few papers in the literature have studied the effects of certain micro-foundations on forecasting accuracy. A few examples are

Kolasa and Rubaszek (2014), who studied the effects of financial frictions on forecasting accuracy during both moderate times and turbulent times, or Del Negro and Schorfheide (2013) who studied whether extending the Smets-Wouters model with financial frictions can improve forecasting accuracy for the American economy during the Great Recession.

Thus, this paper asks three fundamental questions: does extending a stylized New Keynesian model with money in any way improve forecasting accuracy for key macroeconomic variables? What specifications (or combinations of forecasts) provide the best accuracy? How do the Great Moderation and the Great Recession influence forecasting accuracy?

The main findings of this paper are as follows. The estimation results confirm the findings in previous papers, favoring both portfolio adjustment and policy effects. Although the estimate of the coefficient corresponding to nonseparability is positive, the confidence interval nevertheless includes the value zero. Besides the model with no money effects along the model with money featuring all three effects, three further models are considered in the forecasting exercises, each one excluding one effect at a time. The forecasts are compared both for point forecasts and density forecasts. In the case of output with point forecasts, the best results are obtained when the portfolio adjustment effect is eliminated. Moreover, for both inflation and the interest rate, including money in the different specifications leads to more accurate forecasts during the Great Recession. The results for density forecasts are very similar. For the particular case of output, along the model without portfolio adjustment effects, the pool combining the model without money and the model without portfolio adjustment effects perform the best. Nevertheless, while the forecasts obtained by models featuring money do offer some advantages, they appear to be generally inefficient and have a badly calibrated forecast density.

2. The Model

The paper builds on the state-of-art model proposed by Andres, Lopez-Salido and Nelson (2009). The main advantage of this model is that it encompasses all of the most significant previous alternatives, as seen in Nelson (2002), Ireland (2004) or Andres, Lopez-Salido and Valles (2006). More specifically, the model encompasses the nonseparability effect, the direct effect and the policy effect.

The problem of the representative household consists in finding the optimal choice of consumption C_t, labor supply (or the hours worked) N_t, money supply M_t and bond holdings B_t in order to maximize the lifetime utility given by:

$$\max_{C_t, N_t, M_t, B_t} E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[\psi\left(\frac{C_t}{C_{t-1}^h}, \frac{M_t}{e_t P_t} \right) - \frac{N_t^{1+\varphi}}{1+\varphi} \right] - G(\bullet) \tag{1}$$

The variable a_t stands for the preference shocks, e_t for the money demand shocks. The parameter β stands for the discount factor, φ is the inverse of Frisch labor elasticity and h is the degree of habit formation. There is nonseparability across consumption and real balances in the preferences which results in allowing for money to enter the IS equation through the so-called nonseparability channel.

The function $G(.)$ is given by:

$$G(\bullet) = \frac{d}{2} \left\{ \exp\left(c\left\{ \frac{M_t / P_t}{M_{t-1} / P_{t-1}} - 1 \right\} \right) + \exp\left(-c\left\{ \frac{M_t / P_t}{M_{t-1} / P_{t-1}} - 1 \right\} \right) - 2 \right\} \tag{2}$$

Here the parameters c and d stands for portfolio adjustment costs. This allows for the direct effects of money through portfolio adjustment costs channel as proposed by Nelson (2002).

The intertemporal budget constraint is given by:

$$\frac{M_{t-1} + B_{t-1} + W_t N_t + T_t + D_t}{P_t} = C_t + \frac{B_t / r_t + M_t}{P_t} \tag{3}$$

where T_t are the lump-sum transfers, D_t stand for the firms' dividends, W_t are the nominal wages, P_t the prices and, finally, r_t is the gross interest rate.

C_t stands for the CES aggregator of the different goods which are consumed, and is given by:

$$C_t = \int_0^1 \left(C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4}$$

I turn now to the production side. There is a continuum of producing firms which are indexed by $j \in [0,1]$.

The production function for firm j is given by:

$$Y_t(j) = z_t N_t(j)^{1-\alpha} \tag{5}$$

Where $Y_t(j)$ is the output, $N_t(j)$ stands for the labor demand or the hours hired by the firm j while z_t are the technology shocks. The parameter $1-\alpha$ is the elasticity of output relative to labor (hours worked).

The representative firm sells the result of the production in a monopolistically competitive environment sector and sets the prices according to the Calvo mechanism, resulting in the following aggregate price index:

$$P_t = \left[\theta\left(P_{t-1}\pi_{t-1}^{\omega}\right)^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{6}$$

where $1-\theta$ measures the degree of Calvo rigidity (how probable it is that a producer will reset the prices in a given period), while ω is the degree of price indexation for the firms that do not optimize the prices in a certain period. P_t* is the optimal price as set by the firms that reoptimize the prices.

Here Y_t is a CES aggregator of the differed goods produced and is given by:

$$Y_t = \int_0^1 \left(Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{7}$$

The market clearing conditions are given below. For the goods market, the condition is that:

$$Y_t = C_t \tag{8}$$

$$N_t = \int_0^1 N_t(j)dj \tag{9}$$

The monetary authority sets the nominal interest rate with the following monetary rule:

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + (1-\rho_r)\rho_\pi \ln(\pi_t/\pi) + (1-\rho_r)\rho_y \ln(y_t/y) + (1-\rho_r)\rho_\mu \ln(\mu_t/\mu) + \varepsilon_t \tag{10}$$

where $\mu_t = M_t/M_{t-1}$ is the rate of money growth while ε_t are the monetary policy shocks. This specification of the policy rule embeds the policy effect through the reaction of the Central Bank to the growth rate of nominal supply of money.

The log-linear model is summarized in Appendix A, which I present succinctly here. The first equation in Appendix A, equation 1, is a typical Euler equation for consumption where the aggregate constraint is also used. Due to households being forward-looking as well as habit formation, the equation features the output at both forward and backward lags. When there is nonseparability (namely $\psi_2 \neq 0$), the real balances determine the value of the current output. The second equation, equation 2, is a typical New Keynesian Phillips curve, where real balances matter for current inflation through the marginal costs of the firm when $\psi_2 \neq 0$. For both curves, the Euler curve for consumption and the Phillips curve, real balances enter as deviations from money demand shocks e_t.

There are two interpretations for the presence of money in the New Keynesian Phillips curve, for which see also Castelnuovo (2012). In the first interpretation, due mainly to Ravenna and Walsh (2006), money acts through the cost-channel, being understood as a proxy for the lending rate of the banks. In the second interpretation, money enters the Phillips curve due to the presence in the marginal costs, with the latter occurring through the effects of the real balances on the household decisions with respect to the labor supply and thus on wages.

The dynamics of real balances is presented in equation 4, which is the money demand equation. Real money demand is determined by lead and lags of output as well the future expected value of real balances and the current opportunity costs of holding money. Together with the fact that real balances enter as deviations from money demand shocks e_t, it implies that one must consider the changes in real money balances over the ones that originate from the effort by the FED to absorb the money demand shocks (FED answers to money demand shocks by varying the money supply in order to keep the key interest rate constant). Furthermore, even when $\psi_2 = 0$, that is when there is separability in the utility function, the money demand equation will still feature leads and lags, as far as $\delta \neq 0$, namely when the portfolio adjustment costs are significant. As Andres, Lopez-Salido and Nelson (2009) or Castelnuovo (2012) underline, there is also a "direct effect" of the money supply, since real balances lead the movements in the natural real interest rate.

Equation 5 describes a Taylor rule augmented with the nominal growth rate of the money supply (as described in equation 6). This is not something new, as equivalent rules have been also used by Sims and Zha (2006), Andres, Lopez-Salido and Nelson (2009) or Canova and Menz (2011).

The model is closed by considering AR(1) processes for the four stochastic processes, see equation 7, namely for preference shocks ε_{a_t}, money demand shocks ε_{e_t}, technology shocks ε_{z_t} and monetary policy shocks ε_{r_t}.

3. Data and Estimation

3.1. Data selection

I selected quarterly macroeconomic data from the US. The data series are real output, inflation, the nominal interest rate (short-term) and real money balances. The sample spans from 1966 to the fourth quarter of 2013. The beginning of the sample is common to other studies, including Smets and Wouters (2007) or Castelnuovo (2012). The variables are measured through their usual correspondents as follows[1]: output is given by the real GDP in constant prices, inflation by the quarterly percent change in the GDP deflator, the interest rate by the federal funds rate while the real balances are obtained by dividing the M2 stock by the GDP deflator. For the case of both the M2 and the federal funds rate, I use the quarterly averages (the averages over 3 months). The output, inflation and the real money balances are seasonally adjusted using the Census X12 procedure, a rather standard way of proceeding. Finally, both output and real money balances are measured in per-capita terms; that is, they are obtained by dividing the original series through the civilian non-institutional population aged over 16.

It is worth discussing the issue of dealing with the trend present in the data. There are various approaches taken in the DSGE literature, and a thorough discussion of this topic is outside the scope of this paper. However, in the context of the literature on estimating NK models augmented with money, most of the reference papers dealing with the estimation of such models chose to detrend the series. Some papers worked applied the detrending only to the series of output and money, like Canova and Menz (2011) who used the linear trend, while other considered the Hodrick-Prescott filter (HP, hereafter), see Castelnuovo (2012), with the smooth parameter calibrated to the standard choice of 1600. I followed a rather mixed approach, which however proved efficient in estimating the models. Namely, the output and the money were filtered with the linear trend, while the inflation and the interest rate with the HP filter. While, some papers considered only demeaning the inflation and the interest rate, this paper follows Castelnuovo (2012) and takes into account the change in the trend that characterize these two series during the post-WWII sample.

