

# Price and wage inflation inertia under time-dependent adjustments

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May, 2013

## Abstract

We derive and estimate a small-scale DSGE model augmented with price and wage adjustment governed by a time-dependent mechanisms. By using positively sloping hazard functions, we micro-found price and wage inflation intrinsic persistence, as we derive price and wage Phillips curves characterized by both forward and backward terms for inflation. Our estimation confirms upward-sloping hazard functions. Finally, we compare the empirical performance of our model to several popular alternatives based on different price and wage adjustment mechanisms, including Calvo pricing. By comparing log-marginal likelihoods of different estimations, we find that our model clearly outperforms these alternatives.

JEL classification: E31, E32, E52, C11.

Keywords: time-dependent price/wage adjustments, Calvo pricing, intrinsic inflation inertia, hybrid Phillips curves, model comparison.

## 1 Introduction

Our paper derives and estimates by using Bayesian techniques a small-scale model, which generalizes Erceg *et al.* (2000; EHL from now on) to time-dependent price and wage adjustments *à la* Sheedy (2007). Time-dependent models imply that a price or wage change will be more likely to be observed when last price reset happened many periods ago, i.e., the probability to reset a price is time-dependent. This mechanism can be formalized by using a hazard function, which shows the relation between the probability to post a new price and the time elapsed since the last reset: if the hazard function has a positive slope the likelihood to adjust a price is an increasing function of the time (Sheedy, 2007).<sup>1</sup>

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<sup>1</sup>Calvo pricing model is a particular case where the hazard function is flat, i.e. the probability to reset a price is exogenously randomly assigned to all the firms independently of the last time they have reset their prices.

A price adjustment with a non-constant hazard function is considered by many papers, including Taylor (1980), Goodfriend and King (1997), Dotsey *et al.* (1999), Wolman (1999), Guerrieri (2001, 2002), Mash (2004). These models are based on state or time dependent assumptions and focus on price dynamics. We follow Sheedy (2007), based on time-dependent pricing and positive hazard functions, because his approach seems to be more able to fit the macroeconomic figures—in particular to explain inflation persistence.<sup>2</sup> Differently from him, we also consider wage setting. Evidence of hazard function with positive slope for wages in the U.S. is supported by the micro study of Barattieri *et al.* (2010).

Specifically, the attractiveness of time-dependent models with positive hazard functions is that they can provide micro-foundations for a Phillips curve exhibiting “intrinsic persistence,”<sup>3</sup> which is a stylized economic fact hard to formalize in New Keynesian DSGE models (Fuhrer, 2011). Thus, time-dependent models are somehow alternative to the assumption of price indexation to the previous inflation rate. In fact, indexation implies the so-called “hybrid” New Keynesian Phillips curve where current inflation depends on both lagged and expected future inflation. The presence of a lagged term permits to model inflation as an auto-regressive process, where past inflation is source of structural intrinsic persistence.

The model is estimated for U.S. economy with Bayesian estimation techniques. After writing the model in state-space form we evaluate the likelihood function using the Kalman filter. The posterior distribution of the structural parameters is obtained combining priors with the likelihood function. The estimation of the model is performed using informative priors and, as robustness check, non-informative priors for the parameters affecting the slope of the hazard function.

In a similar paper Benati (2009) analyses different models to build inflation persistence including Sheedy (2007).<sup>4</sup> He finds evidence of positive-sloping hazard functions, but, by considering the Great Moderation sub-sample, he also finds that the parameters encoding the hazard slope have dropped to zero in last thirty years. He concludes that these parameters depend on the monetary regime referring to the switch in the way to conduct monetary policy discussed in Clarida *et al.* (2000). However he only focuses on price inflation: we generalize his approach by considering wage dynamics and possible time-dependent adjustment process in the labor markets. Then, by considering a sub-sample, we check if the hazard function remains strictly positive during the Great Moderation in our framework.

Finally, following Rabanal and Rubio-Ramirez (2005),<sup>5</sup> we compare the performance of our model to others based on alternative specifications for price and wage adjustments. Our goal is to test the improvement in explaining the data, in terms of marginal likelihood, due to our mechanism to micro-found inflation persistence. Specifically, as alternatives we consider flat hazard functions (price

<sup>2</sup>See Sheedy (2007) for a detailed discussion.

<sup>3</sup>Following Fuhrer (2011) by “intrinsic persistence” we refer to the inertia that does not depend on the real activity, but it is proper of the inflation process, whereas we refer to “inherited persistence” as the inertia inherited by the driving process, i.e. output gap or real marginal cost.

<sup>4</sup>Specifically, Benati (2009) analysed Fuhrer and Moore (1995), Galí and Gertler (1999), Blanchard and Galí (2007), Sheedy (2007), Ascari and Ropele (2009).

<sup>5</sup>For a wider analysis on model comparison see also Fernández-Villaverde and Rubio-Ramirez (2004), Lubik and Schorfheide (2006), Riggi and Tancioni (2010).

and wage Phillips curves *à la* Calvo) with indexation, which is a popular assumption to take account for inflation persistence (see Christiano *et al.*, 2005; Galí and Gertler, 1999).

The main contributions of our paper are five and can be summarized as follows. First, we derive an analytical solution for the wage Phillips curve with time-dependent adjustment *à la* Sheedy (2007). Second, we estimate a model with time-dependent adjustments for prices and wages for the U.S. and show that hazard functions have positive slopes. Third, by considering a sub-sample, we find that the parameters encoding intrinsic persistence remain significantly different from zero also during the Great Moderation. Fourth, by comparing marginal likelihoods, we find that our model outperforms alternative specifications for price and wage adjustments, i.e. Calvo with indexation and Calvo augmented by Galí-Gertler mechanism for prices and wages. Fifth, we successfully test the robustness of our empirical results by considering both informative and non-informative priors for the parameters affecting the intrinsic component of inflation inertia.

The rest of the paper is organized as follows. In the next section, after introducing Sheedy mechanism, we consider a simple small-scale model characterized by price and wage Phillips curve able to account for inflation persistence. Section 3 presents our model estimations and compares them to EHL and its extension with different kind of inflation indexation. A final section concludes.

## 2 The model

Our model generalizes EHL (2000) by assuming that price and wage adjustments are governed by a time-dependent mechanism. As we differ from EHL (2000) only for the derivation of the Phillips curves, the description of the model is not detailed. We report the log-linear deviations from the steady state. For a full derivation of model, we refer to EHL (2000).

### 2.1 Hazard function and Phillips curves

According to Sheedy (2007),<sup>6</sup> the probability to adjust a price is not random as in Calvo specification, but depends on the time elapsed since last price reset. This means that the probability to change a price is not equal among firms, but it is positive function of the time. Formally, price and wage adjustments are defined by using a hazard function, which expresses the relationship between the probability to reset a price and the duration of price stickiness. The hazard function is specified as follows:

$$\alpha_i = \alpha + \sum_{j=1}^{\min(i-1, n)} \varphi_j \left[ \prod_{k=i-j}^{i-1} (1 - \alpha_k) \right]^{-1}, \quad (1)$$

where  $\alpha_i$  is the probability to change a price which last reset was  $i$  periods ago;  $\alpha$  is the initial value of the hazard function,  $\varphi_j$  is its slope;  $n$  is the number of

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<sup>6</sup>In what follows we consider the same parametrization of the hazard function used in Sheedy (2007). Anyway, Sheedy (2010) uses a different kind of hazard derived from a reparametrization of the original one. However, both hazard functions lead to the same Phillips curve specification.

parameters that control the slope – for  $n = 1$ , the slope is governed by only one parameter,  $\varphi_j = \varphi$ .<sup>7</sup>

