Euro- US Real Exchange Rate Dynamics: How Far Can We Push General Equilibrium Models?

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Abstract

Empirical evidence suggest that the real exchange rate is highly persistent and volatile implying a well known failure of the PPP assumption. There is also large evidence on the international lack of risk sharing; there is very low and in most cases even negative relationship between real exchange rate and relative consumption. This paper addresses the problem of general equilibrium models to match with the behaviour of real exchange rate and its correlation with relative consumption. We do so by developing a two country general equilibrium model with non-traded goods, home bias, incomplete markets and partial degrees of pass through as well as nominal rigidities both in goods and labour markets. We combine this comprehensive framework with a data consistent characterisation of technology shocks. Motivated by data, we assume permanent technology shocks in traded goods sector and persistent temporary shocks in non-traded goods sector. We found that existence of non-traded goods sector has important implications on both real exchange rate dynamics and its negative relationship between relative consumption. Our model matches with the persistence of real exchange rate but not the volatility. We argue that the ability of a general equilibrium model to account for the features of the data is closely related to the predominant driving source of the fluctuations.

Keywords: Real Exchange Rates, Non-traded goods, Nominal Rigidities, Incomplete asset markets, Imperfect Exchange Rate Pass Through, Consumption-Real Exchange Rate Anomaly

JEL classifications: F31, F32, F41.

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1 Introduction

This paper addresses the problem of general equilibrium models to match the behaviour of real exchange rate (RER). We do so by developing a model that contains rich set of rigidities that can deliver deviations from purchasing power parity (PPP), as well as more data-consistent features such as permanent total factor productivity shocks in tradable goods sector and partial exchange rate pass through.

The observed characteristics of the RER contradict the traditional PPP assumption. According to PPP theory the nominal exchange rate is solely determined by the relative prices between countries, so RER should be equal to one. However, in reality the RER is highly persistent and volatile implying a well known failure of the PPP assumption. An important attempt to replicate the behaviour of RER using DSGE models is the paper by Chari, Kehoe, McGrattan (2002, CKM hereafter). Their model accommodates failure of PPP through home bias and local currency pricing (LCP) assumptions with the purpose of capturing the quantiative properties of US-Europe RER . When the preferences are biased towards home produced goods, the terms of trade will have an impact on RER fluctuations. In the absence of home bias, a change in relative price of exports would not change the relative prices across countries. The LCP assumption combined with the price stickiness causes deviations from law of one price (LoOP). This is because when the prices are set in the currency of the buyer, there is no pass through to prices arising from changes in nominal exchange rate. Their main finding is that, in a model with complete markets, separable preferences, high degree of risk aversion and sufficient price stickiness, the monetary shocks can account for the volatility of the RER. Although generating substantial persistence in the model, they still fail to capture the observed persistence of RER. Moreover, in their paper, they try to address the observed lack of international risk sharing. The evidence shows that there is very low and in most cases even negative relationship between RER and relative consumption. CKM's model fails to generate the negative cross correlation between relative consumption and RER even when they remove the complete markets assumption. This result is known as consumption- RER anomaly which is also known as Backus-Smith puzzle since the early work of Backus and Smith (1993) [1]. With perfect international risk sharing there is perfect correlation between RER and ratio of marginal utilities. The traditional suggestion to ac-

[1] Kollman (1995) also reported the puzzle and rejected the complete markets assumption at international level.

count for this puzzle, is allowing trade in noncontingent bonds. When markets are incomplete, agents can increase their wealth by accumulating bonds, hence risk is not shared perfectly. More explicitly, because risk sharing holds only in expected first differences, there will be fluctuations in current account dynamics implying wealth effects. With this regard, CKM's result shows that market incompleteness is not sufficient to generate necessary wealth effects, to account for lack of risk sharing.

Ever since the findings of CKM, there has been ongoing research to capture the stylised facts about RER dynamics in an international real business cycle framework. These models match with some features of the data but at the cost of others. Our paper offers a re-assessment of the previous models through a comprehensive and encompassing theoretical set-up with a special focus on a data consistent characterisation of technology shocks.

In order to account for the negative co-movement between the relative consumption and RER in a general equilibrium set-up, Corsetti *et al* (2008) introduced distribution services along with non-traded goods and market incompleteness in a two country general equilibrium framework. The introduction of distribution services provide extra variability in the RER, as each country has to use non-tradable distribution services to produce tradable goods. They show that with correct parameterisation, the model can generate large wealth effects when a productivity shock hits the economy. Benigno and Thoenissen (2008) also explore the Backus-Smith puzzle by constructing a perfectly competitive, flexible price model with non-traded sector and incomplete markets in which the LoOP holds for traded goods. Their results are in line with the Balassa-Samuelson proposition. The existence of non-traded goods can account for lack of risk sharing depending on the source of fluctuations. Following a supply side improvement in the traded sector the movements in internal relative prices can generate the negative cross correlation. According to Balassa-Samuelson proposition, a productivity improvement in the traded sector increases relative consumption while the RER appreciates as a consequence of the increase in non-traded goods prices. Tuesta (2013) also presents a framework with limited international asset market structure, non-traded goods and distribution services. As in CKM, he allows for imperfect competition and nominal rigidities. He also, emphasizes the importance of the existence of non-traded goods sector for model to account for international lack of risk sharing. With similar objective, Selaive and Tuesta (2003) develop a model with non-traded goods, imperfect pass through along with incomplete markets assumption. They highlight the importance of net foreign asset position of a country in explaining the lack of risk sharing. Their results are again consistent with Balassa-Samuelson proposition[2] . In their sensitivity analysis they show the importance of calibration of the substitution parameter between home and foreign produced traded goods to break the relationship between the relative consumption and RER. The higher the elasticity, the more responsive the net foreign asset position of a country to the changes in terms of trade. Corsetti *et al* (2012) take the analysis into another dimension and investigates the lack of risk sharing at different frequencies both theoretically and empirically. They show that, among OECD countries, the degree of uninsurable risk is higher at business cycle and lower frequencies. They also present an analytical explanation of how degree of risk sharing differs across frequencies. In their companion paper (Corsetti *et al* (2011)) they show that among OECD countries lack of risk sharing is more severe at tradable component of RER. They also derive the sufficient conditions to obtain a perfect risk sharing in a model with non-traded goods sector.[3]

Benigno and Thoenissen (2003) build a two country New-Keynesian model with two production sectors in both countries in order to understand the effects of supply side improvements on the behaviour of RER. In their model set-up, in addition to LCP and home bias assumptions, the existence of non-traded goods sector creates deviations from PPP as non-traded goods are not subject to international arbitrage. This is the classical Balassa-Samuelson explanation for PPP failure. As non-traded and traded goods productivity differ, relative prices will be different too, causing RER fluctuations. They calibrate their model to match with the features of UK and Euro Area economies and they find that total factor productivity (TFP) improvements in traded sector causes RER depreciation which is at odds with Balassa-Samuelson proposition. For their calibration, the impact of terms of trade depreciation over-weighs the internal relative price appreciation. A further emphasis on multi-sector structure is the paper by Dotsey and Duarte (2008) in which they show the importance of non-traded goods to increase the variability of RER. They calibrate their model by using US data with the assumption of perfectly symmetric two countries. Their

[2]There are many other papers which highlight the critical role of non-traded goods sector to address the anomaly in a general equilibrium framework. For instance, Benigno and Thoennissen (2005) explore the implications of non-traded goods sector combined with incomplete markets structure. They argue that the failure of model of CKM to replicate the observed lack of risk sharing is result of abstracting the model from non-traded goods sector.

[3]There are several other papers which address the anomaly through different methods such as Devereux *et al* (2011). They test the consumption-risk sharing empirically by using professional forecasts and find no evidence in favour of positive relationship between the relative consumption and RER.

model accounts for the persistence of the RER correctly but not its volatility[4].

Most of the New Keynesian models test the performance of their models at business cycle frequencies by using HP filter. As a different approach, Ahmad *et al* (2013) investigate the behaviour of RER in a model with non-traded goods, home bias, LCP and incomplete markets for both filtered and unfiltered data. The detrended model generates the correct volatility of RER but fails to capture the persistence and the unfiltered model matches with the persistence but not the volatility. Rabanal and Tuesta (2012) explore the RER fluctuations at different frequencies in a two country general equilibrium model where all goods can be tradable. They motivate their work by showing that the most of the RER variability can be attributable to low frequencies. Their model captures the observed volatility of RER, but generates excessive persistence[5]. Similarly, Yung (2007) evaluates the performance of new open economy macroeconomic models to match with data moments at different frequencies by developing a two-country, two-goods model. He checks the performance of the model for different pricing regimes; PCP and LCP. Compared with data, the model fails to create "hump-shaped spectral density of exchange rates" and generates too much volatility at high frequencies independent from the degree of pass through. On the role of pricing regime, Dotsey and Duarte (2011) develop a two country, three sector model to analyse the implications of degree of pass through on aggregate variables. They demonstrate that extreme cases of pass through do not have considerable consequences with respect to fluctuations of aggregate variables. They only imply differences in the behaviour of the terms of trade.

There is also a growing literature on the estimation of similar models with Bayesian estimation techniques, hence obtaining parameter values from estimations rather than calibration. One of the first attempts on the estimation of two country DSGE models by using Bayesian methods is Lubik and Schorfheide (2005). They develop a general equilibrium framework with imperfect exchange rate pass through and nominal rigidities and estimate it on US and Euro Area data. However, their model does not perform well to explain the exchange rate

[4] There are many other papers accommodating non-traded goods sector to investigate different issues in international macroeconomics. For instance, Liu and Pappa (2008) focus on the gains from international monetary policy coordination in a model with non-traded goods sector. They show that, there are gains from coordination when trading structures differ across countries under a PCP setting. Or, Devereux *et al* (2007) develop a small open economy model with non-traded goods and LCP, to investigate the implications of alternative monetary policy rules on emerging countries.

[5] In their sensitivity analysis, they introduce high portfolio adjustment costs to lower the persistence of the RER.

dynamics. More recently, Rabanal and Tuesta (2010) estimate a two country model for US and Euro Area and found that LCP and incomplete markets assumptions play an important role in RER variability. In Rabanal and Tuesta (2013), they develop a two country model where the LoOP holds and estimate it on US and Euro Area data. They stress the implications of existence of non-traded sector and distribution services on RER variability. Their model is able to capture the RER persistence but not the volatility. [6]

The relevant literature discussed above fails to address all the features of the data. Some generate the correct volatility (persistence) of RER but fails to capture its persistence (volatility) or its negative correlation with relative consumption. Our work is motivated by this mismatch between the findings of general equilibrium models with the features of the data. The model we develop is a two country general equilibrium model with multiple sectors. We introduce various elements that create persistence and volatility as well as wealth effects to account for the dynamics of the RER and its relation with relative consumption. As far as we are aware of, this is the first attempt which incorporates all the channels we have introduced here. As such, our model serves to re-evaluate the ability of these channels to explain the behaviour of RER and its correlation with other variables. As we will mention below, it also analyses the role played by the way TFP shocks are modelled in capturing the properties of the data.

Our multi-sector structure arises from the empirical evidence in two dimensions. First, the share of non-traded goods in consumption baskets are higher than traded ones. A model without non-traded goods would omit an important feature of the data. Second, in their recent paper Betts and Kehoe (2008) demonstrate that despite the significant role of traded goods price movements, changes in relative price of non-traded goods has still a strong relation to RER variability; they show that about one third of its volatility can be attributable to the relative price of non-traded goods to traded goods [7]. In fact, the role of tradable sector price movements on RER variability is based on the influential paper by Engel (1999). Engel (1999) showed that the fluctuations of RER are driven by traded goods price movements rather than the relative price of non-traded goods to traded goods across countries. We capture the deviations from

[6] There are several other contributions on the estimation of new open economy macroeconomic models by using similar methods. See, for instance, Adolfson *et al* (2007), Eiji *et al* (2012), Justiano and Preston (2010) among many others.

[7] Berka and Devereux (2013) also examines the behaviour of RER and the sources of departures from PPP. They empirically showed that the departure from PPP are greater for the non-traded goods and the fluctuations of RER is strongly correlated with relative price of non-traded goods to traded goods at both aggregate and disaggregate level.

LoOP through the assumption of limited pass through to prices from nominal exchange rate movements. Our modelling strategy of pricing regime follows the work of Betts and Devereux (2000) . Their focus is on the effects of monetary and fiscal policy shocks on the dynamics of real and nominal exchange rates under different pricing regimes. As in their model, we assume that certain fraction firms can engage in pricing to market; for the rest, the prices are invoiced in the currency of the producer. This set-up also enables to capture the high correlation between nominal and RER as can be seen in the data. In addition, we allow for home bias in preferences in line with the empirical evidence[8]. When home produced traded goods and imported goods are equally preferred, the price increase in one will raise the demand, hence the price of other; leaving the relative price index of two countries unchanged. However when there is preference bias, the changes in prices will not offset each other. Therefore, in our model as a result of home bias in preferences, terms of trade will have an influence on the dynamics of RER. We also introduce an incomplete asset market structure to be able to address the consumption-RER anomaly thoroughly. The model incorporates nominal rigidities both in goods and labour markets following Calvo (1983) with indexation as well as external habit formation in consumption to increase the persistence produced by the model.

The importance of this assessment is not only presenting a theoretical structure that introduces a rich set of distortions, but also combining this with a data consistent characterisation of technology shocks. The open economy New Keynesian models mostly assume stationary stochastic process for productivity, regardless the true stochastic behaviour of the data they are concerned with. Our specification of productivity innovations allow country and sector specific technical progress which are persistent in growth rates [9].

In our model, we stress the importance of TFP improvements in shaping the dynamic properties of the macroeconomic variables in light of the data. Figure 1 plots the evolution of sectoral total factor productivities in Euro Area and US[10]. It is clear that traded sector both in Euro Area and US has a stochastic

[8]See Obstfeld and Rogoff (2000a) for an empirical discussion on home bias.

[9]Rabanal (2009) also introduces country and sector specific permanent technology shocks to explain the inflation differentials in the European Economic Monetary Union by focussing on the sectoral inflation diversions. Alternatively, Rabanal *et al* (2011) find co-integration between the TFP processes of US and its trading partners. They highlight the importance of introducing co-integrated TFP processes across countries in a two country, two goods model to capture the RER volatility.

[10]We constructed TFP growth series from the TFP estimates of Euklems for the period 1981-2007. It is assumed that agriculture, mining and manufacturing are traded and the remaining are non-traded. However, as the series are of annual frequency we constructed

trend implying permanent effects. On the other hand, the non-traded sector productivity in both countries hardly changed across the time span. Motivated by this fact, we introduce permanent technology shocks in traded sector and persistent transitory shocks in non-traded sector. We specify the productivity process through estimating a VAR for TFP series. Our findings suggest a unit root shock in Euro Area traded sector and a rate of growth shock in US traded sector.

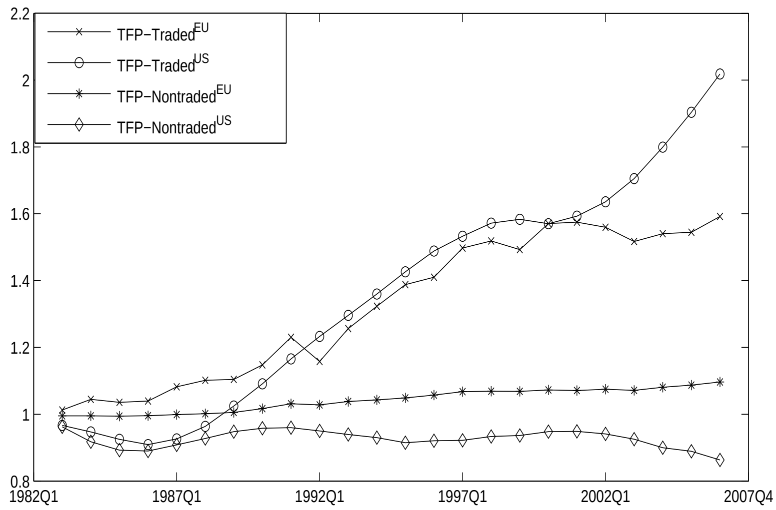

Figure 1: **Sectoral Total Factor Productivity for US and Euro Area**

We focus on the business cycle and high frequency features of the data by using HP filter for business cycle frequencies and first differences for high frequencies. Although we stationarise the model, we use the level variables from our model and filter them in a consistent way with the data. Canova (1998) show that different de-trending methods extract different information from the data. So using different methods to de-trend the model and data would give misleading data-model comparison. In addition, Andrle (2008) argue that innovations with permanent effects have implications on all frequencies, meaning neither business cycle nor high frequencies are independent from the trends. Hence the information obtained from our stationary model does not match with the information HP filtered or first differenced data provides. Therefore, we HP filter both data and the model for business cycle analysis and first difference the

quarterly series by using Chow, Lin (1971) method. The details on the construction of TFP series will be discussed in Section 4.

data and the model to analyse high frequency properties.

We calibrate the model for the US and Euro Area and examine the performance of our model compared to data. We find that non-traded goods sector along with permanent technology shocks have an important role for model to account for the consumption-RER anomaly. With correct specification of technology shocks, the model accounts for the Backus-Smith puzzle but at the cost of other moments. Our model is able to generate a strong negative cross correlation independent of the pricing regime even under complete markets. However, this result is conditional on the fluctuations of the variables to be driven by the technology shocks which are originated in the traded goods sector. Regarding the second moments of RER, our model does a good job in capturing the persistence of RER but, despite the features we have introduced, the model fails to replicate its volatility. We show that these results are very sensitive to calibrated values as well as the predominant driving source of the fluctuations in our robustness analysis. Our Monte Carlo simulations on the calibration of the shock processes, presents substantial differences from the results that we obtained from our benchmark calibration proving the important role of the specification of the innovations. Finally, we analyse the dynamic responses of the RER and its components following supply side innovations. We find that the overall dynamic adjustment of the RER is in line with Balassa-Samuelson explanation.

The remainder of the paper is organised as follows: In the next section we present the model. In Section 3 we discuss the implications of our theoretical framework. In Section 4 we describe the calibration. In Section 5 we discuss the performance of our model by comparing data and model moments and in Section 6 we present the dynamic adjustment of selected variables following TFP improvements. In Section 7 we present sensitivity analysis and, finally, Section 8 concludes. There is a detailed technical appendix to the paper where we derive the steady state, log-linearised form and stationarisation of the model.

2 The Model

The general equilibrium model we present here is a two country model; Home (H) and Foreign (F), which are populated by a mass of infinitely lived households with a fraction of (n) and $(1-n)$ of the world total, respectively. We will denote the foreign country variables by an asterisk (*).

Households can consume non-traded goods, domestically produced traded goods and imported goods. We define consumption baskets in constant elasticity of substitution (CES) aggregation form as the parameter of elasticity of substitution has important implications on the dynamics of the model. We allow for external habit formation to generate further persistence in the fluctuations. The asset markets are incomplete at the international level. The production function is in constant returns to scale form with labour being the only input. Motivated by the findings of Christiano, Eichenbaum, Evans (2005) on the role of wage rigidities in capturing the inflation inertia, we assume monopolistic competition and nominal rigidities in both goods and labour markets. Both wage and price Phillips Curves are modelled in the spirit of Gali and Gertler (1999) to generate further persistence. In particular, with this specification the wage and price Phillips Curves are not purely forward looking; we assume that a certain fraction of price/wage setters are making the decisions with a forward looking manner á la Calvo while the rest index the prices/wages to the past. Monetary policy is specified by a Taylor type interest rate feedback rule.

To be able test for the sources of RER fluctuations fully, in our model neither PPP nor LoOP hold. The multi-sector structure allows us to test for the importance of Balassa-Samuelson proposition considering the controversial results in the literature [11]. Among tradable goods, we allow a preference bias towards domestically produced tradable goods, thus the terms of trade variability has an impact on RER fluctuations. We also assume that a certain fraction of exporting firms can set the prices in the currency of the buyer as modelled in Betts and Devereux (2000). The limited exchange rate pass through along with the home bias ensures fluctuations in RER arising from the changes in tradable goods prices in line with the findings of Engel (1999) .

There are country and sector specific permanent technology shocks as well as preference and monetary shocks. We allow technology shocks to be persistent in growth rates, thus the technical improvement will diffuse slowly. When the process follows a unit root, the economy reaches to its new level immediately. But, when technology shocks are persistent in growth rates as in our model, the changes are gradual. Our specification follows the findings of Lindé (2009), in which he argues that this specification fits better to empirical evidence by focussing on the response of hours to a positive permanent technology shock.