[1] The source for the data series is the freely available FRED database at the Federal Reserve Bank of St. Louis.

3.2. Methodology

The estimation in this paper follows the Bayesian approach. Since this approach has become rather generally accepted within the macroeconomic field, I will not argue in its favor any further. I would like however to underscore that, in the context of estimating models with money, very few papers have followed this approach, a notable exception being Castelnuovo (2012).

Castelnuovo (2012) applies both full sample estimation and estimation along a sliding window, motivated by his willingness to study the instability of the structural parameters of NK models augmented with money. In contrast, this paper uses a recursive estimation, where, starting from an initial sample, one observation is added at a time, in order to obtain recursive estimations of the structural models and further perform recursive forecasts. This approach thus somewhat parallels the one in Castelnuovo (2012), though the samples are recursively increased by one observation at a time.

The initial samples spans from 1966 quarter 2 to 1993 quarter 4. For each new estimation, one observation is added such that in the second estimation the sample consists of observations from 1966 quarter 2 to 1994 quarter 1 and so forth until the full sample is covered, up to and including the final observation, i.e. 2013 quarter 4.

Before discussing the setting of the prior distributions, I briefly present and motivate the calibration of some parameters. The choice of calibrated parameters is typical of the DSGE literature, and I follow mainly Smets and Wouters (2007) as well as Castelnuovo (2012). The discount factor β is calibrated to 0.9925 while the steady state gross quarterly interest rate $\bar{r}$ is set at 1.0138. The elasticity of output with respect to capital α is calibrated at 0.33. Finally, the elasticity of substitution between goods, parameter ε, is set at 6.

While ψ_1, ψ_2, γ_1 and γ_2 are compound parameters (that is, composed of deep parameters, including the partial derivatives of the utility function, see Appendix A), estimating them as such is too difficult a task, and thus I follow the literature and set them as free parameters which are directly estimated without taking into account their convoluted character (see also Ireland (2004), Andres, Lopez-Salido and Valles (2006) or Castelnuovo (2012)).

In order not to bias the estimation with respect to the key parameters related to money, namely ψ_2, ρ_μ and δ_0, they are set such that the estimation does not discard both the possibility of a separable utility being given (as implied by $\psi_2=0$) and of no policy effects (as implies by $\rho_\mu=0$). Thus, following Castelnuovo (2012), I assume that $\psi_2 \sim N(0,0.5)$ and that $\rho_\mu \sim Gamma(0.8,0.4)$. As for the portfolio adjustment costs, based on the results in Andres et al. (2009) and Castelnuovo (2012), it is assumed that $\delta_0 \sim Gamma(6,2.85)$.

A few more parameters are worth commenting on. First of all, the parameter ψ_1, which is related to the impact of money on both output and inflation when there is nonseparability. Following Ireland (2004) and Castelnuovo (2012), the prior distribution is set as $\psi_1 \sim Gamma(0.8,0.1)$. Two more parameters are related to the money demand elasticities, namely γ_1, the elasticity to output, for which I set $\gamma_1 \sim Gamma(0.5,0.25)$ and γ_2 respectively, the (semi)-elasticity to the nominal interest rate, for which I set $\gamma_2 \sim Gamma(0.2,0.15)$. Two further parameters are calibrated since they could not be estimated directly at the same time with δ_0. Following the literature, see Andres et al. (2009) or Castelnuovo (2012), I set the parameter *c* to 1 since we cannot identify both *c* and *d* separately.

The model is written in state space form and I include a measurement equation so that we can relate the observable variables to the theoretical variables (latent processes). The vector of observable variables is given by:

$$\left[y_t^{obs}, m_t^{obs}, \pi_t^{obs}, r_t^{obs}\right] \tag{11}$$

The endogenous variables, as presented in Appendix A, are given by the following vector:

$$\left[y_t, m_t, \pi_t, r_t\right] \tag{12}$$

The vector or exogenous shocks is given by:

$$\left[a_t, e_t, z_t\right] \tag{13}$$

while the vector of innovations is the following:

$$\left[\varepsilon_{a_t}, \varepsilon_{e_t}, \varepsilon_{z_t}, \varepsilon_{r_t}\right] \tag{14}$$

The estimation also considers measurement shocks added to each of the measurement equations.

3.3. Estimation Results

While this paper does not focus entirely on the estimations of the models, I discuss those results here. In contrast to other papers, given the focus of this paper on forecasting, recursive estimations are performed where one additional observation is added at a time. In order to synthesize the results, I present the minimum, mean and maximum of the recursive estimates. Furthermore, given the interest in what kind of microfoundations drive forecasting accuracy, I estimate not only the model with all three effects (nonseparability, portfolio adjustments and policy effects), called "all", and the model without any such effects (the model without money), called "nom", but also simplified models without one effect at a time (either without nonseparability, called "nopsi", or without portfolio adjustments, called "nodel", or without the policy effect, called "nomu"). Appendixes B.1 to B.5 show the results of the estimations.

I also compare the models along the sample in which the recursive estimation is made by comparing each model’s Marginal Likelihoods and the Bayesian Factors for each particular estimation sample, see Appendix B.6. Two clear results emerge. First of all, the model without any effects and the model without portfolio adjustment costs perform the worst. Towards the end of the sample, the model without any effects performs the worst. Furthermore, the model featuring all three effects tends to perform the best along the full sample, though up to the end of the Great Moderation the performance of this model is approximately equal to the one by the models without policy effects, "nomu", and the model with separability, "nopsi".

Thus, it can be seen that the comparison favors the model with all three types of effects, along the model with separability and the model with no policy effects. Are the results in line with the estimation of the coefficients? For the case of the nonseparability effect, although the parameter ψ is estimated at a slightly positive value, the confidence interval includes the zero value. This finding is essentially similar to that of Castelnuovo (2012), who also suggests an unimportant role for the nonseparability.

With respect to the portfolio adjustment parameter, a high estimate is found, with $\delta_0=2.85$ for model "all". Although lower than the one in Castelnuovo (2012) or the one in Andres et al. (2009), it is of comparable magnitude. The significance of the portfolio adjustment parameter is further supported by the fact that the model without portfolio effects has the worst performance along the model with no effects, as indicated by the Marginal Likelihood and the implied Bayes Factors.

As for the policy effects, as indicated by the coefficient ρ_μ, I again found a significant coefficient, with ρ_μ=0.31 for the model featuring all effects, higher than the one in Castelnuovo (2012). Nevertheless, not surprisingly, the model featuring all effects is superior to the model without policy effects, which is contrary to the results in Castelnuovo (2012) or Ireland (2004).

A few more parameters are related to money, namely ψ_1 related to the impact of money on output and inflation, γ_1 related to the money-output elasticity as well as the money-interest rate semi-elasticity parameter γ_2. The parameter ψ_1 is estimated at a close value to the one in the literature, namely at 0.59, while the money-output elasticity γ_1 is estimated at 0.75 which is lower but close to previous findings in the literature. There are some departures from the previous literature with respect to the estimation of the money-interest rate semi-elasticity, as this was estimated for the model featuring all the effects at 0.72, much larger than in previous papers, such as Andres et al. (2009).

As for the other parameters, they are generally in line with the findings in the literature, for example in Smets and Wouters (2007) and related papers, with some minor departures. For example, the habit formation parameter is estimated at 0.97, a larger value than the usual findings in the literature. A slightly higher value was also estimated for the Calvo parameter, at 0.78. Furthermore, the coefficient attached to inflation in the Taylor rule is a bit higher than the usual findings in the literature, though recent papers also indicate some higher values corresponding to a more aggressive reaction function at least for the post-1982 sample, for instance Lubik and Schorfheide (2004) or Benati and Surico (2009). At the same time, the confidence interval overlaps with the usual confidence intervals found for the inflation coefficient. To be more precise, the coefficient attached to inflation basically corresponds to the estimation in Lubik and Schorfheide (2004).

4. Forecasting Assessment

This section focuses on the forecast accuracy of the various models, namely of "all", "nom", "nopsi", "nodel" and "nomu", as well as the various pools based on these models with a equal weight.

The forecasting procedure was as follows. As indicated in the previous section, the models were recursively estimated starting from an initial sample between 1966Q2 and 1993Q4, adding one observation at a time until completing the full sample. The evaluation sample spans from 1994Q1 to 2013Q4. To be more precise, the first set of forecasts is generated for the sample 1994Q1 and 2013Q4, corresponding to the models estimated for the period 1966Q2 and 1993Q4. Thus, I can compute the forecast errors using 80 observations for the case of the 1-step ahead forecasts and using 48 observations for the case of 32-step ahead forecasts. I focus on three key macroeconomic variables, namely output, inflation and the interest rate.

The forecasts are evaluated using various approaches: point evaluations using the mean forecast errors (MFE, hereafter) and root mean squared forecast errors (RMSFE, hereafter), and density evaluations using the log-predictive scores (LPS, hereafter) or the probability integral transform (PIT, hereafter) graphs.