By using (1), Sheedy derives a price Phillips curve that depends on both expected and past inflation. Formally:<sup>8</sup>

$$\pi_t^p = \psi_p \pi_{t-1}^p + \beta [1 + (1 - \beta) \psi_p] E_t \pi_{t+1}^p - \beta^2 \psi_p E_t \pi_{t+2}^p + k_p (x_t + \zeta_t), \quad (2)$$

where  $\pi_t^p$  is the price inflation rate and  $x_t$  is the real marginal cost;  $\beta$  is the stochastic discount factor,  $\zeta_t$  is a price mark-up shock; the coefficients  $\psi_p$  and  $k_p$  are function of the parameters characterizing the hazard function:

$$\begin{cases} \psi_p = \frac{\varphi_p}{(1-\alpha_p) - \varphi_p [1-\beta(1-\alpha_p)]} \\ k_p = \frac{(\alpha_p + \varphi_p) [1-\beta(1-\alpha_p) + \beta^2 \varphi_p]}{(1-\alpha_p) - \varphi_p [1-\beta(1-\alpha_p)]} \eta_{cx} \end{cases} \quad (3)$$

Parameters  $\varphi_p$  and  $\alpha_p$  characterize the hazard function: the former controls the slope and the latter the starting level (i.e.,  $\varphi$  and  $\alpha$  in (1));  $\eta_{cx} = \frac{1-\phi}{1-\phi+\phi\varepsilon_p}$  is the elasticity of a firm's marginal cost with respect to average real marginal cost, where  $1 - \phi$  is the labor share and  $\varepsilon_p$  is the elasticity of substitution between workers. The elasticity  $\eta_{cx}$  is derived from a simple Cobb-Douglas production function without capital:

$$y_t = a_t + (1 - \phi) n_t, \quad (4)$$

where  $y_t$  denotes output,  $a_t$  is the technology shock and  $n_t$  is the amount of hours worked.

The real marginal cost is given by:

$$x_t = \omega_t + n_t - y_t, \quad (5)$$

where  $\omega_t$  denotes the real wage.

By definition, the real wage dynamics is described by:

$$\omega_t - \omega_{t-1} = \pi_t^w - \pi_t^p. \quad (6)$$

The marginal rate of substitution,  $mrst_t$ , between consumption and hours worked is given by:

$$mrst_t = \sigma y_t + \gamma n_t - g_t, \quad (7)$$

where  $\sigma$  denotes the relative risk aversion coefficient and  $g_t$  denotes a preference shifter shock. Since the labor market is characterized by imperfect competition the difference between the real wage and the marginal rate of substitution is equal to the wage mark-up:

$$\mu_t^w = \omega_t - mrst_t. \quad (8)$$

One novelty of our paper is to derive a New Keynesian wage Phillips curve that exhibits intrinsic inflation persistence from the hazard function. Formally:<sup>9</sup>

$$\pi_t^w = \psi_w \pi_{t-1}^w + \beta [1 + (1 - \beta) \psi_w] E_t \pi_{t+1}^w - \beta^2 \psi_w E_t \pi_{t+2}^w - k_w \mu_t^w, \quad (9)$$

<sup>7</sup>For the sake of simplicity, we follow Sheedy (2007) using  $n = 1$ .

<sup>8</sup>For the complete derivation see Sheedy (2007).

<sup>9</sup>Equation (9) is derived in Appendix A.

with

$$\begin{cases} \psi_w = \frac{\varphi_w}{(1-\alpha_w)-\varphi_w[1-\beta(1-\alpha_w)]} \\ k_w = \frac{(\alpha_w+\varphi_w)[1-\beta(1-\alpha_w)+\beta^2\varphi_w]}{(1-\alpha_w)-\varphi_w[1-\beta(1-\alpha_w)]} \Xi_w \end{cases}, \quad (10)$$

where  $\pi_t^w$  is the wage inflation,  $\psi_w$  and  $k_w$  are coefficients depending on the hazard parameters (as in the case for prices,  $\varphi_w$  and  $\alpha_w$  control respectively the slope and the initial level of the hazard function);  $\Xi_w = \frac{1}{1+\varepsilon_w\gamma}$ , where  $\varepsilon_w$  denotes the elasticity of substitution between workers and  $\gamma$  is the inverse of the Frisch labor supply elasticity.

## 2.2 Closing the model: IS curve and Taylor rule

The model is closed by introducing the demand side of the economy and the monetary policy rule. The demand side (IS curve) is obtained by log-linearizing the Euler equation around the steady-state, formally:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1}^p) - \frac{1}{\sigma} (E_t g_{t+1} - g_t), \quad (11)$$

where  $i_t$  is the nominal interest rate set by the central bank.

Monetary policy is modelled as a simple Taylor rule:

$$i_t = \rho_r i_{t-1} + (1 - \rho_r) (\delta_\pi \pi_t^p + \delta_x y_t) + v_t, \quad (12)$$

where  $\rho_r$  captures the degree of interest rate smoothing,  $\delta_\pi$  and  $\delta_x$  measure the response of the monetary authority to the deviation of inflation and output from their steady-state values;  $v_t$  is a monetary policy shock.

All the shocks considered in the model follow an AR(1) process:

$$\begin{cases} a_t = \rho_a a_{t-1} + \varepsilon_t^a, \\ g_t = \rho_g g_{t-1} + \varepsilon_t^g, \\ v_t = \rho_v v_{t-1} + \varepsilon_t^v, \\ \zeta_t = \rho_\zeta \zeta_{t-1} + \varepsilon_t^\zeta, \end{cases} \quad (13)$$

where  $\varepsilon_t^j \sim N(0, \sigma_j^2)$  are white noise shocks uncorrelated among them and  $\rho_j$  is the parameter measuring the degree of autocorrelation for each shock, for  $j = \{a, g, v, \zeta\}$ .

## 3 Empirical analysis

We estimate our model by Bayesian techniques. Our choice is motivated by the fact that Bayesian methods outperform GMM and maximum likelihood in small samples.<sup>10</sup> After writing the model in state-space form, the likelihood function is evaluated using the Kalman filter, whereas prior distributions are used to deliver additional non-sample information into the parameters estimation: once a prior distribution is elicited, posterior density for the structural parameters can be obtained reweighting the likelihood by a prior. The posterior is computed via numerical integration by making use of the Metropolis-Hastings algorithm

<sup>10</sup>For an exhaustive analysis of Bayesian estimation methods see Geweke (1999), An and Schorfheide (2007) and Fernández-Villaverde (2010).

for Monte Carlo integration; for the sake of simplicity all structural parameters are assumed to be independent from each other.

We use four observable macroeconomic variables: real GDP, price inflation, real wage, nominal interest rate. The dynamics is driven by four orthogonal shocks, including monetary policy, productivity, preference and price mark-up; since the number of observable variables is equal to the number of exogenous shocks the estimation does not present problems deriving from stochastic singularity.<sup>11</sup> The estimation of the model is performed by using informative priors and, as robustness check, non-informative priors for the parameters characterizing the slope of the hazard function.

We aim to test if the model exhibits positive hazard function, i.e. time-dependent price/wage adjustments holds. After estimating our model for the full sample (1960:1-2008:4), we also consider a smaller one (1982:1-2008:4), representative of the Great Moderation, in order to investigate if a positive hazard function still holds in a period characterized by small volatility of the shocks and more aggressive central bankers in fighting inflation. By considering only time-dependent price adjustment and flexible wages, Benati (2009) showed that during the Great Moderation, the parameters encoding the structural component of inflation persistence have dropped to zero.

Finally, we evaluate the empirical performance of our time-dependent Phillips curves to alternative specifications commonly used in literature. We consider the traditional forward-looking Phillips curves derived in EHL (2000) extended with price and wage indexation, which is often claimed as one main assumption to account for inflation persistence. Model comparison is based on log-marginal likelihood. In order to apply this methodology, we will show how models compared here are nested.