[11] For instance, Benigno and Thoenisson (2003) suggest that a positive productivity shock to tradable sector results in RER depreciation contradicting with Balassa-Samuelson explanation. On the other hand, there are many other contributions in which internal relative price movements cause RER appreciation such as Benigno and Thoenissen (2006), Ghironi and Melitz (2005) and Tuesta (2013).

2.1 Households

The preferences over intertemporal decisions are identical across countries . The representative home household, (i), receives utility from consumption (C_t^i), and disutility from supplying labour (L_t^i). Preferences are separable between these two arguments. We allow external habit formation in consumption through hC_{t-1} with $h > 0$, so the marginal utility of consumption in period t depends on aggregate consumption in $(t-1)$. τ_t represents the preference shock which affects the discount rate hence the intertemporal decision of the household. The representative household's lifetime utility function may be expressed as:

$$U_t^i = E_t \sum_{t=0}^{\infty} \beta^t \tau_t \left[\frac{(C_t^i - hC_{t-1})^{1-\rho}}{1-\rho} - \chi \frac{(L_t^i)^{1+\eta}}{1+\eta} \right], \; 0 < \beta < 1 \tag{1}$$

where E_t denotes the expectations operator conditional on time t information, β is the discount factor, $1/\rho$ is the intertemporal elasticity of substitution and the parameter η is the inverse of the Frisch elasticity of labour supply. The preference shock, τ, follows an AR(1) process in logs:

$$ln(\tau_t) \; = \; \rho^{\tau} \; ln(\tau_{t-1}) \; + \; \varepsilon_t^{\tau}, \qquad \qquad \varepsilon_t^{\tau} \sim N(0, \sigma_{\tau}^2) \tag{2}$$

Households in each country consume a domestically traded good, an imported good and a non-traded good. The consumption index, which consists consumption of non-traded $(C_{N,t})$ and traded goods $(C_{T,t})$, has the following CES aggregation form:

$$C_t^i = \left(\alpha^{\frac{1}{\nu}} (C_{T,t}^i)^{\frac{\nu-1}{\nu}} + (1-\alpha)^{\frac{1}{\nu}} (C_{N,t}^i)^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}} \tag{3}$$

where α and $(1-\alpha)$ represent the share of traded goods consumption and non-traded goods consumption in steady state home consumption expenditure respectively, and parameter ν is the elasticity of substitution between tradable and non-tradable goods. The share of traded goods in the foreign consumption basket is different form the home country $\alpha^* \neq \alpha$, while the substitution elasticity is same.

Traded goods consumption, $C_{T,t}$, is divided between home tradable, $C_{H,t}$, and foreign tradable goods consumption, $C_{F,t}$.

$$C_{T,t}^i = \left(\omega^{\frac{1}{\theta}} (C_{H,t}^i)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} (C_{F,t}^i)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{4}$$

where ω is the weight of home produced tradable goods, and θ is the elasticity of substitution between home and foreign tradable goods. The elasticity of substitution between the two types is same across countries but the preference weights may differ ($\omega \neq \omega^*$). The value of ω is important, as it shows the degree of home bias in preferences. $\omega > 0.5$ implies a bias towards home produced tradable goods relative to imported goods.

As a result of monopolistic competition, each firm produces differentiated goods. Home country producers are indexed by $h \epsilon [0, 1]$ and foreign country producers are indexed by $f \epsilon [0, 1]$. The consumption sub-indices for each variety of goods are:

$$C_{j,t} = \left[\int_0^1 c_{j,t}(h)^{\frac{\phi-1}{\phi}} dh\right]^{\frac{\phi}{\phi-1}} \ , \ C_{j^*,t} = \left[\int_0^1 c_{j^*,t}(f)^{\frac{\phi-1}{\phi}} df\right]^{\frac{\phi}{\phi-1}} \tag{5}$$

where $j = N, H, H^*$, $j^* = N^*, F^*, F$ and $\phi > 1$ is the elasticity of substitution across brands. $c_{j,t}(h)$ represents the consumption of differentiated goods produced in home country in each sector and $c_{j,t}(f)$ represents the consumption of differentiated goods in produced in foreign country in each sector.

The aggregate consumer price index in home country is then:

$$P_t = \left(\alpha(P_{T,t})^{1-\nu} + (1-\alpha)(P_{N,t})^{1-\nu}\right)^{\frac{1}{1-\nu}} \tag{6}$$

And the price index for tradable goods is given by

$$P_{T,t} = \left(\omega(P_{H,t})^{1-\theta} + (1-\omega)(P_{F,t})^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{7}$$

$P_{H,t}, P_{F,t}, P_{N,t}$ are price indices for home traded, foreign traded and non-traded goods respectively defined as:

$$P_{j,t} = \left[\int_0^1 p_{j,t}(h)^{1-\phi} dh\right]^{\frac{1}{1-\phi}} \ , \ P_{j^*,t} = \left[\int_0^1 p_{j^*,t}(f)^{1-\phi} df\right]^{\frac{1}{1-\phi}} \tag{8}$$

where $j = N, H, H^*$, $j^* = N^*, F^*, F$

The optimal allocation of nominal expenditure of the representative household in home country for each differentiated good yields the following demand functions:

$$c_{N,t}(h) = \left(\frac{p_{N,t}(h)}{P_{N,t}}\right)^{-\phi} C_{N,t} \ , \tag{9}$$

$$c_{H,t}(h) = \left(\frac{p_{H,t}(h)}{P_{H,t}}\right)^{-\phi} C_{H,t} \ , \qquad c_{F,t}(f) = \left(\frac{p_{F,t}(f)}{P_{F,t}}\right)^{-\phi} C_{F,t}$$

where

$$C_{N,t} = (1-\alpha)\left(\frac{P_{N,t}}{P_t}\right)^{-\nu} C_t \ , \tag{10}$$

$$C_{H,t} = \omega\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\theta} C_{T,t} \ , \qquad C_{F,t} = (1-\omega)\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\theta} C_{T,t}$$

with

$$C_{T,t} = \alpha\left(\frac{P_{T,t}}{P_t}\right)^{-\nu} C_t$$

The optimal demand functions of the representative household in the foreign country have the same structure with the home demands:

$$c^*_{N,t}(f) = \left(\frac{p^*_{N,t}(f)}{P^*_{N,t}}\right)^{-\phi} C^*_{N,t} \ , \tag{11}$$

$$c^*_{H,t}(f) = \left(\frac{p^*_{H,t}(h)}{P^*_{H,t}}\right)^{-\phi} C^*_{H,t} \ , \qquad c^*_{F,t}(f) = \left(\frac{p^*_{F,t}(f)}{P^*_{F,t}}\right)^{-\phi} C^*_{F,t}$$

where

$$C^*_{N,t} = (1-\alpha^*)\left(\frac{P^*_{N,t}}{P^*_t}\right)^{-\nu} C^*_t \ , \tag{12}$$

$$C^*_{F,t} = \omega^*\left(\frac{P^*_{F,t}}{P^*_{T,t}}\right)^{-\theta} C^*_{T,t} \ , \qquad C^*_{H,t} = (1-\omega^*)\left(\frac{P^*_{H,t}}{P^*_{T,t}}\right)^{-\theta} C^*_{T,t}$$

with

$$C^*_{T,t} = \alpha^*\left(\frac{P^*_{T,t}}{P^*_t}\right)^{-\nu} C^*_t$$

Note that under symmetric equilibrium prices and consumption for each variety will be equal as the decisions will be identical, hence the consumption and price sub-indices become inconsequential for the model solution.

2.2 Asset market structure and the budget constraint

Households finance their expenditure through labour income and nominal profits. We assume that labour income is subsidised with a constant rate, κ^w. In addition, it is assumed that the international asset markets are incomplete in the sense that households are able to trade non-state-contingent bonds to purchase consumption goods. Here, we follow Benigno (2001) to model incomplete asset market structure. It is assumed that households in home country can hold two kinds of bonds which are denominated in the units of the home currency and the foreign currency. However, the bonds issued by home country are not traded internationally. As discussed in Schmitt-Grohé and Uribe (2003), we avoid the arising non-stationarity in the model from incomplete market structure by introducing an additional cost for the households in the home country when they trade in the foreign asset market. The cost function $\Theta(.)$ ensures a stationary distribution of wealth between countries. [12]

$$\begin{aligned} P_t C_t^i + \frac{B_{H,t}^i}{(1+i_t)} + \frac{S_t B_{F,t}^i}{(1+i_t^*)\Theta(S_t B_{F,t}/P_t)} \leq\ & B_{H,t-1}^i + S_t B_{F,t-1}^i \\ & +(1+\kappa^w)\, W_t^i L_t^i + \int_0^1 \Pi_t^i di \end{aligned} \tag{13}$$

$B_{H,t}^i$ and $B_{F,t}^i$ are the household i's holdings of the domestic and foreign currency denominated nominal risk-free bonds. The nominal interest rates on these bonds in time t are i_t and i_t^* respectively. S_t is the nominal exchange rate defined as the home currency price of buying one unit of foreign currency, W_t^i is the nominal wage rate and $\mathit{\Pi}_t^i$ is the profit income received from the ownership of shares of domestic firms.

The households in home country make the intertemporal decision by maximising (1) subject to (13). The optimum consumption-saving decision is characterised by:

$$\tau_t (C_t - hC_{t-1})^{-\rho} = \beta(1+i_t) E_t \left[\tau_{t+1} (C_{t+1} - hC_t)^{-\rho} \left(\frac{P_t}{P_{t+1}} \right) \right] \tag{14}$$

In addition, the optimal portfolio choice describes the uncovered interest rate parity (UIP) condition:

$$\begin{aligned} & \tau_t \, (C_t \, - \, hC_{t-1})^{-\rho} = \\ \beta\,(1 \, + \, i_t^*)\, \Theta & \left(\frac{S_t \, B_{F,t}}{P_t} \right) E_t \left[\tau_{t+1} \, (C_{t+1} \, - hC_t)^{-\rho} \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{S_{t+1}}{S_t} \right) \right] \end{aligned} \tag{15}$$

The situation of foreign households is analogous.

[12] In order to ensure a well-defined steady state in the model, the following restrictions are imposed on the cost function: $\Theta(.)$ is a differentiable decreasing function in the neighbourhood of steady state level of net foreign assets and when the net foreign assets are in the steady state level ($B_{F,t}^i = 0$), the cost function is equal to 1 ($\Theta(0) = 1$). See Benigno (2001) for details.

2.3 Production technologies and price setting behaviour

There is continuum of monopolistically competitive firms in each country producing differentiated goods for traded and non-traded sectors with labour being the sole production factor. Production has the standard constant returns to scale functional form:

$$Y_{H,t}(h) = A_{H,t}L_{H,t}(h) \tag{16}$$

and

$$Y_{N,t}(h) = A_{N,t}L_{N,t}(h) \tag{17}$$

where $Y_{H,t}(h)$ is the home produced traded output and $Y_{N,t}(h)$ is the non-traded output, $A_{H,t}$ and $A_{N,t}$ are sector specific exogenous technology shocks, $L_{H,t}$ and $L_{N,t}$ are the total labour employed in each sector. The technology in each sector has the following stochastic process:

$$ln(A^j_{t+1}) - ln(A^j_t) = \rho^{a^j}(ln(A^j_t) - ln(A^j_{t-1})) + \varepsilon^{a^j}_t \tag{18}$$

where $0 \leq \rho^{a^j} < 1$, $\varepsilon^{a^j}_t \sim N(0, \sigma^2_{a^j})$ and $j = H, N$. The sectoral and country specific shocks can be correlated as can be seen from the variance-covariance matrix, $V(\varepsilon^a)$, of the productivity innovations:

$$V(\varepsilon^a) = \begin{bmatrix} \sigma^2_{a^H} & \sigma_{a^H,a^N} & \sigma_{a^H,a^F} & \sigma_{a^H,a^{N*}} \\ \sigma_{a^N,a^H} & \sigma^2_{a^N} & \sigma_{a^N,a^F} & \sigma_{a^N,a^{N*}} \\ \sigma_{a^F,a^H} & \sigma_{a^F,a^N} & \sigma^2_{a^F} & \sigma_{a^F,a^{N*}} \\ \sigma_{a^{N*},a^H} & \sigma_{a^{N*},a^N} & \sigma_{a^{N*},a^F} & \sigma^2_{a^{N*}} \end{bmatrix}$$

Since the technology shocks have long run impact, the trended variables should be normalised in order to obtain a stationary equilibrium.[13] The specification of the stochastic process in the model allows the permanent technology shocks to be persistent in growth rates. Notice that when $\rho^{a^j} = 0$, the shock process reduces to a simple unit root process so that following a shock the economy reaches to the new potential level immediately. On the other hand, when $0 < \rho^{a^j} < 1$, the impact of the shock is gradual. And a value smaller than zero, $\rho^{a^j} < 0$, would imply a temporary shock process.

In each sector, pricing decisions are staggered à la Calvo (1983). It is assumed that in every period $(1 - \xi_p)$ of firms can reset their prices. A non-traded good producer who is able to re-optimise the price in the current period will choose the price $\tilde{p}^N_t(h)$ that maximises the expected present discounted value of profits

[13] Derivation of trends can be found in Appendix D.

$$max \sum_{k=0}^{\infty}(\beta\xi_p)^k \ E_t \left[\left(\frac{U_C(C_{t+k})}{U_C(C_t)}\frac{P_t}{P_{t+k}}\right) \ \tilde{y}_{t,t+k}^{Nd}(h) \ ((1+\kappa^p)\tilde{p}_t^N(h) - P_{N,t}MC_{N,t})\right] \tag{19}$$

subject to the downward sloping demand curve

$$\tilde{y}_{t,t+k}^{Nd}(h) = \left(\frac{\tilde{p}_t^N(h)}{P_{N,t}}\right)^{-\phi} C_{N,t} \tag{20}$$

where $\tilde{y}_{t,t+k}^{Nd}(h)$ denotes the total individual demand for non-traded goods produced by producer h at time $t+k$ and $MC_{N,t} = \frac{W_t}{A_{N,t}P_{N,t}}$ is the real marginal cost for the non-tradable goods producer.

The first order condition associated with the above profit maximising problem is given by:

$$\sum_{k=0}^{\infty}(\beta\xi_p)^k E_t \left[\left(\frac{U_c(C_{t+k})}{U_c(C_t)}\frac{P_t}{P_{t+k}}\right) \tilde{y}_{t,t+k}^{Nd}(h) \left((1+\kappa^p)\tilde{p}_t^N(h) - \frac{\phi}{\phi-1}P_{N,t}MC_{N,t}\right)\right] = 0 \tag{21}$$

We introduce a constant subsidy, κ^p, which producers receive for their production in each sector. The subsidy ensures a perfectly competitive equilibrium in steady state as we set $(1+\kappa^p) = \frac{\phi}{\phi-1}$. Hence when all prices are flexible ($\xi_p \to 0$), prices will be equal to marginal cost.

Firms producing in the tradable goods sector can sell their products in the domestic country or export them. An important feature of the model is the failure of law of one price in traded goods, such that exports exhibit partial local currency pricing (LCP). Among the exporting firms, a fraction (d) of firms can price discriminate and set the prices in the currency of the buyer while $(1-d)$ of the firms set the prices in the currency of the seller (PCP). The price index for exports can be written in the following form:

$$P_{H,t}^* = P_{LCP^*,t}^d (P_{H,t}/S_t)^{(1-d)} \tag{22}$$

Note that in the limiting case of perfect pass-through ($d=0$) all firms set prices in the producers' local currency. Hence LoOP holds for traded goods. With $d=1$ on the other hand, all exports are expressed in the currency of the buyer implying complete pricing-to-market behaviour.

In the traded sector, the optimal pricing rule for a firm selling tradable goods in the home country has similar pricing rule with the firm producing non-traded goods. The optimal pricing decision can be written as:

$$\sum_{k=0}^{\infty}(\beta\xi_p)^k E_t\left[\left(\frac{U_c(C_{t+k})}{U_c(C_t)}\frac{P_t}{P_{t+k}}\right)\tilde{y}^{Hd}_{t,t+k}(h)\left((1+\kappa^p)\tilde{p}^H_t(h)-\frac{\phi}{\phi-1}P_{H,t}MC_{H,t}\right)\right] = 0 \quad (23)$$

where $\tilde{y}^{Hd}_{t,t+k}(h)$ denotes the total individual domestic demand for traded goods produced by producer h at time $t+k$ and $MC_{H,t} = \frac{W_t}{A_{H,t}P_{H,t}}$ is the real marginal cost for the tradable goods producer, selling goods in the home country. As the $(1-d)$ part of the traded goods are freely traded by consumers across countries, the optimal price setting of producer currency priced exports is equivalent to the price chosen for the domestic market.

On the other hand, a proportion d of firms can set the prices in the currency of the market they sell the goods. In this case, the LCP firms consider the marginal cost in the foreign currency for foreign country and in home currency for home country and make the pricing decision accordingly. The optimum pricing rule in the traded sector for an exporting firm engaged in LCP is given by:

$$\sum_{k=0}^{\infty}(\beta\xi_p)^k E_t\left[\left(\frac{U_c(C_{t+k})}{U_c(C_t)}\frac{P_t}{P_{t+k}}\right)\tilde{y}^{Hd*}_{t,t+k}(h)\left((1+\kappa^p)\tilde{p}^{LCP}_t(h)-\frac{\phi}{\phi-1}P_{LCP^*,t}MC_{H^*,t}\right)\right] = 0 \quad (24)$$

where $\tilde{y}^{Hd*}_{t,t+k}(h)$ denotes the total individual foreign demand for traded goods produced by producer h at time $t+k$ and $MC_{H^*,t} = \frac{W_t}{A_{H,t}P_{LCP^*,t}S_t}$ is the real marginal cost of exports, priced in the local currency.

Following Gali and Gertler (1999), we assume price indexation through co-existence of two types of firms. A fraction $(1-\varsigma_p)$ of firms re-set prices in a forward looking behaviour, as in the Calvo model described above. The remaining ς_p of firms re-set prices in a backward looking behaviour by using lagged inflation to forecast current inflation. These fractions are assumed to be equal across sectors. The evolution of log-linearised aggregate price setting can be expressed as follows:

$$p_t = \xi_p p_{t-1} + (1-\xi_p)\breve{p}_t \quad (25)$$

$\breve{p}_t$ is the index for re-set prices which is in log-linear form given by:

$$\breve{p}_t = (1 - \varsigma_p) p_t^f + \varsigma_p p_t^b \tag{26}$$

where p_t^f is the price set by forward looking firms, p_t^b is the price set by backward looking firms based on the following rule-of-thumb:

$$p_t^b = \breve{p}_{t-1} + \pi_{t-1} \tag{27}$$

This price setting structure generates a higher inflation persistence in the model. The New Keynesian Phillips Curve is not purely forward looking any more; it is a combination of future expected inflation and lagged inflation.

2.4 Labour supply and the wage setting

There is monopolistic competition among households in the labour market, in the sense that households offer differentiated labour services to each sector indifferently. Labour is mobile between non-traded and traded sectors, but not across countries. Hence, the total labour supply is sum of sectoral supply of labour.

$$L = L_H + L_N \tag{28}$$

As described in Erceg *et al* (2000), an "employment agency" combines individual household's supply in the following Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 L_t^j(i)^{\frac{\sigma_w - 1}{\sigma_w}} di \right]^{\frac{\sigma_w}{\sigma_w - 1}} \tag{29}$$

where $j = H, N$ and $\sigma_w > 1$ is the elasticity of substitution among differentiated labour services which is assumed to be same in the two sectors.

