An Economic Model for the Interpretation of Business Cycles & the Efficiency of Monetary Policy

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Abstract

This paper attempts to investigate business cycles, assuming that both the national income and the interest rate on loans are determined jointly in the product market and the banking sector. For this reason, a second order accelerator model in discrete time is combined with a two-stage Cournot game with scope economies for the oligopolistic banking sector. In addition, the model is calibrated to assess the ability of our system to interpret the cyclical path of national income over time and the possibility of the latter's convergence towards its steady-state. Performing a simulation process, we present the implications of different permanent shocks of monetary policy on national income diachronically.

Key Words: Business Cycles, Bank Behavior, Second-Order Accelerator, Monetary Policy, Scope Economies

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1. Introduction

The present paper establishes a model for the interpretation of business cycles, concentrating on the assumption of the simultaneous determination of national income and interest rate on loans in both the product market and the banking sector. To begin with the product market, we extend the Samuelson's (1939) multiplier-accelerator model, incorporating the second order accelerator for fixed investment in discrete time (Hillinger 1992, 2005) in it. On the other hand, banking sector is described by a two-stage Cournot model with scope economies. Our purpose is the investigation of the effects of different shocks of monetary policy on the time path of national income.

Business cycles are eminently dynamic phenomena to which many different definitions have attributed. Generally, they are considered as the periodic but irregular movement in economic activity, measured in terms of GDP or other macroeconomic variables. The first attempt to modeling business cycles was done by Tinbergen (1930), who built a model of industrial investment cycle. Assuming a time lag between the inception and the completion of an investment project in shipping industry, in fact this model introduced the time to build approach. Frisch (1933) emphasizes on the macroeconomic aspect of business cycles. He claims that random shocks are able to disturb economic activity.

Theories of business cycles can be divided into five schools (Arnold 2002), namely *Keynesian Economics*, *Monetarism*, *New Classical Economics*, *Real Business Cycles* and *New Keynesian Economics*. Our interest is on *Keynesian Economics* that involve models that interpret economic disturbances from the aspect of aggregate demand. The centerpiece in such theories is the notion of the accelerator. Keynes (1936) explains the occurrence of business cycles via disturbances in private consumption and private investment. Samuelson (1939) constructs a multiplier-accelerator model of income determination. Hillinger (1992, 2005) derives a second order accelerator model for fixed investment and inventories in continuous time. Hillinger & Weser (1988) and Weser (1992) use this model to study the aggregation problem in business cycles theory.

From the other schools of business cycles, *Monetarism*, *New Classical Economics* and *Real Business Cycles* argue that there is no need for governmental intervention as economy is inherently stable. More specifically, according to Monetarism, the disturbances in economic activity are triggered by random shocks (Laidler 1976). On the other hand, both the supporters of *New Classical Economics* and the supporters of

Real Business Cycles adopt the assumption of rational expectations. The difference between these approaches is that the former emphasizes on the importance of monetary shocks (Lucas 1975) while the latter (Kydland & Prescott 1982) argues that real shocks are more significant. Finally, *New Keynesian Economics* shift their interest from the cause of economic disturbances to their implications (Tobin 1993).

Moving now into the banking sector, several researches adopt the Industrial Organization approach to banking, treating banks as firms that attempt to maximize their profits. The Klein-Monti model (1971) was the first to introduce this concept. Dalla & Varelas (2013) examine the effects of monetary policy on the optimal behavior of a monopolistic bank. Under the assumptions of symmetric costs and symmetric conduct, Freixas & Rochet (2008) show that, in the context of a Cournot game with finite number of banks, an increase in the interbank rate results in an increase in both the optimal interest rate on loans and deposits. In addition, Toolsema & Schoonbeek (1999) apply a similar model for the cases of asymmetry in the cost function (Cournot game) and asymmetry in the banking conduct (Stackelberg game).

Yamazaki & Miyamoto model (2004) constitutes an extension of the above models, introducing the notion of scope economies in a two-stage Cournot game. Applying this model and assuming an overlapping generation model, Varelas (2007) analyzes the effects of monetary policy via the interbank rate on the bank clients' consumption. In the same manner, Dalla et al. (2014) emphasize on the effects of monetary policy via the minimum reserve requirements on the interest rate spread.

Our paper is structured as follows. Section 2 presents the theoretical model while section 3 provides the solution. Section 4 and section 5 include the calibration and the simulation process respectively. Section 6 examines the efficiency and the implications of monetary policy on the time path of national income. Section 7 concludes.

2. The Theoretical Model

Our structural model consists of (15) equations. Relations (1) ~ (7) describe the product market. In particular, they compose a second order accelerator model in discrete time for the case of a closed economy. The price level is assumed stable over time. On the other hand, the oligopolistic banking sector is determined by equations (8) ~ (15). In fact, this is a two stage Cournot game with scope economies, assuming that there are just two banks, A and B, that operate both on the markets for loans and deposits. We continue, presenting this structural model.

$$I_t - I_{t-1} = c\left(I_t^* - I_{t-1}\right), \quad 0 < c < 1 \quad \mathbf{(1)}$$

$$I_t^* = b(K_t^* - K_{t-1}) + dr_{Lt} \quad b > 0, d < 0 \quad \mathbf{(2)}$$

$$I_t = K_t - K_{t-1} \quad \mathbf{(3)}$$

$$C_t = a_0 + a_1 Y_{t-1}, \quad a_0 > 0, 0 < a_1 < 1 \quad \mathbf{(4)}$$

$$Y_t = AK_t, \quad A > 0 \quad \mathbf{(5)}$$

$$Y_t = C_t + I_t + \Delta_t \quad \mathbf{(6)}$$

$$\Delta_t = \delta K_{t-1}, \quad \delta > 0 \quad \mathbf{(7)}$$

$$L_t = L_{At} + L_{Bt} \quad \mathbf{(8)}$$

$$D_t = D_{At} + D_{Bt} \quad \mathbf{(9)}$$

$$r_{Lt} = r_{Lt}(L_t, Y_t) = \mu_1 Y_t - b_1 L_t, \quad \mu_1, b_1 > 0 \ \& \ r_{Lt}(L_t)' < 0 \quad \mathbf{(10)}$$

$$r_{Dt} = r_{Dt}(D_t) = \beta_1 + \gamma D_t, \quad \beta_1, \gamma > 0 \ \& \ r_{Dt}(D_t)' > 0 \quad \mathbf{(11)}$$

$$\Pi_{it}(L_{it}, D_{it}) = r_{Lt}(L_t, Y_t) \cdot L_{it} + r \cdot M_{it} - r_{Dt}(D_t) \cdot D_{it} - C_{it}(L_{it}, D_{it}), \quad i = A, B \quad \mathbf{(12)}$$

$$M_{it} = (1-a) \cdot D_{it} - L_{it}, \quad i = A, B, \quad a \in (0,1) \quad \mathbf{(13)}$$

$$C_{it}(L_{it}, D_{it}) = \theta(D_{it}) \cdot L_{it} + \varphi D_{it}, \quad \theta(D_{it}) > 0, \varphi > 0, i = A, B \quad \mathbf{(14)}$$

$$\theta\left(D_{it}\right) = \kappa D_{it} + m, \quad \kappa \in \Re \ \ m > 0, i = A, B \quad \mathbf{(15)}$$

