The Labour Market, the Business Cycle and the Shadow Economy

Emilio Colombo*
UNIV. MILANO-BICOCCA

Lorenzo Menna
UNIV. MILANO-BICOCCA

Patrizio Tirelli
UNIV. MILANO-BICOCCA

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Abstract

We build two sector DSGE with search and matching frictions, wage and price stickiness, that accounts for both the official and the shadow economy. We analyse employment fluctuations along both the the intensive and the extensive margin. We find that productivity and investment specific shocks imply a large reallocation effect of employment across the two sectors of the economy. As a result employment adjustment occurs mainly through the extensive margin. The effect on labour force participation critically hinges on whether the shock is symmetric or asymmetric across the two sectors. Government spending, monetary policy and risk premium shocks do not have such reallocative effect.

Keywords: DSGE models, search matching frictions, shadow economy

*Corresponding author: University of Milano-Bicocca Piazza Ateneo Nuovo 1, Edificio U6, 20126 Milano, Italy, e-mail: emilio.colombo@unimib.it. Ph: +390264483167 Fax: +390264483085.

1 Introduction

It is well known that over the business cycle, employment and unemployment are much more volatile than output. It is also well known that standard RBC models are not able to explain this evidence. During the last decades a relevant strand of the literature has addressed these issues by introducing search and matching frictions in RBC models using the Mortensen and Pissarides (1994) framework.[1]

While outperforming the standard model in several ways, the inclusion of search and matching frictions is still unable to match the stylised facts outlined above. The main reason lies in the excessive pro-cyclical movement in wages that dampens employment fluctuations over the cycle. A rapidly growing literature has emerged to take on this puzzle introducing wage stickiness as a solution Gertler and Trigari (2009). A key question in this setting is whether wage stickiness induces employment to adjust along the extensive or the intensive margin (through hours worked).

More recently Erceg and Levin (2014) forcefully argue that cyclical factors account for the major part of the decline in the U.S. labor force participation rate observed after 2007.

This paper contributes to this literature by providing an innovative aproach: we analyse search and matching frictions, wage and price stickiness within a two sector DSGE model that accounts for both the official and the shadow economy. Our approach allows to overcome several problems of the literature: first it provides a more realistic characterisation of the economy, second it deals with both the intensive and the extensive margins in employment adjustment.

We find that productivity and investment specific shocks imply a large reallocation effect of employment across the two sectors of the economy. As a result employment adjustment occurs mainly through the extensive margin. Note that we obtain large variation in employment and in unemployment without assuming unrealistically high values of preference for leisure (typical of models with work-leisure choice) or unrealistically high productivity of home activities (typical of models with home production).

Moreover we show that the effect on labour force participation critically depends on whether the shock is symmetric or asymmetric across the two sectors. With symmetric shocks labour force participation is countercyclical while with asymmetric shocks (only to the official economy) it is procyclical.

Government spending, monetary policy and risk premium shocks do not imply such reallocative effect.

The remainder of the paper is structured as follows: section 2 describes the model, section presents the results, section concludes.

[1] Notable examples are Andolfatto (1996), Merz (1995, 99).

2 The model

Our model is a standard DSGE model that incorporates search and matching frictions in the labour market as well as labour force participation decisions.[2]. Differetly from all the other contributions in the literature ours is a two sectors model that accounts for both the official (o) and the underground (s) economy. In both sectors perfectly competitive firms produce wholesale goods which are then sold to monopolistically competitive retail firms. Retail prices are sticky.

Following Zenou (2008) the labour market is characterised by search frictions in the official sector of the economy and by perfect competition in the unofficial sector.[3] There is a representative household, who has a large family structure. A fraction of the members in the household are employed in the official sector, the rest are either unemployed or employed in the unofficial sector. Employed individuals inelastically supply one unit of labor. In line with Zenou (2008), only unemployed individuals can enter a new match with an employer in the official sector. We do not explicitly model flows between unemployment and employment in the informal sector, but we impose a stock equilibrium condition where in each period the outside option of an individual employed in the unofficial sector is equal to the value attached to unemployment status.

Consumption purchases are subject to monetary transaction costs that motivate a demand for money[4]:

$$s(v), \quad s'(v) > 0 \text{ for } v > v^* \tag{1}$$

where v^j defines sectoral money velocity. The features of $s(v^j)$ are such that a satiation level of money balances ($v^* > 0$) exists where the transaction cost vanishes and, simultaneously, a finite demand for money is associated to a zero nominal interest rate. Following Schmitt-Grohe and Uribe (2004) the transaction cost is parameterized as follows:

$$s(v) = Av + \frac{B}{v} - 2\sqrt{AB} \tag{2}$$

The government finances an exogenous stream of expenditures by levying distortionary taxes and by printing money.

2.1 Preferences

There is a continuum of mass 1 of households who gather a continuum of mass one of family members characterised by the lifetime utility:

$$U_t^i = E_t \sum_{k=o}^{\infty} \beta^k \left\{ \ln\left(c_{t+k}^i\right) - \chi \frac{h_t^{i(1+\phi)}}{1+\phi} l^i \right\}$$

[2]The literature on this field has expanded rapidly during recent years: Gertler and Trigari (2009), Arseneau and Chugh (2008, 12), Faia (2009) and Erceg and Levin (2014) are notable examples.

[3]Other contributions assume a non-segmented, fully competitive labour market (Amaral and Quintin (2006) and Pratap and Quintin (2006)). This assumption is supported by Maloney (1999, 2004) and Pratap and Quintin (2006) who provide evidence against labour market segmentation

[4]See Sims (1994), Guerron-Quintana (2009).

where χ is a parameter that regulates the disutility of work and ϕ defines the Frisch elasticity. Households members consume and, for each sector, own the firms, hold physical capital, and choose their investment. Following earlier contributions (Merz, 1995; Andolfatto, 1996), we assume that household members perfectly share the risk of sectoral employment and unemployment outcomes. As a result consumption and investment decisions are identical across individuals.Their flow budget constraint is:[5]

$$\begin{aligned} & c_t\left(1+s(v_t)\right)+\frac{P_t^{R,o}}{P_t}k_t^o+\frac{P_t^{R,s}}{P_t}k_t^s+\frac{M_t-M_{t-1}}{P_t}+\frac{\frac{B_t}{R_t}-B_{t-1}}{P_t}= \\ = \; & \left(1-\tau_t^w\right)\frac{P_t^{R,o}}{P_t}w_t^o h^o l_t^o+\frac{P_t^{R,o}}{P_t}b_t^u u_t+\frac{P_t^{R,o}}{P_t}\left(1+\left(1-\tau_t^k\right)r_t^{k.o}-\left(1-\tau_t^k\right)\delta\right)k_{t-1}^o \quad (3) \\ & +\frac{P_t^{R,s}}{P_t}w_t^s h^s l_t^s+\frac{P_t^{R,s}}{P_t}\left(1+r_t^{k.s}-\delta\right)k_{t-1}^s+\frac{P_t^{R,o}}{P_t}\Pi_t^o+\frac{P_t^{R,s}}{P_t}\Pi_t^s \quad (4) \end{aligned}$$

where B_t is a nominally riskless government bond that pays one unit of currency in period $t+1$ and R_t is the gross nominal interest rate. Then we define a number of sectoral variables: the retail price $P_t^{R,j}$, the capital stock k_t^j, the return on capital $r_t^{k.j}$, profits Π_t^j, the product wage w_t^j, the number of employed individuals l_t^j. Fiscal variales are defined as follows: τ^c is a consumption tax, τ_t^w and τ_t^k are the labor- and capital- income tax rates, t_t denotes real fiscal transfers, b^u is the unemployment subsidy defined in terms of the official sector consumption bundle.

