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Rational Expectations vs. Imperfect Knowledge Economics: What Does Really Drive the Polish Zloty?

Abstract

A review of the contemporary mainstream literature on exchange rate modelling clearly indicates that the rational expectations hypothesis (REH) is almost invariably taken as a point of reference in empirical investigations. Even so, almost twenty years after Kenneth Rogoff pointed to the fundamental purchasing power parity puzzle the causes of the long-lasting mean-reversion of the real exchange rates still appear uncertain.

REH implications for the empirical models of currencies in Central-East European transition countries are hard to overrate. In these models, the REH is usually a routine assumption, purchasing power parity is assumed for tradables prices, and the discussion basically concentrates around the appreciative expectations implied by the Balassa-Samuelson mechanism and demand adjustments.

This paper tests the rationality of expectations occurring in the Polish foreign exchange market. The empirical analysis is conducted within the framework of the Roman Frydman and Michael Goldberg model where heterogeneous economic agents are assumed to have imperfect knowledge (IKE hypothesis). The modelling strategy consists of (i) the formulation of different assumptions about the persistence of the nominal exchange rate, prices and interest rates, and (ii) the verification of competing cointegrated VAR scenarios congruent with RE and IKE hypotheses. The final outcomes of the paper are the following (i) the REH is rejected in favour of the IKE alternative and (ii) the risk premium is identified as a predominant factor in the Polish zloty swinging in the free float regime.

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1. Introduction

Despite the vast and still growing number of papers dealing with purchasing power parity (hereafter PPP), the hypothesis about international price arbitrage or – following the interpretation adopted in models belonging to New Open Economy Macroeconomics – the hypothesis about the convergence of currency utilities and perfect risk sharing may still give rise to serious reservations. A literature review implies that empirical studies into PPP[1] still seek to solve the puzzle that Kenneth Rogoff formulated almost 20 years ago – the deviations of real exchange rates (RER) from the PPP level and high estimates of RERs' half-lives (3-5 years) are still difficult to explain through nominal rigidities and market frictions.

The empirical studies on PPP hypothesis and PPP model have several strands, but very broadly they can be divided into studies making a direct attempt to confirm the PPP hypothesis and those relaxing some of the overly restrictive assumptions of the law of one price. The first and historically earliest strand that seems to be falling into obsolescence today consists of the linear tests of real rates' stationarity and of the analyses of (vector) error correction models (VEC) used to test the hypotheses that common stochastic trends drive nominal rates as well as domestic and foreign prices. Conclusions deriving from the direct tests of the PPP hypothesis and the PPP model are now considered stylized facts. Firstly, the univariate unit root tests (URT) offer better prospects of the PPP being accepted when long-time series are used and the panel URTs are based on large and homogenous panel data sets. Secondly, the analyses of standard VEC models with nominal rates and domestic and foreign prices frequently confirm that the variables are cointegrated, but very rarely provide arguments for imposing long-term homogeneity (proportionality) or symmetry restrictions which are central to the PPP model. Thirdly, the use of the long-span time series or large and homogenous panel data sets, or the scope of research restricted to the so called weak-form of the PPP model does not lead to the solution of Rogoff's puzzle.

A review of the literature confirms that studies where some assumptions of the PPP hypothesis and model were given up are more promising for explaining the PPP paradox. This research strand consists of (i) analyses where the existence of non-zero transaction costs is acknowledged and RERs are allowed to adjust non-linearly to constant equilibrium level and (ii) analyses taking account of smooth changes in the real rate equilibrium level, and (iii) the most recent analyses allowing simultaneously for smooth shifts in RERs and non-linear adjustments. Despite the increasing number of studies that confirm the non-linearity of the real exchange rates, the conclusion about Rogoff's

[1] In the paper, a distinction is made between the PPP hypothesis which is verified by testing real rates' stationarity, and the PPP model where the fluctuations of nominal rates arise from domestic and foreign prices.

puzzle having been finally solved seems premature. Although the non-linear univariate and panel unit root tests result in a more frequent rejection of real rates' difference-stationarity, the rejection is by no means a rule. Furthermore, the estimates of the half-lives they produce are significantly smaller from the consensus values, but only in periods when RERs' deviations from the parity are the greatest.

Because of the ambiguity of the most recent univariate and panel URTs and of the mainstream literature's failure to provide an explicit explanation of the PPP puzzle, the question must be raised about the adequacy of the theoretical and empirical framework in which PPP analyses are conducted. The question is quite touchy, because the assumption about the rationality of the representative homogeneous economic agent still determines the properties of most theoretical models and causes that studies into purchasing power parity 'have to' seek 'any form' of RERs' stationarity. The rational expectations hypothesis (REH) raises more and more doubts. The global financial crisis (hereafter a *subprime crisis*) triggered by the fall of the Lehmann Brothers appears to be one of the most meaningful proofs that the REH has limited usefulness for describing the process through which agents form their expectations. Frydman and Goldberg (2007) (FG) have presented a powerful critique of rational expectations, proposing instead an imperfect knowledge economics hypothesis (IKE) that acknowledges the psychological determinants of investors' decisions. The differences between the IKE and the REH are substantial, because the FG model both recognizes and explains the causes of long-lasting swings of the nominal and real exchange rates.

In this paper, an attempt is made to establish which hypothesis – rational expectations or imperfect knowledge economics – is more precise in describing the Polish foreign exchange market in the free float period 1999:01-2011:06. The paper is structured as follows. The next section presents an overview of some studies on the PPP hypothesis and the PPP model with rational expectations. Section 3 outlines the FG model and discusses its implications. Section 4 presents a brief history of the Polish foreign exchange market, the preliminary results of the linear and non-linear unit root tests, and the estimates obtained from a second order logistic smooth transition autoregressive model of the zloty/euro real exchange rate. The next two sections are of empirical nature. In section 5, the cointegration analysis of a three-dimensional vector autoregressive model (VEC) with nominal rates and tradables prices for Poland and the Eurozone is discussed. The following specific questions are investigated: (i) do common stochastic trends drive the three nominal variables?, (ii) are there any $I(2)$-symptoms in the standard VEC model with cointegrating vector fulfilling the homogeneity restriction?, (iii) does the cointegration analysis of the VEC model allowing for near-$I(2)$-ness of the nominal variables provide grounds for performing an empirical analysis of the FG model? Because the answer to the first question is 'no' and 'yes' to the next two, in section 6 a cointegration analysis of the VEC model extended to domestic and foreign interest rates is performed. Two variants of the VEC model are considered and discussed in detail: one structured following the predictions of the standard Dornbusch-type monetary model with rational expectations and the other using the assumptions of the FG model.

2. A review of the literature: some stylized facts and bothering doubts

The empirical studies on purchasing power parity form two main strands: one involves direct attempts to confirm the PPP hypothesis and in the other some assumptions of the law of one price are omitted from analyses[2]. The historically earliest strand consisted mainly of investigations employing ADF-type unit root tests with an auxiliary regression:

$$\Delta q_t = \mu + (\rho_1 - 1) q_{t-1} + \sum_{s=1}^{S} \gamma_s \Delta q_{t-s} + \varepsilon_t \,, \tag{1}$$

where: $q = b - p + p^*$ - the real exchange rate, b - the nominal exchange rate (the price of a foreign currency unit), p - domestic prices, p^* - foreign prices, ρ_1 - AR(1) parameter, $0 < \rho_1 < 1$, $\varepsilon \sim i.i.d.(0, \sigma^2)$, $t = 1,...,T$. The expected result of the tests is a rejection of RER's first-difference stationarity, $q_t \sim I(1)$, in favour of an alternative implying mean-reversion, $q_t \sim I(0)$. However, as the PPP hypothesis was surprisingly rarely confirmed in the post Bretton-Woods free float era, discussions focused on the issue of the low statistical power of standard UR tests. This stage in the research can be wrapped up by concluding that for the PPP hypothesis to be confirmed long-span time series or large panel data sets are necessary. It is also at this the stage that the PPP puzzle is formulated – even when large data sets are used and RERs' difference-stationarity is rejected, the estimates of half-lives ($\mathrm{HL} = \ln 0.5 / \ln \rho_1$) point to real rates' persistence that is difficult to interpret (panels in: Frankel and Rose 1996; Oh 1996; long time series in: Frankel 1986; Lothian and Taylor 1996, 2000).

In the recent PPP studies employing univariate URTs two approaches are the most frequent. The direct reason for the first one is the criticism over tests with long-span time series that highlights differences between currency regimes (Hegwood and Papell 1998; Lopez et al. 2005) or the smooth shifts of the PPP level (deterministic trends in: Sollis 2005; Cushman 2008; Fourier functions in: Su et al. 2011; Chang et al. 2012; Yilanci and Eris 2013). Cushman and Michael (2011) make reference

[2] For extensive reviews of the early investigations see Froot and Rogoff (1995), Rogoff (1996), Sarno and Taylor (2002), Taylor (2002) and MacDonald (2007) and a more recent monograph by James et al. (2012).

to the BEER-type model (e.g. Clark and MacDonald 1999), where RER deviates from the PPP because of the impact of medium- and short-term factors on the real exchange rate. They argue that if some of the factors are stationary around nonlinear trends (the trends do not cancel out in the model), the real exchange rate is also stationary around a nonlinear trend. Arguments in favour of the tests based on the Fourier functions have the same underpinning and also provoke a polemic, because it is not clear how their results should be finally interpreted. In particular, a rejection of the null hypothesis means that a set of factors affecting the real exchange rate in shorter time horizons exists, which can be approximated with any selected nonlinear trend; yet, the real exchange rate is not mean-reverting. If, however, the null hypothesis cannot be rejected the real exchange rate is not mean-reverting either, but then it may be affected by the same short- and medium-term factors, the net effect of which should be approximated by a different nonlinear trend. Therefore, regardless of the outcome, questions need to be asked about what determines RERs and the structure of cause-effect relations between them and, last but not least, in what respect this kind of analysis is superior to analyses performed with fully specified models.

The second approach derives from theoretical models discussed by Dumas (1992) and Secru et al. (1995). The models clearly indicate that non-zero transportation costs imply the existence of an inner regime where the real rate may be generated by $I(1)$ process, as well as the existence of lower and upper regimes in which adjustments towards the PPP level can be expected to occur. This means that a null hypothesis about RER's difference stationarity should be tested against its alternatives implying global stationarity with non-linear mean-reversion. Pavlidis et al. (2011) have recently demonstrated that non-zero transportation costs lead to adjustments that can be approximated with the second order logistic function, thus confirming the usefulness of the widely used unit root test with exponential STAR alternative (hereafter KSS) that Kapetanios et al. (2003) have proposed.

RERs' non-linear mean-reversion is widely covered in the literature. Even so, Stephen Norman's conclusion 'that N[onlinear] M[ean] R[eversion] is a resolution to the PPP puzzle' (Norman 2010, p. 936) seems premature because it has not been confirmed either by his research involving the generalized impulse response functions (only less than one-third HL estimates were shorter than 3 years) or by the 'direct' KSS-type unit root tests. Kapetanios et al. (2003), Bahmani-Oskooee et al. (2007), Kim and Moh (2010) and Cuestas and Regis (2013) have considered the ESTAR alternative and consequently have rejected the unit root hypothesis in approximately half of the analysed cases. The illustrative applications of the KSS-type UR tests where the unit root hypothesis was not rejected in the general case have also been presented by Sollis (2009), Christidou and Panagiotidis (2010), Alloy et al. (2011), Zhou and Kutan (2011), Su et al. (2011) and Yilanci and Eris (2013).

Despite all these doubts, the increasingly frequent use of non-linear tests for analysing purchasing power parity is not very surprising. However, the review of the most recent literature seems to substantiate the criticism that the non-linear alternative hypotheses in the UR tests may be overly arbitrary. For instance, in some studies the ESTAR-type adjustments are combined with changes in the RER's equilibrium level approximated by means of the Fourier functions (Christopoulos and León-Ledesma 2010; Yilanci and Eris 2013; He and Chang 2013). Both types of non-linearity are used in UR tests for the panels of real exchange rates (He et al. 2014), also as part of the sequential panel selection method proposed by Chortareas' and Kapetanios' (2009) that has been widely used in recent years (e.g. He and Chang 2013). Surprisingly, even though the alternative hypotheses are very 'capacious', unit root rejections are still so rare that it is difficult to conclude that the thesis about RERs' non-linear adjustments towards nonlinearly changing equilibrium is bringing us closer to the final solution of the Rogoff's puzzle.