In order to assess whether the Great Moderation and the Great Recession influence in any ways the forecasting accuracy of the various models, I split the evaluation sample into two: a sample corresponding to the end of the Great Moderation, namely between 1994Q1 and 2007Q3, and a sample that includes both the Great Recession and the recovery period after it, namely between 2007Q4 and 2013Q4.

4.1. Point Forecasts

Appendix C presents the results for the Mean Forecast Errors for samples corresponding to both the Great Moderation and the Great Recession periods. With respect to the Great Moderation, the models lead to qualitatively the same results: the output is underpredicted, while inflation and the interest rate show no statistically significant bias effects. These results make sense in the context of the Great Moderation, as, given the lack of major recessions, on average the output has been higher than otherwise predicted. But they also potentially signal a bias in the models. For the Great Recession period, there is again some bias in the output prediction, which is logical given the effects of the recession, overpredicted by all types of models.

Based on Appendix C, namely on MFE, we find some improvements in terms of forecasting due to the inclusion of money for either inflation or interest rate, and for the "nodel" model, for the case of output. The results hold for both the Great Moderation and Great Recession assessment samples and their aftermaths. Namely, some improvements in forecast accuracy for output are obtained in the case of the "nodel" model, that is, for the model without portfolio adjustments, while models featuring money in various specifications help forecast the inflation and interest rate. These findings suggest some role for money in helping the accuracy of forecasts of the DSGE models.

I further compare the forecast accuracies of the models using the second order moments, that is, the RMSEs. Appendix D shows the results for the two evaluation samples of the Great Moderation and the Great Recession and their aftermaths. As underscored in the notes attached to the tables in Appendix D, for the baseline model (the model with no effects, "nom") the RMSEs are reported as given, while for the other models they are reported as ratios, where a value above one indicates a worse performance than the baseline model. To judge the statistical significance of the differences (ratios), I also report the Diebold-Mariano test results.

For the Great Moderation sub-sample, I found that, in most cases, the baseline model outperforms the models featuring money in different forms for the case of output, except the model "nodel" (without portfolio adjustment effects) for several forecast horizons. Furthermore, there are no notable differences between the baseline model and alternative models with money for both inflation and the interest rate, except again a few forecast horizons for the case of forecasting the interest rate with the model without portfolio adjustment effects, "nodel".

For the Great Recession and its aftermath, the results do not change in the case of output, with only the model without portfolio adjustment effects outperforming the baseline model. Only in the case of inflation and interest rate do the results differ somewhat drastically: the baseline model is outperformed by various alternative specifications at different horizons, except the higher forecast horizons. The results for the Great Recession point to some utility in including money in DSGE models, which did not appear that obvious from the simple comparison of the MFEs.

A final test which we apply to the point forecasts is the unbiasedness or the efficiency test, in the way suggested by Clements and Hendry (1998). The test implies that one runs the following regression for each model, variable and forecast horizon:

$$x_\tau = \alpha_0 + \alpha_1 x_\tau^F + \eta_t \tag{15}$$

where x_τ stand for the actuals while x_τ^F represent the forecasted values. An efficient/unbiased forecast implies that the constant term α_0 should equal zero, the slope coefficient α_1 should be around one while the fit of the model should be relatively high. Appendix F shows the results of the unbiasedness/efficiency test, considering the forecasts at both 1-step ahead and 4-steps ahead. I also use the Wald test for the null that $\alpha_0=0$ and $\alpha_1=1$ which is a χ^2 test. The tests results are further complemented by the plots of forecasts versus actuals, as seen from Appendix G.

Although the fit of the model for output forecasts is high, and the coefficients seem close to expected values, nevertheless the Wald test rejects the null of unbiased forecasts. The null of unbiased forecasts is also rejected in the case of inflation. However, the forecasts for the interest rate seem efficient in general, as shown by Wald tests.

The analysis of the point forecasts accuracy already leads us to several interesting conclusions. Although the estimation pointed, as in previous studies, to statistically significant portfolio adjustment costs, it seems that the best improvements in terms of forecasting are obtained by eliminating this channel while keeping the nonseparability and the policy effect for both MFEs for all variables and RMSEs for the case of output only. At the same time, I also found that, in terms of RMSEs, for both inflation and the interest rate the forecasts are much more accurate for the Great Recession and its aftermath for various alternatives with money, while MFEs including money leads to better results for both evaluation periods.

4.2. Density Forecasts

In this section, I focus on the density forecasts and discuss the performance of forecasts in terms of log predictive scores (LPS) and probability integral transform (PIT). The main role of this analysis is to check whether the forecasts are meaningful from an uncertainty perspective.

In the case of LPS, the comparison is done again for both sum-samples, corresponding to, respectively, the Great Moderation and the Great Recession and its aftermath. The LPSs are computed following the methodology proposed by Adolfson et al. (2007). We denote by $p(Y_{t+h}|t,i)$ the predictive density for a forecast on the basis of a model M_i, at time t, at h-steps ahead, while the predictive score is given by $p(y_{t+h}|t,i)$. Since I follow Adolfson et al. (2007), it is assumed that the predictive density is Gaussian and thus that we can approximate its moments on the basis of the sample of draws from the predictive density. Thus, we can compute the average LPS for a h-step ahead forecast using model M_i with:

$$S_{i,h} = \frac{1}{R}\sum_{t=P+1}^{P+R} \ln p(y_{t+h}|t,i) \quad (16)$$

where $P+1$ is the point in time at which the first forecast is formulated while R denotes the number of h-step ahead forecasting rounds.

Appendix E shows the results for both sub-samples. The figures for the baseline model are the average values of LPSs, while for the alternative models I present the difference between the average value of LPSs and the one for the baseline model. I further test whether the differences between the LPSs are statistically different with the help of the test proposed by Amisano-Giacomini (2007). In computing the LPSs, I also considered a few forecast pools, namely "11000", which combines the model "nom" with "all", "10100", which combines "nom" with "nopsi", "10010" which combines "nom" with "nodel", "10001" which combines "nom" with "nomu" and "00111" which combines "nopsi", "nodel" and "nomu".

In the case of the Great Moderation, for each of the variables considered and each of the alternative models and pools, the forecasts are either not statistically different in terms of LPSs, or the baseline model outperforms them. There is one exception to this rule, namely the pool "10010", which combines "nom" with "nodel" and leads to better results than the baseline model, but only for output and a few forecast horizons. There is also a forecast horizon for which the model with all effects "all" outperforms the baseline model.

The changes for the Great Recession and its aftermath are again quite significant. This time, both the "nodel" and the same pool "10010" outperform the baseline model for a few horizons in the case of output. For both inflation and the interest rate, the baseline model is outperformed by several alternative specifications with money (for inflation, only for 1-step ahead forecasts).

In order to understand where the differences in the LPSs originate from, I also investigated the histogram in PITs, which are informative whether the density forecasts are well calibrated (meaning that they are unbiased which implies a null MFE, and effective, implying an proper width for the predictive density). The results are shown in Appendix F.

PITs were introduced to economics literature by Diebold et al. (1998), but they have started to be used in the DSGE literature only recently, see Herbst and Schorfheide (2012). Following the usual approaches in the literature, the unit interval is divided into 10 subintervals, and one investigates whether the fraction of PITs is close to 10% in each of these subintervals. Whether or not the PITs are equally distributed can be used to draw implications on how well the density forecast is distributed. Thus, for equally distributed PITs across the subintervals the density is well calibrated, for PITs with a concentration in the lower/upper bins the model overpredicts/underpredicts a variable, while for PITs with a concentration in the middle/outer area of the subintervals the density forecast is too diffuse/tight.

While the previous statistics have been discussed along the two sub-samples, for simplicity, the PITs are discussed for the full evaluation sample, namely for 1994Q1 to 2013Q4.
A few interesting results emerge. First of all, in the case of output, there is a concentration in both the upper and lower parts of the bins, supporting the results from the MFEs, namely that the forecasts of output are biased. However, no such bias is found in the case of inflation or the interest rate (which parallels the findings based on MFEs). When considering the width of the density forecasts, it turns out that for the case of the output, the density forecasts are too tight, as the PITs are concentrated in the outer bins, while for the interest rate and inflation, the density forecasts are rather too diffuse, given their concentration in the middle bins.

I further analyze the PITs with the help of a goodness-for-fit χ^2 test. Since the results are based on the derived forecasts for the same sample evaluation, the evaluation sample is again 1994Q1 to 2013Q4. The role of the goodness-for-fit test is to check whether the distribution of the PITs is uniform. Here I follow the approach suggested by Wallis (2003) in testing for the uniformity of the distribution of PITs with the help of the Pearson χ^2 test. The results are shown in Appendix G.

Except a few cases, for lower horizons for output and inflation, it appears that we can confidently reject the null of a uniform distribution for PITs. This appears more obviously with the higher horizons.

5. Conclusion

This paper studied whether including money in various specifications in an otherwise standard DSGE model leads to better forecasts for key macroeconomic variables such output, inflation and the interest rate. Three key questions were asked: whether including money in a DSGE model leads to better forecasts, whether there are particular specifications that lead to better forecasts and whether the Great Moderation or the Great Recession matter in any way for forecasting accuracy.

The answer is positive to all three questions, though the role of money is far from being clear. More specifically, in the case of output, for both point and density forecasts, the forecasts accuracies are better when the portfolio adjustment effect is eliminated, while the other two effects (nonseparability and policy effects) are kept. The results are the same for the Great Moderation and the Great Recession. At the same time, for both inflation and the interest rate, some role was found for money in forecasts made during the Great Moderation in terms of MFEs. At the same time, including money in various specifications leads to better forecasts during the Great Recession in terms of MFEs, RMSEs and LPSs.