Household preferences over the goods produced in the economy are defined as follows.

$$c_t^j=\left(\int_0^1 c_t^j\left(z^j\right)^{\frac{\sigma^j-1}{\sigma^j}}dz^j\right)^{\frac{\sigma^j}{\sigma^j-1}} \quad (5)$$

and the associated price index is

$$P_t^{Rj}=\left(\int_0^1\left(P_t^{Rj}\left(z\right)\right)^{1-\sigma^j}dz\right)^{\frac{1}{1-\sigma^j}}$$

It follows that demand functions for individual goods within each consumption bundle are:

$$c_t\left(z^j\right)=\left(\frac{P_t^{Rj}\left(z^j\right)}{P_t^{Rj}}\right)^{-\sigma^j}c_t^j$$

The total consumption bundle is

$$c_t=\left[\left(1-\alpha_c\right)^{\frac{1}{\varepsilon}}\left(c_t^o\right)^{\frac{\varepsilon-1}{\varepsilon}}+\left(\alpha_c\right)^{\frac{1}{\varepsilon}}\left(c_t^s\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (6)$$

and

$$P_t=\left[\left(1-\alpha_c\right)\left(\left(P_t^{R,o}\left(1+\tau^c\right)\right)\right)^{1-\varepsilon}+\left(\alpha_c\right)\left(P_t^{R,s}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \quad (7)$$

[5]We drop the superscript i.

defines the consumption price index. Demand functions for the sectoral consumption bundles are:[6]

$$c_t^o = (1-\alpha_c)\left(\frac{P_t^{R,o}(1+\tau^c)}{P_t}\right)^{-\varepsilon} c_t$$

$$c_t^s = \alpha_c\left(\frac{P_t^{R,s}}{P_t}\right)^{-\varepsilon} c_t$$

In aggregate terms:

$$P_t c_t = P_t^{R,o} c_t^o (1+\tau^c) + P_t^{R,s} c_t^s$$

2.2 Labour market

Following Zenou (2008) individuals who are not hired in the official sector either take official sector unemployment status, that earns them the (real) unemployment subsidy, $\frac{P_t^{R,o}}{P_t} b^u$, and allows ongoing search for next-period hire in the official sector, and employment in the unofficial sector at the competitive real wage rate $\frac{P_t^{R,s}}{P_t} w_t^s$,[7] where b^u, w_t^s denote real variables in terms of the official and unofficial sectoral price deflators.

Following Christiano, Trabandt, and Walentin (2011) and Zhang (2011), to facilitate model tractability we assume that employment agencies post vacancies in the official labour market at the official output cost f_{pv}^{EA} and bargain with workers both the official sector product wage rate w_t^o and the number of hours worked h^o. Subsequently they combine individual labour supplies into a labour input which is supplied to official sector wholesale producers at the competitive price P_t^{EA}, defined in terms of official sector goods. In the formal sector new matches per unit of time are determined by a standard matching technology

$$M_t^o = m^o (u_t)^{\varkappa} (V_t)^{1-\varkappa}$$

where V_t, u_t respectively define the number of vacancies in the official sector and unemployment and m^o is a parameter that defines the effieicncy of the matching technology. The probability that a vacancy z_t^V be filled therefore is:

$$z_t^V = \frac{M_t^o}{V_t} = m^o \left(\frac{u_t}{V_t}\right)^{\varkappa}.$$

Similarly, the probability that an unemployed individual gets a job in the official sector, z_t^{un}, is:

$$z_t^{un} = \frac{M_t^o}{u_t} = m^o \left(\frac{V_t}{u_t}\right)^{1-\varkappa}$$

[6] In the officiel sector the consumption tax drives a wedge between the retail price set by firms and the corresponding consumption price.

[7] Note that w_t^j, b, p_t^{EA}, f_{pv}^{EA} denote real variables in terms of the consumption price index (7).

Both probabilities are taken as given by employment agencies and household members. Real profits of the representative employment agency are defined as follows:

$$\Pi^{EA} = \left(P_t^{EA} - \frac{W_t^o}{P_t^o} \right) l_t^o h_t^o - f_{pv}^{EA} V_t - \frac{Z}{2} \left(\frac{W_t^o}{W_{t-1}^o} - 1 \right)^2 l_t^o \tag{8}$$

where $W_t^o = P_t^o w_t^o$ define nominal wages. Note that eq (8) includes a (Rotemberg type) cost for adjusting nominal wages which allow us to model wage stickiness.[8]

Employment follows the law of motion:

$$l_t^o(s) = \rho^s l_{t-1}^o(s) + z_t^V V_t(s) \tag{9}$$

where ρ^s $(0 < \rho^s < 1)$ defines the exogenous probability that a match survives up to next period.

Employment agencies maximise (8) subject to (9) obtaining the following hiring condition:

$$\frac{f_{pv}^{EA}}{z_t^V} = \left(P_t^{EA} - w_t^o \right) h_t^o - \frac{Z}{2} \left(\frac{w_t^o}{w_{t-1}^o \pi_t^o} - 1 \right)^2 + \beta \frac{\pi_{t+1}^o \lambda_{t+1}}{\pi_{t+1} \lambda_t} \frac{f_{pv}^{EA}}{z_{t+1}^V} \rho^S \tag{10}$$

where $\frac{f_{pv}^{EA}}{z_t^V}$ defines the marginal cost of hiring a worker[9] and the r.h.s. of (10) the marginal benefit, including both the price margin $P_t^{EA} - w_t^o$, the wage adjustment cost, and the discounted savings on posting a future vacancy, which are proportional to the match survival rate ρ^s.

2.2.1 Nash bargaining over wages and hours

The nominal wage and the number of hours are negotiated by workers and employment agencies within a Nash bargaining framework.[10] We assume simultaneous bargaining over W and h.[11]

The Bellman equation that describes the value of a new hire for the employment agency is:

$$v_t^{EA} = \left(P_t^{EA} - \frac{W_t^o}{P_t^o} \right) h_t^o - \frac{Z}{2} \left(\frac{W_t^o}{W_{t-1}^o} - 1 \right)^2 + \beta \frac{\pi_{t+1}^o \lambda_{t+1}}{\pi_{t+1} \lambda_t} v_{t+1}^{EA} \rho^S$$

where v_t^{EA} is defined in official goods while in nominal terms $V_t^{ea} = P_t^o v_t^{EA}$

[8] This specification is fairly common in the literature (Arseneau and Chugh, 2012) and is equivalent up to a first order approximation to a Calvo specification with the advantage of being computationally easier.

[9] We assume that $f_{pv}^{EA} V_t$ is purchased in the goods market, as such it enters the official sector aggregate resource constraint below.

[10] In our framework bargaining occurs over nominal wages for wage stickiness to have a bite. Note that in the absence of wage adjustment costs bargaining over nominal wages is equivalent to bargaining over real ones.

[11] Alternative specifications as right to manage or right to work would yield similar results (see Arseneau and Chugh (2012)).

Analogously the Bellman equations that describe the value for an individual of being employed and unemployed are:

$$v_t^{lo} = (1-\tau_t^w)\,\lambda_t p_t^o w_t^o h_t^o - \chi_t \frac{h_t^{o(1+\phi)}}{1+\phi} + \beta\left[\rho v_{t+1}^{lo} + (1-\rho)\,v_{t+1}^u\right] \tag{11}$$

$$v_t^u = \lambda_t p_t^o b + \beta\left[z_{t+1}^{un} v_{t+1}^{lo} + \left(1-z_{t+1}^{un}\right) v_{t+1}^u\right] \tag{12}$$

Note that the option value of being employed in the unofficial sector does not enter (15) because we impose the stock equilibrium condition:

$$v_t^u = v_t^s \tag{13}$$

where

$$v_t^s = \lambda_t p_t^s w_t^s h_t^s - \chi_t \frac{h_t^{s(1+\phi)}}{1+\phi} + \beta v_{t+1}^s \tag{14}$$

defines the value to the individual of being employed in the unofficial sector.

