

# Analysis of the Government Expenditure Multiplier under Zero Lower Bound: the Role of Public Investment<sup>1</sup>

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## Abstract

In times of economic downturn fiscal authorities face the need to stimulate the output by most efficient instruments which are in the fiscal policy toolkit. In the paper of Eggertsson, Krugman (2012) authors illustrate that the multiplier of government expenditures can be significantly higher under zero lower bound comparing to the case of the positive interest rate. This work follows Eggertsson, Krugman (2012) by exploring the fiscal multiplier within a borrower-saver framework. However, it incorporates the productive government expenditure along with the standard utility-enhancing type of public spendings. We consider public investment in order to compare an effect of productive expenditures on the value of the fiscal multiplier under zero lower bound and in the case of positive interest rate. Two types of government expenditures are distinguished as the value of the expenditure multiplier highly depends on the structure of public expenditure and on the interest rate framework. It was found that both the share and productivity of public investment lower the value of the multiplier under zero lower bound and increase it in the “normal” case. The results, therefore, point out in favour of old Keynesian wasteful government expenditure under zero lower bound.

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# 1 Introduction

The recent economic crisis has renewed an attention to the question of fiscal policy efficiency, especially government spending, in stimulating economic activity in the recession. The interest in the fiscal stimulus has been highly increased in the recent years because the short-term nominal interest rate has reached very low values in a lot of countries (e.g. US, Canada and countries of the European Union). Thus, further stimulating interest rate cuts have become impossible. Moreover, the examination of fiscal policy is linked to the special framework of the zero lower bound. This framework is crucial for the analysis and evaluation of the stimulating policies performance as the prescriptions for fiscal authorities in “normal” circumstances (with positive interest rate) can differ dramatically from those, obtained taking into account zero lower bound (Eggertsson, 2011).

A new insight to the analysis of the fiscal multiplier can be brought by considering productive as well as utility-enhancing type of government expenditures. The former was shown to gain a higher share in the total government expenditures during the recession period (Leeper et al., 2010; Bachmann, Sims, 2012). Therefore, it should be taken into account, as the results can be altered, when government expenditures begin to affect directly the productivity.

Starting from the work of Eggertsson (2011), followed by Christiano, Eichenbaum, Rebelo (2011) and Eggertsson, Krugman (2012), authors point out in favor of government expenditures as a stabilization tool used in recession. The paper Eggertsson (2011) illustrates, first of all, how different is the effect of macroeconomic policies when zero lower bound is considered as opposed to the normal case. Secondly, it shows, that the most efficient are those policy measures, that are aimed at stimulating aggregate demand, not aggregate supply. More precisely, some tax cuts (taxes on labor income, taxes on capital), which are known to have an expansionary impact on the economy in the positive interest rate case, can even worsen the recession under zero lower bound. It happens due to an upward-sloping demand curve, which occurs when nominal interest rate is at the zero bound. While it is shown that temporary increase in government expenditures and temporary cuts in sales taxes or investment tax credit can perform very well in decreasing output gap. Moreover, Eggertsson illustrates that government expenditures are especially efficient under zero lower bound, contributing to a series of papers, that illustrated the fiscal multiplier being even higher in these circumstances (Hall,

2009).

Christiano, Eichenbaum, and Rebelo (2011) extended this framework to the medium-size DSGE model, coming to the same conclusion as Eggertsson (2011) and obtaining the multiplier of 1.6 with a peak at 2.3 after 5 quarters (for the government shock which lasts for 12 quarters).<sup>1</sup>

A similar framework has been widely used in the analysis of fiscal policy and different types of financing of the stimulating policies. In the paper of Eggertsson, Krugman (2012), for example, authors consider borrowers (liquidity constrained agents). The authors highlights the efficiency of the government spending when zero lower bound is binding, showing that the multiplier in this case is higher than one.

However, it may not be the case, as it was shown by Roulleau-Pasdeloup (2013), if productive government spending along with utility enhancing public expenditures are considered. Author has illustrated that in the case of excess savings liquidity trap government spending multiplier can be lower than in "normal" case of positive interest rate if public investment is taken into account and enters into the stimulus package. As it was shown by Bachmann, Sims (2012) the structure of government expenditures is different in normal times and recessions, with public investment taking a bigger part at the downturn. Roulleau-Pasdeloup (2013) shows that for high share of productive spending in total expenditures (bigger than 0.641<sup>2</sup>) it is possible for private consumption to be crowded out by productive government spending. Moreover, even with an equal share of productive spending author obtained a smaller government multiplier in the type of the recession compared to the normal times. It was shown that this multiplier is decreasing with the share of productive government expenditures and can become negative for high values. In both papers the same context is considered: excess-savings liquidity trap and an upward-sloping aggregate demand curve is analyzed. However, the difference of results can be explained by the fact that in Eggertsson (2011) it is assumed that aggregate demand is effected more by government spending than the aggregate supply. An increase in aggregate demand and its rightward shift occurs in both cases due to the same reasons. However, in Roulleau-Pasdeloup (2013) productive expenditures having a high share in total expenditures enhance the higher positive effect of public spending on the aggregate supply which in the case of upward-sloping demand curve is contractionary.

In this work we focus on the topic of government spending multiplier, introducing public investment in the Borrower-saver framework (Eggertsson, Krugman,

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<sup>1</sup>Christiano, Eichenbaum and Rebelo (2011), p. 110

<sup>2</sup>Roulleau-Pasdeloup (2013), p. 21

2012). It is interesting to analyze the value of the multiplier in this framework: the introduction of debt constrained consumers is known to increase spending multiplier (Galí, López-Salido, Vallés, 2007; Eggertsson, Krugman, 2012), while productive government expenditures tend to work in the opposite direction, when zero lower bound is considered (Rouilleau-Pasdeloup, 2013). With an aim to analyze the interaction of these assumptions the case of zero lower bound and the case of positive nominal interest rate are compared in the model with productive and utility enhancing government spendings. The magnitude of the multiplier is found to be higher in the zero lower bound case, despite the introduction of productive expenditures. Both the share of productive expenditures and its productivity affect negatively the multiplier under zero lower bound and positively in the case of positive nominal interest rate. Moreover, it was found that fiscal multiplier can become negative, when fiscal stimulus consists only of public investment. However, for this to be true a sufficiently high share of borrowers in the economy is needed. While the threshold level of productive investment share is surprisingly decreasing with the share of debt constrained consumers. An impact of other parameters on the multiplier is analyzed as well, underlining the difference between the two cases.

## 2 The model

### 2.1 Households

Households are represented by a continuum of mass 1 with exogenous share of savers  $\chi_s$  and  $1 - \chi_s$  of borrowers. Agents differ in their time preference: borrowers are less patient and value more their current consumption in comparison to the future one. While savers are standard Ricardian agents, who are more patient and prefer to smooth their consumption over time. Thus, the only difference between agents is the difference in the discount factor: savers have a higher discount factor  $\beta(s) = \beta > \beta(b)$  where  $\beta(i) \in (0, 1)$  and  $i = s, b$ .

Consumer of each type maximizes an expected present value of utility in all future periods. While a utility function is additively separable between consumption  $C_t$ , hours worked  $h_t$  and utility-enhancing government expenditures  $G_t^U$  (as in Christiano et al., 2011).

$$E_0 \sum_{t=0}^{\infty} \beta^t(i) [U^i(C_t(i)) - v_t^i(h_t(i)) + \vartheta_t^i(G_t^U)] \quad (1.1)$$

with  $\vartheta()$  – a concave function and  $i = s, b$

Consumption of differentiated good is represented by Dixit-Stiglitz aggrega-

tor with each firm  $j$  producing its own type of good:

$$C_t = \left( \int_0^1 c_t(j) \frac{\theta - 1}{\theta} dj \right)^{\theta/\theta-1} \quad (1.2)$$

with  $\theta$  being an elasticity of the demand.

And corresponding price index is set as follows:

$$P_t = \left( \int_0^1 p_t(j)^{1-\theta} dj \right)^{1/\theta-1} \quad (1.3)$$

As utility function is additively separable, consumer's maximization problem can be broken down into static problem of optimal choice of the set of products consumed each period and dynamic problem. Solving the static problem of minimization of total expenditures, the following aggregate demand function for each good can be obtained<sup>1</sup>:

$$c_{i,jt} = C_{it} \left( \frac{p_{jt}}{P_t} \right)^{-\theta}$$

Or aggregating it by two types of consumers:

$$c_{jt} = C_t \left( \frac{p_{jt}}{P_t} \right)^{-\theta}$$

As for the aggregate consumption, it can be presented as a weighted sum of per capita consumption of two types of agents, which can be interpreted as per capita consumption in the economy:

$$C_t = \chi_s C_t^s + (1 - \chi_s) C_t^b \quad (1.4)$$

In order to define intertemporal choice of consumption and labor supply, the utility maximization problem is solved subject to the following budget constraint and debt limit constraint.

$$B_t^i(i) + W_t P_t h_t(i) + \int_0^1 \Pi_t(i) = (1 + i_{t-1}) B_{t-1}(i) + P_t C_t(i) + P_t T_t(i) \quad (1.5)$$

$$(1 + r_t) \frac{B_t(i)}{P_t} \leq D_t \quad (1.6)$$

Thus, process of borrowing is realized by sell and purchase of one period

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<sup>1</sup>For greater details see Appendix 1.1

riskless nominal bonds,  $B_t(i)$  with  $i_t$  being a nominal return on the bond in the current period. Despite the borrowed amount, each agent receives a labor income for hours worked  $h_t(i)$  and is paid hourly real wage  $W_t$ . Together with a profit from firm ownership distributed equally among agents the total income is spent on the consumption of a set of goods, on repaying (in case of a borrower) the debt with interest included and on paying lump-sum taxes, that can differ for each type.

The second equation is a debt limit constraint. It postulates that real value of debt, taking into account the real interest paid  $r_t$  on this debt, can't be higher, than an exogenous real debt limit  $D_t$ .