Note that, as a result of perfect labour mobility, the nominal wages will be equalised across sectors. The aggregate nominal wage index is defined as:

$$W_t = \left[\int_0^1 W_t(i)^{1-\sigma_w} di \right]^{\frac{1}{1-\sigma_w}} \tag{30}$$

The cost minimisation problem of producers gives the downward sloping labour demand curve. The total demand for household i's labour services by all firms is:

$$L_t(i) = \left[\frac{W_t(i)}{W_t}\right]^{-\sigma_w} L_t \tag{31}$$

Households set the wages in a Calvo style staggered way. In a given period a constant fraction of $(1 - \xi_w)$ of households are able to adjust their wages. To choose the optimum wage $\widetilde{W}_t(i)$, households maximise the expected lifetime utility (1) subject to the budget constraint (13) and the labour demand curve (31). The first order condition for this nominal wage setting problem is:

$$\sum_{k=0}^{\infty}(\beta\xi_w)^k E_t \left[L_{t+k}(i)U_C(C_{t,t+k})\left((1+\kappa^w)\frac{\widetilde{W}_t(i)}{P_{t+k}} - \frac{\sigma_w}{\sigma_w - 1}MRS_{t,t+k}\right)\right] = 0 \tag{32}$$

where $MRS_{t,t+k}$ is the marginal rate of substitution between consumption and labour in period $t+k$ for the household resetting the wage in period t, i.e. $MRS_{t,t+k} \equiv -\frac{U_L(L_{t,t+k})}{U_C(C_{t,t+k})}$. When wages are flexible ($\xi_w \to 0$), the real wage multiplied by the subsidy will be equal to the mark-up over the marginal rate of substitution:

$$(1+\kappa^w)\frac{W_t}{P_t} = \frac{\sigma_w}{\sigma_w - 1}MRS_{t,t+k} \tag{33}$$

To offset the effect of monopolistic distortion, similar to price setting mechanism, we assume that the amount of subsidy is equal to the monopolistic distortion: $(1+\kappa^w) = \frac{\sigma_w}{\sigma_w - 1}$.

As in the price setting case, it is assumed that in each period, among re-optimising households, a fraction $(1 - \varsigma_w)$ behave in a forward looking manner while the remaining (ς_w) set wages in a backward looking manner.

The aggregate log-linearised wage index evolves according to:

$$w_t = \xi_w w_{t-1} + (1 - \xi_w)\breve{w}_t \tag{34}$$

$\breve{w}_t$ is an index for re-set wages which can be defined in log-linear form as:

$$\breve{w}_t = (1 - \varsigma_w)w_t^f + \varsigma_w w_t^b \tag{35}$$

where w_t^f is the wage setted by forward looking households and w_t^b is the wage setted by backward looking households. Backward looking households are rule-of-thumb wage setters such that:

$$w_t^b = \breve{w}_{t-1} + \pi_{t-1}^w \tag{36}$$

2.5 Current account

As the foreign currency denominated bonds can be internationally traded, it is possible to analyse the dynamics of current account by using the budget constraint. The evolution of the net foreign asset position of the home country can be written as:

$$\frac{S_t B_{F,t}}{P_t(1+i_t^*)\Theta(S_t B_{F,t}/P_t)} - \frac{S_t B_{F,t-1}}{P_t} = \frac{P_{H,t}Y_{H,t}^d}{P_t} + \frac{(1-n)\,S_t P_{H,t}^* Y_{H,t}^{d*}}{n\,P_t} - \frac{P_{T,t}C_{T,t}}{P_t} \tag{37}$$

where $Y_{H,t}^d$ represents the domestic demand for home produced traded goods and $Y_{H,t}^{d*}$ is the demand for home produced traded goods of the foreign country consumers.

2.6 Monetary policy

In this model, the monetary authority follows a Taylor rule in each country, that targets consumer price inflation and the deviation of output from its steady state value:

$$i_t = \Gamma_{i-1} i_{t-1} + (1-\Gamma_{i-1})\Gamma_{\pi_t}\pi_t + (1-\Gamma_{i-1})\Gamma_{y_t}(y_t - \bar{y}) + \varepsilon_{m,t} \tag{38}$$

$$i_t^* = \Gamma_{i-1}^* i_{t-1}^* + (1-\Gamma_{i-1}^*)\Gamma_{\pi_t}^*\pi_t^* + (1-\Gamma_{i-1}^*)\Gamma_{y_t}^*(y_t^* - \bar{y}^*) + \varepsilon_{m,t}^* \tag{39}$$

where

$$\varepsilon_{m,t} \sim N(0,\sigma_m^2), \qquad \varepsilon_{m,t}^* \sim N(0,\sigma_{m^*}^2)$$

are monetary policy shocks.

2.7 Equilibrium conditions

In order to have equilibrium in the model demand should be equal supply in each market. This leads to the following market clearing conditions in goods market for home and foreign country respectively:

$$Y_{N,t} = C_{N,t}, \qquad Y_{N,t}^* = C_{N,t}^* \tag{40}$$

$$Y_{H,t} = C_{H,t} + \left(\frac{1-n}{n}\right) C^*_{H,t}, \qquad Y_{F,t} = C^*_{F,t} + \left(\frac{n}{1-n}\right) C_{F,t} \tag{41}$$

Therefore, non-traded goods production and consumption will be equal for markets to clear. In traded goods sector, the production will be equal to consumption of traded goods in domestic country and consumption of exports.

Finally, aggregate output can be defined as:

$$Y_t = C_t + \left(\frac{1-n}{n}\right) C^*_{H,t} - C_{F,t}, \tag{42}$$

$$Y^*_t = C^*_t + \left(\frac{n}{1-n}\right) C_{F,t} - C^*_{H,t} \tag{43}$$

In total, the domestic production will be equal to sum of total domestic consumption and exports minus the imported goods consumption. Notice that the aggregate variables are normalised with respect to country size.

2.8 Some Definitions and Decomposition of RER

As the international variables have a key role for our analysis, we present the definition of some variables we will be using in the following sections.

The home country's terms of trade, the price of imports relative to price of exports of the home country, is defined as:

$$ToT_t = \frac{P_{F,t}}{S_t \, P^*_{H,t}} \tag{44}$$

Thus, an increase in ToT implies a deterioration of terms of trade, as imports become more expensive.

Exports of the home country are given by:

$$NX_t \;=\; \frac{(1-n)\, P^*_{H,t}\, S_t\, C^*_{H,t} \;-\; n\, P_{F,t}\, C_{F,t}}{P_t} \tag{45}$$

The RER is the ratio of foreign CPI to home CPI adjusted by the nominal exchange rate:

$$RER_t \;=\; \frac{P^*_t\, S_t}{P_t} \tag{46}$$

Real exchange rate appreciates when RER falls.

As discussed earlier, there are three channels which cause deviations from PPP in our model: the existence of a non-traded sector, home bias towards domestically produced traded goods, and imperfect exchange rate pass through to prices. In order to show the role of these channels on the movements of RER, the RER can be decomposed into its components. To be tractable, we will show the log-linear form of the components. The following three components sum up to the RER and they are obtained by plugging in the definition of CPI and tradable price index to RER. So, in log-linear form, the fluctuations of the RER will be the combination of the following three components:

$$rerptm_t = (1-\omega^*)(s_t + p_t^{H^*} - p_t^H) \; + \; \omega^*(s_t + p_t^{F^*} - p_t^F) \tag{47}$$

$$rertot_t = (\omega - (1-\omega^*))(p_t^F - p_t^H) \tag{48}$$

$$rerint_t = (1-\alpha^*)(p_t^{N^*} - p_t^{T^*}) \; - \; (1-\alpha)(p_t^N - p_t^T) \tag{49}$$

The first channel is *rerptm* which captures the limited pass-through to prices from changes in the exchange rate. As long as some exporting firms set the prices in the currency of the buyer, the LoOP will not hold among traded goods. The pricing to market channel has implications in transitional dynamics. In the long run, when prices are flexible, this channel has no effect as the elasticity of substitution between varieties across countries is assumed to be equal.

The second component, *rertot*, shows the effects of preference bias towards domestically produced goods. As a result of home bias, changes in the terms of trade have an impact on the dynamics of real exchange rate. The higher the home bias, the higher the correlation between ToT and RER.

The final channel, *rerint*, captures the traditional Balassa-Samuelson explanation of the RER fluctuations. The presence of non-traded sector causes failure of the PPP as the non-traded sector is not subject to international arbitrage. Hence, the fluctuations in internal relative prices can account for RER variability.

Note that although pricing to market channel only have consequences in the short run, the relative price of non-traded goods and home bias can have long run implications. This is because of the fact that the relative prices are affected by the productivity of sectors and countries. In a model with country and sector specific permanent productivity shocks, because the technology grows at different rates in each sector and in each country, the relative prices will change permanently both across countries and across sectors within a country. As such, permanent productivity differentials between traded and non-traded sectors will have permanent effects on internal relative prices as suggested by Balassa-Samuelson model. And different TFP growth in traded sector among countries will have an impact on the terms of trade permanently. We take into account the trends which are carried by this two channels following a permanent positive technology shock realization when we stationarise the model in Appendix D.

3 Discussion

The theoretical framework we developed, contains many features that are proposed as potential sources of RER fluctuations in the literature. Moreover, the model presented has a rich structure to account for the so called Backus-Smith puzzle or consumption-RER anomaly as in CKM. Some of these channels are worth more consideration in order to have a complete interpretation of our findings.

The goods market and labour market frictions help to generate persistence and volatility in the dynamics of the model. Nominal rigidities have been used in many models in international macroeconomics to explain deviations from PPP such as the traditional Mundell-Fleming model, which is an open economy version of the IS-LM model and Dornbusch model (1976) which adds perfect foresight assumption to the Mundell-Fleming framework. More recently, Obstfeld and Rogoff (1995) carried similar analysis into a micro-founded general equilibrium framework. In a nutshell, when the prices are sticky, they do not adjust to changes in nominal exchange rate. As only a certain fraction of firms can adjust the prices in each period, the prices of goods respond less to an innovation. The wage rigidity works in a similar way. When wages are sticky, households are not able to adjust their wages instantly, thus the nominal marginal cost becomes less responsive to shocks making inflation even less responsive. We have also introduced backward looking rule-of-thumb price and wage setters as well as forward looking ones to generate the observed persistence in inflation[14] and RER dynamics. The persistent prices increases the persistency of RER as well. In addition, under nominal rigidities it is expected to have close relationship between nominal and real exchange rate as can be seen from equation (46). When only certain fraction firms can adjust prices each period, the changes in nominal exchange rate cannot be corrected by the changes in prices fully. Our

[14] See, for instance, Gali and Gertler (1999) for an empirical discussion about the inertia in inflation.

approach is consistent with the empirical evidence on the high correlation between nominal and real exchange rate along with the small reaction of the prices to the changes in nominal exchange rate (See, for instance, Engel (1993), Engel and Rogers (1996) and Parsley and Wei (1999)).

Our incomplete international market setting is important for our model to account for the Backus-Smith puzzle. When agents can access a complete set of contingent claims, the RER will be equal to the ratio of the marginal utility of consumption across countries. However under incomplete markets there is no perfect insurance any more, agents can increase their wealth through accumulating bonds. This wealth effect breaks the relationship between RER and relative consumption. The wedge between these two depends on the fluctuations of the current account.

We allow for external habit formation of consumption which increases the persistence in consumption. Because agents consumption decision is dependent on the previous periods consumption level, with habits incentives to smooth the consumption in reaction to shocks is higher, implying wealth effects. However, our incomplete market structure dampens the persistence impact on RER arising from habits. Under complete markets, as the real exchange rate will be equal to the ratio of marginal utilities across countries, the existence of habits in consumption decision will increase the persistence of RER. But, when the international asset markets are incomplete, the relation between the RER and relative consumption holds only in expected first differences not in levels. So the impact of habit channel on RER decreases.

There are three channels that result in failure of PPP in our model as discussed previously. The terms of trade will have an impact on the dynamics of the RER when the preferences are biased towards home produced goods. Recall the equation (48) which defines the impact of terms of trade on RER:

$$rertot_t = (\omega - (1 - \omega^*))(p_t^F - p_t^H)$$

The parameter ω indicates the degree of home bias in our model. When $\omega = 0.5$, the terms of trade will have no impact on the fluctuations of RER. The mechanism is as follows. With $\omega = 0.5$, consumers are indifferent between the consumption of imported and home produced traded goods. When, say, the price of domestically produced traded goods increase, because consumers do not have a preference bias, the demand and the price of imported goods will increase proportionately, leaving the relative prices unchanged. On the other hand, if preferences are biased towards the domestically produced goods, $\omega > 0.5$, the increase in price of imports will not be sufficient to offset the initial price increase in domestically produced traded goods ($\Delta p_t^H > \Delta p_t^F$), leading to a RER appreciation. So, the higher the degree of home bias the higher the influence of terms of trade on the variability of RER. The fluctuations of terms of trade depends on the value of elasticity of substitution between home and foreign traded goods. The changes in relative prices will be lower with a high elasticity of substitution; meaning high substitutability between home and foreign produced traded goods. As the movements in the relative prices identify the fluctuations of the terms of trade and, hence, the RER, a low (high) elasticity is expected to increase (decrease) the volatility of the terms of trade and RER.

The second channel which cause deviations from PPP arise from the multi-sector structure of the model. Even if the law of one price holds among traded goods changes in internal relative prices will cause deviations from PPP. This is the so called Balassa-Samuelson effect and can be explained through the equation (49):

$$rerint_t = (1-\alpha^*)(p_t^{N^*} - p_t^{T^*}) - (1-\alpha)(p_t^N - p_t^T)$$

When the productivity of traded goods sector and non-traded goods sector are driven by different stochastic processes, total factor productivity improvements will have different effects on RER dynamics. A productivity improvement in the traded sector ($p_t^H \downarrow$), on one hand, deteriorates the terms of trade, as exports become cheaper, and causes RER to depreciate if the preferences are biased towards home produced goods. But on the other hand, it increases the labour demand in that sector which in turn increase the nominal wages. As the nominal wages are common across sectors, this increase will raise the cost of production in non-traded goods sector, implying a rise in non-traded goods prices ($p_t^N \uparrow$). Hence the RER can appreciate if the effect of change in internal relative prices dominates the terms of trade impact.

The existence of non-traded goods sector can also account for the lack of international risk sharing (or Backus-Smith puzzle). Following a positive supply side shock at home country, the consumers from home country become richer. The increase in income, raises the relative consumption when there is home bias in preferences. As RER will decrease (Balassa-Samuleson effect) while relative consumption increase, it is possible to obtain a negative relationship between the relative consumption and RER following a positive TFP shock in traded sector. But, notice that the behaviour of the RER depends on where the shock has originated. A TFP improvement in non-traded goods sector would cause RER to depreciate (increase) and relative consumption to increase, implying a positive cross correlation between RER and relative consumption. In addition, the value of elasticity of substitution between traded and non-traded goods as well as the value of elasticity of substitution between home and foreign produced traded goods have crucial implications on the dominance of the Balassa-Samuelson effect [15]. Regarding the elasticity of substitution between non-traded and traded goods, a low elasticity implies higher internal relative price movements. The higher the movements in internal relative prices the higher the volatility of RER. In case of a positive supply side shock, the low elasticity between non-traded and traded goods will boost the appreciation in RER while relative consumption is increasing, thus it will decrease the correlation between RER and relative consumption. On the other hand, the depreciation in terms of trade can offset the decrease in RER arising from internal price movements depending on the elasticity of substitution between home and foreign produced traded goods. To account for the international lack of risk sharing, a high elasticity of substitution (smaller changes in relative prices of tradable goods) is necessary as it would imply a low volatility of terms of trade and a low impact on RER through terms of trade fluctuations [16].

Depending on the degree of pass through, nominal exchange rate fluctuations have different implications on the dynamics of the RER[17]. Recall the pricing to market component of RER to understand the mechanism fully:

$$rerptm_t = (1-\omega^*)(s_t + p_t^{H^*} - p_t^H) + \omega^*(s_t + p_t^{F^*} - p_t^F)$$

Take the case of a nominal exchange rate depreciation. Under PCP, a nominal exchange rate depreciation will make home produced traded goods cheaper than the foreign produced traded goods, hence the demand for home produced goods will rise. This is the classical expenditure switching effect. As the export prices are set in the currency of the producer, the depreciation will only increase the home currency price of imports implying a terms of trade deterioration. The magnitude of the change in the terms of trade will depend on the value of the substitution parameter which in turn will affect the RER variability through the home bias. On the other hand, when the prices are set in the currency of the buyer, the law of one price does not hold any more. A depreciation will

[15] See, Corsetti *et al* (2011) for an analytical representation of the existence of non-traded goods sector as a possible solution to Backus-Smith puzzle.

[16] Benigno and Thoenissen (2008) present an incomplete market structure with a price elasticity between non-traded and traded goods smaller than one and a price elasticity between home and foreign produced traded goods higher than one. Their model accounts for the lack of risk sharing when there are positive productivity improvements in traded goods sector.

[17] Our discussion follows the detailed analysis of Betts and Devereux (2000) on the implications of degree of pass through.

not change the relative prices, but it will change the relative income in favour of the exporting firms. The home currency price of exports will increase hence the terms of trade will improve. Because the depreciation does not change the relative prices of tradable goods when firms engage in pricing to market, fluctuations in terms of trade will be independent from the value of the elasticity of substitution parameter. So, the changes in terms of trade will only arise from the nominal exchange rate fluctuations. In addition, with LCP, the correlation between RER and nominal exchange rate will be higher as relative prices are not affected from the nominal exchange rate fluctuations. This is because of the fact that, when the exports are invoiced in the buyer's currency the changes in nominal exchange rate will not change the export prices keeping the CPI of both countries unchanged. Hence, as can be noticed from the equation (46), fluctuations of RER will track the changes in nominal exchange rate. Note that pricing to market behaviour will have an impact only in case of staggered prices; it has consequences in transitional dynamics. In long run, all prices will be flexible since in steady state the model approaches to the perfect competition equilibrium. Finally, take the case of intermediate degrees of pass through. When the number of firms that engage in PCP and LCP are equal ($d = 0.5$), the increase in home currency price of imports (from PCP) will be equal to the increase in home currency price of imports (from LCP), leaving the terms of trade at its initial level.

One of the characteristics of the model is the rich shock structure with country specific preference and monetary shocks as well as country and sector specific permanent productivity shocks. We also allow permanent technology shocks to be persistent in growth rates, so that the shocks diffuse into the economy slowly. The parameterisation of the shock processes have a big impact on the dynamics of the model. Take a model where fluctuations are driven by preference shocks. The link between RER and relative consumption growth can be broken even under complete markets since a positive demand shock increases the relative consumption and causes RER to appreciate as in the Mundell-Fleming model[18]. In fact, the correlations between variables will depend on the predominant source of fluctuations of the model. For instance, as in Benigno, Thoenissen (2008), the ability of model to capture lack of risk sharing can be attributed to the Balassa-Samuelson explanation in a model where the sources of fluctuations are supply side improvements in the traded sector. A relatively

[18]See, Stockman and Tesar (1994), Benigno and Thoenissen (2004), Corsetti *et al* (2008) and Corsetti *et al* (forthcoming) for an analysis on the importance of preference shocks to account for the features of the data.

low elasticity of substitution between non-traded and traded goods combined with a high trade elasticity can result in a negative correlation. The supply side improvements have also important implications in terms of income effects. When a unit root shock hits the economy, consumers adjust their consumption immediately, so the shock does not have an impact on the current account dynamics. However, persistent productivity innovations create incentives for further consumption smoothing, implying income effects. Intuitively, when the shock is persistent in growth rates, consumers will borrow this period as a result of higher income expectations in the next period. This mechanism provides extra uninsurable risk on top of incomplete market structure. Although agents' income increase, because they expect higher income next period, they borrow this period instead of lending as they would do in perfect risk sharing case.

4 Calibration

In this section we discuss the baseline calibration of the model which is reported in Table 1. We will discuss the sensitivity of the results to the calibrated values in Section 7. The parameter values are chosen such that the home country is USA and the foreign country is the Euro Area.

We set the discount rate to 0.99 so that steady state real interest rate is 4% per year. The two countries are assumed to have the same size ($n = 1 - n = 0.5$). Following estimations of Smets and Wouters (2002), the external habit formation parameter h is set to 0.55. As the structure of the technological progress is allowed to have permanent effects, we set the coefficient of relative risk aversion, ρ, to 1 in order to have balanced growth path in the model. The value of inverse of the Frisch elasticity of labour supply implies a Frisch elasticity equal to 0.2 ($\eta = 5$)[19]. Following Benigno (2001), we set the cost of intermediation in international asset market to 0.001.

For the share of traded goods in total consumption basket, we choose a value of 0.4 so that the share of non-traded goods is higher than the share of traded goods [20]. We set the degree of home bias to 0.9 based on the calculations of CKM for the share of imports of US from Europe. The calibration of elasticity

[19] Although, business cycle literature often assumes high elasticity of labour supply, the low labour elasticity is consistent with the micro evidence. See Gali, Gertler and López-Salido (2007) for further discussion.