Though the role of money appears partially positive in obtaining more accurate forecasts, the point forecasts generally appear inefficient, while in most cases, the density forecasts are poorly calibrated.

This paper also contributes to the recent literature on what kind of micro-foundations can lead to better forecasts. Several recent papers focused on the case of financial frictions, like Del Negro and Schorfheide (2013) or Kolasa and Rubaszek (2014), finding a positive role for financial frictions in forecasting the key macroeconomic variables during the Great Recession. The results in this paper parallel these results, though the focus was on introducing money using various specifications, leading to our conclusion that extending a DSGE model with money can lead to better forecasts for output, inflation and the interest rate during the Great Recession, while certain specifications of money did also improve the forecasts of output during the Great Moderation.

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Appendix A. Log-linearized DSGE model

$$y_t = \frac{\phi_1}{\phi_1+\phi_2} y_{t-1} + \frac{\beta\phi_1+\phi_2}{\phi_1+\phi_2} E_t y_{t+1} - \frac{1}{\phi_1+\phi_2}\left(r_t - E_t\pi_{t+1}\right) - \frac{\beta\phi_1}{\phi_1+\phi_2} E_t y_{t+2} + \frac{\psi_2}{\psi_1(1-\beta h)(\phi_1+\phi_2)}(m_t - e_t)$$
$$- \frac{\psi_2(1+\beta h)}{\psi_1(1-\beta h)(\phi_1+\phi_2)} E_t(m_{t+1} - e_{t+1}) + \frac{\psi_2 \beta h}{\psi_1(1-\beta h)(\phi_1+\phi_2)} E_t(m_{t+2} - e_{t+2}) + \frac{(1-\beta h\rho_a)(1-\rho_a)}{(1-\beta h)(\phi_1+\phi_2)} a_t \quad (1)$$

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda mc_t \quad (2)$$

$$mc_t = (\chi+\phi_2) y_t - \phi_1 y_{t-1} - \beta\phi_1 E_t y_{t+1} - \frac{\psi_2}{\psi_1(1-\beta h)}(m_t - e_t)$$
$$+ \frac{\psi_2 \beta h}{\psi_1(1-\beta h)} E_t(m_{t+1} - e_{t+1}) - \frac{\beta h(1-\rho_a)}{(1-\beta h)} a_t - (1+\chi) z_t \quad (3)$$

$$(1+\delta_0(1+\beta)) m_t = \gamma_1 y_t - \gamma_2 r_t + [\gamma_2(\bar{r}-1)(h\phi_2 - \phi_1) - h\gamma_1] y_{t-1} - [\gamma_2(\bar{r}-1)\beta\phi_1] E_t y_{t+1}$$
$$+ \delta_0 m_{t-1} + \left[\frac{\psi_2(\bar{r}-1)\beta h \gamma_2}{\psi_1(1-\beta h)} + \delta_0\beta\right] E_t m_{t+1} - \frac{(\bar{r}-1)\beta h(1-\rho_a)}{(1-\beta h)} \gamma_2 a_t \quad (4)$$
$$+ \left\{1 - (\bar{r}-1)\gamma_2\left[\frac{\psi_2 \beta h \rho_e}{\psi_1(1-\beta h)} + 1\right]\right\} e_t$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\rho_y y_t + \rho_\pi \pi_t + \rho_\mu \mu_t) + \varepsilon_r \quad (5)$$

$$\mu_t = m_t - m_{t-1} + \pi_t \quad (6)$$

$$\zeta_t = \rho_\zeta \zeta_{t-1} + \varepsilon_{\xi_t} \text{ with } \zeta \in \{a, e, z\} \text{ and } \varepsilon_{\xi_t} \sim N(0, \sigma_{\varepsilon_\xi}), \xi \in \{a, e, z, r\} \quad (7)$$

The compound parameters are presented below:

$$\psi_1 = -\frac{U_1}{\bar{y}^{(1-h)} U_{11}}, \psi_2 = -\frac{U_{12}}{\bar{y}^{(1-h)} U_{11}} \frac{m}{e}, \gamma_f = \beta\theta\{\theta + \omega[1-\theta(1-\beta)]\}^{-1}$$

$$\gamma_b = \omega\{\theta + \omega[1-\theta(1-\beta)]\}^{-1}, \lambda = (1-\theta)(1-\beta\theta)(1-\omega)\xi, \chi = \frac{\varphi+\alpha}{1-\alpha}$$

$$\xi = \frac{1-\alpha}{1+\alpha(\varepsilon-1)}\{\theta + \omega[1-\theta(1-\beta)]\}^{-1}, \phi_1 = \frac{(\psi_1^{-1}-1)h}{1-\beta h}$$

$$\phi_2 = \frac{\psi_1^{-1} + (\psi_1^{-1}-1)\beta h^2 - \beta h}{1-\beta h}, \delta_0 = -\frac{c^2 d}{U_{22} m^2}$$

Appendix B. Estimation Results

B.1. Results for the model without money

Parameter	Prior Mean	Distribution	Standard Deviation	Min	Posterior Mean	Max
ψ_1	0.80	Gamma	0.10	0.578	0.610	0.711
h	0.70	Beta	0.10	0.971	0.983	0.989
θ	0.65	Beta	0.10	0.724	0.815	0.900
ω	0.50	Beta	0.15	0.531	0.604	0.862
φ	1.00	Gamma	0.25	0.618	1.005	1.445
γ_1	0.50	Gamma	0.25	0.353	0.437	0.961
γ_2	0.20	Gamma	0.15	0.066	0.151	0.244
ρ_R	0.50	Beta	0.10	0.400	0.583	0.640
ρ_y	0.15	Gamma	0.05	0.050	0.141	0.161
ρ_π	1.50	Gamma	0.25	1.732	2.114	2.251
ρ_a	0.75	Beta	0.10	0.693	0.803	0.870
ρ_e	0.75	Beta	0.10	0.958	0.972	0.983
ρ_z	0.75	Beta	0.10	0.604	0.772	0.808
σ_a	0.01	Inverted Gamma	1.5	0.005	0.010	0.014
σ_e	0.01	Inverted Gamma	1.5	0.008	0.009	0.010
σ_z	0.01	Inverted Gamma	1.5	0.019	0.028	0.092
σ_r	0.01	Inverted Gamma	1.5	0.008	0.009	0.010

B.2. Results for the model with money (full specification)

Parameter	Prior Mean	Distribution	Standard Deviation	Min	Posterior Mean	Max
ψ_1	0.80	Gamma	0.10	0.563	0.592	0.636
ψ_2	0.00	Normal	0.50	-0.020	0.007	0.052
h	0.70	Beta	0.10	0.964	0.973	0.989
θ	0.65	Beta	0.10	0.731	0.785	0.878
ω	0.50	Beta	0.15	0.537	0.586	0.621
φ	1.00	Gamma	0.25	0.971	1.037	1.102
γ_1	0.50	Gamma	0.25	0.608	0.750	0.901
γ_2	0.20	Gamma	0.15	0.574	0.726	0.860
δ_0	6.00	Gamma	2.85	6.029	6.722	7.434
ρ_R	0.50	Beta	0.10	0.557	0.591	0.654
ρ_y	0.15	Gamma	0.05	0.122	0.139	0.155
ρ_π	1.50	Gamma	0.25	2.047	2.258	2.476
ρ_μ	0.80	Gamma	0.40	0.227	0.314	0.410
ρ_a	0.75	Beta	0.10	0.765	0.806	0.884
ρ_e	0.75	Beta	0.10	0.925	0.946	0.960
ρ_z	0.75	Beta	0.10	0.704	0.801	0.832
σ_a	0.01	Inverted Gamma	1.5	0.009	0.013	0.021
σ_e	0.01	Inverted Gamma	1.5	0.018	0.020	0.022
σ_z	0.01	Inverted Gamma	1.5	0.017	0.024	0.066
σ_r	0.01	Inverted Gamma	1.5	0.008	0.009	0.011

B.3. Results for the model with *δ=0*

Parameter	Prior Mean	Distribution	Standard Deviation	Min	Posterior Mean	Max
ψ_1	0.80	Gamma	0.10	0.680	0.732	0.800
ψ_2	0.00	Normal	0.50	-1.281	-0.921	-0.467
h	0.70	Beta	0.10	0.883	0.946	0.989
θ	0.65	Beta	0.10	0.751	0.813	0.902
ω	0.50	Beta	0.15	0.446	0.589	0.641
φ	1.00	Gamma	0.25	0.933	1.101	1.285
γ_1	0.50	Gamma	0.25	0.427	0.485	0.574
γ_2	0.20	Gamma	0.15	0.277	0.365	0.427
ρ_R	0.50	Beta	0.10	0.561	0.611	0.675
ρ_y	0.15	Gamma	0.05	0.104	0.125	0.138
ρ_π	1.50	Gamma	0.25	1.807	2.046	2.244
ρ_μ	0.80	Gamma	0.40	0.221	0.402	0.516
ρ_a	0.75	Beta	0.10	0.811	0.837	0.873
ρ_e	0.75	Beta	0.10	0.959	0.972	0.981
ρ_z	0.75	Beta	0.10	0.666	0.810	0.881
σ_a	0.01	Inverted Gamma	1.5	0.008	0.017	0.026
σ_e	0.01	Inverted Gamma	1.5	0.008	0.009	0.010
σ_z	0.01	Inverted Gamma	1.5	0.017	0.029	0.107
σ_r	0.01	Inverted Gamma	1.5	0.008	0.009	0.011

