Estimation of a New Keynesian Model with Trend Inflation for Eastern European Countries

Monika Junicke

May 21, 2014

Abstract

I develop a simple two-country DSGE model with a non-zero steady state inflation to study the behaviour of the Eastern European central banks. The Bayesian analysis that I carry out suggests that this model performs better than the benchmark New Keynesian framework since the existence of trend inflation improves the fit to the data significantly. Furthermore, using a posterior odds test, I show that the hypothesis according to which central banks systematically target CPI inflation rather than PPI inflation is empirically rejected for all the investigated Eastern European countries. This result is robust accross several Taylor-type rules, and is in line with a number of theoretical contributions claiming that PPI inflation targeting performs better than CPI inflation targeting in terms of welfare loss. Studying the conduct of monetary policy for small EECs, my estimations suggest that the Czech National Bank includes the nominal exchange rate into its monetary policy, whereas the Hungarian and Polish central banks do not.

1 Introduction

The Bayesian estimation of the DSGE models has recently attracted the attention of an increasing number of economists investigating whether the predictions of the DSGE models match the statistical properties of the empirical data, which are the transmission channels for the exogenous shocks, and what is the behaviour of the central bank. In this paper, I study the performance of a simple small open economy (SOE) model, and investigate the conduct of monetary policy in Eastern European countries.

There are two innovative aspects that I consider in this paper. First, the most important novel feature of my analysis is assessing to what extent the assumption of a zero steady state inflation influences the model's empirical fit, which sheds new light on the importance of introducing trend inflation into these type of models. Previous contributions mainly point out the importance of the trend inflation on theoretical grounds (e.g., Ascari and Ropelle, 2007). Empirical estimations of models with trade inflation are rare in the literature. One of the few exceptions is by Cogley and Sbordone, 2008, who estimate in isolation the NKPC with a time varying trend inflation. Unlike these authors, I focus on a complex DSGE model that I estimate using a bayesian methodology

Second, in order to estimate the Phillips curve more accurately, I do not treat the marginal cost as a latent variable, as it is common in the Bayesian literature, but I use real unit labour cost data as a proxy for it. Using the resulting framework, I show that the backward looking component in the Phillips curve is important, and the model performs significantly better when accounting for non-zero steady state inflation.

Beside analysing how important the innovations on the supply side of the model are, my interest lies in estimation of a suitable monetary policy rule for both a large and a small economy. When considering SOE monetary policy, a number of theoretical contributions argue that PPI inflation targeting performs better than CPI inflation targeting in terms of welfare loss. Nonetheless, the empirical literature mainly concentrates on simple rules with CPI inflation targeting. Using a posterior odds test, I analyse whether the central banks of some selected Eastern European countries (EEC) systematically target CPI inflation rather than PPI inflation. My results suggest that this hypothesis is empirically rejected for all the investigated Eastern European countries.

Lastly, I analyse to what extent the central bank of a small Eastern European country, such as the Czech Republic, Hungary or Poland, responds to variations in the exchange rate. This question, originally posed by Lubik and Schorfheide

(2007) with regard to Australia, Canada, New Zealand and the UK, is particularly interesting in the context of the emerging EEC, since these countries are potential candidates to join the Eurozone. The decision on whether to join the Eurozone may well be determined at least in part by the gains and losses these countries may suffer from abandoning a flexible exchange rate regime. I identify Germany as the large economy, since this country represents the largest trading partner of all of the selected EEC.[1] The results are mixed. I show that the Czech central bank is likely to target exchange rates, whereas its Hungarian and Polish counterparts are not.

The paper includes two large sections. Section 3 describes the theoretical model, which builds on the New Keynesian literature with non zero inflation trend such as Ascari and Ropele (2007). I first generalise the model for a SOE, in which I introduce an incomplete pass-through and a home bias in the representation of consumer preferences. Altough it is still common in the New Keynesian literature to assume that the law of one price and the PPP hold, both assumptions strongly contradict the well-estiblished empirical evidence.

My results are discussed in Section 4, where I describe the Bayesian methodology adopted to estimate the structural parameters of the model, particularly those specifying the Phillips curve. This approach also allows me to analyse the implications of modifying the Phillips curve to account for a non-zero steady state inflation. For robustness, my findings are derived using different monetary policy rules. Among them, however, I show that a simple monetary policy rule capturing the essential features of the optimal one, as derived by, *e.g.*, Steinsson (2003), significantly improves the fit to the data.

2 Literature Review

The structure of the model closely relates to Galí and Monacelli (2005), Tuesta and Rabanal (2006) and De Paoli (2009). Furthermore, I add some assumptions to the model that are motivated by the empirical evidence. I assume incomplete pass-through following Monacelli (2003) and home bias in consumption, which leads to deviation from power purchasing parity. The intratemporal elasticity of substitution between domestic and foreign goods differs from unity, allowing the SOE central bank to manipulate the terms of trade, which now relates to the relative domestic price. The reason for introducing these variations to the benchmark

[1] Germany attracts between 25 to 30 percent of the total exports from each of the EEC. The fact that this trade partnership is not reciprocal allows me to conclude that Germany behaves as a large economy relative to the EEC.

model is twofold. On the one hand, Devereux and Engel (2003) show that optimal monetary policy, in case of less than perfect (incomplete) pass-through of the exchange rate to the local currency prices, should involve some consideration of exchange rate volatility. On the other hand, although it is typically assumed in the literature that the elasticity of substitution between domestic and foreign goods is one (as in, *e.g.*, Corsetti and Pesenti (2001), Devereux and Engel (2003), and Obstfeld and Rogoff (2002)), empirical estimations suggest larger elasticities. Using this result, Sutherland (2006) argues that the central bank should add targeting of the exchange rate to monetary policy.

The supply side is characterised by a hybrid New Keynesian Phillips Curve, which is derived using a rule of thumb following Galí and Gertler (1999). A similar Phillips Curve specification is also used by Benigno and Lopez-Salido (2002), who analyse the effect of asymmetric supply shocks across countries within a monetary union. Additionally, I follow Ascari and Ropele (2007) and log-linearise this Phillips curve around a non-zero steady state, and show that this assumption improves the fit of the model significantly. The monetary policy is spesified as by using different Taylor type rules for both, the closed and the open economy. The aim is twofold. First, different rules serve the robustness check for my results. However, I can also identify the best suitable monetary policy rule.

There is a large literature using Bayesian techniques to estimate monetary policy rules in DSGE models. The first important work in this field is Smets and Wouters (2003), who estimate structural parameters of a closed economy model using Euro Area data. This work has since been extended for the SOE model. Lubik and Schorfheide (2005) create a two symmetric country model and estimate it using U.S. and Euro Area data. Using a similar dataset, Tuesta and Rabanal (2006) estimate and compare models with complete and incomplete financial markets. In their later paper, Lubik and Schorfheide (2007) estimate how central banks in Australia, Canada, New Zealand and UK respond to exchange rate changes, estimating composite structural parameters. Similarly, Adolfson, Laséen, Lindé, and Villani (2008) and Liu (2006) investigate similar questions while assuming incomplete pass-through, using data for Sweden and New Zealand, respectively. Justiniano and Preston (2010) identify the optimal policy rule within a generalized class of Taylor-type rule, which they estimate using data from Australia, Canada and New Zealand. They show that these rules do not respond to the nominal exchange rates. Negro and Schorfheide (2009) also study the effect of changes in the monetary policy rule, using data for Chile.

The use of Bayesian techniques to estimate the NKPC have so far only yielded mixed results. Schorfheide (2008) reviews the identification and the estimates for the Phillips curve coefficients, obtained using U.S. data. He demonstrates how estimates of marginal costs treated as latent variables or measured in terms of an output gap vary widely with observable marginal costs, measured by unit labour costs. He concludes that the estimated values are more robust if marginal costs are explicitly included. Galí and Gertler (1999) estimate a NKPC with unit labour costs as a proxy for marginal costs as well as output gap, using the general method of moments (GMM). They show that using unit labor costs delivers better estimates than using the output gap. Similarly, Sbordone (2002) argues that estimating the NKPC with the output gap is successful as long as the output gap is a good measure of marginal costs and Cogley and Sbordone (2008) use this proxy to estimate the NKPC with time varying trend inflation.

The rest of the paper is organised as follows. In Section 3, I specify the model assuming two countries that may or may not differ in size. After describing the demand and supply side of the model in details, I specify the monetary policy rules as nominal interest rate rules for each country, and log-linearise the model around its non-zero inflation steady state. In Section 4, I describe the estimation methodology, the dataset, and the choice of prior. I also present the estimation results and the model fit following from the Bayesian estimation, and analyse the impulse responses. Section 5 concludes.

3 The Model

I first specify the model for two generic countries. The framework encapsulates both the scenario where there are two symmetric countries or where there are two countries that differ in size and openness. I then specify the model for a SOE, which interacts with a large economy. Section 3.1 describes in detail the household preferences, its optimisation problem as well as total and aggregate demand for both domestic and foreign country. Section 3.2 describes the supply side of the model. The whole model is log-linearised around its steady state in Section 3.3, and monetary policy rules in simple form are described in more detail in the last subsection.

