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Optimal Taxation

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\* Views expressed are those of the author and do not necessarily reflect official positions of De Nederlandsche Bank.

Working Paper No. 383

June 2013

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# Imperfect Competition and Optimal Taxation\*

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First version May 2012, this version June 2013

## Abstract

This paper provides optimal labor and dividend income taxation in a general equilibrium model with oligopolistic competition and endogenous firms' entry. In the long run the optimal dividend income tax corrects for inefficient entry. The dividend income tax depends on the form of competition and nature of the sunk entry costs. In particular, it is higher in market structures characterized by competition in quantities with respect to those characterized by price competition. Oligopolistic competition leads to an endogenous countercyclical price markup. As a result offsetting the distortions over the business cycle requires deviations from full tax smoothing.

JEL Classification Numbers: E62, L13.

Keywords: Firms' Entry, Market Structures, Market Distortions, Optimal Dividend Income Tax.

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\*I thank Russel Cooper, Martin Ellison, Christian Hellwig, Pierre Lafourcade, Andy Levin, Vivien Lewis, Tommaso Monacelli, Giorgio Motta, Marco Hoeberichts, Damjan Pfajfar, Raffaele Rossi, Andreas Shabert, Neeltje van Horen and Jouko Vilmunen for useful comments. I would also like to thank seminar participants at the Bank of Finland, the University of Leuven, Lancaster University, Tilburg University and the Dutch National Bank. Correspondence: Andrea Colciago, De Nederlandsche Bank, Westeinde 1, 1017 ZN Amsterdam, Netherlands. e-mail: a.colciago@dnb.nl.

# 1 Introduction

A recent macroeconomic literature emphasizes the importance of the creation of new firms for the propagation of business cycle fluctuations. Bilbiie, Ghironi and Melitz (2012, BGM henceforth), Jaimovich and Floetotto (2010) and Colciago and Etro (2010 a and b), among others, show that accounting for firms' dynamics helps improving the performance of dynamic general equilibrium models at replicating the variability of the main macroeconomic variables in response to exogenous disturbances.<sup>1</sup> Most of these studies are characterized by imperfect competition in the goods market. As a result an inefficient number of producers (and products) may arise in equilibrium, leading to welfare losses for the society. For this reason, policy measures aimed at removing market distortions could be desirable.

This paper provides optimal Ramsey dividend and labor income taxation in a framework characterized by alternative, imperfectly competitive, market structures. The economy features distinct sectors, each one characterized by many firms supplying goods that can be imperfectly substitutable to a different degree, taking strategic interactions into account and competing either in prices (Bertrand competition) or in quantities (Cournot competition). As in BGM (2012) the entry of a new firm in the market amounts to the creation of a new product. Sunk entry costs allow to endogenize entry and the (stock market) value of each firm in each sector. Preferences of agents are characterized by love for variety, such that spreading a given nominal consumption expenditure over a larger number of goods leads to an increase in utility.

The degree of market power, as measured by the price markup, depends endogenously on the form of competition, on the degree of substitutability between goods and on the equilibrium number of firms. Importantly, the price markup is countercyclical. During an economic boom profits opportunities attract firms into the market. This strengthens competition and, via strategic interactions, reduces price markups. An early references on the procyclicality of the number of firms' in the U.S. is Chatterjee and Cooper (1993), while a more recent one is Lewis and Poilly (2012). The countercyclicality of the price markup is consistent with the empirical findings by Bills (1987), Rotemberg and Woodford (2000) and Galí et al. (2007). Nevertheless notice that aggregate profits remain strongly procyclical, as in the data.

As emphasize in BGM (2007), the market equilibrium is characterized by two distortions: a *Labor Distortion* and an *Entry Distortion*. As in other models with an imperfectly competitive goods market, the presence of a price markup leads to a wedge between the marginal product of labor and the marginal rate of substitution between consumption and hours. Oligopolistic competition renders this wedge time varying. The *Entry Distortion* operates through the intertemporal firms creation margin and leads to an inefficient number of firms in equilibrium. A positive dividend income tax is optimal in case of excessive

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<sup>1</sup>Early contributions to this literature are Chatterjee and Cooper (1993), Devereux *et al.* (1996) and Devereux and Lee (2001). More recent developments are instead Bergin and Corsetti (2008) and Faia (2012).

entry and viceversa.

To minimize the welfare cost associated to these distortions, the Ramsey optimal fiscal policy, both in the short and in the long run, is provided. The Government levies taxes on dividend income and labor income and issues state contingent bonds to finance an exogenous stream of public spending.

In the long run the dividend income tax removes the Entry Distortion, as in Chugh and Ghironi (2011). The magnitude, and sign, of the dividend income tax depends on the form of the entry cost and on the form of competition, and it is higher in market structures characterized by lower competition. In particular, it is higher under Cournot Competition with respect to Bertrand or monopolistic competition. The long run labor income tax is positive under all market structures considered. As a result the Labor Distortion cannot be removed, and the efficient long run allocation cannot be achieved.

As emphasized by Chugh and Ghironi (2011), this suggest an analogy between optimal taxation in the present setting and that in the standard RBC model. As well know, the latter framework is characterized by a zero long run capital income tax, whereas the labor tax finances government spending and interest payment on debt. Such a policy delivers efficiency along the investment margin, but disregards the social cost of labor distortion. Similarly, in the current framework the dividend income tax, which is a form of capital taxation, off-sets the distortion along the intertemporal (entry) margin.

Over the business cycle, Chugh and Ghironi (2011) show that optimal tax rates are constant under monopolistic competition. This is not the case in oligopolistic market structures. Due to the countercyclicality of the price markup, the distortions affecting the economy are time-varying. Counteracting these distortion requires, thus, non constant tax rates.

Finally, notice that besides the form of competition, also the form of the sunk entry costs matters for optimal taxation purposes. For this reason, together with alternative forms of competition, two forms of the entry costs are considered. One features a constant entry cost measured in units of output, the other one features entry costs in terms of labor.

As a result the present framework features as special cases two models in the entry literature which also focus on optimal taxation problems: Coto Martinez et al. (2007) and Chugh and Ghironi (2011). Coto Martinez et al. (2007) consider an environment characterized by monopolistic competition under constant sunk entry costs. They find that the long run equilibrium is characterized by an inefficiently low number of firms. For this reason it is optimal to subsidize dividend income. Further they find that tax smoothing is optimal. Chugh and Ghironi (2011) consider a framework with monopolistic competition and sunk entry cost in terms of labor. In this case the optimal long run dividend income tax is zero and taxes are constant over the business cycle. By neglecting strategic interactions and considering the appropriate form of the entry costs the provided framework reduces to either one of these models. For this reason it can be regarded as a general framework where to study optimal taxation problems under various form of imperfect competition. Another paper closely related to the present one is that by Lewis (2010). She studies optimal fiscal policy under

oligopolistic competition and endogenous entry, but in a static environment.

The remainder of the paper is laid as follows. Section 2 presents the model; section 3 defines the market equilibrium; section 4 characterizes the efficient allocation; section 5 discusses the distortion associated to the market equilibrium; section 6 provided the Ramsey optimal fiscal; section 7 concludes. Technical details are left in the Appendix.

## 2 The Model

The economy features a continuum of atomistics sectors, or industries, on the unit interval. Each sector is characterized by different firms producing a good in different varieties, using labor as the only input. In turn, the sectoral goods are imperfect substitutes for each other and are aggregated into a final good. Households use the final good for consumption and investment purposes. Oligopolistic competition and endogenous firms' entry is modeled at the sectoral level.

At the beginning of each period  $N_{jt}^e$  new firms enter into sector  $j \in (0, 1)$ , while at the end of the period a fraction  $\delta \in (0, 1)$  of market participants exits from the market for exogenous reasons.<sup>2</sup> As a result, the number of firms in a sector  $N_{jt}$ , follows the equation of motion:

$$N_{jt+1} = (1 - \delta)(N_{jt} + N_{jt}^e) \tag{1}$$

where  $N_{jt}^e$  is the number of new entrants in sector  $j$  at time  $t$ . Following BGM (2012) I assume that new entrants at time  $t$  will only start producing at time  $t + 1$  and that the probability of exit from the market,  $\delta$ , is independent of the period of entry and identical across sectors. The assumption of an exogenous constant exit rate is adopted for tractability, but it also has empirical support. Using U.S. annual data on manufacturing, Lee and Mukoyama (2007) find that, while the entry rate is procyclical, annual exit rates are similar across booms and recessions.

Below alternative forms of competition between the firms within each sector are considered. In particular, the focus is on the traditional monopolistic competition setting and the approach based on oligopolistic competition developed by Jaimovich and Floetotto (2008) and Colciago and Etro (2010 a and b). As in Ghironi and Melitz (2005) and BGM(2012) who gave new life to an interesting literature on the role of entry in macroeconomic models, I introduce sunk entry costs to endogenize the number of firms in each sector. The nature and the form of the entry cost will be specified below, where I will also consider alternative specifications. The household side is standard. They supply labor to firms and choose how much to save in riskless bonds and in the creation of new firms through the stock market.

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<sup>2</sup>As discussed in BGM (2012), if macroeconomic shocks are small enough  $N_{j,t}^e$  is positive in every period. New entrants finance entry on the stock market.

## 2.1 Firms and Technology

The final good is produced aggregating a continuum of measure one of sectoral goods according to the function

$$Y_t = \left[ \int_0^1 Y_{jt}^{\frac{\omega-1}{\omega}} dj \right]^{\frac{\omega}{\omega-1}} \quad (2)$$

where  $Y_{jt}$  denotes output of sector  $j$  and  $\omega$  is the elasticity of substitution between any two different sectoral goods. The final good producer behave competitively. In each sector  $j$ , there are  $N_{jt} > 1$  firms producing differentiated goods that are aggregated into a sectoral good by a CES aggregating function defined as

$$Y_{jt} = \left[ \sum_{i=1}^{N_{jt}} y_{jt}(i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (3)$$

where  $y_{jt}(i)$  is the production of good  $i$  in sector  $j$ ,  $\varepsilon > 1$  is the elasticity of substitution between sectoral goods. As in Colciago and Etro (2010 a), I assume a unit elasticity of substitution between goods belonging to different sectors. This allows to realistically separate limited substitutability at the aggregated level, and high substitutability at the disaggregated level. Each firm  $i$  in sector  $j$  produces a differentiated good with the following production function

$$y_{jt}(i) = A_t h_{jt}^c(i) \quad (4)$$

where  $A_t$  represents technology which is common across sectors and evolves exogenously over time, while  $h_{jt}^c(i)$  is the labor input used by the individual firm for the production of the final good. The unit intersectoral elasticity of substitution implies that nominal expenditure,  $EXP_t$ , is identical across sectors. Thus, the final producer's demand for each sectoral good is

$$P_{jt} Y_{jt} = P_t Y_t = EXP_t. \quad (5)$$

where  $P_{jt}$  is the price index of sector  $j$  and  $P_t$  is the price of the final good at period  $t$ . Denoting with  $p_{jt}(i)$  the price of good  $i$  in sector  $j$ , the demand faced by the producer of each variant is

$$y_{jt}(i) = \left( \frac{p_{jt}(i)}{P_{jt}} \right)^{-\varepsilon} Y_{jt} \quad (6)$$

where  $P_{jt}$  is defined as

$$P_{jt} = \left[ \sum_{i=1}^{N_{jt}} (p_{jt}(i))^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (7)$$

Using (6) and (5) the individual demand of good  $i$  can be written as a function of aggregate expenditure,

$$y_{jt}(i) = \frac{p_{jt}^{-\varepsilon}}{P_{jt}^{1-\varepsilon}} EXP_t \quad (8)$$

As technology, the entry cost and the exit probability are identical across sectors, in what follows the index  $j$  is disregarded to considered a representative sector.

## 2.2 Households

Consider a representative agent with utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - v \frac{H_t^{1+1/\varphi}}{1+1/\varphi} \right\} \quad v, \varphi \geq 0 \quad (9)$$

where  $\beta \in (0, 1)$  is the discount factor,  $H_t$  are hours worked and  $C_t$  is the consumption of the final good. The representative agent enjoys labor and dividend income. The household maximizes (9) by choosing hours of work and how much to invest in bonds and risky stocks. The timing of investment in the stock market is as in BGM (2012) and Chugh and Ghrioni (2011). At the beginning of period  $t$ , the household owns  $x_t$  shares of a mutual fund of the  $N_t$  firms that produce in period  $t$ , each of which pays a dividend  $d_t$ . Denoting with  $V_t$  the value of a firm, it follows that the value of the portfolio held by the household is  $x_t V_t N_t$ . During period  $t$ , the household purchases  $x_{t+1}$  shares in a fund of these  $N_t$  firms as well as the  $N_t^e$  new firms created during period  $t$ , to be carried into period  $t+1$ . Total stock market purchases are thus  $x_{t+1} V_t (N_t + N_t^e)$ . At the very end of period  $t$ , a fraction of these firms disappears from the market.<sup>3</sup>

Following production and sales of the  $N_t$  varieties in the imperfectly competitive goods markets, firms distribute the dividend  $d_t$  to households. The household's total dividend income is thus  $D_t = x_t d_t N_t$ , which is taxed at the rate  $\tau_t^d$ . The variable  $w_t$  is the market real wage, and  $\tau_t^h$  is the tax rate on labor income. The household's holdings of the state-contingent one-period real government bond that pays off in period  $t$  are  $B_t$ ; and  $B_{t+1}^j$  are end-of-period holdings of government bonds that pay off in state  $j$  in period  $t+1$ , which has purchase price  $1/R_t^j$  in period  $t$ . The Flow budget constraint of the household is

$$\sum_j \frac{1}{R_t^j} B_{t+1}^j + C_t + x_{t+1} V_t (N_t + N_t^e) = (1 - \tau_t^h) w_t H_t + B_t + x_t V_t N_t + (1 - \tau_t^d) x_t d_t N_t$$

<sup>3</sup>Due to the Poisson nature of exit shocks, the household does not know which firms will disappear from the market, so it finances continued operations of all incumbent firms as well as those of new entrants.