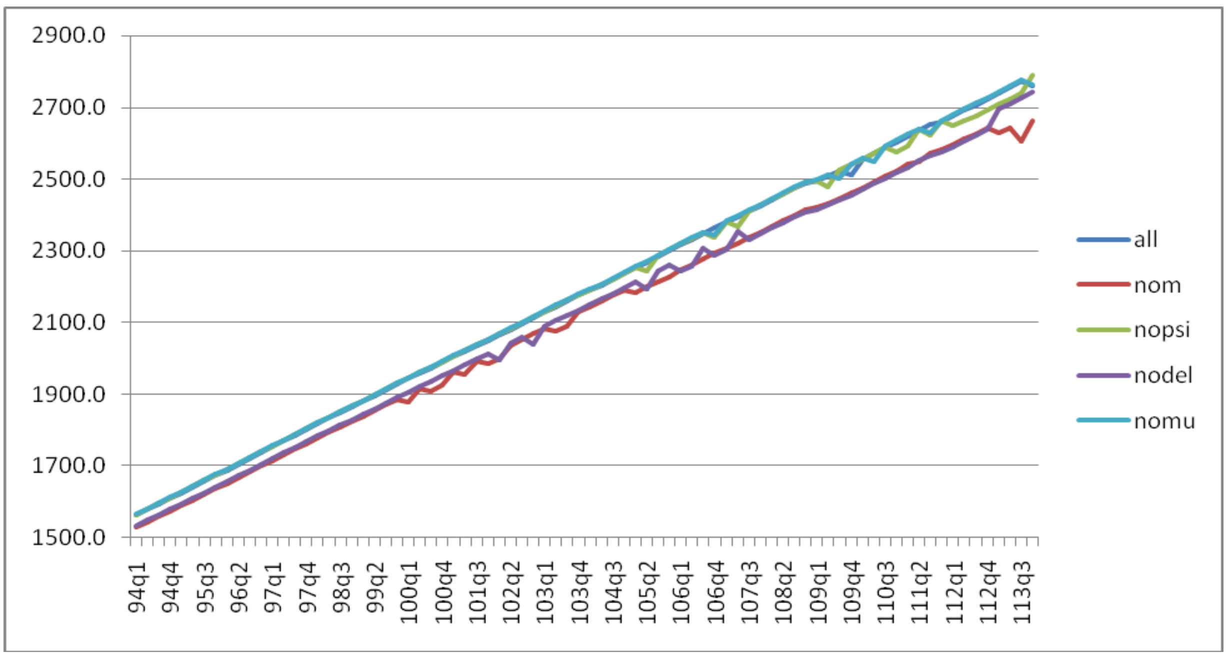
B.4. Results for the model with *μ=0*

Parameter	Prior Mean	Distribution	Standard Deviation	Min	Posterior Mean	Max
ψ_1	0.80	Gamma	0.10	0.573	0.599	0.645
ψ_2	0.00	Normal	0.50	-0.008	0.016	0.059
h	0.70	Beta	0.10	0.967	0.975	0.989
θ	0.65	Beta	0.10	0.772	0.816	0.886
ω	0.50	Beta	0.15	0.521	0.584	0.618
φ	1.00	Gamma	0.25	0.949	1.001	1.071
γ_1	0.50	Gamma	0.25	0.583	0.708	0.905
γ_2	0.20	Gamma	0.15	0.447	0.568	0.705
δ_0	6.00	Gamma	2.85	6.251	6.902	7.492
ρ_R	0.50	Beta	0.10	0.547	0.585	0.628
ρ_y	0.15	Gamma	0.05	0.127	0.141	0.156
ρ_π	1.50	Gamma	0.25	2.051	2.202	2.401
ρ_a	0.75	Beta	0.10	0.778	0.810	0.855
ρ_e	0.75	Beta	0.10	0.923	0.942	0.956
ρ_z	0.75	Beta	0.10	0.727	0.787	0.816
σ_a	0.01	Inverted Gamma	1.5	0.008	0.012	0.015
σ_e	0.01	Inverted Gamma	1.5	0.019	0.020	0.023
σ_z	0.01	Inverted Gamma	1.5	0.019	0.027	0.075
σ_r	0.01	Inverted Gamma	1.5	0.008	0.009	0.010

B.5. Results for the model with $\psi=0$

Parameter	Prior Mean	Distribution	Standard Deviation	Min	Posterior Mean	Max
ψ_1	0.80	Gamma	0.10	0.578	0.604	0.645
h	0.70	Beta	0.10	0.967	0.977	0.989
θ	0.65	Beta	0.10	0.731	0.786	0.910
ω	0.50	Beta	0.15	0.532	0.590	0.632
φ	1.00	Gamma	0.25	0.951	1.031	1.096
γ_1	0.50	Gamma	0.25	0.605	0.717	0.816
γ_2	0.20	Gamma	0.15	0.543	0.735	0.931
δ_0	6.00	Gamma	2.85	5.889	7.237	8.392
ρ_R	0.50	Beta	0.10	0.558	0.595	0.635
ρ_y	0.15	Gamma	0.05	0.123	0.139	0.158
ρ_π	1.50	Gamma	0.25	1.932	2.230	2.405
ρ_μ	0.80	Gamma	0.40	0.205	0.311	0.413
ρ_a	0.75	Beta	0.10	0.764	0.800	0.828
ρ_e	0.75	Beta	0.10	0.925	0.943	0.958
ρ_z	0.75	Beta	0.10	0.637	0.798	0.830
σ_a	0.01	Inverted Gamma	1.5	0.008	0.011	0.014
σ_e	0.01	Inverted Gamma	1.5	0.019	0.021	0.024
σ_z	0.01	Inverted Gamma	1.5	0.017	0.024	0.160
σ_r	0.01	Inverted Gamma	1.5	0.008	0.009	0.011

B.6. Marginal Likelihood



Appendix C. Mean Forecast Error (MFE)

C.1. Great Moderation sub-sample: 1994Q1-2006Q4

				output					
	1	2	3	4	8	12	16	24	32
nom	0.0030***	0.0065***	0.0106***	0.0151***	0.0298***	0.0395***	0.0449***	0.0459***	0.0411***
all	0.0033***	0.0072***	0.0116***	0.0163***	0.0310***	0.0400***	0.0449***	0.0461***	0.0413***
nopsi	0.0033***	0.0072***	0.0117***	0.0163***	0.0311***	0.0402***	0.0452***	0.0463***	0.0412***
nodel	0.0029***	0.0062***	0.0100***	0.0142***	0.0281***	0.0380***	0.0437***	0.0456***	0.0413***
nomu	0.0031***	0.0070***	0.0113***	0.0158***	0.0305***	0.0397***	0.0447***	0.0460***	0.0413***
				inflation					
nom	-0.0002	-0.00022	-0.00024	-0.00025	-0.00016	-0.00005	-0.00001	0.00031	0.00006
all	-0.00002	0.00003	0.00009	0.00016	0.00042	0.00050	0.00047	0.00069	0.00033
nopsi	-0.00003	0.00002	0.00009	0.00016	0.00038	0.00047	0.00046	0.00070	0.00032
nodel	0.00003	0.00006	0.00010	0.00015	0.00031	0.00043	0.00043	0.00076*	0.00046
nomu	-0.0001	-0.00019	-0.00019	-0.00019	-0.00008	0.00002	0.00003	0.00035	0.00008
				interest rate					
nom	-0.0015	-0.0023	-0.0027	-0.0027	-0.0021	-0.0011	-0.0006	-0.0002	-0.0046
all	-0.0016	-0.0022	-0.0024	-0.0023	-0.0016	-0.0006	-0.0001	0.0002	-0.0044
nopsi	-0.0016	-0.0022	-0.0024	-0.0024	-0.0017	-0.0006	-0.0001	0.0002	-0.0043
nodel	-0.0012	-0.0017	-0.0019	-0.0020	-0.0017	-0.0009	-0.0004	0.0001	-0.0042
nomu	-0.0016	-0.0022	-0.0024	-0.0024	-0.0018	-0.0009	-0.0004	-0.0001	-0.0046

*Note: I report the MSFE all models; ***, ** and * denote the 1%, 5% and 10% significance levels of the test of the null that the MSFE is equal to zero. Newey-West method is used to correct for the autocorrelation of the forecast errors. Negative values indicate that, on average, the forecasts are above the actual values.*