Relations (1) ~ (3) constitute a second order accelerator model for fixed investment (Hillinger 1992, 2005) in discrete time. More specifically, equation (1) represents the partial adjustment mechanism for net investment. It shows that net investment is adjusted towards the desired level of investment $\left(I_t^*\right)$ partially. The coefficient $c \in \left(0,1\right)$ is the speed of adjustment. The closer to the one is the value of *c*, the faster is the adjustment of net investment in the present period. Conversely, if $c = 0$, there is no adjustment. The main intuition behind the partial adjustment is the existence of adjustment costs. More specifically, if adjustment costs do not exist $(c=1)$, net investment (I_t) adjusts fully to its desired level (I_t^*), that is $I_t = I_t^*$. In this model, the presence of adjustment costs results in the partial adjustment of net investment (I_t) towards its desired level (I_t^*), which means that $\boldsymbol{I_t - I_{t-1} = c\left(I_t^* - I_{t-1}\right)}$.

Relation (2) is a behavioral equation. It expresses the desired level of net investment (I_t^*) as a positive function of the difference between the desired level of capital (K_t^*) and the actual value of capital with a one-period lag (K_{t-1}) and a negative function of the interest rate on loans (r_{Lt}). At this point, it is necessary to mention that the introduction of the time pattern of the investment expenditure leads to a time lag in the transformation of this expenditure into capital. In particular, we follow the time –to- build and time- to- plan approaches according to which the costs of investment projects are incurred with time lags and become productive only when the project is complete. It should be clear that the existence of adjustment lags is in no sense an indication of irrational behavior. There are costs incurred if the various lags are shortened and other costs if they are lengthened. Under the assumption of a finite time path, we presume that the desired level of capital is stable. This allows the notation of the desired level of capital with K^* for the rest of our analysis.

Equation (3) is the definition of net investment. Net investment (I_t) is defined as the change in stock of capital $(K_t - K_{t-1})$. Moreover, relations (4) and (5) describe the consumption function and the production function respectively. Consumption is expressed as a function of the value of national income with a period lag (Y_{t-1}). The parameter a_0 denotes the autonomous consumption and it is positive while the parameter a_1, that denotes the marginal propensity to consume, takes values in the interval (0,1). On the other hand, total product (Y_t) is given as a positive function of the actual value of capital (K_t), where $A>0$ denotes the parameter of technology. In fact, this kind of production function is called “AK” model and assumes that the only factor of production is capital. Consequently, there is no substitution with labor. In such a model, the parameter of technology (*A*) is equal to both the average and the marginal product of capital.

Relation (6) constitutes the identity of national income. As we can see, national income $\left(Y_t\right)$ is equal to the sum of private consumption $\left(C_t\right)$ and private gross investment $\left(I_t + \Delta_t\right)$. According to equation (7), depreciation (Δ_t) is defined as a positive fraction $\left(\delta\right)$of capital with a time lag $\left(K_{t-1}\right)$.

Equations (8) and (9) describe the total volumes of loans and deposits respectively, where L_{it} and D_{it} denote the individual amounts of loans and deposits of each bank. In addition, equation (10) is the inverse demand function for loans. The interest rate on loans $\left(r_{Lt}\right)$ is expressed as a negative function of total quantity for loans $\left(L_t\right)$ and a positive function of national income $\left(Y_t\right)$. The parameters μ_1 & b_1 are positive. This functional form of (10) constitutes a specification of the inverse demand function for loans in Varelas (2007) and Dalla et al. (2014). In particular, in this paper we determine the shifting factor of demand function, that is the national income. Therefore, an increase in national income shifts the demand function right ceteris paribus, while a decrease in national income leads to a reduction in the demand for loans, shifting the corresponding curve left ceteris paribus.

Similarly, the inverse supply function for deposits is given by equation (11). The interest rate on deposits $\left(r_{Dt}\right)$ is a positive function of the total amount of deposits $\left(D_t\right)$. Equation (12) describes the profit function of the individual bank. The profit of bank i is calculated as the difference between total revenues and total cost of this bank. In particular, total revenues derive from the interest rate on loans $\left(r_{Lt}\right)$ and the exogenous interbank rate $\left(r\right)$ if the net position of the bank $\left(M_{it}\right)$ is positive. On the other hand, total cost originates from the interest rate on deposits $\left(r_{Dt}\right)$ paid on depositors and the cost function $\left(C_{it}\left(.\right)\right)$. Indeed, the exogenous interbank rate $\left(r\right)$ is also included in total costs if the net position of the bank $\left(M_{it}\right)$ is negative.

Equation (13) presents the net position of bank i in the interbank market which is assumed to be linear. The fraction of reserve requirements $\left(a \in \left(0,1\right)\right)$ constitutes an exogenous instrument of monetary policy. On the contrary, the cost function of bank i is given by equation (14) and is assumed to be non-linear, continuous and differentiable of any order. The function $\theta(D_{it}) > 0$ denotes the marginal cost of loans while $\varphi > 0$ is the average cost of deposits.

Equation (15) describes the functional form of marginal cost of loans $(\theta(D_{it}))$. The first derivative of this function with respect to the quantity of deposits of bank i is equal to κ and can take any real value. The sign of κ determines the kind of scope economies (Baumol et al. 1982). According to the definition of scope economies, scope economies exist when the joint offer of deposits and loans by a universal bank is more efficient than their separate offer by specialized banks, that is when $\theta'(D_{it}) = \partial^2 C_{it}(L_{it}, D_{it}) / \partial D_{it}^2 < 0$. In particular, if κ is negative, $\theta'(D_{it})$ is also negative and consequently there are economies of scope. On the other hand, if κ is positive, $\theta'(D_{it})$ is also positive, so there are diseconomies of scope. Finally, if κ is equal to zero, $\theta'(D_{it})$ is also equal to zero and no economies of scope exist. Parameter m takes positive values such that $\theta(D_{it})$ to be always positive independently from the sign of κ. This functional form of $\theta(D_{it})$ satisfies the assumption $\theta''(D_{it}) = 0$ (Varelas 2007, Dalla et al. 2014).