Next subsection presents the data used and prior distributions. Subsection 3.2 provides the estimation for the baseline model. Subsection 3.3 evaluates our time-dependent model against alternative specifications.

### 3.1 Data and prior distributions

In our estimations, we use U.S. quarterly data. All the time series used come from FRED2 database maintained by the Federal Reserve Bank of St. Louis. The *real gross domestic product* is used as measure of the output; the *effective Fed funds rate* is used for the nominal interest rate. Price inflation is measured using the *GDP implicit price deflator* taken in log-difference. Real wage is obtained dividing the nominal wage, measured by the *compensation per hour in nonfarm business sector*, by the *GDP implicit price deflator*. All the variables have been demeaned; output and real wage are detrended by using the Baxter and King's bandpass filter.

Our choices about prior beliefs are as follows. The coefficients of the Taylor rule are centered on a prior mean of 1.5 for inflation and 0.125 for the output gap and follow a Normal distribution. These values are quite standard in the literature. The smoothing parameter is assumed to follow a Beta distribution with mean 0.6 and standard deviation equal to 0.2. The inverse of Frisch elasticity is a tricky parameter to estimate: our choice is based on a Gamma distribution

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<sup>11</sup>Problems deriving from misspecification are widely discussed in Lubik and Schorfheide (2006) and Fernández-Villaverde (2010).

with mean 2 and standard deviation 0.375. For the hazard function coefficients we perform an “informative estimation” by using as priors coefficients estimated from single equation GMM;<sup>12</sup> we assign a Normal distribution to  $\varphi_p$  and  $\varphi_w$  with standard deviation equal to 0.2, whereas  $\alpha_p$  and  $\alpha_w$  follow a Beta distribution with standard deviation 0.1. As robustness check, following Benati (2009), we also estimate the model by using non-informative priors for the parameters affecting the slope of the hazard function, instead of those derived from the GMM estimations. Differently from him, we use a Uniform distribution with support  $[-1, 1]$ : the choice of this large interval is motivated by the fact that we want to investigate if the hazard slope is positive, negative or zero. We need to calibrate some parameters in order to avoid identification problems.<sup>13</sup> Since we consider a production function without capital, it is difficult to estimate  $\beta$  and  $\phi$ , which are set to 0.99 and 0.33, respectively. Similarly, we fix  $\varepsilon_p = 6$  and  $\varepsilon_w = 8.85$ , implying a price and wage mark-up equal to 1.20 and 1.12. Price elasticity is calibrated following Sheedy (2007), to be coherent with the hazard priors derived from his GMM estimation. Wage elasticity is derived as in Galí (2011) by using  $\varepsilon_w = [1 - \exp(-\gamma u^n)]^{-1} = 8.85$ , where we assume  $\gamma = 2$  and a natural unemployment rate  $u^n$  equal to 6%, as the average rate of the period considered. Finally, all the autoregressive coefficients of the shocks follow a Beta distribution with mean 0.5 and standard deviation equal to 0.2. The prior for the shocks standard errors is a Gamma with mean 0.1 and standard deviation 0.05.

## 4 Estimation results

Our baseline model consists of six equations, describing: the production function (4); the real marginal cost (5); the real wage dynamics (6); the marginal rate of substitution (7); the dynamic IS (11); the Taylor rule (12). Two additional equations close the model: the price and wage Phillips curve. In our baseline estimation we consider the time-dependent form for both price and wage equation, i.e. equations (2) and (9). Shocks dynamics are described by (13).

Our estimations are reported in Table 1, which also summarizes the 90% probability intervals and our assumptions about the priors. The table describes the results for the full sample and the Great Moderation. We report posterior estimation of the shocks and structural parameters, obtained by Metropolis-Hastings algorithm, when informative priors for the hazard slope are used.

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<sup>12</sup>We estimate  $\varphi_w$  and  $\alpha_w$  by using GMM as Sheedy (2007) does for the price adjustment (details are provided by Appendix B). For the hazard characterizing price adjustment we directly use as priors the GMM estimates of Sheedy (2007).

<sup>13</sup>The identification procedure has been performed by using the Identification toolbox for Dynare, which implements the identification condition proposed by Iskrev (2010a, 2010b). For a review of identification issues arising in DSGE models see Canova and Sala (2009).

Table 1 – Prior and posterior distributions<sup>14</sup>

	Prior distribution			Posterior distribution (full sample)			Posterior distribution (Great Moderation)		
	Density	Mean	St. Dev.	Mean	5%	95%	Mean	5%	95%
$\sigma$	Gamma	1.0	0.375	4.344	3.533	5.290	3.824	2.825	4.774
$\gamma$	Gamma	2.0	0.375	2.687	2.100	3.232	2.425	1.806	3.000
$\delta_\pi$	Normal	1.5	0.25	1.240	1.036	1.428	1.739	1.349	2.123
$\delta_x$	Normal	0.125	0.05	0.230	0.171	0.287	0.192	0.118	0.264
$\rho_r$	Beta	0.6	0.2	0.712	0.665	0.762	0.743	0.682	0.803
$\alpha_p$	Beta	0.132	0.1	0.013	0.001	0.027	0.032	0.001	0.069
$\varphi_p$	Normal	0.222	0.2	0.196	0.157	0.234	0.179	0.108	0.254
$\alpha_w$	Beta	0.318	0.1	0.166	0.104	0.231	0.192	0.060	0.341
$\varphi_w$	Normal	0.126	0.2	0.240	0.208	0.275	0.275	0.231	0.316
$\rho_a$	Beta	0.5	0.2	0.633	0.529	0.742	0.625	0.474	0.790
$\rho_g$	Beta	0.5	0.2	0.878	0.847	0.908	0.887	0.846	0.928
$\rho_v$	Beta	0.5	0.2	0.230	0.121	0.337	0.495	0.354	0.647
$\rho_\zeta$	Beta	0.5	0.2	0.800	0.734	0.865	0.784	0.696	0.874
$\sigma_a$	Gamma	0.1	0.05	0.021	0.013	0.029	0.016	0.008	0.023
$\sigma_g$	Gamma	0.1	0.05	0.037	0.030	0.043	0.031	0.024	0.037
$\sigma_v$	Gamma	0.1	0.05	0.002	0.002	0.002	0.001	0.001	0.001
$\sigma_\zeta$	Gamma	0.1	0.05	0.016	0.009	0.022	0.018	0.009	0.027

In the full sample case, the estimated hazard function is upward-sloping, since  $\varphi_p$  and  $\varphi_w$  are both positive. Thus, Sheedy mechanism seems to be able to account for inflation inertia for both prices and wages. The duration of a price spell is 3.8 quarters, whereas wages appear to be less sticky, since their duration is 1.9 quarters.<sup>15</sup> The response of monetary authority to inflation and output gap is in line with the Taylor principle; the estimated degree of interest rate smoothing is 0.71. By considering the Great Moderation subsample, differently from Benati (2009), we find that hazard function continues to exhibit positive slope, since both  $\varphi_p$  and  $\varphi_w$  are positive. This result gives us evidence that a pricing mechanism based on hazard function still holds also in a period characterized by a central bank more concerned in fighting inflation, as highlighted by the higher estimated coefficient for  $\delta_\pi$ . Therefore, three out four shocks present a smaller standard deviation in last thirty years. In Figure 1 we plot prior distribution, posterior distribution and posterior mode of the estimated parameters.

Figure 1 - Prior distribution (grey curve), Posterior distribution (green curve)

<sup>14</sup>The posterior distributions are obtained using Metropolis-Hastings algorithm; the procedure is implemented using the Matlab-based Dynare package. Mean and posterior percentiles come from two chains of 250,000 draws each from Metropolis-Hastings algorithm, where we discarded the initial 30% draws.