The FOCs (First Order Conditions) for the household problem are represented by a standard Euler equation for bonds holdings

$$\frac{1}{C_t} = \beta R_t^j \frac{1}{C_{t+1}^j}$$

an asset pricing equation

$$V_t = \beta(1 - \delta) \frac{C_t}{C_{t+1}} E_t [(1 - \tau_{t+1}^d) \pi_{t+1}(\theta, N_{t+1}) + V_{t+1}] \quad (10)$$

and the condition for optimal labor supply

$$v C_t H_t^{\frac{1}{\varphi}} = (1 - \tau_t^h) w_t$$

### 2.3 Endogenous Entry

Upon entry firms faces a sunk cost, defined as  $f_t$ . In each period entry is determined endogenously to equate the value of firms to the entry costs. In what follows I consider two popular forms of the entry cost  $f_t$ , defined as *Form 1* and *Form 2*, respectively. *Form 1*, adopted, inter alia, by Jaimovich and Floetotto (2008) and Coto-Martinez et al. (2007), features a constant entry cost measured in units of output,  $f_t = \psi$ .

*Form 2*, adopted by Bilbiie, Ghironi and Melitz (2007), features an entry cost equal to  $\eta/A_t$  units of labor, with  $\eta > 0$ . Notice that, under this specification, technology shocks affect the productivity of the workers that produce goods and also of the workers that create new businesses.

### 2.4 Government

The Government faces an exogenous expenditure stream  $\{G_t\}_{t=0}^{\infty}$  in real terms. To finance this stream it issues real state contingent bonds,  $B_t^j$ , where the superscript  $j$  refer to the state of nature, and collects taxes on labor and dividend income. Also it provides entry-subsidies,  $\tau_t^s$ . Its period-by-period budget constraint is given by

$$\sum_j \frac{1}{R_t^j} B_{t+1}^j + \tau_t = G_t + B_t \quad (11)$$

where  $\tau_t = \tau_t^h w_t H_t + \tau_t^d N_t d_t$  are total tax revenues. The Government consumes the same index of goods faced by the household and optimizes the composition of its expenditure across goods.<sup>4</sup> Public spending evolves exogenously over time.

<sup>4</sup>Hence It follows that  $g_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} G_t$ .

## 2.5 Strategic Interactions

In each period, the same expenditure for each sector  $EXP_t$  is allocated across the available goods according to the standard direct demand function derived from the expenditure minimization problem of the household and the Government. It follows that the direct individual demand faced by a firm,  $y_t(i)$ , can be written as

$$y_t(i) = Y_t \left( \frac{p_t(i)}{P_t} \right)^{-\theta} = \frac{p_t(i)^{-\theta}}{P_t^{1-\theta}} Y_t P_t = \frac{p_t(i)^{-\theta} EXP_t}{P_t^{1-\theta}} \quad i = 1, 2, \dots, N_t \quad (12)$$

Inverting the direct demand functions, the system of inverse demand functions can be derived:

$$p_t(i) = \frac{y_t(i)^{-\frac{1}{\theta}} EXP_t}{N_t} \quad i = 1, 2, \dots, N_t \quad (13)$$

$$\sum_{i=1}^{N_t} y_t(i)^{\frac{\theta-1}{\theta}}$$

which will be useful in the remainder of the analysis. Firms cannot credibly commit to a sequence of strategies, therefore their behavior is equivalent to maximize current profits in each period taking as given the strategies of the other firms. Each good is produced at the constant marginal cost common to all firms. A main interest of this paper is in the evaluation of the efficiency of equilibria characterized by popular forms of competition by firms such as competition in prices and quantities. Firms take as given their marginal cost of production and the aggregate nominal expenditure.<sup>5</sup> Under different forms of competition we obtain equilibrium prices satisfying

$$p_t(i) = \mu(\theta, N_t) \frac{W_t}{A_t} \quad (14)$$

where  $\frac{W_t}{A_t}$  is the marginal cost and  $\mu(\theta, N_t) > 1$  is the markup function. In the next sections the mark up function under alternative forms of market competition is characterized.

### 2.5.1 Price Competition

Consider competition in prices. In each period, the gross profits of firm  $i$  can be expressed as:

$$\Pi_t [p_t(i)] = \frac{\left[ p_t(i) - \frac{W_t}{A_t} \right] p_t(i)^{-\theta} EXP_t}{\left[ \sum_{j=1}^{N_t} p_t(j)^{1-\theta} \right]} \quad (15)$$

<sup>5</sup>Of course, both of them are endogenous in general equilibrium, but it is reasonable to assume that firms do not perceive marginal cost and aggregate expenditure as affected by their choices.

Firms compete by choosing their prices. We consider two alternative approaches to this problem. The first one is the traditional monopolistic competition approach, which neglects strategic interactions between firms. The second one is the Bertrand approach, where strategic interactions are taken into consideration.

The outcome of profit maximization under monopolistic competition is well known. Each firm  $i$  chooses the price  $p_t(i)$  to maximize profits taking as given the price of the other firms, neglecting the effect of its price choice on the sectoral price index. The symmetric equilibrium price is  $p_t = \mu^{MC}(\theta) W_t/A_t$ , which is associated to the constant price markup  $\mu^{MC}(\theta) = \frac{\theta}{(\theta-1)}$ . The latter does not depend on the extent of competition, but just on the elasticity of substitution between goods.

Under Bertrand competition, each firm  $i$  chooses the price  $p_t(i)$  to maximize profits taking as given the price of the other firms. The first order condition for any firm  $i$  is:

$$p_t(i)^{-\theta} - \theta \left( p_t(i) - \frac{W_t}{A_t} \right) p_t(i)^{-\theta-1} = \frac{(1-\theta)p_t(i)^{-\theta} \left( p_t(i) - \frac{W_t}{A_t} \right) p_t(i)^{-\theta}}{\sum_{i=1}^{N_t} p_t(i)^{1-\theta}}$$

Notice that the term on the right hand side is the effect of the price strategy of a firm on the price index: higher prices reduce overall demand, therefore firms tend to set higher mark ups compared to monopolistic competition. The symmetric equilibrium price  $p_t$  must satisfy

$$p_t = \mu^B(\theta, N_t) \frac{W_t}{A_t}$$

where the mark up reads as

$$\mu^B(\theta, N_t) = \frac{1 + \theta(N_t - 1)}{(\theta - 1)(N_t - 1)} \quad (16)$$

The mark up is decreasing in the degree of substitutability between products  $\theta$  and in the number of firms. Importantly, when  $N_t \rightarrow \infty$  the markup tends to  $\mu^{MC}(\theta)$ , the standard one under monopolistic competition.<sup>6</sup>

### 2.5.2 Quantity Competition

Consider now competition in quantities in the form of Cournot competition. Using the inverse demand function (13), the profit function of a firm  $i$  can be

<sup>6</sup>Since total expenditure  $EXP_t$  is equalized between sectors, we assume that it is also perceived as given by the firms. Under the alternative hypothesis that the sum between public and private consumption,  $C_t + G_t$ , is perceived as given, we would obtain the higher mark up:

$$\tilde{\mu}^B(\theta, N_t) = \frac{\theta(N_t - 1)}{(\theta - 1)(N_t - 1) - 1}$$

which leads to similar qualitative results. This case would correspond to the equilibrium mark up proposed by Yang and Heijdra (1993).

expressed as a function of its output  $y_t(i)$  and the output of all the other firms:

$$\begin{aligned}\Pi_t [y_t(i)] &= \left[ p_t(i) - \frac{W_t}{A_t} \right] y_t(i) = \\ &= \frac{y_t(i)^{\frac{\theta-1}{\theta}} EXP_t}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\theta-1}{\theta}}} - \frac{W_t y_t(i)}{A_t}\end{aligned}\quad (17)$$

Assume now that each firm chooses its production  $y_t(i)$  taking as given the production of the other firms. The first order conditions:

$$\left( \frac{\theta-1}{\theta} \right) \frac{y_t(i)^{-\frac{1}{\theta}} EXP_t}{\sum_i y_t(i)^{\frac{\theta-1}{\theta}}} - \left( \frac{\theta-1}{\theta} \right) \frac{y_t(i)^{\frac{\theta-2}{\theta}} EXP_t}{\left[ \sum_i y_t(i)^{\frac{\theta-1}{\theta}} \right]^2} = \frac{W_t}{A_t}$$

for all firms  $i = 1, 2, \dots, N_t$  can be simplified imposing symmetry of the Cournot equilibrium. This generates the individual output:

$$y_t = \frac{(\theta-1)(N_t-1)A_t EXP_t}{\theta N_t^2 W_t} \quad (18)$$

Substituting into the inverse price, one obtains the equilibrium price  $p_t = \mu^C(\theta, N_t) \frac{W_t}{A_t}$ , where

$$\mu^C(\theta, N_t) = \frac{\theta N_t}{(\theta-1)(N_t-1)} \quad (19)$$

is the markup under competition in quantities. For a given number of firms, the mark up under competition in quantities is always larger than the one obtained before under competition in prices, as well known for models of product differentiation (see for instance Vives, 1999). Notice that the mark up is decreasing in the degree of substitutability between products  $\theta$  and in the number of competitors. In the Cournot equilibrium, the markup remains positive for any degree of substitutability, since even in the case of homogenous goods, we have  $\lim_{\theta \rightarrow \infty} \mu^Q(\theta, N_t) = N_t/(N_t-1)$ .<sup>7</sup> Finally, only when  $N_t \rightarrow \infty$  the markup tends to  $\mu^{MC}(\theta)$ , the markup under monopolistic competition.

### 3 Market Equilibrium

This section contains the conditions characterizing the market equilibrium (ME). Merging the household flow budget constraint with the government budget leads to

$$Y_t^c + N_t^e V_t = w_t H_t + N_t \pi_t \quad (20)$$

<sup>7</sup>This allow to consider the effect of strategic interactions in an otherwise standard setup with perfect substitute goods within sectors. See Colciago and Etro (2010 b)

where  $Y_t^c = C_t + G_t$  denotes the sum between private and public consumption of the final good. Notice that the sum between labor income and profits income equals aggregate GDP. The Euler equation for firms' shares reads as

$$V_t = \beta(1 - \delta)E_t \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( (1 - \tau_{t+1}^d) \pi_{t+1} + V_{t+1} \right) \quad (21)$$

while the set of Euler equations for bond holdings provide the definition of the stochastic discount factor as  $\beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-1}$ . The real wage can be derived from the equilibrium pricing relation as

$$w_t = \frac{A_t p_t}{\mu(\theta, N_t) P_t} = \frac{\rho_t}{\mu_t} A_t$$

where in the symmetric equilibrium  $\rho_t = \frac{p_t}{P_t} = N_t^{1/(\theta-1)}$ . The first order condition for labor supply is

$$v C_t H_t^{\frac{1}{\varphi}} = (1 - \tau_t^l) \frac{\rho_t}{\mu_t} A_t \quad (22)$$

also we must consider the equation determining the dynamics of the number of firms

$$N_{t+1} = (1 - \delta) (N_t + N_t^e) \quad (23)$$

It remains to impose the entry condition and the clearing of the market. To do so the analysis differentiates according to the form of the entry cost.

*Form 1.* When the latter is measured in, constant, units of output, the labor input is entirely employed for the production of the final good, thus the clearing of the labor market requires  $H_t = N_t h_t^c$ . The demand faced by firm  $i$  reads as  $y_t = \frac{Y_t}{N_t \rho_t}$ . In this case firm  $i$ 's profits are

$$\pi_t = \left( 1 - \frac{1}{\mu_t} \right) \rho_t y_t = \left( 1 - \frac{1}{\mu_t} \right) \frac{Y_t}{N_t}$$

Aggregating profits over firms and summing to labor income delivers GDP as  $\rho_t A_t H_t$ . Since the entry condition is simply  $V_t = \psi$  the resource constraint (24) becomes

$$Y_t^c + N_t^e \psi = \rho_t A_t H_t \quad (24)$$

Notice that GDP here coincides with the production of the final good. The Euler equation for firms share translates into

$$\psi = \beta(1 - \delta)E_t \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( (1 - \tau_{t+1}^d) \left( 1 - \frac{1}{\mu_{t+1}} \right) \frac{Y_t}{N_{t+1}} + \psi \right) \quad (25)$$

*Form 2.* When the entry cost is measured in units of labor, the economy amounts to one which features two sectors, one where  $N_t^e \frac{\eta}{A_t}$  units of labor are used to produce new firms, the other one where  $N_t h_t^c = H_t - N_t^e \frac{\eta}{A_t}$  units of

labor are used to produce the final good. This implies that the set up of a new firm reduces the labor input available for the production of the final good. In this setting the individual demand faced by firms  $i$  is  $y_t = \frac{Y_t^c}{N_t \rho_t}$  and firm level profits can be written as<sup>8</sup>

$$\pi_t = \left(1 - \frac{1}{\mu_t}\right) \rho_t y_t = \left(1 - \frac{1}{\mu_t}\right) \frac{Y_t^c}{N_t}$$