3.1 Demand Side

I consider two countries; country H, also called home or domestic country, and F, the foreign country. A continuum of agents of unit mass populate the world economy, where the population in the segment $[0, n)$ belongs to country H and the population in the segment $(n, 1]$ belongs to country F. Consumption C is a

Dixit-Stiglitz aggregator of home and foreign goods. Home consumers preferences are represented by

$$C_t = \left[\gamma^{\frac{1}{\theta}} \left(C_{H,t} \right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} \left(C_{F,t} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \qquad (1)$$

with the intratemporal elasticity of substitution between domestic and foreign goods, θ, not necessary equal to one. Sutherland (2006) surveys the empirical literature and concludes that the majority of the empirical evidence suggests $\theta \in (5, 10)$.

The parameter γ introduces a home bias in consumption, and its value is given by

$$1 - \gamma = (1-n)\,\lambda \quad \rightarrow \quad \gamma = 1 - (1-n)\,\lambda, \qquad (2)$$

where $(1-n)$ is the relative size of country F and $\lambda \in [0,1)$ is the degree of openness of country H. If $\lambda = 0$, the domestic economy is autarkic and only domestic goods are consumed. Furthermore, as the size of the economy (n) increases, consumers buy relatively more domestic goods and imports become less relevant. Therefore, in a large economy, where $n \rightarrow 1$, people mainly consume domestically produced goods, whereas for a small open economy where $n \rightarrow 0$, international trade is more important. A small economy is also more strongly influenced by foreign innovations.

Similar preferences are specified for the foreign consumer:

$$C_t^* = \left[(\gamma^*)^{\frac{1}{\theta}} \left(C_{H,t}^* \right)^{\frac{\theta-1}{\theta}} + (1-\gamma^*)^{\frac{1}{\theta}} \left(C_{F,t}^* \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \qquad (3)$$

where the parameter γ^* is determined by the size and openness of the foreign economy, that is

$$\gamma^* = n\lambda. \qquad (4)$$

Note that the specifications of γ and γ^* imply that the power purchasing parity (PPP) does not hold in this model.

The sub-indices for domestic consumption of domestic (respectively, imported) goods $C_{H,t}$ (resp., $C_{F,t}$) are

$$C_{H,t} = \left(\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n [C_{H,t}(i)]^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

$$C_{F,t} = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 [C_{F,t}(i)]^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{6}$$

Analogously, for foreign consumption $C_{H,t}^{*}$ (resp., $C_{F,t}^{*}$) it holds

$$C_{H,t}^{*} = \left(\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n \left[C_{H,t}^{*}(i)\right]^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{7}$$

$$C_{F,t}^{*} = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 \left[C_{F,t}^{*}(i)\right]^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{8}$$

where $C_{H,t}$ is the home consumption of domestically produced goods and $C_{F,t}$ is the home consumption of imported goods. Analogously, $C_{H,t}^{*}$ is foreign consumption of domestic exports (goods produced in the home country) and $C_{F,t}^{*}$ is foreign consumption of goods produced abroad. Finally, $C_t(i)$ is the total consumption of a generic good (i).[2] The parameter ε is the elasticity of substitution between the differentiated goods produced in one country and holds unchanged across countries.

I assume that the consumption choices of all households from one country are identical. From the consumption maximisation problem of the representative domestic household, I obtain the domestic demand function for a domestic good $C_{H,t}(i)$ and foreign good $C_{F,t}(i)$ as follows

$$C_{H,t}(i) = \frac{1}{n} C_{H,t} \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \quad C_{F,t}(i) = \frac{1}{1-n} C_{F,t} \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon}. \tag{9}$$

Analogously, it holds that the foreign demand for a domestic good $C_{H,t}^{*}(i)$ and

[2] Generally, starred variables are expressed in foreign currency, unstarred in domestic currency. However, this rule does not apply to consumption, which is expressed in real terms: in this case, it is only used to distinguish between consumption at home and abroad.

for a foreign good $C^*_{F,t}(i)$ are respectively given by

$$C^*_{H,t}(i) = \frac{1}{n} C^*_{H,t} \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}} \right)^{-\varepsilon} \qquad C^*_{F,t}(i) = \frac{1}{1-n} C^*_{F,t} \left(\frac{P^*_{F,t}(i)}{P^*_{F,t}} \right)^{-\varepsilon} . \tag{10}$$

The aggregate domestic demand for domestic good and for foreign goods (imports) can be written in terms of aggregate world consumption

$$C_{H,t} = \gamma C_t \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} \qquad C_{F,t} = (1-\gamma) C_t \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} , \tag{11}$$

and the aggregate foreign demand function for domestic goods (in other words, exports from the point of view of the home country) and for goods produced abroad can be written as

$$C^*_{H,t} = \gamma^* C^*_t \left(\frac{P^*_{H,t}}{P^*_t} \right)^{-\theta} \qquad C^*_{F,t} = (1-\gamma^*) C^*_t \left(\frac{P^*_{F,t}}{P^*_t} \right)^{-\theta} . \tag{12}$$

By manipulation of the demand functions, the consumption-based price indices for domestic and foreign country can be expressed respectively as

$$P_t = \left[\gamma \left(P_{H,t} \right)^{1-\theta} + (1-\gamma) \left(P_{F,t} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} , \tag{13}$$

and

$$P^*_t = \left[\gamma^* \left(P^*_{H,t} \right)^{1-\theta} + (1-\gamma^*) \left(P^*_{F,t} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} . \tag{14}$$

The price sub-index $P_{z,t}$ ($P^*_{z,t}$) for goods produced in country $z \in \{H, F\}$ can be expressed in the domestic (foreign) currency as

$$P_{H,t} = \left(\frac{1}{n} \int_0^n \left[P_{H,t}(i) \right]^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \qquad P_{F,t} = \left(\frac{1}{1-n} \int_n^1 \left[P_{F,t}(i) \right]^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \tag{15}$$

$$P^*_{H,t} = \left(\frac{1}{n} \int_0^n \left[P^*_{H,t}(i) \right]^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \qquad P^*_{F,t} = \left(\frac{1}{1-n} \int_n^1 \left[P^*_{F,t}(i) \right]^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \tag{16}$$

with the producer price index of the domestically produced goods $P_{H,t}$ and the importer price index for the goods from foreign country $P_{F,t}$ both expressed in the domestic currency. Analogously, $P^*_{F,t}$ ($P^*_{H,t}$) is the producer price index in foreign country (price of the imported goods from the point of view of consumers abroad) in foreign currency.

3.1.1 The Law of one Price and the Real Exchange Rate

There is strong empirical evidence that the law of one price (LOP) does not hold, which could be because of different producer pricing or because importers face monopolistic competition similar to producers and therefore charge a mark-up over their price. Hence, it is very common in New Open Economy Macroeconomic Models (NOEM) to assume incomplete pass-through. In this paper I follow Monacelli (2003) and assume that the law of one price holds when the goods arrive "at the dock", but setting the price in domestic currency causes a deviation from the LOP. This is explained in more detail in Section 3.2.2, where it is shown that the domestic retailers set the price of the imported good in monopolistic competition. The LOP gap is defined as

$$\Psi_t = S_t \frac{P^*_{F,t}}{P_{F,t}}, \tag{17}$$

where the nominal exchange rate S_t denotes the price of the foreign currency in terms of the domestic currency.[3] Additionally, given the different degrees of home bias in consumption between the two countries, *i.e.* $\gamma \neq \gamma^*$, it follows from equation (13) that the PPP does not hold, and the CPI in each country differs, formally

$$P_t \neq S_t P^*_t.$$

Hence, the real exchange rate differs from one, and I can express it as the price of foreign goods in term of domestic goods

$$RS_t = \frac{S_t P^*_t}{P_t}. \tag{18}$$

Note that a decrease in the nominal exchange rate S_t and analogously, *ceteris paribus*, in the real exchange RS_t implies an appreciation of the domestic currency.

The terms of trade, which is given by a ratio between importer and domestic producer prices is also expressed in terms of relative prices

$$TOT_t = \frac{P_{F,t}}{P_{H,t}} = \frac{\tilde{P}_{F,t}}{\tilde{P}_{H,t}}, \tag{19}$$

[3] Note however that for the domestic price, from the point of view of domestic producer, the low of one price holds, because he gets the price "at the dock".

where $\tilde{P}_{F,t} = P_{F,t}/P_t$ and $\tilde{P}_{H,t} = P_{H,t}/P_t$. Combining the last equation with equation (13) shows that the relative domestic price can be easily expressed as a function of the terms of trade

$$\tilde{P}_{H,t} = \left[\gamma + (1-\gamma)(TOT_t)^{1-\theta}\right]^{-\frac{1}{1-\theta}}.$$

The relationship between domestic and CPI inflation is given by the relationship between domestic relative prices of the current and past period

$$\frac{\Pi_{H,t}}{\Pi_t} = \frac{\tilde{P}_{H,t}}{\tilde{P}_{H,t-1}}. \tag{20}$$

and the relationship between imported and CPI inflation can be expressed as the relationship between relative prices of the imports in domestic currency of the current and past period

$$\frac{\Pi_{F,t}}{\Pi_t} = \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t-1}}. \tag{21}$$

3.1.2 The Household Optimisation Problem

Both domestic and foreign economies consist of a continuum of identically infinite-lived agents. The preferences of the domestic representative agent is given by the instantaneous utility function of the same form as

$$U_t(C,N) = \frac{\varepsilon_t C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}, \tag{22}$$

where the function (U) is separable in consumption (C) and working hours (N), so that $U_{C,N} = 0$, and where the preference shock ε_t affects the rate of intertemporal substitution in consumption for domestic households, similar to the one in Tuesta and Rabanal (2006). The utility function is also time-separable and the parameters σ and η are both positive CRRA (constant relative risk aversion) parameters determining the elasticity of substitution.

