Analysis of Stock Market and Housing Market Crises in a Continuous Time New Keynesian Model

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Abstract

We develop a canonical model to analyse the interdependence between monetary policy and financial markets in the context of the recent financial crisis. We apply a dynamic stochastic full equilibrium New-Keynesian model which is prone to instability and occasionally financial market turmoil. In particular, we address the commence and crush of a stock market and of a housing market bubble and observe the transmission to domestic markets and its contagious effects on foreign markets. Afterwards, we analyse how monetary policy makers respond to those crises, first, conservative central bankers, represented by a classical Taylor Rule and second, applied central bankers, represented by a modified Taylor Rule which takes into account financial markets. We find that abrupt policy reactions lead to severe inflation, however they are able to avoid a severe economic recession.

Secondly, we apply these findings to the case of the United States and Canada and compare real monetary policy to our findings of the model.
Keywords: New Keynesian model, Philipps Curve, Taylor Rule, Stochastic Differential Equations.

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1 Introduction

In the past years most OECD countries have experienced the worst financial crises since the Great Depression. The recent financial crisis emphasises how important financial markets are to ensure macroeconomic stability. International financial markets are highly interconnected and have greater impact on real markets than was assumed as hitherto. De facto only little is known about the amount and even the direction of interactive effects of monetary policy and international financial markets. We want to study these spillover effects from financial crises to the real economy and analyse options and challenges of different monetary policy approaches. In particular, we compare policy makers reactions to financial turmoil by changing the amount of importance of domestic and foreign financial markets in their decision making. We also believe, that they should keep in mind differences of markets and the size of their countries and treat each market in each country individually in their decision making process. In a world still largely dominated by national policymaking, this is a considerable hurdle to the development of optimal policy.

We believe a better understanding of the influence of spillover effects of financial markets as well as abroad monetary policy is crucial to conduct efficient (monetary) policy. However recent studies of the financial crisis either use techniques from finance or macroeconomics. We try to build a bridge between both for a better understanding of the effects on real markets. In light of integration, we study monetary policy extensively by analysing different approaches.

We argue that academic research can help guide central banks' efforts by analysing how much emphasis they should place on financial markets. For this purpose this paper is to explore the consequences of treating the interaction between financial markets, monetary policy, and the real economy in a globalised world seriously by developing a fully dynamic theoretical modelling framework. We are particularly interested in shedding light on the relationship between financial markets, financial crises and monetary policy, which can be characterised by a substantial degree of simultaneity.

Our paper makes several contributions to the literature. First, we follow Ball (1998) and derive the Taylor Rule within the model by employing the nominal interest rate and the exchange rate as monetary policy target. Moreover, we follow Bekaert, Cho, and Moreno (2010) and employ a financial market sector. Therefore the inclusion of financial markets in the policy rule is derived within the model. Faia and Monacelli (2007) provided empirical evidence that the inclusion in the Taylor Rule is significant and has an impact in the decision making process. Similarly, Belke and Klose (2010) estimate Taylor rules for the European Central Bank (ECB) and the Federal Reserve (Fed) and include asset prices as additional monetary policy targets.

Moreover, we account for simultaneity between monetary policy and financial markets by incorporating financial markets (i.e., markets for foreign exchange, bonds, and stocks). The issue of simultaneity was empirically analysed by Bjornland and Leitemo (2009), Rigobon (2003), and Rigobon and Sack (2003). A theoretical explanation is given by Hildebrand (2006). Second, we combine finance research with macroeconomic theory by switching into a a continuous-time framework. This allows us to employ advanced techniques from finance literature, such as Jump-Diffusion processes to model the financial market. Technically, we transform the New Keynesian model in stochastic differential equations and compute solutions by using advanced numerical schemes. This interlinkage of economics, finance and mathematics is rather unique. In a last step, we estimate model parameters using Bayesian estimation techniques.

Our open-economy model starts with the New Keynesian (NK) approach of e.g. Blinder (1997), Clarida, Gali, and Gertler (1999), Romer (2000), and Woodford (1999). We are in line with Clarida, Gali, and Gertler (2002), who incorporate the exchange rate in a NK two-country model. Extensions by Gali and Monacelli (2005) and Engels (2009) adopt Calvo Pricing. Leitemo and Soderstrom (2005) include exchange rate uncertainty in a NK model and analyse different monetary policy rules. We adopt the extended open-economy model, including an exchange rate, and develop a system of equations for each country.

Inclusion of the financial market as a sector was for example explained in an extended NK model of Bekaert, Cho, and Moreno (2010). Another attempt to include an advanced financial market was undertaken by Brunnermeier and Sannikov (2012). However, they do not approach the NK framework and apply a rather simple model. Other strains concentrate on the credit channel, respectively financial constraints. Among others Cï¿½rdia and Woodford (2010), Curdia and Woodford (2008), and Woodford (2010) include the credit channel in the NK model. A second strain concentrates on modelling a banking sector with own targets. Christiano, Motto, and Rostagno (2010) and Gertler and Kiyotaki (2010) create this market by including a further institution as an intermediary.

The switch into continuous time is in line with papers by Asada et al. (2006), Chen et al. (2006a,b), and Malikane and Semmler (2008a,b) and was not applied by other NK approaches. However, by establishing ties between macroeconomic and finance research our approach is unique.

2 The Standard Equations: Phillips Curve , Investment and Savings Curve, Exchange rate

2.1 Households

We apply the model we derived in Hayo and Niehof (2013) and extend it by a more advanced IS curve and financial market to analyse monetary policy transmission under various circumstances. Our core model is based on the New-Keynesian three equation model proposed by Ball (1998) (respectively Allsopp and Vines (2000), Blanchard (2007, 2008), Clarida, Gali, and Gertler (1999), and Gali and Monacelli (2005)). Inclusion of a financial market follows Bekaert, Cho, and Moreno (2010). Furthermore, in line with Asada et al. (2006), Chen et al. (2006a,b), and Malikane and Semmler (2008a,b) we switch to a continuous time framework.

Generally, in line with Rotemberg and Woodford (1998) we consider a standard analytical framework with a representative household that is endowed withe a continuum of goods-specific skill to be supplied to a differentiated product industry. The following derivation follows Razin and Yuen (2002) in terms of notation etc.

The household also operates as consumer with access to domestic and foreign goods. Aggregate supply follows Ball (1998) and Blanchard (2007, 2008) and is evolved from a Calvo pricing equation. In line with Ball (1998) and Woodford (2005) a fraction of firms follow a 'rule of thumb' rather than rational behaviour. Furthermore, we follow Jensen (2002) and Ramon and Vazquez (2006) and apply AR(1) processes as disturbances.

We assume that the economy is inhabited by a continuum number of consumers/ producers $j \in [0,1]$. We start by considering a consumption index (a production index), such as that of Dixit and Stiglitz (1977), C_t (P_t) which consists of domestic goods $c_t(j)$ produced by firm j and foreign goods $c_t(j)^*$ produced by a foreign firm j, $p_t(j)$ $(p_t^*(j))$ the prices of the individual goods, n a fraction of goods which are produced domestically and θ gives the price elasticity of demand for individual goods.

$$C_t = \left[\int_0^n c_t(j)^{\frac{\theta-1}{\theta}} dj + \int_n^1 c_t^*(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

$$P_t = \left[\int_0^n p_t(j)^{1-\theta} dj + \int_n^1 (\epsilon p_t^*(j))^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

Households try to minimise the costs of achieving the level of the composite consumption good by finding the least expensive combination of individual goods $c_t(j)$. This minimisation problem can be written as

$$\min_{c_t(j),c_t^*(j)} \int_0^n p_t(j)c_t(j)dj + \epsilon \int_n^1 p_t^*(j)c_t^*(j)dj$$

where ϵ is the nominal exchange rate. Solving this equation by forming a Langrangian and deriving the First Order Conditions reveals the typical characteristic of a Dixit-Stiglitz consumption index, namely

$$c(j) = C_t \left(\frac{p_t(j)}{P_t}\right)^{-\theta}$$

which describes the demand for good j. For $\theta \to \infty$ individual goods become closer substitutes and individual firms have less market power.