3. Solution of the Model

As it was mentioned above, equations (1) to (7) compose the structural system of equations for the product market. The combination of these relations implies the reduced form in product market, which is described by the following non-homogeneous second order difference equation:

$$AY_t + \left[c(b+1) - (1 + Aa_1 + \delta)\right]Y_{t-1} + (1-c)Y_{t-2} = Aa_0 + cbY^* + Acdr_{Lt} \quad \textbf{(16)}$$

Both national income (Y_t) and the interest rate on loans (r_{Lt}) are endogenous variables that are determined in the product market as well as in the banking sector. The desired level of national income (Y^*) is assumed to be stable over time due to the fact that the desired level of capital is also stable.

We continue with the solution of banks' maximization problem. The maximization problem of bank i can be stated as:

$$\max \ \Pi_{it}\left(L_{it}, D_{it}\right) = r_{Lt}\left(L_t\right) \cdot L_{it} + r \cdot M_{it} - r_{Dt}\left(D_t\right) \cdot D_{it} - C_{it}\left(L_{it}, D_{it}\right) \quad \textbf{(17)}$$

Substituting relations (8) ~ (11) and (13) ~ (14) in the previous relation, we get:

$$\begin{aligned} \max \ \Pi_{it}\left(L_{it}, D_{it}\right) = & \left[\mu_1 Y_t - b_1\left(L_{it} + L_{jt}\right) - r - \theta\left(D_{it}\right)\right] L_{it} + \\ & + \left[r\left(1-\alpha\right) - \beta_1 - \gamma\left(D_{it} + D_{jt}\right) - \varphi\right] D_{it}, \quad i, j = A, B \text{ with } i \neq j \end{aligned} \quad \textbf{(18)}$$

Following Dalla et al. (2014), we induce our analysis in the context of a two stage Cournot game. In the first stage, the banks choose the level of deposits simultaneously, while in the second stage they determine the volume of loans simultaneously. Assuming that the equilibrium constitutes a subgame perfect equilibrium and that the second stage is a well defined Nash equilibrium, we apply the backward induction method.

To begin with the solution of the second stage, the objective function of each bank can be stated as:

$$\begin{aligned} \max_{L_{it}} \ \Pi_{it}\left(L_{it}, D_{it}\right) = & \left[\mu_1 Y_t - b_1\left(L_{it} + L_{jt}\right) - r - \theta\left(D_{it}\right)\right] L_{it} + \\ & + \left[r\left(1-\alpha\right) - \beta_1 - \gamma\left(D_{it} + D_{jt}\right) - \varphi\right] D_{it}, \quad i, j = A, B \text{ with } i \neq j \end{aligned} \quad \textbf{(19)}$$

From the solution of the first order conditions of the profit maximization problem (19) for each bank, we obtain the optimal volume of loans for the individual bank in this second stage subgame. This has as follows:

$$L_{it} = \frac{\mu_1 Y_t - r - 2\theta\left(D_{it}\right) + \theta\left(D_{jt}\right)}{3b_1}, \quad i, j = A, B \ \& \ i \neq j \quad \textbf{(20)}$$

According to the backward induction method, the next step is the solution of the first stage of the game. Each bank maximizes its profit function with respect to the individual volume of deposits. The objective function is derived from the substitution of relation (20) in (18) and has the following functional form:

$$\max_{D_{it}} \Pi_{it}\left(D_{it}\right)=\frac{1}{b_1}\left[\frac{\mu_1 Y_t-2\theta\left(D_{it}\right)+\theta\left(D_{jt}\right)-r}{3}\right]^2+\left[r\left(1-a\right)-\beta_1-\gamma\left(D_{it}+D_{jt}\right)-\phi\right]D_{it}, \quad \textbf{(21)}$$

$i, j = A, B \;\&\; i \neq j$

From the solution of the first order conditions of (21) for the two banks with respect to the corresponding volume of deposits and the use of (15), we get the equilibrium level of deposits for these banks:

$$D^*_{At}=D^*_{Bt}=D^*_{st}=\frac{4\kappa\left(\mu_1 Y_t-m-r\right)-9b_1\left(r\left(1-a\right)-\beta_1-\varphi\right)}{4\kappa^2-27b_1\gamma} \quad \textbf{(22)}$$

Equation (22) shows that the equilibrium volumes of deposits for the two banks are equal. In fact, this was expected as each stage constitutes a symmetric Cournot game. At this point, it is necessary to derive the second order condition for a maximum. This has as follows:

$$\frac{\partial^2 \Pi_{it}\left(D_{it}\right)}{\partial D_{it}^2}=\frac{8}{9b_1}\kappa^2-2\gamma<0, \quad i=A,B \quad \textbf{(23)}$$

Regarding the optimal level of loans, the substitution of relation (15) and (20) in (8) and the fact that $D^*_{At}=D^*_{Bt}=D^*_{st}$ imply the optimal total level of loans, which is described by equation (24):

$$L^*_t=\frac{2\left(\mu_1 Y_t-r-\kappa D^*_{st}-m\right)}{3b_1} \quad \textbf{(24)}$$

Then, substituting (22) in (24) and then the resulting equation into the inverse demand function of loans (10), we obtain the equilibrium interest rate on loans, which depends on national income:

$$r^*_{Lt}=\Omega_1 Y_t+\Omega_2, \quad \Omega_1,\Omega_2\in\Re \quad \textbf{(25)}$$

where

$$\Omega_1 = \frac{1}{3}\left[1 + \frac{8\kappa^2}{4\kappa^2 - 27b_1\gamma}\right]\mu_1 \;\&\; \Omega_2 = \frac{2}{3}\left\{r + m + \kappa\left[\frac{-4\kappa\left(m+r\right) - 9b_1\left(r\left(1-a\right) - \beta_1 - \varphi\right)}{4\kappa^2 - 27b_1\gamma}\right]\right\}$$

The next step in our analysis is the combination of the macroeconomic aspect of this model (product market) with the microeconomic aspect (banking conduct). Our ultimate purpose is the interpretation of business cycles in terms of national income. To achieve this, we combine the reduced form in the product market (16) with the endogenous equilibrium interest rate on loans (25). In particular, after the substitution of (25) in (16), we get:

$$A\left(1 - cd\Omega_1\right)Y_t + \left[c\left(b+1\right) - \left(1 + Aa_1 + \delta\right)\right]Y_{t-1} + \left(1-c\right)Y_{t-2} = Aa_0 + cbY^* + Acd\Omega_2 \quad \textbf{(26)}$$

Equation (26) is a second order difference equation with constant coefficients. It is assumed that all the coefficients are nonzero. The general solution[3] of (26) is given by the sum of the general solution of the corresponding homogeneous difference equation and a particular solution of (26). To begin with the former, it can be interpreted as the reflection of the deviation of national income from its equilibrium. The functional form of this general solution depends on the sign of the discriminant of the characteristic equation. We can distinguish between three cases.