## Savers

It is assumed, that for the saver the debt limit constraint is not binding, so savers maximize their objective function subject to the presented above constraints with respect to consumption, hours worked and the amount lent. After solving this problem<sup>2</sup> the following optimality conditions can be obtained:

$$U_c^s(C_t^s) = \beta(1 + i_t) E_t[U_c^s(C_{t+1}^s) \frac{P_t}{P_{t+1}}] \quad (1.7)$$

$$W_t = \frac{v_h^s(h_t^s)}{U_c^s(C_t^s)} \quad (1.8)$$

First equation is an Euler equation, defining intertemporal substitution between consumption today and consumption tomorrow. Marginal utility of current consumption should be equal to the expected marginal utility of future consumption, taking into account the discount factor and the real value of money tomorrow.

Second equation defines labor supply, postulating that marginal gain, obtained from hours worked in terms of consumption good, should be equal to marginal disutility of labor. Or putting it in another way, real wage has to be equal to the marginal rate of substitution between consumption and labor.

## Borrowers

In the case of borrowers debt limit constraint is binding and borrower's consumption is defined by the budget constraint in real terms taking into account that

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<sup>2</sup>For greater details see Appendix 1.1

debtors borrow up to the debt limit<sup>3</sup>.

$$C_t^b = -D_{t-1} + \frac{D_t}{1+r_t} + W_t^b h_t^b + \int_0^1 \Pi_t^b \frac{1}{P_t} - T_t^b \quad (1.8)$$

Thus, consumers of this type spend all their disposable income together with what is left from the money borrowed after repaying last period debt and interest on it. The fact, that borrowers' consumption is defined by the current income allows to consider this type of agents as standard Keynesian type.

For the labor supply the same equation as for the savers can be obtained from the utility maximization problem:

$$W_t = \frac{v_h^b(h_t^b)}{U_c^b(C_t^b)} \quad (1.9)$$

## 2.2 Firms

There is a continuum of firms measure one, where a fraction  $\lambda$  can set their prices at all periods while  $1 - \lambda$  firms sets the prices one period in advance.

Firms choose their optimal price by maximizing the infinite sum of profits subject to the demand function and the production function.<sup>4</sup>

$$E_t \sum_{t=0}^{\infty} \phi_t [p_t(j) y_t(j) - W_t P_t h_t(j)] \quad (1.10)$$

$$\text{s.t. } y_t(j) = Y_t \left( \frac{p_{jt}}{P_t} \right)^{-\theta}$$

$$y_t(j) = (G_t^P)^\zeta (h_t(j))^\eta$$

Where  $W_t = W(s)_t^{\chi_s} W(b)_t^{\chi_b}$  and  $\phi_t = \chi^s \phi_{1t}^s - (1 - \chi^s) \phi_{2t}^s$  is used as a discount factor and  $\phi_{1t}^s$  and  $\phi_{2t}^s$  are Lagrangian multipliers from utility maximization problem.

It is assumed that production of each type of good is obtained from a combination of labor  $h_t(j)$  needed to produce good of type  $j$  and productive government expenditures. The second type of public expenditures, productive spendings  $G_t^P$ , which are included directly in the productive function, can be interpreted (as in Agénor, 2005) as government investment in the infrastructure. The relative weights of these two inputs  $\zeta$  and  $\eta$  are assumed to satisfy the constant return of scale condition  $\zeta + \eta = 1$ .

<sup>3</sup>For greater details see Appendix 1.1

<sup>4</sup>For greater details see Appendix 1.3

## 2.3 Monetary policy

Central bank follows Taylor rule, taking into account non negative restriction on the nominal interest rate:

$$1 + i_t = \max\{1, (1 + r_t^n)(1 + \Pi_t)^{\phi_\pi}\} \quad (1.11)$$

Where  $r_t^n$  is natural interest rate,  $\Pi_t$  is the inflation rate and  $\phi_\pi > 1$  determines the degree of aggressiveness of monetary policy in setting interest rate with respect to changes in the inflation.

## 2.4 Fiscal policy

Fiscal policy is conducted by the government, who chooses total government expenditures  $G_t$  together with an amount and allocation of taxes among consumers. Two types of government expenditures are introduced: utility-enhancing spendings  $G_t^U$  (government consumption) and government investment  $G_t^P$  (productive spendings). Public spending is financed by taxes  $T_t^s$  and  $T_t^b$ , paid by savers and borrowers correspondingly, and by issuing government bonds  $B_t^g$ . Interest on public bonds  $i_t$  equals to the interest on private borrowings, so there won't be any possibilities for the arbitrage.

Taking the assumption about two types of public expenditures into account, government budget constraint in real terms can be written as follows:

$$G_t^U + G_t^P + \frac{B_{t-1}^g P_{t-1}}{P_{t-1} P_t} (1 + i_{t-1}) = \frac{B_t^g}{P_t} + \chi_s T_t^s + (1 - \chi_s) T_t^b$$

Or after simplifying:

$$G_t + D_{t-1}^g (1 + r_{t-1}) = D_t^g + T_t \quad (1.12)$$

with  $D_t^g = \frac{B_t^g}{P_t}$  being real value of debt and  $T_t = \chi_s T_t^s + (1 - \chi_s) T_t^b$

This equation postulates that current government expenditures and debt repayment are financed by issuing new debt and by taxes on both types of consumers.

Total government expenditures  $G_t$  has the same structure as private consumption, represented by the Dixit-Stiglitz aggregator:

$$G_t = \left( \int_0^1 g_t(j)^{\theta-1/\theta} dj \right)^{\theta/\theta-1} \quad (1.13)$$

The demand for particular good, as well as for private consumption, is defined from the minimization problem of expenditures subject to the definition of the

aggregate index, presented above:<sup>5</sup>

$$g_{jt} = G_t \left( \frac{p_{jt}}{P_t} \right)^{-\theta} \quad (1.14)$$

The sequence of government expenditures, taxes and debt, chosen by fiscal authorities each period, should satisfy intertemporal budget constraint. This constraint postulates, that on the infinite horizon the current value of debt should be covered by present value of future surpluses and future debt.

$$D_t^g = \sum_{\tau=0}^{\infty} \frac{T_{t+\tau} - G_{t+\tau}}{(1+r)^\tau} + \lim_{N \rightarrow \infty} \frac{D_{t+N}^g}{(1+r)^N} \quad (1.15)$$

The sustainability of the government debt is ensured by the transversality condition (no Ponzi game condition), precising that the discounted value of public debt equals to zero in the infinite horizon:

$$\lim_{N \rightarrow \infty} \frac{D_{t+N}^g}{(1+r)^N} = 0$$

In each period of time present value of future government surpluses should be equal to the current level of the public debt.

### 3 Market equilibrium conditions

Aggregate market clearing condition of the **goods market** postulates that all output produced is purchased by two types of consumers and the government:

$$Y_t = C_t + G_t \quad (1.16)$$

Where

$$C_t = \chi_s C_t^s + (1 - \chi_s) C_t^b$$

Equilibrium on the **bond market** is ensured by the following equation:

$$B_t^g + (1 - \chi_s) B_t^b = -\chi_s B_t^s \quad (1.17)$$

It determines that negative borrowings of the savers (savings), taking into account the share of savers in the economy, cover both private and public borrowings. All the government bonds are assumed to be bought by savers.

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<sup>5</sup>For greater details see Appendix 1.1

For the equilibrium on the **labor market**, the labor supply should be equal to labor demand for each type of consumers, which can be represented as follows:<sup>1</sup>

$$W_t \left( \frac{h_t^b}{h_t^s} \right)^{1-\chi_s} = \frac{\nu_h^s(h_t^s)}{U_c^s(C_t^s)}$$

$$W_t \left( \frac{h_t^s}{h_t^b} \right)^{\chi_s} = \frac{\nu_h^b(h_t^s)}{U_c^b(C_t^b)}$$

Where  $W_t = W(s)_t^{\chi_s} W(b)_t^{\chi_b}$  and  $\chi_b = 1 - \chi_s$ .

However, according to the Walras' Law only two market equilibrium conditions (for example (1.16) and (1.17)) are needed to ensure equilibrium on these markets.

### 3.1 Equilibrium dynamics

In order to analyze the effect of the deleveraging shock on inflation and output in the short run, the model needs to be log-linearized around the steady state with zero inflation. The steady state values are denoted by bar and the deviations from the steady state by hat. It should be noted, that the steady state value of real debt limit is assumed to be equal to the low level of debt  $\bar{D} = D^{low}$ . This means that the economy is not in the steady state before the shock and comes to it after debt limit falls from the high value to the low one. Below the main log-linearized equations of the model are presented.<sup>2</sup>

#### Log-linear model

**Borrower's budget constraint:**  $\widehat{C}_t^b = \widehat{Y}_t^b + \beta \gamma_D \widehat{D}_t - \gamma_D \widehat{D}_{t-1} + \gamma_D \pi_t - \beta (i_t - E_t \pi_{t+1} - \bar{r})$

**Saver's Euler equation:**  $\widehat{C}_t^s = E_t \widehat{C}_{t+1}^s - \sigma (i_t - E_t \pi_{t+1} - \bar{r})$

**Aggregate consumption:**  $\widehat{C}_t = \chi_s \widehat{C}_t^s + (1 - \chi_s) \widehat{C}_t^b$

**Savers' labor supply:**  $\widehat{W}_t = \omega^s \widehat{h}_t^s(i) + \sigma^{s-1} \widehat{C}_t^s$

**Borrowers' labor supply:**  $\widehat{W}_t = \omega^b \widehat{h}_t^b(i) + \sigma^{b-1} \widehat{C}_t^b$

**Labor market clearing condition:**  $\widehat{h}_t = \chi_s \widehat{h}_t^s + (1 - \chi_s) \widehat{h}_t^b$

**Production function:**  $\widehat{Y}_t = \eta \widehat{h}_t + \zeta g^p$

**Phillips curve:**  $\pi_t = \kappa(1 + \varphi) \widehat{Y}_t - \kappa \zeta g_t^p - \varphi \kappa \widehat{G}_t + E_{t-1} \pi_t$