The representative agent maximises its discounted stream of instantaneous utility functions over current and future periods

$$U = E_t \sum_{t=0}^{\infty} \beta^t \left[U_t(C,N)\right], \tag{23}$$

where $\beta \in (0,1]$ is the subjective discount factor, by choosing $\{C_t, N_t\}_{t=0}^{\infty}$. She also holds international bonds B_t denominated in the national currency which yields a gross return of R_t at the end of the period.

Her budget constraint, is given by

$$B_t + W_t N_t + T_t + D_t \geq P_t C_t + E_t \left[Q_{t,t+1} B_{t+1} \right]. \tag{24}$$

Note that

$$P_t C_t = \int_0^n P_H(i) C_H(i) \, di + \int_n^1 P_F(i) C_F(i) \, di.$$

where $C_H(i)$ $(C_F(i))$ is consumption of domestic (foreign) good i, given its price $P_H(i)$ $(P_F(i))$, and P_t is the overall consumer price index. Notice also that the agent consumes all goods at any time t. The nominal bonds denominated in domestic currency at the end of the period t are denoted by B_t.[4] W_t is the nominal wage, and T_t and D_t are the lump sum transfer and the profits of the companies held by the household, respectively. $E_t[Q_{t,t+1}]$ is the dynamic stochastic discount factor between period t and $t+1$, for which it holds

$$E_t[Q_{t,t+1}] = \frac{1}{R_t},$$

where R_t is the gross return on a riskless one year nominal bond, with a yield assumed to be small. Furthermore, for sufficiently small values of i_t, it holds that $\log(R_t) \approx i_t$, where i_t is the riskless short term nominal interest rate.

From the first order condition of the maximisation problem of the domestic representative household, I obtain the Euler equation for the domestic economy

$$E_t \left(\left[\frac{C_t}{C_{t+1}} \right]^{-\sigma} \Pi_{t+1} \frac{\varepsilon_t}{\varepsilon_{t+1}} \right) = \beta R_t \tag{25}$$

with domestic CPI inflation given by $\Pi_{t+1} = P_{t+1}/P_t$. Following Steinbach et al. (2009), the expression $\varepsilon_{t+1}/\varepsilon_t$ can be also interpreted as a risk premium on asset holding, *i.e.*, the wedge between the interest rate set by central bank and the actual return on assets. The domestic households labour supply is given by

$$\tilde{W}_t = \frac{N_t^\eta}{C_t^{-\sigma}}. \tag{26}$$

where $\tilde{W}_t$ is the real domestic wage. I assume that labour is immobile across coun-

[4] It holds that $B_t + B_t^* = 0$, so the world-wide stock of international bonds equal zero for all periods.

tries. Assuming that the foreign household faces the same maximisation problem, the Euler equation and the labour supply for a foreign economy are derived analogously.

3.1.3 The Asset Market Structure

In my model, I ignore the transaction costs and assume that financial markets are such that consumers from either country have access to both domestic and foreign bonds. The market price of a domestic riskless bond equals the expected nominal return of the bond, and is given by $1/R_t = E_t\left[Q_{t,t+1}\right]$. Similarly for a foreign bond expressed in domestic currency, it holds that $S_t/\left(R_t^*\right) = E_t\left[S_{t+1}Q_{t,t+1}\right]$. With no possibility of arbitrage, the expected returns of these two bonds must be equal, and the two equations can be combined. Therefore, the uncovered interested parity holds and is expressed as

$$E_t\left[Q_{t,t+1}\left\{R_t - \frac{R_t^* S_{t+1}}{S_t}\right\}\right] = 0,$$

where S_t is the nominal exchange rate, expressed as the price of foreign currency in terms of domestic currency. Using the last equation together with (18), the uncovered interest parity equation can be written as the expected change in the real exchange rate and the ratio between domestic and foreign real interest rate

$$\frac{R_t}{R_t^*}E_t\left[\frac{\Pi_{t+1}^*}{\Pi_{t+1}}\right] = E_t\left[\frac{RS_{t+1}}{RS_t}\right]. \tag{27}$$

Under the assumption of complete securities markets with no uncertainty, consumption risk is perfectly shared and the stochastic discount factor, expressed in the same currency, is equal across the countries. Using the Euler equation (25) and its equivalent for the foreign country, and recalling that the constant subjective discount factor β is shared by both countries, delivers

$$E_t\left[\frac{\varepsilon_t}{\varepsilon_{t+1}}\left(\frac{C_t}{C_{t+1}}\right)^{-\sigma}\frac{S_t}{S_{t+1}}\Pi_{t+1}\right] = E_t\left[\frac{\varepsilon_t^*}{\varepsilon_{t+1}^*}\left(\frac{C_t^*}{C_{t+1}^*}\right)^{-\sigma}\Pi_{t+1}^*\right]. \tag{28}$$

Using again equation (18), (28) can be rewritten as a function of the real exchange rate RS_t:

$$E_t\left[\frac{RS_{t+1}}{RS_t}\right] = \frac{E_t\left[\left(\frac{C_{t+1}^*}{C_{t+1}}\right)^{-\sigma}\frac{\varepsilon_{t+1}^*}{\varepsilon_{t+1}}\right]}{\left(\frac{C_t^*}{C_t}\right)^{-\sigma}\frac{\varepsilon_t^*}{\varepsilon_t}}$$

Given the fact that this equation holds in all periods t, including the steady state condition of zero net foreign assets and the ex-ante identical environment, I obtain the optimal risk sharing, under complete financial markets:

$$RS_t = \kappa_c \left(\frac{C_t^*}{C_t} \right)^{-\sigma} \frac{\varepsilon_t^*}{\varepsilon_t}.$$

The constant κ_c is determined by the initial market equilibrium for state-contingent bonds, which reflects the initial wealth differences. Without loss of generality, following Galí and Monacelli (2005), I can assume that the initial distribution of wealth is such that $\kappa_c = 1$ and the risk sharing equation can be written analogously to the one in Tuesta and Rabanal (2006)[5]

$$RS_t = \left(\frac{C_t^*}{C_t} \right)^{-\sigma} \frac{\varepsilon_t^*}{\varepsilon_t}. \tag{29}$$

This equation reflects the fact that if power purchasing parity (PPP) holds, *e.g.* $RS_t = 1$, the marginal utility of consumption, *i.e.* the consumption level is equal across the countries. However, deviations from PPP imply different consumption levels across the two countries caused by the changes in the real exchange rate. Hence, the ratio of marginal utilities across the two countries is equal to the ratio of aggregate prices denote here by the real exchange rate.

3.1.4 Aggregate Demand for Domestic and Foreign Goods

The total demand for a domestically produced good i consists of the weighted average of n domestic and $(1-n)$ foreign demand

$$Y_t(i) = nC_{H,t}(i) + (1-n)\, C_{H,t}^*(i).$$

Using (9) and (11) together with (10) and (12), it is possible to express the demand for good i in terms of price dispersion and the real exchange rate, where the prices are expressed in domestic currency

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left(\tilde{P}_{H,t} \right)^{-\theta} \left[\gamma C_t + \frac{1-n}{n} \gamma^* C_t^* \left(RS_t \right)^{\theta} \right]. \tag{30}$$

Thus, an appreciation of the currency leads to a decrease in output of domestic good i, $Y_t(i)$. Furthermore, the aggregate demand for domestic output can be

[5] This result holds in the case of symmetric perfect foresight steady state and symmetric initial relative net asset position. Further details in section 3.3.

written as a sum of the amounts produced domestically of good i

$$Y_t = \left(\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n \left[Y_t(i) \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Plugging this into equation (30) together with (15), the aggregate demand in the domestic country yields

$$Y_t = \left(\tilde{P}_{H,t} \right)^{-\theta} \left[\gamma C_t + \frac{1-n}{n} \gamma^* C_t^* \left(RS_t \right)^{\theta} \right], \quad (31)$$

hence the demand for a home-produced good is inversely related to an appreciation of the exchange rate. The reason is that foreign consumption decreases in terms of the home currency. Therefore, the degree to which appreciation influences the domestic production of good i depends on size of the foreign economy and its (domestic) openness as much as of the price dispersion given by the elasticities of substitutions. The higher the elasticity of substitution between domestic and foreign goods, the more sensitive the output of the domestic economy to the changes in the currency.