The household seeks to maximise his discounted sum of expected utilities by being subject to a period-by-period budget constraint. Using a constant relative risk aversion utility function (CRRA), the representative household's lifetime utility can be written as

$$U = E\sum_{t=0}^{\infty} \beta^t \left(u\left(C_t, \frac{M_t}{P_t}\right) - \int_0^n v(h_t(j))dj\right)$$

where β is the subjective discount factor, M denotes the money supply. $h(j)$ is the supply of type j-labour to the production of the good of variety jl

$$P_tC_t = \int_0^n p_t(j)c_t(j)dj + \epsilon_t \int_n^1 p_t^*(j)c_t^*(j)dj$$

$$P_t\Pi_t = \int_0^n w_t(j)h_t(j)dj + \int_0^n \Pi_t(j)dj$$

$$P_tC_t + \left(\frac{i_t}{1+i_t}\right) M_t + B_t + \epsilon_t B_t^* = M_{t-1} + (1 - i_{t-1})B_t + \epsilon_{t-1,t}(1 + i_{t-1}^*)B_t^* + P_t\Pi_t$$

where B (B^*) is the domestic (foreign)-currency value of domestic (foreign) borrowing, $\epsilon_{t,t-1}$ the forward rate and i (i^*) interest rates. $w(j)$ is the wage per unit labour of type j and $\Pi(j)$ profit income from firms of type j. With perfect capital mobility, covered interest parity prevails:

$$1 + i_t = (1 + i_t^*)\left(\frac{\epsilon_{t,t-1}}{\epsilon_t}\right)$$

For simplicity, consumer utility is assumed to be separable between consumption and real money balances. Maximising the expected utility under the budget constraint gives the Euler equation for the optimal temporal allocation of consumption In total we solve

$$
aE\sum_{t=0}^{\infty}\beta^t\left(u\left(C_t,\frac{M_t}{P_t}\right)-\int_0^n v(h_t(j))dj\right)
$$
$$
-\sum_{t=0}^{\infty}\lambda_t\left(P_tC_t+\left(\frac{i_t}{1+i_t}\right)M_t+B_t+\epsilon_tB_t^*-(M_{t-1}+(1-i_{t-1})B_t+\epsilon_{t-1,t}(1+i_{t-1}^*)B_t^*+P_t\Pi_t\right)
$$

with the following First-Order-Conditions

$$
\begin{aligned}
C_t &: u_c\left(C_t,\frac{M_t}{P_t}\right)-\lambda_tP_t=0\\
C_{t+1} &: E\left(\beta u_c\left(C_{t+1},\frac{M_{t+1}}{P_{t+1}}\right)\right)-\lambda_{t+1}P_{t+1}=0\\
C_t^* &: u_c\left(C_t^*,\frac{M_t}{P_t}\right)-\lambda_tP_t^*=0\\
C_{t+1}^* &: E\left(\beta u_c\left(C_{t+1}^*,\frac{M_{t+1}}{P_{t+1}}\right)\right)-\lambda_{t+1}P_{t+1}^*=0\\
B_t &: -\lambda_t+E\left(\lambda_{t+1}(1+i_t)\right)=0\\
B_t^* &:= -\lambda_t\epsilon_t+E\left(\lambda_{t+1}\epsilon_{t,t+1}(1-i_t^*)\right)
\end{aligned}
$$

This gives the Euler-equation

$$
\beta(1+i_t)P_tE\left(\frac{u_c\left(C_t,\frac{M_t}{P_t}\right)}{P_{t+1}}\right)=\beta(1+i_t^*)\frac{\epsilon_{t,t+1}}{\epsilon_t}P_t^*E\left(\frac{u_c\left(C_t^*,\frac{M_t}{P_t}\right)}{P_{t+1}^*}\right)
$$

This yields the condition for the choice of labour supply and for the consumption-saving choice

$$
\begin{aligned}
\frac{v_h(h_t(j))}{u_c(C_t)} &= \frac{w_t(j)}{P_t}\\
\frac{u_c(C_t)}{u_c(C_{t+1})} &= \beta(1+r^*)
\end{aligned}
$$

where r^* is the world real interest rate.

2.2 Firms

Firms minimise their costs by choosing the lowest possible level of labour subject to producing the firm specific good $c_t(j)$

$$\min_{h_t(j)} \int_0^1 \frac{w_t(j)}{p_t(j)} h_t(j) df$$

given the production function for the firm specific good

$$y_t(j) = A_t f(h_t(j))$$

where A_t is a random productivity shock. Applying Lagrangian and taking First-Order-Condition reveals the real marginal cost

$$s_t(j) = \frac{w_t(j)}{P_t A_t f'\left(f^{-1}\left(\frac{y_t(j)}{A_t}\right)\right)}$$

Taking the condition for the choice of labour supply this can be rewritten as

$$s(y, C, A) = \frac{v_h\left(f^{-1}\left(\frac{y}{A}\right)\right)}{u_c(C) A f'\left(f^{-1}\left(\frac{y}{A}\right)\right)}$$

Trade-wise, price-making firms face world demand for their products so that

$$c(j) = C_t \left(\frac{p_t(j)}{P_t}\right)^{-\theta}$$

$$y_t(j) = Y_t^w \left(\frac{p_t(j)}{P_t}\right)^{-\theta}$$

where $y(j)$ is the quantity of good j supplied by the firm to meet the world demand and $Y^w = Y^h + Y^f$, the index for all goods produced around the world with production indices for domestic and foreign goods

$$Y^h = \int_0^n \left(\frac{p_t(j) y_t(j)}{P_t}\right) dj$$

$$Y^f = \int_0^n \left(\epsilon \frac{p_t^*(j) y_t(j)}{P_t}\right) dj$$

In a second step, firms maximise profits, given by income from selling the individual good $c_t(j)$ minus the costs of producing this product, $\pi_t c_t(j)$, by setting their prices

$p_t(j)$ for their individual goods subject to the demand curve for their individual good given and the assumption that prices are sticky. Following Calvo (1983), in each period, a fraction γ of firms is not able to change its price and has to stick to the price chosen in the previous period. This can be solved by maximising the profit under the condition for the choice of labour supply. In the former case, the price is marked up above the marginal cost by $\frac{\theta}{\theta-1}$.

$$\frac{p_{1t}}{P_t} - \frac{\theta}{\theta-1} s(y_{1t}, C_t, A_t) = 0$$

In the second case the price will be chosen to maximise expected discount profit.

$$E\left(\left(\frac{1}{1+i_{t-1}}\right) Y^w P_t^{\theta-1} \left(\frac{p_{2t}}{P_t} - \frac{\theta}{\theta-1} s(y_{2t}, c_t, A_t)\right)\right) = 0$$

This reveals the optimal price setting rule. In the simple case of $\gamma = 1$ (with P_t price index) this can be expressed as

$$\begin{aligned} P_t &= \left(n\left(\gamma p_{1t}^{1-\theta} + (1-\gamma) p_{2t}^{1-\theta}\right) + (1-n)(\epsilon_t p_t^*)^{1-\theta}\right)^{\frac{1}{1-\theta}} \\ \frac{p_t}{P_t} &= \frac{\theta}{1-\theta} s(Y_t^n, C_t^n, A_t) \end{aligned}$$