**Resource constraint:**  $\widehat{Y}_t = \widehat{C}_t + \widehat{G}_t$

**Taylor rule:**  $i_t = \max(0, r_t^n + \phi_\pi \pi_t)$

**Government budget constraint:**  $\widehat{G}_t + \widehat{D}_{t-1}^g (1 + \bar{r}) = \widehat{D}_t^g + \overline{D}^g (i_t - E_t \pi_{t+1} - \bar{r}) + \widehat{T}_t$

Where  $\bar{r} = \log \beta^{-1}$ ,  $\pi_t = \log \frac{P_t}{P_{t-1}}$ ,  $i_t = \log(1 + i_t)$ ,  $\gamma_D = \frac{\bar{D}}{\bar{Y}}$ ,  $\widehat{C}_t^s = \log \frac{C_t^s}{\bar{Y}}$   
 $\widehat{C}_t^b = \log \frac{C_t^b}{\bar{Y}}$ ,  $\widehat{h}_t^b = \log \frac{h_t^b}{\bar{Y}}$ ,  $\overline{D}^g = \log \frac{D^g}{\bar{Y}}$ ,  $\kappa = \left( \frac{\lambda}{1-\lambda} \right) (\omega + \sigma^{-1})$ ,  $\varphi = \frac{\sigma^{-1}}{\sigma^{-1} + \omega}$

<sup>1</sup>For greater details see Appendix 1.3

<sup>2</sup>For greater details see Appendix 2

$$\omega^s = \frac{\bar{v}_{hh}^s \bar{h}}{\bar{v}_h^s}, \omega^b = \frac{\bar{v}_{hh}^b \bar{h}}{\bar{v}_h^b}, \sigma^s = -\frac{\bar{U}_c^s}{U_{cc}^s \bar{Y}}, \sigma^b = -\frac{\bar{U}_c^b}{U_{cc}^b \bar{Y}}, \omega^s = \omega^b = \omega > 0, \sigma^s = \sigma^b = \sigma > 0$$

Considering model in log-deviations from the steady state, it is convenient to represent two types of expenditures as shares of the total government spendings.

Thus, in log-deviations from the steady state:  $\widehat{G}_t = g_t^P + g_t^U$

Let  $\psi \in (0, 1)$  be a share of of government investment in total government spending. Then we have:  $g_t^P = \psi \widehat{G}_t$  and  $g_t^U = (1 - \psi) \widehat{G}_t$

The resource constraint can, thus, be expressed as follows:

$$\widehat{Y}_t = \widehat{C}_t + \widehat{G}_t = \widehat{C}_t + g_t^U + g_t^P = \zeta g_t^P + \eta \widehat{h}_t$$

## 4 The derivation of the fiscal multiplier and comparative statics

### 4.1 Short run and long run dynamics

To simplify the analysis, economy is split into the "short run" and the "long run" period. An unexpected shock occurs in the short run, while in the long run economy returns to the steady state with a low value of debt limit.

For the long run equilibrium of the model from Phillips curve  $\pi_t = \kappa(1 + \varphi)\widehat{Y}_t - \kappa\zeta g_t^P - \varphi\kappa\widehat{G}_t + E_{t-1}\pi_t$  it is clear, that output gap equals to zero in the long run  $\widehat{Y}_L = 0$  when the economy returns to flexible price equilibrium. Moreover, applying that in the long run interest rate equals to its steady state value - natural interest rate  $i_L = r_L^n = \bar{r}$ , we obtain zero long run inflation  $\pi_L = 0$ .

In the short run the shock occurs, lowering the borrowing limit. Thus, in order to analyze an effect of the shock, the model is rewritten, applying that all variables in the current period  $t$  are assumed to be the short run variables, while next period variables  $t + 1$  are at their long run value.

The budget constraint of a borrower transforms into the following<sup>1</sup>:

$$\widehat{C}_S^b = \widehat{Y}_S^b - \widehat{D} + \gamma_D \pi_S - \gamma_D \beta (i_t - \pi_L - \bar{r}) - \widehat{T}_S^b \quad (2.1)$$

where  $\widehat{D} = \frac{\beta D^{high} - \bar{D}}{\bar{Y}}$

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<sup>1</sup>For greater details see Appendix 3.1

The borrower's consumption has the following representation<sup>2</sup>:

$$\widehat{Y}_S^b = \mu \widehat{Y}_S + \sigma^{-1}(\omega^{-1}\chi_b^{-1}\chi_s - 1)\widehat{G}_S - (1+\omega)\zeta\eta^{-1}g_S^P + \omega^{-1}\chi_b^{-1}\chi_s\sigma^{-1}(\widehat{C}_L^s - \sigma(i_S - E_t\pi_L - \bar{r})) \quad (2.2)$$

with  $\mu = (1 + \omega^{-1})(\omega\eta^{-1} + \sigma^{-1}) - \sigma^{-1}\omega^{-1}\chi_b^{-1} > 0$ .<sup>3</sup>

Thus, the deviation of borrower's consumption from the steady state in the short run positively depends on the deviation of his disposable income and current inflation, as it reduces real value of debt. While real interest rate, paid for borrowings, and the deleveraging shock itself (the drop of the debt limit from its high value to steady state) have a negative impact on the consumption. Moreover, there appears to be an additional negative impact of the productive government expenditures on the borrowers' consumption, coming from the substitution effect between public investment and labor. This negative effect falls with the higher weight of the labor in the production function  $\eta$ .

As for the consumption of the saver, given by Euler equation in terms of short run and long run variables, it looks as follows:

$$\widehat{C}_S^s = \widehat{C}_L^s - \sigma(i_S - \pi_L - \bar{r}) \quad (2.3)$$

As opposed to the borrowers consumption, saver's consumption depends on the expected future consumption and doesn't depend on the disposable income. This once again illustrates the different nature of the two types of consumers and the fact, that fiscal policy, in terms of taxes, doesn't affect the consumption choice of savers. As it will be shown later Ricardian equivalence holds for savers.

## 4.2 Aggregate demand and the effect of the deleveraging shock

To derive the aggregate demand, first, the expressions of consumption of two types of agents in the short run are substituted into the new resource constraint. Second, implying the fact that  $\widehat{C}_L^s = 0$  and  $\pi_L = 0$ , the following expression for the aggregate demand in the short run can be obtained:<sup>4</sup>

$$\widehat{Y}_S = -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \mu\chi_b}(i_S - \bar{r}) - \frac{\chi_b}{1 - \mu\chi_b}\widehat{D} - \frac{\chi_b}{1 - \mu\chi_b}\widehat{T}_S^b +$$

<sup>2</sup>For greater details see Appendix 2.3

<sup>3</sup>For the positivity of  $\mu$  there should be a sufficient number of constrained consumers, or more precisely their share should satisfy the following condition  $\chi_b > \frac{\sigma^{-1}\omega^{-1}}{(1 + \omega^{-1})(\omega\eta^{-1} + \sigma^{-1})}$  which for the calibrated parameters is equal to 0.32

<sup>4</sup>For greater details see Appendix 3.3

$$+\frac{\chi_b\gamma_D}{1-\mu\chi_b}\pi_S + \frac{1+\sigma^{-1}\omega^{-1}\chi_s-\chi_b\sigma^{-1}}{1-\mu\chi_b}\widehat{G}_S - \frac{(1+\omega)\zeta\eta^{-1}\chi_b}{1-\mu\chi_b}g_S^P \quad (2.4)$$

Comparing to the baseline model of Eggertsson and Krugman (2012), now there appears to be a positive effect of total government spending which looks as before, but with a different  $\mu$  and an additional negative impact of productive government spending on the aggregate demand. The new value of  $\mu$ , parameter measuring the sensitivity of borrower's income to the change in the output, has become a little bit higher as now labor has to be increased by more than 1 in order to increase output by one, due to the decreasing returns to scale of the production function with respect to labor. It varies from 0.13 to 1.13 instead of 0 and 1 with an increase of borrowers in the economy from  $\frac{1}{3}$  to  $\frac{1}{2}$  correspondingly. Moreover, a condition on the share of borrowers is introduced as in the baseline model in order to ensure, that aggregate output will fall in response to an increase in the nominal interest rate. For this to be true  $1-\mu\chi_b > 0$ , and taking into account the change in the  $\mu$  it will mean that  $\chi_b > 0.64$ .<sup>5</sup>

The negative impact of government investment disappears, when there are no borrowers in the economy,  $\chi_s \rightarrow 1$ , as it comes from an impact of this part of spendings on borrowers' labor income. This effect decreases with the higher elasticity of the output with respect to the labor  $\eta$  and increases with the weight of public investment in the production function.

Applying the definition of the natural interest rate to the equation, the **IS curve** can be rewritten as follows:

$$\widehat{Y}_S = -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1-\mu\chi_b}(i_S - r_S^n) \quad (2.5)$$

Thus, with a fall in the nominal interest rate, savers are encouraged to consume more, than under a previous level of the interest rate. The higher consumption of savers results in the higher income of both savers and borrowers. Due to the fact that borrowers are liquidity constrained, they consume all their additional income, increasing the demand and, thus, the output once again.

The natural rate is an interest rate in the flexible price equilibrium, when output gap is equal to zero.

Where

$$r_S^n = \bar{r} - \frac{\chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}\widehat{D} + \frac{\gamma_D\chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}\pi_S -$$

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<sup>5</sup>For this condition to be satisfied  $\chi_b < \frac{1+\sigma^{-1}\omega^{-1}}{(1+\omega)(\eta^{-1}+\sigma^{-1}\omega^{-1})}$ , or using the calibration presented below  $\chi_b < 0.67$

$$-\frac{\chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}\widehat{T}_S + \frac{1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b\sigma^{-1}}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}\widehat{G}_S - \frac{(1 + \omega)\zeta\eta^{-1}\chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}g_S^P$$

This rate depends negatively on the value of the deleveraging shock, because due to the cut in consumption of borrowers natural interest rate should fall to stimulate the consumption of savers.

In the long run, as it was already mentioned, natural rate equals to the steady state value  $\bar{r}$ . In the short run it increases with inflation and government expenditures, while deleveraging shock and taxes diminish natural interest rate.