Note that all values above are defined in utils, while in nominal terms: $V_t^{lo} = \frac{v_t^{lo}}{\lambda_t} P_t$ and $V_t^u = \frac{v_t^u}{\lambda_t} P_t$.

Wages and hours are set to maximise the product:

$$\left(V_t^{EA}\right)^{1-\vartheta}\left(V_t^{lo} - V_t^u\right)^{\vartheta} \tag{15}$$

where ϑ identifies the relative bargaining power of each party.

Nash bargaining implies that 15 is maximised by:

$$\frac{\varpi_t}{(1-\varpi_t)}\left(P_t^{EA} h_t^o - w_t^o h_t^o - \frac{Z}{2}\left(\frac{w_t^o}{w_{t-1}^o \pi_t^o} - 1\right)^2 + \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{f_{pv}^{EA}}{z_{t+1}^V} \frac{\pi_{t+1}^o}{\pi_{t+1}} \rho\right) =$$

$$(1-\tau_t^w)\,w_t^o h_t^o - \chi_t \frac{h_t^{o(1+\phi)}}{(1+\phi)\,\lambda_t p_t^o} - b + \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\varpi_{t+1}}{(1-\varpi_{t+1})} \frac{\pi_{t+1}^o}{\pi_{t+1}} \left(\rho - z_{t+1}^{un}\right) \frac{f_{pv}^{EA}}{z_{t+1}^V}$$

$$-\vartheta \frac{(1-\varpi_t)}{\varpi_t}\left((1-\tau_t^w)\,w_t^o - \frac{\chi_t \left(h_t^o\right)^{\phi}}{\lambda_t p_t^o}\right) = (1-\vartheta)\left(P_t^{EA} - w_t^o\right)$$

where

$$\varpi_t = \frac{\vartheta}{\vartheta - (1-\vartheta)\,\frac{\partial V_t^{ea}}{\partial W_t^o} \Big/ \frac{\partial\left(V_t^{lo} - V_t^u\right)}{\partial W_t^o}}$$

2.3 Firms

In each sector $j\,(o, s)$, perfectly competitive (flex-price) firms produce wholesale goods Ij and sell them to retail producers Rj that differentiate products and are subject to price rigidity.

2.3.1 Wholesale producers

Wholesale producers have access to the production technology:

$$y_t^j = (\exp \theta_t^j) \left(k_{t-1}^j\right)^{\alpha^j} \left(h_t^j l_t^j\right)^{1-\alpha^j}$$

where y_t^j, k_t^j, h_t^j respectively define sector-specific output, capital and labour inputs, and θ_t^j captures a sectoral productivity shock, which displays the following time path:

$$\theta_t^o = \rho^\theta \theta_{t-1}^o + \xi_t^o; \quad \xi_t^o \text{ i.i.d.}$$

Factor demands are:

$$w_t^s = (1-\alpha^s)\, \theta_t^s \left(\frac{k_t^s}{h_t^s l_t^s}\right)^{\alpha^s} \tag{16}$$

$$P_t^{EA} = (1-\alpha^o)\, \theta_t^o \left(\frac{k_t^o}{h_t^o l_t^o}\right)^{\alpha^o} \tag{17}$$

$$r_t^j = \alpha^j \theta_t^j \left(\frac{k_t^j}{h_t^i l_t^j}\right)^{-\left(1-\alpha^j\right)} \tag{18}$$

Intermediate sector real marginal costs, mc_t^{Ij}, are:

$$mc_t^{I,o} = \left(\frac{r_t^o}{\alpha^o}\right)^{\alpha^o} \left(\frac{\left(P_t^{EA}\right)}{(1-\alpha^o)}\right)^{1-\alpha^o} \tag{19}$$

$$mc_t^{I,s} = \left(\frac{r_t^s}{\alpha^s}\right)^{\alpha^s} \left(\frac{w_t^s}{(1-\alpha^s)}\right)^{1-\alpha^s}$$

2.3.2 Retail producers

Retail producers turn intermediate goods into differentiated retail products. They are subject to a fixed production costs fc^j such that their profits are zero in steady state. We assume a sticky price specification based on Rotemberg (1982) quadratic cost of nominal price adjustment:

$$\frac{\varphi}{2}\left(\pi_t^{Rj} - 1\right)^2 \tag{20}$$

where $\varphi \geq 0$ is a measure of price stickiness, $\pi_t^{Rj} = \frac{P_t^{Rj}}{P_{t-1}^{Rj}}$ denotes the sectoral gross inflation rate.

In a symmetrical equilibrium the price adjustment rule satisfies:

$$\left(\frac{(1-\sigma^j)}{\sigma^j} + mc_t^{I,j}\right)\frac{\sigma^j}{\varphi} + \beta\left[\left(\frac{\lambda_{t+s}}{\lambda_t}\right)\frac{y_{t+1}^j}{y_t^j}\left(\pi_{t+1}^{Rj} - 1\right)\left(\pi_{t+1}^{Rj}\right)\right] = \left(\pi_t^{Rj} - 1\right)\pi_t^{Rj} \tag{21}$$

where $\frac{P_t^{Ij}}{P_t^{Rj}}$ defines real marginal costs in terms of the sectoral retail price. Consumption price inflation is:

$$\pi_t = \frac{P_t}{P_{t-1}}$$

2.4 Households decisions

The intertemporal Euler equation is:

$$\lambda_t = \beta E_t \left(\lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right) \tag{22}$$

where

$$\lambda_t = \frac{u_c\left(c_t\right)}{1 + s(\frac{c_t}{m_t}) + \frac{c_t}{m_t} s'(\frac{c_t}{m_t})} \tag{23}$$

In condition (23) the monetary transaction cost introduces a wedge between the marginal utility of consumption, $u_c\left(c_t\right) = \frac{1}{c_t - b c_{t-1}}$, and the marginal utility of wealth, λ_t^u, where $m_t = \frac{M_t}{P_t}$.

Households portfolio equilibrium requires that capital demand is driven by:

$$\lambda_t = \beta E_t(\lambda_{t+1}[(1-\delta) + r_{t+1}^{k,s}]) \tag{24}$$

and

$$\lambda_t = \beta E_t(\lambda_{t+1}[(1-\tau_{t+1}^k) r_{t+1}^{ko} + \delta \tau_{t+1}^k + (1-\delta)])$$

the implicit money demand function is $s(v) = Av + \frac{B}{v} - 2\sqrt{AB}; 1 - \frac{1}{R_t} = s'(v_t)\left(v_t\right)^2$

$$1 - \frac{1}{R_t} = s'(v_t)\left(v_t\right)^2 \tag{25}$$

2.5 Government

The government supplies an exogenous, stochastic[12] and unproductive amount of public good g_t (defined in terms of the official sector good) and unemployment benefits. Government financing is obtained through an income tax, money creation and issuance of one-period, nominally risk free bonds. The government flow budget constraint is then given by

$$\frac{B_{t-1}}{P_t^o} + g_t + b_t^u u_t = \left(\tau_t^w w_t^o h_t^o l_t^o \ + \tau_t^k \left(r_t^{ko} - \delta\right) k_{t-1}^o + c_t^o \tau^c\right) + \frac{M_t - M_{t-1}}{P_t^o} + \frac{B_t}{R_t P_t^o} + \tau_t^{LS} \tag{26}$$

where τ_t^{LS} defines lump sum taxes.