2.3 The New Keynesian Phillips Curve

In order to obtain a tractable solution, we log-linearise the equilibrium conditions around the steady state. For the derivation of the New Keynesian Phillips curve in a general equilibrium framework, one uses firms' optimal price setting rule. In the steady state $\beta(1+r^*) =$, particularly, in a *deterministic* steady state there is $A_t = A$, $\epsilon_t = \epsilon$, $p_t^* = p^*$, $C_t = C$. Every variable having a hat the proportional deviation of itself from its deterministic state ($\hat{x_t} = \log(x_t/x) \approx \frac{x_t - x}{x}$).

$$\begin{aligned} s(y, C, A) &= \frac{v_h\left(f^{-1}\left(\frac{y}{A}\right)\right)}{u_c(C) A f'\left(f^{-1}\left(\frac{y}{A}\right)\right)} \\ \hat{s}_t - \hat{s}_t{}^n &= \omega(\hat{y}_t - \hat{Y}_t{}^n) + \sigma^{-1}(\hat{C}_t - \hat{C}_t{}^n) \\ \omega &= \left(\frac{v_{hh}(y/A)}{v_h f'} - \frac{f''(f^{-1}(.))(y/A)}{f'(f^{-1}(.)) f'(.)}\right) \\ \sigma &= -\left(\frac{u_{cc} c}{u_c}\right) \end{aligned}$$

Applying this equation to the price settings

$$
\begin{aligned}
\frac{p_{1t}}{P_t} - \frac{\theta}{\theta-1} s(y_{1t}, C_t, A_t) = 0 \\
E\left(\left(\frac{1}{1+i_{t-1}}\right) Y^w P_t^{\theta-1} \left(\frac{p_{2t}}{P_t} - \frac{\theta}{\theta-1} s(y_{2t}, c_t, A_t)\right)\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
\log(p_{1t}) &= \log(P_t) + \omega(\hat{y_{1t}} - \hat{Y_t}^n) + \sigma^{-1}(\hat{C}_t - \hat{C_t}^n) \\
\log(p_{2t} &= E(\log(P_t) + \omega(\hat{y_{2t}} - \hat{Y_t}^n) + \sigma^{-1}(\hat{C}_t - \hat{C_t}^n) \\
\log(P_t) &= n(\gamma \log(p_{1t} + (1-\gamma)\log(p_{2t})) + (1-n)\log(\epsilon_t p_t^*)
\end{aligned}
$$

We define the inflation rate as $\pi_t = \log(P_t/P_{t-1})$ and the real exchange rate as $e_t = \epsilon_t \frac{P_t^*}{P_t}$. Log-linearising the demand function

$$
\begin{aligned}
c(j) &= C_t \left(\frac{p_t(j)}{P_t}\right)^{-\theta} \\
y_t(j) &= Y_t^w \left(\frac{p_t(j)}{P_t}\right)^{-\theta}
\end{aligned}
$$

yields

$$
\hat{y_{jt}} = \hat{Y_t}^W - \theta(\log(p_{jt} - \log(P_t))
$$

Substituting the equations and rearranging terms leads to

$$
\begin{aligned}
\log(p_{1t} &= \log(P_t) + \left(\frac{\omega}{1+\theta\omega}\right)(\hat{Y_t}^W - \hat{Y_t}^n) + \sigma^{-1}\left(\frac{1}{1+\theta\omega}\right)(\hat{C}_t - \hat{C_t}^n) \\
\log(p_{2t}) &= E\left(\log(P_t) + \left(\frac{\omega}{1+\theta\omega}\right)(\hat{Y_t}^W - \hat{Y_t}^n) + \sigma^{-1}\left(\frac{1}{1+\theta\omega}\right)(\hat{C}_t - \hat{C_t}^n)\right) \\
&= E(\log(p_{1t})
\end{aligned}
$$

Taking into account the price index and the unanticipated rate of inflation this implies

$$\log(P_t) - E(\log(P_t)) = \left(\frac{\gamma}{1-\gamma}\right)(\log(p_{1t} - \log(P_t)) + \left(\frac{1-n}{n}\right)\left(\left(\frac{1}{1-\gamma}\right)\log(e_t) - E(\log(e_t))\right)$$

Replacint $\log(p_{1t})$ yields in the open economy NKPC

$$\pi_t - E(\pi_t) = \left(\frac{\gamma}{1-\gamma}\right)\left(\left(\frac{n\omega}{1+\theta\omega}\right)(\hat{Y}_t^h - \hat{Y}_t^n) + \left(\frac{(1-n)\omega}{1+\theta\omega}\right)(\hat{Y}_t^f - \hat{Y}_t^n\right) + \left(\frac{1-n}{n}\right)\left(\left(\frac{1}{1-\gamma}\right)\log(e_t) - E(\log(e_t))\right)$$

For simplicity's sake we write

$$\pi_t = \pi_{t-1} + \alpha_y y_{t-1} - \alpha_e(e_{t-1} - e_{t-2}) + \eta_t \tag{1}$$

where π is the rate of inflation, y the output gap and e the exchange rate, α_is are weighing parameters and η is a standard AR(1) process.

2.4 The Investment and Savings Curve

The aggregate demand curve follows Ball (1998) and starts with a simple open-economy. Furthermore, following Allsopp and Vines (2000) and Clarida, Gali, and Gertler (1999) we account for foreign output. As we aim to study financial market and monetary policy spillover effects in times of crises we extend our previous approach in Hayo and Niehof (2013) and follow Stracca (2010) by including asset prices in the investments and savings (IS) curve.

Techinically, the IS curve is derived by deriving the Euler equation and log-linearising around the steady state.

$$\beta(1+i_t)P_t E\left(\frac{u_c\left(C_t, \frac{M_t}{P_t}\right)}{P_{t+1}}\right) = \beta(1+i_t^*)\frac{\epsilon_{t,t+1}}{\epsilon_t}P_t^* E\left(\frac{u_c\left(C_t^*, \frac{M_t}{P_t}\right)}{P_{t+1}^*}\right)$$

Re-arranging the equations yields the IS curve

$$y_t = \lambda_y y_{t-1} - \lambda_i(i_{t-1} - \pi_{t-1}) - \lambda_{y^*} y_{t-1}^* - \lambda_e e_{t-1} + \lambda_p p_{t-1} + \epsilon_t \tag{2}$$

where y denotes the output gap, i the interest rate, π the inflation rate, e the exchange rate and p a linear system of financial market equations. ϵ follows a standard AR(1) process and λ's are weighing parameters.

2.5 The Exchange Rate

Focusing on the short run, we allow exchange rate adjustment to incorporate the uncovered interest parity condition. However, the short-run oriented purchasing power parity may not hold in this set-up. In line with Ball (1998), McCallum (1994) and Batini and Nelson (2000), the exchange rate is a function of the nominal interest rate and inflation. To allow for an explicit analysis of exchange rate bubbles, we follow the more approach of Batini and Nelson (ibid.) and add another state variable φ to reflect the potential burst of a bubble (see equation (3)). This implies that there is an explosive time series parameter such that a closed solution cannot be computed. A detailed description of the computation, length and values of the additional variable is provided by Batini and Nelson (ibid.)

$$e_t = \theta_e e_{t-1} + \theta_i(i_t - \pi_t) - \theta_{i^*}(i_t^* - \pi_t^*) + \psi_t + \varphi_t \tag{3}$$

In line with previous equations we define the error term ψ_t as an AR(1) process.

2.6 The Financial Markets

Inclusion of the financial market is rather new. We follow Bekaert, Cho, and Moreno (2010) and model assets in discrete time at first and switch to a continuous time afterwards. The pricing kernel is based on the IS equation (thus on consumption).