Aggregating profits over firms and summing to labor income delivers GDP as  $GDP_t = \left(1 - \frac{1}{\mu_t}\right) Y_t^c + \frac{\rho_t}{\mu_t} A_t H_t$ . Also recall that the entry condition implies  $V_t = f_t = \frac{\eta}{A_t} w_t = \eta \frac{\rho_t}{\mu_t}$ . In this case, the resource constraint reads as

$$Y_t^c + N_t^e \eta \rho_t = \rho_t A_t H_t \quad (26)$$

The Euler equation (21) for the value of the firm reduces, instead, to

$$\eta \frac{\rho_t}{\mu_t} = \beta(1 - \delta) E_t \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( (1 - \tau_{t+1}^d) \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{Y_{t+1}^c}{N_{t+1}} + \eta \frac{\rho_{t+1}}{\mu_{t+1}} \right) \quad (27)$$

**Definition 1 (Market Equilibrium)** *Given the exogenous processes  $\{A_t, G_t\}_{t=0}^{\infty}$  and processes  $\{\tau_t^d, \tau_t^l\}_{t=0}^{\infty}$ , the Market Equilibrium (ME) consists of an allocation  $\{C_t, H_t, N_t, N_t^e, B_{t+1}\}_{t=0}^{\infty}$  which satisfies equations (22), (11), (23), the definition of the price markup and the definition of the love for variety to which we must add equations (24) and (25) in the case of entry costs in Form 1, or equations (27) and (26) in the case of entry costs in Form 2.*

## 4 Efficient Equilibrium

This section outlines a scenario where a benevolent Social Planner (SP) maximizes households' lifetime utility by choosing quantity directly. In doing this, the SP is subject to the same technological constraints described in the previous sections. The SP maximizes (9) with respect to  $\{C_t, N_{t+1}, N_t^e, H_t\}_{t=0}^{\infty}$ . The choice is subject to two constraints. The first one is given by the dynamics of the number of firms, equation (23), the second one is the resource constraint, which is represented by equation (26) in the case of Form 2 of the entry costs and by (24) when the entry cost is measured in terms of output. The social

<sup>8</sup>To see this notice that

$$P_t (C_t + G_t) = \int_0^{N_t} p_{it} y_{it} di$$

in a symmetric equilibrium it follows that

$$P_t (C_t + G_t) = N_t p_t y_t$$

thus

$$y_t = \frac{P_t (C_t + G_t)}{p_t N_t} = \frac{(C_t + G_t)}{\rho_t N_t}.$$

planner takes into account the effect of the number of varieties,  $N_t$ , on the relative price,  $\rho_t$ , which is a primitive of the problem. The FOC with respect to  $H_t$  is independent of the form of the entry cost and reads as

$$\chi C_t H_t^{\frac{1}{\psi}} = \rho_t A_t \quad (28)$$

this condition simply states that the Planner equates the marginal rate of substitution between hours and consumption to the marginal product of labor, which depends on the number of varieties in the economy.

The Planner's Euler equation depends instead on the form of the entry cost. In the case of Form 1, it reads as

$$1 = \beta(1 - \delta) E_t \frac{C_t}{C_{t+1}} \left[ \frac{\epsilon(N_{t+1}) \frac{Y_{t+1}}{N_{t+1}}}{\psi} + 1 \right] \quad (29)$$

while in the case of Form 2 it amounts to

$$1 = \beta(1 - \delta) E_t \frac{C_t}{C_{t+1}} \left[ \eta \frac{\rho_{t+1}}{\rho_t} + \frac{\epsilon(N_{t+1}) \frac{Y_{t+1}^c}{N_{t+1}}}{\eta \rho_t} \right] \quad (30)$$

where  $\epsilon(N_{t+1}) = \frac{\rho_{N,t+1} N_{t+1}}{\rho_{t+1}}$  is the benefit of variety in elasticity form. Under CES preferences the latter is a constant equal to  $\frac{1}{\epsilon-1}$ , hence, in the remainder, we simply denote it with  $\epsilon$ . These equations state that at the social optimum the marginal rate of substitution between present and future consumption must be identical to the intertemporal marginal rate of transformation (IMRT). In what follows I discuss the nature of the IMRT and its dependence on the form of the entry cost.

To gain intuition, consider a non-stochastic version of the economy. In this case the marginal rate of substitution between present and future consumption equals  $\frac{1}{\beta} \frac{C_{t+1}}{C_t}$ . To analyze the IMRT and to provide concepts that will be used in the next sections we build on the analysis by Chugh and Ghironi (2011) and BGM (2007).

The nature of the IMRT depends on the specific form of the entry cost. However, under both formulations, the transformation of current resources into future ones can be understood by considering firms as the capital stock of the economy. As such, the creation of new firms contributes to tomorrows' consumption through two channels, a production channel (PC) and an investment channel (IC). To each of this channel is associated a partial intertemporal rate of transformation through which current consumption can be transformed in future consumption. The overall rate of transformation is the sum of the partial rates.

Consider Form 1 of the entry cost. One unit of consumption today can be transformed into  $\frac{1}{\psi}$  new firms. A fraction  $(1 - \delta)$  of these firms will produce in the next period. Each of the additional  $\frac{1-\delta}{\psi}$  firms contribute to time t+1 consumption through the production process for  $\epsilon \frac{Y_{t+1}}{N_{t+1}}$  units. Hence the partial

rate of transformation associated to the production channel reads as  $\frac{(1-\delta)\epsilon^{\frac{Y_{t+1}}{N_{t+1}}}}{\psi}$ , and we denote it by  $PC - IMRT_1^{SP}$ , where the superscript  $SP$  denotes that it is the socially efficient rate, while the subscript 1 means that it is the rate associated to entry costs in form 1. Recall that new firms represent investment and can be transformed directly into  $(1 - \delta)$  units of tomorrows' consumption ( $\frac{1-\delta}{\psi}$  firms each with a consumption value  $\psi$ ). Hence the intertemporal rate of transformation associated to the investment channel,  $IC-IMRT_1^{SP}$ , is  $(1 - \delta)$ . The efficient intertemporal marginal rate of transformation,  $IMRT_1^{SP}$ , is the sum between these two components and reads as  $IMRT_1^{SP} = \frac{(1-\delta)\epsilon^{\frac{Y_{t+1}}{N_{t+1}}}}{\psi} + (1 - \delta)$ .

In the case of Form 2 of the entry cost the creation of a new firms requires units of labor. Foregoing one unit of consumption leads to the creation of  $\frac{1}{\eta\rho_t}$  new firms. This implies two differences with respect to the previous case. The first one is that the  $IC - IMRT_2^{SP}$  is not constant over time (and states of nature), but depends on the ratio between the benefit of variety into two adjacent periods,  $\frac{\rho_{t+1}}{\rho_t}$ . In other words, the cost of creating a firm in terms of the final good is not constant over time. The reason is that the love for variety affects the marginal productivity of labor and thus the opportunity cost of creating a new firm in terms of the final good. The second one is that at time  $t+1$  the creation of  $N_{t+1}^e$  new firms requires  $\frac{\eta}{A_t}N_{t+1}^e$  units of labor, which are diverted from the production of the final good. As a result the  $PC-IMRT^{SP}$  depends on the sum between private and public consumption and not on GDP. In particular, the net gain in terms of consumption coming from the production channel is  $\rho_{N,t+1} \frac{A_{t+1}(H_t - \frac{\eta}{A_t}N_{t+1}^e)}{N_{t+1}} = \frac{\rho_{N,t+1}}{\rho_{t+1}} \frac{\rho_{t+1}(A_{t+1}H_t - N_{t+1}^e\eta)}{N_{t+1}} = \epsilon^{\frac{Y_{t+1}^c}{N_{t+1}}}$ . Hence, the  $PC-IMRT_2^{SP}$  is  $\frac{(1-\delta)\epsilon^{\frac{Y_{t+1}^c}{N_{t+1}}}}{\eta\rho_t}$ . Overall, the efficient intertemporal marginal rate of transformation associated to this form of the entry cost reads as  $IMRT_2^{SP} = \frac{(1-\delta)\epsilon^{\frac{Y_{t+1}^c}{N_{t+1}}}}{\eta\rho_t} + \frac{\rho_{t+1}}{\rho_t}$ . Below we used the partial rate of transformation just defined to analyze the distortions characterizing the market allocation. To conclude this section we provide a definition of the socially efficient equilibrium.

**Definition 2 (Efficient Equilibrium)** *The Efficient Equilibrium consists of an allocation  $\{C_t, h_t, N_t, N_t^e\}_{t=0}^\infty$  satisfying equations (23), (28) together with equations (26) and (30) in the case of entry cost in terms of labor, and equations (24) and (29) in the case of entry costs in terms of output, for given  $N_0$  and  $\{A_t, G_t\}_{t=0}^\infty$ .*

## 5 Market Distortions

The market allocation features two distortions. To identify them it is convenient to set, for the moment being, fiscal instruments to zero,  $\tau_t^d = \tau_t^h = 0$ . In the next sections we reintroduce fiscal instruments and design them in order to

minimize the welfare losses associated to the distortions that we are about to discuss.

The first distortion is referred to as the *Labor Distortion*. In the competitive equilibrium labor is supplied up to the point that the following condition is satisfied

$$\chi C_t H_t^{1/\varphi} = \frac{\rho_t}{\mu_t} A_t \quad (31)$$

A comparison between equation (28) and equation (31) reveals that in the decentralized equilibrium the marginal rate of substitution between hours and consumption,  $\chi C_t h_t^{1/\varphi}$ , is lower than the marginal rate of transformation between hours and output,  $\rho_t A_t$ . As in other models with an imperfectly competitive goods market, this wedge is due to the presence of a price markup. Oligopolistic competition renders this wedge time varying. Ruling out the labor distortion requires  $\mu_t = 1$ . However, since in this case firms would not recover the entry cost, the allocation would be degenerate.<sup>9</sup>

The second distortion involved in the decentralized allocation is an *Entry Distortion*. This wedge operates through the intertemporal firms creation margin and could lead to an inefficient number of firms in equilibrium. To illustrate the entry distortion I will refer to the intertemporal marginal rates of transformation associated to the production and the investment channel introduced above. By comparing the social rates of transformation to the market counterparts we will be able to identify the sources of the distortions involved in the ME.

Denote with  $\text{PC-IMRT}^{ME}$  the intertemporal rate of transformation associated to the production channel in the ME. The latter depends on the level of the markup, which provides the private incentive to create new firms, while the social rate depends on the love for variety effect. As emphasized by Coto-Martinez et al. (2007), there are two externalities of opposite sign which could lead to a difference between  $\text{PC-IMRT}^{ME}$  and  $\text{PC-IMRT}^{SP}$ . A higher number of firms increases the welfare that a household obtains by spending a given nominal amount. Given the individual firm neglects this effect, entry is too low with respect to the social optimum. The second effect is a business stealing effect. A higher number of firms in equilibrium reduces individual profits. Since individual firms do not take this effect into account when deciding about entry, the resulting number of firms in equilibrium is too high. Notice that the business stealing effect is stronger under oligopolistic competition with respect to the monopolistic competition case, since entry erodes the markup.

Next consider the investment channel, and define  $\text{IC-IMRT}^{ME}$  the relative marginal rate of transformation. This channel is absent in static entry model such as that considered by Coto-Martinez et al. (2007), but it is specific to the dynamic entry process which characterizes our setting, and other models adopting the BGM entry framework. As mentioned earlier, investing into new firms is a mean to save for the future, hence the rate at which firms can be

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<sup>9</sup>In a first best environment, if lump sum taxes were available, a labor income subsidy equal to  $\tau_t^h = 1 - \frac{1}{\mu_t}$  would eliminate the Labor Distortion.

directly transformed into consumption into two adjacent period affects investment decisions. Suppose that the units of consumption required to create a new firm are lower in the private equilibrium with respect to those required in the efficient equilibrium. In this case the ME could be characterized by over-entry and thus by a level of consumption too low with respect to the optimal one. To provide the conditions to rule out the entry distortion we distinguish according to the form of the entry cost.

*Form 1.* The rate of transformation associated to the production channel is  $PC - IMRT_1^{ME} = \frac{1-\delta}{\psi} \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{Y_{t+1}}{N_{t+1}}$ . The latter is identical to  $PC - IMRT_1^{SP}$  if

$$\left(1 - \frac{1}{\mu_{t+1}}\right) = \epsilon \quad (32)$$

i.e., if the so called Lerner index equals the benefit of variety<sup>10</sup>. Since both the planner and the market can transform firms into consumption at the same cost,  $\psi$ , in terms of output the  $IC - IMRT_1^{ME}$  is not distorted. As a result condition (32) is necessary and sufficient to rule out the entry distortion.

Since  $\epsilon = \frac{1}{\theta-1}$  the condition above implies that the price markup must be constant over time,  $\mu_t = \mu$ . As a result it cannot be satisfied under oligopolistic competition. Notice also that, although monopolistic competition leads to a constant markup  $\mu = \frac{\theta}{\theta-1}$ , the condition for efficiency is not satisfied since  $1 - \frac{1}{\mu} < \epsilon$ .