Combining (30) and (31), the total demand for good i, written in terms of domestic aggregate output is

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t, \quad (32)$$

which depends directly on the aggregate domestic output, the price of good i relative to the overall domestic price level, as well as the elasticity of substitution between domestic goods.

Analogously, the total demand for a foreign produced good i is

$$Y_t^*(i) = n C_{F,t}(i) + (1-n) C_{F,t}^*(i)$$

and can be rewritten as

$$Y_t^*(i) = \left(\frac{P_{F,t}^*(i)}{P_{F,t}^*} \right)^{-\varepsilon} \left(\tilde{P}_{F,t}^* \right)^{-\theta} \left[\frac{n}{1-n} (1-\gamma) C_t \left(RS_t \right)^{-\theta} + (1-\gamma^*) C_t^* \right], \quad (33)$$

where $\tilde{P}_{F,t}^* = P_{F,t}^* / P_t^*$ is the relative foreign producer price index expressed in foreign currency.

The aggregate demand for foreign output can be written as a sum of the foreign production of all goods

$$Y_t^* = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 \left[Y_t^* (i) \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

Using (33) together with (16), the aggregate demand for foreign output is given by

$$Y_t^* = \left(\tilde{P}_{F,t}^* \right)^{-\theta} \left[\frac{n}{1-n} (1-\gamma) C_t (RS_t)^{-\theta} + (1-\gamma^*) C_t^* \right]. \tag{34}$$

Thus, combining (33) and (34), I obtain total demand for the foreign good i in terms of foreign aggregate output

$$Y_t^* (i) = \left(\tilde{P}_{F,t}^* \right)^{-\varepsilon} Y_t^*. \tag{35}$$

3.1.5 Large Economy versus Small Open Economy

I can rewrite the key equations by assuming that the size of the foreign economy (domestic) market is sufficiently large that it is hardly influenced by the SOE. In this sense, analogous to Galí and Monacelli (2005), the large economy behaves as if it is autarkic and its associated economic variables are exogenous from the point of view of the SOE. Using the definition (2) and (4), and assuming that the domestic economy is small, *i.e.*, $n \to 0$, the aggregate consumption of domestic and foreign goods given by (1) and (3) becomes

$$C_t = \left[(1-\lambda)^{\frac{1}{\theta}} (C_{H,t})^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}} (C_{F,t})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} .$$

For the foreign large economy, defined as the rest of the world, the quantity of imports from the SOE are so mariginal that we can assume:

$$C_t^* = C_{F,t}^*.$$

Given (13), the relative domestic price index equation yields

$$P_t = \left[(1-\lambda) (P_{H,t})^{1-\theta} + \lambda (P_{F,t})^{1-\theta} \right]^{\frac{1}{1-\theta}} . \tag{36}$$

Note that for the foreign large economy there is no dispersion between producer and consumer price index, formally

$$P_t^* = P_{F,t}^*. \tag{37}$$

and it follows from equation (34) that the aggregate demand for goods produced in large foreign economy is given as $Y_t^* = C_t^*$.

Thus, the LOP gap (17) can be written in terms of real exchange rate and the terms of trade

$$\Psi_t = \frac{RS_t}{\tilde{P}_{F,t}}. \tag{38}$$

The total demand for a generic domestic good i given (30)

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left(\tilde{P}_{H,t}\right)^{-\theta} C_t \left[1 - \lambda + \lambda RS_t^{\theta - \frac{1}{\sigma}}\right] \tag{39}$$

depends on the openness of the domestic economy λ, the price dispersion between domestic producer and consumer price indexes and the real exchange rate RS_t. The real depreciation of the exchange rate leads to an increase in production of good i, the domestic good is cheaper and therefore the consumption of the good abroad increases. Analogous to the closed economy case, the higher the dispersion between the price of a particular good i and the domestic price index caused by the price stickiness, the lower the demand for good i. Additionally, for the SOE, there is a wedge between producer and consumer price indexes, which lowers domestic output.

The aggregate demand for domestic goods, assuming all the conditions associated with a SOE yields

$$Y_t = \left(\tilde{P}_{H,t}\right)^{-\theta} C_t \left[1 - \lambda + \lambda RS_t^{\theta - \frac{1}{\sigma}}\right]. \tag{40}$$

3.2 Firm Optimisation: The Phillips Curve

The supply side of the domestic economy consists of two parts. There are producers and import retailers, both setting prices in the manner described by Calvo (1983) and Galí and Gertler (1999). Each producer (resp., retailer) belongs to one of two types of firms. A measure $1 - \omega$ (resp., $1 - \omega^F$) set the price optimally, and are labelled f. A measure ω (resp., ω^F) set the price according to a rule-of-thumb, and are labelled b. Firms may face two different situations: i) either they are allowed to set their price with probability $1 - \alpha$ (resp., $1 - \alpha^F$); ii) or they are not allowed to do so with probability α (resp., α^F). Hence, at each time

t, a measure $(1-\omega)(1-\alpha)$ (resp., $[1-\omega^F][1-\alpha^F]$) sets the price optimally; a measure $\omega(1-\alpha)$ (resp., $\omega^F[1-\alpha^F]$) sets the price according to a rule-of-thumb; a measure α holds the price unchanged.

3.2.1 Price Setting Mechanism for Final Goods Producers

Let us first consider one of the $(1-\omega)(1-\alpha)$ firms in country H that, at time t, are allowed to set their price optimally. Each producer in this group sets price $P_t^f(i)$ to maximise its expected stream of profits $G_t(i)$

$$\begin{aligned} &\max_{P_t^f(i)} \sum_{j=0}^{\infty} \alpha^j E_t \left[Q_{t,t+j} \left(P_t^f(i)\, Y_{t+j}(i) - W_{t+j} N_{t+j}(i) \right) \right], \\ &\text{subject to:} \quad Y_{t+j}(i) = A_{t+j} N_{t+j}(i), \quad \text{and} \quad Y_{t+j}(i) = \left(\tfrac{P_t^f(i)}{P_{H,t+j}} \right)^{-\varepsilon} Y_{t+j}, \end{aligned} \tag{41}$$

where A_t is total factor productivity, and the constraints respectively represent the production technology, and the demand function (32). The first order condition for the SOE producers delivers the optimal choice of the forward looking price $P_t^f(i)$

$$\begin{aligned} \frac{P_t^f(i)}{P_t} \sum_{j=0}^{\infty} (\alpha\beta)^j E_t \left[(C_{t+j})^{-\sigma} Y_{t+j} \frac{P_t}{P_{t+j}} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\varepsilon} \right] \\ = \frac{\varepsilon}{\varepsilon-1} \sum_{j=0}^{\infty} (\alpha\beta)^j E_t \left[\left(\frac{Y_{t+j}}{A_{t+j}} \right)^{\eta+1} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\varepsilon} \right]. \end{aligned}$$

Denoting by $\tilde{P}_t^f(i) = P_t^f(i)/P_t$ the relative forward looking price of domestic firm i, the last equation can be rewritten in terms of difference equations

$$\tilde{P}_t^f(i) = \frac{J_t}{H_t}, \tag{42}$$

with

$$J_t = \mu V_t \left(\frac{Y_t}{A_t} \right)^{\eta+1} + \alpha\beta E_t \left[(\Pi_{H,t+1})^{\varepsilon} J_{t+1} \right] \tag{43}$$

and

$$H_t = C_t^{-\sigma} Y_t + \alpha\beta E_t \left[(\Pi_{H,t+1})^{\varepsilon} (\Pi_{t+1})^{-1} H_{t+1} \right], \tag{44}$$

where $\mu = \varepsilon/(\varepsilon-1)$ is the domestic mark-up. I also introduce the mark-up shock

V_t

$$\log \frac{V_t}{\bar{V}} = \rho_v \log \frac{V_{t-1}}{\bar{V}} + \varepsilon_{v,t},$$

where $\bar{V}$ is the steady state value of the mark-up innovation and $\varepsilon_{v,t}$ is an i.i.d. shock. Given equilibrium on the labour market, the first expression in (43) can be written in terms of real marginal costs $\widetilde{MC}_t$ and the relative domestic price $\tilde{P}_{H,t}$

$$\left(\frac{Y_t}{A_t}\right)^{\eta+1} = C_t^{-\sigma} Y_t \widetilde{MC}_t \tilde{P}_{H,t},$$

where $\widetilde{MC}_t = MC_t / P_{H,t}$. The forward looking price therefore depends on domestic and CPI inflation, and the relative domestic price.

The remaining $\omega(1-\alpha)$ domestic firms set prices at time t according to the rule of thumb, indexing it to the last observed price index. In terms of the rate of domestic producer inflation $\Pi_{H,t-1}$

$$P_t^b = \Pi_{H,t-1} X_{t-1}, \tag{45}$$

where X_{t-1} denotes an index of the prices set at date $t-1$, given by

$$X_t \equiv \left[(1-\omega) P_t^{f(1-\varepsilon)} + \omega P_t^{b(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}. \tag{46}$$

The aggregate producer price level then follows the law of motion

$$P_{H,t} = \left[(1-\alpha) X_t^{1-\varepsilon} + \alpha \left(P_{H,t-1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{47}$$

The set of equations (42)-(47) constitute the hybrid New Keynesian Phillips curve, which characterises the producer side of country H. The set of equations leading to the Phillips curve for country F is derived analogously. The hybrid NKPC for country H, because of the dispersion between PPI and CPI, can be written as a function of the consumer price index and the terms of trade, as shown by, Benigno and Benigno (2003).