Duffie and Kan (1996) shows that state variable dynamics and pricing kernel processes in affine term structure models must be linear. Furthermore, shocks must be conditionally normal. We apply shocks as AR(1) processes, therefore they fall into the class of conditionally normal error terms. Moreover, all equations are linear in their dynamics and thereby fulfil the requirements for affine term structure models as well. We assume

$$E(M_{t+1}, R_{t+1}) = 1$$

for the pricing kernel process M, and a n-asset R. If $M > 1$ the no-arbitrage condition is fulfilled (Bekaert, Cho, and Moreno 2010). Logarithmising the pricing kernel yields to

$$ln(M_{t+1}) = -i_t - \frac{1}{2}\Lambda_t' D_t \Lambda_t - \Lambda_t' \epsilon_{t+1}$$

Λ_t is a vector (of the length of the number of equations in the model), which mainly contains of the dependend variables (inflation, output, etc.) and furthermore, of the inverse of the elasticity of intertemporal substitution (derived in the maximising problem of the households), as well as a households hapit do shift consumption from one period to another.

In case D does not change overt time this yields into a Gaussian price of risk model. Furthermore, taking $\Lambda_t = \Lambda_0$ we attain a homoscedastic model (Cox, Ingersoll Jr, and Ross 1985) which fits our class of model derived by the IS curve.

Because our derivation of the IS curve assumed a particular preference structure, the pricing kernel is given by the intertemporal consumption marginal rate of substitution of the model

$$m_{t+1} = ln(\psi_t) - \sigma y_{t+1} + (\sigma + \eta) y_t - \eta y_{t+1} + (g_{t+1} - g_t) - \pi_{t+1} \tag{4}$$

σ is the inverse of the intertemporal rate of substitution, η is the habit of shifting consumption into another period, ψ is the demand shifting factor (all are elasticities in the preferences of the households, that is part of $u\left(C_t, \frac{M_t}{P_t}\right)$. g_t is a logarithmised demand shock. Logarithmic asset prices and are modelled as affine functions of the state variables

$$p_t = \beta + \beta_p p_{t-1} + \beta_i (i_{t-1} - \pi_{t-1}) + \beta_y y_t + \beta_e e_t + \xi_t \tag{5}$$

Note that p might be a vector including different financial market instruments. Bekaert, Cho, and Moreno (2010) show that the macro variables and the term spreads follow a first-order VAR with complex cross-equation restrictions.

We follow our assumption of simultaneity and highly interacted markets and augment the standard representation of stocks and bonds to model the financial sector p. In particular, we model the stock market as a Stochastic Volatility model as shown by Heston (1993). This setting extends the approach of Black and Scholes (1973) and takes into account non-lognormal distribution of the asset returns, leverage effect, important mean-reverting property of volatility and it remains analytically tractable. To take the assumption of highly interacted markets we include the foreign stock market, as well as house prices, exchange rate, output and interest rate in the drift term. For example, including the output gap in the stock market is in line with Cooper and Priestley (2009) and Vivian and Wohar (2013), a general approach of incorporating for macroeconomic factors in stock returns was provided by Pesaran and Timmermann (1995). Moreover, Fama and Farber (1979) give evidence for common factors in bond and stock markets.

In line with Bayer et al. (2010) house prices are modelled as stochastic differential equation taking into account a local risk, a national risk and an idiosyncratic risk. This allows to model housing prices in an asset pricing environment. Similarly as before we account for macroeconomic variables in the drift term. In line with empirical findings of Adams and Füss (2010), Agnello and Schuknecht (2011), Capozza et al. (2002), and Hirata et al. (2012) we include the real interest rate, output gap and the derived asset from the stock market in the drift term to account for the interconnectness. To analyse call prices we apply the extended Black-Scholes formula (as in Kou (2002)).

In total we have a system of seven financial market equations, namely two stock market equations, two bond market equations, two calls and the exchange rate[1].

$$
\begin{aligned}
dS &= (S_t((r-\lambda\mu) + \rho_b b_t + \rho_b^* b_t^* + \rho_s^* S_t^* + \rho_i i_t + \rho_y y_t + \rho_e e_t + \rho_\pi \pi_t)dt \\
&\quad + \sqrt{V_t} dW_S(t) + \sum_{i=1}^{dN_t} J(Q_i) \qquad (6)\\
dV_t &= \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dW_V(t) \\
dh_t &= (\gamma_h h_t + \gamma_S S_t + \gamma_s^* S_t^* + \gamma_h^* h_t^* + \gamma_y y_t - \gamma_i(i_t - \pi_t))dt \\
&\quad + \sigma_1 dW_h^1(t) + \sigma_2 dW_h^2(t) + \sigma_3 dW_h^3(t) \qquad (7)\\
e_t &= \theta_e e_{t-1} + \theta_i(i_t - \pi_t) - \theta_{i^*}(i_t^* - \pi_t^*) + \psi_t + \varphi_t \qquad (8)
\end{aligned}
$$

where $J(Q)$ is the Poisson jump-amplitude, Q is an underlying Poisson amplitude mark process ($Q = \ln(J(Q)+1)$) and $N(t)$ is the standard Poisson jump counting process with jump density λ and $E(dN(t)) = \lambda dt = Var(dN(t))$. dW_s and dW_v denotes Brownian motions. Furthermore, β denotes the long-term mean level, α the speed of reversion and σ the instantaneous volatility. e_t is the exchange rate equation derived above.

[1] We maintain the simple exchange rate of Ball (1998) to stick to the classical New Keynesian framework.

2.7 The Monetary Policy Rule

In line with Ball (1998), Leitemo and Soderstrom (2005), Lubik and Schorfheide (2007), Lubik and Smets (2005), Svensson (2000), and Taylor (1993) we apply an open-economy Taylor-Rule, which also takes into account financial markets. Given exchange rate e and interest rate i an optimal policy rule minimises the loss function

$$Var(y) + \mu_1 Var(\pi) + \mu_3 Var(y^*) + \mu_4 Var(p)$$

The variation in the parameters μ_i defines the range of efficient policies. Regarding implementation of our rule, we follow Ball (1998) and substitute the exchange rate equation into the IS curve, the NKPC and the Financial Market equation.

$$\begin{aligned}
y_{t+1} &= \lambda_y y_t + \frac{\lambda_i}{\theta_i}(e_t - \theta_e e_{t-1} + \theta_i^*(i_t^* - \pi_t^*)) - \lambda_y^* y_t^* - \lambda_e e_t \\
\pi_{t+1} &= \pi_t - \alpha_y y_t - \alpha_e(e_t - e_{t-1}) \\
p_{t+1} &= \beta + \beta_p p_t + \frac{\beta_i}{\theta_i}(e_t - \theta_e e_{t-1} + \theta_i^*(i_t^* - \pi_t^*)) + \beta_y y_t + \beta_e e_t
\end{aligned}$$

As we take into account i and e one can define the state variables of the model by two expressions corresponding to terms on the right-hand sides of equations

$$\begin{aligned}
\lambda_y y + \frac{\lambda_i}{\theta_i}\theta_i^*(i_t^* - \pi_t^*) - \lambda_y^* y_t^* - \frac{\lambda_i}{\theta_i}\theta_e e_{t-1} \\
\pi_t - \alpha_y y + \alpha_e e_{t-1} \\
\beta + \beta_p p + \frac{\beta_i}{\theta_i}\theta_e e_{t-1} + \frac{\beta_i}{\theta_i}\theta_i^*(i_t^* - \pi_t^*) + \beta_y y
\end{aligned}$$

Combining these parts and re-arranging the parameters yields to the Taylor-Rule equation

$$(1 - \omega_e)\, e_t + \omega_i i_t = \beta + \beta_p p_{t-1} + \omega_e e_{t-1} + \omega_\pi \pi_t + \omega_y y_t + \omega_{y^*} y_t^* \tag{9}$$

Thus, our Taylor rule accounts for domestic and foreign output, the exchange rate, inflation and the financial market. It facilitates the analysis of spillover effects between financial markets and monetary policy as well between foreign and domestic policy. Moreover, by including the financial sector consisting of various markets, we account for a direct relationship between monetary policy and financial markets. Rigobon (2003) and Rigobon and Sack (2003) proof empirically that the inclusion of this part is useful.