Firstly, if the discriminant is positive, the characteristic roots are real and linearly independent. When both roots are positive, the movement of national income is monotonic. Conversely, the path of income is improper oscillatory if the sign of both roots is negative. In any case, income converges towards its long-run equilibrium if, and only if, both roots are less than one in absolute values. Otherwise, income diverges from its equilibrium. It should be noted that if one of the roots is less than one in absolute values while the other is greater that one in absolute values, the path of income depends on the absolute value of the root with the greater absolute value. The latter is called “dominant root” and results in a divergent time path.

Secondly, when the discriminant is equal to zero, the characteristic equation possesses a real root with multiplicity two. National income converges towards its long-run equilibrium if, and only if, the absolute value of the characteristic root is less than one. Otherwise, the motion of income is divergent. Regarding the kind of this

[3] See the appendix for a thorough deduction of the model’s general solution.

movement, the convergence is monotonic if the value interval of the characteristic root is the (0,1), while is (improper) oscillatory if the value interval is (-1,0). Similarly, the divergence will be monotonic if the value interval of the root is the (1,+∞), while it will be oscillatory if the root's value belongs to the interval (-∞,-1).

Finally, under the existence of a negative discriminant, the characteristic roots are complex conjugate numbers. In this case, national income shows trigonometric oscillations with period $2\pi / \omega$. The convergence or divergence of national income from the equilibrium depends on the modulus or absolute value of the complex numbers, that takes values in the interval (0,1). In particular, when the modulus is greater than one, the amplitude of the trigonometric oscillations is increasing, leading to a divergence from the steady-state. On the other hand, if the modulus is less than one, the amplitude is decreasing and income converges toward the steady-state. Finally, if the modulus is equal to one, the resulting trigonometric oscillations have constant amplitude.

The stability of our system can also be ensured by the satisfaction of a set of necessary and sufficient conditions (Gandolfo 1995). In the case of our model, these conditions are given by the following inequalities:

$$\frac{A\left(1-cd\Omega_1-a_1\right)+cd-\delta}{A\left(1-cd\Omega_1\right)}>0,\ A\left(1-cd\Omega_1\right)\neq 0 \quad \textbf{(27)}$$

$$\frac{A\left(1-cd\Omega_1\right)-\left(1-c\right)}{A\left(1-cd\Omega_1\right)}>0,\ A\left(1-cd\Omega_1\right)\neq 0 \quad \textbf{(28)}$$

$$\frac{A\left(1-cd\Omega_1+a_1\right)-c\left(b+2\right)+\delta+2}{A\left(1-cd\Omega_1\right)}>0,\ A\left(1-cd\Omega_1\right)\neq 0 \quad \textbf{(29)}$$

The satisfaction of the conditions (27) ~ (29) requires both the nominators and the common denominator of the corresponding fractional equations to have the same sign and be nonzero. The latter is assumed nonzero as a coefficient of (26).

Regarding the particular solution of (26), it can be interpreted as the equilibrium level of national income. Applying the method of undetermined coefficients, we get:

$$\bar{Y}_t=\frac{Aa_0+Acd\Omega_2+cbY^*}{A\left(1-a_1-cd\Omega_1\right)+cb-\delta},\ A\left(1-a_1-cd\Omega_1\right)+cb-\delta\neq 0 \quad \textbf{(30)}$$

Equation (30) requires the term $A\left(1-a_1-cd\Omega_1\right)+cb-\delta$ to be nonzero. This holds in the case of stability of our system due to the inequality (27). So, the stability of our system implies the acceptance of (30) as the equilibrium state of income. Moreover, the interpretation of (30) as the steady-state of income entails that its value should be positive. Consequently, both the terms $Aa_0+Acd\Omega_2+cbY^*$ and $A\left(1-a_1-cd\Omega_1\right)+cb-\delta$ should be of the same sign. Relating this remark with the fact that the first stability condition implies that the terms $A\left(1-a_1-cd\Omega_1\right)+cb-\delta$ and $A\left(1-cd\Omega_1\right)$ should be of the same sign, we result that the terms $Aa_0+Acd\Omega_2+cbY^*$ and $A\left(1-cd\Omega_1\right)$ should also have the same sign. This notification is very useful for the simulation of our model.

The aim of this model is the determination of business cycles. For this reason, we concentrate on the case of periodic trigonometric oscillatory movement of national income that converges towards the steady-state. This kind of motion is described by the following equation which presents the general solution of our model:

$$Y_t = r^t\left(A_1\cos\omega_t + A_2\sin\omega_t\right) + \frac{Aa_0+Acd\Omega_2+cbY^*}{A\left(1-a_1-cd\Omega_1\right)+cb-\delta},\ A_1, A_2 \in \Re \quad \textbf{(31)}$$

where A_1, A_2 are arbitrary constants which can be determined using initial conditions, *r* denotes the modulus or absolute value of the conjugate complex characteristic roots and $A(1-a_1-cd\Omega_1)+cb-\delta \neq 0$.

4. Calibration

In this section, we present the numerical values that are given to the parameters in order to simulate our model and examine the effects of monetary policy. We follow the method of Karpetis & Varelas (2005, 2012), assigning random values to the parameters and taking into consideration their value intervals in the theoretical model. Firstly, we describe the values of the macroeconomic parameters and the policy variables and then the values of the parameters of banking sector.

To begin with the speed of adjustment (c), its value interval is the (0,1). We choose the value 0.4 that corresponds to a slow adjustment of net investment towards its desired level. In addition, the behavioral parameter *b* is set equal to 1. Given the desired level of capital, $b=1$ means that an increase in the present period's capital leads to a proportional decrease in the desired level of net investment with one period lead ceteris paribus. The parameter *d*, that shows the negative relation between the interest rate on loans and the desired level of net investment, is set equal to -0.3<0. Regarding the parameters of the consumption function (4), we set the autonomous consumption $\left(a_0\right)$ equal to 20 and the marginal propensity to consume $\left(a_1\right)$ equal to 0.75. The latter shows that a marginal increase in the national income with a period lag results in an increase in aggregate consumption by 0.75 units and a decrease in savings by 0.25 units. This is the minimum value that is assigned to marginal propensity to consume in empirical researches.

Concerning the parameters of technology, *A* is set equal to 2 while the depreciation rate of capital (δ) is assigned to 0.2. Setting the desired level of capital equal to 150 $\left(K^*=150\right)$, the resulting desired level of national income from (5) is equal to 300 units $\left(Y^*=300\right)$.