<sup>15</sup>The durations ( $D$ ) of price and wage stickiness are computed by using the following relation:  $D = \frac{1-\varphi}{\alpha+\varphi}$  (see Sheedy, 2007).

and Posterior mode (dotted line) of the estimated parameters.

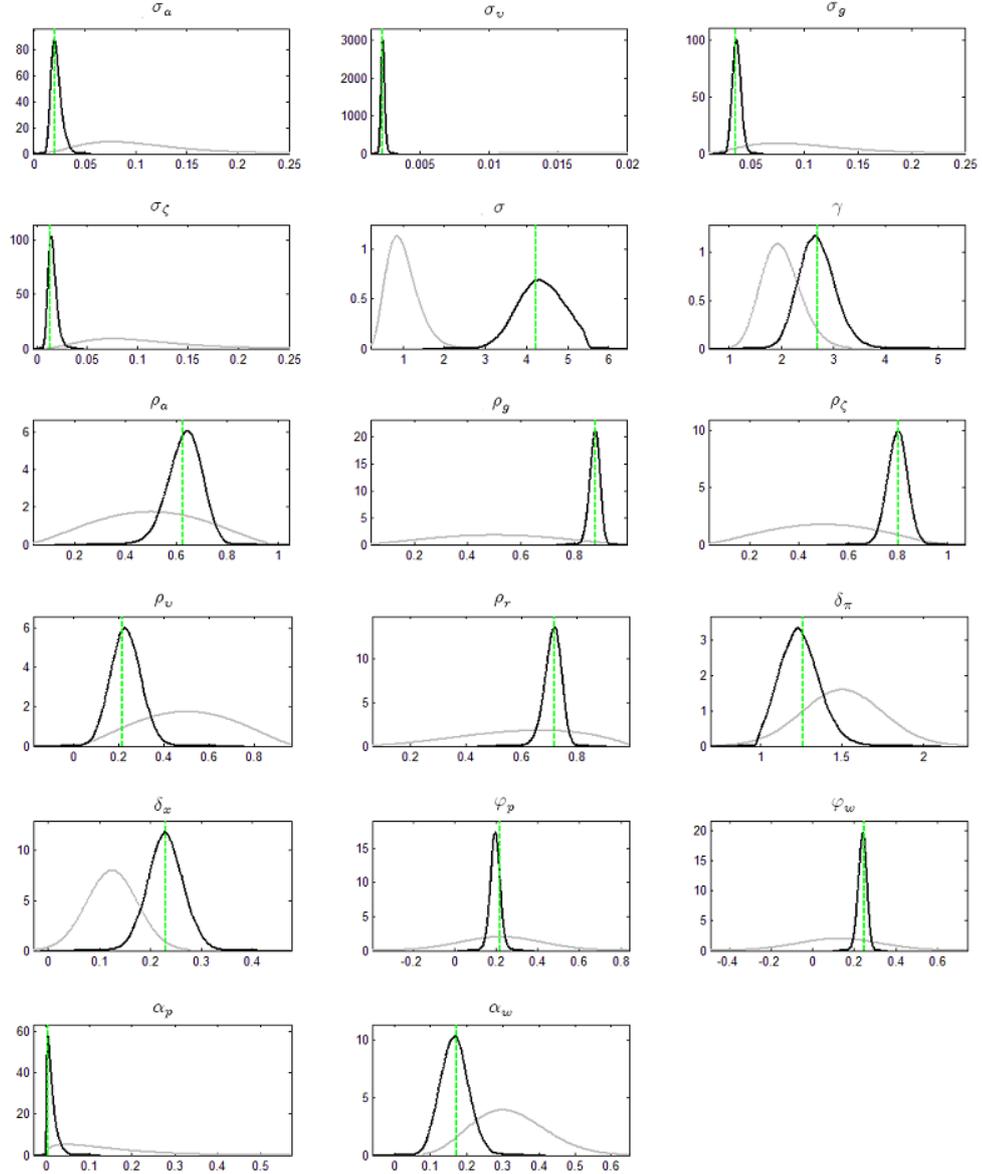


Table 2 reports the results from the estimation based on non-informative priors for the parameters  $\varphi_p$  and  $\varphi_w$ , whereas the prior distributions for the remaining parameters are the same used previously.

Table 2 - Prior and posterior distributions under non-informative priors

	Prior distribution			Posterior distribution (full sample)			Posterior distribution (Great Moderation)		
	Density	Mean	St. Dev.	Mean	5%	95%	Mean	5%	95%
$\sigma$	Gamma	1.0	0.375	4.361	3.555	5.306	3.851	2.853	4.826
$\gamma$	Gamma	2.0	0.375	2.685	2.099	3.251	2.405	1.807	3.031
$\delta_\pi$	Normal	1.5	0.25	1.235	1.029	1.418	1.704	1.288	2.107
$\delta_x$	Normal	0.125	0.05	0.230	0.173	0.288	0.193	0.120	0.266
$\rho_r$	Beta	0.6	0.2	0.713	0.665	0.761	0.744	0.683	0.805
$\alpha_p$	Beta	0.132	0.1	0.013	0.001	0.028	0.041	0.001	0.089
$\varphi_p$	Uniform	0	0.57	0.195	0.153	0.233	0.162	0.073	0.257
$\alpha_w$	Beta	0.318	0.1	0.165	0.105	0.228	0.185	0.071	0.319
$\varphi_w$	Uniform	0	0.57	0.240	0.205	0.275	0.271	0.203	0.324
$\rho_a$	Beta	0.5	0.2	0.636	0.527	0.748	0.627	0.443	0.809
$\rho_g$	Beta	0.5	0.2	0.879	0.849	0.910	0.886	0.845	0.927
$\rho_v$	Beta	0.5	0.2	0.230	0.119	0.336	0.494	0.346	0.633
$\rho_\zeta$	Beta	0.5	0.2	0.799	0.733	0.863	0.781	0.692	0.874
$\sigma_a$	Gamma	0.1	0.05	0.021	0.014	0.029	0.017	0.009	0.024
$\sigma_g$	Gamma	0.1	0.05	0.037	0.030	0.043	0.031	0.024	0.037
$\sigma_v$	Gamma	0.1	0.05	0.002	0.002	0.002	0.001	0.001	0.001
$\sigma_\zeta$	Gamma	0.1	0.05	0.016	0.009	0.023	0.020	0.009	0.035

The “non-informative” estimation confirms our results about the hazard function, which is still characterized by positive slope, both in full sample and during the Great Moderation; the estimated parameters for the hazard slope are very similar to the ones estimated under "informative" priors. This result shows as the hazard function mechanism is robust to a change of policy.

## 4.1 Time-dependent Phillips vs. alternatives

In this section we aim to compare the empirical performance of our time-dependent Phillips curves to different specifications for price and wage adjustments. In our framework this can be easily done as the model encompasses several alternatives. Simply by setting  $\varphi_p = 0$  and  $\varphi_w = 0$ , we obtain flat hazard functions, and therefore, price and wage Phillips curves *à la* Calvo. Moreover, different kinds of indexation can be introduced by minimal manipulations. In the following we show how to derive the EHL (2000) Phillips curves from our model and augment them by indexation and then we compare these alternatives to our baseline model in terms of log-marginal density.

### 4.1.1 Alternative price-setting mechanisms: EHL with indexation

It is easy to verify that the price Phillips curve (2) nests the EHL case. Assuming  $\varphi_p = 0$ , we get:

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p (x_t + \zeta_t) \quad (14)$$

where  $\lambda_p = \frac{\alpha_p [1 - \beta(1 - \alpha_p)]}{1 - \alpha_p} \eta_{cx}$ . Equation (14) can be also augmented by indexation:

$$\pi_t^p = \frac{\iota_p}{(1 + \iota_p \beta)} \pi_{t-1}^p + \frac{\beta}{1 + \iota_p \beta} E_t \pi_{t+1}^p + \lambda_p^\iota (x_t + \zeta_t) \quad (15)$$

where  $\iota_p$  denotes the degree of price indexation to last period's inflation, and  $\lambda_p^\iota = \frac{\lambda_p}{(1+\iota_p\beta)}$ .