Thus, taking into account Taylor rule, two possible regimes emerge. In the case of a relatively small shock natural interest rate will remain positive and nominal interest rate will be able to fall to stimulate the demand and offset negative impact of the shock. On the other hand, if the shock of the debt limit is high enough, natural interest rate can become negative and zero lower bound would become binding. In this case, output gap will be negative, with output falling below its potential level. Eggertsson and Krugman has illustrated, that in this case AD curve becomes upward-sloping, as inflation now increases output due to its negative effect on the real value of debt.<sup>6</sup>

In order to evaluate an impact of the shock on the output and prices one needs to combine AD and AS curves, to take into account the response of the aggregate supply.

The aggregate supply in the short run is set by the **Phillips curve**, which will have the following representation:

$$\pi_t = \kappa(1 + \varphi)\widehat{Y}_t - \kappa\zeta g_t^P - \varphi\kappa\widehat{G}_t + E_{t-1}\pi_t \quad (2.6)$$

$$\text{where } \kappa = \frac{1}{\eta} \frac{\lambda}{1 - \lambda} (1 + \omega), \quad \varphi = \frac{\eta\sigma^{-1}}{1 + \omega}$$

Besides the change in parameters, now prices are affected by productive part of government expenditures as well as by the total expenditures. Productive expenditures are included separately in the Phillips curve due to its effect on aggregate supply. This negative impact  $\kappa\zeta$  in the Phillips curve comes from the fact that increasing productive government expenditures raise marginal product of labor, decreasing, therefore, marginal costs and, thus, prices.

Combining AS and AD equations we can obtain an expression of the output gap in the short run:<sup>7</sup>

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<sup>6</sup>Eggertsson and Krugman (2012) assume, that the slope of the AD curve, when it is upward-sloping, is higher, than the slope of AS curve to ensure the equilibrium stability. For this to be true the following condition must be satisfied:  $\frac{1 - \mu\chi_b}{\chi_b\gamma_D} > \kappa$ , thus  $\chi_b < \frac{1}{\kappa\gamma_D + \mu}$ . The values used in the calibration presented below satisfy this condition.

<sup>7</sup>For greater details see Appendix 3.3

$$\begin{aligned}
\widehat{Y}_S = & -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}(i_s - \bar{r}) - \frac{\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{D} - \\
& -\frac{\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{T}_S^b + \frac{1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b(\sigma^{-1} + \gamma_D\varphi\kappa)}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{G}_S - \\
& -\frac{\kappa\zeta\gamma_D\chi_b + (1 + \omega)\zeta\eta^{-1}\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}g_S^p \\
\widehat{Y}_S = & -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}(i_s - \bar{r}) - \frac{\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{D} - \\
& -\frac{\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{T}_S^b + \\
& +\frac{1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b(\sigma^{-1} + \gamma_D\varphi\kappa) - \psi\zeta\chi_b(\kappa\gamma_D + (1 + \omega)\eta^{-1})}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{G}_S \quad (2.7)
\end{aligned}$$

Negative impact of productive government expenditures on the demand is supplemented by a negative effect of productive expenditures on prices and both of them disappear, when there are no borrowers in the economy. Productive expenditures affect output gap, because current consumption of borrowers depends positively on the current level of inflation, which decreases real value of debt. Thus, this additional impact on the output disappears, when there are only savers in the economy.

Depending on the size of the deleveraging shock two cases can be considered: when nominal interest rate remains positive and when zero lower bound constraint becomes binding. An upward-sloping demand curve appears as in the model in the case of the zero lower bound. So an additional condition on the share of borrowers  $\chi_b$  should be satisfied to precise, that AD curve is steeper than the AS curve, in order to ensure an equilibrium stability.<sup>8</sup>

### 4.3 Comparison of the fiscal multiplier under two cases

#### Case of a positive nominal interest rate

If zero lower bound doesn't bind the Central Bank follows the Taylor rule for nominal interest rate:

$$i_t = r_t^n + \phi_\pi \pi_t$$

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<sup>8</sup>For this to be true the following condition should be satisfied:  $\frac{1 - \mu\chi_b}{\chi_b\gamma_D} > (1 + \varphi)\kappa$ .

Thus  $\chi_b < \frac{1}{\kappa\gamma_D(1 + \varphi) + \mu}$ . For the calibration presented below this condition transforms into  $\chi_b < 0.52$ .

with  $\phi_\pi > 1$

Substituting the definition of positive nominal interest rate and the definition of the inflation in the short run from the Phillips curve in the IS curve equation, the following expression for the output gap can be obtained:<sup>9</sup>

$$\widehat{Y}_S = -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \mu\chi_b}(i_S - r_S^n) \quad (2.8)$$

$$\widehat{Y}_S = \frac{\phi_\pi(\kappa\zeta\psi + \varphi)(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)}{1 - \mu\chi_b + \phi_\pi\kappa(1 + \varphi)(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)}\widehat{G}_t \quad (2.9)$$

With the following fiscal multiplier:

$$\text{mult}_G = \frac{\phi_\pi(\kappa\zeta\psi + \varphi)(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)}{1 - \mu\chi_b + \phi_\pi\kappa(1 + \varphi)(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)} \quad (2.10)$$

The parameter values used for calibration are:<sup>10</sup>  $\beta = 0.99$ ,  $\kappa = 0.54$ ,  $\varphi = 0.31$ ,  $\sigma = 2$ ,  $\omega = \frac{1}{2}$ ,  $\gamma_D = 1$ ,  $\chi_b = \frac{1}{3}; \frac{1}{2}$ . These values specify the parameters, which are functions of structural parameters  $\kappa = 0.33$  and  $\phi_\pi = 1.5$  as in Christiano et al. (2011), the value of elasticity of output with respect to public productive investment  $\zeta = 0.08$  and  $\psi = 0.5$  as in Roulleau-Pasdeloup (2013). Using this calibration the values of 0.4 and 0.36 for  $\chi_b = \frac{1}{2}$  and  $\chi_b = \frac{1}{3}$ , correspondingly, can be obtained. These estimates are lower, than in the paper of Roulleau-Pasdeloup (2013): author has found a fiscal multiplier to be equal to 0.57 in normal times when zero lower doesn't bind and stimulus package composes equally of utility-enhancing and productive expenditures.<sup>11</sup>

This brings up a question of how sensible is the value of the fiscal multiplier in the "normal" case to the share of borrowers in the economy. When there are only savers in the economy the value of the multiplier equals to 0.38, thus, being higher than in the case when economy consists by a third of borrowers and lower when there is a half of borrowers in the society. By taking partial derivative and analyzing how it changes in the sign the threshold value of  $\chi_b$  can be found. It is equal to 0.13: for higher values multiplier is increasing with respect to the share of borrowers and for lower - it is decreasing. This explains the magnitudes presented above. However, it should be noted, that the corner case of  $\chi_b = 0$  and values of  $\chi_b < 0.32$  don't satisfy the condition needed to have  $\mu$  being positive. Thus, for possible values of the borrowers' share fiscal multiplier is increasing with respect to this share.

<sup>9</sup>For greater details see Appendix 3.3

<sup>10</sup>These values are used for example in Krugman, Eggertsson (2012)

<sup>11</sup>Roulleau-Pasdeloup (2013)

The effect of all other parameters was analyzed and the summary of this analysis is presented in the **Table 1** below. Although a comparison of the two cases is presented in the separate section it is worth mentioning some key relationships between parameters and the fiscal multiplier. In the context of positive interest rate and, thus, standard downward-sloping AD curve, fiscal multiplier is increasing with price rigidities. As with a lower price flexibility increase in the aggregate demand would lead to a lower inflation, which is contractionary under "normal" circumstances. Moreover, increase in the share of productive expenditures and the degree of their productivity augment multiplier. The increase in public investment gives an additional push to aggregate supply, while the rise in aggregate supply is expansionary in the normal case of downward-sloping AD curve.

**Table 1. Impact of parameters' values on the size of the fiscal multiplier under positive interest rate**

	$\kappa$	$\chi_b$	$\varphi$	$\gamma_D$	$\sigma$	$\omega$	$\zeta$	$\psi$
<b>sign</b>	-	+/-	+	-	+	-	+	+
$\chi_b \uparrow$	-	+	+	+	-	+	+	+

### Case of a zero lower bound

If deleveraging shock is sufficiently high, zero lower bound becomes binding due to the deflation effect. The following expression of the short run output gap can be derived in this case:<sup>12</sup>

$$\widehat{Y}_S = \Gamma - \frac{\chi_b}{1 - \chi_b(\mu + \gamma_D \kappa(1 + \varphi))} \widehat{D} - \frac{\chi_b}{1 - \chi_b(\mu + \gamma_D(\kappa + \varphi - 1))} \widehat{T}_S^b + \frac{1 + \sigma^{-1} \omega^{-1} \chi_s - \chi_b(\sigma^{-1} + \gamma_D \varphi \kappa)}{1 - \chi_b(\mu + \gamma_D \kappa(1 + \varphi))} \widehat{G}_S - \frac{\kappa \zeta \gamma_D \chi_b + (1 + \omega) \zeta \eta^{-1} \chi_b}{1 - \chi_b(\mu + \gamma_D \kappa(1 + \varphi))} g_S^p \quad (2.11)$$

$$\text{with } \Gamma = \frac{\chi_s(\sigma + \omega^{-1}) + \chi_b \gamma_D \beta}{1 - \chi_b(\mu + \gamma_D \kappa(1 + \varphi))} \bar{r} \quad (2.12)$$

This expression shows that there exist two effects of productive spendings on the output: through total government expenditures and a separate negative effect. For the analysis of the fiscal multiplier an other representation is more convenient, presenting an impact of total government expenditures, taking into account that

<sup>12</sup>For greater details see Appendix 3.5

$$g_t^P = \psi \widehat{G}_t.$$

Thus, it becomes clear, that government spending multiplier, putting aside the method of financing (assuming that an increase is financed by taxes on savers or debt) can be written as follows:

$$\text{mult}_G = \frac{1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b(\sigma^{-1} + \gamma_D\varphi\kappa) - \psi\zeta\chi_b(\kappa\gamma_D + (1 + \omega)\eta^{-1})}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}$$

It should be noted, that although multiplier of government expenditures (both productive and utility enhancing) seems to be positive, an additional negative effect comes from an impact of productive spending. This effect is a combination of the negative impact public investment has on the aggregate demand and on prices. Both these effects, as it was already mentioned, disappear, when there are no borrowers in the economy. First negative effect  $(1 + \omega)\zeta\eta^{-1}$  comes from an effect of productive expenditures on the wage and hours worked through increased marginal product of labor, making workers more productive. The negative effect on price comes from a negative effect of productive expenditures on marginal cost. It affects output gap through borrowers, as their current consumption depends positively on the current level of inflation, which is negatively affected by the production part of the expenditures.