Moving now to the policy variables, we designate both the fraction of minimum reserve requirements (α) and the interbank rate (r) equal to 0.1. Later in this paper, we are going to change this value, in order to examine the effects of alternative monetary policies on national income over time. The following table summarizes the values that are given to the macroeconomic parameters and the policy parameters of our model.

c	b	d	a_0	a_1	A	δ	K^*	Y^*	α	r
0.4	1	-0.3	20	0.75	2	0.2	150	300	0.1	0.1

Table 1: Calibration of the macroeconomic parameters and the parameters of policy

Regarding the parameters of the banking sector's aspect of our model, we begin with the inverse demand function for loans. We set μ_1 equal to 1.5>0 to show that a marginal increase in national income result in an increase of the interest rate on loans by 1.5 units. Moreover, we assign the value 0.5 to the parameter b_1, so the slope of the demand function for loans is equal to -0.5. The parameters of the inverse supply function for deposits, β_1 & γ, are determined to 30 and 1.2 respectively. The latter indicates that the slope of the supply function for deposits is positive and equal to 1.2.

Turning to the parameters of cost function, the average cost of deposits is chosen equal to 10 $(\varphi = 10)$ while the parameter *m* is set equal to 200.

The remaining parameter κ, the sign of which determines the kind of scope economies, is crucial in our analysis. In order to calibrate it, we take into consideration the second order condition for a maximum (23). So, given the values of the parameters $b_1 = 0.5 \,\&\, \gamma = 1.2$, the satisfaction of (23) requires the following inequality to hold:

$$P(\kappa) = \frac{16}{9}\kappa^2 - 2.4 < 0 \quad \textbf{(32)}$$

Table 2 presents the sign of the polynomial $P(\kappa)$ for different value intervals of κ. It is understandable, though, that (32) holds only if the value interval of κ is the $(-1.162, 1.162)$. Consequently, the parameter κ is assumed equal to -1.15 for the rest of our analysis. Choosing a negative value of κ, in fact we accept the existence of economies of scope.

	Economies of Scope		**No Scope Economies**	**Diseconomies of Score**	
κ	$(-\infty, -1.162)$	$(-1.162, 0)$	0	$(0, 1.162)$	$(1.162, +\infty)$
$P(\kappa)$	+	-	-	-	+
$P(\kappa) = 0$ for $\kappa = -1.162 \,\&\, \kappa = 1.162$					

Table 2: Sign table of P(κ)

The following table shows the given values to the banking sector's parameters.

μ_1	***b₁***	β_1	γ	**φ**	***m***	**κ**
1.5	0.5	30	1.2	10	200	-1.15

Table 3: Calibration of the banking sector's parameters

5. Simulation[4]

Using the values discussed in the previous section, we solve our model in order to confirm the dynamic properties of the system. Starting from the derivation of the general solution of the homogeneous difference equation that corresponds to (26), the latter is given by:

$$2.0036296Y_t - 1.9Y_{t-1} + 0.6Y_{t-2} = 0 \quad \textbf{(33)}$$

The characteristic equation that is obtained by (33) is a second order equation and has the following functional form:

$$2.0036296\mu^2 - 1.9\mu + 0.6 = 0 \quad \textbf{(34)}$$

The solution of (34) requires the calculation of the discriminant, which is equal to $\Delta = -1.19871 < 0$. Due to the fact that the discriminant is negative, it is inferred that national income follows a trigonometric oscillatory path. The characteristic roots are conjugate complex numbers whose values are:

$$\mu_1 = 0.4741395 + 0.2732181i \ \& \ \mu_2 = 0.4741395 - 0.2732181i \quad \textbf{(35)}$$

where $i = \sqrt{-1}$ the imaginary unit.

Using the notation of Chiang (1984), we assume that $h = 0.4741395$ & $u = 0.2732181$. Then, the modulus or absolute value of the complex oots is given by:

$$r = \sqrt{h^2 + u^2} = \sqrt{(0.4741395)^2 + (0.2732181)^2} = 0.547226 < 1 \quad \textbf{(36)}$$

As it was discussed before, the convergence or divergence of national income from its steady-state depends on the modulus of the complex characteristic roots. Due to the fact that the latter is equal to $0.547226 < 1$, national income follows a trigonometric oscillatory path with decreasing amplitude, so it convergences towards its steady-state.

The general solution of (33) has the following mathematical form:

$$Y_t = (0.547226)^t \left(A_1 \cos \omega t + A_2 \sin \omega t\right),\ A_1, A_2 \in \Re \quad \textbf{(37)}$$

[4] The simulation results that are presented in this section were derived using the program Wolfram Mathematica 9.0.

Using the following trigonometric functions (Chiang 1984), we get:

$$\cos\omega = \frac{h}{r} = \sqrt{3}/2$$

$$\sin\omega = \frac{u}{r} = 1/2$$

From the trigonometric tables, we infer that the angle with $\cos\omega = \sqrt{3}/2 \,\&\, \sin\omega = 1/2$ is the angle of 30^{o} or $\omega = \pi/6$ rads. Consequently, equation (37) can be stated as:

$$Y_t = (0.547226)^t \left(A_1 \cos\frac{\pi}{6}t + A_2 \sin\frac{\pi}{6}t \right),\ A_1, A_2 \in \Re \quad \textbf{(38)}$$

The equilibrium level of national income (initial steady state), as this is obtained after substituting the numerical values of the parameters in equation (30), constitutes a partial solution of the non-homogeneous difference equation and is equal to $\overline{Y}_t = 155.524$. Therefore, the general solution of our model (equation 31) is given by:

$$Y_t = (0.547226)^t \left(A_1 \cos\frac{\pi}{6}t + A_2 \sin\frac{\pi}{6}t \right) + 155.524,\ A_1, A_2 \in \Re \quad \textbf{(39)}$$

Assuming that $Y_0 = 150 \,\&\, Y_1 = 200$, we obtain the numerical values of the constants A_1 and A_2. These are $A_1 = -5.524 \,\&\, A_2 = 172.199$. Hence, the general solution of our model can be determined totally as follows:

$$Y_t = (0.547226)^t \left((-5.524)\cos\frac{\pi}{6}t + (172.119)\sin\frac{\pi}{6}t \right) + 155.524 \quad \textbf{(40)}$$

Equation (40) describes the motion of national income over time. It can be clearly seen, though, that national income shows trigonometric oscillations that converge towards the steady-state with period equal to $2\pi/\omega = 2\pi/(\pi/6) = 12$. Figure 1 depicts the path of national income over time.