Similarly, equation (9) nests the EHL case for  $\varphi_w = 0$ :

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w \mu_t^w \quad (16)$$

where  $\lambda_w = \frac{\alpha_w[1-\beta(1-\alpha_w)]}{1-\alpha_w} \Xi_w$ . It can be augmented by indexation:

$$\pi_t^w = \iota_w \pi_{t-1}^w - \iota_w \beta \pi_t^p + \beta E_t \pi_{t+1}^w - \lambda_w \mu_t^w \quad (17)$$

where  $\iota_w$  denotes the degree of wage indexation to last period's inflation.

#### 4.1.2 Galí-Gertler setting

A further specification to account for inflation persistence has been introduced by Galí and Gertler (1999). They proposed a modification of the Calvo mechanism by introducing partial indexation due to a backward looking rule of thumb. The Phillips curves are specified as follows:

$$\pi_t^p = \frac{\xi_p}{\Lambda_p} \pi_{t-1}^p + \frac{\beta(1-\alpha_p)}{\Lambda_p} E_t \pi_{t+1}^p + \lambda_p^\xi (x_t + \zeta_t) \quad (18)$$

$$\pi_t^w = \frac{\xi_w}{\Lambda_w} \pi_{t-1}^w + \frac{\beta(1-\alpha_w)}{\Lambda_w} E_t \pi_{t+1}^w - \lambda_w^\xi \mu_t^w \quad (19)$$

where  $\xi_p$  measures the degree of price indexation to past inflation,  $\xi_p$  denotes the degree of wage indexation to past inflation,  $\Lambda_p = 1 - \alpha_p + \xi_p [\alpha_p + (1 - \alpha_p) \beta]$ ,  $\Lambda_w = 1 - \alpha_w + \xi_w [\alpha_w + (1 - \alpha_w) \beta]$ ,  $\lambda_p^\xi = \frac{\alpha_p(1-\xi_p)[1-\beta(1-\alpha_p)]}{\Lambda_p}$  and  $\lambda_w^\xi = \frac{\alpha_w(1-\xi_w)[1-\beta(1-\alpha_w)]}{\Lambda_w(1+\varepsilon_w\gamma)}$ .

#### 4.1.3 Model comparisons

As shown above, our formalization nests different models of price and wage adjustment. Differences only depend on the Phillips curve parameterization. By different assumptions on  $\varphi_p$ ,  $\varphi_w$ ,  $\iota_p$ ,  $\iota_w$ ,  $\xi_p$ ,  $\xi_w$ , we can consider positive hazard functions or flat hazard functions augmented by two different kind of indexation. We compare our baseline (BASE) to two alternative scenarios:<sup>16</sup>

1. EHL model with indexation (EHLind), by considering (15) and (17);
2. EHL model with indexation *à la* Galí-Gertler (GG), by considering (18) and (19).

The measure used to compare the models is the log-marginal likelihood, which is a measure of the fit of a model in explaining the data. The aim is to evaluate if the way in which is modeled price and wage adjustment affects the fit a model. The model with the highest log-marginal likelihood better explains the data. Table 3 reports our results.

<sup>16</sup>We omit the comparison with a model characterized by simple forward-looking Phillips curves *à la* Calvo since this model has not intrinsic persistence. Anyway, Rabanal and Rubio-Ramirez (2005) showed that this model exhibits quite the same performance of a model with indexation.

Table 3 - Log-marginal data densities and Bayes factors for different models<sup>17</sup>

Model	Log-marginal data density	Bayes factor vs. BASE
BASE	3443.6	
BASE (non-info)	3441.1	$\exp[-2.5]$
EHLind	3396.8	$\exp[-46.8]$
GG	3390.0	$\exp[-53.6]$

The difference, in terms of marginal likelihood, between Galí-Gertler specification and EHL augmented by indexation is minimal. According to Jeffreys' scale of evidence,<sup>18</sup> this difference must be considered as “slight” evidence in favor of EHLind with respect to GG. However, our model clearly outperforms both the alternative considered: in particular, Bayes factor gives “very strong” evidence in favor of our specification. This means that the pricing method based on hazard functions seems to capture better inflation inertia. Under “non-informative” priors we observe a small decrease of the marginal likelihood: this happens since under diffuse priors there is an increase of model complexity and this penalizes the marginal data density (this effect dominates the improvement in model fit).

## 5 Conclusions

We have built and estimated a model that considers both price and wage adjustment governed by the time-dependent mechanism described by Sheedy (2007). By making use of a hazard function, we have derived price and wage Phillips curves that are able in micro-founding price and wage inflation intrinsic persistence—as they are characterized by both forward and backward terms for inflation. We have estimated our model with Bayesian techniques. The estimation of our model has confirmed that a hazard function upward-sloping emerges. Differently from Benati (2009), who only considers price inflation, we find that the hazard function slope does not change with the policy regime, i.e. during the Great Moderation era. Finally, we have compared the empirical performance in fitting the data of our model to those of others based on popular alternative mechanisms for price and wage adjustment. By comparing log-marginal likelihoods of different estimations, we have found that our model clearly outperforms alternatives.

## Appendix A – Wage Phillips curve derivation

Following Sheedy (2007), we assume that wages are set according to a time-dependent mechanism: the probability to change a wage depends positively

<sup>17</sup>For the computation of the marginal likelihood for different model specifications we used the modified harmonic mean estimator, based on Geweke (1999). The Bayes factor is the ratio of posterior odds to prior odds (see Kass and Raftery, 1995).

<sup>18</sup>Jeffreys (1961) provided a scale for the evaluation of the Bayes factor indication. Odds ranging from 1:1 to 3:1 give “very slight evidence;” odds ranging from 3:1 to 10:1 constitute “slight evidence;” odds ranging from 10:1 to 100:1 constitute “strong to very strong evidence;” odds greater than 100:1 give “decisive evidence.”

on the time elapsed since last wage reset. This adjustment process can be formalized by using a hazard function.<sup>19</sup>

Assuming that  $\Gamma_t \subset \Theta$  denotes the set of households that post a new wage at time  $t$ , we can define the duration of wage stickiness as:

$$D_t(j) \equiv \min \{l \geq 0 \mid j \in \Gamma_{t-l}\} \quad (20)$$

where  $D_t(j)$  is the duration of a wage spell for household  $j$  which last reset was  $l$  periods ago.

We now introduce the hazard function, which expresses the relationship between the probability to post a new wage and the wage duration. The hazard function is defined by a sequence of probabilities:  $\{\alpha_l\}_{l=1}^{\infty}$ , where  $\alpha_l$  represents the probability to reset a wage which remained unchanged for  $l$  periods. This probability is defined as:  $\alpha_l \equiv \Pr(\Gamma_t \mid D_{t-1} = l - 1)$ .

Each hazard function is related to a survival one, which expresses the probability that a wage remains fixed for  $l$  periods. As for the hazard, the survival function is defined by a sequence of probabilities:  $\{\varsigma_l\}_{l=0}^{\infty}$ , where  $\varsigma_l$  denotes the probability that a wage fixed at time  $t$  will be still in use at time  $t+l$ . Formally, the survival function is defined by:

$$\varsigma_l = \prod_{h=1}^l (1 - \alpha_h) \quad (21)$$

with  $\varsigma_0 = 1$ . Following Sheedy (2007), we assume that the hazard function satisfies two restrictions:

$$\begin{cases} \alpha_1 < 1, \text{ meaning that is allowed a degree of wage stickiness;} \\ \alpha_{\infty} > 0, \text{ with } \alpha_{\infty} = \lim_{l \rightarrow \infty} \alpha_l. \end{cases} \quad (22)$$

The hazard function can be reparameterized by making use of a set of  $n+1$  parameters and rewritten as (1), where  $\{\varphi_l\}_{l=1}^n$  is a set of  $n$  parameters that control the hazard slope, whereas parameter  $\alpha$  controls its initial level.