C.2. Great Recession and its aftermath: 2007Q1-2012Q4

				output					
	1	2	3	4	8	12	16	24	32
nom	-0.0039***	-0.0079***	-0.0122***	-0.0168***	-0.0284***	-0.0324***	-0.0329***	-0.0323**	-0.0317**
all	-0.0038***	-0.0082***	-0.0130***	-0.0177***	-0.0291***	-0.0326***	-0.0333***	-0.0328**	-0.0319**
nopsi	-0.0038***	-0.0079***	-0.0127***	-0.0174***	-0.0290***	-0.0327***	-0.0332***	-0.0325**	-0.0318**
nodel	-0.0035***	-0.0072***	-0.0114***	-0.0158***	-0.0277***	-0.0330***	-0.034***	-0.0338**	-0.0327**
nomu	-0.0037***	-0.0080***	-0.0128***	-0.0176***	-0.0290***	-0.0326***	-0.0331**	-0.032**	-0.0318**
				inflation					
nom	0.0002	0.0002	0.0001	0.0001	-0.0000	-0.0001	-0.0001	-0.0001	-0.0001
all	0.0002	0.0001	0.00002	-0.00004	-0.0002	-0.0002	-0.0002	-0.0001	-0.0001
nopsi	0.0001	0.0001	0.0000	-0.00004	-0.0002	-0.0002	-0.0002	-0.0001	-0.0001
nodel	0.0000	0.0001	0.0000	-0.00001	-0.0001	-0.0002	-0.0001	-0.0001	-0.0001
nomu	0.0002	0.0002	0.0001	0.00008	-0.0001	-0.0001	-0.0001	-0.0002	-0.0001
				interest rate					
nom	0.0017	0.0023	0.0025	0.0022	0.0008	0.00006	-0.00006	0.00004	0.0001
all	0.0012	0.0019	0.0021	0.0019	0.0007	0.0001	0.000004	0.0001	0.0001
nopsi	0.0012	0.0018	0.0019	0.0018	0.0006	-0.00007	-0.0001	-0.0000	0.0003
nodel	0.0012	0.00189	0.0019	0.0020	0.0008	0.00006	-0.00006	-0.0001	0.000006
nomu	0.00173	0.0024	0.0024	0.0021	0.0007	-0.00009	-0.00000	-0.00009	0.0001

*Note: I report the MSFE all models; ***, ** and * denote the 1%, 5% and 10% significance levels of the test that the MSFE is equal to zero. Newey-West method is used to correct for the autocorrelation of the forecast errors. Negative values indicate that, on average, the forecasts are below the actual values.*

Appendix D. Root Mean Squared Forecast

D.1. Great Moderation sub-sample: 1994Q1-2006Q4

				output					
	1	2	3	4	8	12	16	24	32
nom	0.0058	0.0095	0.0138	0.0182	0.0332	0.0421	0.0468	0.0475	0.0417
all	1.0335**	1.0558***	1.0596***	1.0561***	1.0316***	1.0105**	1.0005	1.0034	1.0040
nopsi	1.0480***	1.0608***	1.0667***	1.0609***	1.0375***	1.0145***	1.0064*	1.0070*	1.0017
nodel	0.9995	0.9829	0.9642	0.9552***	0.9575**	0.9735***	0.9838	0.9948	1.0047
nomu	1.0264***	1.0455***	1.0417***	1.0343***	1.0181**	1.0026	0.9953	1.0022	1.0051
				inflation					
nom	0.0012	0.0014	0.0014	0.0015	0.0016	0.00162	0.001559	0.001601	0.00161
all	0.9701	0.9807	0.9760	0.9887	0.9922	0.9870	1.0077	1.0832	1.0125
nopsi	0.9699	0.9681	0.9765	0.9778	0.9707	0.9785	1.0227	1.0691	1.0475
nodel	1.0230	1.0131	1.0222	1.0360	0.9992	0.9812	0.9675	1.0742	1.0456
nomu	0.9967	0.9937	1.0201	1.0111	1.0043	0.9991	0.9974	1.0226	1.0073
				interest rate					
nom	0.0057	0.0085	0.0100	0.0109	0.0121	0.0129	0.0132	0.0142	0.0142
all	1.0216	1.0164	1.0126	1.0030	1.0032	1.0026	1.0080	0.9943	0.9901
nopsi	1.0173	1.0084	1.0105	1.0011	0.9941	0.9796	0.9932	0.9982	0.9864
nodel	0.8965***	0.9156**	0.9506	0.9721	1.0188*	1.0039	0.9918	0.9863	0.9905
nomu	0.9808	0.9838	0.9895	0.9900	0.9918	0.9939	1.0002	1.0029	1.0011

*Note: I report the RMSFE in levels for the baseline model (nom) and in ratios for the remaining ones. ***, ** and * denote the 1%, 5% and 10% significance levels of the Diebold Mariano test.*

D.2. Great Recession and its aftermath: 2007Q1-2012Q4

				output					
	1	2	3	4	8	12	16	24	32
nom	0.0065	0.012445	0.018138	0.023544	0.037418	0.044604	0.04698	0.047705	0.047059
all	0.9908	0.999915	1.016423	1.02218	1.030785**	1.008499	1.008606	1.009282**	1.006135*
nopsi	0.9868	0.988893	1.005834	1.013232	1.022911	1.012535	1.008621	1.002178	1.00341
nodel	0.9733*	0.958606**	0.96342**	0.963281**	0.96211**	0.977725	0.999582	1.0244**	1.018242**
nomu	0.9820	1.001196	1.014647	1.020827*	1.023584**	1.009099	1.007886**	1.002716	1.005848*
				inflation					
nom	0.0021	0.0023	0.0023	0.0025	0.0024	0.0022	0.0021	0.0020	0.0020
all	0.9503*	0.9529	0.9578	0.9708	0.9451	0.9618*	0.9850**	0.9873	1.0139**
nopsi	0.9622**	0.9709	0.9483	0.9746	0.9546	0.9583**	0.9710**	1.0004	0.9951
nodel	0.9904	1.0023	0.9985	1.0021	0.9735**	0.9872	0.9792	0.9834	1.0223*
nomu	0.9658*	0.9575	0.9582	0.9768	0.9615	0.9779	0.9668	0.9836**	1.0006
				interest rate					
nom	0.0051	0.0076	0.0091	0.0100	0.0111	0.0112	0.0111	0.0105	0.0105
all	0.9788	0.9687*	0.9686	0.9922	1.0134	0.9975	0.9957	0.9974	1.0040
nopsi	0.9900	0.9654	0.9704	0.9902	0.9998	1.0020	0.9877**	1.0006	0.9993
nodel	0.940*	0.9492*	0.9648	0.9970	1.0034	1.0195	1.0219**	0.9948	1.00003
nomu	1.0022	0.9940	0.9777**	0.9907	0.9960	0.990	0.9935	0.9928	1.0039

*Note: I report the RMSFE in levels for the baseline model (nom) and in ratios for the remaining ones. ***, ** and * denote the 1%, 5% and 10% significance levels of the Diebold Mariano test.*

Appendix E. Average Log Predictive Scores

E.1. Great Moderation sub-sample: 1994Q1-2006Q4

				output					
	1	2	3	4	8	12	16	24	32
nom	191.52	162.77	141.95	126.13	86.31	62.51	48.35	37.54	33.19
all	-2.06***	-2.62***	-2.97***	-2.69***	-2.13***	-0.06	0.95*	0.87	0.12
nopsi	-1.86***	-2.32***	-2.92***	-2.70***	-2.56***	-0.88**	-0.81	-0.94**	0.00
nodel	0.55	1.27	1.80	2.18	1.23	-0.15	-1.31**	-2.03*	-1.16***
nomu	-1.44***	-2.05***	-2.07***	-1.59***	-0.85*	0.63	1.07*	0.16	-0.03
p11000	-1.00***	-1.27***	-1.44***	-1.31***	-1.03***	-0.01	0.51*	0.51	0.07
p10100	-0.90***	-1.12***	-1.42***	-1.31***	-1.22***	-0.40*	-0.37	-0.42**	0.02
p10010	0.31	0.69	0.96*	1.18*	0.74	0.07	-0.47**	-0.88*	-0.55***
p10001	-0.69***	-1.00***	-1.01***	-0.78***	-0.40*	0.34	0.56	0.12	-0.01
p00111	-0.86**	-0.97*	-0.97	-0.59	-0.59	0.04	-0.14	-0.84**	-0.38
				inflation					
nom	263.06	249.98	239.20	229.17	203.51	184.33	165.97	129.63	93.59
all	0.44	-0.79	-0.91	-0.99	-0.53	-0.49	-0.32	-0.45*	-0.21
nopsi	0.38	-0.75	-1.11*	-0.76	-0.16	-0.28	-0.39	-0.27	-0.35*
nodel	-0.26	-0.58	-0.68	-0.61	-0.71	-1.00*	-0.76*	-1.22***	-0.74***
nomu	-0.13	-0.24	-0.68***	-0.54**	-0.68***	-0.75***	-0.50**	-0.71***	-0.49***
p11000	0.24	-0.36	-0.42	-0.46	-0.24	-0.22	-0.14	-0.22*	-0.10
p10100	0.21	-0.34	-0.53*	-0.35	-0.05	-0.12	-0.18	-0.13	-0.17
p10010	-0.10	-0.26	-0.31	-0.28	-0.33	-0.48*	-0.36*	-0.60***	-0.36***
p10001	-0.06	-0.11	-0.33***	-0.26**	-0.34***	-0.37***	-0.24**	-0.35***	-0.24***
p00111	0.02	-0.50	-0.80*	-0.61	-0.49	-0.66	-0.54*	-0.72***	-0.52***
				interest rate					
nom	182.48	166.28	156.73	150.09	133.13	118.98	106.82	82.01	59.38
all	-0.85	-0.92	-0.77	-0.62	-0.44	-0.47	-0.59	0.14	0.10
nopsi	-0.81	-0.64	-0.55	-0.22	-0.19	0.41	-0.06	0.00	0.27
nodel	0.68	1.09	0.67	0.12	-1.06	-0.56	-0.36	0.13	-0.09
nomu	0.46*	0.39	0.25	0.41	0.33	0.26	0.04	-0.14	-0.10
p11000	-0.40	-0.43	-0.35	-0.29	-0.20	-0.22	-0.28	0.08	0.05
p10100	-0.39	-0.30	-0.25	-0.09	-0.07	0.23	-0.01	0.01	0.14
p10010	0.36	0.61	0.41	0.12	-0.50	-0.26	-0.16	0.08	-0.04
p10001	0.23*	0.20	0.14	0.22	0.18	0.14	0.02	-0.07	-0.05
p00111	0.14	0.33	0.18	0.14	-0.28	0.06	-0.11	0.01	0.03