[12] We assume that the logarithm of government consumption is normal and i.i.d.

2.6 Capital accumulation

Sectoral capital accumulation is driven by

$$k_t^o = (1-\delta)\, k_{t-1}^o + y_t^o - c_t^o\,(1+s(v_t)) - g_t - \frac{\xi_p}{2} y_t^o\,(\pi_t^o - 1)^2 - f_{pv}^{EA} V_t \tag{27}$$

$$k_t^s = (1-\delta)\, k_{t-1}^s + y_t^s - c_t^s\,(1+s(v_t)) - \frac{\xi_p}{2} y_t^s\,(\pi_t^s - 1)^2 \tag{28}$$

2.7 Labor resource constraint

Finally, the labour resource constraint is:

$$1 - u_t = l_t^o + l_t^s \tag{29}$$

2.8 Calibration

Parameters characterising the official economy and households preferences are fairly standard. The values chosen for the household subjective discount factor, $\beta = 0.99$, the capital income share $\alpha^o = 0.36$, the capital depreciation rate, $\delta = 0.02$, follow the literature. The degree of price stickiness, $\varphi^o = 4.37$, and the price-elasticity parameter $\sigma^o = 6$ are taken from Schmitt-Grohe and Uribe (2004). The elasticity of substitution between official and shadow consumption bundles, is set at 1.5 as in Batini et al. (2011). Turning to firms operating in the shadow economy, to capture the relatively low capital intensity in their production function we have chosen the capital share parameter, $\alpha^s = 0.28$, as in Koreshkova (2006); we have also assumed that firms operating in the unofficial retail sector have limited market power, $\sigma^s = 20$. To the best of our knowledge, there is no evidence about nominal rigidities in the unofficial sector. We therefore take as benchmark the values adopted for the degree of price stickiness in the official sector.[13]

Labour market parameters are selected as follows. Hobijn and Sahin (2009) find that monthly separation rates in OECD countries range between 2 and 0.7%. We therefore set $\rho^s = 0.95$, implying a 5% quarterly separation rate. Following Colgiago and Rossi (2014) we calibrate m, f_{pv}^{EA} to obtain a job finding rate[14] $z^{un} = 0.7$ and a vacancy filling rate $z^V = 0.9$, and we set $b^u = 0.4w^o$ parametrised to the US economy. The Nash bargaining parameter ϑ is assigned value 0.5, which is standard in this literature.

Finally we close the model by calibrating α_c at the value that would imply a near-zero value of the shadow economy if the official one was untaxed. The tax rates are chosen to match those of the US economy: consumption tax $\tau^c = 0.077$, capital tax $\tau^k = 0.184$, labour tax $\tau^w = 0.154$, debt to gdp ratio 60% and government spending is set to 16% of gdp.

With those tax rates the size of the shadow economy raises to 8.5% which is around the consensus estimate for the US (Schneider and Buehn, 2007).

[13] On theoretical grounds it is not obvious that the proportional output cost associated to price revisions should be different across the two sectors. We also experimented with $\varphi^s = 2.18$, and our results were entirely confirmed.

[14] Hobijn and Sahin (2009) document that monthly job-finding rates in the OECD seem to range between 56% and 2.6%.

3 Results

The figures report impulse response functions following relevant shocks. We have analysed the effects of shocks to productivity, the risk premium, government spending, monetary policy and investment. For productivity and investment specific shocks we have considered both the case where they are symmetric (i.e. affecting both the official and the shadow economy), and the case where they are asymmetric (i.e. affecting only the official economy). Unless where variables are already defined as percentages, impulse responses are calculated as percentage deviation from the steady state values. The dashed line in the figure describes the behaviour of the one sector (official) economy while the continuous line identifies the behaviour of the two sector (official and shadow) economy. Following a symmetric productivity shock output and investment increase in both sectors; the official economy, being relatively more labour intensive, commands a higher increase in wages. As a consequence the shadow economy absorbs a part of the employment increase and also the drop in unemployment turns out to be lower than in a single sector economy case. The participation rate drops as well as hours worked, albeit the adjustment in the intensive margin is much lower than the one in extensive margin.

In the case of an asymmetric productivity shock (to the official economy) the transmission mechanism differs. The increase in official output shock determines an increase in employment and wages in the official economy (the latter traded with less hours in the bargaining framework). The widening gap between the value of being employed and the value of being unemployed, as a consequence employment is reallocated from the shadow to the official economy magnifying the official employment response and increasing wages in the shadow economy too. Asymmetric productivity shocks are therefore associated with an increase in the participation rate. Investment shocks have an effect similar to productivity shocks, on the contrary government spending, monetary policy and risk premium shocks do not imply reallocative effects between the two sectors of the economy.

4 Conclusions

In this paper we analyse search and matching frictions, wage and price stickiness within a two sector DSGE model that accounts for both the official and the shadow economy. Our approach allows to deal with both the intensive and the extensive margins in employment adjustment.

We find that productivity and investment specific shocks imply a large reallocation effect of employment across the two sectors of the economy. As a result employment adjustment occurs mainly through the extensive margin. Note that we obtain large variation in employment and in unemployment without assuming unrealistically high values of preference for leisure (typical of models with work-leisure choice) or unrealistically high productivity of home activities (typical of models with home production).

Moreover we show that the effect on labour force participation critically depends on whether the shock is symmetric or asymmetric across the two sectors. With symmetric shocks labour force participation is countercyclical while with asymmetric shocks (only to the official economy) it is procyclical.

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Figure 1: Productivity shock: symmetric