2.8 The complete model

Our model in discrete time, as derived above, can be summarised as

$$
\begin{aligned}
y_t &= \lambda_y y_{t-1} - \lambda_i (i_{t-1} - \pi_{t-1}) - \lambda_{y^*} y^*_{t-1} - \lambda_e e_{t-1} + \epsilon_t \\
\pi_t &= \pi_{t-1} + \alpha_y y_{t-1} - \alpha_e (e_{t-1} - e_{t-2}) + \eta_t \\
e_t &= \theta_e e_{t-1} + \theta_i (i_t - \pi_t) - \theta_{i^*} (i^*_t - \pi^*_t) + \psi_t (+\varphi_t) \\
p_t &= \beta + \beta_p p_{t-1} + \beta_i (i_{t-1} - \pi_{t-1}) + \beta_y y_t + \beta_e e_t + \xi_t \\
i_t &= \frac{1}{\omega_i} \left((1 - \omega_e) e_t + \beta + \beta_p p_{t-1} + \omega_e e_{t-1} + \omega_\pi \pi_t + \omega_y y_t + \omega_{y^*} y^*_t \right)
\end{aligned}
\tag{10}
$$

Reflecting our own approach (Hayo and Niehof 2013) and work by Asada et al. (2006), Chen et al. (2006a,b), and Malikane and Semmler (2008a,b) we switch to a continuous time framework by taking first differences of the equations and using stochastic differential equations to model the shocks as Brownian motions. For reasons of simplicity, instead of using the combined terms, we rename the parameters as they were before. Prior parameters can be recovered by solving a system of linear equations. In particular

$$
\begin{aligned}
dy &= (\lambda_y y - \lambda_i i + \lambda_i \pi - \lambda_{y^*} y^* - \lambda_e e) dt + \sigma_y dW^y_t \\
d\pi &= (\alpha_\pi \pi + \alpha_y y - \alpha_e e) dt + \sigma_\pi dW^\pi_t \\
de &= (\theta_e e + \theta_i i - \theta_\pi \pi - \theta_{i^*} i^* + \theta_{\pi^*} \pi^*) dt + \sigma_e dW^e_t \\
dp &= (\omega_p p + \omega_e e + \omega_y y + \omega_i i - \omega_\pi \pi + \beta) dt + \sigma_p p dW^p_t \\
di &= (\gamma_i i + \gamma_\pi \pi + \gamma_y y - \gamma_{y^*} y^* + \gamma_e e + \gamma_p p) dt
\end{aligned}
\tag{11}
$$

Note that p can be specified as a vector including various assets, each of which can be included linearly and also priced differently.

Regarding stability we rely on Lyapunov techniques (Khasminskii 2012). We can re-write our continuous time model as

$$
X(t) = X(t_0) + \int_0^1 AX(t)dt + \int_0^1 \sigma(t)X(t)dW(t) \tag{12}
$$

where A is a matrix (therefore, a linear function) and b is a vector. X represents our dependent variables (output gap, inflation, exchange rate etc.). For the sake of simplicity we normalised the time frame from zero to one but any interval t_0, T is possible as well. A Lyapunov function is a scalar Function $V(t, x)$ defined on $\mathbb{R}^n$ that is continuous, positive definite, $V(0) > 0$ for all $x \neq 0$, and has continuous first-order partial derivatives at every point of D. The differential generator is defined as

$$LV(t,x) = \frac{\partial V(t,x)}{\partial t} + \sum_{i=1}^{l}(Ax)_i \frac{V(t,x)}{\partial t} + \frac{1}{2}tr(b'Xb) \tag{13}$$

The trivial solution of a stochastic differential equation is called stable, if the differential generator of the Lyapunov function is negative definite $LV < 0$ for all values in a neighbourhood $D\backslash\{0\}$. We consider the Euklidean Norm

$$V(x) = \|x\|_2^2 = \left(\sqrt{\sum |x_i|^2}\right)^2 \tag{14}$$

as Lyapunov function and apply the differential operator. Given the parameters evolved in the upcoming chapter this yields into

$$\frac{1}{(e^2 + p^2 + (p^*)^2 + \pi^2 + \pi^*)^2 + i^2 + (i^*)^2 + y^2 + (y^*)^2)^{\frac{3}{2}})} Terms \tag{15}$$

$Terms$ depends both, on the depended variables as well as on the parameters. Note that the numerator converges slower towards zero than the denominator, therefore we depend on a proper choice of parameters regarding the stability for sufficient small values in a neighbourhood of 0. This also shows that the trivial solution is weakly stable. However, in our case stability of the trivial solution is guaranteed.

In economic terms, this means that if an economy drifts away from its steady state, it will return or, alternatively, it will not move very far away. With graphical means the zero solution is as follows

Note that we concentrate our dynamic analysis on short-run adjustments. Within this time frame, there is no guarantee that the variables will actually return to their starting values. If we analyse scenarios which are too far away from our neighbourhood stability is no longer guaranteed and trajectories show no tendencies towards long-term equilibrium.

3 Studying Effects of Financial Market Turmoils

To shed light on financial crises and the contagious effects on domestic and foreign real economies we study a crush in the stock and in the housing market. We adopt a modern economies cooperative behaviour and simulate highly connected countries instead of the classic vice versa relationship. For example Canada and the United States are a typical case of these economies.

To study monetary policy rules reactions to financial distres, we first apply the classical Taylor Rule, which takes into account domestic and foreign output gap, inflation and exchange rate. In a second attempt we allow for adaption of financial markets in the decision rule to compare advantages and disadvantages of both policy rules. Blue lines represent central bankers who incorporate financial markets, black lines represent classic central bankers. Moreover, solid lines represent the domestic economy and dashed lines the foreign economy.

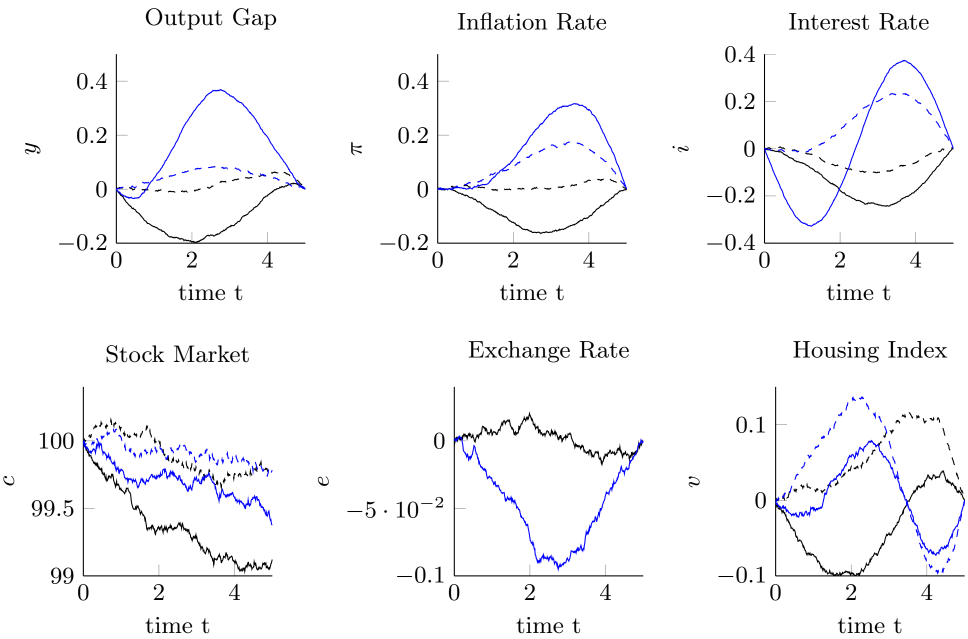
In particular, we start by analysing a minor financial market distress, which, taking into account the neutrality of money, should not effect the real economy much. Afterwards we simulate a bubble's onset and subsequent collapse on (domestic) stock and housing market and analyse monetary policy reactions, which first neglect and second incorporate financial markets in their policy rule. Technically, we take the mean of 100,000 simulations with 0,01 time steps. We use a normalised Euler-Maruyama scheme to simulate the trajectories of the stochastic differential equations.