By making use of (21), we can rewrite the non-linear recursion (1) for the wage adjustment probabilities as a linear recursion for the corresponding survival function:

$$\varsigma_l = (1 - \alpha)\varsigma_{l-1} - \sum_{h=1}^{\min(l-1, n)} \varphi_h \varsigma_{l-1-h} \quad (23)$$

The parameters  $\{\varphi_l\}_{l=1}^n$  control the slope of the hazard function in the following way:

$$\begin{cases} \varphi_l = 0, \text{ for all } l = 1, \dots, n \text{ the hazard is flat (Calvo case);} \\ \varphi_l \geq 0, \text{ for all } l = 1, \dots, n \text{ the hazard is upward-sloping;} \\ \varphi_l \leq 0, \text{ for all } l = 1, \dots, n \text{ the hazard is downward-sloping.} \end{cases} \quad (24)$$

Let  $\theta_{lt} \equiv \Pr(D_t = l)$  denote the proportion of households earning at time  $t$  a wage posted at period  $t-l$ . The sequence  $\{\theta_{lt}\}_{l=0}^{\infty}$  denotes the distribution

<sup>19</sup>We refer to Sheedy (2007) for the proofs relative to the hazard function mentioned here. See in particular his Appendix A.2 and A.5.

of the duration of wage stickiness at time  $t$ . This distribution evolves over the time according to:

$$\begin{cases} \theta_{0t} = \sum_{l=1}^{\infty} \alpha_l \theta_{l-1,t-1} \\ \theta_{lt} = (1 - \alpha_l) \theta_{l-1,t-1} \end{cases} \quad (25)$$

If the hazard function satisfies the restrictions (22) and the evolution over the time of the distribution of wage duration evolves as in (25), then a) from whatever starting point, the economy always converges to a unique stationary distribution  $\{\theta_l\}_{l=0}^{\infty}$ . Hence  $\theta_{lt} = \theta_l = \Pr(D_t = l)$ ,  $\forall t$ ; b) let consider (1) and assume that the economy has converged to  $\{\theta_l\}_{l=0}^{\infty}$ , the following three relations are obtained:

$$\begin{cases} \theta_l = \left( \alpha + \sum_{h=1}^n \varphi_h \right) s_l \\ \alpha^e = \alpha + \sum_{l=1}^n \varphi_l \\ D^e = \frac{1 - \sum_{l=1}^n l \varphi_l}{\alpha + \sum_{l=1}^n \varphi_l} \end{cases} \quad (26)$$

where  $\alpha^e$  denotes the unconditional probability of wage reset and  $D^e$  represents the duration of wage stickiness.

Our supply side of the economy is fairly standard (see, e.g., Galí, 2008: Chapter 6). It is composed by a continuum of monopolistically competitive firms indexed on the unit interval  $\Omega \equiv [0, 1]$ . The production function of the representative firm  $i \in \Omega$  is described by a Cobb-Douglas without capital:  $Y_t(i) = A_t N_t(i)^{1-\alpha}$ , where  $Y_t(i)$  is the output of good  $i$  at time  $t$ ,  $A_t$  represents the state of technology,  $N_t(i)$  is the quantity of labor employed by  $i$ -firm and  $1 - \alpha$  is the labor share. The quantity of labor used by firm  $i$  is defined by:

$$N_t(i) = \left[ \int_{\Omega} N_t(i, j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \quad (27)$$

where  $N_t(i, j)$  is the quantity of  $j$ -type labor employed by firm  $i$  in period  $t$  and  $\varepsilon_w$  denotes the elasticity of substitution between workers. Cost minimization with respect to the quantity of labor employed yields to labor demand schedule:

$$N_t(i, j) = \left( \frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_t(i) \quad (28)$$

where  $W_t(j)$  is the nominal wage paid to  $j$ -type worker and  $W_t$  is the aggregate wage index defined in the following way:

$$W_t = \left[ \int_0^1 W_t(j)^{1-\varepsilon_w} dj \right]^{\frac{1}{1-\varepsilon_w}} \quad (29)$$

We consider a continuum of monopolistically competitive households indexed on the unit interval  $\Theta \equiv [0, 1]$ . Each household supplies a different type of labor  $N_t(j) = \int_{\Omega} N_t(i, j) di$  to all the firms. The representative household  $j \in \Theta$

chooses the quantity of labor  $N_t(j)$  to supply, in order to maximize the following separable utility:

$$U(C_t(j), N_t(j)) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ g_t \frac{C_t^{1-\sigma}(j)}{1-\sigma} - \frac{N_t^{1+\gamma}(j)}{1+\gamma} \right] \right\} \quad (30)$$

where  $E_0$  is the expectation operator conditional on time  $t = 0$  information,  $\beta$  is the stochastic discount factor,  $\sigma$  denotes the relative risk aversion coefficient and  $\gamma$  is the inverse of the Frisch labor supply elasticity. Finally,  $g_t$  is a preference shock which is assumed to follow an  $AR(1)$  stationary process. The household faces a standard budget constraint specified as follows in nominal terms:

$$P_t(j) C_t(j) + E_t [Q_{t+1,t} B_t(j)] \leq B_{t-1}(j) + W_t(j) N_t(j) + T_t(j) \quad (31)$$

where  $P_t(j)$  is the price of good  $j$ ,  $B_t(j)$  denotes holdings of one-period bonds,  $Q_t$  is the bond price,  $T_t$  represents a lump-sum government nominal transfer. Finally,  $C_t(j)$  represents the consumption of household  $j$  and it is described by a CES aggregator:  $C_t(j) = \left( \int_{\Theta} C_t(i, j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$ , where  $C_t(i, j)$  denotes the quantity of  $i$ -type good consumed by household  $j$  and  $\varepsilon_p$  is the elasticity of substitution between goods.

In our framework households are wage-setters. In setting wages, each maximizes (30) internalizing the effects of labor demand (28) and taking account of (31). Households are subject to a random probability to reset price, but, according to our time-dependent mechanism, a wage change will be more likely to be observed when last price reset happened many periods ago. Formally, suppose that at time  $t$  a household sets a new wage, denoted by  $W_t^*$ ,<sup>20</sup> if the household still earns this wage at time  $\tau \geq t$  then its relative wage will be  $W_t^*/W_\tau$  and the household utility can be written as  $U [W_t^*/W_\tau; C_{\tau|t}; N_{\tau|t}]$ ,<sup>21</sup> by considering the survival function, the household will then choose its optimal reset wage by solving:

$$\max_{W_t^*} \sum_{\tau=t}^{\infty} \varsigma_{\tau-t} E_t \left\{ \left( \prod_{s=t+1}^{\tau} \frac{\pi_s^p}{I_s} \right) U \left[ \frac{W_t^*}{W_\tau}; C_{\tau|t}; N_{\tau|t} \right] \right\} \quad (32)$$

where  $\pi_s^p = P_s/P_{s-1}$  is the gross price inflation rate and  $I_s = i_s/i_{s-1}$  is the gross nominal interest rate. This maximization is subject to the budget constraint (31) and the labor demand schedule (28). Equation (32) yields the following first-order condition:

$$\sum_{\tau=t}^{\infty} \varsigma_{\tau-t} E_t \left( \frac{W_t^*}{W_\tau} \right)^{-\varepsilon_w} \left( \prod_{s=t+1}^{\tau} \frac{\pi_s}{I_s} \right) \left[ U_c(C_{\tau|t}, N_{\tau|t}) \frac{N_{\tau|t}}{P_\tau} (1 - \varepsilon_w) + \right. \\ \left. -\varepsilon_w U_n(C_{\tau|t}, N_{\tau|t}) \frac{N_{\tau|t}}{W_\tau} \frac{W_\tau}{W_t^*} \right] = 0 \quad (33)$$

where  $U_c(C_{\tau|t}, N_{\tau|t})$  is the marginal utility of consumption and  $-U_n(C_{\tau|t}, N_{\tau|t})$  is the marginal disutility of labor. Considering that the marginal rate of substitution between consumption and leisure is  $MRS_{\tau|t} = -\frac{U_n(C_{\tau|t}, N_{\tau|t})}{U_c(C_{\tau|t}, N_{\tau|t})}$ , and the

<sup>20</sup>Since each household solves the same optimization problem, henceforth index  $j$  are omitted.