3.2.2 Price Setting Mechanism for Importing Retailers

Following Monacelli (2003), I assume that for retailers, who import differentiated goods into the domestic economy, the law of one price holds "at the dock". Similar to the domestic producers, domestic importing retailers also face a downward sloping demand curve. Under monopolistic competition, they set their prices, in terms of domestic currency, accordingly. The deviation between the prices of the imported good in domestic and foreign currency therefore generates a LOP gap.

Consider the $\alpha^F \omega^F$ share of local retailers importing good j at a cost $S_t P^*_{F,t}(i)$, and setting the price of the imported good in a domestic currency to maximise their profits

$$\begin{aligned} &\max_{P_t^f(i)} \sum_{j=0}^{\infty} \alpha_F^j E_t \left[Q_{t,t+j} \left(P_t^{F,f}(i) - S_t P_t^{F*}(i) \right) C_{F,t+j}(i) \right], \\ &\text{subject to:} \quad C_{F,t}(i) = \tfrac{1}{1-n} \left(\tfrac{P_t^{F,f}(i)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t}. \end{aligned} \tag{48}$$

$P_t^{F,f}(i)$ is the price of the imported good in domestic currency set by a forward looking retailer, $P_t^{F*}(i)$ is the price of the same good in the currency of the producer and α^F is the probability that this price holds unchanged the next period. In generally, it is assumed that the parameter α^F can differ from those associated with producers, denoted by α. The problem is solved analogously to the one solved by the domestic producer. The first order condition delivers the optimal choice of the relative forward looking price, $\tilde{P}_t^{F,f}(i) = P_t^{F,f}(i)/P_t$

$$\tilde{P}_t^{F,f}(i) = E_t \left[\frac{\mu \sum_{j=0}^{\infty} (\alpha^F \beta)^j (C_{t+j})^{-\sigma} C_{F,t+j} \Psi_{t+j} V_{t+j}^F \left(\prod_{k=1}^{j} \Pi_{F,t+k} \right)^{\varepsilon} \tilde{P}_{F,t+j}}{\sum_{j=0}^{\infty} (\alpha^F \beta)^j (C_{t+j})^{-\sigma} C_{F,t+j} \left(\prod_{k=1}^{j} \Pi_{F.t+k} \right)^{\varepsilon} \left(\prod_{k=1}^{j} \Pi_{t+k} \right)^{-1}} \right],$$

where I also use equation (38). In terms of difference equation, the first order condition delivers

$$\tilde{P}_t^{F,f}(i) = \frac{J_t^F}{H_t^F}, \tag{49}$$

with

$$J_t^F = \mu V_t^F C_t^{-\sigma} C_{F,t} \Psi_t \tilde{P}_{F,t} + (\alpha^F \beta) E_t \left[(\Pi_{F,t+1})^{\varepsilon} J_{t+1}^F \right], \tag{50}$$

where V_t^F is the importers mark up shock, with analogous characteristics as the producer's; and

$$H_t^F = C_t^{-\sigma} C_{F,t} + (\alpha^F \beta) E_t \left[(\Pi_{F,t+1})^{\varepsilon} (\Pi_{t+1})^{-1} H_{t+1}^F \right]. \tag{51}$$

The remaining $\omega^F (1 - \alpha^F)$ importers set their prices at time t according to the rule of thumb by indexing them to the last observed rate of import inflation

$\Pi_{F,t-1}$

$$P_t^{F,b} = \Pi_{F,t-1} X_{F,t-1} \tag{52}$$

where $X_{F,t-1}$ denotes an index of the prices of imported goods set at date $t-1$, given by

$$X_{F,t} \equiv \left[(1-\omega) P_t^{F,f(1-\varepsilon)} + \omega \left(P_t^{F,b} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{53}$$

Assuming that all firms face the same shock, I can write the aggregate importer price level

$$P_{F,t} = \left[(1-\alpha) X_{F,t}^{1-\varepsilon} + \alpha \left(P_{F,t-1} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{54}$$

Equations (49) to (54) characterise the import price inflation hybrid NKPC.

3.3 Steady State and Log-linearised Form of the Model

Before the actual estimation, the equations characterising the non-policy part of the model should be log-linearised around the steady state, assuming that $n \to 0$. Monetary policy is described in more details in the next section. In this section, I assume a perfect-foresight steady state for both economies with zero income growth and stable technology. Furthermore, I normalise the steady state nominal exchange rate to unity, formally $S = 1$. One additional assumption about the steady state: prices of imports increase at the same rate as prices of domestically produced goods. Therefore, inflation is the same across both countries, so that the real exchange rate in steady state is stable. This restriction is reasonable because any equilibrium with an explosive exchange rate would not be sustainable.

Since in the steady state all prices change at the same rate, and the price of the imports increases at the same rate as the price of the domestically produced goods, I can normalise the price indices by imposing $\bar{P}_H = \bar{P}_F$.[6] Therefore, from equation (13), it follows that the consumer and producer price index are equal, formally, $\bar{P} = \bar{P}_H$. Inflation, as well as the relative prices, do not change and it holds that

$$\bar{\Pi}_H = \bar{\Pi}_F = \bar{\Pi} = \bar{\Pi}^*.$$

Furthermore, denoting growth factors by G, from the definition of real exchange rate it follows that

$$G_{RS} = 1.$$

[6] This assumption follows De Paoli (2009), the price indices in steady state are normalised such as $\bar{P}_H = \bar{P}_F$ and $\bar{P}_H^* = \bar{P}_F^*$, so that the producer prices are in the steady state the same for both countries.

which in conjunction with (27) leads to

$$R = R^*.$$

Together with (17), I can then write

$$G_{RS} = G_S = G_\Psi = 1.$$

Note that, since in steady state production per capita is equal across countries, and recalling that the nominal exchange rate equals one, then it must be that price indices are also equal across countries. Hence, considering also the definition of the real exchange rate, it follows that

$$RS = \Psi = S = 1$$

Using (29) in steady state, consumption is equalised across countries, and given by $\bar{C} = \bar{C}^*$. Market clearing implies $\bar{Y} = \bar{C}$ and $\bar{Y}^* = \bar{C}^*$. Given the previous results, it holds that $\bar{Y} = \bar{Y}^*$, so the domestic and foreign country have the same per capita income. Therefore, as long as the production technology is the same for both countries, $\bar{N} = \bar{N}^*$.

The structural equations characterising the non-policy part of the model can be written in the (log-)linearised form around their steady state. Linearising equation (36) defines the relationship between producer and importer relative price

$$1 = (1 - \lambda)\, \tilde{p}_{H,t} + \lambda \tilde{p}_{F,t}. \tag{55}$$

The relationships between relative producer price and inflation and relative importer price and inflation are given respectively by

$$\tilde{p}_{H,t} - \tilde{p}_{H,t-1} = \hat{\pi}_{H,t} - \hat{\pi}_t \tag{56}$$

and

$$\tilde{p}_{F,t} - \tilde{p}_{F,t-1} = \hat{\pi}_{F,t} - \hat{\pi}_t. \tag{57}$$

The LOP gap (38) and the real exchange rate (18), written in first difference, are given respectively by

$$\hat{\Psi}_t = \widehat{rs}_t - \tilde{p}_{F,t} \tag{58}$$

and

$$\Delta \widehat{rs}_t = \Delta \hat{s}_t + \hat{\pi}^*_t - \hat{\pi}_t + \varepsilon_{rs,t}, \tag{59}$$

where I add $\varepsilon_{rs,t}$, an unobservable shock, to capture possible measurement error in the data and to relax the potentially tight cross-equation restrictions in the model.

The domestic Euler equation (25) can be rewritten in terms of deviations from the steady state as

$$\hat{c}_t = E_t\left[\hat{c}_{t+1}\right] - \frac{1}{\sigma}\left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right] + E_t\left[\Delta\epsilon_{t+1}\right]\right), \tag{60}$$

where I have used again the approximation $log\left(R_t\right) \approx \hat{\imath}_t$. The term $\Delta\epsilon_{t+1} = \log\varepsilon_{t+1} - \log\varepsilon_t$ is the first differences of the structural preference shock. The linearisation of the uncovered interest parity delivers (27)

$$\left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right) - \left(\hat{\imath}^*_t - E_t\left[\hat{\pi}^*_{t+1}\right]\right) = E_t\left[\widehat{rs}_{t+1}\right] - \widehat{rs}_t. \tag{61}$$

The UIP equation describes the relationship between real interest rate and real exchange rate.