*Form 2.* In this case both the partial intertemporal rates of transformation described above could be distorted, and thus two conditions need to be imposed to reinstate efficiency. Recall that  $PC - IMRT_2^{ME} = \frac{1-\delta}{\eta} \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{\mu_t}{\rho_{t+1}} \frac{Y_{t+1}^c}{N_{t+1}}$ , while  $PC - IMRT_2^{SP} = \frac{(1-\delta)}{\eta} \frac{\epsilon}{\rho_{t+1}} \frac{Y_{t+1}^c}{N_{t+1}}$ . The latter are identical if

$$\mu_t \left(1 - \frac{1}{\mu_{t+1}}\right) = \epsilon \quad (33)$$

Also  $IC - IMRT_2^{ME} = (1-\delta) \frac{\rho_{t+1}}{\rho_t} \frac{\mu_t}{\mu_{t+1}}$  while  $IC - IMRT_2^{SP} = (1-\delta) \frac{\rho_{t+1}}{\rho_t}$ . In this case the condition for efficiency is

$$\frac{\mu_t}{\mu_{t+1}} = 1 \quad (34)$$

Combining conditions (33) and (34) we recover those emphasized by BGM

<sup>10</sup>The Lerner Index is defined as follows

$$\begin{aligned} LI &= \frac{p(i) - MC(i)}{p(i)} = \frac{\frac{p(i)}{P} - \frac{MC(i)}{P}}{\frac{p(i)}{P}} = \\ &= \frac{\rho - \frac{\rho}{\mu}}{\rho} = 1 - \frac{1}{\mu} \end{aligned}$$

(2007), namely

$$\epsilon = \mu_{t+1} - 1 \text{ and } \frac{\mu_t}{\mu_{t+1}} = 1$$

which implies that the price mark up should be constant over time and that the benefit of variety in elasticity form should equal the market power as measured by the net markup. Under oligopolistic competition both conditions fail. Contrary to the previous case, these conditions are satisfied under monopolistic competition, as emphasized by BGM (2007) and Chugh and Ghironi (2011). Thus, under monopolistic competition the entry margin is not distorted.

The analysis shows that the conditions for efficiency depend on three factors: i) preferences, which affect the nature of the benefit of variety; ii) the form of the entry costs, which affect the marginal rate of transformation associated to the investment channel, IC-IMRT and iii) the nature of competition, which determines the form of the price markup and through this way distorts the marginal rate of transformation associated to the production channel, PC-IMRT, and the supply of labor.

Importantly, under oligopolistic competition, the entry margin is always distorted, no matter the form of the entry cost. This is not the case under monopolistic competition, which implies an efficient number of firms under entry costs in form 2, as emphasized by BGM (2007) and Chugh and Ghironi (2011).

Notice that the economic environments in Chugh and Ghironi (2011) and Coto Martinez et al. (2007), can be considered as special cases of the one outlined here. In particular, the present setup collapses to that considered by Coto Martinez et al. (2007) when entry costs are in form 1 and the strategic interactions between firms are neglected. It coincides, instead, with that considered by Chugh and Ghironi (2011) under entry costs in form 2 and strategic interaction are neglected.

The next outlines the fiscal policy aimed at minimizing welfare losses due to the market distortions when the Government can raise revenues solely by imposing distortionary taxes and issuing state contingent bonds.

## 6 Ramsey Optimal Fiscal policy

As emphasized by Chugh and Ghironi (2011) a primal approach cannot be applied to solve for the optimal policy. To see this consider the Euler equation for firms' shares, which in its general form reads as

$$f_t = \beta(1 - \delta)E_t \left( \frac{C_{t+1}}{C_t} \right)^{-1} ((1 - \tau_{t+1}^d) \pi_{t+1} + f_{t+1})$$

Expected future dividend income taxes cannot be removed from this equation using other equilibrium conditions. As a result the set of allocations that the Planner can select cannot uniquely be characterized by means of the so called implementability constraint. Further, computing a first order condition with respect to  $E_t \tau_{t+1}^d$  would leave  $\tau_{t+1}^d$  indeterminate from the point of view of time-t.

To resolve these indeterminacy issue I adopt the solution proposed by Chugh and Ghironi (2011). In particular, it is assumed that the planner chooses a state contingent schedule for the time  $t+1$  dividend income tax rate  $\tau_{j,t+1}^d$ , where  $j$  indexes the state of the economy. This schedule is in the time  $t$  information set. Also we assume that the Planner commits to the schedule, meaning that the state contingent tax rate is implemented with certainty at time  $t+1$ .

**Definition 3 (Ramsey Equilibrium)** *Given the government expenditure  $\{G_t\}_{t=0}^\infty$  and the initial conditions  $\{b_0, N_0\}$  the allocations associated to the optimal fiscal policy  $\{\tau_t^h, \tau_{j,t+1}^d\}_{t=0}^\infty$  are derived by solving*

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - v \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right\}$$

*The choice variables are  $C_t, N_t, H_t, N_t^e$ , and  $\tau_{t/t+1}^d$ . The allocation is restricted by four constraints, two of them depend on the form of the entry cost. The constraints which are independent of the form of the entry cost are equation (23), determining the dynamic of the number of firms, and the implementability constraint, which reads as*

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ 1 - v h_t^{\frac{1}{\varphi}} \right] = \frac{b_0}{C_0} + \frac{1}{C_0} [(1 - \tau_0^d) d_0 + V_0] N_0 s_0$$

*The allocation is further restricted by equations (24) and (25) in the case of entry cost of form 1 and by equations (26) and (27) in the case of entry cost in form 2.*

The first order conditions for the Ramsey problem are reported in the Appendix, for both forms of the entry cost. Throughout the analysis, it is assumed that the policy maker can credibly commit himself, but the initial period ( $t = 0$ ) is ignored. In deriving the Ramsey policy, the well known problem that the policy maker's decision rules will be different for the first period in which the policy is implemented is neglected. This is justified by the fact that the interest is that of making statements about the deterministic steady state as well as about business cycle fluctuations around it, while the transition path from the initial values towards the steady state is not analyzed. For the analysis, it is further assumed that the initial values for the predetermined variables are equal to their values in the deterministic Ramsey steady state.

Since part of the following analysis is numerical, the calibration of structural parameters follows. The time unit is meant to be a quarter. The discount factor,  $\beta$ , is set to the standard value for quarterly data 0.99, while the rate of business destruction,  $\delta$ , equals 0.025 to match the U.S. empirical level of 10 per cent business destruction a year. Steady state productivity is equal to  $A = 1$ . The baseline value for the entry cost is set to  $\eta = \psi = 1$ . The baseline value for the intrasectoral elasticity of substitution is  $\theta = 6$ , which is in line with the typical calibration for monopolistic competition and delivers markups levels belonging

to the empirically relevant range.<sup>11</sup> In what follows equilibrium allocations under Cournot, Bertrand and Monopolistic Competition for each entry cost configuration are compared.

This is done holding parameters fixed, in order to understand the role of the different market structures. To this end, the following calibration strategy for the utility parameter  $v$  is adopted. The value of  $v$  is such that steady state labor supply is equal to one under monopolistic competition. In this case the Frish elasticity of labor supply reduces to  $\varphi$ , to which we assign a value of four as in King and Rebelo (2000). Next the values of  $v$  and  $\varphi$  are held constant under both Bertrand and Cournot competition. Turning to fiscal parameters, the ratio of Government spending over GDP equals 0.22, as estimated by Schmitt-Grohè and Uribe (2005), and the ratio of Government debt to output equals 0.5 on an annual basis, in line with the U.S. postwar average. There are two exogenous processes in the economy, that for Government spending and that for technology. They are both assumed to be AR (1) processes in log deviations from the steady state:

$$\begin{aligned}\log \frac{G_t}{G} &= \rho_g \log \frac{G_{t-1}}{G} + \varepsilon_t^g \\ \log \frac{A_t}{A} &= \rho_a \log \frac{A_{t-1}}{A} + \varepsilon_t^a\end{aligned}$$

The autoregressive coefficient for the technology process is  $\rho_a = 0.979$  and the standard deviation of the disturbance,  $\sigma_a$ , is 0.0072, as in the RBC model by King and Rebelo (2000). The parameterization of the Government spending process follows Chari and Kehoe (1999) in setting  $\rho_g = 0.97$  and  $\sigma_g = 0.027$ .

## 6.1 Ramsey Steady State

By using the dividend income tax, the Ramsey Planner removes the inefficiency along the entry margin. This result has first been identified by Chugh and Ghironi (2011). In what follows it is shown that the level of the optimal dividend income tax rate and optimal entry subsidy differ according to the form of the entry cost and the form of competition. Proposition 1 provides the main result of this section.

**Proposition 4 (Optimal long-run dividend income taxes)** *Optimal long-run dividend income taxes depend on the form of the entry costs*

$$\begin{aligned}(i) \text{ Form 1: } \tau^d &= 1 - \frac{\epsilon}{1 - \frac{1}{\mu}} \\ (ii) \text{ Form 2: } \tau^d &= 1 - \frac{\epsilon}{\mu - 1}\end{aligned}$$

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<sup>11</sup>Oliveira Martins and Scarpetta (1999) provide estimates of price mark ups for US manufacturing industries over the period 1970-1992. In broad terms most of the sectoral markups defined over value added are in the range 30-60 per cent, while when defined over gross output they are in the range 5-25 per cent. In the latter case, high mark ups, over 40 per cent, are observed in few sectors.

**Proof.** In the Appendix. ■

The dividend income tax affects uniquely the PC-IMRT. The latter depends on the form of the entry cost and for this reason optimal tax rates differ accordingly. Consider condition (i). Profits are taxed when the private incentive to create a new firm, as measured by the Lerner Index, that is  $\left(1 - \frac{1}{\mu}\right)$ , is larger than the social incentive to introduce a new variety,  $\epsilon$ . In this case the number of firms in equilibrium is too large with respect to optimal and thus a positive dividend income tax is required. A profit income subsidy is, instead, optimal in the opposite situation. Notice that the number of firms is not the optimal one even if the inefficiency along the entry margin is removed. The reason is that the Ramsey Planner cannot remove the labor distortion. As a result steady state hours will be lower in the Ramsey steady state with respect to the those observed in the long run allocation reached by the Social Planner. In other words the Ramsey Planner disregards the social cost of the labor distortion. However, as shown below, the Planner implements the efficient level of units of effective labor per firm,  $\frac{\rho H}{N}$ . Notice that in the standard neoclassical growth model the Ramsey Planner would target the efficient ratio between capital and hours. This further emphasizes the analogy between the stock of capital in the neoclassical model and the stock of firms in the economy we have outlined. The optimal dividend income tax rate under Bertrand competition is

$$\tau_{\text{Bertrand}}^d = \frac{1}{N} - \frac{1}{\theta - 1}$$

while under Cournot competition we obtain

$$\tau_{\text{Cournot}}^d = \frac{1}{N} - \frac{1}{(\theta - 1)^2}$$

Notice that the optimal tax rate in the case of monopolistic competition is

$$\tau_{\text{Monopolistic}}^d = -\frac{1}{\theta - 1},$$

Under oligopolistic competition the optimal dividend income tax could take the form of a subsidy, depending on the parameterization of the model. On the contrary, removing the entry distortion under monopolistic competition requires a subsidy no matter the parameterization of the model. This means that monopolistic competition always leads to a suboptimal steady state number of firms. Figure 3 displays the Ramsey steady state number of firms, hours, output per firm and optimal tax rates as a function of the entry cost under Bertrand, Cournot and monopolistic competition. Allocations are compared to the efficient ones. First, for any given value of the entry cost, the Ramsey Planner implements the same allocation for hours and the number of firms across market structures.

These differ from the efficient ones, since there are less firms and hours worked are lower at the steady state. As mentioned above the Ramsey Planner

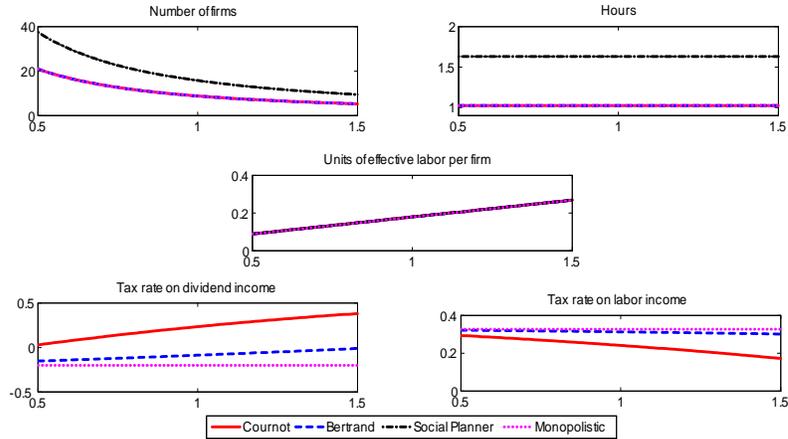


Figure 1: Ramsey Steady State under Entry Costs in Form 1. Entry cost ( $\psi$ ) on the horizontal axis.

targets the level of units of effective labor per firm, which are at their efficient level. This, however, requires a different combination of optimal taxes across market structures. The market equilibrium under Cournot competition would be characterized by excessive entry. Hence, restoring efficiency along the entry margin requires a positive dividend income tax, no matter the level of the entry cost. As in Coto-Martinez et al. (2007), the number of firms under monopolistic competition is too low and dividend income should be subsidized to promote entry. Bertrand competition falls between these two cases, since we observe a dividend income subsidy in the case of a low entry cost and a tax in the case of a high entry cost. As stated by Vives (1984), Bertrand competition can be regarded as a more competitive market structure with respect to Cournot and for this reason it is judged as more efficient.<sup>12</sup> This analysis suggests that, for given entry costs, the dividend income tax should be lower in markets characterized by more competitive market structures.

Condition (ii) is isomorphic to that obtained by BGM (2007) and Chugh and Ghironi (2011) under the case of monopolistic competition. In this case profits should be taxed whenever the net markup exceeds the benefit of variety in elasticity form. However, under monopolistic competition condition (ii) is automatically satisfied and the market equilibrium displays efficiency along the entry margin. This is not the case under oligopolistic competition. Under

<sup>12</sup>Vives (1984) provide the following intuitive explanation to support this view. In Cournot competition each firm expects the others to cut prices in response to price cuts, while in Bertrand competition the firm expects the others to maintain their prices; therefore Cournot penalizes price cutting more. One should expect Cournot prices to be higher than Bertrand prices.