In case that monetary policy does not react on stock market changes a minor slump causes a drop in the output gap of $0,2$ at most. To stabilise the economy monetary policy makers carefully reduce the nominal interest rate by the same amount. In total, the nominal interest has its minimal point after three quarters of the simulated observation period. This causes minor inflationary pressure. Hence, interest rate increases and output and inflation adjust to equilibrium. The exchange rate appreciate exiguous to the drop in the output gap. As a consequence, spillover effects are transferred only by a small amount.

As the economies are positively connected the drop in the the stock market causes a slump of half the amount in the foreign economy. Therefore, the output gap reacts negligible. As a result to a combination of the appreciated exchange rate and reduced output gap monetary policy makers drop the nominal interest rate by about $0,1$ percentage points.

In case that monetary policy does *always* include financial market movements a slump in the stock market causes monetary policy makers to react hastily and drop the nominal interest rate by 0.25 percentage points within one quarter of the simulated observation period. As a consequence, the output gap reacts stronger to increased borrowing conditions and the expected drop turns into a boom. Henceforth, inflation increases and monetary policy makers re-react to the inflationary pressure by hastily increasing the interest rate, which causes the output to decrease. As a consequence of increased out-

Figure 1: **Stock Market Slumps**



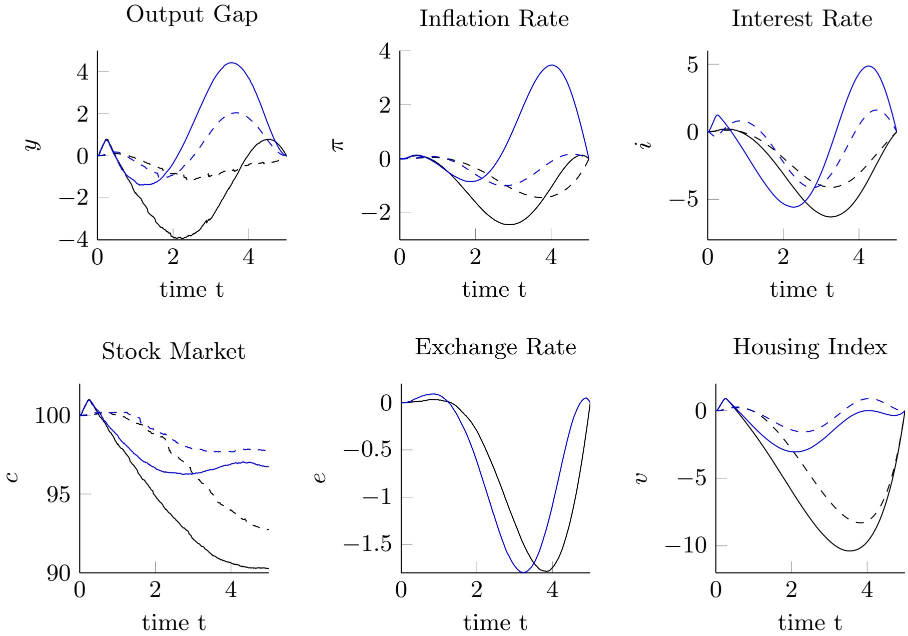
put the (domestic) exchange rate depreciates first and appreciates with the increased nominal exchange rate and decreasing output gap.

In this case the monetary policy causes more reactions on foreign markets. As economies are well connected the high output causes positive spillovers in the foreign country. Particularly, the positive effect of real variable prevails the negative spillover from the stock market and the exchange rate. Therefore, output gap increases which causes inflationary pressure such that foreign monetary policy makers increase the nominal exchange rate by 0.25 percentage points.

Contrary to our previous considerations, we now analyse the commence and collapse of a stock market bubble. Starting from the equilibrium we observe stable times until the swell of a stock market bubble. After a time of persistence it bursts and lead to a stock market crash. In both cases of Taylor Rule analysis we establish well connected countries. This obviously causes huge effects of contagion.

In case that there is no immediate reaction from monetary policy the effects of the financial market crisis spill over to the real economy. As stock market assets also reflect a firms' value the burst of a bubble on the stock market causes the output to drop beyond potential. The minimal turning point of four percent of the output gap is within half of the period. A negative output gaps causes a recessionary gap as it reflects higher growth in supply than in demand. As a negative output gap and a recessionary gap

Figure 2: **Stock Market Crisis**



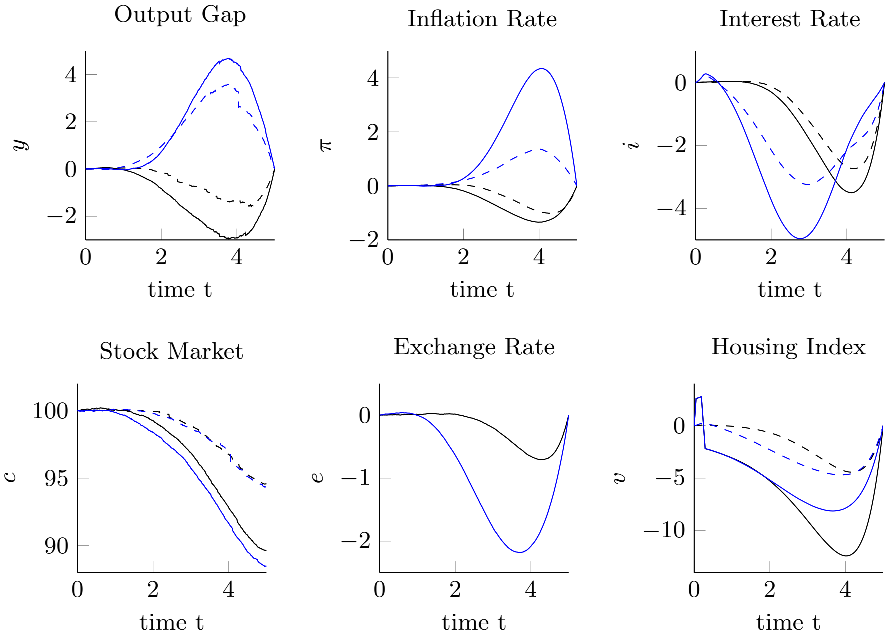
causes conflicting monetary policy reactions, central bankers react tardy towards the upcoming recession. First reactions are after one quarter of the simulated period. The minimal turning point of the interest rate is in the third quarter of the observation period. As a consequence, the financial crises strengthens and turns to a real market crisis.

As markets are well connected the stock market crush spills over to foreign stock markets as well. Henceforth, foreign real markets are hit by recessionary pressure. Output moves beyond potential and causes a recessionary gap as well. In general reactions are afterwards similar to those in the domestic economy. Monetary policy reacts tardily and amplifies the crisis.

In case that central bankers do incorporate financial markets in their decision rule the crises proceeds relatively moderate. A severe drop of the nominal interest rate of five percentage points within the first third of the simulated period yields into a drop in the output gap of $1,5$ percentage points. However, the severe cut in the nominal interest rate causes high inflationary pressure. Inflation increases over five percent. Along with recovering economy and increasing prices the interest rate increases up to five percentage point in the last third of the analysed period.