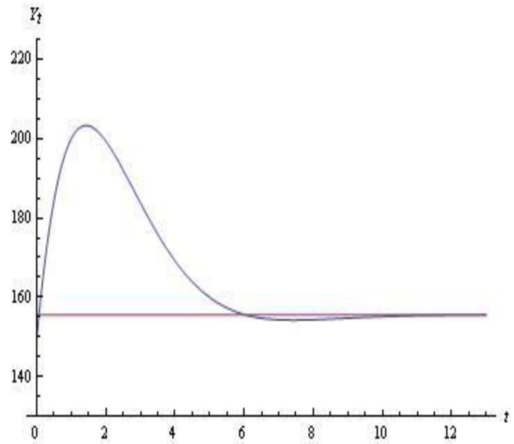

Figure 1: The motion of national income over time

The stability of our system can be also ensured by the stability conditions, as these are expressed by the inequalities (27) ~ (29). Substituting the values of the parameters in the aforementioned inequalities, we get:

$$(27) \Rightarrow \frac{A(1-cd\Omega_1-a_1)+cd-\delta}{A(1-cd\Omega_1)}=0.351178>0$$

$$(28) \Rightarrow \frac{A(1-cd\Omega_1)-(1-c)}{A(1-cd\Omega_1)}=0.700543>0$$

$$(29) \Rightarrow \frac{A(1-cd\Omega_1+a_1)-c(b+2)+\delta+2}{A(1-cd\Omega_1)}=2.24774>0$$

The satisfaction of the stability conditions ensures the convergence of national income towards the steady-state.

Concluding, our model achieves to interpret the existence of business cycles in terms of national income in the case of small economies of scope $(\kappa=-1.15)$. Table 4 presents the values of the basic variables of the present model for 25 periods.

6. Monetary Policy Implications[5]

In this section we examine the effects of monetary policy on national income. Taking into consideration that our model is deterministic, which implies full information, no uncertainty and perfect foresight, we present the transition path of this variable from

[5] The results of monetary policy were deduced using the program Matlab R2008a.

t	Y_t	C_t	I_t	K_t	Δ_t	r_{Lt}
1	200	132.5	52.5001	100	15	213.728
2	199.334	170	9.33359	99.6668	20	213.718
3	183.729	169.5	-5.70445	91.8646	19.9334	213.482
4	169.138	157.797	-7.03136	84.5692	18.3729	213.261
5	159.982	146.854	-3.78581	79.9909	16.9138	213.123
6	155.672	139.986	-0.312248	77.8362	15.9982	213.057
7	154.33	136.754	2.00817	77.1648	15.5672	213.037
8	154.348	135.747	3.16735	77.1738	15.433	213.037
9	154.767	135.761	3.57117	77.3833	15.4348	213.044
10	155.158	136.075	3.6068	77.5792	15.4767	213.05
11	155.404	136.369	3.51965	77.7021	15.5158	213.053
12	155.52	136.553	3.42637	77.76	15.5404	213.055
13	155.556	136.64	3.36406	77.778	15.552	213.056
14	155.556	136.667	3.33293	77.7778	15.5556	213.056
15	155.544	136.667	3.32209	77.7722	15.5556	213.055
16	155.534	136.658	3.32113	77.7669	15.5544	213.055
17	155.527	136.65	3.32347	77.7636	15.5534	213.055
18	155.524	136.645	3.32597	77.7621	15.5527	213.055
19	155.523	136.643	3.32765	77.7616	15.5524	213.055
20	155.523	136.642	3.32848	77.7616	15.5523	213.055
21	155.523	136.642	3.32877	77.7617	15.5523	213.055
22	155.524	136.643	3.3288	77.7619	15.5523	213.055
23	155.524	136.643	3.32874	77.762	15.5524	213.055
24	155.524	136.643	3.32867	77.762	15.5524	213.055
25	155.524	136.643	3.32863	77.762	15.5524	213.055

Table 4: Values of the basic variables for 25 periods.

the initial steady-state to a new steady state after a permanent shock of monetary policy. Table 5 summarizes the various changes in monetary policy that are examined. Both the restrictive and the expansionary monetary policy concern equal increases and decreases in minimum reserve requirements (α) and interbank rate (r) respectively.

Monetary Policy	***α***	***r***
Initial	0.1	0.1
Monetary Policy via *α*		
Restrictive	**0.15**	0.1
Expansionary	**0.05**	0.1
Monetary Policy via *r*		
Restrictive	0.1	**0.15**
Expansionary	0.1	**0.05**

Table 5: Alternative monetary policies

6.1 Monetary Policy via the Minimum Reserve Requirements

Let the economy lie on the steady-state, as this was deduced in the previous section. At t=1, Central Bank decides to implement restrictive monetary policy, using the minimum reserve requirements as an instrument, in order to restrict the economic activity. This policy involves an increase in α from 0.1 to 0.15. In fact, this is a permanent shock of monetary policy that results in the convergence of the system towards a new steady-state.

Concentrating on the implications of the aforementioned shock on national income, figure 2.1 presents the transition path of national income from the initial steady-state. Starting from the level of 155.524 at t=1, the increase in α leads to a sharp reduction of national income to the level of 155.5236 at t=3. Then, national income continues decreasing with slower rate, reaching at the new steady-state, at the level of 155.523, at t=6.

Moving now into the case of expansionary monetary policy via the minimum reserve requirements, Central Bank reduces the fraction of minimum reserve requirements to 0.05. Such an expansionary policy attempts to the amplification of the aggregate demand and economic activity as a whole. Let the shock of monetary policy occur at t=1. The reduction in α results in the increase of national income from 155.524 to 155.52435 at t=3. Following a degressive increasing path over time, the variable reaches at 155.525 which constitutes the new steady-state at t=6 (Figure 2.2).

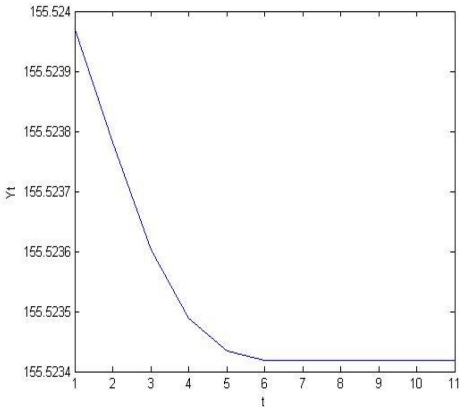

Figure 2.1: Transition path of national income after an increase in α by 50%.

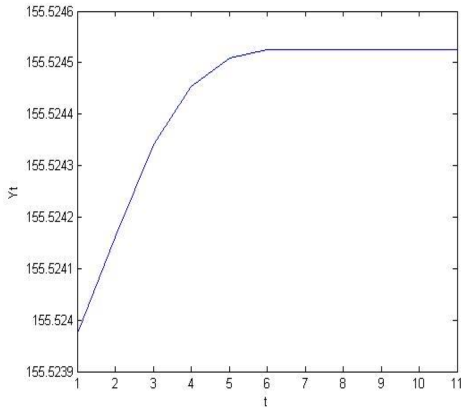

Figure 2.2: Transition path of national income after a decrease in α by 50%.