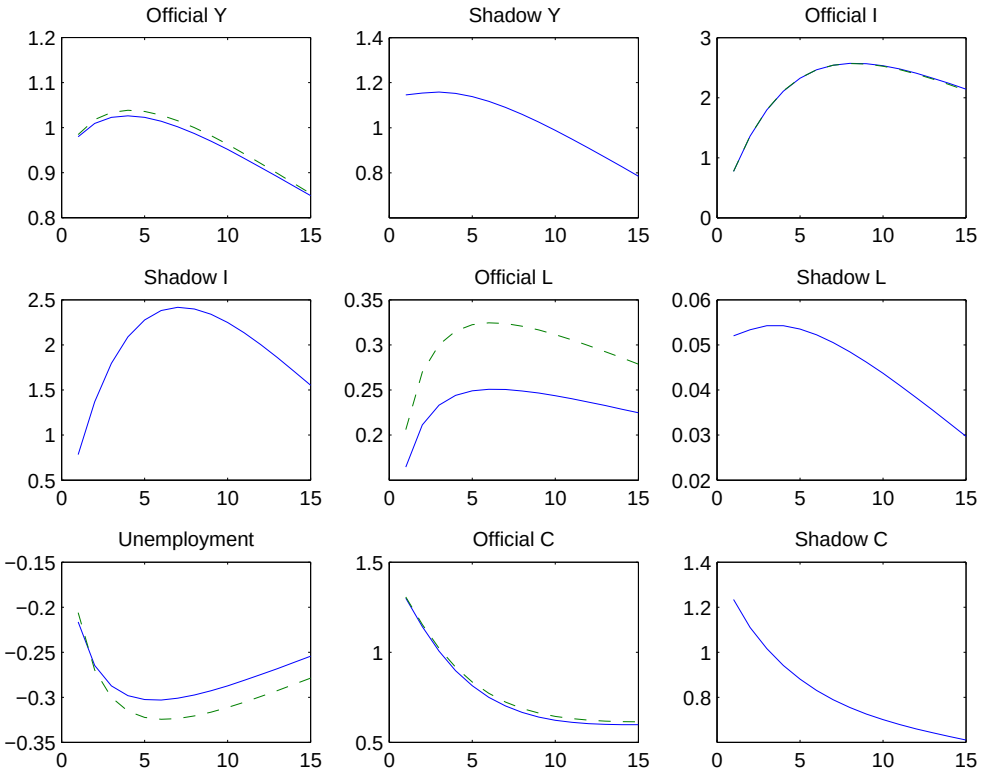

Figure 2: Productivity shock: symmetric

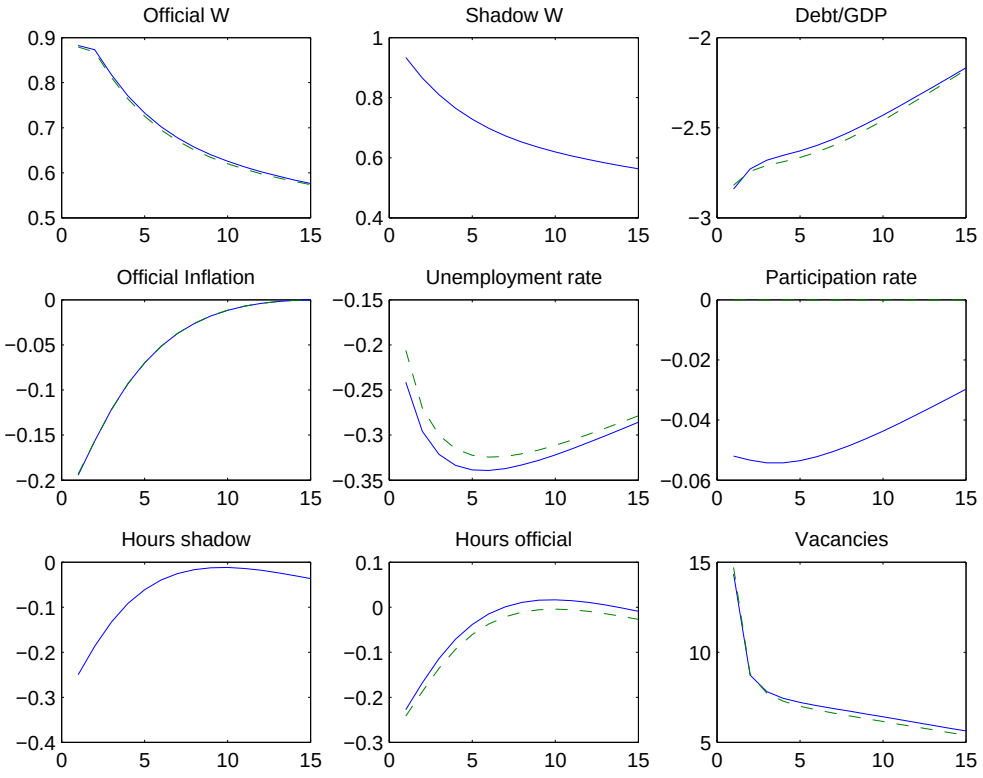

Figure 3: Productivity shock: asymmetric

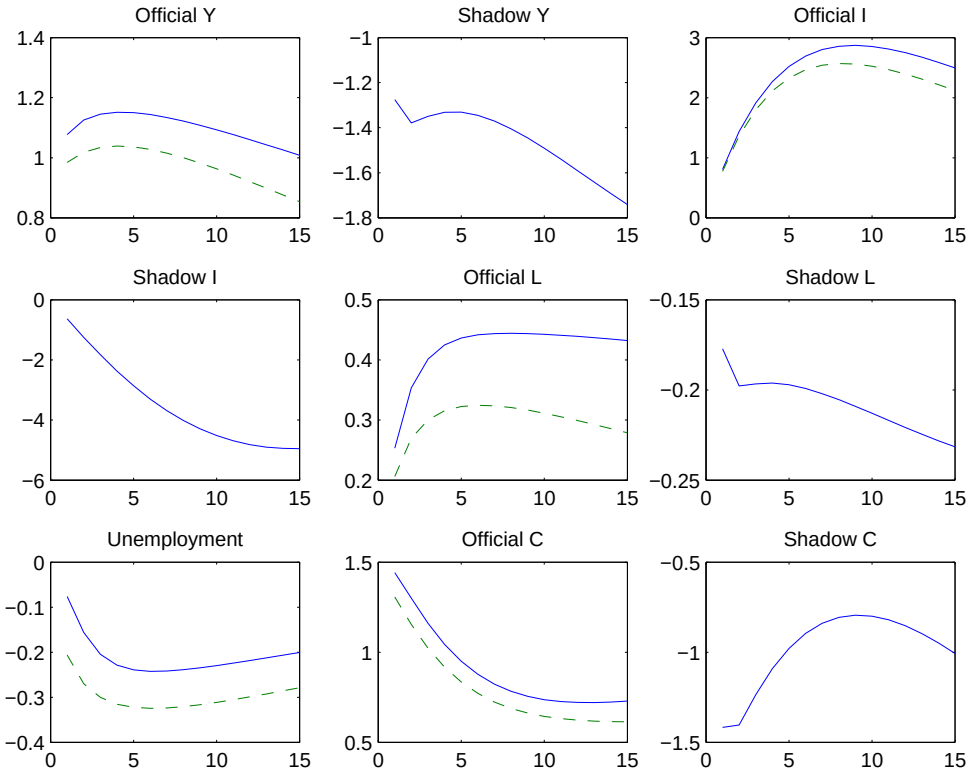

Figure 4: Productivity shock: asymmetric