What the linear and non-linear univariate and panel tests of the PPP hypothesis have in common is that two hypotheses, i.e. long- and short-term homogeneity restrictions, are maintained. The long-term homogeneity restriction means that the verification of the PPP hypothesis amounts to a simultaneous verification of (i) the hypothesis that common trends are present in the processes generating nominal exchange rate, domestic and foreign prices:

$$(\beta_1 b_t - \beta_2 p_t + \beta_3 p_t^* + \mu) \sim I(0) \tag{2}$$

and (ii) the hypothesis that the equilibrium parameters are equal, $\beta_1 = \beta_2 = \beta_3 = 1$; the short-term homogeneity restriction is imposed on the parameters on the first differences of three nominal variables, $\gamma_s^B = \gamma_s^P = \gamma_s^{P*} = \gamma_s$, $s = 1,...,S-1$. The results of the earliest cointegration analyses of the PPP model provide solid arguments for questioning these restrictions – whether the presence of common stochastic trends driving nominal exchange rates and prices will be confirmed is not obvious *but* the confirmation of long-term homogeneity seems an exception to a rule. One of the first explanations of why problems occur in the construction of a PPP model has been presented by Patel (1990), who pointed to transportation costs causing disturbances in arbitrage. However, the assumption about transaction costs being neutral to arbitrage is only one of a series of restrictive assumptions made in PPP analyses. As a result, some researchers have adopted MacDonald's (1993) interpretation, according to which the presence of common trends in the three nominal variables confirms a so-called weak-form of the PPP model (MacDonald 2007, pp. 52-57). Although the weak-form PPP models are much more flexible than the UR tests, the results they yield are still unsatisfactory – in many cases the half-lives estimates still take 'puzzling' values of 3-5 years (MacDonald 1993, 1995).

Alternative approaches to exchange rate modelling consist in the modifications of the PPP model and lead to the analysis of different variants of the monetary model. In the approaches referring to the Dornbusch-type models, processes in a commodity market that stays in equilibrium when purchasing power parity holds and processes in capital markets that clear under uncovered interest rate parity are analysed within the same empirical model (Juselius 1991, 1995; Johansen and Juselius 1992) [3]:

$$(\beta_1 b_t - \beta_2 p_t + \beta_3 p_t^* + \beta_4 i_t - \beta_5 i_t^* + \mu) \sim I(0), \tag{3}$$

where i and i^* denote domestic and foreign nominal interest rates. Hall et al. (2013) have recently observed that an arbitrary long-term homogeneity restriction may lead to model misspecification and biased estimates if the nominal rate is affected by factors other than prices. The model they have considered has time-varying coefficients:

$$(b_t - \beta_{2t} p_t + \beta_{3t} p_t^* + \mu_t) \sim I(0) \tag{4}$$

where μ_t stands for shocks uncorrelated with prices. Changes in parameters β_{2t} and β_{3t} are caused by misspecification errors and can be described by means of a set of stochastic linear equations containing a set of coefficient-drivers. Given that, some of the drivers may affect the parameters as a result of omitted variables and the effect of other drivers may arise from ignored non-linearity, Hall et al. (2013) argue that the long-term homogeneity and thereby the PPP hypothesis should be inferred from the 'bias-free' components of parameters β_{2t} and β_{3t}. The estimates that the exchange rate models have yielded for the currencies of 14 developed countries point out that this approach is correct – if the specification errors are concentrated out, the 'bias-free' components are correctly signed and are close to unity, which, as Hall et al. (2013) indicate, strongly supports the PPP hypothesis.

[3] Juselius and MacDonald (2004), (2006) have proposed extending model (3) by assuming long-term homogeneity restriction and allowing for short- and long-term domestic and foreign interest rates in the model. The affined BEER-type models usually take into consideration a broader set of the medium- and short-term drivers of the RER, but with arbitrarily assumed long- and short-term homogeneity restrictions.

3. The IKE hypothesis and CVAR scenarios for the PPP model

A brief overview of empirical PPP analyses provides conclusions in support of the thesis that the non-linear models, the various variants of the monetary model and the BEER-type, eclectic medium-term models allow overcoming some of overly restrictive assumptions of the PPP model. The same overview provides an equally clear indication that the rational expectations hypothesis implying the existence of some form of RER's stationarity remains the 'untouchable' assumption. The perspective changes when, following Frydman and Goldberg (2007), the REH assumption about full predeterminacy of expectations is overruled. According to the fundamental assumptions of the FG model, heterogeneous individuals use different forecasting strategies that as well as varying in time cannot be prespecified in advance (Frydman and Goldberg 2007, also: Frydman et al. 2008; Juselius 2010, 2011, 2013). Economic agents are rational in that they try not to miss any opportunities for profits. This means that in relatively long subperiods the forecasting strategy adjustments are mainly determined by psychological factors that contribute to the occurrence of 'trend followers' in the foreign exchange markets and conservative changes in investment strategies.

Frydman and Goldberg (2007) have considered the Dornbusch-type sticky-price monetary model of exchange rate, where the *i*-th investor seeks alternative forecasting strategies to maximise profits:

$$F_{i,t}^{IKE}(b_{t+1}^{i}) = \beta'_{(k)i,t}x_{(k)i,t} + \rho_{1,i}b_t\,, \tag{5}$$

where: $x_{(k)i,t}$ - the vector of fundamentals, $\beta_{(k)i,t}$ - parameters, $k = 1,\ldots, K$. Strategies are modified because of oscillations in fundamentals and variations in parameter values:

$$\begin{aligned} E_{i,t}^{IKE}(b_{t+1}^{i}) - E_{i,t-1}^{IKE}(b_{t}^{i}) &= \beta'_{(k)i,t-1}\Delta x_{(k)i,t} + (\Delta\beta'_{(k)i,t})x_{(k)i,t} = \\ &= (\beta'_{(k)i,t-1}\mu_{(k)}^{x_i} + \beta'_{(k)i,t-1}\nu_{(k)}^{x_i}) + (\Delta\beta'_{(k)i,t})x_{(k)i,t} \end{aligned} \tag{6}$$

where: $E_{i,t}^{IKE}(b_{t+1}^{i})$ - a semi-reduced form of equation (5), $E_{i,t}^{IKE}(b_{t+1}^{i}) = \beta'_{(k)i,t}x_{(k)i,t}$; $\beta'_{(k)i,t-1}(\Delta x_{(k)i,t})$ – a status quo forecast, $\beta'_{(k)i,t-1}\mu_{(k)}^{x_i}$ – a baseline drift, $\mu_{(k)}^{x_i}$ – drifts in the production and monetary aggregate, $\beta'_{(k)i,t-1}\nu_{(k)}^{x_i}$ – a deviation from the baseline drift, $\nu_{(k)}^{x_i}$ – supply and monetary shocks, $(\Delta\beta'_{(k)i,t})x_{(k)i,t}$ – forecast revision. Forecast corrections are by assumption caused by changes in parameter values are smaller than changes caused directly by the baseline drift:

$$\left|(\Delta\beta'_{(k)i,t})x_{(k)i,t}\right| < \left|\beta'_{(k)i,t-1}\mu_{(k)}^{x_i}\right| \tag{7}$$

and changes in $\beta_{(k)}$ in two adjacent periods do not change the sign of the baseline drift.

The main difference between predictions yielded by the REH and IKE variants of the monetary model is that in the latter individuals' expectations are not formulated at the PPP level. The steady state conditions in the FG model vary in time (Frydman et al. 2008):

$$\bar{b}_t^{IKE} = \bar{b}_t^{REH} + \frac{\eta + \varphi\lambda}{\varphi + (1-\rho_1)(\eta + \varphi\lambda)}(F_t^{IKE}(b_{t+1}) - F_t^{REH}(b_{t+1})), \tag{8}$$

$$\bar{q}_t^{IKE} = \bar{q}^{REH} + \frac{\eta}{\varphi + (1-\rho_1)(\eta + \varphi\lambda)}(F_t^{IKE}(b_{t+1}) - F_t^{REH}(b_{t+1})), \tag{9}$$

where: η, φ, λ, ρ_1 - FG model's parameters, $\eta, \varphi, \lambda > 0$.

The implications of relaxing the RE hypothesis are difficult to overestimate. Firstly, the IKE world allows considering different dynamics of relative prices' adjustments and of nominal exchange rates' adjustments to equilibrium. This means that the FG model is free of the paradox of large half-lives. Secondly, taking the IKE hypothesis as a starting point does not mean that the RE hypothesis is rejected *a priori*. The latter is nested in the general IKE hypothesis and is verifiable against the IKE alternative. However, if there are strong enough reasons for rejecting the RE hypothesis, the methods of PPP analysis should be redefined. Frydman and Goldberg (2007) assume that 'unbounded' swings may occur when the nominal and real exchange rates are approaching the PPP level for a time, but after parity is reached both rates continue to increase or decrease. This situation is not inconsistent with the equilibrium conditions, because in the IKE world investors change their forecasting strategies that divert one-way drifts of exchange rates. Frydman et al. (2008) point to a whole range of factors (psychological, political) which are likely to cause forecast revisions, attributing the greatest significance to uncertainty premium that increases as various gap effects grow stronger. They finally argue that real rates may be generated by near-*I*(2) processes, whereas the RE hypothesis implies RER's near difference stationarity at most. More specifically, the data generation process $q_t = q_{t-1} + \mu_t + \varepsilon_t$ embraces non-constant drift μ_t, the changes in which reflect revisions of forecasts, $\mu_t = \rho_t \mu_{t-1} + \varepsilon_t^{\omega}$ (Juselius 2010). The IKE hypothesis implies that in some subperiods the values of ρ_t are close to unity. Because the exchange rates' swings may be longer in duration a standard vector error correction model VEC-*I*(1):

$$\Delta y_{(m)t} = \Pi y_{(m)t-1} + \textstyle\sum_{s=1}^{S-1} \Gamma_s \Delta y_{(m)t-s} + \mu_{(m)} + \varepsilon_{(m)t} \tag{10}$$

should be replaced with a much more flexible VEC-*I*(2) model:

$$\Delta^2 y_{(m)t} = \Pi y_{(m)t-1} + \Gamma \Delta y_{(m)t-1} + \sum_{s=1}^{S-2} \Phi_s \Delta^2 y_{(m)t-s} + \mu_{(m)} + \varepsilon_{(m)t}, \tag{11}$$

where: Π - the total multipliers matrix, $\Gamma = -(I - \sum_{s=1}^{S-1}\Gamma_s)$ - the matrix of medium-term multipliers, $\Gamma:[M \times M]$, $\Phi_s = -\sum_{j=s+1}^{S-1}\Gamma_j$ - short-term parameters, $\Phi_s:[M \times M]$, $\mu_{(m)}$ - the constant term.