As financial markets and real economies are well connected the stock market crisis causes severe spillovers to the foreign country. In general we observe a minor time lag.

Figure 3: **House Market Crisis**



Afterwards the transmission is similar to the origin country of the crisis, however, the effects are of smaller amount. Our last scenario tackles the actual origin of the recent financial and economic crisis. We employ a housing market bubble and observe the spillover effects first, to the financial markets in general and as a second reaction, to the real market. In terms of the transmission process this is similar to the scenario described before. In case that monetary policy makers do not directly react on the burst of the bubble the stock market crisis hits the real economy with a time lag after the burst. To stabilise the economy monetary policy makers drop the nominal interest rate by four percent.

In case that monetary policy directly reacts to the housing market bubble there is no effect on the real economy. Quite the contrary, the improved borrowing conditions lead into a boom and high inflationary pressure.

4 Employing Empirically Estimated Parameters

To compare our model to findings of real world economies we estimate parameters for the United States and Canada. As we are using continuous time equations we rely on stochastic estimation (approximate Bayesian computation, see Beaumont, Zhang, and Balding (2002)). To cover for the financial crisis we split the sample in two parts, one before before and the other after outburst of the financial crisis.

Wright (2008) explains the fundamental idea of Bayesian Model Averaging (which was set out by Leamer (1985) at first). Consider a set of n models $M_1, ..., M_n$. The i-th moment is indexed by a parameter vector. The researcher knows that one of these models is the true model, but does not know which one. The researcher has prior beliefs about the probability that the i-th model is the true model which we write as $P(M_i)$. He observes data D, and updates his beliefs to compute the posterior probability that the ith model is the true model:

$$P(M_i|D) = \frac{P(D|M_i)P(M_i)}{\sum_{j=1}^{n} P(D|M_j)P(M_j)}$$

Each model implies a forecast. In the presence of model uncertainty, our overall forecast weights each of these forecasts by the posterior for that model. This gives the minimum mean square error forecast. The researcher needs only to specify the set of models, the model priors, $P(M_i)$ and the parameter priors. Our discrete model has the form

$$y = X_i \beta_i + \epsilon$$

To obtain a closed form solution we assume that the regressors are exogenous.

Two inputs are crucial to obtaining plausible results through MCMC estimations: first, the choice of priors and, second, the choice of initial values. Our choice of prior distributions for New Keynesian models is similar to decisions by, among others Smets and Wouters (2007), Negro et al. (2007) or Lindé (2005).

We follow Kimmel (2007), Wright (2008) or Jones (2003) and choose normal distributions for financial instruments. The financial parameters take the natural conjugate g-prior specification, so that for each prior for a financial parameter conditional on σ^2 is $N(0, \sigma^2(X_i' X_i)^{-1})$. Zellner and Palm (2004) shows one can then calculate the required likelihood of the model analytically, therefore, the prior for the parameters are proper priors.

In Bayesian Analysis, one cannot use improper priors for model-specific parameters, because improper priors are unique only up to an arbitrary multiplicative constant and so their use would lead to an indeterminacy of the model posterior probabilities (Kass and Raftery 1995), therefore we used informative priors for each parameter[2].

[2] Note that in some cases New Keynesian literature applied improper priors to compare them to their results. In this case they applied the uniform distribution with zero mean and standard deviation of one

An overview of the priors is given in the first four columns of Table (1). We run 50,000 simulations to obtain our results, with an average acceptance ratio of about 40-50%.

Data are obtained from the Bureau of Economic Analysis, the Federal Reserve St. Louis, the US Bureau of Labor Statistics, Statistics Canada, Datastream and OECD database. We employ quarterly data starting in 1971:Q1 to 2013:Q1. The output gap is obtained as the transitory component after applying the HP filter to logged quarterly GDP. The monetary policy interest rate is the short-term money market rate. The inflation series is constructed as $400(CPI_t/CPI_{t-1} - 1)$. Regarding financial variables we employ major stock indices S&P and TSX. Instead of the bond market we include the housing market, represented by changes in housing prices.

Table 1: Priors and Posteriors for the extended NK model (USA & Canada)

	Model	Prior	Mean	Variance	Posterior					
					Complete Set		Crisis		Pre-Crisis	
					USA	CA	USA	CA	USA	CA
λ_y	0.95	B	1.00	0.10	0.96	1.00	0.78	0.90	0.39	1.00
λ_i	0.99	B	1.00	0.10	0.82	1.00	0.91	0.99	0.72	0.99
λ_y^*	0.70	B	1.00	0.10	0.02	0.51	0.41	0.13	0.98	0.87
λ_p	0.60	B	1.00	0.10	1.00	0.55	0.80	0.94	0.67	-0.62
λ_e	0.60	B	1.00	0.10	0.17	0.94	0.99	0.99	0.51	0.41
α_π	1.00	0.99	-	-	0.43	0.68	1.42	1.00	1.18	1.65
α_y	0.40	IG	0.50	0.50	0.52	1.23	0.43	0.22	1.99	1.10
α_e	0.80	IG	0.50	0.50	1.31	1.26	1.03	0.91	1.57	1.83
θ_e	0.40	B	1.00	0.10	-0.39		0.85		-0.25	
θ_i	0.20	B	1.00	0.10	1.25	0.75	1.05	0.37	0.91	0.23
θ_π	0.10	B	1.00	0.10	0.27	0.21	0.71	-0.83	-0.16	0.35
ω_h	0.80	N	0.00	5.00	0.82	1.03	0.80	0.54	0.88	0.78
ω_h^*	0.50	N	0.00	5.00	0.23	-0.28	0.02	-0.13	0.89	0.58
ω_s	0.30	N	0.00	5.00	0.84	1.07	0.18	0.51	-0.41	-0.63
ω_e	0.60	N	0.00	5.00	0.41	0.60	0.62	1.02	0.61	2.39
ω_y	0.40	N	0.00	5.00	1.04	0.53	-0.37	0.52	0.30	0.93
ω_i	0.50	N	0.00	5.00	-0.60	0.39	0.81	1.41	0.23	-0.36
ω_π	0.30	N	0.00	5.00	-0.60	0.39	0.81	1.41	0.23	-0.36
ω_i^*	0.50	N	0.00	5.00	-0.02	0.94	1.35	0.57	1.48	1.34
ω_π^*	0.30	N	0.00	5.00	-0.02	0.94	1.35	0.57	1.48	1.34