[20] This value is in line with other studies; for instance Dotsey and Duarte (2008) set the share of traded goods in total consumption basket to 0.44 and similarly Devereux, Lane and Xu (2006) set it to 0.45. For an empirical discussion see Stockman and Tesar (1995).

of substitution between traded and non-traded goods as well as between home and foreign produced traded goods has important consequences on the dynamics of the model. To be able evaluate the performance of our model, we choose the value of the substitution parameters in accordance with the literature. The elasticity of substitution between traded and non-traded goods is set to 0.44 as in Stockman and Tesar (1995) and the elasticity of substitution between traded goods is set to 1.5 as in CKM. There are several studies in which they estimate the value of elasticity of substitution between traded and non-traded goods such as Mendoza (1991). He found a value of 0.77 which is still below one. On the other hand, there is a big heterogeneity in the literature on the calibration of elasticity of substitution between traded goods. The calibration of CKM is based on the findings of Backus, Kehoe and Kydland (1994). However, as discussed in Obstfeld Rogoff (2000a), there are estimates up to 5-6 as well as very low values, such as 0.94 in Rabanal and Tuesta (2010).

We take the value of elasticity of substitution among differentiated labour services from Erceg *et al* (2000) implying a 13% mark-up. We assume 4 quarters of contract duration for both wages and prices. We set the proportion of backward looking agents to 0.4 so that still the forward looking behaviour dominates the dynamics of price and wage inflation. The Taylor rule coefficients are taken from the estimates of Rabanal and Tuesta (2013). These coefficients have an important role in the dynamics of the model as the model is closed by the monetary policy rule. We follow the work of Rabanal and Tuesta (2013) since their estimations are based on the US and the Euro Area data.

Regarding the stochastic processes, the standard deviations of monetary and preference shocks as well as the autocorrelation coefficient of preference shocks are specified again from Rabanal and Tuesta (2013). The monetary and preference shocks has approximately the same relative standard deviation to technology shocks as in their estimations [21] In order to specify the evolution of the technological progress, we estimated a VAR for TFP series for the home and foreign traded and non-traded sectors. The details of calculation can be found in Appendix A. Briefly, we took disaggregated data for sectors of the Euro Area and US of about 30 sectors and then aggregated them by large sectors. The data for this calculation is in annual frequency and covers the period 1981-2007. Through calculating value added shares of sectors, we constructed TFP growth series from the TFP estimates of Euklems. It is assumed that agriculture, mining

[21] We calculated the standard deviations of preference and monetary shocks relative to technology shocks from the estimations of Rabanal and Tuesta (2013) and set the standard deviations of our shocks with a similar relative magnitude.

and manufacturing are traded sectors and the remaining are non-traded sectors. However, as the series are of annual frequency we constructed quarterly series by using Chow, Lin (1971) method. As a reference, we took quarterly real output series from the OECD for the period 1981:Q1-2007:Q4. We approximated the output in traded sector by the real output in *industry* and non-traded by *market services*. Industry accommodates sectors of mining, manufacturing, electricity, gas and water. While market services accommodate retail and wholesale trade, transportation, communication, information, accommodation, restaurants, financial intermediation, insurance and real estate sectors. For Euro Area data, we use the total of Germany, France, Italy and Spain as this covers most of the output. We conducted ADF test on the interpolated series and found that the technology shocks in traded goods sector for both countries have permanent effects. We specified the process of the shocks through VAR estimates. The permanent TFP shock in traded sector is persistent in growth rates in the US and it is an exact unit root in the Euro Area. On the other hand, the technological improvement in non-traded goods sector is a very persistent transitory shock in both countries. We took the correlations and spillovers of the shocks from our estimations of VAR as well[22].

5 Quantitative Properties of the Calibrated Model

We now examine the performance of our simulated model compared with the data. We present the incomplete markets and complete markets versions of different pricing regimes. The LCP case refers to $d = 1$, PCP refers to $d = 0$ and partial LCP refers to $d = 0.5$[23].

The moments are calculated by assuming the US as home country and the Euro Area as foreign country. Our data covers the period between 1990:Q1 and 2012:Q3. Because the model is log-linearised around the deterministic steady

[22]We do not take into consideration the values which are very close to zero in the variance-covariance matrix.

[23]The empirical evidence on exchange rate pass through supports the partial exchange rate pass through as a modelling strategy. Recently, Campa and Goldberg (2005) provide evidence in favour of intermediate degrees of pass through among OECD countries especially in the short run. They argue that the exchange rate pass through is gradual as also emphasised by Engel (1999) and Goldberg and Knetter (1997). On the other hand, Obstfeld and Rogoff (2000b) highlight the empirical support on the expenditure-switching effect of pass through (implying PCP) while Engel and Rogers (1996) and Gopinath *et al* (2001) provide evidence in favour of pricing to market behaviour in international markets. Moreover, Engel (1999) and CKM argue that the deviations from PPP can be attributable to the international price differentials between traded goods (failure of LoOP).

Table 1: Calibrated Values for US and Euro Area

Parameter	Value	Description
β	0.99	discount factor
$n = 1 - n$	0.5	relative country size
$h = h^*$	0.55	habit persistence
ρ	1	coefficient of risk aversion
η	5	inverse Frisch elasticity of labour supply
δ	0.001	cost of intermediation
$\alpha = \alpha^*$	0.4	share of traded goods in total consumption
$\omega = \omega^*$	0.72	degree of home bias
ν	0.44	elasticity of substitution: tradable and non-tradable goods
θ	1.5	elasticity of substitution: home and foreign tradable goods
σ_w	4	elasticity of substitution across types of labour
$\xi_p = \xi_p^* = \xi_w = \xi_w^*$	0.75	Calvo prices/wages
$\varsigma_p = \varsigma_p^* = \varsigma_w = \varsigma_w^*$	0.4	price/ wage index
Taylor Rule Coefficients		
Γ_i	0.88	interest rate smoothing- US
Γ_{i^*}	0.76	interest rate smoothing- Euro Area
Γ_y	0.90	response to output-US
Γ_{y^*}	0.56	response to output- Euro Area
Γ_π	2.05	response to inflation- US
Γ_{π^*}	2.72	response to inflation-Euro Area
AR Coefficients- Shocks		
ρ_τ	0.88	Preference- US
ρ_{τ^*}	0.87	Preference- Euro Area
ρ_{aT}	0.8	Permanent technology shock: tradable sector- US
ρ_{aN}	0.9	Temporary technology shock: nontradable sector- US
ρ_{aT^*}	0	Permanent technology shock: tradable sector- Euro Area
ρ_{aN^*}	0.9	Temporary technology shock: nontradable sector- Euro Area
$\rho_{aT^*,T}$	0.26	Correlation between shock in traded sector of EU and US
Standard Deviation of Shocks		
ε_τ	0.0191	Preference- US
ε_{τ^*}	0.0189	Preference- Euro Area
ε_m	0.0011	Monetary- US
ε_{m^*}	0.0016	Monetary- Euro Area
ε_{aT}	0.016	Technology shock: tradable sector- US
ε_{aN}	0.011	Technology shock: nontradable sector- US
ε_{aT^*}	0.029	Technology shock: tradable sector- Euro Area
ε_{aN^*}	0.006	Technology shock: nontradable sector- Euro Area
ε_{aT^*,aN^*}	0.00012	Covariance of technology shocks: Euro Area

state, we take logs of the data as well so that the results will be consistent. We used the following series for our analysis: We took the nominal exchange rate series form IMF, IFS data base, which is defined as US Dollars per Euro in market rate. To calculate the RER, we used the CPI of the US and the Euro area. The CPI of the US is taken from the OECD's Main Economic Indicators database and the CPI of the Euro Area is taken from the EUROSTAT. We used Engel's (1999) approach[24] to decompose the aggregate price index into its traded and non-traded goods components. We use the producer price index (PPI) as a measure of tradable goods prices. The PPI series for both US and Euro Area are obtained from the OECD's Main Economic Indicators database for total manufacturing. The terms of trade is obtained from the Bureau of Labour Statistic (BLS) by taking the ratio of import prices from EU to the aggregate export price index of US. Both price indices are in US dollars and as there is no price information about imports from the Euro Area, we use EU as an approximate[25]. The series of output and consumption for both US and Euro Area were taken from the OECD's Main Economic Indicators database. Output is the total GDP by expenditure and consumption is the private final consumption expenditure. We use seasonally adjusted index series, in national currency. Net exports are constructed by dividing the trade balance of US with Euro Area by the nominal GDP of US. The trade balance is taken from the BLS. While computing the net exports we kept both series in levels. Finally, the correlation between relative consumption and RER is calculated by using the logged series in the following way: $Corr[rer_t \, , \, (c_t^{US} - c_t^{EA})]$. Further details on the description of data are given in the Appendix A. In our analysis, the standard deviation of the variables are shown relative to standard deviation of US GDP.

Having permanent technology shocks in traded sector causes non-stationarity in the model. In order to have a well behaved steady state, we transform the

[24]The internal RER is constructed by using the following decompositon:

$$InternalRER = [ln(CPI^{US}) - ln(PPI^{US})] - [ln(CPI^{EA}) - ln(PPI^{EA})]$$

[25]The reported moments of bilateral terms of trade is an approximation as a result of data limitations. The data on import prices of Euro Area in terms of country of origin is not available. In order to get the closest approximation to bilateral trade, we used the difference between price of imports from EU and aggregate export price index of US. We also constructed the series by using aggregate import prices of Euro Area but the volatility of terms of trade became extremely high. As an alternative, we calculated the difference between aggregate import and export price indices of US. This calculation had a very close volatility to the volatility of the calculated series that we report here.

trended variables. The derivation of trends and transformation of the model can be found in Appendix D. For our analysis, we are interested in the performance of the model for both business cycle and higher frequencies. We will use HP filter[26] for the business cycle frequencies and first differencing for high frequencies[27]. However, our stationary model does not necessarily fit with the HP filtered or first differenced series. Andrle (2008) discuss that in reality, cyclical dynamics are not independent from trends (*trend-cycle interactions*). He argued that the permanent shocks have an impact on all frequencies; implying neither HP filtering nor 1st differencing removes the affect of permanent shocks from the observed series. Therefore, as the stationarised model extracts the dynamics induced by permanent shocks, the properties of de-trended model may not match with the HP filtered or first differenced data properties. In addition, Canova (1998) shows that different de-trending methods have different implications for the covariance structure of the data. The ability of the theoretical model to replicate the features of the observed series is dependent on the use of consistent filtering methods since the moments that filtered data provides differ with respect to the choice of filtering method. With this regard, while simulating the model we used level variables; for the business cycle frequencies, we are comparing both HP filtered model and the data and for the higher frequencies, we are using the first difference of level variables along with the 1st differenced data moments. Hence, the statistics obtained from the actual data and from the model has compatible characteristics.

For the business cycle frequencies, the second moments generated by the model in comparison with the data are reported in Table 2. The statistics obtained from the data shows that both real and nominal exchange rate are very volatile and persistent; they are about 5 times more volatile than GDP with autocorrelation of 0.7. The terms of trade is also highly persistent with autocorrelation of 0.78; but the volatility obtained from our HP filtered data for terms of trade is less than one. The volatility of internal relative prices generated by our data matches with the calculation of Benigno, Thoenissen (2003) for the UK and Euro Area data. Its persistence is not as high as other international variables. Looking at the business cycle statistics, both consumption and output has autocorrelation about 0.9 indicating very high persistency. Consumption is

[26]We choose 1600 for the smoothing parameter of HP filter as we are working with quarterly data.

[27]First differencing extracts the long-run information from the series leaving high frequency features of the data. On the other hand, HP filter removes the movements lower than business cycle frequency; i.e. the information obtained from the HP filtered data corresponds to frequencies around 8-32 quarters.

0.84 as volatile as GDP, similar to the calculation of other studies [28].

Our model matches with data to a certain extent. Regarding persistence, the model performs relatively better overall. Particularly, the model does a reasonable job in replicating the persistence of RER and consumption. The persistence of output and nominal exchange rate is slightly higher than the data, but still reasonably close to the observed value. The persistence of terms of trade in polar cases of pass through is about right. However, its volatility is too high in extreme cases of pass through. But remember that our calculation of bilateral terms of trade is an approximation as a result of data constraints. Other studies mostly reported values above 1 and in some cases even above 2 (Benigno and Theonisson (2008) 2.12; Benigno and Theonisson (2003) 1.2; Corsetti *et al* (2008) 1.68; Dotsey and Duarte (2011) 1.96). So our finding is relatively acceptable. Nevertheless, it is worth to mention that, none of the mentioned studies use bilateral terms of trade calculation. On the other hand, the volatility of terms of trade is higher than RER in both PCP and LCP regimes which is at odds with the data. Our model gets closer to data in partial LCP case. The volatility of terms of trade can be reduced by increasing the value of elasticity of substitution between home and foreign produced traded goods. Motivated by the discussion of Imbs and Méjean (2009) [29], we increase the value of elasticity of substitution between foreign and domestic goods to 5. Expectedly, with large elasticity the volatility of terms of trade is smaller than RER in all pricing regimes. We further test the sensitivity of the volatility of terms of trade to the values of elasticity of substitution in our robustness exercise.

The model matches the volatility of neither the RER nor the nominal exchange rate. In fact, our model generates the correct variability of RER but at the cost of really high output volatility [30]. As we report the standard deviations relative to GDP, the RER variability appears to be too little. Note that the relative RER volatility we obtain from our model is similar to the findings of other studies. For instance; it is 1.5 in Dotsey, Duarte (2008) or 1.01 in Tuesta (2013). The internal relative price movements are, on the other hand, too volatile compared to data, but still below RER volatility. In addition, our

[28] For instance, it is 0.79 in CKM, 0.94 in Corsetti *et al* (2008) and 0.76 in Benigno, Thoenissen (2008).

[29] Imbs and Méjean (2009) discuss that the diversion between micro and macroeconomic studies about the value of elasticity of substitution between home and foreign produced traded goods arise from the difference between aggregation. By allowing heterogeneity among varieties, they obtain a value up to 5 which is in line with the micro studies.

[30] This problem arises in Rabanal and Tuesta (2010) as well.

model produces excessive consumption volatility in spite of the habit formation in consumption decisions. This is a consequence of our benchmark calibration; the high share of non-traded goods combined with high degrees of home bias increases the volatility of consumption as in a closed economy model. Also the high substitutability between home and foreign produced traded goods implies a higher consumption variability. We checked the sensitivity of the consumption volatility to the calibration of these parameters. We set the share of non-traded goods almost equal to zero ($\alpha = \alpha^* = 0.9$), decrease the degree of home bias ($\omega = \omega^* = 0.7$) and we set the elasticity of substitution between home and foreign traded goods to a near Cobb-Douglas ($\theta = \theta^* = 0.9999$). With this parameterisation, the volatility of consumption decreases to 0.86 relative to GDP which is almost equal to the volatility obtained from the data (0.84). A possible extension to get the correct volatility, would be introduction of capital to the model set-up as well as habit formation.

Our findings do not present a substantial difference between pricing regimes with the only exception being the behaviour of terms of trade. This result confirms the findings of Dotsey and Duarte (2011). In our setting, the high share of non-traded goods in preferences as well as nominal rigidities reduces the impact of the pass through. Even so, the correlation between the terms of trade and other variables will depend on the degree of pass through. Here, we are focussing on the correlation between the RER and the terms of trade. The correlation between terms of trade and RER in business cycle frequencies is positive, confirming both the Mundell-Flemming explanation and the empirical evidence presented by Obstfeld and Rogoff (2000b). Consistently, our model produces positive co-movement under PCP and partial LCP regimes. Despite of matching the correct signs in these two pricing regimes, the model does not perform well in terms of magnitude.

The model does a good job in producing high correlation between real and nominal exchange rate in both frequencies. It also captures negative correlation between net exports and output as well as RER and output. Under incomplete markets, this a natural result of the UIP condition. An increase in output would increase the interest rates which in turn appreciates the RER hence, decrease the net exports. But in our HP filtered model, counter-cyclical behaviour of net exports is not as severe as in data and only generated under LCP and partial LCP with complete markets. The net exports are appear to be acyclical in PCP and partial LCP with incomplete markets. The spillover between traded shocks along with the expenditure switching channel dampens the income effects. In fact when we abstract the model from the spillover between shocks, the model generates high counter-cyclical behaviour both in LCP and partial LCP regimes. Also the negative correlations we obtain, are not sensitive to the asset market structure which again emphasises the wealth effects arising in the our model set-up and calibration.

As a result of the characteristics of our sample period, the correlation between both foreign and domestic consumption and output in our data is relatively high among the studies using US an Euro Area data. For instance, the correlation between foreign and home consumption and output in CKM is 0.27 and 0.52 respectively. In Rabanal, Tuesta (2013), these values are 0.18 and 0.3 respectively. Although our model generates lower correlations for both variables what we obtain is still moderate regarding the calculations of other studies. In our model setting, the predicted correlation is consequence of the spillover between the shocks across countries. When we remove the spillover effect, the correlation is very low and negative in some cases. The high share of non-traded goods in consumption baskets along with the high degree of home bias also reduce the co-movement of consumption. In fact, the existence of non-traded goods itself decreases the co-movement of consumption across countries. To see the quantitative implications of these two parameters, we set the share of non-traded goods to 0.9 ($\alpha = \alpha^* = 0.9$), and removed the home bias from preferences ($\omega = \omega^* = 0.5$). The cross correlation between consumption and output increased substantially ($C - C^* = 0.81$ and $Y - Y^* = 0.73$). We further fix the substitution parameter between home and foreign produced traded goods to Cobb-Douglas and, expectedly, it increased the cross correlation of consumption even more ($C - C^* = 0.89$).

The higher frequency properties of the data and the model are shown in Table 3. Because first differencing leaves the high frequency features of the data, the volatility of variables are much higher and the persistence are much lower compared to HP filtered moments. Broadly, the model does relatively better in explaining business cycle frequencies. The first differenced model generates too much persistence and too little volatility compared with the data. Looking at the correlations of the variables, the first differenced moments differ from the HP filtered moments in two dimensions. First, the correlation between the relative consumption and RER becomes negative in data. However, as argued in Corsetti *et al* (2012), Backus-Smith puzzle is more related to low and business cycle frequencies rather than high frequency. Therefore, we do not rely on the first differenced data/model moments in our consumption-risk sharing analysis. Second, the correlation between the RER and terms of trade, in short run, is negative. This is consistent with the previously discussed empirical studies on the lack of pass through to prices in short run. Our model generates the correct sign in LCP regime both in complete and incomplete markets settings. Our business cycle and high frequency analysis proves that, the choice of pricing regime as a modelling strategy depends on the frequency that researcher wants to address.

Looking at both Table 2 and Table 3, an evident results is the indifference of the variable moments to complete/incomplete market structures. This is a consequence of our model set up, as there are various channels that create income and wealth effects apart from the incomplete market structure. Importantly, our model accounts for the consumption-RER anomaly regardless the degree of pass through and market completeness. In our HP filtered data, the correlation between RER and relative consumption is positive but very close to zero. The low cross-correlation is in line with the empirical evidence on lack of risk sharing presented in other papers[31]. Our results show that even the PCP model with complete markets is sufficient to break the link between RER and relative consumption. The negative correlation, the model produces is in fact too negative confirming large wealth effects arising from model set-up. The main driving source of the fluctuations in our model is the tradable sector shock which is originated in home country (see Table 4). Because this shock is persistent in growth rates, as discussed previously it generates further wealth effects. Furthermore, the low elasticity between non-traded and traded goods, high share of non-traded goods in consumption combined with relatively high trade elasticity, which in turn dampens the changes in terms of trade, amplifies the dominance of Balassa-Samuelson channel in our model. To see the impact arising from Balassa-Samuelson effect we set the share of non-traded goods to almost zero ($\alpha = \alpha^* = 0.9$), and simulate the model for incomplete markets set-up. In the absence of non-traded goods, the cross correlation between RER and relative consumption is positive regardless the degree of pass through ($PCP = 0.68, \;\; LCP = 0.64, \;\; Par.LCP = 0.66$). So, above all, our result is very important considering the attempts to explore the Backus-Smith puzzle in international business cycle models. We show that with correct parametrization, a theoretical framework which accommodates non-traded goods can produce the lack of risk sharing at both medium and high frequencies. This result is consistent with the findings of Benigno and Thoenissen (2005, 2008), Tuesta (2013) and also with the discussion presented in Corsetti *et al* (2011).

[31]See Corsetti *et al* (2012) for an empirical list of the correlation between RER and relative consumption for various countries: out of 19 countries, 17 countries display negative consumption-RER correlation against US at business cycle frequencies.