The optimal risk sharing from equation (29) becomes

$$\widehat{rs}_t = \sigma\left(\hat{c}_t - \hat{c}^*_t\right) + \epsilon^*_t - \epsilon_t, \tag{62}$$

where the difference between the world and the domestic preference shock $\left(\epsilon^*_t - \epsilon_t\right)$ captures the deviations from optimal risk sharing. The risk sharing equation describes the link between real exchange rate and consumption. Assuming complete markets, both equations hold, making Euler Equation for the domestic country redundant. The risk sharing equation ensures that the marginal utility is the same in both countries. Assuming eveyone in the world shares the same prefences, the level of consumption is the same across the countries. Because the UIP holds, the domestic real interest rate moves along with the interest rate abroad. The relationship between the domestic variables is therefore complete, and there is no need of further specification such as that given by the Euler Equation, which is typically used to obtain these relationships.

The good market clearing condition, represented by (40), yields

$$\hat{y}_t = -\theta\tilde{p}_{H,t} + \hat{c}_t + \lambda\left(\theta - \frac{1}{\sigma}\right)\widehat{rs}_t. \tag{63}$$

The log-linearisation of the supply side is given in more details in Appendix A and leads to a hybrid NKPC with a non-zero steady state inflation

$$\hat{\pi}_{H,t} = \chi^f E_t\left[\hat{\pi}_{H,t+1}\right] + \chi^b\hat{\pi}_{H,t-1} + \kappa_{mc}\left(\widehat{mc}_t + v_t\right) + \chi^\pi\left(\hat{h}_t - \left(\hat{y}_t - \sigma\hat{c}_t\right)\right) \tag{64}$$

where the real marginal costs $\widehat{mc}_t = \widehat{mc}_t^{nom} - \hat{p}_{H,t}$ are expressed by

$$\widehat{mc}_t = \eta \hat{y}_t + \sigma \hat{c}_t - (\eta + 1) a_t - \tilde{p}_{H,t} \tag{65}$$

and

$$\hat{h}_t = \left(1 - \alpha\beta\bar{\Pi}^{\varepsilon-1}\right)(\hat{y}_t - \sigma\hat{c}_t) + (\alpha\beta)\bar{\Pi}^{\varepsilon-1}E_t\left[\varepsilon\hat{\pi}_{H,t+1} - \hat{\pi}_{t+1} + \hat{h}_{t+1}\right]. \tag{66}$$

Analogously, the NKPC for imported prices can be log-linearised to obtain

$$\hat{\pi}_{F,t} = \chi_F^f E_t\left[\hat{\pi}_{F,t+1}\right] + \chi_F^b \hat{\pi}_{F,t-1} + \kappa_F\left(\widehat{\Psi}_t + v_t^F\right) + \chi_F^\pi\left(\hat{h}_t^F - \left(\hat{c}_t^F - \sigma\hat{c}_t\right)\right) \tag{67}$$

with

$$\hat{h}_t^F = \left(1 - \alpha^F\beta\bar{\Pi}^{\varepsilon-1}\right)\left(\hat{c}_t^F - \sigma\hat{c}_t\right) + \left(\alpha^F\beta\right)\bar{\Pi}^{\varepsilon-1}E_t\left[\varepsilon\hat{\pi}_{F,t+1} - \hat{\pi}_{t+1} + \hat{h}_{t+1}^F\right] \tag{68}$$

and, from (11),

$$\hat{c}_{F,t} = \hat{c}_t - \theta\tilde{p}_{F,t}. \tag{69}$$

For country F, the producer's NKPC is log-linearised analogously to the producer's NKPC of the SOE. The large economy works as in autarky, (imports and exports of this country can be seen as negligible,) so that this NKPC is identical to the one for closed economy. The market clearing condition is

$$\hat{y}_t^* = \hat{c}_t^*, \tag{70}$$

the Euler Equation is

$$\hat{c}_t^* = E_t\left[\hat{c}_{t+1}^*\right] - \frac{1}{\sigma}\left(\hat{\imath}_t^* - E_t\left[\hat{\pi}_{t+1}^*\right] + E_t\left[\Delta\varepsilon_{t+1}^*\right]\right), \tag{71}$$

the Phillips curve with a backward looking and non-zero inflation component is

$$\hat{\pi}_t^* = \chi_f^* E_t\left[\hat{\pi}_{t+1}^*\right] + \chi_b^*\hat{\pi}_{t-1}^* + \kappa_{mc}^*\left(\widehat{mc}_t^* + v_t^*\right) + \chi_\pi^*\left[\hat{h}_t^* + (\sigma - 1)\hat{y}_t^*\right], \tag{72}$$

where

$$\hat{h}_t^* = \left(1 - \alpha\beta\bar{\Pi}^{\varepsilon-1}\right)(\hat{y}_t^* - \sigma\hat{c}_t^*) + (\alpha\beta)\bar{\Pi}^{\varepsilon-1}E_t\left[(\varepsilon - 1)\hat{\pi}_{t+1}^* + \hat{h}_{t+1}\right] \tag{73}$$

and the marginal costs are

$$\widehat{mc}_t^* = (\eta + \sigma)\hat{y}_t^* - (1 + \eta)a_t^*. \tag{74}$$

To estimate this model all that is needed now is a monetary policy rule. In this paper, I use simple interest rate rules of a Taylor type with producer inflation targeting and consumer inflation targeting, and strict exchange rate targeting. The monetary policy rules are described in more details below.

3.4 Monetary Policy Rules

To close the model, I need to specify the policy chosen by the monetary authority. For estimation purposes, most of the recent papers, *e.g.* Smets and Wouters (2003), use a generalised Taylor rule, where the central bank systematically responds to the changes in inflation, output and, in the case of a SOE, to the exchange rate. Analysing the effect of simple rules has some advantages relative to the optimal monetary policy, as they are more likely to be used in practice because they are more easily implemented. Additionally, their parameters are more robust to the model specification than the structural parameters of the optimal rule.

This paper compares a number of different simple targeting rules of theTaylor type for both economies. For the relatively large closed economy, three monetary policy rules are analysed. The first one is a common Taylor rule with an interest rate smoothing component, $\rho_i^*\hat{\imath}_{t-1}^*$, which is typically used in the literature to improve the fit of the empirical estimation as it incorporates observed interest rate persistence. The rule has the following form

$$\hat{\imath}_t^* = \rho_i^*\hat{\imath}_{t-1}^* + \phi_\pi^*\hat{\pi}_t^* + \phi_y^*\hat{y}_t^* + \varepsilon_{u,t}^*, \tag{75}$$

where $\varepsilon_{u,t}^*$ is an exogenous monetary policy shock. Alternatively, following Smets and Wouters (2003), the central bank also responds to the speed of inflation $\Delta\pi_t^*$.

$$\hat{\imath}_t^* = \rho_i^*\hat{\imath}_{t-1}^* + \phi_\pi^*\hat{\pi}_t^* + \phi_y^*\hat{y}_t^* + \phi_{\Delta\pi}^*\Delta\hat{\pi}_t^* + \varepsilon_{u,t}^*, \tag{76}$$

The third analysed rule takes the form of the optimal monetary policy rule identified using a welfare loss function from, *e.g.*, Steinson (2003), where I approximate the optimal behaviour of the central bank

$$\hat{\imath}_t^* = \rho_i^*\hat{\imath}_{t-1}^* + \phi_\pi^*\hat{\pi}_t^* + \phi_y^*\hat{y}_t^* + \phi_{\Delta 1}^*\Delta\hat{\pi}_t^* + \phi_{\Delta 2}^*\Delta\hat{\pi}_{t+1}^* + \phi_{\Delta y}^*\Delta\hat{y}_t^* + \varepsilon_{u,t}^*. \tag{77}$$

The aim of using three different rules is to find out whether the European central bank targets acting as the large economy in this model, conducts monetary policy using a simple Taylor rule (75) or incorporates any of the additional terms in (76) and (77). When modeling the economies of the Czech Republic, Hungary or Poland I choose the rule that best fits for each case.

For these small economies I again specify the three main monetary policy rules, but modified for a SOE . The first one is similar to (76), where in addition to the traditional Taylor Rule, the central bank targets the change in inflation and in the exchange rate

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \phi_\Delta \Delta\hat{\pi}_t + \phi_S \Delta\hat{s}_t + \varepsilon_{u,t}. \quad (78)$$

The second rule is analogous to the rule of optimal type (77)

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \phi_{\Delta 1} \Delta\hat{\pi}_t + \phi_{\Delta 2} \Delta\hat{\pi}_{t+1} + \phi_{\Delta y} \Delta\hat{y}_t + \phi_S \Delta\hat{s}_t + \varepsilon_{u,t}. \quad (79)$$

Alternatively, I also assume that the central bank targets exchange rate strictly, following Adolfson, Laséen, Lindé, and Villani (2008)

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_S \Delta\hat{s}_t + \varepsilon_{u,t}. \quad (80)$$

I am interested in answering two main questions regarding the monetary policy rule for a SOE. First, there are many studies that modify the simple instrumental rule to match the needs of a small open economy. Although the theoretical work emphasises that a targeting PPI inflation performs better in terms of welfare loss, the empirical literature usually assumes a simple rule with consumer inflation targeting. In fact, by moving the interest rate, the central bank can either target producer domestic inflation or CPI inflation. However, Galí and Monacelli (2005) as well as Sutherland (2002) point out that if the economy's non-stochastic steady state is at its optimum and no (or only very small) cost push distortions are present, the optimal monetary policy is pure domestic inflation targeting (e.g., $\hat{\pi}_{H,t} = 0$). Strict producer-price targeting has a smoother effect on domestic variables without any distortion to the foreign economy. However, Sutherland also argues that when cost push shocks have larger variance, CPI targeting may obtain better results.