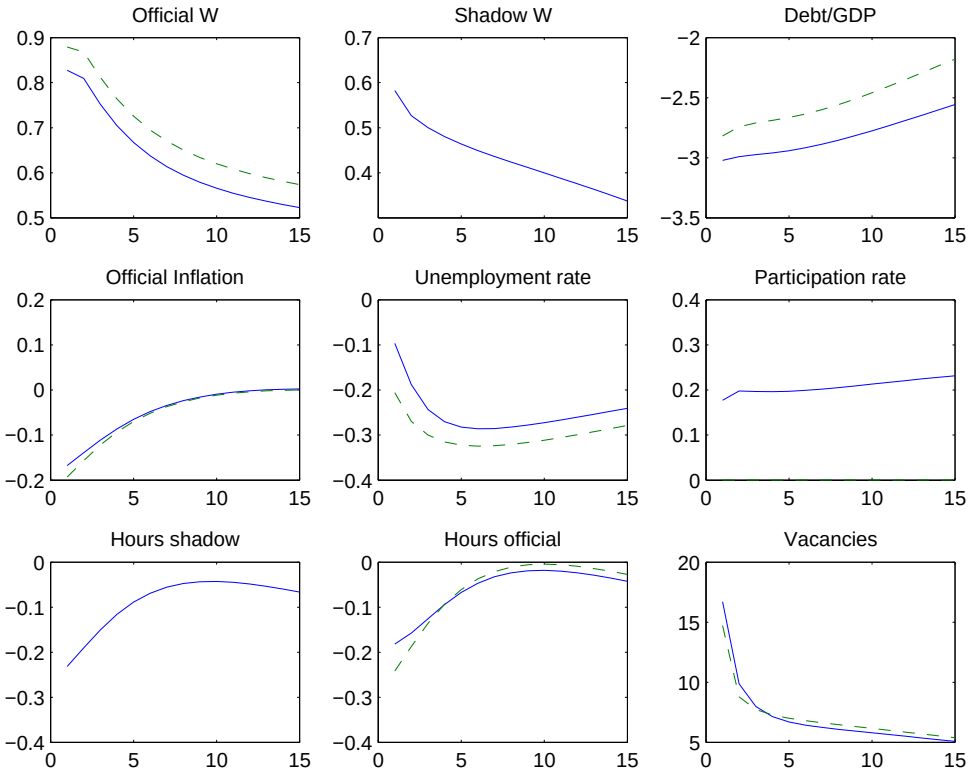

Figure 5: Monetary policy shock

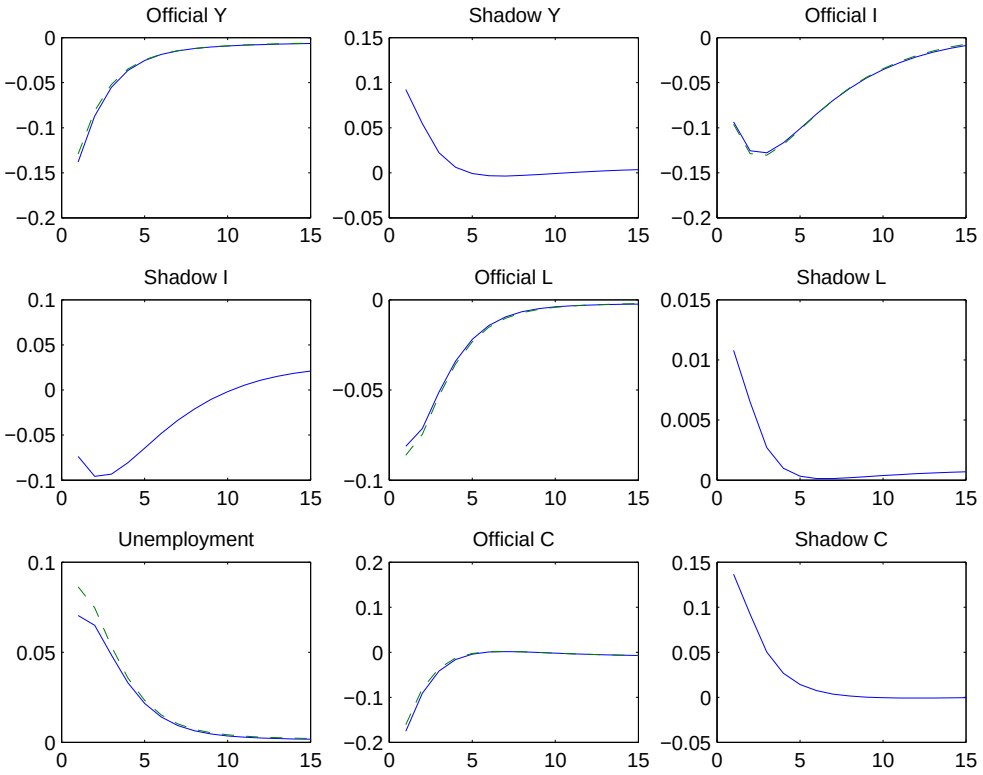

Figure 6: Monetary policy shock

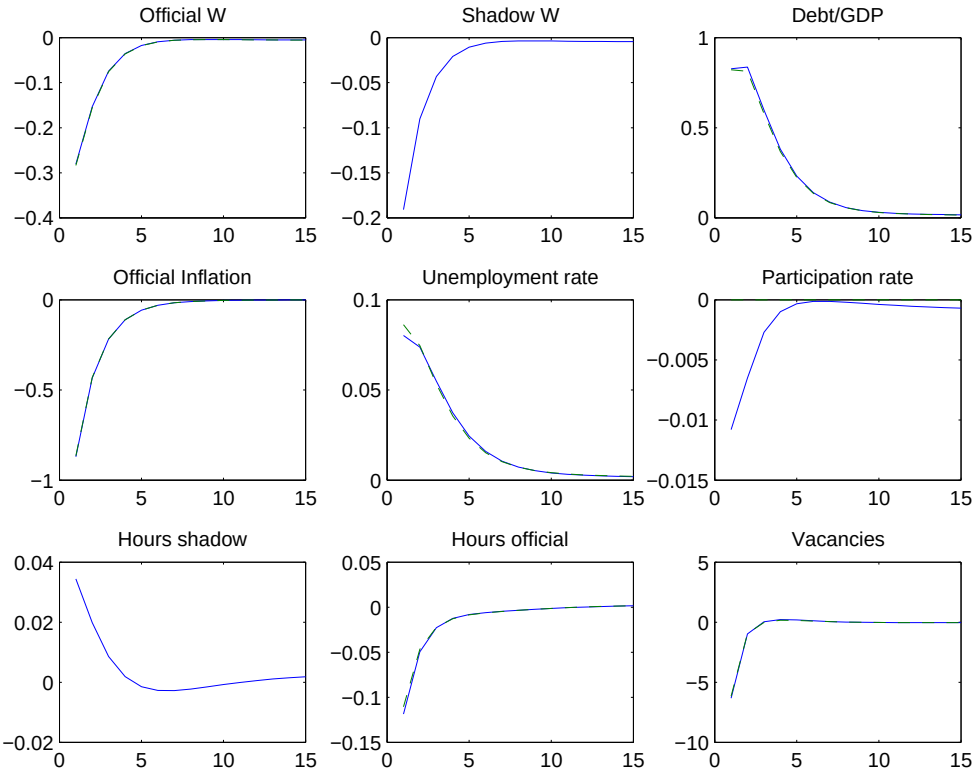

Figure 7: Government shock

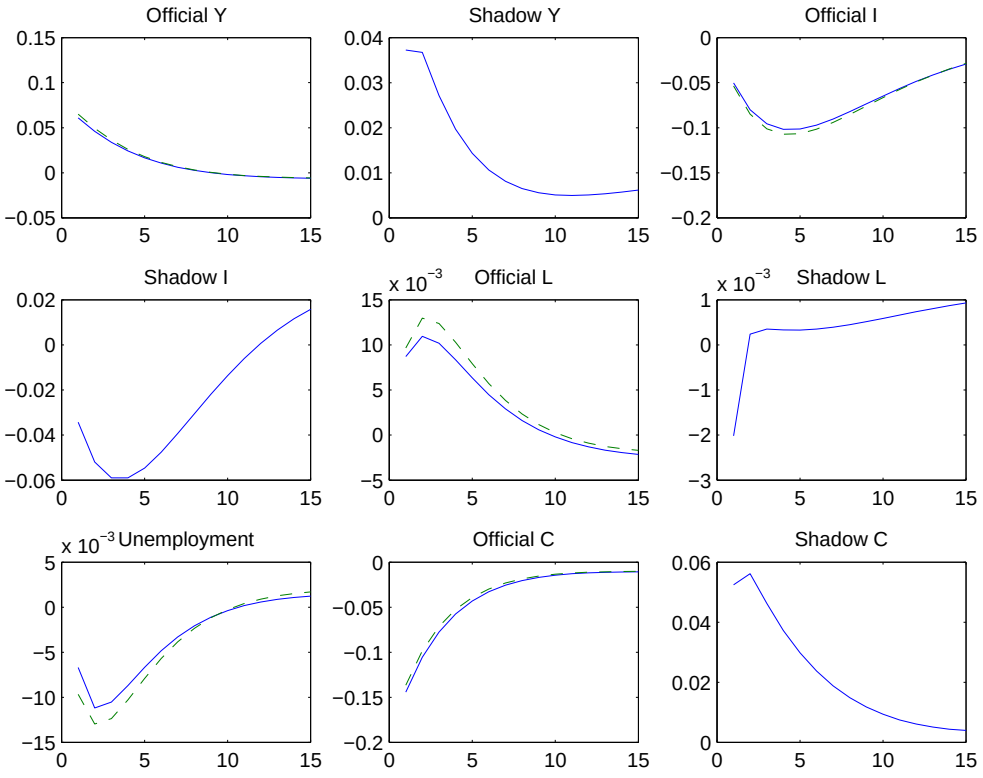