Table 2: Selected HP-filtered Moments

	Data	**PCP**		**LCP**		**Par.LCP**	
		Comp.	Incomp.	Comp.	Incomp.	Comp.	Incomp.
Std.dev.Rel.Y							
C	0.8438	1.025	1	1	1	1.0035	1
RER	5.2801	1.0824	1.0428	1.2616	1.2629	1.1648	1.125
TOT	0.9116	1.387	1.3321	1.2724	1.2629	0.9462	0.8285
NER	5.1890	1.379	1.3892	1.4838	1.5333	1.4265	1.4285
Int.RER	0.6850	0.8781	0.875	0.8745	0.9111	0.8745	0.875
Autocorrelations							
Y	0.88	0.92	0.93	0.93	0.93	0.93	0.93
C	0.91	0.92	0.91	0.92	0.92	0.92	0.92
RER	0.71	0.65	0.72	0.65	0.68	0.65	0.7
TOT	0.78	0.73	0.79	0.69	0.7	0.94	0.94
NER	0.69	0.73	0.8	0.74	0.77	0.74	0.78
Int.RER	0.64	0.95	0.95	0.96	0.95	0.95	0.95
Cross-Correlations							
RER-NER	0.9960	0.89	0.9	0.9	0.9	0.9	0.9
NX-Y	-0.5045	0.06	0.1	-0.29	-0.12	-0.08	0.0016
RER-Y	-0.1022	-0.44	-0.47	-0.49	-0.46	-0.46	-0.46
*C-C**	0.6332	0.037	0.32	0.41	0.38	0.39	0.35
*Y-Y**	0.7934	0.35	0.37	0.36	0.37	0.48	0.37
RER-TOT	0.2267	0.68	0.65	-0.547	-0.659	0.19	0.0127
RER-Rel.C	0.0091	-0.3	-0.53	-0.28	-0.44	-0.3	-0.49

Table 3: Selected 1st Differenced Moments

	Data	**PCP**		**LCP**		**Par.LCP**	
		Comp.	Incomp.	Comp.	Incomp.	Comp.	Incomp.
Std.dev.Rel.Y							
C	0.8	1.022	1.0833	1	1	1.015	1.0833
RER	7.8	1.9469	1.8583	2.2307	2.13	2.106	2.06
TOT	1.2	2.1967	2.1	2.1538	2.06	0.7633	0.8333
NER	7.7666	2.2727	2.19	2.3846	2.28	2.29	6.58
Int.RER	1.1333	0.8333	0.9333	0.7692	0.76	0.8396	0.9166
Autocorrelations							
Y	0.4357	0.69	0.77	0.71	0.71	0.71	0.75
C	0.6249	0.69	0.66	0.7	0.69	0.69	0.66
RER	-0.0832	0.003	0.005	0.03	-0.03	0.001	-0.017
TOT	0.0958	0.1	0.1252	0.07	-0.01	0.89	0.89
NER	-0.0657	0.08	0.1301	0.11	0.069	0.095	0.09
Int.RER	-0.1456	0.8321	0.8493	0.93	0.93	0.89	0.9
Cross-Correlations							
RER-NER	0.9935	0.96	0.95	0.96	0.96	0.96	0.96
NX-Y	-0.2607	-0.05	0.2	-0.19	-0.27	-0.04	0.03
RER-Y	-0.0628	-0.13	-0.43	-0.29	-0.5	-0.21	-0.46
*C-C**	0.5700	0.27	0.25	0.35	0.33	0.31	0.29
*Y-Y**	0.6268	0.36	0.35	0.34	0.33	0.36	0.35
RER-TOT	-0.0535	0.89	0.86	-0.9	-0.91	0.009	0.06
RER-Rel.C	-0.05	-0.48	-0.54	-0.43	-0.43	-0.46	-0.49

Table 4: Variance Decomposition (in percentages)

Vbl. Name	ε_m	ε_{m^*}	ε_τ	ε_{τ^*}	ε_{aT}	ε_{aN}	ε_{aT^*}	ε_{aN^*}
Y	0.58	0	2.27	0.01	96.8	0.33	0.01	0
C	0.51	0	3.6	0.17	95.32	0.36	0.04	0
$Y*$	0	0.91	0.01	3.74	14.38	0	80.54	0.42
$C*$	0	0.84	0.31	6.14	13.72	0.01	78.56	0.41
RER	0.81	0.95	18.35	16.95	57.47	0.71	4.34	0.41
TOT	0.08	0.06	10.79	8.9	38.17	0.71	41.02	0.25
NER	0.54	0.64	12.63	11.68	58.79	0.51	14.91	0.3
Rel. C	0.45	0.47	4.98	5.16	43.88	0.41	44.41	0.25
NX	1.54	1.48	10.85	7.92	11	1.07	65.13	1.02

Note: We report the variance decomposition for the Partial LCP with incomplete markets version of the model.

Most of the general equilibrium models make their analysis on temporary shocks. To see the impact arising from our TFP process we now simulate our model for country and sector specific temporary productivity shocks. Table 5 shows the data and model moments when all technology shocks are assumed to be temporary, but very persistent [32]. We only report the incomplete market version, as our results do not imply any significant difference between the two market structures. The model does much better in producing the volatility of RER as well as nominal exchange rate especially in LCP case. Regarding the persistence, local currency pricing model fits the data best although the differences are quantitatively very small between pricing regimes. On the other hand, when the shocks are temporary the model generates excess consumption volatility and the cross-correlation between home and foreign consumption and output is very low. This is due to high share of non-traded goods as well as the difference in shocks' structure. In addition when the shocks are temporary the fluctuations are mainly driven by preference shocks, not technology shocks any more [33] . Therefore, the impact of spillover effect on the correlation of

[32] Now all technology shocks assumed to follow the following process:

$$ln(A^j_{t+1}) = \rho^{a^j} ln(A^j_{t-1}) + \varepsilon^{a^j}_t\ , \quad with\ \ \rho^{a^j} = 0.9$$

[33] Under temporary shocks, the variance decomposition shows that, 45% of RER volatility is driven by home country preference shocks and 43% of its volatility is driven by foreign country preference shocks. Similarly, preference shocks explain 58% of output fluctuations and 67% of consumption fluctuations.

variables across countries is much lower. The volatility of consumption is also mostly driven by preference shocks, increasing its variability. For further exploration on the wealth effects and the role of non-traded goods to account for Backus Smith anomaly, we set the share on non-traded goods to a sufficiently low value ($\alpha = \alpha^* = 0.9$) and check the cross correlation between RER and relative consumption. In this scenario, shutting down the non-traded goods do not change the negative cross-correlation, our model generates still very large negative relationship. This is because the predominant source of fluctuations are no longer productivity shocks, it is preference shocks. In fact, when we remove the preference shocks from our simulation, the existence of non-traded goods is not sufficient to account for the lack of risk sharing even though the fluctuations are driven by technology shocks; the correlation is almost equal to one.

Table 5: Temporary Shocks: Selected HP-filtered Moments

	Data	**PCP**	**LCP**	**Par.LCP**
Std.dev.Rel.Y				
C	0.8438	1.33	1.14	1.2244
RER	5.2801	4.104	4.44	4.2653
TOT	0.9116	5.0833	5.36	3.9795
NER	5.189	4.875	5	4.9387
Int.RER	0.685	2.33	2.14	2.2244
Autocorrelations				
Y	0.8858	0.82	0.81	0.82
C	0.9178	0.83	0.83	0.83
RER	0.7145	0.78	0.74	0.76
TOT	0.7807	0.81	0.82	0.94
NER	0.6954	0.81	0.79	0.8
Int.RER	0.6446	0.94	0.95	0.94
Cross-Correlations				
RER-NER	0.996	0.89	0.9	0.9
NX-Y	-0.5045	-0.36	-0.36	-0.376
RER-Y	-0.1022	-0.32	-0.42	-0.37
*C-C**	0.6332	-0.14	0.017	-0.07
*Y-Y**	0.7934	0.13	0.08	0.1154
RER-TOT	0.2267	0.83	-0.27	0.39
RER-Rel.C	0.0091	-0.61	-0.56	-0.6

To conclude, a model with non-traded goods sector and permanent technology shocks can account for the consumption-RER anomaly. We show that neither of them can account for the puzzle without the other. In our benchmark model, when we abstract the model from non-traded goods, the risk sharing is close to be perfect as in CKM. In addition, our simulations with the temporary shocks present the importance of predominant source of fluctuations. In a model where preference shocks drive the behaviour of macroeconomic variables, the existence of non-traded goods is inconsequential for the correlation between RER and relative consumption; absence of preference shocks implies almost perfect risk sharing. Hence, in our benchmark simulations the large wealth effects are arising from the combination of the multi-sector set-up and our estimated TFP shock structures. With correct shock specification, the model can account for the Backus Smith puzzle bu at the cost of other moments.

6 Transmission of Productivity Shocks

In this section, we look at the dynamic responses of the RER and its components following supply side innovations. By doing so, we will be able to evaluate the importance of each component on the dynamics of RER. We report the non-stationarised dynamics in order to interpret them relative to the initial steady state. We are using the incomplete markets version of the model with polar cases of pass through ($d = 1 \, and \, d = 0$) in levels. The permanent technology shock to the US traded sector is persistent in growth rates and the non-traded shock is a persistent transitory shock, as mentioned in the calibration.

Figure 2 depicts the impulse responses to a productivity improvement in the traded goods sector in home country. Following the shock, the RER appreciates. The source of appreciation is the increase in internal relative pries conforming Balassa-Samuelson proposition as can be seen from the decrease in internal RER.

The degree of pass through has implications on home bias and pricing to market channels. Notice that, under PCP, the pricing to market channel becomes inconsequential. In the LCP case, however, the pricing to market channel falls when a positive productivity shock in traded sector hits the economy. This is because of the dominant nominal exchange rate appreciation combines with a high degree of home bias and staggered price setting. An exchange rate appreciation decreases the home currency price of exports while relative prices remain the same. As a result, RER appreciates through this channel. As expected, the home bias channel depreciates in both pricing regimes but there is an initial appreciation in the PCP case which is again consequence of nominal rigidities with high degree of home bias. With nominal rigidities, the movements in nominal exchange rate has an impact on the RER fluctuations. As a matter of fact, the incomplete market structure reduces the movements in relative prices through asset accumulation, causing an appreciation of the terms of trade, in addition to the implicit income effect arising from the productivity innovation in traded sector. Recall that, when the shocks are persistent in growth rates, there are further incentives to smooth consumption.

The effects of a positive productivity shock to the non-tradable sector is shown in Figure 3. The RER depreciates when a productivity increase occurs in non-traded sector despite the nominal exchange rate appreciation. This comes from the strong depreciation of the internal RER. Both the terms of trade and the pricing to market channel decrease as the home produced traded goods become more expensive now.

To sum up, the overall dynamic adjustment of the RER is in line with Balassa-Samuelson explanation. RER and internal relative prices move in the same direction as a consequence of relatively high share of non-traded goods in the consumption basket, a low elasticity between non-traded and traded goods and a high elasticity between traded goods. Similar transmission mechanism has also been presented in several general equilibrium frameworks such as Benigno and Thoenissen (2008) and Selaive and Tuesta (2003)[34]. It is worth to emphasise that the transmission mechanism we presented is not only the result of our multi-sector structure but also the strong wealth effects arising from the model set-up and our calibration.

7 Robustness

To test the sensitivity of our results to the calibration, in this section, we are presenting the results of the robustness exercises. We set the parameters be-

[34]Balassa-Samuelson effect has been tested empirically in several papers. Recently, Berka and Devereux (2013) found that the relative price of non-traded goods to traded goods has a big impact on the RER fluctuations and RER is very closely related to GDP per capita among European countries. Their finding is in line with the so called "Penn-effect" and/or Balassa-Samuelson effect. Also, Asea and Mendoza (1994), Canzoneri *et al* (1999), Kakkar (2003) found supportive evidence on the existence of Balassa-Samuelson relationship. On the other hand, there are others which do not find observational evidence; e.g. Strauss (1998), Faria and León-Ledesma (2003).

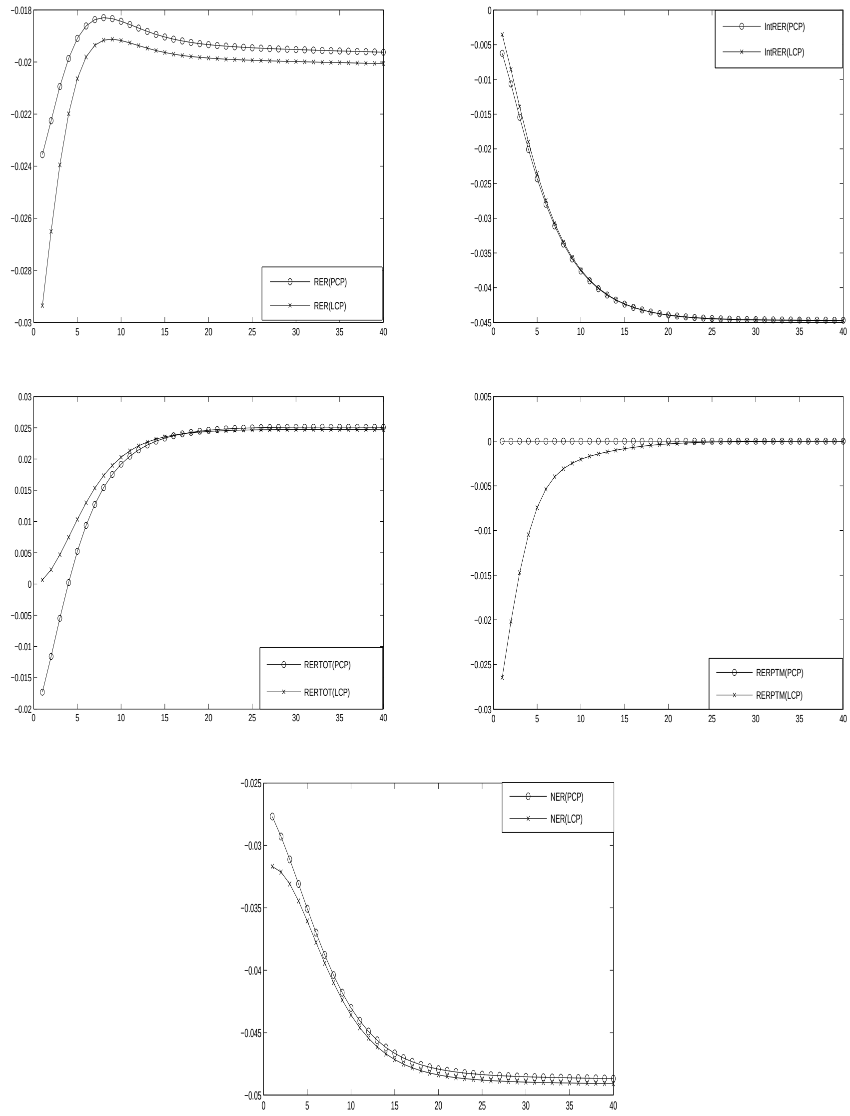

Figure 2: **Positive technology shock to the US Traded Goods Sector**

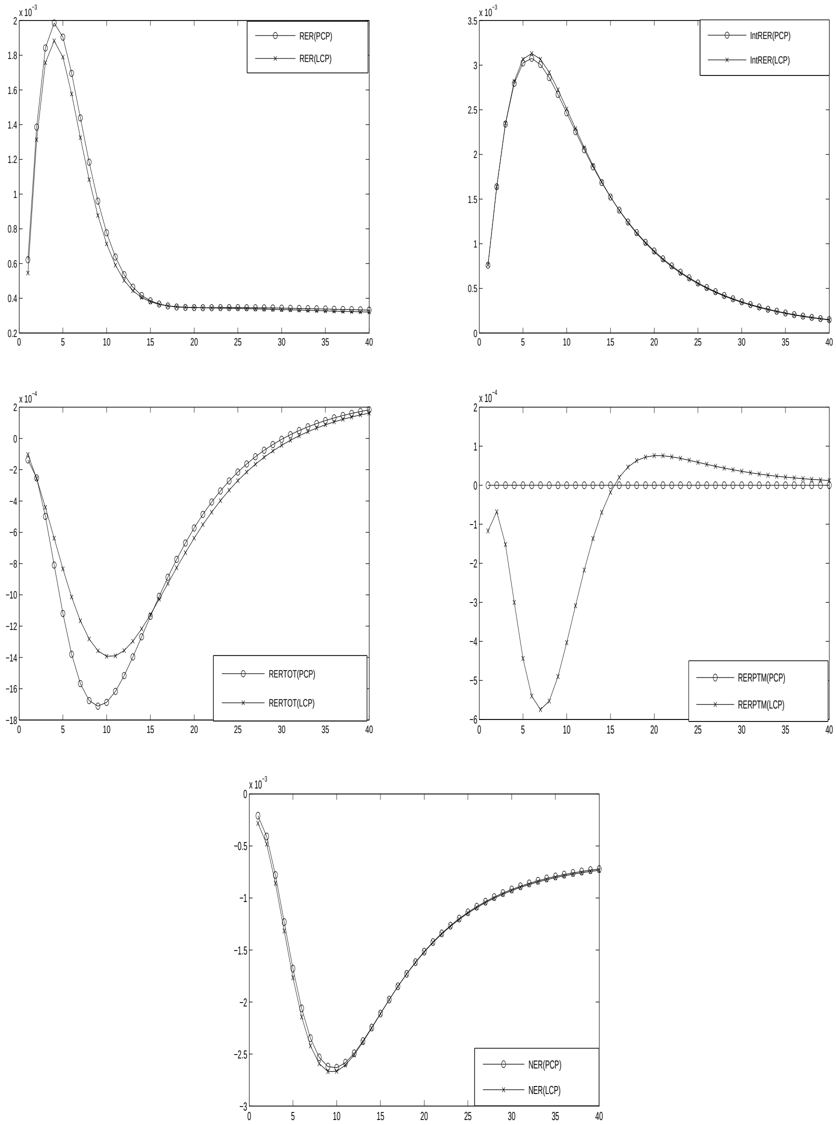

Figure 3: **Positive technology shock to the US Non-Traded Goods Sector**

tween low and high values and obtain a range of model moments. To do so, we run two separate Monte-Carlo experiments. The Monte Carlo simulation generates parameter values by drawing randomly from a uniform distribution over the parameter range. In the first Monte Carlo experiment, we set the preference parameter from low values to high values and maintain the monetary policy parameters and the shock parameters as in our baseline calibration. For the second one, we investigate the importance of parameterisation of the shocks and monetary policy coefficients leaving the other parameters as in our baseline calibration. For both experiments, we report the median values of the calculated moments along with the 10% and 90% percentiles.

The Monte Carlo parameters of the first experiment are listed in the Table 6[35]. For the habit parameter, we choose a very low value which almost removes the habit formation from consumption. And as a high value we choose 0.7. We let the degree of pass through to vary between the polar cases of pass through. The share of traded goods are set such that non-traded goods can have higher or lower share in aggregate consumption basket. The degree of home bias ranges from no home bias to really high degrees of home bias. As discussed previously the calibration of elasticity of substitution between home and foreign produced traded goods is controversial. We set it between a very low and a high value to accommodate the different estimations from the literature. The elasticity of substitution between non-traded and traded goods ranges from a low value of 0.13, based on the estimations of Rabanal and Tuesta (2013), to a value above unity, 1.2. We set the range for the proportion of Calvo wage/price setters from almost fully flexible to almost completely rigid within a period. The range for the proportion of backward looking firms are chosen such that the forward looking behaviour still dominates in Phillips Curve. The proportion of backward looking households ranges from almost zero, based on the empirical estimates of Rabanal and Tuesta (2010), to a high value, 0.66 as in Smets and Wouters (2002).

The results obtained from the Monte Carlo experiment on preference parameters are shown in Table 7. Our experiment shows that the moments generated by our model do not differ substantially from the calculated mean values. However, looking at the intervals, one can realise that the range is quite large for almost all parameters. In fact, the cross correlation of variables in most cases vary from positive values to negative values. This high variability proves the sensitivity of the results to the calibration of the model.

[35] The parameters which are not listed in Table 6 are kept as in the baseline calibration.