In most empirical analyses only the difference-stationarity of $y_{(m)t}$ is considered. This amounts to assuming that $y_{(m)t}$ components are determined by S pushing common stochastic *I*(1) trends and by $V = M - S$ attracting cointegrating vectors. Consequently, the total multipliers matrix Π has reduced rank and can be decomposed into adjustments and cointegrating matrices[4]:

$$\Delta y_{(m)t} = \alpha(\beta' y_{(m)t-1}) + \mu_{(m)} + \varepsilon_{(m)t} \tag{12}$$

with stationary CI(1,1) cointegrating relations $\beta' y_{(m)t} \sim I(0)$, $\alpha, \beta:[M \times V]$. The analysis becomes more complicated, when $y_{(m)t}$ is driven by *I*(2) stochastic trends and model (12) is replaced by its isomorphic *I*(2) transformation:

$$\Delta^2 y_{(m)t} = \alpha(\beta' y_{(m)t-1} + \delta' \Delta y_{(m)t-1}) + \zeta\tau' \Delta y_{(m)t-1} + \varepsilon_{(m)t}, \tag{13}$$

where: δ - the matrix of dynamic equilibrium parameters, τ - the matrix of medium-term equilibrium parameters, $\tau = (\beta, \beta_{\perp 1})$, $\beta_{\perp 1}$ - the submatrix of $\beta_{\perp}$, $\beta_{\perp}$ - the orthogonal complement of β, ζ - the matrix of adjustments parameters, $\delta:[M \times V]$, $\tau:[M \times M - S_2]$, $\zeta:[M \times M - S_2]$, $\beta_{\perp 1}:[M \times S_1]$, S_2 - the number of *I*(2) stochastic trends, S_1 - the number of autonomous stochastic *I*(1) trends. Assuming that the components of $y_{(m)t} \sim I(2)$ are cointegrated, $V = M - S_1 - S_2$ cointegrating vectors can be identified, which in the general case define nonstationary CI(2,1) relations, $\beta' y_{(m)t} \sim I(1)$. The stationary relations are identifiable if the linear combinations $\beta' y_{(m)t}$ cointegrate with first differences of the *I*(2) variables:

$$(\beta' y_{(m)t} + \delta' \Delta y_{(m)t}) \sim I(0). \tag{14}$$

Equation (14) defines the dynamic equilibrium of the VEC-*I*(2) model (polynomial cointegration). Matrix $\tau = (\beta, \beta_{\perp 1})$ identifies V stationary linear combinations of first-differenced variables $\beta' \Delta y_{(m)t} \sim I(0)$ and S_1 medium-run equilibrium relations $\beta_{\perp 1}' \Delta y_{(m)t} \sim I(0)$.

[4] To simplify the notation, the short-term and deterministic components have been left out from formulas (12)-(15).

The dynamic structure of the cointegrating component of the VEC-I(2) model significantly increases the scale of problems faced by the researcher compared with the standard VEC- I(1) model. A case in point is when $V=1$, $S_2=1$ and $S_1=1$. An analysis of the nominal rate equation:

$$\begin{aligned}\Delta^2 b_t = {} & \alpha^B(\beta_1 b_{t-1}+\beta_2 p_{t-1}+\beta_3 p^*_{t-1}+\delta_1\Delta b_{t-1}+\delta_2\Delta p_{t-1}+\delta_3\Delta p^*_{t-1})+\\ & +\zeta_1^B(\beta_1\Delta b_{t-1}+\beta_2\Delta p_{t-1}+\beta_3\Delta p^*_{t-1})+\\ & +\zeta_2^B(\beta_{\perp 1,1}\Delta b_{t-1}+\beta_{\perp 1,2}\Delta p_{t-1}+\beta_{\perp 1,3}\Delta p^*_{t-1})+\varepsilon_{Bt}.\end{aligned} \tag{15}$$

leads to a conclusion that *a priori* restrictions on cointegration vectors representing alternative variants of the monetary model may be difficult to identify. The solution proposed by Juselius (2010) shifts focus from the structuring of the cointegrating vectors to the analysis of the propagation of I(2) and I(1) shocks in the common stochastic trend representation (CST) of the VEC model. The idea behind theory-consistent CVAR scenarios is the following: if the theoretical model is correct, it can be used to identify the direction of I(2) and I(1) shocks diffusion and to find out which variables absorb these shocks. This means that the analysed time series have ‘testable’ regularities allowing discrimination between the variants of the monetary model and thereby between IKE and REH hypotheses.

In the case of difference-stationarity, $y_{(m)t}$ is driven by stochastic trends $u_{(m)i}=\alpha'_\perp\varepsilon_{(m)i}$:

$$y_{(m)t}=\tilde{\beta}_\perp\sum_{i=1}^{t}u_{(m)i}+\mathrm{T}_0(t)+e_{(m)t}\,, \tag{16}$$

where: $\tilde{\beta}_\perp=\beta_\perp(\alpha'_\perp(-(I-\sum_{s=1}^{S-1}\Gamma_s))\beta_\perp)^{-1}$; $\alpha_\perp$ and $\beta_\perp$ - the orthogonal complements of α and β, $\mathrm{T}_0(t)$ - the function of the initial values $y_{(m)0}$, $e_{(m)t}\sim I(0)$. For $y_{(m)t}\sim I(2)$, the CST representation

$$y_{(m)t}=\tilde{\beta}_{\perp 2}\sum_{i=1}^{t}\sum_{j=1}^{i}u_{(m)j}+\tilde{C}_1\sum_{i=1}^{t}u_{(m)i}+\mathrm{T}_0(t)+e_{(m)t}\,, \tag{17}$$

is considered, where: $\tilde{\beta}_{\perp 2}=\beta_{\perp 2}(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}$, $\Psi=\Gamma\beta(\beta'\beta)^{-1}\alpha(\alpha'\alpha)^{-1}\Gamma-\sum_{s=1}^{S-2}\Phi_s\Delta^2 y_{(m)t-s}$, $u_{(m)i}=\alpha'_{\perp 2}\varepsilon_{(m)i}$.

Tab.1 about here

Regularities that can be identified for different assumptions about the number of I(2) and autonomous I(1) stochastic trends are summarized in table 1. The PPP1 scenario concerns a case with nominal variables integrated of order one. Scenarios PPP2-PPP4 refer to the IKE hypothesis. A comparison of the polynomial cointegrating vectors disclose simplifications and inaccuracies in the routine VEC-I(1) models. Firstly, for $y_{(m)t} \sim I(2)$ the omission of dynamic components from the equilibrium relationships may lead to the identification of relations containing a moderate I(1) component. In this case, the estimates of the error correction terms will be very precise then and will 'confirm' strong persistence of the real rates that cannot be explained within the framework of the RE hypothesis. Secondly, the PPP2-PPP4 scenarios predict that non-linear adjustments are already present in the polynomial cointegrating vectors. It is easy to notice that these non-linearities have different interpretation than those in the STAR-type models. In particular, when the currencies of some catching-up economies are analysed, non-linear price adjustments may be easily attributed to disinflationary processes. This case will be dealt more in detail in the next sections.

4. Polish Zloty in the transition period: preliminaries

The late 1980s witnessed the collapse of Poland's centrally planned economy. Permanently imbalanced internal market, technological underdevelopment aggravated by depreciated fixed capital and foreign debt insolvency combined into the most acute economic crisis among Central and Eastern European countries (CEEC) that triggered evolutionary changes in the political system and a rapid modification of the economic system. The exchange rate policy was one of the linchpins of Balcerowicz's Shock Therapy launched in 1990. All this explains why the Polish currency system made a semicircle, evolving from a fixed to fully floating exchange rate regime (for details see Appendix 1). The first objective of the exchange rate policy in the early transition period was to reduce depreciation expectations and then to anchor inflationary expectations. Another important objective was to support price competitiveness in the external sector and to reduce the inflow of speculative capital and sterilization costs. The exchange rate regime was gradually relaxed with falling inflation and expanding foreign capital inflows, which enabled its evolution from a constant nominal rate through crawling peg and crawling band to a free float regime. The latter was officially introduced in April 2000, but in fact it had appeared with the NBP's last major intervention in the foreign exchange market in February 1998.

Fig. 1 about here

A visual inspection of the zloty/euro rate oscillations (Fig. 1) confirms the heterogeneity of the exchange rate regime in Poland – considerable fluctuations of the three real rates in 1999-2011 are significantly different from the earlier smooth trends present in the CPI-based RER and from the mean-reversion of the zloty/euro rate deflated by tradables prices (hereafter PT-based RER). A review of empirical studies shows that while they take into account the heterogeneity of the Polish zloty exchange rate, its impacts may be underestimated. Doubts arise over those studies that extrapolate tendencies observed before the zloty exchange rate was floated into the 2000s and that perceive the appreciation of the CPI- and even PT-based real rates before the subprime crisis as an effect of productivity gains (review in Égert et al. 2006). The view that the supply-side effects are the most important determinant of the zloty/euro real rate is excessively one-sided and its weaknesses are easy to prove. Firstly, in the period immediately following the transition shock the real appreciation of the zloty was caused by a stabilization policy aimed to keep the monthly devaluation rate below observed inflation. Secondly, the systematic liberalisation of the capital market in the late 1990s was attracting an inflow of investment capital, mostly of a speculative character. Thirdly, the belief that an appreciation trend was inherent in real exchange rate of the Polish zloty may have been founded on the misperception of global risks before the subprime crisis. Kelm (2010) has noticed that the conclusion about a steady real appreciation may have resulted from 'an anomaly in appreciation' observed in the period 2007:01-2008:07, immediately before the subprime crisis erupted (see also Kębłowski and Welfe 2012). Under this assumption, the abrupt exchange rate depreciation in the second half of 2008 should be viewed as an equilibrium-restoring process rather than a temporary deviation from the appreciation trend.

That the last interpretation is justified is confirmed by the results of standard linear unit root tests. Kelm (2013, ch. 4) has analysed the order of integration of the CPI- and PT-based zloty/euro exchange rates in three monthly subsamples[5]: (i) 1993:01-1998:12 (the crawling peg and crawling band regimes), (ii) 1999:01-2008:06 (effective and then full free float before subprime crisis), and (iii) 1999:01-2011:06. The results proved meaningful. The CPI-based RER turned out to be trend-stationary whereas its PT-based counterpart was mean-reverting in the crawling-peg and crawling-band regimes. Both RERs were driven by difference-stationary processes in the pre-crisis free-float period. With the period of analysis extended to the subprime crisis, the results of the linear UR tests turned out to be borderline, so they could be interpreted in favour of the PPP hypothesis. The results of non-linear unit root tests performed with the extended sample also support the conclusion that the subprime crisis had a clearing effect on the Polish foreign exchange market. The KSS test and its asymmetric generalization (AKSS) proposed by Sollis (2009) do not reject difference-stationarity of the CPI- and PT-based RERs in the pre-crisis period, and both explicitly point to RERs' global stationarity in the extended sample 1999:01-2011:06.

Tab.2 about here

Fig.2 about here

[5] The standard ADF, DF-GLS, Elliot-Rothenberg-Stock and KPSS linear tests were employed.

The conclusion about RERs' global stationarity is consistent with the currently prevailing view that the PPP hypothesis may be non-linear. However, a deeper analysis of the estimates of the second-order logistic STAR model of the PT-based real exchange rate[6]:

$$\begin{aligned}\Delta q_t^T &= ((\rho_1 - 1)q_{t-1}^T + \textstyle\sum_{s=1}^S \gamma_s \Delta q_{t-s}^T) + \\ &+ ((\tilde{\rho}_1 - 1)q_{t-1}^T + \textstyle\sum_{s=1}^S \tilde{\gamma}_s \Delta q_{t-s}^T) \cdot (1 + \exp(-\theta(q_{t-D} - c_1)(q_{t-D} - c_2)))^{-1} + \varepsilon_t\end{aligned} \tag{18}$$

provokes a number of questions. The estimates of the error correction terms in the outer regime (ECT = –0.273, HL = 2.2 months, see tab. 2) confirm RER's global stationarity, but positive ECT's estimate for the inner regime seems to indicate that a small explosive root is present in the data generation process, which drives the real exchange rate outside the non-arbitrage interval. The outer regime is identified in periods when RER's deviations are the greatest. Moreover, the distribution of q_t^T in that regime shows that mean-reversion occurs after periods of undervalued RER; the non-linearity of the real rate adjustments is confirmed by a relatively small number of 'outliers'. Finally, from the analysis of fig. 2 it follows that RER's fluctuations were determined by one relatively long and three shorter subperiods of one-direction trends predicted by the Frydman and Goldberg (2007) model. The STAR model allows identifying periods of changes in the directions of RER's drifts. In the FG model these changes are attributed to forecasting strategies being revised. If we additionally allow for the fact that in relatively short samples stochastic processes with small explosive roots may be indistinguishable from the $I(2)$ processes, then the cointegration analysis of the VEC- $I(2)$ model accounting for the nominal zloty/euro exchange rate and domestic and foreign prices becomes fully justifiable.