Government multiplier would be positive if  $1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b(\sigma^{-1} + \gamma_D\varphi\kappa) > \psi\zeta\chi_b(\kappa\gamma_D + (1 + \omega)\eta^{-1})$  taking into account the assumption postulated before, that denominator of the multiplier is always positive. This condition corresponds to the case, when positive effect of productive spending on the output through the demand is higher than its effect on the aggregate supply.

The results of calibration give the value of multiplier, varying from 1.96 to 13.8 with increased share of borrowers from  $\frac{1}{3}$  to  $\frac{1}{2}$ . The dramatic increase in the value of the multiplier can be explained by the change in the value of  $\mu$  from 0.13 to 1.13. Although the multiplier seems to be increasing with respect to the share of borrowers an analysis of the sign of partial derivative shows that it changes the sign from negative to positive when their share increases from one third to one half. The threshold level of  $\chi_b$  was found to be equal to 0.34, which explains why the share of borrowers has a negative impact on the multiplier at  $\frac{1}{3}$  and a positive at  $\frac{1}{2}$ . The result presented above can be explained by the fact that negative impact of productive expenditures disappears when there are no borrowers in the economy. So, on the one hand, multiplier is increasing with respect to the share of borrowers, when there are no productive spendings, on the other hand, in the model with productive spendings due to the method of derivation of the aggregate output

negative effect of productive spendings is proportional to the share of borrowers introducing the ambiguity of the effect of  $\chi_b$  on the value of the multiplier.

However, the magnitude of the multiplier and the threshold level depend crucially on the values of parameters used for calibration. Before turning to the effect, these parameters have on the multiplier, it should be noted that, opposed to the positive nominal interest rate case, the multiplier under zero lower bound can become negative for sufficiently high share of productive investment in the total government expenditures. For this to be true all fiscal stimulus should consist of productive expenditures:  $\psi = 1$  for the multiplier to be negative when  $\chi_b = 0.52$ .<sup>13</sup> Roulleau-Pasdeloup (2013) has found that consumption is crowded out by public spendings if productive part of fiscal stimulus is higher than 0.64.<sup>14</sup> For this, the share of productive expenditures bring multiplier below zero the share of borrowers should be higher than 0.65 which doesn't satisfy several restricting conditions on  $\chi_b$  introduced earlier.

As for the parameters' impact on the value of multiplier a negative impact of the both share and productivity of public investment should be underlined. This impact decreases with the rise in the share of constrained consumers. Moreover, from the partial derivatives presented below it can be seen that this negative effect on the multiplier disappears when there are no borrowers in the economy. This can be explained by the method used to derive the multiplier, as productive expenditures affect output only through the inflation (which appears in the expression of the consumption of borrowers) and borrowers' labor income.

The results of the calibration concerning the effects of parameters are presented in the **Table 2**.

**Table 2. Impact of parameters' values on the size of the fiscal multiplier under zero lower bound**

	$\kappa$	$\chi_b$	$\varphi$	$\gamma_D$	$\sigma$	$\omega$	$\zeta$	$\psi$
<b>sign</b>	+	+/-	+	+	-	-	-	-
$\chi_b \uparrow$	+	+	+	+	-	-	-	-
$\chi_b = 0$	0			0			0	0

<sup>13</sup>This is the value of the share of borrowers from the condition  $\chi_b < 0.52$  needed to ensure that AD curve is steeper than AS curve

<sup>14</sup>Roulleau-Pasdeloup (2013), p. 21

This table also illustrates what effects disappear, when there are only savers in the economy. Although the case of  $\chi_b$  being equal to 0 doesn't satisfy the conditions on the share of borrowers  $\chi_b$ , it still shows the transmission channels of parameter effects.

#### 4.4 Comparison of the two cases: the role of zero lower bound

First of all, it should be noted, that the results presented above correspond to the results of Eggretsson, Krugman (2012), Christiano et al. (2011): the value of fiscal multiplier being higher in the zero lower bound, than in the normal case, despite the introduction of productive government spending. However, this result depends crucially on the calibration used and the assumption about the share of productive spendings. The difference with results presented in the work of Roulleau-Pasdeloup (2013), where author has found a lower multiplier under zero lower bound than in "normal" times, can be explained by the introduction of an additional channel, by which fiscal policy proceeds through the presence of borrowers, which partially offsets the negative impact of the productive expenditures and facilitates the higher impact of public expenditures on the aggregate demand than on the aggregate supply.

As for economic characteristics, higher **price flexibility** (higher  $\kappa$ , steeper AS curve) assures higher increase in the output under zero lower bound as it allows prices to change more in response to an increase in the government expenditures leading to a higher increase in prices and to a higher inflation, which is expansionary in this framework. In the "normal" case, and, thus, downward-sloping AD curve, on the contrary less price flexibility (lower  $\kappa$ ) is better for the size of the fiscal multiplier as prices change less.

As for the **share of borrowers**  $\chi_b$  it has non-linear effect on the multiplier, negatively affecting it when this share is below threshold level and affecting it positively when it is above this threshold. This threshold level highly depends on the values of parameters used in the calibration and its relevance depends on whether this threshold level is in the interval provided by conditions on the  $\chi_b$ . Anyway, this non linearity is important to understand how the prerequisite about two types of agents and the introduction of productive government spending in the fiscal stimulus interact with each other.

The results concerning the effect of  $\varphi$ , a coefficient in the AS equation in front of the government expenditures and in front of the output, are the same in the case of zero lower bound and in the normal case. Recalling that Phillips curve

has the following representation:

$$\pi_t = \kappa(1 + \varphi)\widehat{Y}_t - \kappa\zeta g_t^P - \varphi\kappa\widehat{G}_t + E_{t-1}\pi_t$$

$$\text{where } \kappa = \frac{1 - \lambda}{\eta(1 - \lambda)}(1 + \omega), \varphi = \frac{\eta\sigma^{-1}}{1 + \omega}$$

It could be seen that the results concerning the sign can't be compared with the case of zero lower bound without public investment as in the second model it stands for a different effect on the prices. In the baseline model of Eggertsson, Krugman (2012)  $\varphi$  stands for the sensitivity of the prices to the change in government expenditures, and the higher  $\varphi$  the lower are prices for a given level of government expenditure. Which in it's turn increases the real value of debt having a contractionary impact on the economy due to the "paradox of toil". In the model with productive expenditures it is also included in the coefficient in front of the output, changing the slope of the AS curve. The positive effect of  $\varphi$  can be explained by the fact, that in the case of the zero lower bound this effect works as increased flexibility of prices, compensating the negative effect of  $\varphi$  through the change in prices, due to the change in total government expenditures. While in the case of the positive interest rate, with standard downward sloping AD curve, decrease in prices, due to increased aggregate supply in the response to fiscal stimulus, is greater, than jump in prices, due to increase in aggregate demand under values of parameters used. As this result is very sensitive to the calibration a further investigation is needed and possibly other expression for the Phillips curve to specify precisely the role of this parameter in the framework.

The effect of the **debt to GDP ratio**  $\gamma_D$  is different under zero lower bound and in "normal" case. This parameter appears in the expression of the borrower's consumption specifying, how consumption changes in response to the real interest rate. Under zero lower bound its effect is positive, as the increase in prices, which follows an expansionary fiscal policy, would result in a higher consumption of the borrowers due to the lower level of the real debt, while nominal interest rate is zero. This effect is the same, as in the case of the zero lower bound without public investment. In the case of a positive nominal interest rate this parameter is a part of the coefficient in front of the interest rate in the IS curve, which specifies negative relationship between the output gap and the deviation of the nominal interest rate from its steady state value. Thus, in this case the negative impact of  $\gamma_D$  coming from the fact that the higher is the interest rate on the debt of borrowers the lower is their consumption.

An impact of  $\sigma$ , an elasticity of intertemporal substitution, has a different sign under two cases considered. It has a negative impact under zero lower bound.

With a higher  $\sigma$  labor supply is less responsive to the consumption, or putting it in another way with an increase in wage under higher  $\sigma$  workers tend to work more and increasing their consumption by less. In the case of the positive interest rate there is an effect coming from the behaviour of the savers and effect of public spendings on the prices: higher government expenditures increase aggregate supply decreasing price and lowering nominal interest rate, with a higher  $\sigma$  it will lead to a bigger increase in the current consumption of the savers. Of course this is only an impact from aggregate supply side. The other effect of increased government expenditures is captured by the other parameters in the multiplier. In the positive interest rate case an impact of an elasticity of intertemporal substitution is decreasing with respect to the share of borrowers because the positive effect of  $\sigma$  is coming through the savers' channel. In the case of zero lower bound this impact is coming through borrowers and becoming more negative as their share in the economy increases.