Bertrand competition the optimal tax rate is

$$\tau_{\text{Bertrand}}^d = \frac{1}{N}$$

while under Cournot competition is

$$\tau_{\text{Cournot}}^d = \frac{\theta}{N + \theta - 1}$$

Hence entry costs in form 2 always imply a positive dividend income tax rate under oligopolistic competition. Figure 4 displays the Ramsey Steady State number of firms and hours together with optimal tax rate as a function of the entry cost under entry cost in form 2. In this case allocations slightly differ across market structures. Hours and the number of firms are lower with respect to their efficient counterpart, but output per firm is equal to its efficient level. It is interesting to note that under Cournot competition the tax rate on labor income is always lower than that on dividend income, thus in the long run the Government should rely more heavily on dividend income taxation, with respect to labor income taxation, to finance public spending.<sup>13</sup>

## 6.2 Ramsey Dynamics and Optimal Tax Volatility

This section shows the business cycle implications of the Ramsey optimal policy. The dynamics under optimal policy are obtained by solving a first order approximation to the Ramsey equilibrium conditions. As shown in various contributions by Schmitt-Grohè and Uribe, in models characterized by more frictions than the present one, a first order approximation to the equilibrium conditions delivers dynamics that are very close to the exact ones.

Figure 3 depicts percentage deviations from the steady state of key variables in response to a one standard deviation technology shock under entry cost in form 1. For tax rates we report deviations from the steady state level in percentage points. Time on the horizontal axis is in quarters. Solid lines refer to the Efficient (Social Planner) allocation, dashed and dash-dotted lines refer to Ramsey dynamics under Bertrand and the Cournot competition respectively, and dotted lines to the case of Ramsey dynamics under monopolistic competition. The technology shock creates expectations of future profits which lead to the entry of new firms in the market. This is so under both the Ramsey and the

<sup>13</sup>As shown by Chugh and Ghironi (2011) the Ramsey Planner can also remove the entry distortion by using an entry subsidy instead of the dividend income tax. Define  $\tau^s$  the entry subsidy such that the net entry cost is  $(1 - \tau_t^s) f_t$ . It can be shown that optimal subsidies depend on the form of the entry cost as follows

$$(i) \text{ Form 1: } \tau^s = 1 - \frac{1 - \frac{1}{\mu}}{\epsilon}$$

$$(ii) \text{ Form 2: } \tau^s = 1 - \frac{\mu - 1}{\epsilon}$$

The Ramsey Planner will resort to an entry tax in the case of excessive entry or to a subsidy in the case of inefficiently low entry.

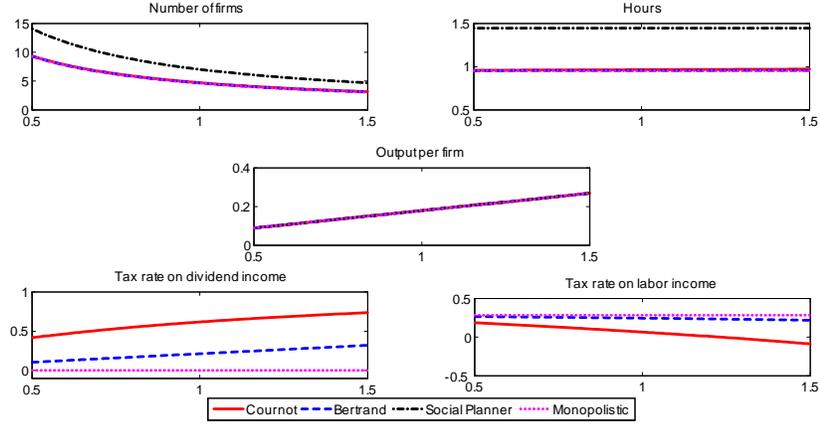


Figure 2: Ramsey Steady State under Entry Costs in Form 2. Entry cost ( $\eta$ ) on the horizontal axis.

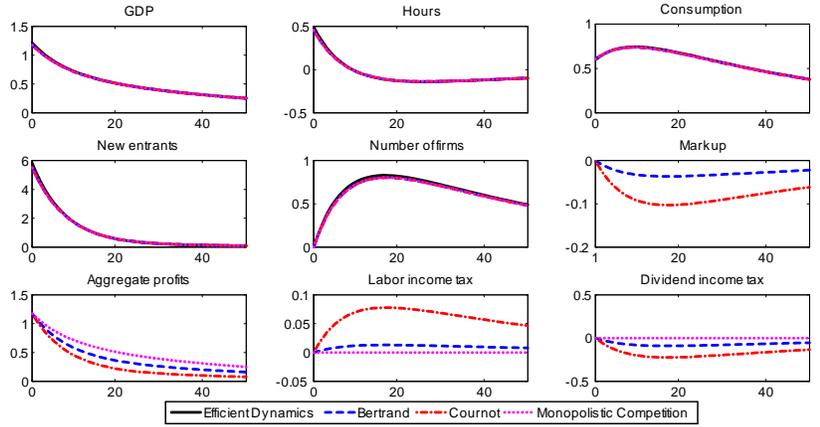


Figure 3: Entry Costs in form 1. Response of the main macroeconomic variables to a one standard deviation shock to technology. Solid lines refer to the social planner allocation, dashed and dash-dotted lines refer to Ramsey dynamics under Bertrand and the Cournot respectively and dotted lines to the case of Ramsey dynamics under monopolistic competition.

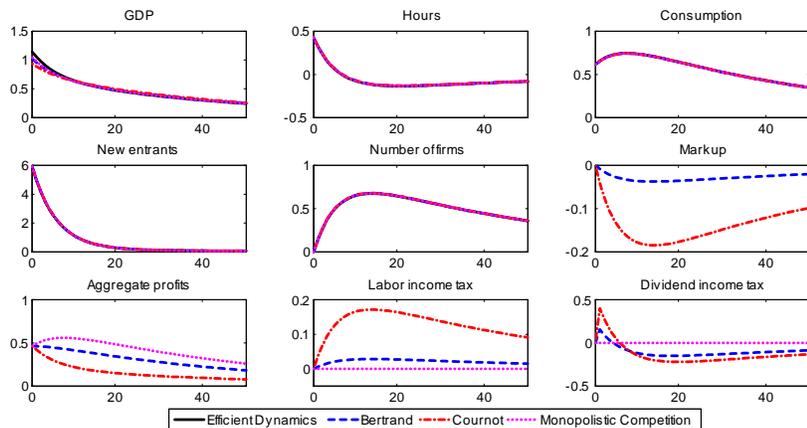


Figure 4: Entry Costs in form 2. Response of the main macroeconomic variables to a one standard deviation shock to technology. Solid lines refer to the social planner allocation, dashed and dash-dotted lines refer to Ramsey dynamics under Bertrand and the Cournot respectively and dotted lines to the case of Ramsey dynamics under monopolistic competition.

efficient equilibrium. Under all market structures the dynamics of output, consumption, hours and the number of firms are very close to the efficient ones. In the non stochastic economy we showed that a necessary condition for efficiency is the costancy of the price markup. The following arguments suggests that a similar result applied to the stochastic case. Under monopolistic competition the markup is constant along the business cycle. As can be seen from Figure 3, this results in constant tax rates. This is not the case under Oligopolistic competition. Under Bertrand and Cournot competition the entry of new firms leads to higher competition which, in turn, leads to a countercyclical price markup. The price markup variability entails a deviation from optimal dynamics which is offset by the Ramsey Planner adjusting the tax rates. In particular, we observe an increase in the labor income tax coupled with a decrease in the dividend income tax. Changes in the tax rates are mild, but stronger under Cournot competition where the markup is characterize by a higher elasticity to the number of firms with respect to Bertrand. The Ramsey policy under entry costs in Form 1 is, thus, characterized by a countercyclical labor income tax rate and by a procyclical dividend income tax.

Figure 4 displays the percentage changes in response to a technology shock under entry costs in form 2. Lines have the same meaning as in the Figure 3. Previous considerations extend to this case. A relevant difference is the impact increase in the dividend income tax, which is reverted after few periods. Notice that similar dynamics, although quantitatively less sizeable, can be observed in

	Y	C	H	N	N <sup>e</sup>	$\tau^h$	$\tau^d$
Monopolistic competition							
$\bar{x}$	1.58	1.01	1.02	8.83	0.22	0.32	-0.2
$\sigma(x)$	1.79	0.88	1.15	0.61	7.49	0	0
$Cor(Y_t, x_t)$	1	0.60	0.87	0.4	0.95	0	0
Bertrand Competition							
$\bar{x}$	1.58	1.01	1.02	8.83	0.22	0.31	-0.08
$\sigma(x)$	1.79	0.88	1.15	0.61	7.49	0.02	0.07
$Cor(Y_t, x_t)$	1	0.60	0.87	0.4	0.95	0.11	-0.4
Cournot Competition							
$\bar{x}$	1.58	1.01	1.02	8.83	0.22	0.24	0.23
$\sigma(x)$	1.79	0.88	1.15	0.61	7.49	0.06	0.17
$Cor(Y_t, x_t)$	1	0.60	0.87	0.4	0.95	0.12	-0.4

Table 1: Mean, Standard deviations and correlations with output of main-macro variables under alternative market structures. Entry Costs in Form 1

the case of a government spending shock.<sup>14</sup>

Next the variability of the main macroeconomic variables in response to the technology and government spending shocks is analyzed. Table 1 displays the mean, the coefficient of variation and the correlation with output of a number of variables of interest under the Ramsey dividend and labor income taxation policy, under entry costs in form 1. For tax rates it is reported the standard deviation in percentage points. Table 2 reports the same statistics for the case of entry costs in form 2.

Under entry cost in form 1 the variability of the main macroeconomic variables under the optimal policy is identical across market structures. However, this is reached by means of a different fiscal policy. As expected from Figure 3, while under monopolistic competition taxes are constant this is not the case under oligopolistic competition. Tax rates are more volatile under Cournot competition with respect to Bertrand, with the dividend income tax more variable than the labor income tax rate. Recall that the elasticity of the price markup to the number of firms is higher under Cournot. As a result minimizing the welfare cost of the distortions over the business cycle requires more variable taxes when firms compete in quantities.

Under entry costs in form 2 allocations and volatilities are no longer identical across market structures. Interestingly, while the overall variability characterizing the economy in response to shocks, as measured by the standard deviation of output, is higher under entry cost in form 1, the variability of tax rates is higher under entry costs in form 2. In particular the standard deviation of the dividend income tax under Cournot competition is sizeable.

<sup>14</sup>Notice, however, that aggregate consumption drops in response to a Government spending shock under all the market structures considered.

	Y	C	H	N	N <sup>e</sup>	$\tau^h$	$\tau^d$
Monopolistic competition							
$\bar{x}$	1.27	0.85	0.95	4.64	0.11	0.28	0
$\sigma(x)/\bar{x}$	1.60	0.91	1.10	0.61	7.90	0	0
$Cor(Y_t, x_t)$	1	0.60	0.85	0.50	0.93	0	0
Bertrand Competition							
$\bar{x}$	1.27	0.86	0.96	4.66	0.12	0.24	0.21
$\sigma(x)/\bar{x}$	1.57	0.91	1.09	0.61	7.90	0.02	0.26
$Cor(Y_t, x_t)$	1	0.60	0.85	0.50	0.93	0.21	0.60
Cournot Competition							
$\bar{x}$	1.25	0.87	0.96	4.68	0.12	0.07	0.62
$\sigma(x)$	1.46	0.91	1.08	0.61	7.91	0.15	0.59
$Cor(Y_t, x_t)$	1	0.60	0.85	0.53	0.91	0.24	0.72

Table 2: Mean, Standard deviations and correlations with output of main-macro variables under alternative market structures. Entry costs in form 2

## 7 Conclusions

This paper proposes an economy where the degree of market power, as measured by the price markup, depends endogenously on the form of competition, on the degree of substitutability between goods and on the number of firms. Imperfect competition leads to distortions in both the goods and the labor market and in both the short and the long run. The optimal long run dividend income corrects for inefficient entry, and it is higher in market structures characterized by lower competition. In particular it is higher under Cournot Competition with respect to Bertrand or monopolistic competition. Whereas optimal taxes over the business cycle are constant under monopolistic competition, this is not the case in an oligopolistic market structure. Also the effect of alternative forms of sunk entry costs for the design of optimal taxation has been considered.

The resulting framework features as special cases two models in the entry literature which also focus on optimal taxation problems in the case of an endogenous dynamics of the number of firms. Coto Martinez et al (2007) consider an environment characterized by monopolistic competition under constant sunk entry costs. Chugh and Ghironi (2011) consider a framework with monopolistic competition and sunk entry cost in terms of labor. By neglecting strategic interactions and considering the appropriate form of the entry costs our model reduces to either one of these models. For this reason it can be regarded as a general framework where to study optimal taxation problems under various form of imperfect competition.

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## Appendix

### Appendix A. Steady State of the Market Equilibrium

#### Entry Costs in Form 1

The steady state number of entrants is

$$N^e = \frac{\delta}{(1-\delta)}N$$

The Euler equation for shares implies

$$V = \frac{\beta(1-\delta)(1-\tau^d)}{[1-\beta(1-\delta)]}\pi$$

Steady state profits are given by<sup>15</sup>

$$\pi = \rho y - wh = \rho \left(1 - \frac{1}{\mu}\right) y = \rho \left(1 - \frac{1}{\mu}\right) \frac{Y}{N\rho} = \left(1 - \frac{1}{\mu}\right) \frac{Y}{N},$$

hence the share of profits over output reads as

$$\frac{\pi N}{Y} = 1 - \frac{1}{\mu}$$

and the value of firms over output is

$$\frac{NV}{Y} = \frac{\beta(1-\delta)(1-\tau^d)}{[1-\beta(1-\delta)]} \left(1 - \frac{1}{\mu}\right)$$

Investment over output is

$$\frac{VN^e}{Y} = \frac{V}{Y} \frac{\delta}{(1-\delta)} N = \frac{NV}{Y} \frac{\delta}{(1-\delta)} = \delta \frac{\beta(1-\tau^d) \left(1 - \frac{1}{\mu}\right)}{[1-\beta(1-\delta)]}$$

The share of consumption over output is

$$\frac{C}{Y} = 1 - g_y - \frac{N^e \psi}{Y}$$

Finally to get the ratio of labor income over output recall that

$$\frac{wH}{Y} = 1 - \frac{\pi N}{Y}$$

In order to fix  $v$  we assume that  $H=1$ .<sup>16</sup> In this case

$$v = (1 - \tau^h) \frac{\frac{w}{Y}}{\frac{C}{Y}}$$

<sup>15</sup>Notice that this is the main difference wrt to cost 2 since in that case profits depend on  $Y^c$ .