Table 6: Monte Carlo Parameter Ranges: Preferences

Parameter	**Range**	**Description**
$h = h^*$	[0.05, 0.7]	habit persistence
$d = d^*$	[0, 1]	degree of pass through
$\alpha = \alpha^*$	[0.2, 0.6]	share of traded goods in total consumption
$\omega = \omega^*$	[0.5, 0.95]	degree of home bias
θ	[0.5, 5]	elasticity of substitution: home and foreign tradable goods
ν	[0.13, 1.2]	elasticity of substitution: tradable and non-tradable goods
$\xi_p = \xi_p^*$	[0.01, 0.9]	Calvo prices
$\varsigma_p = \varsigma_p^*$	[0.2, 0.49]	proportion of bwd. looking firms
$\xi_w = \xi_w^*$	[0.01, 0.9]	Calvo wages
$\varsigma_w = \varsigma_w^*$	[0.1, 0.66]	proportion of bwd. looking households
Monte Carlo draws	200	

The model generates too much persistence for international variables, although the correct values coincides into intervals. Regarding the volatility, the model creates too much output and consumption variability. The volatility of RER generated by the model actually perfectly matches with the volatility obtained from the data. But relative to GDP volatility, our model generates too little RER volatility. Remember that, in our benchmark calibration, the model was generating excess terms of trade volatility relative to RER in the extreme cases of pass through. Our Monte Carlo experiment shows that the volatility we obtained from the partial degrees of pass through version of the benchmark model is robust as the variability terms of trade is below the volatility of RER in Table 7. The median of cross correlations obtained from our model matches with the data in terms of sign but not the magnitude. The correlation between the relative consumption and RER is negative and very low as in our benchmark results.

As a result of the sensitivity of the results to the calibration of preference parameters, we further explore the robustness of our results by making a second Monte Carlo experiment on the shock processes and the monetary policy coefficients. As discussed previously the fluctuations in our model was mainly driven by the productivity shocks in traded goods sector. We now investigate the sensitivity of our results to the persistence of the shocks as well as their standard deviations[36]. The list of parameter values for this Monte Carlo experiment are

[36]While doing this exercise we set the covariance between the shocks to zero.

shown in Table 8. We set the range of the Taylor rule coefficients such that the upper bound refer to a relatively aggressive monetary policy. For the temporary shocks, we set the persistence of the shocks from almost purely transitory to very close to a unit root. On the other hand, for the permanent technology shocks the same parameterisation implies a margin from almost a unit root to a very persistent growth rate shock. The large range of standard deviation of shocks reflects the uncertainty on the actual values of these parameters[37].

The results of the Monte Carlo simulation on the shock processes and the monetary policy coefficients are shown in Table 9. The moments obtained from this experiment is different from the previous results. The model does worse in matching the properties of the data. However, as in our first Monte Carlo experiment, the parameter values vary substantially across the range. Looking at the median values, the model generates neither the volatility nor the persistence of RER. The volatility of consumption is lower than output matching closer with the data. A striking result is the positive cross correlation between the relative consumption and RER. This result confirms the large wealth effects arising from our shock structure in our baseline calibration. Our sensitivity analysis certifies the importance of predominant driving source of the fluctuations for the general equilibrium models to account for the properties of the data. The large range of parameter values obtained from the Monte Carlo simulations proves the sensitivity of the performance of the model to the parameter calibration.

8 Conclusion

Empirical evidence shows that PPP does not hold at international markets and RER is actually highly persistent and volatile. In addition, there is large evidence suggesting lack of risk sharing at international level; the correlation between the RER and relative consumption is very low or even negative in most cases. However, general equilibrium models replicate some of these features of the data but at the cost of others. In this paper, we re-examined the ability of general equilibrium models to match with the observed behaviour of RER and its correlation with the relative consumption.

Specifically, we tested to what extent presenting a general equilibrium model

[37]There is a big diversity in the literature on the calibration of standard deviations of the shocks. For instance, CKM chose the standard deviation of the monetary shocks in order to match with volatility of the GDP which implies a large standard deviation. On the other hand, Ireland (2002) estimated a relatively low standard deviation of monetary policy shock for US, 0.0022.

Table 7: Robustness: Preference Parameters

	Autocorrelations		***Variance***	
	Data	**Model**	**Data**	**Model**
Y	0.88	0.9	0.00003	0.00075
		[0.85, 0.93]		[0.0003, 0.0015]
C	0.91	0.87	0.00002	0.0009
		[0.81, 0.92]		[0.0003, 0.0017]
RER	0.71	0.75	0.0009	0.0009
		[0.68, 0.82]		[0.00038, 0.002]
TOT	0.78	0.86	0.000027	0.0007
		[0.75, 0.93]		[0.00027, 0.0025]
NER	0.69	0.82	0.00089	0.002
		[0.77, 0.86]		[0.00074, 0.003]
Int.RER	0.64	0.91	0.000015	0.0007
		[0.88, 0.94]		[0.00026, 0.0015]

	Cross-Correlations	
	Data	**Model**
RER-NER	0.996	0.8927
		[0.6098, 0.9597]
NX-Y	-0.5045	-0.2697
		[-0.7385, 0.3393]
RER-Y	-0.1022	-0.5437
		[-0.6975, 0.1867]
*C-C**	0.6332	0.1918
		[-0.0196, 0.4757]
*Y-Y**	0.7934	0.2864
		[0.1506, 0.4283]
RER-TOT	0.2267	0.1113
		[-0.4554, 0.6242]
RER-Rel.C	0.0091	-0.6584
		[-0.8108, -0.0391]

Table 8: Monte Carlo Parameter Ranges: Shocks and Monetary Policy

Parameter	**Range**	**Description**
Taylor Rule Coefficients		
$\Gamma_i = \Gamma_{i^*}$	[0.3, 0.9]	interest rate smoothing
$\Gamma_y = \Gamma_{y^*}$	[0.03, 1.6]	response to output
$\Gamma_\pi = \Gamma_{\pi^*}$	[1, 3]	response to inflation
Persistence of Shocks		
$\rho_\tau = \rho_{\tau^*}$	[0.1, 0.9]	Preference
$\rho_{aT} = \rho_{aT^*}$	[0.1, 0.9]	Permanent technology shock: tradable sector
$\rho_{aN} = \rho_{aN^*}$	[0.1, 0.9]	Temporary technology shock: nontradable sector
$\rho_{aT^*,T} = \rho_{aT,aT^*}$	[0.001, 0.27]	Correlation between shocks across traded sectors
Standard Deviation of Shocks		
$\varepsilon_\tau = \varepsilon_{\tau^*}$	[0.01, 0.03]	Preference
$\varepsilon_m = \varepsilon_{m^*}$	[0.001, 0.03]	Monetary
$\varepsilon_{aT} = \varepsilon_{aT^*}$	[0.003, 0.03]	Technology shock: tradable sector
$\varepsilon_{aN} = \varepsilon_{aN^*}$	[0.006, 0.01]	Technology shock: nontradable sector
Monte Carlo draws	100	

Table 9: Robustness: Shocks and Monetary Policy Parameters

	Autocorrelations		***Variance***	
	Data	**Model**	**Data**	**Model**
Y	0.88	0.8	0.00003	0.0011
		[0.69, 0.88]		[0.0003, 0.0032]
C	0.91	0.81	0.00002	0.0009
		[0.71, 0.88]		[0.0003, 0.003]
RER	0.71	0.45	0.0009	0.0023
		[0.3616, 0.5761]		[0.0011, 0.006]
TOT	0.78	0.93	0.000027	0.00059
		[0.92, 0.95]		[0.00024, 0.0011]
NER	0.69	0.54	0.00089	0.003
		[0.44, 0.66]		[0.001, 0.007]
Int.RER	0.64	0.94	0.000015	0.0005
		[0.88, 0.94]		[0.00017, 0.0016]

	Cross-Correlations	
	Data	**Model**
RER-NER	0.996	0.96
		[0.93, 0.98]
NX-Y	-0.5045	-0.4181
		[-0.839, -0.04]
RER-Y	-0.1022	0.209
		[-0.27, 0.6174]
*C-C**	0.6332	0.258
		[-0.0196, 0.4964]
*Y-Y**	0.7934	0.2254
		[0.0452, 0.4891]
RER-TOT	0.2267	0.08
		[-0.1831, 0.3217]
RER-Rel.C	0.0091	0.2442
		[-0.45, 0.5969]

Note: We run the simulations for partial LCP ($d = 0.5$) with incomplete markets version of the model.

with a rich set of frictions that delivers deviations from PPP can account for the well-known puzzles in international macroeconomics on the behaviour of RER. We did so by developing a two country general equilibrium model with non-traded goods, home bias, incomplete markets and partial degrees of pass through as well as nominal rigidities both in goods and labour markets. In addition to our comprehensive theoretical model set-up, we analysed the importance of source of fluctuations through incorporating monetary shocks, preference shocks and country and sector specific technology shocks into our framework. We further emphasised the role of supply side shocks by presenting a data based TFP shock structure. We assumed permanent technology shocks in traded goods sector and persistent temporary shocks in non-traded goods sector, motivated from our data.

We argue that the ability of a general equilibrium model to account for the features of the data is closely related to the predominant driving source of the fluctuations. We show that a model with non-traded goods and permanent productivity shocks can account for the consumption-RER anomaly. Our result shows the importance of the multi-sector structure along with technology shocks to produce sufficient wealth effects for the model to account for limited international risk sharing. In our analysis, we further show that, the arising wealth effects are related to the predominant source of fluctuations. In a model where fluctuations are driven by preference shocks, the link between RER and relative consumption can be broken even in a framework with only two sectors. Our sensitivity analysis also shows the importance of the shock structure in explaining the behaviour of macroeconomic variables. On the other hand, our model fails to generate the observed RER volatility. It does a better job in producing the sufficient persistence. When we change our TFP shock structure to temporary, the volatility of RER gets closer to the data.

These results open interesting avenues for future research. For instance, introducing capital might improve the performance of the model, particularly in reducing the excess consumption volatility. In fact, abstracting capital from the model omits a fundamental part of the data. Furthermore, the sensitivity of our results to the parameterisation of the model stresses the relevance of testing the performance of our model with estimated values rather than relying on calibration only. Above all, a model with estimated shock structures would give a more satisfactory interpretation on the ability of the model in replicating the properties of the data. Our findings shows the importance of modelling the trends consistently with the data. The data-consistent characterisation of technology shocks, improves the ability of our model to account for some features of the data, but at the cost of others. Specifically, our model generates too high consumption and output volatility and too little RER volatility.

Appendices

A Data Appendix

The time span of the data covers the period 1990:Q1-2012:Q3. The following series for the US and the Euro Area are used for the analysis:

- The nominal exchange rate data is taken form IMF, IFS data base. It is defined as US Dollars per Euro in market rate. It is converted to an index by using 2005 as base year.
- To calculate the RER we used the CPI of the US and the Euro area. The CPI of the US is taken from the OECD's Main Economic Indicators database and the CPI of the Euro Area is taken from the EUROSTAT. Both series are for all items and invoiced in national currency. 2005 is taken as 100. We calculate the bilateral RER between euro and the US dollar by multiplying Euro Area CPI with nominal exchange rate and dividing it by US CPI.
- We used Engel's (1999) approach to decompose aggregate price index into its traded and non-traded goods components. We use the producer price index (PPI) as a measure of tradable goods prices. The PPI series for both US and Euro Area are obtained from the OECD's Main Economic Indicators database for total manufacturing in national currency with 2005=100. The internal RER is constructed by using the following decompositon: $InternalRER = [ln(CPI^{US}) - ln(PPI^{US})] - [ln(CPI^{EA}) - ln(PPI^{EA})]$
- Terms of trade is obtained from the Bureau of Labour Statistic (BLS) by taking the ratio of import prices from EU to the aggregate export price index of US. Both price indices are in US dollars and as there is no price information about imports from the Euro Area, we use EU as an approximate. We transformed the base year from 2000 to 2005 for both series.
- The series of output and consumption for both US and Euro Area were taken from the OECD's Main Economic Indicators database. We use seasonally adjusted index series (2005=100), in national currency. Output is the GDP by total expenditure and consumption is the private final consumption expenditure.

- Net exports are constructed by dividing the trade balance of US with Euro Area by the nominal GDP of USA. Both series are in US dollars. The GDP series is taken from the OECD's Main Economic Indicators database which is seasonally adjusted and computed by expenditure approach in current prices. The trade balance taken from the BLS. While computing the net exports we kept both series in levels.

- As explained previously, for the productivity innovations we took disaggregated data for sectors of the Euro Area and US of about 30 sectors and then aggregated them by large sectors. The data for this calculation is in annual frequency and covers the period 1981-2007. Through calculating value added shares of sectors, we constructed TFP growth series from the TFP estimates of Euklems. It is assumed that agriculture, mining and manufacturing are traded and the remaining are non-traded. However, as the series are of annual frequency we constructed quarterly series by using Chow, Lin (1971) method. As a reference, we took quarterly real output series from the OECD for the period 1981:Q1-2007:Q4. We approximated the output in traded sector by the real output in *industry* and non-traded by *market services.* Industry accommodates mining, manufacturing, electricity, gas and water. While market services accommodate retail and wholesale trade, transportation, communication, information, accommodation, restaurants, financial intermediation, insurance and real estate. For Euro Area data, we took Germany, France, Italy and Spain average as this covers most of the output. The series are seasonally adjusted by using ARIMA-X12 package. We took logs and growth rates of real output data to have consistent properties with annual data before interpolation. After the interpolation, we converted the growth rate series into index and tested the stationarity of the series through ADF test. The traded sector series found to be non-stationary implying permanent effects of traded shocks. On the other hand, the non-traded sector shocks found to be transitory. As a next step, we estimated a VAR in order to specify the structure of the shock processes as well as to calculate shock correlations. As the TFP in non-traded is found to be stationary for both Euro Area and US, we kept the series in levels and we first differenced the TFP series of traded sectors in both countries. For the traded sector, the US TFP process found to be persistent in growth rates whereas Euro Area TFP is a simple unit root process.

- The correlation between relative consumption and RER is calculated by using the logged series in the following way:

$$Corr[rer_t =, (c_t^{US} - c_t^{EA})]$$

B Steady State

Since we are using the stationarised variables for our analysis, we can compute the constant steady state which will be used to approximate the model. The steady state levels of the variables will be indicated by overbars.

As the prices are flexible in steady state, we can normalise the prices such that $\bar{P}^j = \bar{P}^{j^*} = 1$ where $j = 1, N, T, H, F, LCP$ and $j^* = 1, N^*, T^*, H^*, F^*, LCP^*$. Applying this, we obtain: $\bar{S} = \overline{RER} = \overline{TOT} = 1$.

In steady state consumption, output, investment, capital and labour supply are constant: $\bar{C} = \bar{C}^*; \bar{Y} = \bar{Y}^*; \bar{X} = \bar{X}^*; \bar{K} = \bar{K}^*; \bar{L} = \bar{L}^*$.

The total world consumption and output is the population weighted average of the steady state consumption and output:

$$\bar{Y}^w = n\bar{Y} + (1-n)\bar{Y}^* = n\bar{C} + (1-n)\bar{C}^* = \bar{C}^w \tag{B.1}$$

We can obtain the steady state interest rate by applying the above steady state conditions to Euler equation (14) which is equal to foreign country steady state interest rate:

$$\frac{1}{1+\bar{i}} = \beta \tag{B.2}$$

This relationship implies $\bar{S}\bar{B}_F/\bar{P} = 0$ from UIP condition (15).

The static demand functions in the steady state can be written as:

$$\bar{C}^N = (1-\alpha)\,\bar{C} \tag{B.3}$$

$$\bar{C}^T = \alpha\,\bar{C} \tag{B.4}$$

$$\bar{C}^F = (1-\omega)\,\alpha\,\bar{C} \tag{B.5}$$

$$\bar{C}^H = \omega\,\alpha\,\bar{C} \tag{B.6}$$

$$\bar{C}^{N^*} = (1-\alpha^*)\,\bar{C}^* \tag{B.7}$$

$$\bar{C}^{T^*} = \alpha^*\,\bar{C}^* \tag{B.8}$$

$$\bar{C}^{H^*} = (1-\omega^*)\,\alpha^*\,\bar{C}^* \tag{B.9}$$

$$\bar{C}^{F^*} = \omega^*\,\alpha^*\,\bar{C}^* \tag{B.10}$$

There are no technological improvements in steady state: $\bar{A}^j = 1$ for all $j = H, N, F, N^*$. Consequently the production functions reduce to:

$$\bar{Y}^N = \bar{L}^N, \quad \bar{Y}^H = \bar{L}^H \tag{B.11}$$

$$\bar{Y}^{N^*} = \bar{L}^{N^*}, \quad \bar{Y}^F = \bar{L}^F \tag{B.12}$$

We assume that: $\bar{W} = \bar{W}^*$. When wages and prices are fully flexible, as the wage contracts depend on expected future inflation, the steady state of wages and prices will be equal. Thus, as a result of normalisation of prices to one, wages will be equal to 1 as well. In the zero inflation steady state the real wage will be equal to the marginal rate of substitution as the wage subsidy offsets the effect of monopolistic distortion:

$$1 = \frac{\bar{W}}{\bar{P}} = \frac{\bar{W}^*}{\bar{P}^*} = \overline{MRS} = \overline{MRS}^* \tag{B.13}$$

where $\overline{MRS} = \kappa\,(\bar{C} - h\bar{C})^{\rho}\,(\bar{L})^{\eta}$

Setting $(1+\kappa^p) = \frac{\phi}{\phi-1}$ and imposing the steady state assumptions gives us the flexible price equilibrium with constant marginal costs in steady state:

$$1 = \overline{MC}^N = \overline{MC}^H = \overline{MC}^{H^*} = \overline{MC}^{N^*} = \overline{MC}^{F^*} = \overline{MC}^F \tag{B.14}$$

The steady state market clearing conditions are:

$$\bar{Y}^N = \bar{C}^N = (1-\alpha)\,\bar{C} \tag{B.15}$$

$$\bar{Y}^{N^*} = \bar{C}^{N^*} = (1-\alpha^*)\,\bar{C}^* \tag{B.16}$$

and

$$\bar{Y}^H = \omega\,\alpha\,\bar{C} + \left(\frac{1-n}{n}\right)(1-\omega^*)\,\alpha^*\,\bar{C}^* \tag{B.17}$$

$$\bar{Y}^F = \omega^*\,\alpha^*\,\bar{C}^* + \left(\frac{n}{1-n}\right)(1-\omega)\,\alpha\,\bar{C} \tag{B.18}$$

We know that in steady state the nominal exchange rate is constant and equal to one, and also the steady state level of net foreign asset position is equal to zero. Consequently, the steady state level of net exports will be equal to zero as well; i.e. imports are equal to exports. Accordingly, the following expression can be obtained by applying the above steady state relationships to (45):

$$(1-\omega^*)\,\alpha^*\,(1-n)\,\bar{C}^* = (1-\omega)\,\alpha\,n\,\bar{C}$$

Using $\bar{C}^* = \bar{C}$,

$$(1-\omega^*)\,\alpha^*\,(1-n) = (1-\omega)\,\alpha\,n \tag{B.19}$$

C Log-Linearised model

In this appendix, we present all the linearised equations in the model. We log-linearise the model around the steady state.