Table 6 summarizes the initial steady-states as well as the new steady-states of the basic variables of our model that achieved after the monetary policy shocks that we have discussed.

Variables	**Initial Steady-State**	**Steady-State after Restrictive Monetary Policy**	**Steady-State after Expansionary Monetary Policy**
$\bar{Y}$	155.524	155.523	155.525
$\bar{C}$	136.643	136.6425	136.6434
$\bar{I}$	3.3286	3.32851	3.32868
$\bar{K}$	77.762	77.7617	77.7623
$\bar{\Delta}$	15.5524	15.5523	15.5525
$\bar{r}_L$	213.055	213.057	213.054

Table 6: Initial steady-state and steady-states after monetary policy via α

6.2 Monetary Policy via the Interbank Rate

Let the economy lie on the steady-state, as this was derived initially. We assume that Central Bank decides to implement monetary policy using the interbank rate (r) as an instrument. Firstly, we examine restrictive monetary policy via an increase in the interbank rate from 0.1 to 0.15 (figure 3.1). Secondly, we investigate expansionary monetary policy via a decrease in *r* from 0.1 to 0.05 (figure 3.2). In any case, monetary policy is realized as a permanent shock that results in a new steady- state. Specifically, in the first case, national income dipped from 155.524 to 155.512 at t=6. On the contrary, in the second case, national income increases at a decreasing rate, reaching at 155.536 at t=6.

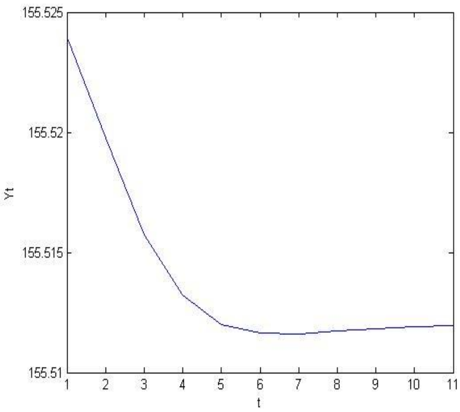

Figure 3.1: Transition path of national income after an increase in *r* by 50%.

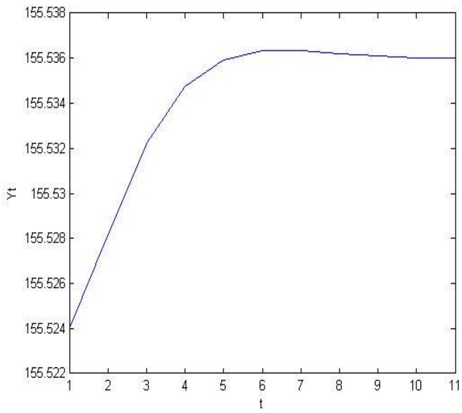

Figure 3.2: Transition path of national income after a decrease in *r* by 50%.

Table 7 shows the initial steady-states as well as the new steady-states of the basic variables of our model after the monetary shocks discussed in this subsection.

Variables	**Initial Steady-State**	**Steady-State after Restrictive Monetary Policy**	**Steady-State after Expansionary Monetary Policy**
$\bar{Y}$	155.524	155.512	155.536
$\bar{C}$	136.643	136.634	136.652
$\bar{I}$	3.3286	3.32679	3.3304
$\bar{K}$	77.762	77.756	77.768
$\bar{\Delta}$	15.5524	15.5512	15.5536
$\bar{r}_L$	213.055	213.09	213.02

Table 7: Initial steady-state and steady-states after monetary policy via *r*

To conclude, our analysis proves that monetary policy is efficient in the context of the present model, for the case of small economies of scope $(\kappa = -1.15)$. But is monetary policy efficient in the case of diseconomies of scope as well? In order to answer to this question, we have to take into consideration the intuition behind the efficiency of monetary policy. More specifically, the efficiency of monetary policy implies that expansionary (restrictive) permanent monetary policy should lead to a new higher (lower) steady-state of national income, regardless the instrument that is implemented. Consequently, the first derivative of (30) with respect to α and r respectively should be negative, given the values of all the parameters except from κ. The next table summarizes the sign of the aforementioned derivatives for the different value intervals of κ.

	Economies of Scope				Diseconomies of Scope		
κ	$(-\infty,-4)$	$(-4,-2.012)$	$(-2.012,-1.77)$	$(-1.77,0)$	$(0,1.77)$	$(1.77,2.012)$	$(2.012,+\infty)$
$\dfrac{\partial \overline{Y}}{\partial a}$	+	+	+	-	+	-	-
$\dfrac{\partial \overline{Y}}{\partial r}$	-	+	+	-	-	+	+
$\partial \overline{Y} / \partial a = 0$ for $\kappa = 0$ (No Economies of Scope) $\partial \overline{Y} / \partial r = 0$ for $\kappa = -4$ (Economies of Scope)							

Table 8: Efficiency of monetary policy

It can be clearly seen that both monetary policy via α and monetary policy via r are efficient only for values of κ in the interval $(-1.77, 0)$, i.e. in the case of small economies of scope. Taking into consideration table 2, we infer that the value interval of κ for which not only the condition (23) is satisfied but also monetary policy is efficient regardless the instrument, given the other parameters' values, is the $(-1.162, 0)$.

7. Conclusion

In this paper we examined business cycles based on the simultaneous determination of national income and interest rate on loans in both the product market and the oligopolistic banking sector. Firstly, we formulated a macroeconomic model which consists of a second order accelerator model for national income in discrete time and a two-stage Cournot game with scope economies for the oligopolistic banking sector. The solution of this model yielded the behavior over time of national income. After calibrating our model, we followed a simulation process to confirm the dynamic properties of our system and its ability to generate cycles. It was inferred that, given the assigned values to the parameters, national income follows a trigonometric oscillatory path of 12 periods, converging towards its steady-state.

Moreover, we examined the effects of different shocks of monetary policy on national income diachronically. We showed that, in the case of small economies of scope, both monetary policy via the interbank rate and monetary policy via the minimum reserve requirements are efficient, leading to a new steady state. However, in the case of diseconomies of scope, the interbank rate is the only efficient instrument of monetary policy for any value of κ in the accepted, according to the second order condition (23), interval $\left(0, 1.162\right)$.

Appendix

The basic difference equation of our model is given by equation (26). The solution of this equation requires the determination of the general solution of the corresponding homogeneous difference equation and the derivation of a particular solution of the non-homogeneous equation (26).