The effect of  $\omega$  (measures the curvature of the disutility of labor), is negative in both cases, increasing in the absolute value with the higher share of borrowers. The higher  $\omega$ , the lower labor supply elasticity) leads to a lower increase in labor supply in response to an increase in the wage, which decreases the labor income of borrowers taking wage as given. An impact of an increase in government expenditures on the wage level is captured by the other combination of parameters). This effect could be seen from an expression of the labor income of borrowers:

$$\widehat{Y}_S^b = \mu \widehat{Y}_S + \sigma^{-1} (\omega^{-1} \chi_b^{-1} \chi_s - 1) \widehat{G}_S - (1 + \omega) \zeta \eta^{-1} g_S^P + \omega^{-1} \chi_b^{-1} \chi_s \sigma^{-1} (\widehat{C}_L^s - \sigma (i_S - E_t \pi_L - \bar{r}))$$

$$\text{with } \mu = (1 + \omega^{-1}) (\omega \eta^{-1} + \sigma^{-1}) - \sigma^{-1} \omega^{-1} \chi_b^{-1} > 0$$

As it was already mentioned, the effect of the share of productive expenditures and the productivity of public investment is different under zero lower bound and positive interest rate as well. As expected in the case of zero lower bound increased share of productive expenditures of their productivity leads to a bigger increase in aggregate supply, comparing to the baseline model without productive investment, lowers the magnitude of the multiplier partially offsetting an increase in the output provided by an increase aggregate demand due to the upward-sloping demand curve. The same effect was shown by Roulleau-Pasdeloup (2013), who has illustrated that in the case of excess savings liquidity trap fiscal multiplier can be lower than in "normal" times and even negative for a sufficiently high share of productive investment. Although the negative effect of productive investment remains in the framework considered here, the magnitude of the multiplier in the

zero lower bound case is still higher than in normal case under calibration used. This could be explained by the introduction of constrained agents what provides an additional transmission channel of the fiscal policy. However, the multiplier still can become negative under sufficiently high share of productive spendings. This threshold level is higher than the one found by Roulleau-Pasdeloup (2013) and decreases with the share of borrowers in the economy. Moreover, these two effects disappear when there are no borrowers in the economy. The result may emerge due to the methodology used for the derivation of the aggregate output, as these two effects come through an expression of the borrowers' consumption.

## 5 Conclusion

In the recent years the way government conducts the stimulating policies has become a topic of the interest since the monetary policy toolkit is limited in the case of the low nominal interest rates.

In this work productive government expenditures were introduced in the simple borrower-saver model to analyze how the assumption of debt constrained agents interacts with productive expenditures, providing an additional push to aggregate supply (which is contractionary in this framework). Introduction of borrowers whose consumption depends on the current income seems to increase the value of the multiplier under zero lower bound comparing to the case where homogeneous agents are considered<sup>1</sup>. Although, both the share and the productivity of public investment decrease the value of the multiplier, pointing out in favor of old Keynesian wasteful government expenditures. In the case of a positive nominal interest rate both public investment productivity and its share in the total expenditures positively affect fiscal multiplier working in the same direction with the introduction of two types of agents in the model. Moreover, an increase in the share of borrowers intensifies the effect of production expenditures on the multiplier value. Positive effect of public investments increases with the share of borrowers in the positive interest rate case. While, negative effect of this type of government expenditures can be partly compensated by the a sufficient number of borrowers in the economy.

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<sup>1</sup>see, for example, Roulleau-Pasdeloup (2013)

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## Appendix

### A.1. Derivation of the model equations

#### 1.1 Derivation of private and public demand

As the steps of derivation are the same for private and public demand here the derivation of the public demand on a differentiated good  $i$  would be presented. Government demand can be obtained analogously.

Consumer  $i$  of each type solves the following minimization problem:

$$\begin{aligned} & \min_{c_t^i(j)} \int_0^1 p_t(j) c_t^i(j) dj \\ & \text{s.t. } C_t^i \geq \left( \int_0^1 c_t^i(j)^{(\theta-1)/\theta} dj \right)^{\theta/(\theta-1)} \\ & \mathcal{L}_t^i = \int_0^1 p_t(j) c_t^i(j) dj + \mu_t^i \left( C_t^i - \left[ \int_0^1 c_t^i(j)^{\theta-1/\theta} dj \right]^{\theta/(\theta-1)} \right) \\ & \frac{\partial \mathcal{L}_t^i}{\partial c_t^i(j)} = p_t(j) - \mu_t^i \left( c_t^i(j)^{-1/\theta} \left[ \int_0^1 c_t^i(j)^{\theta-1/\theta} dj \right]^{1/\theta-1} \right) = 0 \end{aligned}$$

Taking into account that  $C_t^{1/\theta} = \int_0^1 c_t^i(j)^{\theta-1/\theta} dj^{1/\theta-1}$

We get:  $p_t(j) = \mu_t^i c_t^i(j)^{-1/\theta} C_t^{1/\theta} \implies c_t^i(j) = C_t^i \left( \frac{p_t(j)}{\mu_t^i} \right)^{-\theta}$

$$C_t^i = \left[ \int_0^1 \left( \left( \frac{p_t(j)}{\mu_t^i} \right)^{-\theta} C_t^i \right)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)} = \left( \frac{1}{\mu_t^i} \right)^{-\theta} \left[ \int_0^1 p_t^{1-\theta}(j) dj \right]^{\theta/(\theta-1)} C_t^i$$

Thus  $\mu_t^i = \left[ \int_0^1 p_t^{1-\theta}(j) dj \right]^{1/(\theta-1)} = P_t$

Substituting this in the demand function we have:

$$c_t^i(j) = C_t^i \left( \frac{p_t(j)}{P_t} \right)^{-\theta}$$

Or if aggregated by two types of consumers:

$$c_t(j) = C_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta}$$

Using the same procedure government demand can be obtained:

$$g_t(j) = G_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta}$$

## 1.2 Utility maximization problem

$$E_0 \sum_{t=0}^{\infty} \beta^t(i) [U^i(C_t(i)) - v_t^i(h_t(i)) + \vartheta_t^i(G_t^U)]$$

with  $i = s$  or  $b$

$$\text{s.t. } B_t^i(i) + W_t^i P_t h_t(i) + \int_0^1 \Pi_t(i) = (1 + i_{t-1})B_{t-1}(i) + P_t C_t(i) + P_t T_t(i)$$

$$(1 + r_t) \frac{B_t(i)}{P_t} \leq D_t(i)$$

$$\begin{aligned} \mathcal{L}_0^i = E_0 \sum_{t=0}^{\infty} \beta^t(i) [ & U^i(C_t(i)) - v_t^i(h_t(i)) + \vartheta_t^i(G_t^U) + \varphi_{1t}(i)(B_t^i(i) + W_t^i P_t h_t(i) + \int_0^1 \Pi_t(i) - \\ & - (1 + i_{t-1})B_{t-1}(i) - P_t C_t(i) - P_t T_t(i)) + \varphi_{2t}(i)(D_t - (1 + r_t) \frac{B_t(i)}{P_t})] \end{aligned}$$

$$\frac{\partial \mathcal{L}_t^i}{\partial C_t(i)} = 0 \iff U_c^i(C_t(i)) = \varphi_{1t}(i) P_t$$

$$\frac{\partial \mathcal{L}_t^i}{\partial h_t(i)} = 0 \iff v_c^i(h_t(i)) = \varphi_{1t}(i) P_t W_t$$

$$\frac{\partial \mathcal{L}_t^i}{\partial B_t(i)} = 0 \iff \varphi_{1t}(i) - \beta(i) E_t \varphi_{1,t+1}(i) (1 + i_t) - \varphi_{2t}(i) \frac{1 + r_t}{P_t} = 0$$

And the slackness condition:

$$\varphi_{2t}(i) \geq 0, D_t(i) \geq (1 + r_t) \frac{B_t(i)}{P_t}$$

and

$$\varphi_{2t}(i)(D_t - (1 + r_t)\frac{B_t(i)}{P_t}) = 0$$

### Savers' consumption and labor supply

For savers debt constraint is not binding, thus, the Lagrangian multiplier of this constraint  $\varphi_{2t}(s) = 0$  from the slackness condition. Taking this into account and substituting the definition of  $\varphi_{1t}(s)$  from the first order condition into the last one we get the Euler equation for the saver:

$$U_c^s(C_t^s) = \beta(1 + i_t) E_t[U_c^s(C_{t+1}^s) \frac{P_t}{P_{t+1}}]$$

And combining first two first order conditions labor supply function can be obtained:

$$W_t = \frac{\nu_h^s(h_t^s)}{U_c^s(C_t^s)}$$

### Borrowers' consumption and labor supply

In the case of borrowers debt limit constraint is binding, thus, borrowers consumption can be defined by budget constraint where real value of debt is expressed through the exogenous debt limit.

$$C_t^b = -D_{t-1} + \frac{D_t}{1 + r_t} + W_t^b h_t^b + \int_0^1 \Pi_t^b \frac{1}{P_t} - T_t^b$$

Labor supply of borrower can be obtained from the utility maximization problem as in previous case.

$$W_t = \frac{\nu_h^b(h_t^b)}{U_c^b(C_t^b)}$$

## 1.3 Firms

The demand on a good  $j$  consists of private and public demand of this good. Thus, market clearing condition for good  $j$  can be written as follows:

$$y_t(j) = c_t(j) + g_t(j)$$

Taking into account that

$$c_t(j) = C_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta}$$

$$g_t(j) = G_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta}$$

Output of good  $j$  can be written as follows:

$$y_t(j) = (C_t + G_t) \left( \frac{p_t(j)}{P_t} \right)^{-\theta}$$

Or taking into account market clearing condition for the goods market  $Y_t = C_t + G_t$

$$y_t(j) = Y_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta}$$

### Profit maximization problem and price setting

There is a continuum of firms measure one, where a fraction  $\lambda$  can set their prices at all periods while  $1 - \lambda$  firms sets their prices one period in advance.

Firms maximize the infinite sum of profits using  $\phi_t = \chi^s \phi_{1t}^s - (1 - \chi^s) \phi_{2t}^s$  as a discount factor. Where  $\phi_{1t}^s$  and  $\phi_{2t}^s$  are Lagrangian multipliers from utility maximization problem.

$$E_t \sum_{t=0}^{\infty} \phi_t [p_t(j) y_t(j) - W_t P_t h_t(j)]$$

s.t.

$$y_t(j) = Y_t \left( \frac{p_{jt}}{P_t} \right)^{-\theta}$$

$$y_t(j) = (G_t^P)^\zeta (h_t(j))^\eta$$

From the first order condition of this problem we get prices for both types of firms.