5. The strict PPP model

A preliminary cointegration analysis of the PPP model was conducted within the framework of a standard three-dimensional VEC-$I(1)$ system containing the nominal zloty/euro exchange rate and domestic and foreign price indices. The nominal rate oscillations, the ratios of price indices in the domestic and foreign external sectors $p^T - p^{*T}$, and the relative CPI in Poland and the euro area $p - p^*$ are presented in fig. 3. The preliminary conclusions arising from the visual inspection are consistent with those formulated with respect to the CPI- and PT-based RERs. Before the zloty exchange rate was floated (to 1999 inclusive) nominal depreciation had been observed, accompanied

[6] The estimates of the parameters were obtained using a standard procedure encompassing (i) lag selection (here: S=2), (ii) non-linearity tests, (iii) the selection of the transition variable, its delay (here: D=12), and the transition function and (iv) the estimation of the parameters. Detailed estimation results are available on request.

by a rise in relative prices which was similar to that in tradables prices but distinctly faster in the case of the relative consumer price indices. The trends resulted in the appreciation of the CPI-based RER and the stabilization of its PT-based counterpart. Between 2000 and 2007 both relative prices stabilized. From the early days of the subprime crisis in 2008 a widening gap between the relative CPI and the nominal rate could be observed, as well as the convergence of oscillations in relative tradables prices and the nominal rate.

Fig. 3. about here

The $I(1)$ cointegration analysis was performed with eight variants of the VEC model that differed from each other in terms of deflators (CPIs or tradables prices), estimation samples (a pre-crisis period 1999:01-2008:06 or a full 1999:01-2011:06 sample) and deterministic variables (the cointegration space with or without a trend). The first stage of estimation was routine: outliers were neutralised with dummies[7], the optimal lag (S=3) was defined, and following that normality, autocorrelation and heteroskedasticity tests were performed. An assumption consistent with Greenslade et al. (2002) recommendations was made that the next stages of the cointegration analysis should take account of the successive sequences of (i) cointegration tests, (ii) weak exogeneity tests, and (iii) over-identifying restrictions tests. However, this strategy was not applied in the $I(1)$ analysis, as none of the eight models presented convincing arguments in favour of rejecting the no-cointegration null hypothesis. Because this outcome contradicts the results that other authors obtained for the pre-crisis period (Barlow 2003; Sideris 2006), analysis was extended to the characteristic equation $\left|I - \Pi_1 \lambda^{-1} - \Pi_2 \lambda^{-2} - ... - \Pi_S \lambda^{-S}\right| = 0$, where λ s denote characteristic roots of the companion matrix in the VAR process.

Tab. 3 about here

Table 3 provides information about the five largest unit roots in alternative VAR models. In seven models the largest characteristic roots are outside the unit circle and the moduli of the next two roots are close to unity. Why explosive roots are present is not easy to explain, but a closer analysis of alternative VAR models estimated for shorter periods reveals almost explicit connection between the building-up explosive tendencies and the deepening 'anomaly in appreciation' in the pre-crisis period 2007:01-2008:07. It can be argued therefore that the reason for explosive roots to appear may have been the speculative strengthening of the long-lasting, one-direction appreciation drift before

[7] The discussion in the next section of the paper concentrates on the PPP model with tradables prices. For the model to have the correct stochastic properties the period of the Argentine crisis in the equation of the nominal rate (2001:06=1, 0 elsewhere) must be addressed, 'a blip-dummy' in the peak of the subprime crisis in the domestic prices equation must be taken into account (2009:02=1, 2009:03= –1, 0 elsewhere), and the deflation period in the external prices equation must be allowed for (2008:01-2009:01).

the subprime crisis. The presence of such a baseline drift is a key assumption in the Frydman-Goldberg model and a strong argument for using the VEC-*I*(2) model as a more flexible approximation of the properties of the data generation process.

Tab. 4 about here

A sequential approach to testing cointegration in the VEC-*I*(2) model was put forward by Johansen (1995) and Paruolo (1996). The results of the *Trace* cointegration test in the VEC-*I*(2) model $y_{(m)} = [b, p^T, p^{*T}; t]'$ are summarized in table 4. The conclusions are clear-cut: at standard significance levels of 0.05 or 0.10 there are no grounds for rejecting the no-cointegration null hypothesis, $\{V = 0, S_2 = 0\}$. However, if the interpretation is 'favourable' to the PPP model, the probability value (0.123) in the test of the hypothesis $\{V = 0, S_2 = 0\}$ can still be considered too small, which justifies analysing $\{V = 1, S_2 = 1\}$.

Tab. 5 about here

The estimation results of the VEC-*I*(2) model with proportionality restriction are presented in tab. 5. Normalization of the polynomial cointegrating vector with respect to the domestic prices:

$$\Delta p_t^T = 0.063(b_{t-1} - p_{t-1}^T + p_{t-1}^{*T}) - 0.990\Delta b_t - 0.010\Delta p_t^{*T} - 0.00004t + 0.0063 \qquad (19)$$

leads to the following conclusions. Firstly, the relation between inflation Δp_t^T and the real exchange rate $(b_{t-1} - p_{t-1}^T + p_{t-1}^{*T})$ can be interpreted as an internal error correction mechanism: as a result of prices p^T rising above the PPP level the price dynamics declines, but a foreign price increase or the weakening of the Polish zloty drives domestic inflation upwards; in either case, long-term equilibrium is restored by internal error correction. Finding an equally convincing interpretation of the other components of equation (19) would be problematic. While it might be argued that the transmission between foreign and domestic price inflation is negligible and that the deterministic trend is a reflection of disinflation in the period under consideration, the identification of factors causing a rise in the nominal depreciation of Δb resulting in a proportional decrease in Δp^T is difficult. Equation (19) transformed into $\Delta p_t^T = 0.063 b_{t-1} - 0.990\Delta b_t + \ldots$ provides arguments in support of the statement that the 'enforced' proportionality restriction may excessively increase the parameter by the lagged nominal rate b and that a negative estimate by Δb has a compensatory character. This reasoning can be additionally enhanced by a thesis about the occurrence of local currency pricing strategies in the Polish foreign exchange market, but both these interpretations would ultimately be *ex post* speculations rather than solid arguments in favour of the 'amorphous' structure of relation (19).

Despite the ambiguous interpretation of the polynomial cointegrating vector, an analysis of the adjustment parameters reveals interesting properties of the mechanisms affecting domestic prices and the nominal rate. In particular, it is possible to identify which mechanism makes the nominal rate deviate from the equilibrium path. This is important, inasmuch as the occurrence of error equilibrium increasing mechanism (hereafter EEI; for details see: Juselius 2010) indirectly confirms the presence of $I(2)$ stochastic trends in the data generation process.

By omitting insignificant error correction parameters, the following short-term equations of the domestic prices and exchange rate are obtained:

$$\begin{aligned}\Delta^2 p_t^T &= -\underset{(8.0)}{0.026}(p_{t-1}^T - b_{t-1} - p_{t-1}^{*T} + 15.88\Delta p_{t-1}^T + 15.72\Delta b_{t-1} + 0.154\Delta p_{t-1}^{*T} \\ &\quad + \underset{(5.7)}{0.0006}\,t - 0.0993) - \underset{(4.1)}{0.388}(\Delta p_{t-1}^T - \Delta b_{t-1} - \Delta p_{t-1}^{*T} + \underset{(5.7)}{0.0006}) + ST_t^P,\end{aligned} \tag{20}$$

$$\begin{aligned}\Delta^2 b_t &= -\underset{(3.0)}{0.039}(p_{t-1}^T - b_{t-1} - p_{t-1}^{*T} + 15.88\Delta p_{t-1}^T + 15.72\Delta b_{t-1} + 0.154\Delta p_{t-1}^{*T} \\ &\quad + \underset{(5.7)}{0.0006}\,t - 0.0993) + ST_t^B.\end{aligned} \tag{21}$$

It easy to see that both nominal variables adjust to trajectory (19), but the values and signs of the error correction terms are intuitive only in equation (20). Domestic prices p^T and domestic inflation Δp^T rising above the equilibrium path (19) slow down domestic inflation (i.e. decrease $\Delta^2 p^T$ with parameters equal, respectively, –0.026 and –0.800; $-0.026 \cdot 15.88 - 0.388 = -0.800$). The impacts of the nominal exchange rate's oscillations are not symmetric. An increase in b accelerates inflation (a parameter of 0.026), but the nominal depreciation of the zloty produces definitely weaker effects (a parameter of 0.021).

The character of the nominal rate's adjustments to the equilibrium path is much more complex. An increase in nominal depreciation Δb is accompanied by a decline in $\Delta^2 b$ (*a parameter* of 0.613). This mechanism is one of the first two which cause that the nominal rate really adjusts to the cointegrating vector (19). Let us note that the net effect of increasing nominal exchange rate is ambiguous, because the increase accelerates the pace of nominal depreciation $\Delta^2 b$ (a parameter of 0.039) and the rate of depreciation Δb (a parameter of 0.064=1/15.72), and thereby decreases $\Delta^2 b$. Summing up, an analysis of equation (21) confirms that the nominal rate 'departs' (which is typical of the IKE hypothesis) from the equilibrium path (19) and that the nominal rate simultaneously stabilises along the same trajectory (19).

The last error correction mechanism affects the nominal exchange rate through domestic prices: an increase in p^T and/or Δp^T decreases the rate of nominal depreciation $\Delta^2 b$. This property of the VEC-*I*(2) system cannot be unambiguously explained in the framework of the strict PPP model. A working hypothesis can be formulated, though, that the structure of equation (20) confirms the presence of a mechanism that links fluctuations in the nominal exchange rate and the nominal interest rates, provided that the latter are co-determined by fluctuations in domestic prices and inflation (*via* monetary policy rule). Accepting this interpretation means that the natural extension of the above research would involve adding domestic and foreign interest rates to the VEC model and analysing purchasing power parity and uncovered interest rate parity within the same VEC model.

6. Does the IKE apply to the Polish zloty?

The lack of cointegration between the nominal exchange rate and domestic and foreign tradables prices is not a sufficient argument for rejecting the RE hypothesis. It is easy to challenge research results presented in the two previous sections for their being obtained from a relatively short sample where the nominal rate fluctuations may have been produced not only by arbitrage in the commodity market, but also by adjustments in the capital market. Allowing for these adjustments naturally leads to a joint analysis of the PPP and UIP models and of a working hypothesis according to which sustainable disequilibria in the current and capital accounts of the balance of payments have the same source.

The so-called capital-enhanced equilibrium exchange rate model (CHEER; a term proposed by Ronald MacDonald) is useful not only because the model (3) provides a much more flexible framework for conducting empirical analyses, but also because it allows discriminating between the RE and IKE hypotheses. When variables are difference stationary, the structure of condition (3) is consistent with the predictions of the standard monetary model. Juselius (2010) has presented a CVAR scenario (REH5, see tab. 6), where the *I*(2)-ness of the nominal rate and of prices and the difference-stationarity of nominal interest rates do not provide grounds for rejecting the RE hypothesis. A serious drawback of the REH5 scenario is its internal inconsistency and overly restrictive assumption about the difference-stationarity of the nominal interest rates. Juselius (2010) has therefore outlined also a ‘competing’ scenario designed on the IKE assumptions and allowing for the *I*(2)-ness of all variables. A comparison of both scenarios’ polynomial cointegrating vectors (see tab. 6) leads to a clear-cut conclusion: the non-stationarity of $(b_t - p_t + p_t^*) + \omega_1(i_t - i_t^*)$ and the stationarity of $(b_t - p_t + p_t^*) + \omega_1(i_t - i_t^*) - \omega_2(\Delta p_t - \Delta p_t^*)$ support the IKE hypothesis.

Tab. 6 about here

The CHEER model with the zloty/euro nominal rate is estimated here divided into several stages. First, an assumption is made that all nominal variables are difference-stationary. The periods taken for analysis are the same as in the PPP model: 1999:01-2008:06 and 1999:01-2011:06. Special attention is given to the period 1999:01-2009:09, because using a somewhat longer sample Kelm (2010) obtained stable estimates of the cointegrating vectors in the CHEER model with short-term interest rates and a proxy of the risk premium. In the CHEER model, the indices of domestic and foreign tradables price are used and, interchangeably, (i) nominal interest rates on ten years' bonds (I^L, I^{*L}) and (ii) three-month interbank rates WIBOR 3M and EURIBOR 3M (I^S, I^{*S}). The analysed model was the VAR model:

$$y_{(m)} = [b, p^T, p^{*T}, i\ , i^*; \iota'_{(k)}]' , \tag{22}$$

where: $i\ = \ln(1 + I/1200)$, $i^* = \ln(1 + I^*/1200)$, I, I^* - an annual nominal interest rate on assets denominated in the zloty and euro (%); $\iota_{(k)}$ - deterministic variables.