ω_h	0.30	N	0.00	5.00	0.84	-0.27	0.59	0.03	-0.22	0.13
ω_h^*	0.20	N	0.00	5.00	-0.01	-0.35	0.07	0.19	-0.41	1.15
ω_s	0.50	N	0.00	5.00	-0.26	0.92	0.06	0.32	0.54	0.85
ω_s^*	0.30	N	0.00	5.00	0.73	0.41	0.05	-0.06	-0.05	-0.09
ω_e	0.60	N	0.00	5.00	1.82	1.38	1.13	0.79	0.97	0.10
ω_y	0.50	N	0.00	5.00	0.12	0.46	-0.06	0.74	2.07	1.17
ω_i	0.50	N	0.00	5.00	0.01	0.65	0.28	0.51	-0.03	-0.39
ω_π	0.40	N	0.00	5.00	1.34	1.14	0.65	-0.32	-0.67	0.87
γ_y	0.90	B	1.00	0.10	1.62	1.55	2.33	1.29	1.24	1.39
γ_y^*	0.50	IG	0.50	0.50	0.84	0.53	1.25	0.46	0.79	0.77
γ_π	0.30	IG	0.50	0.50	0.21	0.47	0.75	0.53	0.58	-1.16
γ_e	0.90	IG	0.50	0.50	1.42	1.69	1.08	0.69	2.44	1.92
γ_h	0.40	IG	0.50	0.50	1.20	0.53	0.69	0.81	1.43	2.77
γ_h^*	0.20	IG	0.50	0.50	0.61	0.36	0.24	0.34	0.21	-0.57
γ_s	0.40	IG	0.50	0.50	0.37	1.04	0.54	0.67	0.10	1.35
γ_s^*	0.20	IG	0.50	0.50	0.11	0.40	0.73	-0.14	-0.45	0.96
γ_i	0.60	IG	0.50	0.50	0.75	0.94	0.91	0.97	0.37	-0.19
K	0.30	-	-	-	-	-	-	-	-	-
κ	1.50	-	-	-	-	-	-	-	-	-
λ	4.00	-	-	-	-	-	-	-	-	-
μ	-0.05	-	-	-	-	-	-	-	-	-
r	0.40	-	-	-	-	-	-	-	-	-
ρ	0.40	-	-	-	-	-	-	-	-	-
σ	0.40	-	-	-	-	-	-	-	-	-
σ_v	0.20	-	-	-	-	-	-	-	-	-
θ	0.005	-	-	-	-	-	-	-	-	-
θ_2	0.01	-	-	-	-	-	-	-	-	-
σ_y	0.10	B	1.00	0.10	1.11	1.34	1.74	-0.84	0.33	0.38
σ_π	0.10	B	1.00	0.10	-0.27	-0.02	-0.99	0.99	0.41	0.49
σ_e	0.10	B	1.00	0.10	-0.38	-0.70	0.98	-0.06	0.10	0.69
σ_h	0.10	B	1.00	0.10	-0.46	0.50	0.66	0.96	0.50	0.98
σ_s	0.10	B	1.00	0.10	-0.35	-0.29	0.52	-0.33	-0.91	0.98
σ_c	0.10	B	1.00	0.10	-0.62	-0.75	0.32	0.53	0.32	0.43[3]

[3] Parameters of the theoretical model are identical for both countries and therefore displayed only

Our main purpose is to study monetary policy, thence we concentrate the analysis on the monetary policy factors. In general, estimations are consistent with the parameters in the analysis before. However, there are huge differences in some parameters before and after the crisis, especially regarding the financial markets. First, output gap and inflation seemed to be more important before the crisis than afterwards, whereas the exchange rate became more important after the crisis than before. Housing markets gained more importance after the crisis than before, too.

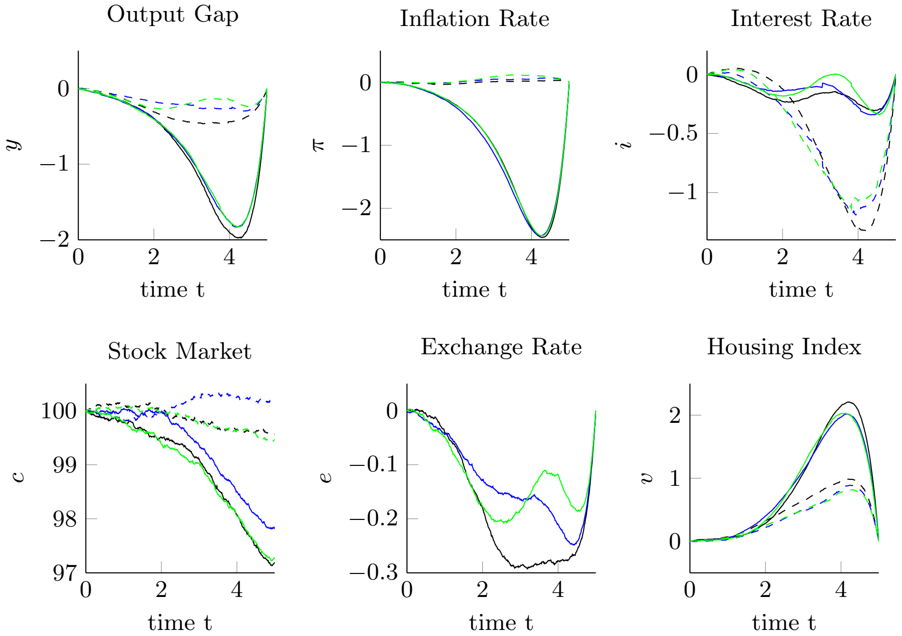
Furthermore, our estimations reveal some findings about the ineffectiveness of the monetary policy rate during the crisis. The parameter is lower after the crisis, in case of Canada it even became negative. However, most parameters do not differer much before and after the crisis. This is shown in trajectories of the estimation results. Black lines represent the full sample, green lines represent the data before the crisis and blue lines after the crisis. We simulate a minor stock market drop. This drops the output gap with some minor time gap, however the interest rate drop is later and almost identical for all three estimation results. We also observe spillover effects, however not with the same amount as in the scenario. A drop of two percent of the output gap causes a drop in Canadian output of $0,5$ percent. Most differences are within the exchange rate. The amount of depreciation is highest after the crisis.

5 Conclusions

In this paper, we extend the well-known, open economy New Keynesian model of Clarida, Gali, and Gertler (1999) and Lubik and Schorfheide (2007) in two important ways. First, we include a well-developed financial sector and, second, we apply stochastic differential equations and move the analysis to a continuous-time framework. We employ classic research from the field of finance and model the financial sector by including the market for foreign exchange, the housing market, and the stock market, both in the domestic as well as in the foreign economy, thereby acknowledging that these markets are driven by different aspects of the economy. Applying stochastic differential equations allows us to rely on established research as provided by Merton (1973). In particular, we

once. Note that B represents the Beta distribution, G the Gamma distribution and N the Normal distribution

Figure 4: **Estimation Results (considering a minor stock market drop)**



specify the financial markets as Jump-Diffusion Processes and belonging Black-Scholes equations (Black and Scholes 1973) for call prices. Furthermore, we employ Lyapunov techniques (Khasminskii 2012) to analyse the stability of the solutions and steady-state properties. Thus, in our analysis, we combine New Keynesian macroeconomic analysis, classic finance research, and standard mathematical procedures.

Our main research quest is to analyse effects of financial crises on real markets by comparing theoretical and empirical results. We find theoretical and empirical evidence for spillover effects of monetary policy. Furthermore, we observe that including financial markets in the Taylor rule mitigates the effect of a financial crisis for the at the expense of an overshooting in inflation.

Our continuous time estimation compares our findings with real-world data from the United States and Canada. Employing quarterly data over the period 1971:Q1: to 2013:Q1, we use estimates based on Bayesian estimation techniques to derive the model's parameters. To analyse the changes of monetary policy in the financial crisis we split the dataset and run the simulations for each separately. The simulation results support our findings from the theoretically parameterized model. We find spillover effects from monetary policy if conducted in the United States but only very small effects if the policy is initiated by the Bank of Canada. Moreover, US monetary policy appears to have a larger effect on Canada than Canadian monetary policy itself. This finding is consistent with evidence reported by Hayo and Neuenkirch (2012) on how monetary policy communication impacts financial markets in the United States and Canada. Furthermore, we find evidence that the Taylor rule did not change with respect to incorporating stock prices. In line with our theoretical analysis this might have amplified the effects of the crisis.

Our study has some interesting policy implications. We find evidence that monetary policy actions spill over to other countries. The impact and size of the effect depend on, first, the linkage between the markets and, second, the structure of the markets. Policymakers, particularly those of very open and well interacted countries, should take into account that spillovers could have effects that (depending on the degree of interaction) might even be larger than domestic policies. In contrast, we find evidence in support of the conventional wisdom that big countries and relatively closed economies can design and engage in policy primarily based on domestic factors. We also discover evidence that monetary policy in one country can substantially affect financial markets in other countries, even trigger booms and busts. In particular taking into account the financial market directly mitigates an upcoming crisis.

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