For the firms, who set their price freely:

$$p_t(1) = \frac{\theta}{\theta - 1} \frac{W_t}{[\eta (G_t^P)^\zeta (h_t(j))^{\eta-1}]} P_t$$

And for those who set their price one period in advance:

$$p_t(2) = E_{t-1} \left( \frac{\theta}{\theta - 1} \frac{W_t}{[\eta (G_t^P)^\zeta (h_t(j))^{\eta-1}]} P_t \right)$$

Where  $M = \frac{\theta}{\theta - 1}$  is a markup over marginal cost.

## A.2. Log-linearisation of the model

### 2.1 Linearisation of the demand side

Let us consider first **the budget constraint of the borrower**:

$$C_t^b = - \left( \frac{1 + i_{t-1}}{1 + r_{t-1}} \right) \frac{P_{t-1}}{P_t} D_{t-1} + \frac{D_t}{1 + r_t} + C - T_t^b$$

Log-linearizing it around steady state when  $D_t = D^{low} = \bar{D}$  and  $Y_t = \bar{Y}$

$$\begin{aligned} (C_t^b - \bar{C}^b) &= - (D_{t-1} - \bar{D}) + \bar{D} \frac{1 + i_{t-1}}{1 + r_{t-1}} \left( \log \frac{P_t}{P_{t-1}} - \log 1 \right) + \\ &+ \frac{1}{1 + \bar{r}} (D_t - \bar{D}) + (Y_t^b - \bar{Y}^b) - \bar{D} \frac{1}{1 + \bar{r}} \left( \log(1 + i_t) + E_t \log \frac{P_{t+1}}{P_t} - \log(1 + \bar{r}) \right) \end{aligned}$$

where  $Y_t^b = W_t h_t^b$ .

In the steady state the real interest rate is given by the discount factor of the patient consumer so that  $\beta = \frac{1}{1 + \bar{r}}$ , inflation at steady state is zero  $\frac{P_t}{P_{t-1}} = 1$  and that  $1 + r_t = (1 + i_t) \frac{P_t}{P_{t+1}}$ . Moreover, we have  $1 + i_{t-1} = 1 + r_{t-1}$  at the steady state.

$$\widehat{C}_t^b = \widehat{Y}_t^b + \beta \widehat{D}_t - \widehat{D}_{t-1} + \gamma_D \pi_t - \gamma_D \beta (i_t - E_t \pi_{t+1} - \bar{r})$$

where  $\widehat{C}_t^b = \log \frac{C_t^b}{\bar{C}^b}$ ,  $\widehat{Y}_t^b = \log \frac{Y_t^b}{\bar{Y}^b}$ ,  $\bar{r} = \log \beta^{-1}$ ,  $\pi_t = \log \frac{P_t}{P_{t-1}}$ ,  $i_t = \log(1 + i_t)$  and  $\gamma_D = \frac{\bar{D}}{\bar{Y}}$  and  $\widehat{Y}_t^b = \widehat{W}_t + \widehat{h}_t^b$

Combining first order conditions of the utility maximization problem of a saver the following **Euler equation of the saver** can be obtained:

$$U_c^s(C_t^s) = \beta (1 + i_t) E_t U_c^s(C_{t+1}^s) \left( \frac{P_t}{P_{t+1}} \right)$$

Log-linearizing this equation and dividing it by  $U_{cc}^s \bar{Y}$  we get:

$$\frac{(C_t^s - \bar{C})}{\bar{Y}} = \frac{E_t (C_{t+1}^s - \bar{C})}{\bar{Y}} + \frac{U_c^s}{U_{cc}^s \bar{Y}} (r_t - \bar{r})$$

$$\widehat{C}_t^s = E_t \widehat{C}_{t+1}^s - \sigma (i_t - E_t \pi_{t+1} - \bar{r})$$

where  $\widehat{C}_t^s = \log \frac{C_t^s}{\bar{C}}$  and  $\sigma = -\frac{\bar{U}_c^s}{\bar{U}_{cc}^s \bar{Y}}$

Log-linearizing **aggregate consumption** which is set as  $C_t = \chi_s C_t^s + (1 - \chi_s) C_t^b$  we get:

$$(C_t - \bar{C}) = \chi_s (C_t^s - \bar{C}) + (1 - \chi_s) (C_t^b - \bar{C})$$

Dividing by  $\bar{Y}$  we get:

$$\widehat{C}_t = \chi_s \widehat{C}_t^s + (1 - \chi_s) \widehat{C}_t^b$$

where  $\widehat{C}_t = \log \frac{C_t}{\bar{Y}}$ ,  $\widehat{C}_t^s = \log \frac{C_t^s}{\bar{Y}}$ ,  $\widehat{C}_t^b = \log \frac{C_t^b}{\bar{Y}}$

While log-linearized **aggregate output** is

$$\widehat{Y}_t = \widehat{C}_t + \widehat{G}_t$$

Thus we get:

$$\widehat{Y}_t = \chi_s \widehat{C}_t^s + (1 - \chi_s) \widehat{C}_t^b + \widehat{G}_t$$

## 2.2 Derivation of the Phillips curve

From the first order condition of this problem we get prices for both types of firms:

$$p_t(1) = \frac{\theta}{\theta - 1} \frac{W_t}{[\eta(G_t^P)^\zeta (h_t(j))^{\eta-1}]} P_t$$

$$p_t(2) = E_{t-1} \left( \frac{\theta}{\theta - 1} \frac{W_t}{[\eta(G_t^P)^\zeta (h_t(j))^{\eta-1}]} P_t \right)$$

Where  $M = \frac{\theta}{\theta - 1}$  is a markup over marginal cost.

Log-linearizing this equation we get:

$$\log p_{1t} = \widehat{W}_t + \log P_t + (1 - \eta) \widehat{h}_t - \zeta g_t^P$$

Where  $\widehat{W}_t = \log \frac{W_t}{\overline{W}}$ ,  $\widehat{h}_t = \log \frac{h_t}{\overline{Y}}$

Then an unexpected inflation can be expressed as follows:

$$\pi_t - E_{t-1} \pi_t = \log P_t - E_{t-1} \log P_t = \frac{\lambda}{1 - \lambda} (\log p_t(1) - \log P_t) = \frac{\lambda}{1 - \lambda} (\widehat{W}_t + (1 - \eta) \widehat{h}_t - \zeta g_t^P)$$

From first order conditions we get labor supply for two types of consumers:

$$W_t = \frac{\nu_h^s(h_t^s)}{U_c^s(C_t^s)}$$

$$W_t = \frac{\nu_h^b(h_t^b)}{U_c^b(C_t^b)}$$

Log-linearizing this conditions we obtain:

$$\widehat{W}_t = \omega^s \widehat{h}_t^s(i) + \sigma^{s-1} \widehat{C}_t^s$$

$$\widehat{W}_t = \omega^b \widehat{h}_t^b(i) + \sigma^{b-1} \widehat{C}_t^b$$

Where  $\omega^s = \frac{\overline{v}_{hh}^s h}{v_h^s}$ ,  $\omega^b = \frac{\overline{v}_{hh}^b h}{v_h^b}$ ,  $\sigma^s = \frac{\overline{U}_c^s}{U_{cc}^s \overline{Y}}$ ,  $\sigma^b = \frac{\overline{U}_c^b}{U_{cc}^b \overline{Y}}$ ,  $\widehat{h}_t^b(i) = \log \frac{h_t^b}{\overline{Y}}$

It is assumed that  $\omega^s = \omega^b = \omega$  and  $\sigma^s = \sigma^b = \sigma$ .

Aggregating these conditions we get:

$$\widehat{W}_t = \omega \widehat{h}_t + \sigma^{-1} \widehat{C}_t$$

$$\pi_t - E_{t-1}\pi_t = \frac{\lambda}{1-\lambda}(\omega \widehat{h}_t + \sigma^{-1} \widehat{C}_t + (1-\eta)\widehat{h}_t - \zeta g_t^P)$$

$$\widehat{h}_t = \frac{1}{\eta} \widehat{Y}_t - \frac{\zeta}{\eta} g_t^P$$

$$\text{From market equilibrium condition: } \widehat{C}_t = \widehat{Y}_t - \widehat{G}_t = \widehat{Y}_t - g_t^U - g_t^P$$

$$\pi_t = \frac{\lambda}{1-\lambda} \left[ (1+\omega) \frac{1}{\eta} \widehat{Y}_t - \frac{\zeta}{\eta} (1+\omega) g_t^P + \sigma^{-1} (\widehat{Y}_t - g_t^P - g_t^U) \right] + E_{t-1}\pi_t$$

$$\pi_t = \frac{\lambda}{1-\lambda} \left[ ((1+\omega) \frac{1}{\eta} + \sigma^{-1}) \widehat{Y}_t - \frac{\zeta}{\eta} (1+\omega) g_t^P - \sigma^{-1} G_t \right] + E_{t-1}\pi_t$$

$$\pi_t = \kappa (1+\varphi) \widehat{Y}_t - \kappa \zeta g_t^P - \varphi \kappa \widehat{G}_t + E_{t-1}\pi_t$$

$$\text{where } \kappa = \frac{1}{\eta} \frac{\lambda}{1-\lambda} (1+\omega), \quad \varphi = \frac{\eta \sigma^{-1}}{1+\omega}$$

### 2.3 Derivation of labor income of the borrower

$$\widehat{Y}_t^b = \widehat{W}_t + \widehat{h}_t^b$$

Combining Euler equation of the saver together with

$$\text{Labor supply of the borrower: } \widehat{h}_t^b = \omega^{-1} \widehat{W}_t - \sigma^{-1} \omega^{-1} \widehat{C}_t^b$$

$$\text{Borrower's consumption from the resource constraint: } \widehat{C}_t^b = \chi_b^{-1} \widehat{Y}_t - \chi_b^{-1} \chi_s \widehat{C}_t^s - \chi_b^{-1} \widehat{G}_t$$

$$\text{From the production function: } h_t = \eta^{-1} \widehat{Y}_t - \zeta \eta^{-1} g_t^P$$