<sup>21</sup> $C_{\tau|t}$  and  $N_{\tau|t}$  denote respectively the level of consumption and the labour supply at time  $\tau$  of a household which last wage reset was in period  $t$ .

steady-state wage mark-up is  $\mu_w = \frac{\varepsilon_w}{\varepsilon_w - 1}$ , equation (33) can be rearranged and expressed in terms of the optimal wage reset as:

$$W_t^* = \left[ \frac{\mu_w \left( \sum_{\tau=t}^{\infty} \varsigma_{\tau-t} \beta^{\tau-t} MRS_{\tau|t} P_{\tau} \right)}{\sum_{\tau=t}^{\infty} \varsigma_{\tau-t} \beta^{\tau-t}} \right] \quad (34)$$

Assuming that the economy has converged to  $\{\theta_l\}_{l=0}^{\infty}$ , then the wage level (29) can be expressed as a weighted-average of the past reset wages:

$$W_t = \left( \sum_{l=0}^{\infty} \theta_l W_{t-l}^{*1-\varepsilon_w} \right)^{\frac{1}{1-\varepsilon_w}} \quad (35)$$

By log-linearizing (34) and (35) around a steady-state (characterized by zero wage inflation), we get:<sup>22</sup>

$$w_t^* = \sum_{\tau=t}^{\infty} \left( \frac{\beta^{\tau-t} \varsigma_{\tau-t}}{\sum_{j=0}^{\infty} \beta^j \varsigma_j} \right) [w_{\tau} - \Xi_w \mu_{\tau}^w] \quad (36)$$

$$w_t = \sum_{l=0}^{\infty} \theta_l w_{t-l}^* \quad (37)$$

Equations (36) and (37) describe the wage adjustment mechanism. The wage Phillips curve (9) used in the paper is derived by combining them with (23) and (26).

Specifically, by combining (23) with (36), we obtain:

$$w_t^* = \beta(1-\alpha) E_t w_{t+1}^* - \sum_{l=1}^n \beta^{l+1} \varphi_l E_t w_{t+l+1}^* + \left[ 1 - \beta(1-\alpha) + \sum_{l=1}^n \beta^{l+1} \varphi_l \right] (w_t - \Xi_w \mu_t^w) \quad (38)$$

By making use of (26), equation (37) can be recast as follows:

$$w_t = (1-\alpha) w_{t-1} - \sum_{l=1}^n \varphi_l w_{t-1-l} + \left( \alpha + \sum_{h=1}^n \varphi_h \right) w_t^* \quad (39)$$

where we have used the fact that the stationary distribution of the wage duration (26) can be rewritten in recursive way as:

$$\theta_l = (1-\alpha)\theta_{l-1} - \sum_{h=1}^{\min(l-1, n)} \varphi_h \theta_{l-h-1} \quad (40)$$

with  $\theta_0 = \alpha + \sum_{h=1}^n \varphi_h$ .

The general expression for the wage Phillips curve is obtained from (38) and (39):

$$\pi_t^w = \sum_{l=1}^n \psi_l \pi_{t-l}^w + \sum_{l=1}^{n+1} \delta_l E_t \pi_{t+l}^w - k_w \mu_t^w \quad (41)$$

<sup>22</sup>Small-caps letters denote log-deviation from the steady-state.

where the coefficients  $\psi_l$ ,  $\delta_l$  and  $k_w$  have the following parameterization:

$$\begin{aligned}\psi_l &= \frac{\varphi_l + \sum_{h=l+1}^n \varphi_h \left[ 1 - \beta(1 - \alpha) + \sum_{k=1}^{h-1} \beta^{k+1} \varphi_k \right]}{\chi} \\ \delta_1 &= \frac{\beta \left[ (1 - \alpha) - \sum_{h=1}^n \beta^h \varphi_h \left( \alpha + \sum_{k=1}^{h-1} \varphi_k \right) \right]}{\chi} \\ \delta_{l+1} &= \frac{\beta^{l+1} \left[ \varphi_l + \sum_{h=l+1}^n \beta^{h-1} \varphi_h \left( \alpha + \sum_{k=1}^{h-1} \varphi_k \right) \right]}{\chi} \\ k_w &= \frac{\Xi_w \left[ (\alpha + \sum_{h=1}^n \varphi_h) \left[ 1 - \beta(1 - \alpha) + \sum_{h=1}^n \beta^{h+1} \varphi_h \right] \right]}{\chi}\end{aligned}$$

where  $\chi = (1 - \alpha) - \sum_{h=1}^n \varphi_h \left[ 1 - \beta(1 - \alpha) + \sum_{k=1}^{h-1} \beta^{k+1} \varphi_k \right]$ , for  $l = 1, \dots, n$ .

It is easy to check that if we assume that only one parameter controls the slope of the hazard function (i.e.,  $n = 1$ ), the wage Phillips curve (41) becomes that reported in the paper, i.e. (9).

## Appendix B – GMM estimation of the wage Phillips curve

As in Sheedy (2007) we estimate our wage Phillips curve via generalized method of moments (GMM), in order to get priors for the parameters affecting the hazard function. Since it is not easy to find an observable proxy for the wage mark-up, the latter can be expressed as a function of unemployment, as in Galí *et al.* (2011):

$$\mu_t^w = \gamma u_t \quad (42)$$

where  $u_t$  represents the unemployment gap. Therefore (41) becomes:

$$\pi_t^w = \sum_{l=1}^n \psi_l \pi_{t-l}^w + \sum_{l=1}^{n+1} \delta_l E_t \pi_{t+l}^w - k_w \gamma u_t \quad (43)$$

To perform a GMM estimation of (43) we need to use a set of instruments, in order to correctly identify all the coefficients. Let  $z_{t-1}$  represents a vector of observable variables known at time  $t - 1$ : under rational expectations the error forecast of  $\pi_t^w$  is uncorrelated with information contained in  $z_{t-1}$ , then the following orthogonality condition holds:

$$E_t \left[ \left( \pi_t^w - \sum_{l=1}^n \psi_l \pi_{t-l}^w - \sum_{l=1}^{n+1} \delta_l E_t \pi_{t+l}^w + k_w \gamma u_t \right) z_{t-1} \right] = 0 \quad (44)$$

Following Galí and Gertler (1999), since (44) is non-linear in the structural parameters, we normalize the orthogonality condition in the following way:

$$E_t \left[ \left( \chi \pi_t^w - \chi \sum_{l=1}^n \psi_l \pi_{t-l}^w - \chi \sum_{l=1}^{n+1} \delta_l E_t \pi_{t+l}^w + \chi k_w \gamma u_t \right) z_{t-1} \right] = 0 \quad (45)$$