To investigate whether the central bank targets domestic producer inflation instead of CPI inflation, I compare (78) and (79) with the corresponding rules in terms of PPI inflation, simply obtained by replacing $\hat{\pi}_t$ with $\hat{\pi}_{H,t}$, and reported here for conveninence

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_\pi \hat{\pi}_{H,t} + \phi_y \hat{y}_t + \phi_\Delta \Delta\hat{\pi}_{H,t} + \phi_S \Delta\hat{s}_t + \varepsilon_{u,t} \quad (81)$$

and

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_\pi \hat{\pi}_{H,t} + \phi_y \hat{y}_t + \phi_\Delta \Delta\hat{\pi}_{H,t} + \phi_{\Delta 2} \Delta\hat{\pi}_{H,t+1} + \phi_{\Delta y} \Delta\hat{y}_t + \phi_S \Delta\hat{s}_t + \varepsilon_{u,t}. \quad (82)$$

As I show later, in both cases, the difference in the model fit is significant.

Second, following Lubik and Shorfheide (2007), I study to what extent the central banks of the EEC countries respond not only to the changes in inflation and output, but also to the changes in inflation and exchange rate, *e.g.*, whether the parameter ϕ_S plays an important rule. I compare the simple rules (81) and (82) with their equivalents by assuming that $\phi_S = 0$.

3.4.1 Summary of the model and exogenous disturbances

To summarise, the model consists of a non-policy part determined by equations (55) to (74), a monetary policy rule specified above and a set of exogenous shocks, which follow an autoregressive process given in a log-linearised form.

The country-specific TFP for domestic and foreign country are defined respectively by

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_{a,t}, \\ a_t^* &= \rho_{a^*} a_{t-1}^* + \varepsilon_{a,t}^*; \end{aligned}$$

the preference innovations are given for domestic and foreign consumers respectively by

$$\begin{aligned} \epsilon_t &= \rho_e \epsilon_{t-1} + \varepsilon_{e,t}, \\ \epsilon_t^* &= \rho_{e^*} \epsilon_{t-1}^* + \varepsilon_{e,t}^*. \end{aligned}$$

Finally, the cost push for domestic producers and for domestic retailers are expressed by

$$\begin{aligned} v_t &= \rho_v v_{t-1} + \varepsilon_{v,t}, \\ v_t^F &= \rho_{v^F} v_{t-1}^F + \varepsilon_{v^F,t}, \end{aligned}$$

whereas for foreign producers they are

$$v_t^* = \rho_{v^*} v_{t-1}^* + \varepsilon_{v,t}^*.$$

The stochastic AR(1) processes are driven by exogenous shocks, of which seven are white noise, $\varepsilon_{a,t}$, $\varepsilon_{a,t}^*$, $\varepsilon_{e,t}$, $\varepsilon_{e,t}^*$, $\varepsilon_{v,t}$, $\varepsilon_{v^F,t}$, $\varepsilon_{v,t}^*$, plus two exogenous monetary policy shocks, $\varepsilon_{u,t}$ and $\varepsilon_{u,t}^*$, and one measurement error, $\varepsilon_{rs,t}$.

4 Model Estimation and Estimation Results

This section illustrates the estimation of the model, and is divided into three parts. First, I discuss the Bayesian methodology and estimation technique I use in detail. Then, after a brief look at the data, I describe my choice of priors in the context of the existing literature on this field. Finally, I present the estimation results, including the posterior distribution, impulse responses and variance decomposition.

The estimated model consists of a set of equilibrium equations. All equations are log-linearised, and the variables are expressed in terms of the deviation from their respective steady state levels, both for the small and the large economy, as described in a previous sections. The small open economy case is estimated on data from the EEC countries, such as the Czech Republic, Hungary and Poland. The large economy is represented by Germany.

4.1 Methodology

For the empirical analysis of my DSGE model, I adopt a Bayesian estimation approach, which has many advantages. First, the Bayesian approach allows us to incorporate priors based on theoretical considerations or other research. Second, the Bayesian approach is a full information method in contrast to a single equation method such as GMM and therefore it is more likely to produce better estimates.[7] Furthermore, using the estimated log data density of the model, facilitates comparisons of the goodness of fit of different models. Following most of the literature, I use a random walk Metropolis-Hasting algorithm to approximate the posterior distribution of the estimated parameters that I briefly describe below.[8]

Suppose that the aim is to draw a sample from a target density $\pi(\Phi)$. Note that Φ is a $(K \times 1)$ vector of parameters of interest. The target density is a posterior distribution, which is too complex to allow a direct sample. Therefore an indirect method is needed. The steps describing a random Walk Metropolis Hastings Algorithm are the following:

1. Set a prior distribution for each parameter $p(\Phi)$

2. Find the mode of the posterior distribution $\pi(\Phi)$ via numerical maximisation. Denote the estimates of the parameters at the mode by $\Phi^{\max}$, and their covariance matrix, which is the inverse Hessian matrix, by $H^{\max}$

3. To approximate $\pi(\Phi)$, the following algorithm is used:

[7] See Linde (2005).

[8] For more details, see Blake and Mumtaz (2012) and An and Schorfheide (2007).

(a) Specify a candidate density $q\left(\Phi^{G+1}/\Phi^{G}\right)$, where G is an index of draws

(b) Set the initial estimates of the parameters Φ^{G} with $G=0$

(c) Generate a candidate value Φ^{G+1} from the candidate density. I use a random walk version of this algorithm with the candidate density specified as a random walk, such as

$$\Phi^{G+1}=\Phi^{G}+e,$$

where e is a K-vector random walk with a normal distribution

$$e\sim N\left(0,\Sigma\right).$$

(d) Compute the acceptance probability. The candidate Φ^{G+1} is accepted with probability α, given by

$$\alpha=\min\left(\frac{\pi\left(\Phi^{G+1}\right)/q\left(\Phi^{G+1}/\Phi^{G}\right)}{\pi\left(\Phi^{G}\right)/q\left(\Phi^{G}/\Phi^{G+1}\right)};1\right),$$

where the numerator is the target density evaluated at the new draw of the parameters $\pi\left(\Phi^{G+1}\right)$ relative to the candidate density evaluated at the new draw parameters $q\left(\Phi^{G+1}/\Phi^{G}\right)$, and the denominator is the same expression evaluated at the previous draw of the parameters. Again, using a random walk version together with the fact that the normal distribution is symmetric, the acceptance probablity simplifies to

$$\alpha=\min\left(\frac{\pi\left(\Phi^{G+1}\right)}{\pi\left(\Phi^{G}\right)};1\right).$$

Step 3 is repeated M times. The first $(M-J)$ iterations are discarded. The last J draws are instead retained to estimate the posterior marginal distribution. For the results, I use four chains of $M=200,000$ draws, each starting from a different value. From each chain, the last $J=0.55\times M$ draws are used to approximate the empirical distribution of the parameters.

Using the Metropolis-Hastings algorithm, the acceptance rate depends on the variance Σ, which is set manually. It holds that the higher the variance, the more volatile the drawings. Therefore, a lower acceptance is to be expected in this case. On the other hand, if Σ is set too low, the volatility of the drawings is low as well. Therefore, the estimation of the parameters is likely to be close to the prior.

Drawing a random number u from a uniform distribution $u \sim U(0,1)$, it holds that the candidate Φ^{G+1} is accepted if $\alpha > u$, otherwise it is rejected.

The acceptance rate, given as the ratio between the accepted draws and the total number of draws, should lie between 20% and 40%. Some researchers are more specific and suggest that, for multivariate estimations, the acceptance rate should optimally be set to approximately 23%. The convergence of the chains is checked according to a Brooks and Gelman (1998) convergence diagnostic. The visual comparison between chains variance of the main results for selected estimations are shown in Figures 1 and 2 and described in more details in Section 4.3.

I use posterior odds test to compare the performance across models. Assume the null hypothesis that a model M1 is preferred to a model M2. The marginal data density is given for M1 by $\pi_{0,T}$, and for M2 by $\pi_{1,T}$. The posterior odds test is computed as the ratio of the marginal data density of M1 to M2. Following Lubik and Schorfheide (2007), the posterior odds can be interpreted as follows:

- $\frac{\pi_{0,T}}{\pi_{1,T}} > 1$, the null hypothesis is supported;
- $1 > \frac{\pi_{0,T}}{\pi_{1,T}} > 10^{-1/2}$, there is evidence against the null hypothesis, but it is not worth more than a bare mention;
- $10^{-1/2} > \frac{\pi_{0,T}}{\pi_{1,T}} > 10^{-1}$, there is substantial evidence against the null hypothesis;
- $10^{-1} > \frac{\pi_{0,T}}{\pi_{1,T}} > 10^{-3/2}$, there is strong evidence against the null hypothesis;
- $10^{-3/2} > \frac{\pi_{0,T}}{\pi_{1,T}} > 10^{-2}$, there is very strong evidence against the null hypothesis;
- $10^{-2} > \frac{\pi_{0,T}}{\pi_{1,T}}$, there is decisive evidence against the null hypothesis.