<sup>16</sup>As mentioned in the section on calibration I fix  $H=1$  under monopolistic competition and obtain the corresponding value of  $v$ . Under oligopolistic competition I consider the value of  $v$  so obtained and compute the corresponding value of  $H$ .

where both ratios are known. To compute the number of firms notice that

$$V = \frac{\beta(1-\delta)(1-\tau^d)}{[1-\beta(1-\delta)]} \pi$$

Imposing the entry condition and substituting for individual profits

$$N = \frac{\beta(1-\delta)(1-\tau^d) \left(1 - \frac{1}{\mu}\right) \rho}{(1-\tau^s)[1-\beta(1-\delta)] \psi} AH$$

the solution to this equation delivers the number of firms at the steady state. This allows to compute all the other variables. For a given  $H$  the number of firms at the steady state is larger the higher the markup, hence is larger under oligopolistic competition.

### Entry Costs in Form 2

In this case the steady state level of individual profits is

$$\pi = \rho y - wh = \left(\rho - \frac{w}{A}\right) y = \rho \left(1 - \frac{1}{\mu}\right) y = \left(1 - \frac{1}{\mu}\right) \frac{(C+G)}{N} = \left(1 - \frac{1}{\mu}\right) \frac{Y^c}{N}$$

As a result

$$\frac{\pi N}{Y^c} = \left(1 - \frac{1}{\mu}\right)$$

To obtain the share of investment over consumption output notice that

$$V = \frac{\beta(1-\delta)(1-\tau^d)}{[1-\beta(1-\delta)]} \left(1 - \frac{1}{\mu}\right) \frac{Y^c}{N}$$

and

$$\begin{aligned} \frac{VN^e}{Y^c} &= \frac{\beta(1-\delta)(1-\tau^d)}{[1-\beta(1-\delta)]} \left(1 - \frac{1}{\mu}\right) \frac{N^e}{N} = (1-\tau^d) \frac{\delta\beta}{[1-\beta(1-\delta)]} \left(1 - \frac{1}{\mu}\right) \\ &= (1-\tau^d) \frac{\delta(\mu-1)}{\mu(r+\delta)} \end{aligned}$$

To compute shares over aggregate output recall that

$$1 = \frac{Y^c}{Y} + \frac{N^e V Y^c}{Y} = \frac{Y^c}{Y} \left[ 1 + \delta \frac{\beta(1-\tau^d)}{[1-\beta(1-\delta)]} \left(1 - \frac{1}{\mu}\right) \right]$$

thus

$$\frac{Y^c}{Y} = \left[ 1 + (1-\tau^d) \delta \frac{\beta}{[1-\beta(1-\delta)]} \left(1 - \frac{1}{\mu}\right) \right]^{-1}$$

which implies that the share of private consumption over output is

$$\frac{C}{Y} = \frac{Y^c}{Y} - g_y$$

Since  $\frac{WH}{Y} + \frac{\pi}{Y} = 1$  we can compute the ratio between labor income and GDP as

$$\frac{WH}{Y} = 1 - \frac{\pi N Y^c}{Y}$$

Given H, the latter leads to

$$v = \frac{(1 - \tau^h) wH}{CH^{1+\frac{1}{\varphi}}} = \frac{(1 - \tau^h) \frac{wH}{Y}}{\frac{C}{Y} H^{1+\frac{1}{\varphi}}}$$

Labor market equilibrium requires

$$\begin{aligned} H &= H_t^C + H_t^E = Nh + \frac{\eta}{A} N^e = N \frac{y}{A} + \frac{\eta}{A} N^e \\ &= \frac{Y^c}{\rho A} + \frac{\eta}{A} N^e \end{aligned}$$

thus

$$N^e = \frac{AH}{\eta} - \frac{Y^c}{\rho\eta}$$

and

$$N = (1 - \delta) \left( N + \frac{AH}{\eta} - \frac{Y^c}{\eta\rho} \right)$$

or

$$N = \frac{(1 - \delta) AH}{\delta \eta} - \frac{(1 - \delta) Y^c}{\delta \eta\rho}$$

Next consider

$$V = \frac{\beta(1 - \delta)(1 - \tau^d)}{[1 - \beta(1 - \delta)]} \left( 1 - \frac{1}{\mu} \right) \frac{Y^c}{N}$$

substituting for the entry condition delivers

$$(1 - \tau^s) \frac{\eta}{A} w = \frac{\beta(1 - \delta)(1 - \tau^d)}{[1 - \beta(1 - \delta)]} \left( 1 - \frac{1}{\mu} \right) \frac{Y^c}{N}$$

or

$$(1 - \tau^s) \eta \frac{\rho}{\mu} = \frac{\beta(1 - \delta)(1 - \tau^d)}{[1 - \beta(1 - \delta)]} \left( 1 - \frac{1}{\mu} \right) \frac{Y^c}{N}$$

or

$$\frac{Y^c}{\eta\rho} = \frac{(1 - \tau^s)}{\mu} \frac{[1 - \beta(1 - \delta)]}{\beta(1 - \delta)(1 - \tau^d) \left( 1 - \frac{1}{\mu} \right)} N$$

Substituting the latter into the equation of motion for the number of firms delivers an equation that can be solved for N

$$N = \frac{(1 - \delta) AH}{\delta \eta} - \frac{(1 - \tau^s)}{\delta} \frac{[1 - \beta(1 - \delta)]}{\beta(1 - \tau^d)(\mu - 1)} N$$

or

$$N = \frac{\frac{(1-\delta)}{\delta} \frac{AH}{\eta}}{\left(1 + \frac{(1-\tau^e)}{\delta} \frac{[1-\beta(1-\delta)]}{\beta(1-\tau^d)(\mu-1)}\right)}$$

As above a higher markups leads to a higher number of firms in equilibrium for any given H.

## Appendix B. Efficient Equilibrium

### Entry Costs in Form 1

The social Planner problem reads as

$$\max_{\{C_t, N_{t+1}, N_t^e, H_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - v \frac{H_t^{1+1/\varphi}}{1+1/\varphi} \right\}$$

s.t.

$$C_t + G_t + N_t^e \psi = \rho_t A_t H_t$$

and

$$N_{t+1} = (1 - \delta) (N_t + N_t^e)$$

We attach the Lagrange Multiplier  $\lambda_t$  to the first constraint and the multiplier  $\sigma_t$  to the second one. First order conditions are as follows

$$C_t : \frac{1}{C_t} = \lambda_t$$

$$N_{t+1} : \sigma_t = \beta E_t \lambda_{t+1} \rho_{N,t+1} A_t H_t + \beta E_t \sigma_{t+1} (1 - \delta)$$

$$N_t^e : \lambda_t \psi = (1 - \delta) \sigma_t$$

$$H_t : v H_t^{1/\varphi} = \lambda_t \rho_t A_t$$

Combining the first and the third condition delivers

$$\frac{1}{C_t} \frac{\psi}{(1 - \delta)} = \sigma_t$$

Substituting the latter into the third condition we obtain

$$\psi = (1 - \delta) \beta E_t \frac{C_t}{C_{t+1}} (\rho_{N,t+1} A_t H_t + \psi)$$

which can be written as

$$\psi = (1 - \delta) \beta E_t \frac{C_t}{C_{t+1}} \left( \epsilon \frac{Y_t}{N_t} + \psi \right)$$

finally the FOC with respect to hours can be written as

$$v H_t^{1/\varphi} C_t = \rho_t A_t$$

To obtain the steady state we have the following equations

$$\begin{aligned} C + G + N^e \psi &= \rho AH \\ N &= (1 - \delta)(N + N^e) \\ \psi &= (1 - \delta)\beta \left( \epsilon \frac{Y}{N} + \psi \right) \\ vH^{1/\varphi} C &= \rho A \end{aligned}$$

Consider the resource constraint

$$\frac{C}{Y} = 1 - g_y - \frac{N^e \psi}{Y} = 1 - g_y - \frac{\psi}{Y} \frac{\delta}{1 - \delta} N$$

or

$$\frac{C}{Y} = 1 - g_y - \psi \frac{\delta}{1 - \delta} \frac{N}{Y}$$

From the third equation

$$\frac{Y}{N} = \frac{(1 - (1 - \delta)\beta)\psi}{(1 - \delta)\beta\epsilon}$$

Combining we obtain

$$\frac{C}{Y} = 1 - g_y - \frac{\delta}{1 - \delta} \frac{(1 - \delta)\beta\epsilon}{(1 - (1 - \delta)\beta)}$$

Next consider equation

$$H^{1+1/\varphi} = \frac{1}{vC} \rho AH = \frac{Y}{vC}$$

hence we have H as

$$H = \left( \frac{Y}{vC} \right)^{\frac{1}{1+1/\varphi}}$$

Next we want to compute N. Notice that

$$\frac{N}{\rho} = \frac{(1 - \delta)\beta\epsilon}{(1 - (1 - \delta)\beta)} AH$$

given  $\rho = N^{\frac{1}{\theta-1}}$  it follows

$$N^{\frac{\theta-2}{\theta-1}} = \frac{(1 - \delta)\beta\epsilon}{(1 - (1 - \delta)\beta)} AH$$

or

$$N = \left[ \frac{(1 - \delta)\beta\epsilon}{(1 - (1 - \delta)\beta)} AH \right]^{\frac{\theta-1}{\theta-2}}$$

## Entry Costs in form 2

The Social Planner problem can be written as follows

$$\max_{\{C_t, N_{t+1}, N_t^e, H_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - v \frac{H_t^{1+1/\varphi}}{1+1/\varphi} \right\}$$

s.t.

$$C_t + G_t + N_t^e \eta \rho_t = \rho_t A_t H_t$$

and

$$N_{t+1} = (1 - \delta)(N_t + N_t^e)$$

We attach the Lagrange Multiplier  $\lambda_t$  to the first constraint and the multiplier  $\sigma_t$  to the second one. First order conditions are as follows

$$C_t : \frac{1}{C_t} = \lambda_t$$

$$N_{t+1} : \sigma_t = \beta E_t \lambda_{t+1} \rho_{N,t+1} (A_t H_t - N_t^e \eta) + \beta E_t \sigma_{t+1} (1 - \delta)$$

$$N_t^e : \lambda_t \eta \rho_t = (1 - \delta) \sigma_t$$

$$H_t : v H_t^{1/\varphi} = \lambda_t \rho_t A_t$$

Substituting the first condition into the third delivers

$$\frac{1}{(1 - \delta) C_t} \eta \rho_t = \sigma_t$$

Substituting the latter and the definition of  $\lambda_t$  into the other equations we are left with

$$\eta \rho_t = (1 - \delta) \beta E_t \frac{C_t}{C_{t+1}} \left[ \rho_{N,t+1} (A_t H_t - N_t^e \eta) + \eta \rho_{t+1} \right] \quad (35)$$

and

$$v H_t^{1/\varphi} C_t = \rho_t A_t$$

Since  $A_t H_t - N_t^e \eta = \frac{C_t + G_t}{\rho_t}$ , equation (35) can be rewritten as

$$\eta \rho_t = (1 - \delta) \beta E_t \frac{C_t}{C_{t+1}} \left[ \rho_{N,t+1} \frac{C_t + G_t}{\rho_t} + \eta \rho_{t+1} \right]$$

or

$$\eta \rho_t = (1 - \delta) \beta E_t \frac{C_t}{C_{t+1}} \left[ \epsilon \frac{C_{t+1} + G_{t+1}}{N_{t+1}} + \eta \rho_{t+1} \right]$$

To find the steady state we can consider the following equations

$$\eta \rho = (1 - \delta) \beta \left[ \epsilon \frac{Y^c}{N} + \eta \rho \right]$$

$$\begin{aligned}
vH^{1/\varphi}C &= \rho A \\
Y^c + N^e\eta\rho &= \rho AH \\
N &= (1 - \delta)(N + N^e)
\end{aligned}$$

From the first one

$$\frac{Y^c}{\eta\rho} = \frac{(1 - (1 - \delta)\beta)}{(1 - \delta)\beta\epsilon} N$$

The aggregate resource constraint implies

$$\frac{Y^c}{\eta\rho} = \frac{AH}{\eta} - \frac{\delta}{1 - \delta} N$$

Combining

$$N = \frac{\frac{AH}{\eta}}{\frac{(1 - (1 - \delta)\beta)}{(1 - \delta)\beta\epsilon} + \frac{\delta}{1 - \delta}}$$

we get N as a function of H. Notice that we have repeatedly used the steady state version of the equation of motion for the number of firms. Consider again the aggregate resource constraint

$$C + G + N^e\eta\rho = \rho AH$$

or

$$\frac{C}{\rho AH} = 1 - g_y - \frac{N^e\eta\rho}{\rho AH} = 1 - g_y - \eta\rho \frac{\delta}{1 - \delta} \frac{N}{\rho AH}$$

then

$$vH^{1/\varphi} \frac{C}{\rho AH} = \frac{\rho A}{\rho AH}$$

delivers H implicitly as a function of N

$$vH^{1/\varphi} \left( 1 - g_y - \eta\rho \frac{\delta}{1 - \delta} \frac{N}{\rho AH} \right) = \frac{1}{H}$$