The homogeneous equation that is obtained by (26) has the following mathematical form:

$$A\left(1-cd\Omega_1\right)Y_t+\left[c\left(b+1\right)-\left(1+Aa_1+\delta\right)\right]Y_{t-1}+\left(1-c\right)Y_{t-2}=0 \quad \textbf{(A.1)}$$

Let μ^t be a solution of equation (A.1). Substituting into (A.1), we get the so-called characteristic equation:

$$A\left(1-cd\Omega_1\right)\mu^2+\left[c\left(b+1\right)-\left(1+Aa_1+\delta\right)\right]\mu+\left(1-c\right)=0 \quad \textbf{(A.2)}$$

We proceed to the solution of this second order equation, calculating first the discriminant which is equal to:

$$\Delta = c(b+1)\left[c(b+1) - 2(1+Aa_1+\delta)\right] + (1+Aa_1+\delta)^2 - 4A(1-cd\Omega_1)(1-c) \quad \textbf{(A.3)}$$

We can distinguish between three cases:

Firstly, if the discriminant is positive, (A.2) possesses two real and linearly independent roots. These are:

$$\mu_{1,2} = \frac{-\left[c(b+1) - (1+Aa_1+\delta)\right] \pm \sqrt{\Delta}}{2A(1-cd\Omega_1)} \quad \textbf{(A.4)}$$

The general solution of the homogeneous equation (A.2) is given by:

$$Y_t = A_1\left[\frac{-\left[c(b+1)-(1+Aa_1+\delta)\right]+\sqrt{\Delta}}{2A(1-cd\Omega_1)}\right]^t + A_2\left[\frac{-\left[c(b+1)-(1+Aa_1+\delta)\right]-\sqrt{\Delta}}{2A(1-cd\Omega_1)}\right]^t, A_1, A_2 \in \Re \quad \textbf{(A.5)}$$

where $A_1, A_2 \in \Re$ are arbitrary constants which can be determined using initial conditions.

Secondly, if the discriminant is equal to zero, (A.2) has a real root with multiplicity two. This is given by:

$$\mu_0 = -\frac{c(b+1)-(1+Aa_1+\delta)}{2A(1-cd\Omega_1)} \quad \textbf{(A.6)}$$

Then, the general solution of (A.2) is described by:

$$Y_t = (A_1 + A_2 t)\left[\frac{-\left[c(b+1)-(1+Aa_1+\delta)\right]}{2A(1-cd\Omega_1)}\right]^t, A_1, A_2 \in \Re \quad \textbf{(A.7)}$$

where $A_1, A_2 \in \Re$ are arbitrary constants which can be determined using initial conditions.

Thirdly, if the discriminant is negative, the characteristic roots are two conjugates complex numbers. These are:

$$\mu_{1,2} = -\frac{c(b+1)-(1+Aa_1+\delta)}{2A(1-cd\Omega_1)} \pm \frac{\sqrt{|\Delta|}}{2A(1-cd\Omega_1)} i \quad \textbf{(A.8)}$$

In this case, the general solution of (A.2) is:

$$Y_t = r^t \left(A_1 \cos \omega t + A_2 \sin \omega t \right),\ A_1, A_2 \in \Re \quad \textbf{(A.9)}$$

where $A_1, A_2 \in \Re$ are arbitrary constants which can be determined using initial conditions and r denotes the modulus or absolute value of the conjugate complex characteristic roots which is calculated as follows:

$$r = \sqrt{\left[-\frac{c(b+1)-(1+Aa_1+\delta)}{2A(1-cd\Omega_1)} \right]^2 + \left[\frac{\sqrt{|\Delta|}}{2A(1-cd\Omega_1)} \right]^2} = \sqrt{\frac{1-c}{A(1-cd\Omega_1)}} \quad \textbf{(A.10)}$$

Moving now into the particular solution of (26), we apply the general method of undetermined coefficients. Let $\bar{Y}_t = \Gamma$, where $\Gamma \in \Re$ an undetermined coefficient. Substituting $\bar{Y}_t = \Gamma$ in (26) and solving with respect to *Γ*, we obtain:

$$\Gamma = \frac{Aa_0 + Acd\Omega_2 + cbY^*}{A(1-a_1-cd\Omega_1)+cb-\delta},\ A(1-a_1-cd\Omega_1)+cb-\delta \neq 0$$

As it was discussed before, the constraint $A(1-a_1-cd\Omega_1)+cb-\delta \neq 0$ is satisfied in the case of stability due to (27). So, in this case the particular solution of (26) is given by:

$$\bar{Y}_t = \frac{Aa_0 + Acd\Omega_2 + cbY^*}{A(1-a_1-cd\Omega_1)+cb-\delta},\ A(1-a_1-cd\Omega_1)+cb-\delta \neq 0 \quad \textbf{(A.11)}$$

However, in the case of instability, it is also possible that $A(1-a_1-cd\Omega_1)+cb-\delta = 0$. Then, we assume that $\bar{Y}_t = \Gamma t$, where $\Gamma \in \Re$ an undetermined coefficient. Following the same process as before, we result in:

$$\bar{Y}_t = -\frac{Aa_0 + Acd\Omega_2 + cbY^*}{c(b-1)+1-Aa_1-\delta} t,\ c(b-1)+1-Aa_1-\delta \neq 0,\ t = 1,2,... \quad \textbf{(A.12)}$$

The interpretation of (A.12) as the equilibrium level of income, requires that its value is positive. This implies that $Aa_0 + Acd\Omega_2 + cbY^*$ and $c(b-1)+1-Aa_1-\delta$ should have opposite signs.

Finally, if $c\left(b-1\right)+1-Aa_1-\delta=0$, we assume that $\bar{Y}_t=\Gamma t^2$, where $\Gamma\in\Re$ an undetermined coefficient. So, we get:

$$\bar{Y}_t=\frac{Aa_0+Acd\Omega_2+cbY^*}{2\left(1-c\right)}t^2,\ 2\left(1-c\right)\neq 0,\ t=1,2,... \quad \textbf{(A.13)}$$

The constraint $2\left(1-c\right)\neq 0$ is satisfied since $c\in(0,1)$. Indeed, this term is positive. In order to ensure the economic meaning of (A.13), the nominator $Aa_0+Acd\Omega_2+cbY^*$ should also be positive.

The general solution of (26) is obtained adding one of (A.5), (A.7) or (A.9), depending on the sign of the discriminant (A.3), with one of (A.11) ~ (A.13), depending on the stability or instability. To the extent that the trigonometric oscillatory path with decreasing amplitude is the case, the general solution of (26) is given by (31).

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