Figure 8: Government shock

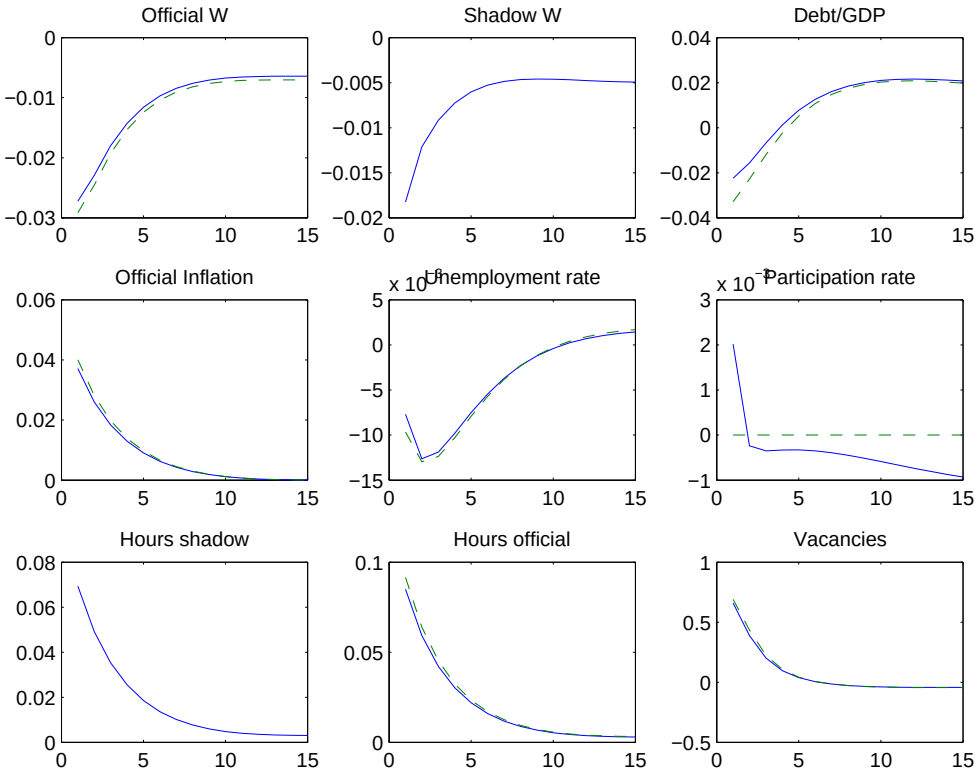

Figure 9: Risk premium shock

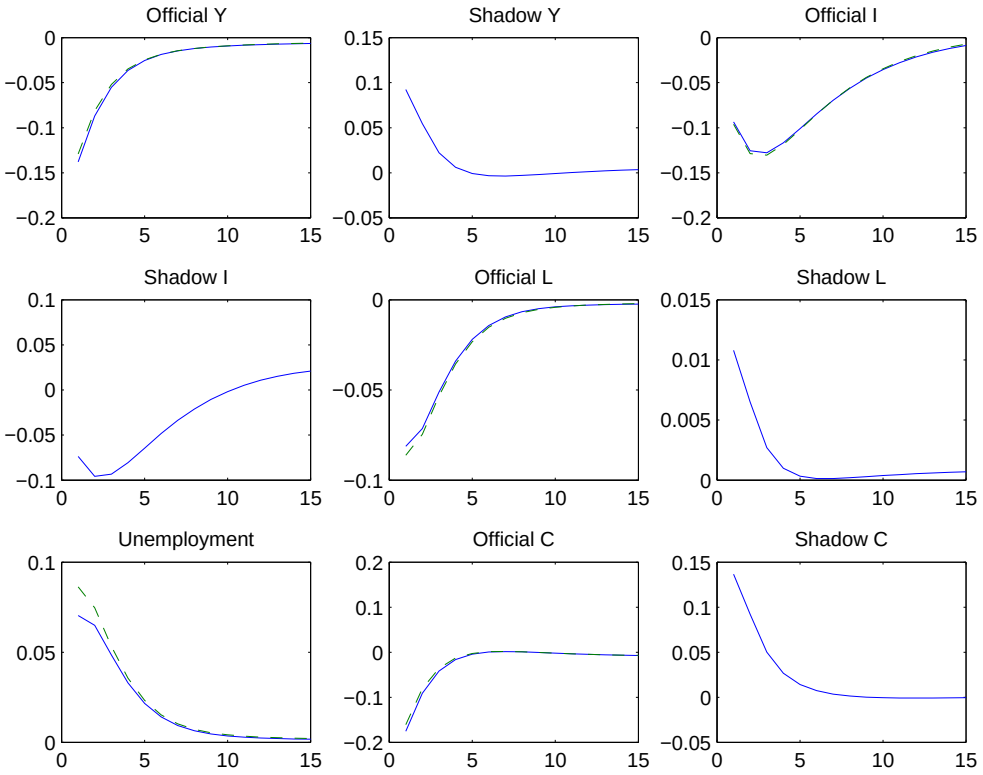

Figure 10: Risk premium shock

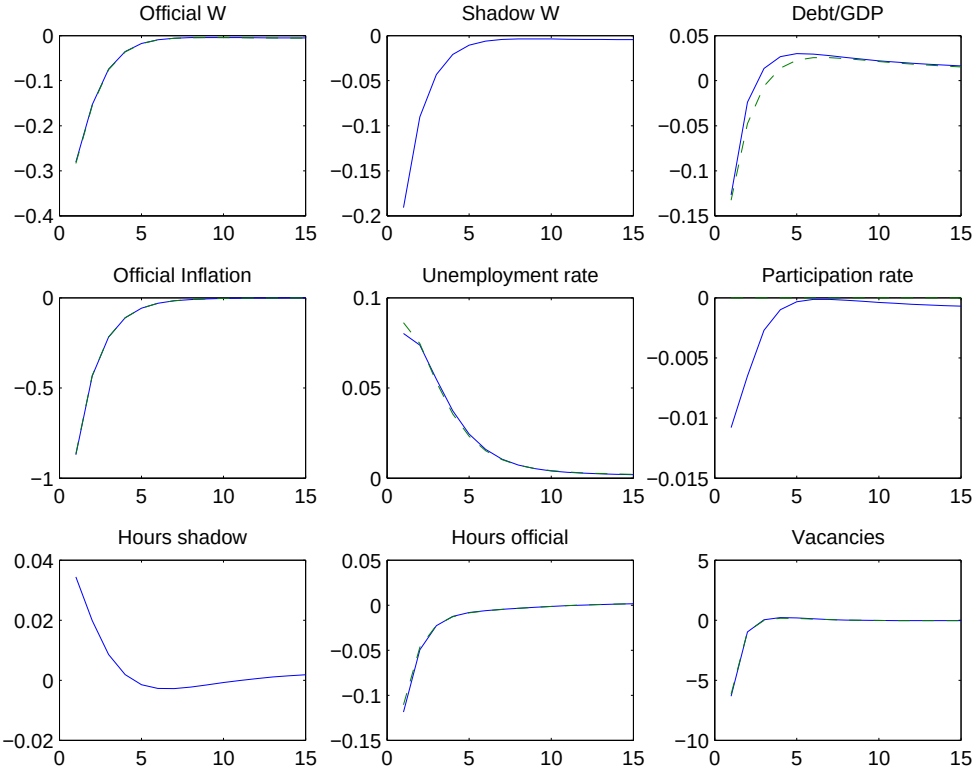

Figure 11: Investment shock: symmetric