Our estimation is made using quarterly U.S. data ranging from 1960:1 to 2011:4: all the data comes from FRED2 database. The wage inflation is measured by the *compensation per hour*, whereas for the unemployment rate we use the *civilian unemployment rate*. The set of instruments is composed by the lags of the following observable variables: wage inflation, unemployment, price inflation, consumer price index, output gap, labor share, spread between ten-year Treasury Bond and three-month Treasury Bill yields. In particular six lags of price inflation, wage inflation and CPI, four lags for the output gap and two lags for the remaining instruments are used. For the sake of simplicity we show only the estimation of (45) when  $n = 1$ . Under the latter assumption, (44) and (45) change as follows:

$$E_t \left\{ \left[ \pi_t^w - \psi_w \pi_{t-1}^w - \beta(1 + (1 - \beta) \psi_w) E_t \pi_{t+1}^w + \beta^2 \psi_w E_t \pi_{t+2}^w + k_w \gamma u_t \right] z_{t-1} \right\} = 0 \quad (46)$$

$$E_t \left\{ \left[ \chi_w \pi_t^w - \chi_w \psi_w \pi_{t-1}^w - \chi_w \beta(1 + (1 - \beta) \psi_w) E_t \pi_{t+1}^w + \chi_w \beta^2 \psi_w E_t \pi_{t+2}^w + \chi_w k_w \gamma u_t \right] z_{t-1} \right\} = 0 \quad (47)$$

where  $\chi_w = (1 - \alpha_w) - \varphi_w [1 - \beta(1 - \alpha_w)]$ .

The structural form of (47) is estimated by imposing  $\beta = 0.99$ ,  $\varepsilon_w = 8.85$  and  $\gamma = 2$ ; the reduced form coefficients (see (10)) are convolution of the structural parameters estimated and they are obtained by substituting these parameters into them; the standard errors are computed using the delta method.<sup>23</sup>

The results for the structural form estimation are reported in Table 4. We show the estimation for the structural parameters  $\varphi_w$  (hazard slope) and  $\alpha_w$  (hazard initial value); moreover, we also report  $D^e$  and  $\alpha_w^e$  (computed as in (26)) and the  $J - stat$ .

Table 4 – Wage Phillips curve estimation (structural form)<sup>24</sup>

$\alpha_w$	$\varphi_w$	$D^e$	$\alpha_w^e$	$J - stat$
0.318*	0.126*	1.964*	0.444*	19.527
(0.050)	(0.030)	(0.146)	(0.033)	[0.813]

Notes: a 6-lag Newey-West estimate of the covariance matrix is used.

Standard errors are shown in parentheses.

For the J-stat the p-value is shown in brackets.

\* denotes statistical significance at 5% level.

All the coefficients estimated are statistically significant and the hazard function is estimated to be upward-sloping. The  $J - stat$  is a test of over-identifying moment condition: in our case we accept the null hypothesis that the over-identifying restrictions are satisfied (the model is “valid”).

We now report the reduced form of (46), obtained by substituting the estimated values of  $\alpha_w$  and  $\varphi_w$  into (10).

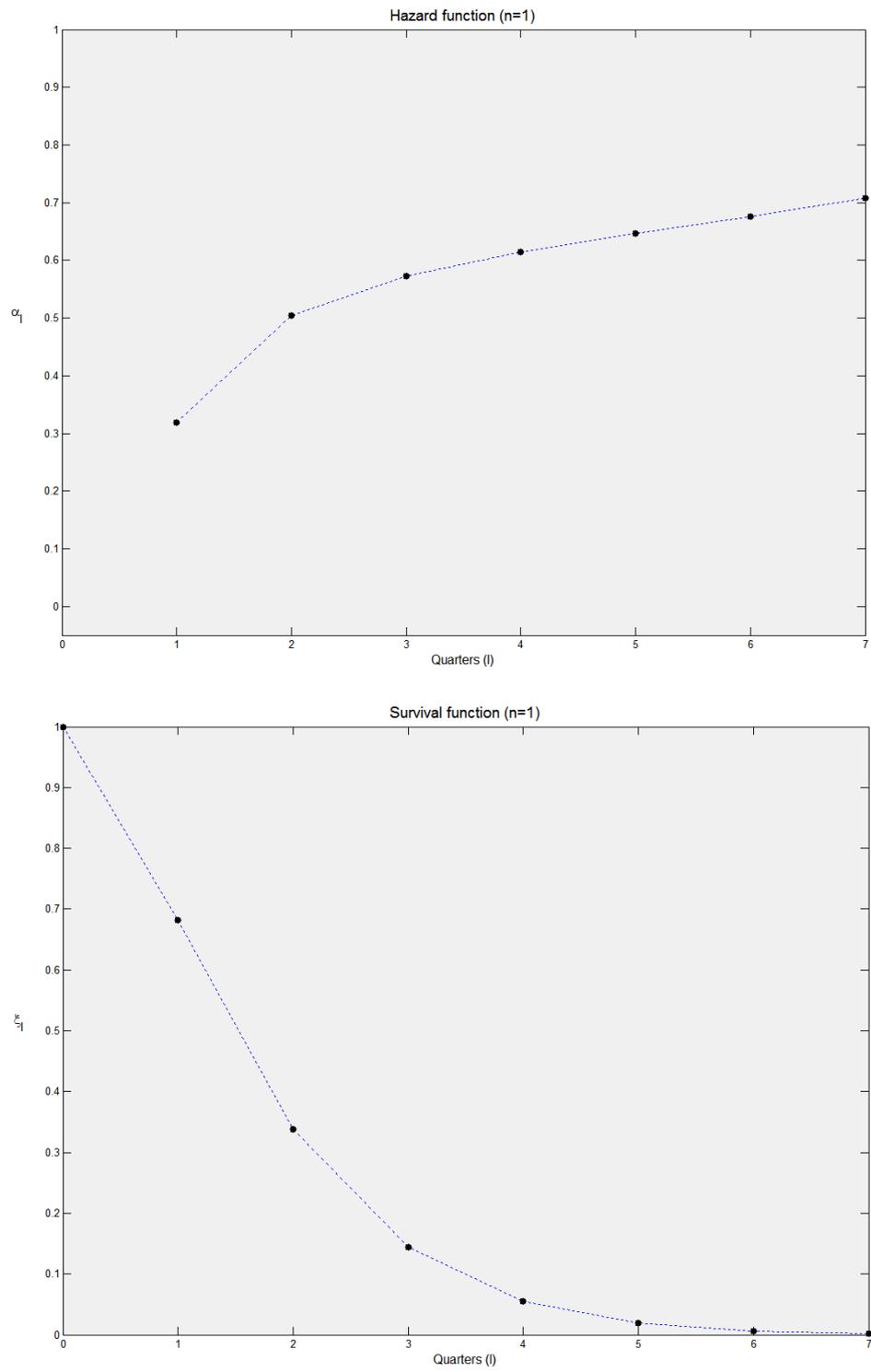
$$\pi_t^w = \begin{matrix} 0.197\pi_{t-1}^w & + & 0.991E_t\pi_{t+1}^w & - & 0.193E_t\pi_{t+2}^w & - & 0.03u_t \\ (0.038) & & (0.000) & & (0.037) & & (0.006) \end{matrix} \quad (48)$$

<sup>23</sup>See Papke and Wooldridge (2005).

<sup>24</sup>The estimation has been performed by using Cliff’s (2003) GMM package for MATLAB available from <https://sites.google.com/site/mcliffweb/programs>.

Also under this specification all the coefficients are statistically significant at 5% level (standard errors computed using delta method are reported in parentheses). Our wage Phillips curve, in line with the underlying theory, is able to capture the well-known negative relation between the unemployment gap and the wage inflation, as highlighted by the negative coefficient measuring the slope of the NKWPC. In Figure 2 we report a graphical representation for the hazard and survival functions deriving from our estimation and computed respectively by using (1) and (23). The hazard clearly shows a positive slope, meaning that a time-dependent mechanism for wage adjustment emerges.

Figure 2 - Hazard and survival function deriving from GMM estimation.



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