- Euler equation and UIP condition[38]

[38]Note that when only state-contingent nominal bonds are traded, i.e. international markets are complete, through households intertemporal decision we obtain a perfect risk sharing condition:

$$rer_t = \frac{\rho}{1-h}[(c_t - hc_{t-1}) - (c_t^* - hc_{t-1}^*)] - (\tau_t - \tau_t^*)$$

$$c_t \;=\; \frac{1}{1+h}\,E_t\,c_{t+1} \;+\; \frac{h}{1+h}\,c_{t-1} \;-\; \frac{1-h}{(1+h)\,\rho}\,(i_t \;-\; E_t\,\pi_{t+1}) \\ -\frac{1-h}{(1+h)\,\rho}\,(\tau_{t+1} \;-\; \tau_t) \quad (C.1)$$

$$c_t^* \;=\; \frac{1}{1+h}\,E_t\,c_{t+1}^* \;+\; \frac{h}{1+h}\,c_{t-1}^* \;-\; \frac{1-h}{(1+h)\,\rho}\,(i_t^* \;-\; E_t\,\pi_{t+1}^*) \\ -\frac{1-h}{(1+h)\,\rho}\,(\tau_{t+1}^* \;-\; \tau_t^*) \quad (C.2)$$

where $\pi_{t+1} = p_{t+1} - p_t$ and $\pi_{t+1}^* = p_{t+1}^* - p_t^*$

$$s_{t+1} - s_t = i_t - i_t^* + \delta b_t \quad (C.3)$$

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- Demand Functions

$$c_t^N = \nu(p_t - p_t^N) + c_t \quad (C.4)$$

$$c_t^T = \nu(p_t - p_t^T) + c_t \quad (C.5)$$

$$c_t^H = \theta(p_t^T - p_t^H) + c_t^T \quad (C.6)$$

$$c_t^F = \theta(p_t^T - p_t^F) + c_t^T \quad (C.7)$$

$$c_t^{N^*} = \nu(p_t^* - p_t^{N^*}) + c_t^* \quad (C.8)$$

$$c_t^{T^*} = \nu(p_t^* - p_t^{T^*}) + c_t^* \quad (C.9)$$

$$c_t^{H^*} = \theta(p_t^{T^*} - p_t^{H^*}) + c_t^{T^*} \quad (C.10)$$

$$c_t^{F^*} = \theta(p_t^{T^*} - p_t^{F^*}) + c_t^{T^*} \quad (C.11)$$

- The Price Indices

– Consumer Price Indices

$$p_t \;=\; \alpha\, p_t^T \;+\; (1\;-\;\alpha)\, p_t^N \tag{C.12}$$

$$p_t^* \;=\; \alpha^*\, p_t^{T^*} \;+\; (1\;-\;\alpha^*)\, p_t^{N^*} \tag{C.13}$$

– Price Indices for Tradable Goods

$$p_t^T \;=\; \omega\, p_t^H \;+\; (1\;-\;\omega)\, p_t^F \tag{C.14}$$

$$p_t^{T^*} \;=\; \omega^*\, p_t^{F^*} \;+\; (1\;-\;\omega^*)\, p_t^{H^*} \tag{C.15}$$

– Export Prices

As some firms engage in local currency pricing, some exports are home currency priced and some are foreign currency priced. The export price indices have the following form:

$$p_t^F \;=\; d^*\, p_t^{LCP} \;+\; (1-d^*)\,(p_t^{F^*} + s_t) \tag{C.16}$$

$$p_t^{H^*} \;=\; d\, p_t^{LCP^*} \;+\; (1-d)\,(p_t^H - s_t) \tag{C.17}$$

$$\tag{C.18}$$

- Production Functions

$$y_t^H = a_t^H + l_t^H, \quad y_t^N = a_t^N + l_t^N \tag{C.19}$$

$$y_t^F = a_t^F + l_t^F, \quad y_t^{N^*} = a_t^{N^*} + l_t^{N^*} \tag{C.20}$$

- Price Setting

Let's define the inflation variables: $\pi_t^N = p_t^N - p_{t-1}^N$, $\pi_t^H = p_t^H - p_{t-1}^H$, $\pi_t^{N^*} = p_t^{N^*} - p_{t-1}^{N^*}$, $\pi_t^{F^*} = p_t^{F^*} - p_{t-1}^{F^*}$ $\pi_t^{LCP} = p_t^{LCP} - p_{t-1}^{LCP}$, $\pi_t^{LCP^*} = p_t^{LCP^*} - p_{t-1}^{LCP^*}$

– home country non-traded sector

$$\pi_t^N \;=\; \gamma^f\, \pi_{t+1}^N \;+\; \gamma^b\, \pi_{t-1}^N + \lambda\, mc_t^N \tag{C.21}$$

where

$$mc_t^N = w_t - p_t^N - a_t^N$$

– home country traded sector

$$\pi_t^H \; = \; \gamma^f \, \pi_{t+1}^H \; + \; \gamma^b \, \pi_{t-1}^H + \lambda \, mc_t^H \tag{C.22}$$

where

$$mc_t^H = w_t - p_t^H - a_t^H$$

– home country locally priced imports

$$\pi_t^{LCP} \; = \; \gamma^{f^*} \, \pi_{t+1}^{LCP} \; + \; \gamma^{b^*} \, \pi_{t-1}^{LCP} + \lambda^* \, mc_t^F \tag{C.23}$$

where

$$mc_t^F = w_t^* - p_t^{LCP} - a_t^F + s_t$$

– foreign country non-traded sector

$$\pi_t^{N^*} \; = \; \gamma^{f^*} \, \pi_{t+1}^{N^*} \; + \; \gamma^{b^*} \, \pi_{t-1}^{N^*} + \lambda^* \, mc_t^{N^*} \tag{C.24}$$

where

$$mc_t^{N^*} = w_t^* - p_t^{N^*} - a_t^{N^*}$$

– foreign country traded sector

$$\pi_t^{F^*} \; = \; \gamma^{f^*} \, \pi_{t+1}^{F^*} \; + \; \gamma^{b^*} \, \pi_{t-1}^{F^*} + \lambda^* \, mc_t^{F^*} \tag{C.25}$$

where

$$mc_t^{F^*} = w_t^* - p_t^{F^*} - a_t^F$$

– foreign country locally priced imports

$$\pi_t^{LCP^*} \; = \; \gamma^f \, \pi_{t+1}^{LCP^*} \; + \; \gamma^b \, \pi_{t-1}^{LCP^*} \; + \lambda^* \, mc_t^{H^*} \tag{C.26}$$

where

$$mc_t^{H^*} = w_t - p_t^{LCP^*} - a_t^H - s_t$$

The corresponding composite parameters are:

$$\gamma^f = \frac{\beta\,\xi_p}{\xi_p + (\varsigma_p(1-\xi_p(1-\beta)))},$$

$$\gamma^b = \frac{\varsigma_p}{\xi_p + (\varsigma_p(1-\xi_p(1-\beta)))},$$

$$\lambda = \frac{(1-\beta\,\xi_p)(1-\xi_p)(1-\varsigma_p)}{\xi_p + (\varsigma_p(1-\xi_p(1-\beta)))}$$

$$\gamma^{f^*} = \frac{\beta\,\xi_{p^*}}{\xi_{p^*} + (\varsigma_{p^*}(1-\xi_{p^*}(1-\beta)))},$$

$$\gamma^{b^*} = \frac{\varsigma_{p^*}}{\xi_{p^*} + (\varsigma_{p^*}(1-\xi_{p^*}(1-\beta)))},$$

$$\lambda^* = \frac{(1-\beta\,\xi_{p^*})(1-\xi_{p^*})(1-\varsigma_{p^*})}{\xi_{p^*} + (\varsigma_{p^*}(1-\xi_{p^*}(1-\beta)))}$$

- Wage Setting

  The wage inflation is: $\pi_t^w = w_t - w_{t-1}$ *and* $\pi_t^{w^*} = w_t^* - w_{t-1}^*$

  - wage dynamics in home country

    $$\pi_t^w \;=\; \gamma^{f,w}\,\pi_{t+1}^w \;+\; \gamma^{b,w}\,\pi_{t-1}^w \;-\; \lambda^w\,mrs_t \qquad \text{(C.27)}$$

    where

    $$mrs_t \;=\; w_t \;-\; p_t \;-\; \eta\, l_t \;-\; (\frac{\rho}{1-h})\,(c_t \;-\; h\,c_{t-1})$$

  - wage dynamics in foreign country

    $$\pi_t^{w^*} \;=\; \gamma^{f,w^*}\,\pi_{t+1}^{w^*} + \gamma^{b,w^*}\,\pi_{t-1}^{w^*} \;-\; \lambda^{w^*}\,mrs_t^* \qquad \text{(C.28)}$$

    where

    $$mrs_t^* \;=\; w_t^* \;-\; p_t^* \;-\; \eta\, l_t^* - (\frac{\rho}{1-h})\,(c_t^* \;-\; h\,c_{t-1}^*)$$

The corresponding composite parameters are:

$$\gamma^{f,w} = \frac{\beta\,\xi_w}{((1+\eta\,\sigma_w)\xi_w) + (\varsigma_w(1-\xi_w(1-\beta)))},$$

$$\gamma^{b,w} = \frac{\varsigma_w}{((1+\eta\,\sigma_w)\xi_w) + (\varsigma_w(1-\xi_w(1-\beta)))},$$

$$\lambda^{w} = \frac{(1-\beta\,\xi_w)(1-\xi_w)(1-\varsigma_w)}{((1+\eta\,\sigma_w)\xi_w) + (\varsigma_w(1-\xi_w(1-\beta)))}$$

$$\gamma^{f,w^*} = \frac{\beta\,\xi_{w^*}}{((1+\eta\,\sigma_w)\xi_{w^*}) + (\varsigma_{w^*}(1-\xi_{w^*}(1-\beta)))},$$

$$\gamma^{b,w^*} = \frac{\varsigma_{w^*}}{((1+\eta\,\sigma_w)\xi_{w^*}) + (\varsigma_{w^*}(1-\xi_{w^*}(1-\beta)))},$$

$$\lambda^{w^*} = \frac{(1-\beta\,\xi_{w^*})(1-\xi_{w^*})(1-\varsigma_{w^*})}{((1+\eta\,\sigma_w)\xi_{w^*}) + (\varsigma_{w^*}(1-\xi_{w^*}(1-\beta)))}$$

- Current Account

$$\begin{aligned}\beta b_t - b_{t-1} = {} & (1-\omega^*)\,\alpha^*\frac{(1-n)}{n}\,(c_t^{H^*} + p_t^{H^*} + s_t - p_t) \\ & + \omega\,\alpha\,(c_t^H + p_t^H - p_t) - \alpha\,(c_t^T + p_t^T - p_t)\end{aligned} \tag{C.29}$$

- Monetary Policy

$$i_t = \Gamma_{i-1} i_{t-1} + (1-\Gamma_{i-1})\Gamma_{\pi_t}\pi_t + (1-\Gamma_{i-1})\Gamma_{y_t} y_t + \varepsilon_{m,t} \tag{C.30}$$

$$i_t^* = \Gamma_{i-1}^* i_{t-1}^* + (1-\Gamma_{i-1}^*)\Gamma_{\pi_t}^*\pi_t^* + (1-\Gamma_{i-1}^*)\Gamma_{y_t}^* y_t^* + \varepsilon_{m,t}^* \tag{C.31}$$

where

$$\varepsilon_{m,t} \sim N(0,\sigma_m^2), \qquad \varepsilon_{m,t}^* \sim N(0,\sigma_{m^*}^2)$$

- Market Clearing

– Non-traded sector

$$y_t^N = c_t^N, \qquad\qquad y_t^{N^*} = c_t^{N^*} \tag{C.32}$$

– Traded sector

$$y_t^H = \omega\, c_t^H + \frac{(1-n)(1-\omega^*)\alpha^*}{n\,\alpha}\, c_t^{H^*} \tag{C.33}$$

$$y_t^F = \omega^*\, c_t^{F^*} + \frac{n\,(1-\omega)\,\alpha}{(1-n)\,\alpha^*}\, c_t^F \tag{C.34}$$

- Output and Labour supply

– output

$$y_t = c_t + \frac{(1-\omega^*)\,\alpha^*\,(1-n)}{n}\, c_t^{H^*} - (1-\omega)\alpha\, c_t^F \tag{C.35}$$

$$y_t^* = c_t^* + \frac{(1-\omega)\,\alpha\, n}{1-n}\, c_t^F - (1-\omega^*)\alpha^*\, c_t^{H^*} \tag{C.36}$$

– labour supply

$$l_t = \alpha\, l_t^H + (1-\alpha)\, l_t^N \tag{C.37}$$

$$l_t^* = \alpha^*\, l_t^F + (1-\alpha^*)\, l_t^{N^*} \tag{C.38}$$

- Definition of International Variables

– net exports

$$nx_t = \left(\frac{(1-\omega^*)\,\alpha^*(1-n)}{n}\right)(c_t^{H^*} + p_t^{H^*} + s_t - p_t) - (1-\omega)\,\alpha\,(c_t^F + p_t^F - p_t) \tag{C.39}$$

– terms of trade

$$tot_t = p_t^F - p_t^{H^*} - s_t \tag{C.40}$$

– real exchange rate

$$rer_t = p_t^* + s_t - p_t \tag{C.41}$$

– decomposition of real exchange rate

* internal relative price movements channel

$$rerint_t = (1-\alpha^*)(p_t^{N^*} - p_t^{T^*}) - (1-\alpha)(p_t^N - p_t^T) \tag{C.42}$$

- * pricing to market channel

$$rerptm_t = (1-\omega^*)(s_t + p_t^{H^*} - p_t^H) + \omega^*(s_t + p_t^{F^*} - p_t^F) \quad \text{(C.43)}$$

- * home bias channel

$$rertot_t = (\omega - (1-\omega^*))(p_t^F - p_t^H) \quad \text{(C.44)}$$

- Shock Processes

  - preference shocks

$$ln(\tau_t) = \rho^{\tau} ln(\tau_{t-1}) + \varepsilon_t^{\tau} \quad \text{(C.45)}$$
$$ln(\tau_t^*) = \rho^{\tau^*} ln(\tau_{t-1}^*) + \varepsilon_t^{\tau^*} \quad \text{(C.46)}$$

  - technology shocks

$$a_t^N = (1+\rho^{a^N})\, a_{t-1}^N - \rho^{a^N}\, a_{t-2}^N + \varepsilon_t^{a^N} \quad \text{(C.47)}$$
$$a_t^H = (1+\rho^{a^H})\, a_{t-1}^H - \rho^{a^T}\, a_{t-2}^H + \varepsilon_t^{a^H} \quad \text{(C.48)}$$

$$a_t^{N^*} = (1+\rho^{a^{N^*}})\, a_{t-1}^{N^*} - \rho^{a^{N^*}}\, a_{t-2}^{N^*} + \varepsilon_t^{a^{N^*}} \quad \text{(C.49)}$$
$$a_t^F = (1+\rho^{a^F})\, a_{t-1}^F - \rho^{a^F}\, a_{t-2}^F + \varepsilon_t^{a^F} \quad \text{(C.50)}$$

D Stationarising the Model

In the model, the specification of technological change allows for different productivity growth in each country and in each sector. As a result of permanent shocks in the model, we need to stationarise the model in order to have a well behaved deterministic steady state. As technology grows at different rates in each sector, permanent productivity differentials between traded and non-traded sectors will have permanent effects on relative prices as suggested by Balassa-Samuelson model. Moreover, total factor productivity growth will have an impact on terms of trade which in turn will affect the non-stationary behaviour of the trended variables.

We will use the previously derived log-linearised equations to obtain the trends of non-stationary variables. For approximation of the model around the deterministic steady state to hold, we need to subtract the trends from the level variables.

For convenience, let's define the relative price variables:

$$x_t^N = p_t^N - p_t \tag{D.1}$$

$$x_t^T = p_t^T - p_t \tag{D.2}$$

$$x_t^H = p_t^H - p_t^T \tag{D.3}$$

$$x_t^F = p_t^F - p_t^T \tag{D.4}$$

$$x_t^{N^*} = p_t^{N^*} - p_t^* \tag{D.5}$$

$$x_t^{T^*} = p_t^{T^*} - p_t^* \tag{D.6}$$

$$x_t^{H^*} = p_t^{H^*} - p_t^{T^*} \tag{D.7}$$

$$x_t^{F^*} = p_t^{F^*} - p_t^{T^*} \tag{D.8}$$

And the real wages can be defined as:

$$rw_t = w_t - p_t \tag{D.9}$$

$$rw_t^* = w_t^* - p_t^* \tag{D.10}$$

The variables which inherit non-stationary behaviour and needed to be transformed are the following: c_t, c_t^*, c_t^T, c_t^N, c_t^H, c_t^F, $c_t^{T^*}$, $c_t^{N^*}$, $c_t^{H^*}$, $c_t^{F^*}$, x_t^N, x_t^T, x_t^H, x_t^F, $x_t^{N^*}$, $x_t^{T^*}$, $x_t^{H^*}$, $x_t^{F^*}$, rw_t, rw_t^*, y_t, y_t^*, y_t^N, y_t^H, $y_t^{N^*}$, y_t^F, l_t^N, l_t^H, $l_t^{N^*}$, l_t^F, tot_t, rer_t, $rerint_t$, $rerptm_t$, $rertot_t$.

We will show how the variables respond to productivity shocks as in León-Ledesma and Mihailov (forthcoming). Deriving the trends requires a bit of algebra. From the firms static optimal choice of allocation of the production between non-traded and traded sectors, one can easily get the standard Balassa-Samuelson result. The internal relative price of non-traded goods to traded goods is dependent on the productivity differentials between sectors: $p_t^N - p_t^T = a_t^H - a_t^N$. Note that we assume perfect labour mobility across sectors in the model, hence the nominal wages will be equalised across sectors.

We can define the equilibrium terms of trade by using the long-run trade balance assumption: $c_t^{H^*} - c_t^F = tot_t = p_t^F - p_t^{H^*} - s_t$. In addition, given that in long run the prices will be flexible and there are no strategic gains from pricing to market practices when the prices are flexible, we can re-define the equilibrium terms of trade: $tot_t = p_t^F - p_t^H$. Therefore, in long run law-of-one-price will hold for traded goods.

We can find the trend of terms of trade through re-writing the optimal demand functions. We need to find the trends of relative prices first. From the above definition of relative price of non traded goods, we have: $x_t^N = p_t^N - p_t$.

We will use the log-linearised form of CPI (C.12):

$$p_t \;=\; \alpha\, p_t^T \;+\; (1-\alpha)\, p_t^N$$

We will send the p_t^N to the left hand side, plug in the price index of for traded goods (C.14) and subtract (αp_t^H) from both sides:

$$p_t^N \;-\; p_t \;=\; \alpha\omega\, p_t^H \;+\; \alpha(1-\omega)\, p_t^F \;-\alpha\, p_t^N \;+\; (\alpha\, p_t^H \;-\; \alpha\, p_t^H)$$

By using Balassa-Samuelson proposition and definition of equilibrium terms of trade, we obtain:

$$p_t^N \;-\; p_t \;=\; \alpha\,(a_t^H \;-\; a_t^N) \;-\; \alpha(1-\omega)\; tot_t$$

We can use this expression to re-write the optimal demand function (C.4):

$$c_t^N \;=\; c_t \;-\; \nu\,\alpha\,(a_t^H \;-\; a_t^N) \;+\; \nu\,\alpha\,(1-\omega)\, tot_t \tag{D.11}$$

We will write the other optimal demand functions in a similar way:

$$c_t^T \;=\; c_t \;+\; \nu\,(1-\alpha)\,(a_t^H \;-\; a_t^N) \;-\; \nu\,(1-\alpha)\,(1-\omega)\, tot_t \tag{D.12}$$

$$c_t^H \;=\; c_t \;+\; \nu\,(1-\alpha)\,(a_t^H \;-\; a_t^N) \;+\; [\theta\,(1-\omega) \;-\; \nu\,(1-\alpha)\,(1-\omega)]\, tot_t \tag{D.13}$$

$$c_t^F \;=\; c_t \;+\; \nu\,(1-\alpha)\,(a_t^H \;-\; a_t^N) \;-\; [\theta\,\omega \;+\; \nu\,(1-\alpha)\,(1-\omega)]\, tot_t \tag{D.14}$$

The corresponding optimal demand functions for foreign country is:

$$c_t^{N^*} \;=\; c_t^* \;-\; \nu\,\alpha^*\,(a_t^F \;-\; a_t^{N^*}) \;-\; \nu\,\alpha^*\,(1-\omega^*)\, tot_t \tag{D.15}$$

$$c_t^{T^*} \;=\; c_t^* \;+\; \nu\,(1-\alpha^*)\,(a_t^F \;-\; a_t^{N^*}) \;+\; \nu\,(1-\alpha^*)\,(1-\omega^*)\, tot_t \tag{D.16}$$

$$c_t^{F^*} \;=\; c_t^* \;+\; \nu\,(1-\alpha^*)\,(a_t^F \;-\; a_t^{N^*}) - \;[\theta\,(1-\omega^*) \;-\; \nu\,(1-\alpha^*)\,(1-\omega^*)]\, tot_t \tag{D.17}$$

$$c_t^{H^*} \;=\; c_t^* \;+\; \nu\,(1-\alpha^*)\,(a_t^F \;-\; a_t^{N^*}) \;+\; [\theta\,\omega^* \;+\; \nu\,(1-\alpha^*)\,(1-\omega^*)]\, tot_t \tag{D.18}$$

In order to derive the trend of terms of trade, first we need to write consumption as a function of changes in productivity and terms of trade. We know that total labour supply is stationary in equilibrium. Using the production functions of each sector, (C.19): $y_t^H = a_t^H + l_t^H, \quad y_t^N = a_t^N + l_t^N$ and substituting these expression to the total labour supply equation (C.37), we obtain: $l_t = \alpha\,(y_t^H - a_t^H) + (1-\alpha)\,(y_t^N - a_t^N)$. We can also normalise total labour supply to 1, so that: $0 = \alpha\,(y_t^H - a_t^H) + (1-\alpha)\,(y_t^N - a_t^N)$.