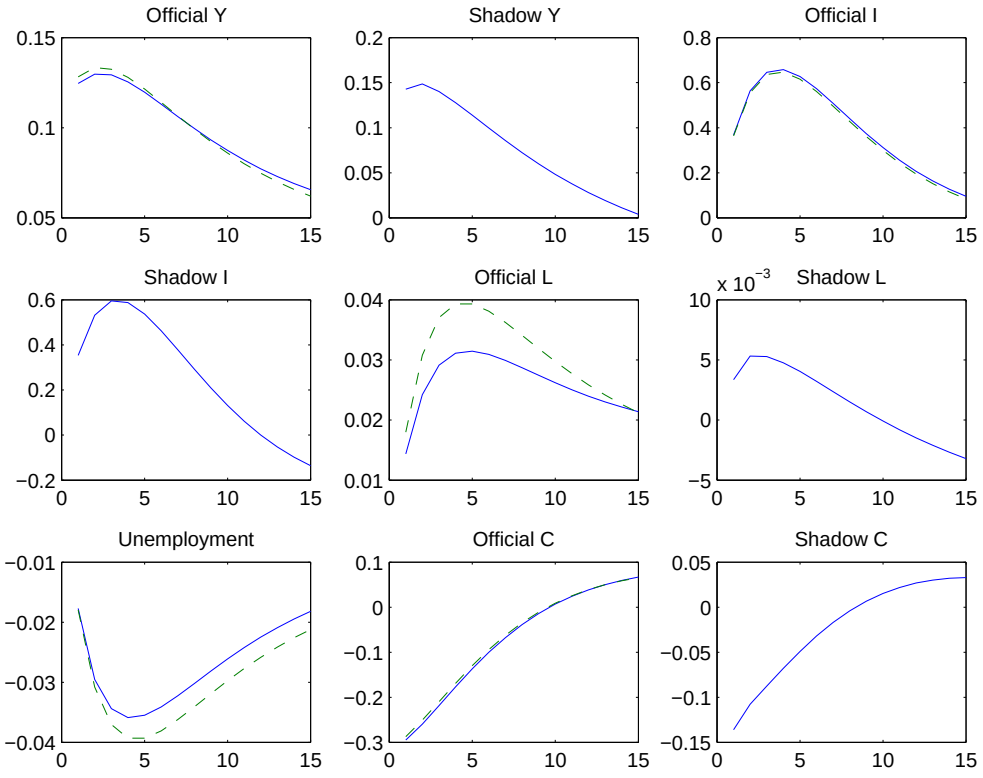

Figure 12: Investment shock: symmetric

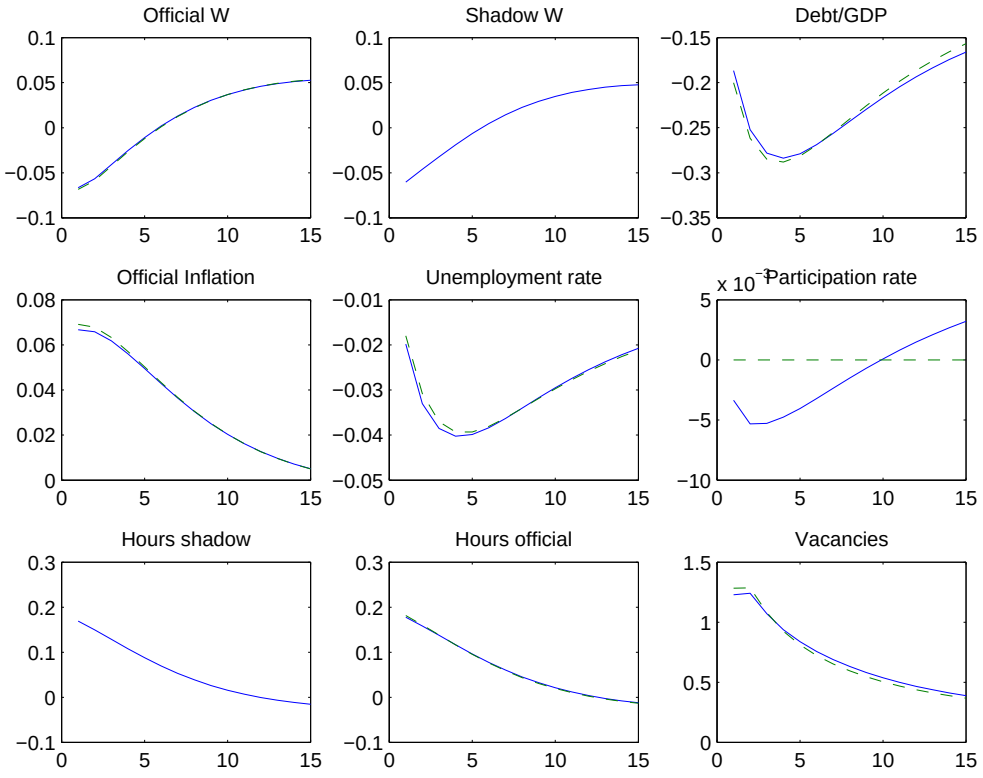

Figure 13: Investment shock: asymmetric

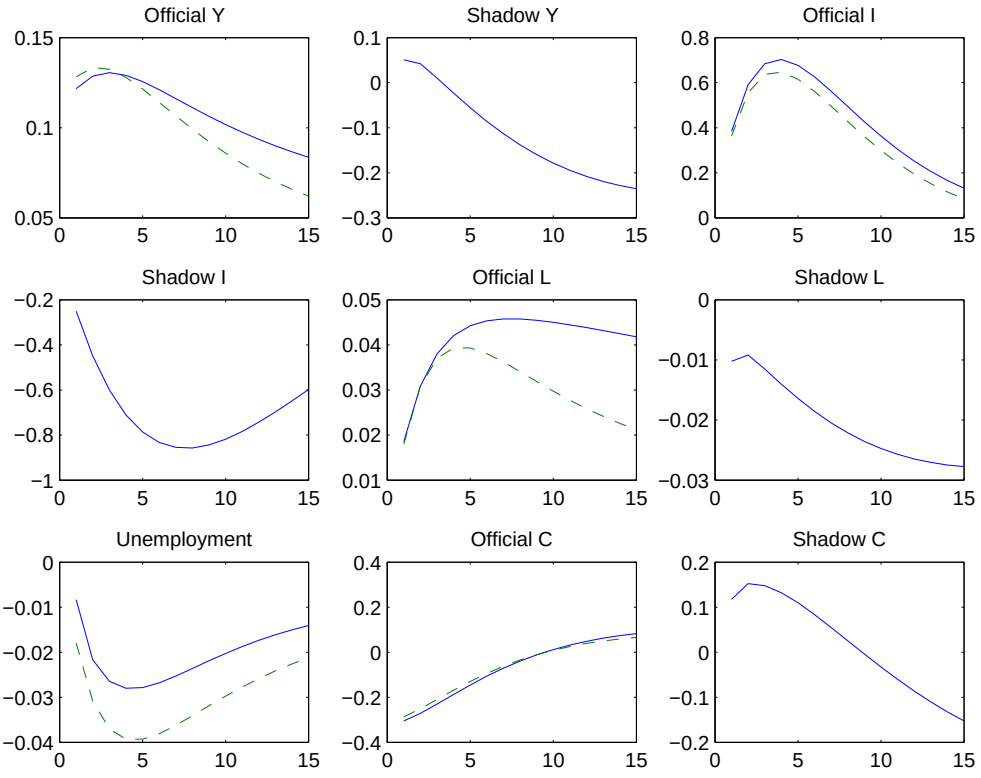

Figure 14: Investment shock: asymmetric