The latter is equivalent to

$$vH^{1+1/\varphi} \left( 1 - g_y - \eta\rho \frac{\delta}{1 - \delta} \frac{N}{\rho AH} \right) = 1$$

Next substitute for N as a function of H in the round bracket and

$$H = \left\{ v \left[ 1 - g_y - \frac{\delta}{1 - \delta} \left( \frac{(1 - (1 - \delta)\beta)}{(1 - \delta)\beta\epsilon} + \frac{\delta}{1 - \delta} \right)^{-1} \right] \right\}^{-\frac{\varphi}{1 + \varphi}}$$

## Appendix C. The Implementability Constraint

This Appendix follows closely Arsenau and Chugh (2012) and Chugh and Ghironi (2012). Consider the household flow budget constraint (in the symmetric equilibrium)

$$\sum_j \frac{1}{R_t^j} B_{t+1}^j + V_t(N_t + N_t^e)x_{t+1} + C_t = (1 - \tau_t^l) w_t H_t + B_t + [(1 - \tau_t^d) d_t + V_t] N_t x_t$$

Multiply both sides by  $\beta^t u_c(c_t)$

$$\begin{aligned} & \sum_j \beta^t u_c(c_t) \frac{1}{R_t^j} B_{t+1}^j + \beta^t u_c(c_t) V_t(N_t + N_t^e)x_{t+1} + \beta^t u_c(c_t) C_t \\ = & \beta^t u_c(c_t) (1 - \tau_t^l) w_t H_t + \beta^t u_c(c_t) b_t + \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t x_t \end{aligned}$$

and sum over dates starting from  $t=0$ , where all term are understood as in expectation as of time 0

$$\begin{aligned} & \sum_{t=0}^{\infty} \sum_j \beta^t u_c(c_t) \frac{1}{R_t^j} B_{t+1}^j + \sum_{t=0}^{\infty} \beta^t u_c(c_t) V_t(N_t + N_t^e)x_{t+1} + \sum_{t=0}^{\infty} \beta^t u_c(c_t) C_t \\ = & \sum_{t=0}^{\infty} \beta^t u_c(c_t) (1 - \tau_t^h) w_t H_t + \sum_{t=0}^{\infty} \beta^t u_c(c_t) b_t + \sum_{t=0}^{\infty} \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t x_t \end{aligned}$$

The euler equation for bonds implies  $u_c(C_t) = \beta R_t^j u_c(C_{t+1}^j)$ , using this in the first term on the LHS

$$\begin{aligned} & \sum_{t=0}^{\infty} \sum_j \beta^{t+1} u_c(C_{t+1}^j) B_{t+1}^j + \sum_{t=0}^{\infty} \beta^t u_c(C_t) V_t(N_t + N_t^e)x_{t+1} + \sum_{t=0}^{\infty} \beta^t u_c(C_t) C_t \\ = & \sum_{t=0}^{\infty} \beta^t u_c(c_t) (1 - \tau_t^h) w_t H_t + \sum_{t=0}^{\infty} \beta^t u_c(C_t) b_t + \sum_{t=0}^{\infty} \beta^t u_c(C_t) [(1 - \tau_t^d) d_t + V_t] N_t x_t \end{aligned}$$

Notice that the term  $\sum_j u_c(C_{t+1}^j) B_{t+1}^j$  can be understood as the payoff of a risk free bond. As such we can cancel out the first summation on the LHS with the respective terms in the second summation in the RHS, leaving just time 0 terms

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t u_c(c_t) V_t(N_t + N_t^e) + \sum_{t=0}^{\infty} \beta^t u_c(c_t) C_t \\ = & \sum_{t=0}^{\infty} \beta^t u_c(c_t) (1 - \tau_t^h) w_t H_t + u_c(c_0) b_0 + \sum_{t=0}^{\infty} \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t \end{aligned}$$

Notice that the clearing of the asset market implies  $x_t = 1$  at all  $t$ . Considering

that  $\frac{u_h(h_t)}{u_c(c_t)} = -(1 - \tau_t^h) w_t$  leads to

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t u_c(c_t) V_t (N_t + N_t^e) + \sum_{t=0}^{\infty} \beta^t u_c(c_t) C_t + \sum_{t=0}^{\infty} \beta^t u_h(h_t) H_t \\ &= u_c(c_0) b_0 + \sum_{t=0}^{\infty} \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t \end{aligned}$$

Next consider

$$u_c(C_t) V_t = E_t \beta (1 - \delta) u_c(C_{t+1}) [(1 - \tau_{t+1}^d) d_{t+1} + V_{t+1}]$$

and plug it into the first summation in the LHS

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^{t+1} (1 - \delta) u_c(C_{t+1}) [(1 - \tau_{t+1}^d) d_{t+1} + V_{t+1}] (N_t + N_t^e) x_{t+1} + \\ & + \sum_{t=0}^{\infty} \beta^t u_c(c_t) C_t + \sum_{t=0}^{\infty} \beta^t u_h(h_t) h_t \\ &= u_c(c_0) b_0 + \sum_{t=0}^{\infty} \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t x_t \end{aligned}$$

considering that

$$(N_t + N_t^e) = \frac{N_{t+1}}{1 - \delta}$$

it follows

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^{t+1} u_c(C_{t+1}) [(1 - \tau_{t+1}^d) d_{t+1} + V_{t+1}] N_{t+1} + \\ & + \sum_{t=0}^{\infty} \beta^t u_c(c_t) C_t + \sum_{t=0}^{\infty} \beta^t u_h(h_t) h_t \\ &= u_c(c_0) b_0 + \sum_{t=0}^{\infty} \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t \end{aligned}$$

Simplifying the first summation on the LHS with the second in the RHS delivers the implementability constraint

$$E_0 \sum_{t=0}^{\infty} \beta^t [u_c(C_t) C_t + u_h(H_t) H_t] = u_c(C_0) B_0 + u_c(C_0) [(1 - \tau_0^d) d_0 + V_0] N_0$$

where we reintroduced the expectation operator.

## Appendix D. The Ramsey Problem

**Entry Costs in form 1. Includes proof of result (i) in Proposition 1.**

The Ramsey problem reads as

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, H_t)$$

subject to

$$N_{t+1} = (1 - \delta)(N_t + N_t^e) : \lambda_{1t}$$

$$C_t + G_t + N_t^e \psi = \rho_t A_t H_t : \lambda_{2t}$$

$$\psi u_{ct} = \beta(1 - \delta) E_t u_{ct+1} \left( \left(1 - \tau_{t+1}^d/t\right) \left(1 - \frac{1}{\mu}\right) \frac{C_{t+1} + G_{t+1}}{N_{t+1}} + \psi \right) : \lambda_{3t}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t E_0 [u_{ct} C_t + u_{ht} H_t] = u_{c0} B_0 + u_{c0} [(1 - \tau_0^d) d_0 + V_0] N_0 : \xi$$

Where  $\lambda_{it}$  define the Lagrange multipliers respectively attached to each constraint and  $\xi$  is the (constant) lagrange multiplier attached to the implementability constraint.

The choice variables are  $C_t$ ,  $N_t$ ,  $H_t$ ,  $N_t^e$ , and either  $\tau_{t/t+1}^d$  or  $\tau_t^s$ . Following Ljungqvist and Sargent (2004) I define

$$V(C_t, H_t, \xi) = u(C_t, H_t) + \xi (u_{ct} C_t + u_{ht} H_t)$$

and

$$\Omega = u_{c0} B_0 + u_{c0} [(1 - \tau_0^d) d_0 + V_0] N_0$$

As a result the Lagrangian function can be written as

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} V(C_t, H_t, \xi) + \lambda_{1t} [(1 - \delta)(N_t + N_t^e) - N_{t+1}] \\ + \lambda_{2t} (\rho_t A_t H_t - C_t - G_t - N_t^e \psi) + \\ \lambda_{3t} \left[ \psi u_{ct} - \beta(1 - \delta) E_t u_{ct+1} \left( \left(1 - \tau_{t+1}^d/t\right) \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{\rho_{t+1} A_{t+1} H_{t+1}}{N_{t+1}} + \psi \right) \right] \end{array} \right\} - \xi \Omega$$

The first order conditions for periods  $t \geq 1$  are

$$C_t : V_c(C_t, H_t, \xi) - \lambda_{2t} + \lambda_{3t} u_{cct} \psi - \lambda_{3t-1} (1 - \delta) u_{cct} \left( \left(1 - \tau_{t/t-1}^d\right) \left(1 - \frac{1}{\mu_t}\right) \frac{\rho_t A_t H_t}{N_t} + \psi \right) = 0$$

$$\begin{aligned} N_{t+1} & : \lambda_{1t} + \beta(1 - \delta) \lambda_{3t} \frac{E_t u_{ct+1} \left(1 - \tau_{t+1}^d/t\right) A_{t+1} H_{t+1}}{N_{t+1}} \left[ \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{\rho_{N_{t+1}} N_{t+1} - \rho_{t+1}}{N_{t+1}} \right. \\ & \quad \left. + \frac{\mu_{N_{t+1}} \rho_{t+1}}{\mu_{t+1}^2} \right] \\ & = \beta(1 - \delta) E_t \lambda_{1t+1} + \beta E_t \lambda_{2t+1} \rho_{N_{t+1}} A_{t+1} H_{t+1} \end{aligned}$$

$$\tau_{t+1/t}^d : (1 - \delta)\beta^{t+1}\lambda_{3t}E_0u_{ct+1} \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{\rho_{t+1}A_{t+1}H_{t+1}}{N_{t+1}} = 0$$

$$N_t^e : \lambda_{1t}(1 - \delta) = \lambda_{2t}\psi$$

$$H_t : V_h(C_t, H_t, \xi) + \lambda_{2t}\rho_t A_t - \lambda_{3t-1}(1 - \delta)u_{ct} \left(1 - \tau_{t/t-1}^d\right) \left(1 - \frac{1}{\mu_t}\right) \frac{\rho_t A_t}{N_t} = 0$$

Since

$$V(C_t, H_t, \xi) = u(C_t, H_t) + \xi(u_{ct}C_t + u_{ht}H_t)$$

it follows

$$V_c(C_t, H_t, \xi) = u_c(C, H) + \xi u_{cct}C_t + \xi u_{ct} = \frac{1}{C_t} - \xi \frac{1}{C_t^2}C_t + \xi \frac{1}{C_t} = \frac{1}{C_t}$$

and

$$V_h(C_t, H_t, \xi) = -vH_t^{1/\varphi} \left[ \xi \left( \frac{1 + \varphi}{\varphi} \right) + 1 \right]$$

Consider now the steady state. The FOC with respect to  $\tau_{t+1/t}^d$  reads as

$$\tau_{t+1/t}^d : (1 - \delta)\beta^{t+1}\lambda_3 \left(1 - \frac{1}{\mu}\right) \frac{C + G}{N} u_c = 0$$

The latter implies that at the steady state  $\lambda_3$  is equal to zero. In the Ramsey steady state the firms entry condition does not restrict the allocation. As a result we can write the steady state version of the FOCs as

$$C_t : \lambda_2 = u_c$$

$$H_t : V_h(C, H, \xi) + u_{ct}\rho A = 0$$

$$N_t^e : \lambda_1 = \frac{\psi}{(1 - \delta)} u_c$$

$$N_{t+1} : \beta\rho_N AH = [1 - \beta(1 - \delta)] \frac{\psi}{(1 - \delta)}$$

The FOC with respect to N can be written as

$$\psi = \beta(1 - \delta) [\rho_N AH + \psi]$$

Since

$$\rho_N = \frac{1}{\theta - 1} \frac{\rho}{N} = \epsilon \frac{\rho}{N}$$

it follows

$$\psi = \beta(1 - \delta) \left[ \epsilon \frac{\rho AH}{N} + \psi \right]$$

The euler equation for asset implies that

$$\psi = \beta(1 - \delta) \left( (1 - \tau^d) \left(1 - \frac{1}{\mu}\right) \frac{\rho AH}{N} + \psi \right)$$

For the latter two equations to be consistent with each other it has to be the case that

$$\epsilon = (1 - \tau^d) \left(1 - \frac{1}{\mu}\right)$$

or

$$\tau^d = 1 - \frac{\epsilon}{\left(1 - \frac{1}{\mu}\right)}$$

which proves point (i) in proposition 1. Importantly, the dividend income tax differs from zero under monopolistic competition. Substituting the optimal dividend income tax into the Euler equation for shares we get

$$\psi = \beta(1 - \delta) \left(\epsilon \frac{Y}{N} + \psi\right)$$

Which implies

$$\frac{Y}{N} = \frac{[1 - \beta(1 - \delta)]}{\beta(1 - \delta)\epsilon} \psi$$

Notice that

$$\frac{C}{Y} = 1 - g_y - \frac{\delta}{1 - \delta} \psi \frac{N}{Y}$$

Next consider the implementability constraint, which can be written as

$$\Omega = \frac{Y}{C} \left( \frac{B}{Y} + \frac{\epsilon}{\left(1 - \frac{1}{\mu}\right)} \frac{\Pi}{Y} + \frac{VN}{Y} \right)$$

where  $\frac{B}{Y}$  is exogenously given and  $\frac{C}{Y}$  has been computed above. The Euler equation with respect to assets implies

$$\frac{VN}{Y} = \frac{\beta(1 - \delta)}{[1 - \beta(1 - \delta)]} \epsilon$$

and also we know

$$\begin{aligned} \frac{\Pi}{Y} &= \left(1 - \frac{1}{\mu}\right) \\ \Omega &= \frac{Y}{C} \left( \frac{B}{Y} + \epsilon + \frac{VN}{Y} \right) \\ H &= \left[ \frac{1 - (1 - \beta)\Omega}{v} \right]^{\frac{\varphi}{1+\varphi}} \end{aligned}$$