Substituting it into the aggregate labor supply together with the resource constraint:

$$\widehat{W}_t = (\omega \eta^{-1} + \sigma^{-1}) \widehat{Y}_t - \omega \eta^{-1} \zeta g_t^P - \sigma^{-1} \widehat{G}_t$$

We obtain:

$$\widehat{Y}_t^b = \mu \widehat{Y}_t + \sigma^{-1} (\omega^{-1} \chi_b^{-1} \chi_s - 1) \widehat{G}_t - (1+\omega) \zeta \eta^{-1} g_t^P + \omega^{-1} \chi_b^{-1} \chi_s \sigma^{-1} (E_t \widehat{C}_{t+1}^s - \sigma (i_t - E_t \pi_{t+1} - \bar{r}))$$

$$\text{where } \mu = (1 + \omega^{-1})(\omega \eta^{-1} + \sigma^{-1}) - \sigma^{-1} \omega^{-1} \chi_b^{-1}$$

## A.3 Analysis of the model

### 3.1 Short run dynamics of the model

Thus the budget constraint of a borrower transforms into:

$$\widehat{C}_s^b = \widehat{Y}_s^b - \widehat{D} + \gamma_D \pi_s - \gamma_D \beta (i_t - \pi_L - \bar{r})$$

where now  $\widehat{D} = \frac{\beta D^{high} - \bar{D}}{\bar{Y}}$  and  $\widehat{Y}_s^b = \mu \widehat{Y}_s + \sigma^{-1} (\omega^{-1} \chi_b^{-1} - 1) \widehat{G}_s - \omega^{-1} \chi_b^{-1} \chi_s \sigma^{-1} (i_s - \bar{r})$

Taking into account that  $\widehat{D}_t$  and  $\widehat{D}_{t-1}$  are deviations from steady state so that

$$\beta \gamma_D \widehat{D}_t - \gamma_D \widehat{D}_{t-1} = \beta \gamma_D \frac{D_t - \bar{D}}{\bar{D}} - \gamma_D \frac{D_{t-1} - \bar{D}}{\bar{D}} = \frac{\bar{D}}{\bar{Y}} \frac{\beta D_t - D_{t-1}}{\bar{D}}$$

In period  $t$  debt limit falls from  $D^{high}$  to  $D^{low} = \bar{D}$ , thus in  $t-1$  we have  $D^{high}$  and  $D^{low}$  in period  $t$ .

Therefore, this part of the budget constraint becomes

$$\frac{\beta D_t - D_{t-1}}{\bar{Y}} = \frac{\beta D^{high} - \bar{D}}{\bar{Y}}$$

The budget constraint of a saver transforms into:

$$\widehat{C}_s^s = \widehat{C}_L^s - \sigma (i_s - \pi_L - \bar{r})$$

### 3.2 Derivation of the output in the short run

$$\begin{aligned} \widehat{Y}_s &= -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b \gamma_D \beta}{1 - \mu \chi_b} (i_s - \bar{r}) - \frac{\chi_b}{1 - \mu \chi_b} \widehat{D} - \frac{\chi_b}{1 - \mu \chi_b} \widehat{T}_s^b + \\ &+ \frac{\chi_b}{1 - \mu \chi_b} \gamma_D \pi_s + \frac{1 + \sigma^{-1} \omega^{-1} \chi_s - \chi_b \sigma^{-1}}{1 - \mu \chi_b} \widehat{G}_s - \frac{(1 + \omega) \zeta \eta^{-1} \chi_b}{1 - \mu \chi_b} g_s^P \end{aligned}$$

Implementing the definition of the natural interest rate:

$$\widehat{Y}_s = -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b \gamma_D \beta}{1 - \mu \chi_b} (i_s - r_s^n)$$

Where

$$r_s^n = \bar{r} - \frac{\chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b \gamma_D \beta} \widehat{D} + \frac{\gamma_D \chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b \gamma_D \beta} \pi_s -$$

$$-\frac{\chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}\widehat{T}_S + \frac{1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b\sigma^{-1}}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}\widehat{G}_S - \frac{(1 + \omega)\zeta\eta^{-1}\chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}g_S^P$$

**Phillips Curve** now will have the following representation:

$$\pi_S = (\kappa + \varphi - 1)\widehat{Y}_S - \kappa\zeta g_S^P - \varphi\widehat{G}_S$$

$$\text{where } \kappa = \frac{\lambda}{1 - \lambda}(1 + \omega)\frac{1}{\eta}, \varphi = \frac{\lambda}{1 - \lambda}\sigma^{-1}$$

Combining AS and AD equations we get:

$$\begin{aligned} \widehat{Y}_S = & -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \mu\chi_b}(i_S - \bar{r}) - \frac{\chi_b}{1 - \mu\chi_b}\widehat{D} - \frac{\chi_b}{1 - \mu\chi_b}\widehat{T}_S^b + \\ & + \frac{\chi_b}{1 - \mu\chi_b}\gamma_D(\kappa(1 + \varphi)\widehat{Y}_S - \kappa\zeta g_S^P - \varphi\widehat{G}_S) + \frac{1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b\sigma^{-1}}{1 - \mu\chi_b}\widehat{G}_S - \frac{(1 + \omega)\zeta\eta^{-1}\chi_b}{1 - \mu\chi_b}g_S^P \end{aligned}$$

$$\begin{aligned} \widehat{Y}_S = & -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}(i_S - \bar{r}) - \frac{\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{D} - \\ & - \frac{\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{T}_S^b + \frac{1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b(\sigma^{-1} + \gamma_D\varphi)}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{G}_S - \frac{\kappa\zeta\gamma_D\chi_b + (1 + \omega)\zeta\eta^{-1}\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}g_S^P \end{aligned}$$

### 3.3 Case of a positive nominal interest rate

Substituting the definition of positive nominal interest rate  $i_t = r_t^n + \phi_\pi\pi_t$  in the expression for output: Implementing the definition of the natural interest rate:

$$\widehat{Y}_S = -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \mu\chi_b}(i_S - r_S^n)$$

Where

$$\begin{aligned} r_S^n = & \bar{r} - \frac{\chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}\widehat{D} + \frac{\gamma_D\chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}\pi_S - \\ & - \frac{\chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}\widehat{T}_S^b + \frac{1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b\sigma^{-1}}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}\widehat{G}_S - \frac{(1 + \omega)\zeta\eta^{-1}\chi_b}{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}g_S^P \\ \widehat{Y}_S = & -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \mu\chi_b}(r_S^n + \phi_\pi\pi_S - r_S^n) = -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \mu\chi_b}\phi_\pi\pi_S \end{aligned}$$

Substituting the definition of the inflation in the short run from the Phillips

curve:

$$\widehat{Y}_S = -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \mu\chi_b}\phi_\pi[\kappa(1 + \varphi)\widehat{Y}_S - \kappa\zeta g_S^P - \varphi\widehat{G}_S]$$

$$\widehat{Y}_S = -\frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \mu\chi_b}\phi_\pi[\kappa(1 + \varphi)\widehat{Y}_S - (\kappa\zeta\psi + \varphi)\widehat{G}_S]$$

$$\widehat{Y}_S = \frac{\phi_\pi(\kappa\zeta\psi + \varphi)(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)}{1 - \mu\chi_b + \phi_\pi\kappa(1 + \varphi)(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)}\widehat{G}_t$$

$$\text{mult}_G = \frac{\phi_\pi(\kappa\zeta\psi + \varphi)(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)}{1 - \mu\chi_b + \phi_\pi\kappa(1 + \varphi)(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)}$$

An impact **the share of productive spending**  $\psi$  has on the fiscal multiplier:

$$\frac{\partial \text{mult}_G}{\partial \psi} = \frac{\phi_\pi\kappa\zeta(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)}{1 - \mu\chi_b + \phi_\pi\kappa(1 + \varphi)(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)}$$

An impact of **productivity of government investment**  $\zeta$  is:

$$\frac{\partial \text{mult}_G}{\partial \zeta} = \frac{\phi_\pi\kappa\psi(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)}{1 - \mu\chi_b + \phi_\pi\kappa(1 + \varphi)(\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta)}$$

### 3.4 The case of a zero lower bound

If deleveraging shock is sufficiently high and zero lower bound becomes binding we get:

$$\begin{aligned}\widehat{Y}_S &= \Gamma - \frac{\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{D} - \frac{\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{T}_S^b + \\ &+ \frac{1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b(\sigma^{-1} + \gamma_D\varphi)}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{G}_S - \frac{\kappa\zeta\gamma_D\chi_b + (1 + \omega)\zeta\eta^{-1}\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}g_S^P\end{aligned}$$

$$\text{with } \Gamma = \frac{\chi_s(\sigma + \omega^{-1}) + \chi_b\gamma_D\beta}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\bar{r}$$

$$\begin{aligned}\widehat{Y}_S &= \Gamma - \frac{\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{D} - \frac{\chi_b}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{T}_S^b + \\ &+ \frac{1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b(\sigma^{-1} + \gamma_D\varphi) - \psi\zeta\chi_b(\kappa\gamma_D + (1 + \omega)\eta^{-1})}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}\widehat{G}_S\end{aligned}$$

$$\text{mult}_G = \frac{1 + \sigma^{-1}\omega^{-1}\chi_s - \chi_b(\sigma^{-1} + \gamma_D\varphi) - \psi\zeta\chi_b(\kappa\gamma_D + (1 + \omega)\eta^{-1})}{1 - \chi_b(\mu + \gamma_D\kappa(1 + \varphi))}$$

An impact of **the share of productive spending**  $\psi$  has on the fiscal multiplier:

$$\frac{\partial \text{mult}_G}{\partial \psi} = \frac{-\zeta \chi_b (\kappa \gamma_D + (1 + \omega) \eta^{-1})}{1 - \chi_b (\mu + \gamma_D \kappa (1 + \varphi))}$$

An impact of **productivity of government**  $\zeta$  investment is:

$$\frac{\partial \text{mult}_G}{\partial \zeta} = \frac{-\psi \chi_b (\kappa \gamma_D + (1 + \omega) \eta^{-1})}{1 - \chi_b (\mu + \gamma_D \kappa (1 + \varphi))}$$