Tab. 7 about here

Table 7 presents the estimates of the moduli of the largest characteristic roots and summarizes the results of cointegration tests. The conclusions are similar to those provided by the strict PPP model: (i) explosive roots are still present in the pre-crisis period, and (ii) 1 to 3 cointegrating vectors can be identified when exchange rate depreciation in the years 2008-2009 is taken into account. The results of the VEC model with long-term interest rates estimated over the entire sample are representative of this stage of research. The nominal rate adjusts to the path:

$$b = p^T - p^{*T} - 40.49(i^L - i^{*L}) + \mu_1 , \tag{23}$$

and the equation

$$p^T = p^{*T} + b + 7.878(i^L - i^{*L}) + \mu_2 \tag{24}$$

is an attractor of domestic prices. ECTs' estimates indicate that the nominal rate adjusts to relation (23) at a rate of 5.1% of *b*'s deviation from (23) in the previous month. Domestic prices' adjustments along (24) develop more slowly, at a rate of 2.4%. In the test of overidentifying restrictions *p*-values are greater than those considered borderline (0.130 and 0.408 in the test with small-sample Bartlett correction), thus providing provide formal arguments for accepting equation (23) as an equilibrium condition for the foreign exchange market in Poland. Moreover, the structure of equations (23) and (24) is the same as of the first equilibrium condition in the REH5 scenario – this result appears to confirm the RE hypothesis. The conclusions are different when the model's properties are analysed more in depth. Fig. 4 presents nominal rate's and domestic prices' deviations from the equilibrium trajectories. The conclusions are unambiguous: the stationarity of $\beta'_{v(m)}y_{(m)t}$ and $\beta'_{v(m)}R_{1t}$ (R_{1t} - the regressors matrix in so-called concentrated model $R_{0t} = \alpha\beta' R_{1t} + \varepsilon_t$, Juselius 2006, pp. 292-293) is at best questionable, and equations (23)-(24) may, in fact, be CI(2,1) cointegrating relations.

Fig.4. about here

Tab. 8 about here

The last ambiguity called for performing a full *I*(2) analysis of the CHEER model in stage two of the research. Assuming that at least two polynomial cointegrating vectors occur, the cointegration test in the VEC model $y_{(m)} = [b, p^T, p^{*T}, i^L, i^{*L}; t]'$ suggests that two *I*(2) trends and one autonomous *I*(1) stochastic trend (tab. 8) are present. The analysis of the strict PPP model in the section above well illustrates the scale of problems involved in the construction of a VEC-*I*(2) system, if the system is to allow a clear economic interpretation of the cointegrating vectors. Therefore, an attempt was made to replace an *I*(2) analysis with a much more transparent *I*(2)-in-*I*(1) approach. A necessary and sufficient condition justifying the use of the *I*(2)-in-*I*(1) analysis in the CHEER model is the long-run homogeneity of the nominal exchange rate and of domestic and foreign prices. In particular, if the homogeneity restriction can be imposed on the β and δ vectors in (14), the *I*(2) analysis of $y_{(m)} = [b, p^T, p^{*T}, i^L, i^{*L}; t]'$ can be replaced with the *I*(1) analysis of the VEC model $y_{(m)} = [q^T, \Delta p^T, \Delta p^{*T}, i^L, i^{*L}; t]'$.

The results of the long-term homogeneity restriction are borderline (the *p*-values are 0.11 and 0.08 for $V = 2$ and 3, respectively). However, if they are interpreted in such a way as to ensure the clarity of the economic interpretation of the model, they provide sufficient arguments for performing an *I*(2)-in-*I*(1) analysis. Cointegration tests and the estimates of the largest characteristic roots explicitly confirm that two cointegrating vectors are present in the model $y_{(m)} = [q^T, \Delta p^T, \Delta p^{*T}, i^L, i^{*L}; t]'$. From the analysis of the adjustment matrix in the unrestricted model it follows that the first cointegrating vector should be normalized with respect to price inflation in the domestic tradables sector and the second one with respect to the real exchange rate, which is the only domestic variable gravitating in this direction. Ultimately, however, the properties of the VEC system with over-identifying restrictions aligned with the PPP3 and IKE6 scenarios turn out to be unsatisfactory (tab. 9, the upper panel), because of the 'persistently' weak exogeneity of the real exchange rate.

Tab. 9 about here

Completely different results are obtained only when the long-term interest rates are replaced by their short-term counterparts and only in the sample 1999:01-2009:09 (tab. 9, the middle panel). The basic drawbacks of the VEC model $y_{(m)} = [q^T, \Delta p^T, \Delta p^{*T}, i^S, i^{*S}; t]'$ is that the over-identifying restrictions test yields a borderline result and, more importantly, that the equilibrium parameters are not stable. Furthermore, extended over the full sample 1999:01-2011:06, analysis shows the model with short-term interest rates to be completely unacceptable (tab. 9, the lower panel). The recursive tests of over-identifying restrictions[8] $\mathrm{LR}(y)$ and $\mathrm{LR}(R_1)$ give conclusive arguments for rejecting the structuralizing restrictions for samples ending between the second half of 2009 and the end-point of the sample 2011:06 (fig. 5).

Fig. 5 about here

A more in-depth analysis of the recursive estimation results allows two complementary working hypotheses to be formulated. Firstly, even a cursory visual inspection of the recursive estimates of the equilibrium parameters shows a rapid increase and then stabilisation of the semi-elasticity by the spread of the real interest rates in the second cointegrating vector (fig. 6). An intuitive and fully justified working hypothesis is therefore one linking the instability of the parameter with the eruption of the worldwide financial crisis and with the sudden increase in global risk. Secondly, parameters' estimates for the reduced sample 1999:01-2009:09 show that in the pre-crisis period the semi-elasticity by the spread smoothly ranged from 20 to 30. It is, therefore, likely that the zloty/euro exchange rate was also affected by risk in that period, but its source must have been different than in the height of the subprime crisis.

Fig. 6 about here

The standard framework for analysing exchange rate's dependency on risk premium is an asset-pricing model with stochastic discount factors (SDF, Smith and Wickens, 2002). Under standard assumptions about probability distributions of the SDFs, the UIP equation is following:

$$\Delta b_{t+1} = i_t^1 - i_t^{*1} + \lambda_t = i_t^1 - i_t^{*1} + \tfrac{1}{2}\{(\pi_{(k)t})'\pi_{(k)t} - (\pi^*_{(k)t})'\pi^*_{(k)t}\} + (\pi_{(k)t} - \pi^*_{(k)t})'\varepsilon_{(k)t+1}, \qquad (25)$$

[8] For details see Dennis (2006).

where the first component of the risk premium λ is the Jensen inequality term (JIT) and the second component of λ indicates that risk fluctuations depend on shocks $\varepsilon_{(k)}$ and the differences between domestic and foreign market prices of risk $\pi_{(k)} - \pi^{*}_{(k)}$ (e.g. Iwata and Wu, 2006).

In exchange rate analyses relation (25) can be used for enhancing empirical models with direct market measures of risk[9]. Alternatively, according to Lucas' (1982) general equilibrium asset-pricing model (GEAP), risk fluctuations may be attributed to the variability of some macrovariables accounting for changes in fiscal and monetary policy. In the literature, few recommendations concerning the selection of variables approximating risk premium can be found. Special attention is usually paid to the impacts of fiscal deficits, the significance of foreign or government debt, and the role of disequilibria in the external sector. In all instances, model extensions are viewed as working hypotheses (e.g. Juselius 1995; Clark and MacDonald 1999; a review in Jongen et al. 2008). Few analyses of the determinants of the Polish zloty exchange rate in the pre-crisis period have also used the GEAP approach. For instance, Kelm (2010) have established that the short-term government debt share of GDP is the most reliable proxy of risk premium. According to the author, the debt increase resulting from larger issues of T-bills is an indication that the government's problems with funding its current expenditures are growing or that investors are losing trust in securities with longer maturity. The short-term debt fluctuations may be alternatively caused by the transmission of global risk. Because it is safer for the government to fund its expenditure by selling long-term securities, more T-bills are likely to be issued when the demand for bonds declines.

Fig. 7 about here

The fluctuations of the real exchange rate q^T and of the proxy of risk $U^{DST} = D^{ST} / F^{ST}$ (D^{ST}, F^{ST} – short-term government debt to GDP ratios in Poland and the euro zone) are presented in fig. 7. There are three comments that the figure provokes on visual inspection. Firstly, the shapes of the RER and of risk proxy are similar enough to allow a working hypothesis about both variables being driven by a common stochastic trend. Secondly, turning points in the risk premium come before turning points in the real rate. Thirdly, the abrupt real depreciation between September 2008 and March 2009 was greater than the increase in risk related to it. The CHEER model should therefore be extended by introducing risk proxy U^{DST}, but also a shift dummy representing 'a structural break' caused by higher global risk in the subprime crisis.

[9] A pioneering study into the zloty/euro exchange rate with risk variability approximated by Credit Default Swaps has been presented by Kębłowski (2011) and Kębłowski and Welfe (2012).

The model $y_t = [q_t^T, \Delta p_t^T, \Delta p_t^{T*}, i_t^S, i_t^{*S}, U_t^{DST}, t]'$ with two samples 1999:01-2009:09 and 1999:01-2011:06 (with a shift dummy for the period 2009:04-2011:06) was analysed in the same way as the previous versions of the CHEER model. Again, the VAR models with three lags proved to be the optimal system. Regardless of what dimension of the cointegration space was assumed, the results of the weak exogeneity tests provide grounds for considering VEC models conditional on the measure of risk at standard levels of significance.

Tab. 10 about here

The estimation results of the respecified CHEER model are presented in tab. 10. The estimates of the cointegrating vectors are stable regardless of the sample. The recursive estimates of the equilibrium parameters indicate only a slight drift in the samples ending in the successive months of years 2005–2007 and then stabilize at levels corresponding to the final estimates (see fig. 8). The differences between CHEER models using the pre-crisis sample or the full sample are either negligible or very moderate. The estimates of the second equilibrium condition point to a growing significance of the internal risk proxy U^{DST} and a weakening influence of real interest rates on the real exchange rate.

The interpretation of the cointegrating vectors is clear-cut: producer price inflation is attracted by the first polynomial cointegrating vector:

$$\Delta p_t^T = \underset{(3.5)}{0.0213}(e_t - p_t^T + p_t^{*T}) - \underset{(2.8)}{0.00002}\,t\,. \tag{26}$$

The structure of the above equilibrium relation supports the hypothesis that prices in the tradables sector of a small and open economy are determined by foreign prices and a nominal exchange rate. What makes equation (26) different from the standard PPP model is non-linear price adjustments within the internal equilibrium correction. In particular, an increase in the nominal exchange rate (depreciation) or in the prices of a foreign tradables sector accelerates the growth of domestic prices, making inflation rise. On the other hand, when domestic prices exceed a PPP-determined level inflation goes down, i.e. domestic prices converge to a level determined by price arbitrage in the tradables sector. The equilibrium equation for the real exchange rate is the following:

$$q_t^T = -\underset{(4.3)}{4.233}((i_t^S - \Delta p_t^T) - (i_t^{*S} - \Delta p_t^{*T})) + \underset{(8.3)}{0.166}U_t^{DST} + \underset{(5.7)}{0.082}\,C(09.04)_t\,. \tag{27}$$

The depreciation of the zloty against the euro initiates a process of adjustments, the intensity of which is much higher than of adjustments in the standard linear PPP models or the CHEER models without risk premium – disequilibrium decreases by around 15 percent per month. There are also moderate symptoms of `. RER fluctuations also run along the long-run equilibrium condition for tradables prices – higher price dynamics Δp^T caused by the nominal depreciation of the zloty in (26) leads to real depreciation in (27) with the parameter of 0.59 (0.95 in the short sample).