Now, by using the market clearing conditions combined with the trade balance condition in equilibrium, $(1-\alpha)\,y_t^N = (1-\alpha)\,c_t^N, \quad \alpha\,y_t^H = \omega\,\alpha c_t^H + (1-\omega)\alpha\,c_t^{H^*}$ [39] we can re-write the log-linearised labour market equilibrium condition as

$$0 = \alpha\,((\omega\,c_t^H + (1-\omega)\,c_t^{H^*}) - a_t^H) + (1-\alpha)\,(c_t^N - a_t^N)$$

from balanced trade condition, this expression is equivalent to

$$\alpha\,a_t^H + (1-\alpha)\,a_t^N = (1-\alpha)\,c_t^N + \alpha\,\omega\,c_t^H + \alpha\,(1-\omega)\,c_t^F + \alpha\,(1-\omega)\,tot_t \tag{D.19}$$

By using the above optimal demand functions, we can solve the expression for c_t:

$$c_t = \alpha\,a_t^H + (1-\alpha)\,a_t^N - \alpha\,(1-\omega)\,tot_t \tag{D.20}$$

Now, let's plug this into (D.14), (D.13), (D.18):

$$\begin{aligned} c_t^H =\ & (1-\alpha)\,(1-\nu)\,a_t^N + [\alpha + (1-\alpha)\,(1-\nu)]\,a_t^H \\ & - [\alpha\,(1-\omega) - \theta\,(1-\omega) + \nu\,(1-\omega)\,(1-\alpha)]tot_t \end{aligned} \tag{D.21}$$

$$\begin{aligned} c_t^F =\ & (1-\alpha)\,(1-\nu)\,a_t^N + [\alpha + (1-\alpha)\,(1-\nu)]\,a_t^H \\ & - [\alpha\,(1-\omega) - \theta\,\omega + \nu\,(1-\omega)\,(1-\alpha)]tot_t \end{aligned} \tag{D.22}$$

$$\begin{aligned} c_t^{H^*} =\ & (1-\alpha^*)\,(1-\nu)\,a_t^{N^*} + [\alpha^* + (1-\alpha^*)\,(1-\nu)]\,a_t^F \\ & + [\alpha^*\,(1-\omega^*) + \theta\,\omega^* + \nu\,(1-\omega^*)\,(1-\alpha^*)]tot_t \end{aligned} \tag{D.23}$$

We can use this expressions to obtain terms of trade:

$$\begin{aligned} tot_t = \frac{1}{\Upsilon}\,\{&(1-\nu)\,[(1-\alpha^*)\,a_t^{N^*} - (1-\alpha)\,a_t^N] + \nu\,[(1-\alpha^*)\,a_t^F - (1-\alpha)\,a_t^H] \\ & + \alpha^*\,a_t^F - \alpha\,a_t^H\} \end{aligned} \tag{D.24}$$

[39]Note that trade balance condition ensures that in equilibrium $\frac{(1-n)(1-\omega^*)\alpha^*}{n\,\alpha} = (1-\omega)\alpha$, even under asymmetric preference structure.

where $\Upsilon = 1 - \alpha^* (1-\omega^*) - \alpha (1-\omega) - \theta (\omega + \omega^*) - \nu[(1-\omega^*)(1-\alpha^*) + (1-\omega)(1-\alpha)]$.

Given that now we have an expression for terms of trade which shows the effects of productivity shocks on terms of trade, we can derive the trends of non-stationary variables. The above expression is simply the trend of terms of trade:

$$t^{tot} = \frac{1}{\Upsilon} \{(1-\nu)[(1-\alpha^*) a_t^{N^*} - (1-\alpha) a_t^N] + \nu [(1-\alpha^*) a_t^F - (1-\alpha) a_t^H] + \alpha^* a_t^F - \alpha a_t^H\} \tag{D.25}$$

By substituting this expression into the equation (D.20), we get the trend of home total consumption:

$$t^c = \alpha a_t^H + (1-\alpha) a_t^N - \alpha (1-\omega) t^{tot} \tag{D.26}$$

Next, combining (D.26) and(D.25) with optimal demand functions, we get:

$$t^{c^N} = t^c - \nu \alpha (a_t^H - a_t^N) + \nu \alpha (1-\omega) t^{tot} \tag{D.27}$$

$$t^{c^T} = t^c + \nu (1-\alpha)(a_t^H - a_t^N) - \nu (1-\alpha)(1-\omega) t^{tot} \tag{D.28}$$

$$t^{c^H} = t^c + \nu (1-\alpha)(a_t^H - a_t^N) + [\theta (1-\omega) - \nu (1-\alpha)(1-\omega)] t^{tot} \tag{D.29}$$

$$t^{c^F} = t^c + \nu (1-\alpha)(a_t^H - a_t^N) - [\theta \omega + \nu (1-\alpha)(1-\omega)] t^{tot} \tag{D.30}$$

Similarly, we can obtain the trends for foreign country demand functions. Let's first define the trend of foreign country consumption:

$$t^{c^*} = \alpha^* a_t^F + (1-\alpha^*) a_t^{N^*} + \alpha^* (1-\omega^*) t^{tot} \tag{D.31}$$

The trends of foreign country optimal demand equations:

$$t^{c^{N^*}} = t^{c^*} - \nu\,\alpha^*\,(a_t^F - a_t^{N^*}) - \nu\,\alpha^*\,(1-\omega^*)\,t^{tot} \tag{D.32}$$

$$t^{c^{T^*}} = t^{c^*} + \nu\,(1-\alpha^*)\,(a_t^F - a_t^{N^*}) + \nu\,(1-\alpha^*)\,(1-\omega^*)\,t^{tot} \tag{D.33}$$

$$t^{c^{F^*}} = t^{c^*} + \nu\,(1-\alpha^*)\,(a_t^F - a_t^{N^*}) - [\theta\,(1-\omega^*) - \nu\,(1-\alpha^*)\,(1-\omega^*)]\,t^{tot} \tag{D.34}$$

$$t^{c^{H^*}} = t^{c^*} + \nu\,(1-\alpha^*)\,(a_t^F - a_t^{N^*}) + [\theta\,\omega^* + \nu\,(1-\alpha^*)\,(1-\omega^*)]\,t^{tot} \tag{D.35}$$

Trend of relative prices:

$$t^{x^N} = \alpha\,(a_t^H - a_t^N) - \alpha\,(1-\omega)\,t^{tot} \tag{D.36}$$

$$t^{x^T} = -(1-\alpha)\,(a_t^H - a_t^N) + (1-\alpha)\,(1-\omega)\,t^{tot} \tag{D.37}$$

$$t^{x^H} = -(1-\omega)\,t^{tot} \tag{D.38}$$

$$t^{x^F} = \omega\,t^{tot} \tag{D.39}$$

$$t^{x^{N^*}} = \alpha^*\,(a_t^F - a_t^{N^*}) + \alpha^*\,(1-\omega^*)\,t^{tot} \tag{D.40}$$

$$t^{x^{T^*}} = -(1-\alpha^*)\,(a_t^F - a_t^{N^*}) - (1-\alpha^*)\,(1-\omega^*)\,t^{tot} \tag{D.41}$$

$$t^{x^{H^*}} = -\omega^*\,t^{tot} \tag{D.42}$$

$$t^{x^{F^*}} = (1-\omega^*)\,t^{tot} \tag{D.43}$$

We will now derive the trend of aggregate output. Combining (C.35) with trade balance condition we get: $y_t = c_t + (1-\omega)\alpha\, tot_t$. Substituting the trends of consumption and terms of trade gives us the trend of output:

$$t^y = (1-\alpha)\,a_t^N + \alpha\,a_t^H \tag{D.44}$$

and similarly the trend of foreign country output is given by:

$$t^{y^*} = (1-\alpha^*)\,a_t^{N^*} + \alpha^*\,a_t^F \tag{D.45}$$

In equilibrium $mrs = 0$ and labour supply is stationary. This implies that, from the equation (C), the trend of real wage should be equal to trend of consumption: $t^{rw} = t^c$ and for foreign country it will be $t^{rw^*} = t^{c^*}$. Note that in order to have balanced growth path we assume that risk aversion parameter, ρ, is equal to 1.

Finally we will derive the trend of RER. As the dynamics of RER is determined by its three components, its trend will be combination of trends of those too. Given that in long run all prices are flexible, the pricing to market behaviour will not have any role in long run dynamics of RER, it only has an impact on transitional dynamics. Therefore we need to derive the trends of home bias channel (C.44) and internal relative price movements channel (C.42). The trends of home bias and internal relative price channels are:

$$t^{rertot} = (\omega - (1-\omega^*))\, t^{tot} \tag{D.46}$$

$$\begin{aligned} t^{rerint} = {} & (1-\alpha^*)\,(a_t^F - a_t^{N^*} + (1-\omega^*)\, t^{tot}) \\ & - (1-\alpha)\,(a_t^H - a_t^N - (1-\omega)\, t^{tot}) \end{aligned} \tag{D.47}$$

We can now write the trend of RER:

$$t^{rer} = t^{rertot} + t^{rerint} \tag{D.48}$$

Since we have the particular normalisation of each variable, now we can list the normalised log-linear equations of the model. The stationarised variables will be denoted by a hat.

- Euler equation and UIP condition

$$\begin{aligned} \hat{c}_t = {} & \frac{1}{1+h} E_t \hat{c}_{t+1} + \frac{h}{1+h} \hat{c}_{t-1} - \frac{1-h}{(1+h)} (i_t - E_t \pi_{t+1}) \\ & - \frac{1-h}{(1+h)} (\tau_{t+1} - \tau_t) + \frac{1}{1+h} (t_{t+1}^c - t_t^c) - \frac{h}{1+h} (t_t^c - t_{t-1}^c) \end{aligned} \tag{D.49}$$

$$\begin{aligned} \hat{c}_t^* = {} & \frac{1}{1+h} E_t \hat{c}_{t+1}^* + \frac{h}{1+h} \hat{c}_{t-1}^* - \frac{1-h}{(1+h)} (i_t^* - E_t \pi_{t+1}^*) \\ & - \frac{1-h}{(1+h)} (\tau_{t+1}^* - \tau_t^*) + \frac{1}{1+h} (t_{t+1}^{c^*} - t_t^{c^*}) - \frac{h}{1+h} (t_t^{c^*} - t_{t-1}^{c^*}) \end{aligned} \tag{D.50}$$

where $\pi_{t+1} = p_{t+1} - p_t$ and $\pi_{t+1}^* = p_{t+1}^* - p_t^*$

$$s_{t+1} - s_t = i_t - i_t^* + \delta b_t \tag{D.51}$$

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- Demand Functions

$$\hat{c}_t^N = \nu(-\hat{x}_t^N) + \hat{c}_t \tag{D.52}$$

$$\hat{c}_t^T = \nu(-\hat{x}_t^T) + \hat{c}_t \tag{D.53}$$

$$\hat{c}_t^H = \theta(-\hat{x}_t^H) + \hat{c}_t^T \tag{D.54}$$

$$\hat{c}_t^F = \theta(-\hat{x}_t^F) + \hat{c}_t^T \tag{D.55}$$

$$\hat{c}_t^{N^*} = \nu(-\hat{x}_t^{N^*}) + \hat{c}_t^* \tag{D.56}$$

$$\hat{c}_t^{T^*} = \nu(-\hat{x}_t^{T^*}) + \hat{c}_t^* \tag{D.57}$$

$$\hat{c}_t^{H^*} = \theta(-\hat{x}_t^{H^*}) + \hat{c}_t^{T^*} \tag{D.58}$$

$$\hat{c}_t^{F^*} = \theta(-\hat{x}_t^{F^*}) + \hat{c}_t^{T^*} \tag{D.59}$$

- The Price Indices
  - Consumer Price Indices

$$0 = \alpha\, \hat{x}_t^T + (1-\alpha)\, \hat{x}^N \tag{D.60}$$

$$0 = \alpha^*\, \hat{x}^{T^*} + (1-\alpha^*)\, \hat{x}^{N^*} \tag{D.61}$$

  - Price Indices for Tradable Goods

$$\hat{x}_t^T = \omega\, \hat{x}^H + (1-\omega)\, \hat{x}^F \tag{D.62}$$

$$\hat{x}^{T^*} = \omega^*\, \hat{x}^{F^*} + (1-\omega^*)\, \hat{x}^{H^*} \tag{D.63}$$

  - Export Prices

    As some firms engage in local currency pricing, some exports are home currency priced and some are foreign currency priced. The export price indices have the following form:

$$p_t^F = d^*\, p_t^{LCP} + (1-d^*)\, (p_t^{F^*} + s_t) \tag{D.64}$$

$$p_t^{H^*} = d\, p_t^{LCP^*} + (1-d)\, (p_t^H - s_t) \tag{D.65}$$

- Production Functions

$$\hat{y}_t^H = \hat{l}_t^H, \quad \hat{y}_t^N = \hat{l}_t^N \tag{D.66}$$

$$\hat{y}_t^F = \hat{l}_t^F, \quad \hat{y}_t^{N^*} = \hat{l}_t^{N^*} \tag{D.67}$$

- Price Setting

  Let's define the inflation variables: $\pi_t^N = p_t^N - p_{t-1}^N$, $\pi_t^H = p_t^H - p_{t-1}^H$, $\pi_t^{N^*} = p_t^{N^*} - p_{t-1}^{N^*}$, $\pi_t^{F^*} = p_t^{F^*} - p_{t-1}^{F^*}$ $\pi_t^{LCP} = p_t^{LCP} - p_{t-1}^{LCP}$, $\pi_t^{LCP^*} = p_t^{LCP^*} - p_{t-1}^{LCP^*}$

  - home country non-traded sector

  $$\pi_t^N \; = \; \gamma^f \, \pi_{t+1}^N \; + \; \gamma^b \, \pi_{t-1}^N + \lambda \, mc_t^N \tag{D.68}$$

  where

  $$mc_t^N = w_t - p_t^N - a_t^N$$

  - home country traded sector

  $$\pi_t^H \; = \; \gamma^f \, \pi_{t+1}^H \; + \; \gamma^b \, \pi_{t-1}^H + \lambda \, mc_t^H \tag{D.69}$$

  where

  $$mc_t^H = w_t - p_t^H - a_t^H$$

  - home country locally priced imports

  $$\pi_t^{LCP} \; = \; \gamma^{f^*} \, \pi_{t+1}^{LCP} \; + \; \gamma^{b^*} \, \pi_{t-1}^{LCP} + \lambda^* \, mc_t^F \tag{D.70}$$

  where

  $$mc_t^F = w_t^* - p_t^{LCP} - a_t^F + s_t$$

  - foreign country non-traded sector

  $$\pi_t^{N^*} \; = \; \gamma^{f^*} \, \pi_{t+1}^{N^*} \; + \; \gamma^{b^*} \, \pi_{t-1}^{N^*} + \lambda^* \, mc_t^{N^*} \tag{D.71}$$

  where

  $$mc_t^{N^*} = w_t^* - p_t^{N^*} - a_t^{N^*}$$

– foreign country traded sector

$$\pi_t^{F^*} = \gamma^{f^*} \pi_{t+1}^{F^*} + \gamma^{b^*} \pi_{t-1}^{F^*} + \lambda^* mc_t^{F^*} \tag{D.72}$$

where

$$mc_t^{F^*} = w_t^* - p_t^{F^*} - a_t^F$$

– foreign country locally priced imports

$$\pi_t^{LCP^*} = \gamma^f \pi_{t+1}^{LCP^*} + \gamma^b \pi_{t-1}^{LCP^*} + \lambda^* mc_t^{H^*} \tag{D.73}$$

where

$$mc_t^{H^*} = w_t - p_t^{LCP^*} - a_t^H - s_t$$

- Wage Setting

  The wage inflation is: $\pi_t^w = w_t - w_{t-1}$ *and* $\pi_t^{w^*} = w_t^* - w_{t-1}^*$

  – wage dynamics in home country

$$\pi_t^w = \gamma^{f,w} \pi_{t+1}^w + \gamma^{b,w} \pi_{t-1}^w - \lambda^w mrs_t \tag{D.74}$$

where

$$mrs_t = r\hat{w}_t - \eta l_t - (\frac{\rho}{1-h})(\hat{c}_t - h\hat{c}_{t-1}) - \frac{h}{1-h}(t_t^c - t_{t-1}^c)$$

– wage dynamics in foreign country

$$\pi_t^{w^*} = \gamma^{f,w^*} \pi_{t+1}^{w^*} + \gamma^{b,w^*} \pi_{t-1}^{w^*} - \lambda^{w^*} mrs_t^* \tag{D.75}$$

where

$$mrs_t^* = r\hat{w}_t^* - \eta l_t^* - (\frac{\rho}{1-h})(\hat{c}_t^* - h\hat{c}_{t-1}^*) - \frac{h}{1-h}(t_t^{c^*} - t_{t-1}^{c^*})$$

- Current Account

$$\begin{aligned}\beta b_t - b_{t-1} = {} & (1 - \omega^*)\alpha^*(1-n)(\hat{c}_t^{H^*} + \hat{x}_t^{H^*} + s_t) \\ & + \omega\alpha n(\hat{c}_t^H + \hat{x}_t^H) - \alpha n(\hat{c}_t^T + \hat{x}_t^T)\end{aligned} \tag{D.76}$$

- Monetary Policy

$$i_t = \Gamma_{i-1} i_{t-1} + (1-\Gamma_{i-1})\Gamma_{\pi_t}\pi_t + (1-\Gamma_{i-1})\Gamma_{y_t}\hat{y}_t + \varepsilon_{m,t} \tag{D.77}$$

$$i^*_t = \Gamma^*_{i-1} i^*_{t-1} + (1-\Gamma^*_{i-1})\Gamma^*_{\pi_t}\pi^*_t + (1-\Gamma^*_{i-1})\Gamma^*_{y_t}\hat{y}^*_t + \varepsilon^*_{m,t} \tag{D.78}$$

where

$$\varepsilon_{m,t} \sim N(0,\sigma^2_m), \qquad \varepsilon^*_{m,t} \sim N(0,\sigma^2_{m^*})$$

- Market Clearing
  - Non-traded sector

$$\hat{y}^N_t = \hat{c}^N_t, \qquad \hat{y}^{N^*}_t = \hat{c}^{N^*}_t \tag{D.79}$$

  - Traded sector

$$\hat{y}^H_t = \omega\,\hat{c}^H_t + \frac{(1-n)(1-\omega^*)\alpha^*}{n\,\alpha}\,\hat{c}^{H^*}_t \tag{D.80}$$

$$\hat{y}^F_t = \omega^*\,\hat{c}^{F^*}_t + \frac{n\,(1-\omega)\,\alpha}{(1-n)\,\alpha^*}\,\hat{c}^F_t \tag{D.81}$$

- Output and Labour supply
  - output

$$\hat{y}_t = \hat{c}_t + \frac{(1-\omega^*)\,\alpha^*\,(1-n)}{n}\,\hat{c}^{H^*}_t - (1-\omega)\alpha\,\hat{c}^F_t \tag{D.82}$$

$$\hat{y}^*_t = \hat{c}^*_t + \frac{(1-\omega)\,\alpha\,n}{1-n}\,\hat{c}^F_t - (1-\omega^*)\alpha^*\,\hat{c}^{H^*}_t \tag{D.83}$$

  - labour supply

$$l_t = \alpha\,\hat{l}^H_t + (1-\alpha)\,\hat{l}^N_t \tag{D.84}$$

$$l^*_t = \alpha^*\,\hat{l}^F_t + (1-\alpha^*)\,\hat{l}^{N^*}_t \tag{D.85}$$

- Definition of International Variables
  - net exports

$$nx_t = (1-\omega^*)\alpha^*(1-n)\,(\hat{c}^{H^*}_t + \hat{x}^{H^*}_t + s_t) - (1-\omega)\alpha n\,(\hat{c}^F_t + \hat{x}^F_t) \tag{D.86}$$

– terms of trade

$$\hat{tot}_t = p_t^F - p_t^{H^*} - s_t - t_t^{tot} \tag{D.87}$$

– decomposition of real exchange rate

∗ internal relative price movements channel

$$re\hat{rint}_t = (1-\alpha^*)(p_t^{N^*} - p_t^{T^*}) - (1-\alpha)(p_t^N - p_t^T) - t_t^{rerint} \tag{D.88}$$

∗ pricing to market channel

$$rerptm_t = (1-\omega^*)(s_t + p_t^{H^*} - p_t^H) + \omega^*(s_t + p_t^{F^*} - p_t^F) \tag{D.89}$$

∗ home bias channel

$$rer\hat{tot}_t = (\omega - (1-\omega^*))(p_t^F - p_t^H - t_t^{tot}) \tag{D.90}$$

– real exchange rate

$$\hat{rer}_t = rer\hat{tot}_t + rerptm_t + re\hat{rint}_t \tag{D.91}$$

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