*Note: I report the log predictive scores in levels for the baseline model (nom2) and in ratios for the remaining ones. ***, ** and * denote the 1%, 5% and 10% significance levels of the Amisano-Giacomini test.*

E.2. Great Recession and its aftermath: 2007Q1-2012Q4

				output					
	1	2	3	4	8	12	16	24	32
nom	100.85	81.81	70.65	62.83	48.75	42.04	40.17	38.62	39.52
all	0.10	0.64	0.00	-0.33	-1.08	-0.41	-1.17*	-0.76*	-0.94
nopsi	0.01	0.55	-0.32	-0.47	-1.42	-1.57	-2.04**	-0.50	-0.88
nodel	0.34	1.10	1.32***	1.50**	1.65*	2.12*	0.57	-2.11***	-2.36**
nomu	0.21	0.35	0.09	-0.27	-0.80	-0.21	-0.87***	-0.65	-0.47
p11000	0.12	0.51	0.10	-0.10	-0.48	-0.14	-0.55*	-0.36*	-0.42
p10100	0.02	0.34	-0.09	-0.18	-0.67	-0.73	-0.95**	-0.23	-0.40
p10010	0.27	0.65	0.69***	0.79**	0.90**	1.17*	0.41	-0.94**	-1.07**
p10001	0.14	0.27	0.14	-0.08	-0.37	-0.07	-0.41***	-0.29	-0.19
p00111	0.30	0.80	0.45	0.32	-0.06	0.30	-0.59	-0.98*	-1.16
				inflation					
nom	132.11	130.31	128.99	127.38	125.78	126.12	126.14	125.86	125.11
all	1.82*	0.99	0.56	-0.06	0.10	-0.16	-0.36	-0.28	-0.30
nopsi	1.20*	0.75	0.91	-0.03	0.10	-0.20	-0.05	-0.34	0.10
nodel	0.38	0.04	0.04	-0.07	0.36	-0.12	-0.19	-0.41	-1.00***
nomu	0.97	1.10	0.75	0.22	-0.09	-0.11	0.12	-0.09	-0.37**
p11000	1.01*	0.61	0.37	0.01	0.08	-0.07	-0.17	-0.13	-0.14
p10100	0.65*	0.46	0.52	0.02	0.07	-0.09	-0.02	-0.16	0.06
p10010	0.25	0.10	0.04	-0.02	0.19	-0.05	-0.08	-0.19	-0.49***
p10001	0.54	0.62	0.43	0.13	-0.03	-0.05	0.07	-0.04	-0.18**
p00111	0.91	0.69	0.62	0.07	0.14	-0.13	-0.03	-0.27	-0.41***
				interest rate					
nom	100.85	94.11	90.53	88.56	85.94	85.17	85.21	86.01	85.78
all	0.50	0.62**	0.55	0.19	-0.58**	-0.31	-0.43	-0.44*	-0.53**
nopsi	0.28	0.49	0.48	0.04	-0.26	-0.30**	-0.06	-0.56*	-0.26
nodel	0.52	0.20	0.02	-0.43	-0.74*	-0.84	-1.14***	-0.94	-1.25**
nomu	0.58**	0.25**	0.43*	0.15	-0.11	0.07	0.02	0.16	-0.19
p11000	0.26	0.32**	0.28	0.10	-0.28**	-0.15	-0.21	-0.21**	-0.26**
p10100	0.15	0.25	0.26	0.03	-0.12	-0.15**	-0.02***	-0.28*	-0.12
p10010	0.27	0.11	0.04	-0.20	-0.36	-0.40	-0.55	-0.45	-0.60**
p10001	0.30**	0.13**	0.22**	0.08	-0.05	0.04	0.02	0.08	-0.09
p00111	0.48	0.33	0.33	-0.07	-0.35	-0.34	-0.37*	-0.43	-0.55**

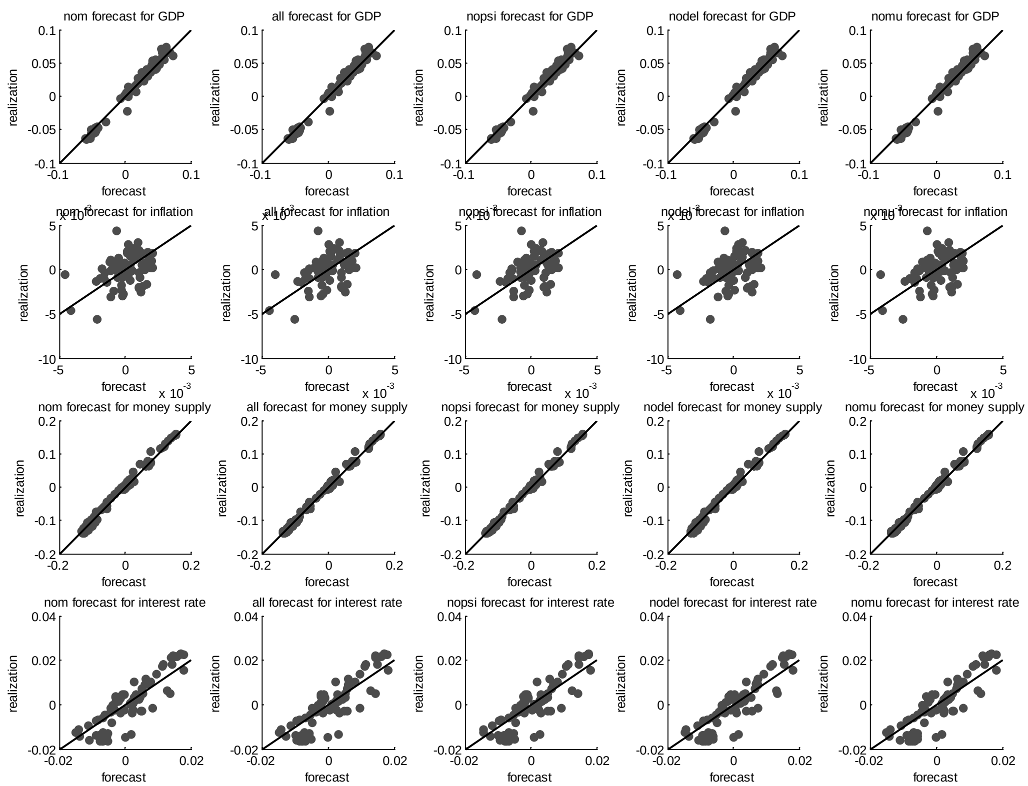
*Note: I report the log predictive scores in levels for the baseline model (nom2) and in ratios for the remaining ones. ***, ** and * denote the 1%, 5% and 10% significance levels of the Amisano-Giacomini test.*