Fig. 8 about here

The results produced by the recursive tests of the over-identifying restrictions and by the parameter constancy tests do not allow rejecting the proposed structure of the cointegrating vectors (fig. 9). The 'purely' long-term deviations of $\beta'_{v(m)}R_{1t}$ from the first cointegrating vector oscillate around zero across the sample, and in the second equilibrium relation they stabilize after 2003 (fig. 10). The 'overall' disequilibria $\beta'_{v(m)}y_{(m)t}$ oscillate around zero too, but their amplitude increases over the subprime crisis. The stochastic properties of the model may provoke some reservations. In particular, the joint Doornik-Hansen test clearly rejects the normality of the errors. However, the univariate tests show that the equations of the real rate (p-value = 0.13) and of domestic and foreign price inflation (p-values = 0.26 and 0.33) meet the normality assumption. The negative result of the joint DH test follows from kurtosis excess in the equations of the nominal interest rates.

Fig. 9 about here

Fig. 10 about here

There is apparently one reason for which the VEC model with cointegrating vectors (26)-(27) can be criticised – the shift-dummy approximating changes in the global risk perception is included on an ad-hoc basis. This solution provokes questions about whether a change in risk pricing can really be abrupt and why the ratio of the short-term debts U^{DST} fails to approximate risk variability over the entire sample 1999:01-2011:06. To find an answer, in the final stage of the investigation the observable 'external' risk proxy for the subprime crisis period was identified. This part of the analysis started with the identity:

$$\lambda^A = \lambda - \lambda^* = (\lambda^{INT} + \lambda^{EXT}) - (\lambda^{*INT} + \lambda^{*EXT}), \qquad (28)$$

which defines fluctuations in the 'aggregate' risk premium λ^A as a net effect of the change in risk carried by assets denominated in the Polish zloty (λ) and the euro (λ^*)[10]. The different sizes and the

[10] Superscripts denote domestic (*INT*) and foreign (*EXT*) sources of the risk changes.

strong dependence of the Polish economy on Euro *zone* economies justify making a simplifying assumption $\lambda^{EXT} = \lambda^{*INT} + \lambda^{*EXT}$, as a result of which equation (28) is reduced to:

$$\lambda^{A} = \lambda^{INT} \cong D^{ST} . \tag{29}$$

However, if global developments produce major variations in risk, equation (28) needs to be rearranged into:

$$\lambda^{A} = \lambda^{INT} + (1 + m^{U} + v^{U})\lambda^{EXT} - (1 + m^{U})(\lambda^{*INT} + \lambda^{*EXT}) , \tag{30}$$

where m^{U} denotes a crisis-induced mark-up in global risk pricing (see eq. (25)) and v^{U} stands for additional mark-up in risk pricing for assets denominated in peripheral currencies. For $\lambda^{EXT} = \lambda^{*}$, global risk transfers into the 'aggregate' risk premium according to the identity:

$$\lambda^{A} = \lambda^{INT} + v^{U}\lambda^{*} \cong D^{ST} + v^{U} F^{ST} \tag{31}$$

which justifies appropriate extensions of the CHEER model.

Fig.11 about here

Tab. 11 about here

A visual inspection confirms that the shift dummy C(09.04) precisely identifies periods when the foreign risk proxy F^{ST} reached its maxima (fig. 10). Accordingly, in the final stage of the analysis of the zloty/euro exchange rate, a cointegration analysis was performed of alternative CHEER models with risk premiums approximated using various combinations of variables U^{DST}, D^{ST}, F^{ST} and $C(09.04)$. Table 11 summarizes estimation results for one of the competing VEC models, $y_{(m)t} = [q_t^T, \Delta p_t^T, \Delta p_t^{T*}, i_t^S, i_t^{*S}, U_t^{DST}, F_t^{ST}]'$. The conclusions are clear-cut again. Firstly, the probability value in the test of over-identifying restrictions is distinctly higher than the probability value in the model with a shift dummy. The downside of the model comprising F^{ST} is the mediocre stochastic properties of the residuals, but in this case, too, error normality is rejected for kurtosis excess, and the borderline results of the autocorrelation tests result from the rapid swing in F^{ST} at the end of 2010. Secondly, replacing the shift dummy with F^{ST} does not have a major effect on the estimates of the equilibrium parameters and adjustment coefficients, so earlier conclusions about the determinants of the zloty/euro exchange rate before and during the subprime crisis remain unchanged.

Conclusions

The review of empirical studies has shown that purchasing power parity continues to serve as the central point of reference in exchange rate analyses. Despite a great number of studies which are being undertaken to solve the PPP puzzle, Rogoff's (1996) conclusion that for most economists purchasing power parity is a long-term model of real exchange rates is still valid. The review has also revealed that in the definite majority of empirical studies agents are assumed to be rational, as a result of which researchers still seek to confirm that real exchange rates are stationary and to explain their persistence. The strategy of analysis changes when the rational expectations hypothesis is replaced with a more general hypothesis of imperfect knowledge economics that Roman Frydman and Michael Goldberg have proposed. Their hypothesis implies that real exchange rates can be generated by both difference-stationary processes and near-$I(2)$ stochastic trends. This means that the IKE world is free of Rogoff's puzzle, because its 'inherent' property is long-lasting swings in the real and nominal exchange rates. Moreover, the rational expectations hypothesis which is a special case of the IKE hypothesis offers an opportunity to test the REH against its IKE alternative.

An attempt has been made in the paper to establish which of the two hypotheses (rational expectations or imperfect knowledge economics) is more precise in describing processes that took place in the Polish foreign exchange market in the free float period. The outcomes are the following. Firstly, the strict PPP model with a nominal rate and tradables prices has been rejected. In the samples ending before the subprime crisis explosive roots were identified, which 'shrank' after the analysis was extended to the period of crisis. This does not change the fact that the nominal rate and tradables prices do not cointegrate. These results raise doubts as to the usefulness of standard unit root tests applied to analyse the zloty/euro exchange rate. They also provide a reason for formulating a somewhat 'unexpected' working hypothesis that the subprime crisis had a balancing influence on the Polish foreign exchange market. The estimates yielded by the vector error correction models with 'enforced' cointegrating vectors offer more arguments against the strict PPP model – the symptoms of near-$I(2)$-ness in the standard VEC-$I(1)$ representations and of misspecification in the VEC-$I(2)$ model are evident. Secondly, the analysis of the Dornbusch-type monetary model with a nominal rate, domestic and foreign prices and nominal interest rates has not lessened the doubts about how useful the RE hypothesis is for describing the Polish foreign exchange market under free float. Assuming that all variables are difference-stationary, the VEC-$I(1)$ model identifies two CI(2,1) cointegrating relations, but the estimates of the error correction terms still show that the nominal exchange rate is fairly persistent. Thirdly, the analysis of the VEC-$I(2)$ model with the pre-crisis sample has identified cointegrating vectors with structures consistent with those predicted by the

Frydman-Glodberg model. On the other hand, the analysis of the estimation results has pointed out that it is not sufficient to limit the specification to the nominal exchange rate, prices and nominal interest rates for a model to be acceptable, because of the instability of its parameters and its inconsistency with the data generation process. In the Frydman-Goldberg model forecasting strategies undergo revision because of building-up gap effects, two of which have been identified in the paper. By accounting in the CHEER model's specification for risk fluctuations arising from domestic disequilibria and for a shift in the perception of global risk ultimately a fully-specified model with satisfying properties is constructed, where the half-life of RER's adjustments to RER's equilibrium path is only 4¼ months.

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Appendix I

Exchange rate regimes in Poland

Date	Exchange rate regime	Change
1990-01	US dollar peg	1 USD = 0,95 PLN
1991-05	Basket of currencies peg	Devaluation against the US dollar: 16.8%
1991-10	Crawling peg	Rate of crawl = 1.8%
1992-02		Devaluation = 12.0%
1992-08		Devaluation = 8.1% Rate of crawl = 1.6%
1994-09		Rate of crawl = 1.5%
1994-11		Rate of crawl = 1.4%
1995-02		Rate of crawl = 1,2%
1995-05	Crawling band	Rate of crawl = 1.2% Crawling band = +/-7.0%,
1995-12		Revaluation = 6.0%
1996-01		Rate of crawl = 1.0%
1998-02		Rate of crawl = 0.8% Crawling band= +/-10.0%
1998-07		Rate of crawl = 0.65%
1998-02		Last large NBP's intervention in the Polish foreign exchange market
1998-09		Rate of crawl = 0.5%
1998-10		Rate of crawl = 0.5% Crawling band = +/-12.5%
1999-03		Rate of crawl = 0.35% Crawling band = +/-15.0%
2000-04	Free float	-

Appendix II: Data sources and time series definitions

Prices:

p consumer price index in Poland (logarithm, 2000=0), source: Main Statistical Office, Poland

p^T producer price index in manufacturing in Poland (logarithm, 2000=0), source: Main Statistical Office, Poland

p^* consumer price index in euro area (logarithm, 2000=0), source: OECD

p^{*T} producer price index in manufacturing in euro area (logarithm, 2000=0), source: OECD

Exchange rates:

b nominal exchange rate (price of 1 EUR in PLN, logarithm, 2000=0), source: Narodowy Bank Polski

$q = q(CPI)$ CPI-based real exchange rate zloty/euro, $q = b - p + p^*$

$q^T = q(PT)$ PT-based real exchange rate zloty/euro, $q^T = b - p^T + p^{*T}$

Interest rates:

I^L nominal interest rates on 10Y bonds denominated in zlotys (%), source: OECD

I^S three-month interbank rate WIBOR 3M (%),source: OECD

$i^J = \ln(1 + I^J / 1200)$, $J = \{L, S\}$

I^{*L} nominal interest rates on 10Y bonds denominated in euros (%),source: OECD

I^{*S} three-month interbank rate EURIBOR 3M (%),source: OECD

$i^{*J} = \ln(1 + I^{*J} / 1200)$, $J = \{L, S\}$

Risk proxies:

D^{ST} short-term government debt to GDP ratio, Poland, $D^{ST} = ST^D / Y$

ST^D nominal short-term government debt in Poland, millions PLN, source: Narodowy Bank Polski

Y nominal gross domestic product in Poland, millions PLN, own monthly estimates on the basis of official quarterly data (source: Main Statistical Office)

F^{ST} short-term government debt to GDP ratio, euro area, $F^{ST} = ST^F / Y^*$

ST^F nominal short-term government debt in euro area, millions EUR, source: Bundesbank

Y^* nominal gross domestic product in euro area, millions EUR, own monthly estimates on the basis of official quarterly data (source: Eurostat)

Tab.1 CVAR scenarios in the PPP model

Scenario	Assumptions	Cointegrating vectors
PPP1	$V=1$ $b_t, p_t, p_t^* \sim I(1)$	$b_t - p_t + p_t^* \sim I(0)$
PPP2	$V=2,\ S_2=1,\ S_1=0$ $p_t, p_t^* \sim I(2),\ b_t \sim I(1)$	$b_t - p_t + p_t^* \sim I(0)$ $b_t - \bar{c}_1 \Delta p_t \sim I(0)$
PPP3	$V=1,\ S_2=1,\ S_1=1$ $p_t, p_t^* \sim I(2),\ b_t \sim I(1)$	$(b_t - p_t + p_t^*) + \bar{c}_2 \Delta p_t \sim I(0)$ $\beta_{\perp 1,1}\Delta b_t + \beta_{\perp 1,2}\Delta p_t + \beta_{\perp 1,3}\Delta p_t^T \sim I(0)$
PPP4	$V=1,\ S_2=1,\ S_1=1$ $b_t, p_t, p_t^* \sim I(2)$	$b_t - \bar{c}_3(p_t - p_t^*) - \bar{c}_4 \Delta p_t \sim I(0)$ and $\bar{c}_3 \neq 1$ $\beta_{\perp 1,1}\Delta b_t + \beta_{\perp 1,2}\Delta p_t + \beta_{\perp 1,3}\Delta p_t^T \sim I(0)$

Tab.2 The STAR model (18), 1999:01-2011:06

Inner regime				Outer regime (change)				Transition parameters		
μ	$\rho_1 - 1$	γ_1	γ_2	$\tilde{\mu}$	$\tilde{\rho}_1 - 1$	$\tilde{\gamma}_1$	$\tilde{\gamma}_2$	θ_2	c_2^1	c_2^2
–0.001 (0.7)	0.025 (1.0)	0.328 (4.3)	–0.291 (3.6)	0.004 (1.0)	–0.298 (4.4)	–0.087 (0.5)	0.227 (1.0)	10.97 (0.5)	–0.098 (10.9)	0.084 (26.2)

Diagnostics:

AR(1)=0.835, AR(2)=0.932, ARCH(1)=0.652, JB=0.553, F(non)=0.633

Notes: Tildes discern estimates' change in the outer regime. *t*-ratios are reported in parentheses. Dots stand for the parameters with *t*-ratios smaller than 2. *P*-values are reported for AR, ARCH, JB and F(non) tests; AR(*s*) – test of the errors autocorrelation of order *s*, ARCH(*s*) – test of the ARCH effect of order *s*, JB – Jarque-Bera normality test, F(non) – test of no remaining nonlinearity in the STAR model.