Finally given H and recalling that

$$\frac{Y}{N} = \frac{[1 - \beta(1 - \delta)](\theta - 1)}{\beta(1 - \delta)} \psi$$

it follows

$$N = \left[ \frac{\beta(1 - \delta)\epsilon}{[1 - \beta(1 - \delta)]\psi} AH \right]^{\frac{\theta-1}{\theta-2}}$$

**Entry Costs in Form 2. Includes proof of result (ii) in Proposition 2.**

In this case the Lagrangian is

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} V(C_t, H_t, \xi) + \lambda_{1t} [(1 - \delta)(N_t + N_t^e) - N_{t+1}] \\ + \lambda_{2t} (\rho_t A_t H_t - C_t - G_t - N_t^e \eta \rho_t) + \\ \eta \frac{\rho_t}{\mu_t} u_{ct} + \\ \lambda_{3t} \left[ -\beta(1 - \delta) u_{ct+1} \left( \left(1 - \tau_{t+1}^d\right) \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{C_{t+1} + G_{t+1}}{N_{t+1}} + \eta \frac{\rho_{t+1}}{\mu_{t+1}} \right) \right] \end{array} \right\} - \xi \Omega$$

As above, the choice variables are  $C_t$ ,  $N_t$ ,  $H_t$ ,  $N_t^e$ , and either  $\tau_{t/t+1}^d$  or  $\tau_t^s$ . The first order conditions are

$$\begin{aligned} C_t &: V_c(C_t, H_t, \xi) + \lambda_{3t} \eta \frac{\rho_t}{\mu_t} u_{cct} \\ &\quad - \lambda_{3t-1} \left[ (1 - \delta) u_{cct} \left( \left(1 - \tau_{t/t-1}^d\right) \left(1 - \frac{1}{\mu_t}\right) \frac{C_t + G_t}{N_t} + \eta \frac{\rho_t}{\mu_t} \right) + \right. \\ &\quad \left. + (1 - \delta) u_{ct} \left(1 - \tau_{t/t-1}^d\right) \left(1 - \frac{1}{\mu_t}\right) \frac{1}{N_t} \right] \\ &= \lambda_{2t} \end{aligned}$$

$$\begin{aligned} N_{t+1} &: \lambda_{1t} + \\ &\quad \beta(1 - \delta) \lambda_{3t} E_t u_{ct+1} \left[ + \frac{\left(1 - \tau_{t+1}^d\right)}{\mu_{t+1}} \left( \frac{\mu_{N,t+1}}{\mu_{t+1}} - \frac{\mu_{t+1} - 1}{N_{t+1}} \right) \frac{Y_{t+1}^c}{N_{t+1}} \right] \\ &\quad + \beta \eta (1 - \delta) \lambda_{3t} E_t u_{ct+1} \left( \frac{\rho_{Nt+1} \mu_{t+1} - \mu_{Nt+1} \rho_{t+1}}{\mu_{t+1}^2} \right) \\ &= \beta(1 - \delta) E_t \lambda_{1t+1} + \beta E_t \lambda_{2t+1} \rho_{Nt+1} (A_{t+1} H_{t+1} - N_{t+1}^e \eta) + \\ &\quad + \beta \eta E_t \lambda_{3t+1} u_{ct+1} \left( \frac{\rho_{Nt+1} \mu_{t+1} - \mu_{Nt+1} \rho_{t+1}}{\mu_{t+1}^2} \right) \end{aligned}$$

$$H_t : V_h(C_t, H_t, \xi) + \lambda_{2t} \rho_t A_t = 0$$

$$N_t^e : \lambda_{1t} (1 - \delta) = \lambda_{2t} \eta \rho_t$$

$$\tau_{t+1/t}^d : (1 - \delta) \beta^{t+1} E_t u_{ct+1} \lambda_{3t} \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{C_{t+1} + G_{t+1}}{N_{t+1}} = 0$$

Notice that

$$\rho_{tN} = \frac{1}{\theta - 1} \frac{\rho_t}{N_t}$$

$$\mu_t^C = \frac{\theta N_t}{(\theta - 1)(N_t - 1)}$$

$$\mu_{Nt}^C = \frac{\theta(\theta - 1)(N_t - 1) - (\theta - 1)\theta N_t}{(\theta - 1)^2(N_t - 1)^2} = \frac{\theta(N_t - 1) - \theta N_t}{(\theta - 1)(N_t - 1)^2} = -\frac{\theta}{(\theta - 1)(N_t - 1)^2}$$

$$\mu_t^B = \frac{1 + \theta(N_t - 1)}{(\theta - 1)(N_t - 1)}$$

$$\mu_{Nt}^B = \frac{\theta(\theta - 1)(N_t - 1) - (\theta - 1)[1 + \theta(N_t - 1)]}{(\theta - 1)^2(N - 1)^2} = \frac{-1}{(\theta - 1)(N - 1)^2}$$

Given  $\lambda_3 = 0$  we can write the steady state system as above

$$C_t : V_c(C, H, \xi) = \lambda_2$$

$$N_{t+1} : \lambda_1 = \beta [(1 - \delta)\lambda_1 + \lambda_2 \rho_N (AH - N^e \eta)]$$

$$H_t : V_h(C, H, \xi) + \lambda_2 \rho A = 0$$

$$N_t^e : \lambda_1 (1 - \delta) = \lambda_2 \eta \rho$$

Since  $\lambda_1 = \frac{\eta \rho}{(1 - \delta)} V_c(C, H, \xi)$  and  $-\frac{V_h(C, H, \xi)}{V_c(C, H, \xi)} = \rho A$  and given the definitions of  $V_c$  and  $V_h$  we get

$$\frac{1}{\beta} = (1 - \delta) \left[ 1 + \epsilon \frac{1}{\eta \rho} \frac{Y^c}{N} \right]$$

Evaluating the Euler equation for assets at the steady state implies

$$\frac{1}{\beta} = (1 - \delta) \left( \frac{(1 - \tau^d)(\mu - 1) Y^c}{\eta \rho} \frac{1}{N} + 1 \right)$$

For the two to be consistent is has to be the case that

$$(1 - \tau^d)(\mu - 1) = \epsilon$$

which proves point (ii) in Proposition 1. Notice that differently from the monopolistic competition case this does not imply  $\tau^d = 0$ . The Euler equation for assets evaluated at the steady state reads as

$$\frac{1}{\beta} = (1 - \delta) \left[ 1 + \frac{\epsilon}{\rho \eta} \frac{Y^c}{N} \right]$$

then

$$\frac{Y^c}{\eta \rho} = \frac{1 - \beta(1 - \delta)}{(1 - \delta)\beta\epsilon} N$$

The aggregate resource constraint implies

$$\frac{Y^c}{\eta \rho} = \frac{AH}{\eta} - N^e$$

using the equation for the dynamics of the number of firms

$$\frac{Y^c}{\eta \rho} = \frac{AH}{\eta} - \frac{\delta}{1 - \delta} N$$

Combing the latter two equations

$$N = \frac{\frac{AH}{\eta}}{\frac{(1-(1-\delta)\beta)}{(1-\delta)\beta\epsilon} + \frac{\delta}{1-\delta}} = \frac{\frac{(1-\delta) AH}{\delta \eta}}{1 + \frac{(1-(1-\delta)\beta)}{\delta\beta\epsilon}}$$

we get  $N$  as a function of  $H$ . This also implies that we can compute the markup, under both Cournot and Bertrand, as a function of  $H$ . Recall that it has to be the case that

$$Y = wH + \Pi$$

since

$$wH = \frac{\rho}{\mu}AH; \quad \Pi = \left(1 - \frac{1}{\mu}\right)Y^c$$

it follows

$$Y = \frac{\rho}{\mu}AH + \left(1 - \frac{1}{\mu}\right)Y^c$$

using the aggregate resource constraint  $Y^c = \rho AH - \eta\rho N^e$  we obtain

$$\begin{aligned} Y &= \frac{\rho}{\mu}AH + \left(1 - \frac{1}{\mu}\right)(\rho AH - \eta\rho N^e) \\ &= \rho AH - \rho A \left(\frac{\mu-1}{\mu}\right) \frac{\eta}{A} N^e \end{aligned}$$

Also notice

$$1 = \left(1 - \frac{1}{\mu}\right) \frac{Y^c}{Y} + \frac{1}{\mu} \frac{\rho AH}{Y}$$

To compute  $\frac{Y^c}{Y}$  and  $\frac{C}{Y}$  consider the euler equation for assets

$$V = \frac{\beta(1-\delta) \frac{\epsilon}{\mu} Y^c}{1 - \beta(1-\delta) N}$$

which implies

$$\frac{VN^e}{Y^c} = \frac{\beta(1-\delta) \frac{\epsilon}{\mu} N^e}{1 - \beta(1-\delta) N} = \frac{\delta\beta \frac{\epsilon}{\mu}}{1 - \beta(1-\delta)}$$

this allows to compute  $\frac{Y^c}{Y}$  as follows

$$Y^c + N^e V = Y$$

then

$$\frac{Y^c}{Y} = 1 - \frac{N^e V}{Y^c} \frac{Y^c}{Y}$$

and finally

$$\frac{Y^c}{Y} = \left(1 + \frac{VN^e}{Y^c}\right)^{-1}$$

From the latter we get  $\frac{C}{Y}$  as

$$\frac{C}{Y} = \frac{Y^c}{Y} - \frac{G}{Y}$$

Knowing  $\frac{Y^c}{Y}$  we can determine  $\frac{\rho AH}{Y}$

$$\frac{\rho AH}{Y} = \mu \left[ 1 - \left( 1 - \frac{1}{\mu} \right) \frac{Y^c}{Y} \right]$$

The FOC for hours

$$-\frac{V_h(C, H, \xi)}{V_c(C, H, \xi)} = \rho A$$

substituting the definitions of variables

$$\frac{v H_t^{1/\varphi} \left[ \xi \left( \frac{1+\varphi}{\varphi} \right) + 1 \right]}{\frac{1}{C}} = \rho A$$

or

$$H^{1/\varphi} = \frac{\frac{1}{v} \frac{\rho A}{C}}{\left[ \xi \left( \frac{1+\varphi}{\varphi} \right) + 1 \right]}$$

Multiplying both sides by H, the latter is equivalent to

$$H = \left[ \frac{\frac{\rho AH}{Y} \frac{Y}{C}}{v \left[ \xi \left( \frac{1+\varphi}{\varphi} \right) + 1 \right]} \right]^{\frac{\varphi}{1+\varphi}}$$

Hence H is both a function of H and  $\xi$ . Next consider the implementability constraint

$$\Omega = \frac{Y}{C} \frac{B}{Y} + (1 - \tau^d) \frac{\pi N}{C} + \frac{VN}{C}$$

As a result

$$\Omega = \frac{Y}{C} \left( \frac{B}{Y} + (1 - \tau^d) \frac{\Pi}{Y} + \frac{VN}{Y} \right) = \frac{Y}{C} \left( \frac{B}{Y} + \frac{\epsilon}{(\mu - 1) Y} \frac{\Pi}{Y} + \frac{VN}{Y} \right)$$

where  $\frac{B}{Y}$  is given and  $\frac{C}{Y}$  is a function of H. Also from

$$V = \frac{\beta(1 - \delta) \frac{\epsilon}{\mu} Y^c}{1 - \beta(1 - \delta) N}$$

we get

$$\frac{VN}{Y} = \frac{\beta(1 - \delta) \frac{\epsilon}{\mu} Y^c}{1 - \beta(1 - \delta) Y}$$

and

$$\frac{\Pi}{Y} = \left( 1 - \frac{1}{\mu} \right) \frac{Y^c}{Y}$$

Hence we can compute  $\Omega$  as a function of  $H$ . Next using the steady state version of the implementability constraint we get

$$\frac{1}{1-\beta} \left[ 1 - vH^{1+1/\varphi} \right] = \Omega$$

or

$$1 - vH^{1+1/\varphi} = (1-\beta)\Omega$$

which implies

$$H = \left[ \frac{1 - (1-\beta)\Omega}{v} \right]^{\frac{\varphi}{1+\varphi}}$$

which is a function solely of  $H$  and can be solved numerically. Given the value  $H$  we can determine the lagrange multiplier  $\xi$

$$\xi = \frac{\varphi}{1+\varphi} \left[ \frac{\frac{1}{v} \frac{\rho AH Y}{Y^c} - 1}{H^{\frac{1+\varphi}{\varphi}}} \right]$$

Recall that  $N$  can be computed as

$$N = \frac{\frac{A}{\eta} \frac{1-\delta}{\delta}}{1 + \frac{(1-(1-\delta)\beta)}{\delta\beta\epsilon}} H$$

which allows to compute the price markup at the ramsey steady state. Also it implies a value for  $\rho$ . Since

$$\eta \frac{\rho}{\mu} A = \frac{\beta(1-\delta) \frac{\epsilon}{\mu} Y^c}{1-\beta(1-\delta) N}$$

we get

$$Y^c = \frac{\eta \rho A N}{\mu} \frac{1-\beta(1-\delta)}{\beta(1-\delta) \frac{\epsilon}{\mu}}$$

In particular notice that

$$\tau^d = 1 - \frac{\epsilon}{(\mu-1)}$$

and

$$\tau^h = 1 - \frac{vCH^{\frac{1}{\varphi}}}{w}$$

## Previous DNB Working Papers in 2013

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