Appendix F. Unbiasedness Test

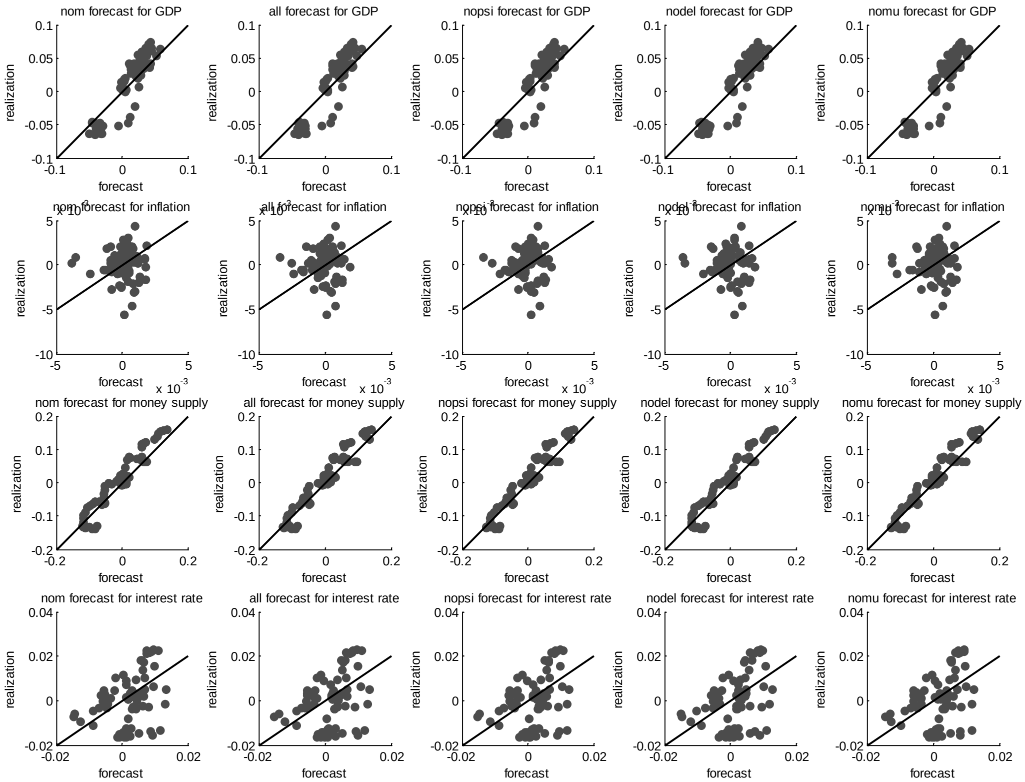
					output				
		1 step ahead					**4 step ahead**		
	α_0 ($S\alpha_0$)	α_1 ($S\alpha_1$)	R^2	χ^2 (prob)		α_0 ($S\alpha_0$)	α_1 ($S\alpha_1$)	R^2	χ^2 (prob)
nom	-0.0002	1.063405	0.982283	31.23132	nom	-0.00017	1.373887	0.841941	29.29547
	0.000701	0.011409		1.65E-07		0.003181	0.069161		4.35E-07
all	-5.1E-06	1.067362	0.982162	36.90222	all	-0.00025	1.44367	0.842461	44.31184
	0.000688	0.011122		9.7E-09		0.003178	0.067249		2.39E-10
nopsi	5.11E-05	1.066285	0.981763	39.3883	nopsi	6.24E-05	1.432563	0.841022	44.25172
	0.000691	0.010591		2.8E-09		0.003158	0.065722		2.46E-10
nodel	-1.4E-05	1.056326	0.982112	24.77325	nodel	-1.1E-05	1.331746	0.849787	27.5037
	0.000708	0.011425		4.17E-06		0.003151	0.063264		1.07E-06
nomu	-3.3E-05	1.064524	0.982142	33.7843	nomu	-0.00043	1.424489	0.842232	37.11743
	0.000684	0.011139		4.61E-08		0.003174	0.06999		8.71E-09
					inflation				
		1 step ahead					**4 step ahead**		
nom	-2.6E-05	0.52018	0.151461	12.7343	nom	6.1E-08	-0.02728	0.000277	60.55596
	0.000178	0.150319		0.001717		0.000256	0.141761		7.08E-14
all	3.04E-05	0.594623	0.19342	7.49303	all	-5.3E-06	-0.02571	0.00021	65.0778
	0.000158	0.151607		0.0236		0.00027	0.151302		7.44E-15
nopsi	2.47E-05	0.582973	0.183547	8.347773	nopsi	-4.1E-06	-0.01196	4.58E-05	74.55351
	0.000161	0.149625		0.015392		0.000271	0.153056		1.11E-16
nodel	2.3E-05	0.515765	0.142066	12.1361	nodel	-1.5E-05	-0.1259	0.00557	91.74184
	0.000177	0.14336		0.002316		0.000272	0.148111		0
nomu	-1.9E-05	0.558292	0.173637	10.2149	nomu	-2.7E-07	-0.03072	0.000328	53.30119
	0.000169	0.153653		0.006051		0.000258	0.148986		2.67E-12
					interest rate				
		1 step ahead					**4 step ahead**		
nom	-0.00063	1.204673	0.773034	3.246984	nom	-0.00028	0.646758	0.125498	1.441339
	0.001055	0.113938		0.197209		0.001998	0.29831		0.486427
all	-0.00081	1.18887	0.766969	3.028176	all	-0.0002	0.646895	0.124185	1.364107
	0.001054	0.113767		0.220009		0.002003	0.308332		0.505578
nopsi	-0.00086	1.184611	0.766576	2.895944	nopsi	-0.00026	0.655215	0.126592	1.281078
	0.00106	0.113115		0.235046		0.002002	0.307844		0.527008
nodel	-0.00057	1.205058	0.817189	4.519714	nodel	-0.00013	0.707065	0.144533	0.952244
	0.000907	0.097423		0.104365		0.002003	0.308151		0.621188
nomu	-0.00065	1.207589	0.780178	3.478286	nomu	-0.00022	0.683055	0.13179	1.116258
	0.001032	0.11191		0.175671		0.001999	0.30509		0.572279

Appendix G. Realizations and Forecasts

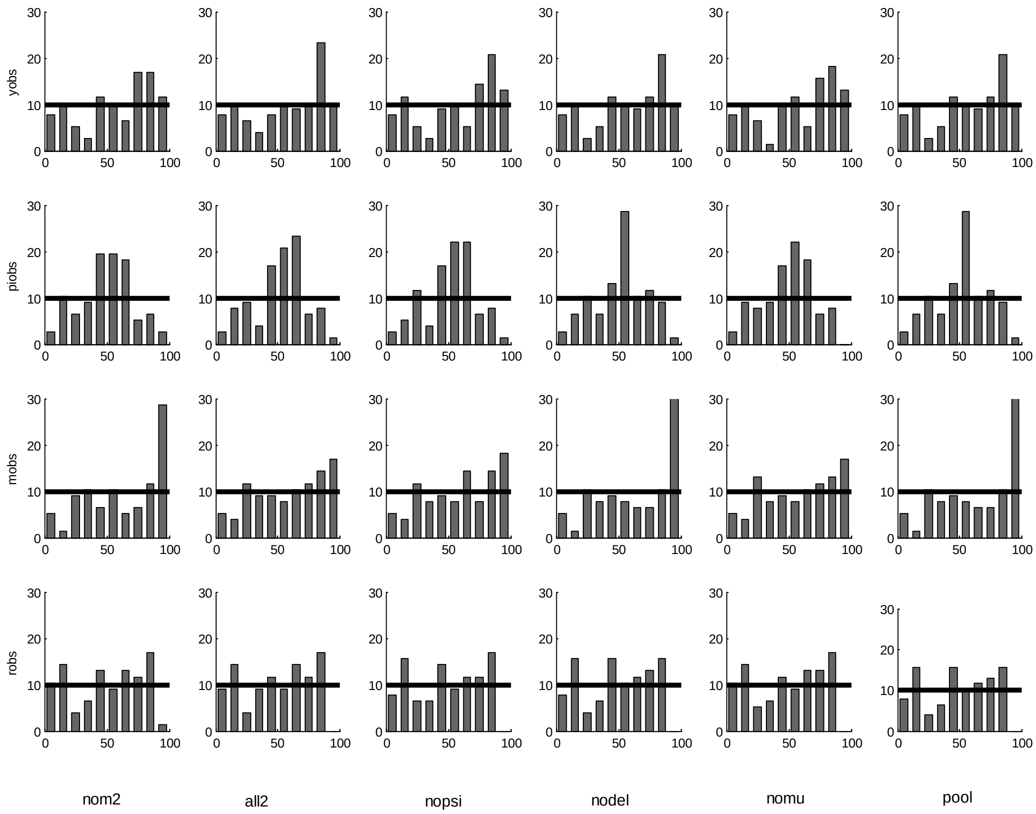
G.1. One Step Ahead Forecasts



G.2. Four Step Ahead Forecasts



Appendix H. PIT Tables (4-step ahead forecasts)



Appendix I. Goodness of Fit χ^2 test

				output					
	1	2	3	4	8	12	16	24	32
nom2	14.4	16.5*	12.8	19.6**	64.6***	128.3***	193.3***	206.9***	200.3***
all2	19.7**	12.6	17.4**	24.0***	70.6***	138.0***	191.8***	218.6***	196.1***
nopsi	12.2	12.3	15.1*	25.3***	72.8***	139.2***	198.9***	215.5***	209.5***
nodel	20.0**	14.9*	9.8	20.6**	46.2***	104.4***	187.1***	237.0***	224.4***
nomu	16.9*	17.1**	12.4	22.6***	64.6***	129.2***	193.3***	218.6***	193.6***
				inflation					
	1	2	3	4	8	12	16	24	32
nom2	16.3*	21.3**	37.1***	40.5***	57.4***	69.5***	78.2***	82.5***	75.3***
all2	27.5***	29.3***	46.3***	53.3***	57.4***	87.1***	95.3***	73.3***	87.8***
nopsi	33.1***	33.5***	47.6***	55.0***	63.8***	75.4***	83.0***	70.2***	69.5***
nodel	25.9***	28.0***	39.4***	51.3***	53.3***	81.7***	92.9***	77.0***	81.2***
nomu	16.9*	23.2***	40.7***	43.9***	49.6***	77.9***	78.7***	78.2***	84.5***
				Interest rate					
	1	2	3	4	8	12	16	24	32
nom2	50.0***	28.3***	21.0**	21.3**	24.4***	21.6**	33.7***	45.6***	62.8***
all2	47.2***	32.5***	19.0**	23.0***	22.9***	29.2***	37.5***	29.6***	51.2***
nopsi	53.1***	28.0***	18. **	23.3***	23.7***	31.7***	39.4***	29.0***	39.5***
nodel	60.9***	30.3***	19.3**	26.0***	18.0**	27.5***	34.7***	29.0***	37.9***
nomu	49.4***	28.0***	19.7**	22.3***	23.3***	21.6**	47.0***	51.1***	54.5***

*Note: I report the value test statistics for each and forecasting horizon. ***, ** and * denote the 1%, 5% and 10% significance levels for the rejection of the null of the Pearson χ^2 test.*