Tab. 3 The largest moduli of characteristic roots in PPP models

$\iota_{(k)}$	Sample:	$y_{(m)} = [b, p, p^*; \iota'_{(k)}]'$				
μ	1999:01-2008:06	1.0092	0.9757	0.9254	0.5122	0.5077
	-2011:06	1.0028	0.9885	0.9276	0.5476	0.4840
$\mu\ .\ t$	-2008:06	0.9938	0.9632	0.9340	0.4744	0.4651
	-2011:06	1.0417	0.9445	0.9445	0.5575	0.5575
$\iota_{(k)}$	Sample:	$y_{(m)} = [b, p^T, p^{*T}; \iota'_{(k)}]'$				
μ	1999:01-2008:06	1.0301	0.9467	0.9467	0.5951	0.5951
	-2011:06	1.0112	0.9512	0.9512	0.5935	0.5935
$\mu\ .\ t$	-2008:06	1.0445	0.9442	0.9442	0.5520	0.5520
	-2011:06	1.0083	0.9271	0.9271	0.5646	0.5646

Tab. 4 Cointegration test in the PPP model, 1999:01-2011:06

v	s_2 3	2	1	0
0	213.81 (0.000)	138.60 (0.000)	81.18 (0.000)	38.76 (0.123)
1	-	73.99 (0.000)	19.80 (0.757)	12.73 (0.760)
2	-	-	10.40 (0.617)	2.95 (0.871)

Notes: *P*-values are reported in parentheses.

Tab. 5 The estimation of the PPP model ($V=1,\ S_1=1,\ S_2=1$), 1999:01-2011:06

	b	p^T	p^{*T}	t	Δb	Δp^T	Δp^{*T}	Δt
$[\beta',\delta']$	–1	1	–1	0.0006 (5.7)	15.72	15.88	0.154	–0.0993
α'	–0.039 (3.0)	–0.026 (8.0)	–0.002 (1.2)					
β'					–1	1	–1	0.0006 (5.7)
ζ_1'					0.032 (0.1)	–0.388 (4.1)	0.146 (2.5)	-
$\beta_{\perp 1}'$					–1.030	1	2.030	–0.0078
ζ_2'					0.056 (0.4)	–0.024 (0.6)	–0.178 (7.1)	-

LR = 0.118

AR(1) = 0.803 AR(2) = 0.103 DH = 0.025

AR(3) = 0.708 AR(4) = 0.645 ARCH(1) = 0.539 ARCH(2) = 0.417

Notes: *t*-ratios are reported in parentheses. Dots stand for the parameters with *t*-ratios smaller than 2. *P*-values are reported for LR, AR, DH and ARCH tests; LR – over-identifying restrictions test, AR(*s*) – test of the errors autocorrelation of order *s*, DH – Doornik-Hansen normality test, ARCH(*s*) – test of the ARCH effect of order *s*.

Tab. 6 CVAR scenarios in the CHEER model

Scenario	Assumptions	Cointegrating vectors
REH5	$V=3,\ S_2=1,\ S_1=1$	$(b_t - p_t + p_t^*) + \omega_1(i_t - i_t^*) \sim I(0)$
	$b_t, p_t, p_t^* \sim I(2)$	$(i_t - \Delta p_t) - a_1 p_t - a_2 b_t \sim I(0)$
	$i_t, i_t^* \sim I(1)$	$(i_t^* - \Delta p_t^*) - a_3 p_t^* - a_4 b_t \sim I(0)$
		$\beta_{\perp 1,1}\Delta b_t + \beta_{\perp 1,2}\Delta p_t + \beta_{\perp 1,3}\Delta p_t^* + \beta_{\perp 1,4}\Delta i_t + \beta_{\perp 1,5}\Delta i_t^* \sim I(0)$
IKE6	$V=3,\ S_2=2,\ S_1=0$	$(b_t - p_t + p_t^*) + \omega_1(i_t - i_t^*) - \omega_2(\Delta p_t - \Delta p_t^*) \sim I(0)$
	$b_t, p_t, p_t^*, i_t, i_t^* \sim I(2)$	$i_t - a_1\Delta p_t - a_2 p_t - a_3 b_t \sim I(0)$
		$i_t^* - a_4\Delta p_t^* - a_5 p_t^* - a_6 b_t \sim I(0)$

Tab. 7 Largest moduli of the characteristic roots and cointegration tests in the CHEER model

$\iota'_{(k)}$	Sample:	$y_{(m)} = [b, p^T, p^{*T}, i^L, i^{*L}, \iota'_{(k)}]'$					V
μ	1999:01-2008:06	1.0362	0.9977	0.9586	0.9586	0.8550	-
	-2009:09	0.9731	0.9731	0.9536	0.9536	0.8493	1 - 2
	-2011:06	0.9999	0.9561	0.9515	0.9515	0.8268	1 - 3
t	1999:01-2008:06	1.0427	0.9505	0.9505	0.8809	0.8809	-
	-2009:09	0.9626	0.9554	0.9554	0.9071	0.9071	1 - 2
	-2011:06	0.9536	0.9536	0.9341	0.9189	0.8607	1
$\iota'_{(k)}$	Sample:	$y_{(m)} = [b, p^T, p^{*T}, i^S, i^{*S}, \iota'_{(k)}]'$					V
μ	1999:01-2008:06	1.0451	0.9972	0.9543	0.9453	0.9453	-
	-2009:09	1.0106	0.9028	0.9028	0.9329	0.9329	1 - 3
	-2011:06	1.0045	0.9881	0.9881	0.9251	0.9251	1 - 2
t	1999:01-2008:06	1.0576	0.9777	0.9387	0.9387	0.6984	-
	-2009:09	1.0053	1.0053	0.9313	0.9313	0.8856	1
	-2011:06	0.9968	0.9968	0.9362	0.9362	0.8747	1

Notes: Results of cointegration *Trace* tests are reported in the column *V*.

Tab. 8 The cointegration test in the CHEER model $y_{(m)} = [b, p^T, p^{*T}, i^L, i^{*L}; t]'$, 1999:01-2011:06

v	s_2	3	2	1	0
2		86.57 (0.077)	58.24 (0.307)	40.86 (0.448)	32.39 (0.374)
3		-	38.62 (0.339)	24.05 (0.479)	15.33 (0.555)
4		-	-	12.86 (0.388)	3.85 (0.761)

Notes: *P*-values are reported in parentheses.

Tab. 9 The estimation of the CHEER model $y_{(m)} = [q^T, \Delta p^T, \Delta p^{*T}, i\ \ , i^*; t]'$

a. Long-term interest rates, 1999:01-2011:06

	q^T	Δp^T	i^L	Δp^{*T}	i^{*L}	t
β_1'	–0.0329	**1**	0	0	0	0.0000
	(4.4)					(0.2)
β_2'	**1**	–42.70	42.70	42.70	–42.70	0.0002
		(9.5)	(9.5)	(9.5)	(9.5)	(0.4)
α_1'	–1.021	**–0.609**	·	0.471	·	
	(2.5)	(4.8)		(5.9)		
α_2'	**–0.023**	.	–0.0003	–0.005	.	
	(2.4)		(2.5)	(2.7)		

LR = 0.403

AR(1) = 0.523 AR(2) = 0.461 DH = 0.000

AR(3) = 0.152 AR(4) = 0.306 ARCH(1) = 0.209 ARCH(2) = 0.046

b. Short-term interest rates, 1999:01-2009:09

	q^T	Δp^T	i^S	Δp^{*T}	i^{*S}	t
β_1'	–0.0102	**1**	0	0	0	0.0001
	(1.8)					(4.0)
β_2'	**1**	–32.58	32.58	32.58	–32.58	0.0016
		(9.5)	(9.5)	(9.5)	(9.5)	(2.7)
α_1'	–2.075	**–0.868**	0.018	–0.419	0.006	
	(3.2)	(5.4)	(2.6)	(3.7)	(2.0)	
α_2'	**–0.056**	.	.	–0.010	.	
	(3.3)			(3.2)		

LR = 0.136

AR(1) = 0.090 AR(2) = 0.190 DH = 0.130

AR(3) = 0.104 AR(4) = 0.308 ARCH(1) = 0.128 ARCH(2) = 0.670

c. Short-term interest rates, 1999:01-2011:06

	q^T	Δp^T	i^S	Δp^{*T}	i^{*S}	t
β_1'	–0.0191	**1**	0	0	0	0.00002
	(2.4)					(1.7)
β_2'	**1**	–62.86	62.86	62.86	–62.86	0.00184
		(7.5)	(7.5)	(7.5)	(7.5)	(1.9)
α_1'	–1.009	**–0.693**	·	–0.456	·	
	(2.2)	(5.3)		(5.2)		
α_2'	–0.012	.	.	–0.004	.	
	(1.8)			(3.1)		

LR = 0.001

AR(1) = 0.235 AR(2) = 0.164 DH = 0.000

AR(3) = 0.136 AR(4) = 0.271 ARCH(1) = 0.005 ARCH(2) = 0.099

Notes: see table 5; dots stand for the parameters with *t*-ratios smaller than 2.

Tab. 10 The estimation of the CHEER model $y_{(m)} = [q^T, \Delta p^T, \Delta p^{*T}, i^S, i^{*S}, U^{DST}; t]'$

a. 1999:01-2009:09

	q^T	Δp^T	i^S	Δp^{*T}	i^{*S}	U^{DST}		t
β_1'	–0.0217	**1**	0	0	0	0		0.00003
	(3.6)							(3.1)
β_2'	**1**	–5.949	5.949	5.949	–5.949	–0.152		0
		(4.5)	(4.5)	(4.5)	(4.5)	(5.7)		
α_1'	–0.950	**–0.741**	0.028	–0.194	0.004	0		
	(2.1)	(6.6)	(5.6)	(2.5)	(2.1)			
α_2'	**–0.148**	.	0.002	0.013	.	0		
	(4.4)		(5.8)	(2.2)				

LR = 0.550

AR(1) = 0.198 AR(2) = 0.127 DH = 0.074

AR(3) = 0.124 AR(4) = 0.308 ARCH(1) = 0.730 ARCH(2) = 0.966

b. 1999:01-2011:06

	q^T	Δp^T	i^S	Δp^{*T}	i^{*S}	U^{DST}	C(09.04)	t
β_1'	–0.0213	**1**	0	0	0	0	0	0.00002
	(3.5)							(2.8)
β_2'	**1**	–4.233	4.233	4.233	–4.233	–0.166	–0.082	0
		(4.3)	(4.3)	(4.3)	(4.3)	(8.3)	(5.7)	
α_1'	–0.590	**–0.725**	0.019	–0.330	0.004	0		
	(1.8)	(8.5)	(5.2)	(5.3)	(2.2)			
α_2'	**–0.150**	.	0.003	0.019	0.0004	0		
	(4.6)		(7.2)	(3.3)	(2.1)			

LR = 0.199

AR(1) = 0.155 AR(2) = 0.030 DH = 0.000

AR(3) = 0.490 AR(4) = 0.240 ARCH(1) = 0.084 ARCH(2) = 0.079

Notes: see table 9.

Tab. 11 The estimation of the CHEER model $y_{(m)} = [q^T, \Delta p^T, \Delta p^{*T}, i^S, i^{*S}, U^{DST}, F^{ST}; t]'$, , 1999:01-2011:06

	q^T	Δp^T	i^S	Δp^{*T}	i^{*S}	U^{DST}	F^{ST}	t
β_1'	–0.0189	**1**	0	0	0	0	0	0.00003
	(3.0)							(3.0)
β_2'	**1**	–3.101	3.101	3.101	–3.101	–0.144	–0.025	0
		(3.4)	(3.4)	(3.4)	(3.4)	(8.4)	(6.1)	
α_1'	–0.308	**–0.679**	0.019	–0.292	.	0	0	-
	(1.0)	(7.9)	(5.2)	(4.8)				
α_2'	**–0.149**	.	0.003	0.024	.	0	0	-
	(4.3)		(7.2)	(3.8)				

LR = 0.301

AR(1) = 0.012 AR(2) = 0.077 DH = 0.000

AR(3) = 0.338 AR(4) = 0.050 ARCH(1) = 0.029 ARCH(2) = 0.088

Notes: see table 9.

Fig. 1 CPI- and PT-based real exchange rates (logarithms)

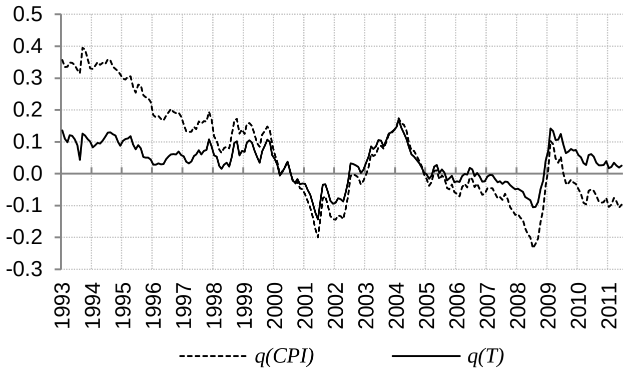


Fig.2 Inner and outer regimes in the STAR model (18)

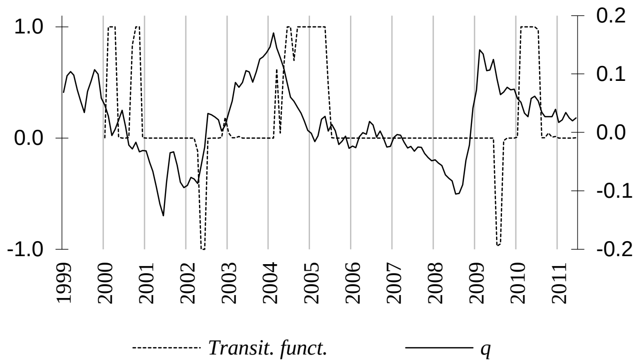


Fig. 3 The nominal zloty/euro exchange rate and relative prices in Poland and the euro area

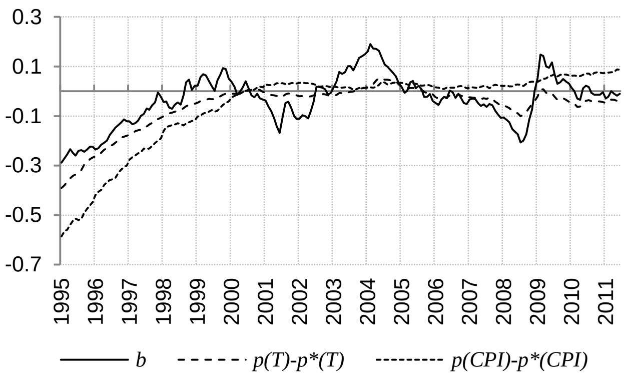


Fig.4 $\beta'_{1(m)}y_{(m)t}$ and $\beta'_{1(m)}R_{1t}$ deviations from the long-term relation (23)

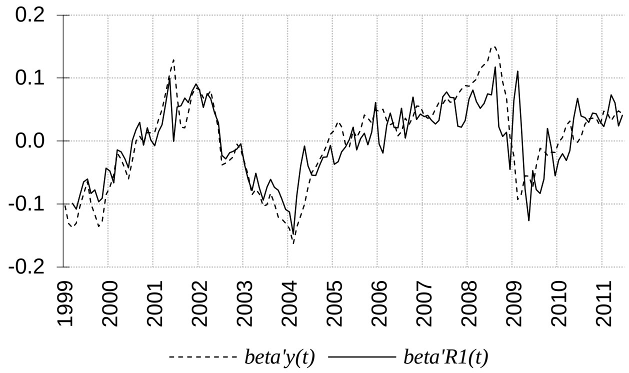
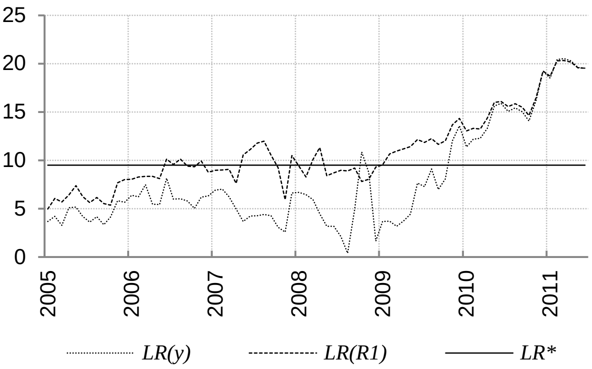


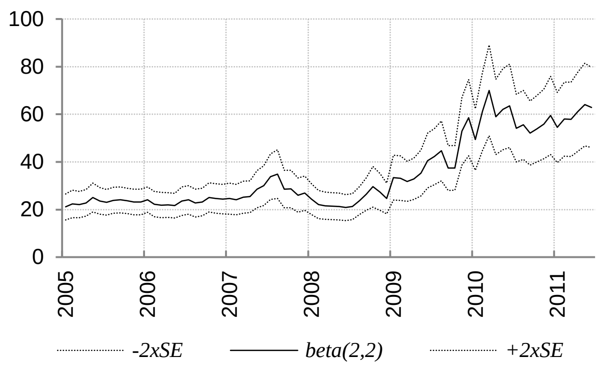
Fig. 5 The recursive LR tests of over-identifying restrictions in the CHEER model

$$y_{(m)} = [q^T, \Delta p^T, \Delta p^{*T}, i^S, i^{*S}; t]'$$



Note: The horizontal line indicates critical value at 0.05 significance level

Fig. 6 Recursive parameter estimates in the $y_{(m)} = [q^T, \Delta p^T, \Delta p^{*T}, i^S, i^{*S}; t]'$ model



Note: The estimates relate to the semi-elasticity by the real interest spread in third variant of the CHEER model (table 9, panel c)

Fig. 7 The real exchange rate q^T and the relative short-term government debt U^{DST}

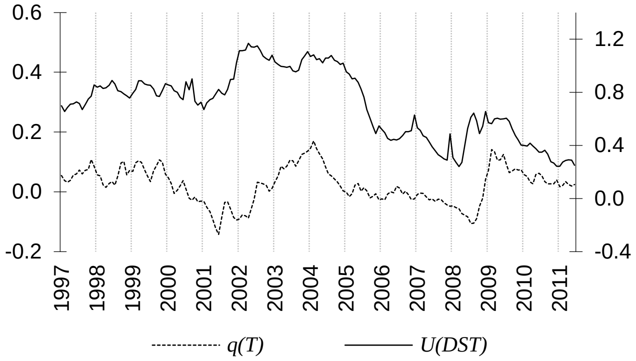


Fig. 8 The recursive estimates of the cointegrating vectors (26)-(27)

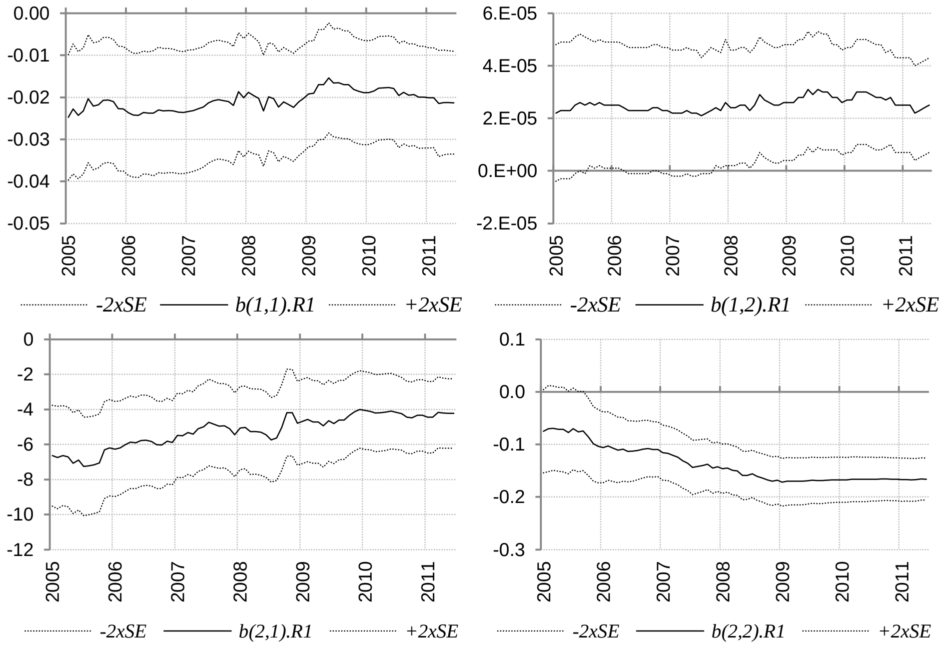
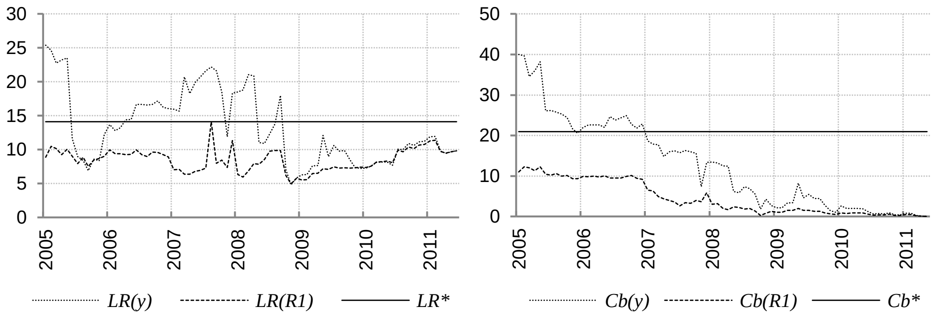


Fig. 9 The recursive tests of over-identifying restrictions (LR) and parameters constancy (Cb) in the $y_{(m)} = [q^T, \Delta p^T, \Delta p^{*T}, i^S, i^{*S}, U^{DST}; t]'$ model



Note: The horizontal lines indicate critical values at 0.05 significance level.

Fig. 10 $\beta'_{1(m)}y_{(m)t}$ and $\beta'_{1(m)}R_{1t}$ deviations from the long-term relations (26)-(27)

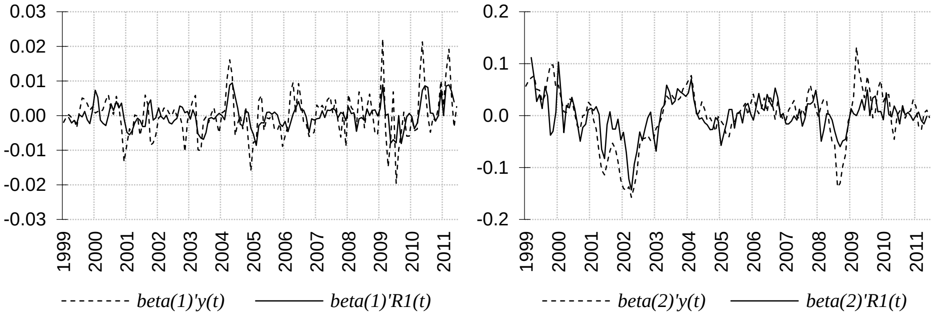


Fig.11 The global risk proxy F(ST) and the shift dummy C(09.04)