4.2 Data and Choice of Prior

For the empirical analysis, I use time series data on inflation, output growth, interest rates, exchange rates and unit labour costs. The sources of the raw data are Datastream and the Fred database. The empirical estimation is based on a data sample over the period 1996 to 2012 for Germany and the Czech Republic, and 1998 to 2012 for Hungary and Poland. All observations are quarterly, seasonally adjusted using the defaults settings of the X12 filter in Eviews 6.

1. Inflation is defined as the log difference of the consumer price index (for Germany) or the producer price index (for EEC) multiplied by 100

2. Output growth is constructed as the log difference of real output, defined as a nominal output divided by a deflator, again multiplied by 100

3. The interest rate is an annualised quarter to quarter call money interest rate monthly average (for Germany), and an annualised quarter to quarter discount rate (for the EEC), divided by four so as to be expressed in quarterly terms.

4. The quarterly change in the exchange rate is computed as a log difference of the bilateral nominal exchange rate between Euro and EEC's currency.

5. Unit labour cost is defined as the percentage change of the ratio between total labour costs and real GDP.

Whereas the first four variables are standard in the literature, I add unit labour costs as a proxy for real marginal costs. I follow Sbordone (2002) and Galí and Gertler (1999), who estimate the NKPC using unit labour costs and show that it is a more appropriate measure for the NKPC than the output gap. However, most of the empirical papers take the marginal costs as a latent variable and, as Schorfheide (2008) describes, the estimation results on the NKPC parameters may vary significantly. Note that the number of time series is lower than number of shocks to prevent problem of stochastic singularity.

Parameter	Distribution	Mean	Standard error
$\sigma\left(\varepsilon_a^*\right)$	Inverse Gamma	1	10
$\sigma\left(\varepsilon_e^*\right)$	Inverse Gamma	1	10
$\sigma\left(\varepsilon_v^*\right)$	Inverse Gamma	1	10
$\sigma\left(\varepsilon_u^*\right)$	Inverse Gamma	1	10
ρ_a^*	Beta	0.8	0.1
ρ_e^*	Beta	0.8	0.1
ρ_v^*	Beta	0.8	0.1
ρ_i^*	Beta	0.5	0.2
ϕ_π^*	Gamma	1.5	0.1
ϕ_y^*	Gamma	0.125	0.05
$\phi_{\Delta 1}^*$	Gamma	0.3	0.1
$\phi_{\Delta 2}^*$	Gamma	0.3	0.1
$\phi_{\Delta y}^*$	Gamma	0.0625	0.05
χ_f^*	Beta	0.5	0.2
χ_b^*	Beta	0.5	0.2
κ_{mc}^*	Gamma	0.1	0.05
χ_π^*	Normal	0	0.05
Π	Gamma	1.005	0.003

Table 1: Prior Distribution for Large Economy

4.2.1 Choice of Prior

I choose Bayesian estimation over maximum likelihood estimation because it permits me to incorporate a prior distribution. Incorporating priors means introducing additional general information about subjective beliefs of the parameter distribution, or information coming from previous econometric studies. In the case that just a small sample of data is available, a prior distribution is additional information that enables more stability in the optimisation algorithm. However, selecting an appropriate prior is one of the most difficult tasks associated with the use of the Bayesian approach.

I use German data to estimate the parameters for the large economy. The selection of the prior distribution follows closely Smets and Wouters (2003), and are represented in Table 1. For parameters that are restricted to the interval $(0, 1)$, I use a Beta distribution. Non-negative parameters are then Gamma distributed. For the autoregressive parameters of the shocks, I use a Beta distribution with a mean of 0.8 and a standard deviation of 0.1. The variances of the shocks are inverse gamma, with prior distribution $\sigma^2 \sim \Gamma^{-1}(1, 10)$. The standard errors are set such that the domain covers a reasonable range of parameter values.

The priors for the interest rate rule coefficients have rather wide confidence intervals. They are distributed around a mean given by the Taylor rule, following Lubik and Schorfheide (2005). Additionally, the prior distribution for the parameter ϕ_π has a lower bound of one, to satisfy the Taylor principle. Priors for the rest of the parameters in the monetary policy rule are Gamma distributed, with mean and standard error as those chosen by Smets and Wouters (2003) and Lubik and Schorfheide (2005).

Following Lubik and Schorfheide (2007), I estimate the composite structural coefficients of the NKPC rather than the underlying primitives, to avoid identification issues. The values of the NKPC parameters χ^b, χ^f and κ_{mc} reported in the literature are controversial. Therefore, the priors chosen here are consistent with the middle case, with a standard deviation large enough to ensure that the estimate is mainly determined by the data. Consistent with Lubik and Schorfheide (2007), the parameters χ^b, χ^f are beta distributed, and the parameter κ_{mc} is gamma distributed. The minimum level of the prior is consistent with the findings by Galí and Gertler (1999). The parameter χ^π is normally distributed around a zero mean, since it might take both positive and negative values. The prior of the inflation trend Π is gamma distributed around the average of the trend value, given by the HP filter, and it is lower-bounded at one. For Germany, the average inflation of the estimated sample corresponds to $\Pi = 1.005$.

The parameters for the SOE have similar priors as those for the closed economy.

Parameter	Distribution	Mean	Standard error
$\sigma(\varepsilon_a)$	Inverse Gamma	1	10
$\sigma(\varepsilon_e)$	Inverse Gamma	1	10
$\sigma(\varepsilon_u)$	Inverse Gamma	1	10
$\sigma(\varepsilon_v)$	Inverse Gamma	1	10
$\sigma(\varepsilon_{v^F})$	Inverse Gamma	1	10
$\sigma(\varepsilon_{rs})$	Inverse Gamma	1	10
ρ_a	Beta	0.8	0.1
ρ_e	Beta	0.8	0.1
ρ_v	Beta	0.8	0.1
ρ_{v^F}	Beta	0.8	0.1
ρ_i	Beta	0.5	0.2
ϕ_π	Gamma	1.5	0.1
ϕ_y	Gamma	0.125	0.05
$\phi_{\Delta 1}$	Gamma	0.3	0.1
$\phi_{\Delta 2}$	Gamma	0.3	0.1
$\phi_{\Delta y}$	Gamma	0.0625	0.05
ϕ_S	Gamma	0.3	0.1
χ^f	Beta	0.5	0.2
χ^b	Beta	0.5	0.2
κ_{mc}	Gamma	0.1	0.05
χ^π	Normal	0	0.05
χ_F^f	Beta	0.5	0.2
χ_F^b	Beta	0.5	0.2
κ_F	Gamma	0.1	0.05
χ_F^π	Normal	0	0.05
Π	Gamma	1.005	0.003

Table 2: Prior Distribution for Small Open Economy

Monetary policy rule	Log Data Density		Posterior odds
	A1	A2	
Rule 1 (75)	−178.15	−173.8	0.013
Rule 2 (76)	−170.67	−161.55	0.000
Rule 3 (77)	−166.39	−156.05	0.000

Table 3: Posterior Odd Test

Note: the table reports posterior odds test for German data on the hypothesis H0: $\chi_\pi^* = 0$ against the alternative $\chi_\pi^* \neq 0$.

The priors for the importer NKPC parameter are set analogously to the producer NKPC. The prior for ϕ_S is gamma distributed, with mean equal to 0.3. The steady state inflation Π is the trend inflation given the HP filter for the observed period. It is the same for the Czech Republic and Germany, and for Hungary and Poland, it corresponds to $\Pi = 1.0153$ and $\Pi = 1.0154$, respectively. The degree of openness λ is set to 0.6 for the Czech Republic, corresponding to the average Import/GDP ratio over the data sample. For Hungary and Poland, it is set to be 0.7 and 0.36, respectively.

Most of the parameters are not imposed to be the same for all countries, but it is merely assumed that they have identical priors. This also mirrors the fact that the countries have a similar economic history and have undergone similar structural changes since the end of the Cold War. Some parameters are identical for all countries. For example, the parameter β, which is fixed and not estimated. Instead I follow the convention and set it at 0.99.

4.3 Estimation Results

The composite structural parameters are estimated in two steps. The first step contains the estimation of the model for the closed economy, obtained using German data. In this part, I focus on three main issues. First, I generally estimate the NKPC for Germany, and show the importance of the backward looking component. Second, I am interested in whether the estimate for χ_π^* is significant. In other words, if the assumption of non-zero inflation in steady state improves the fit to the data. The third issue, which is important for further estimation and analysis of different simple rules, is to find the one rule that fits best the German data.

In the second part, the model for the SOE is estimated, using the data from

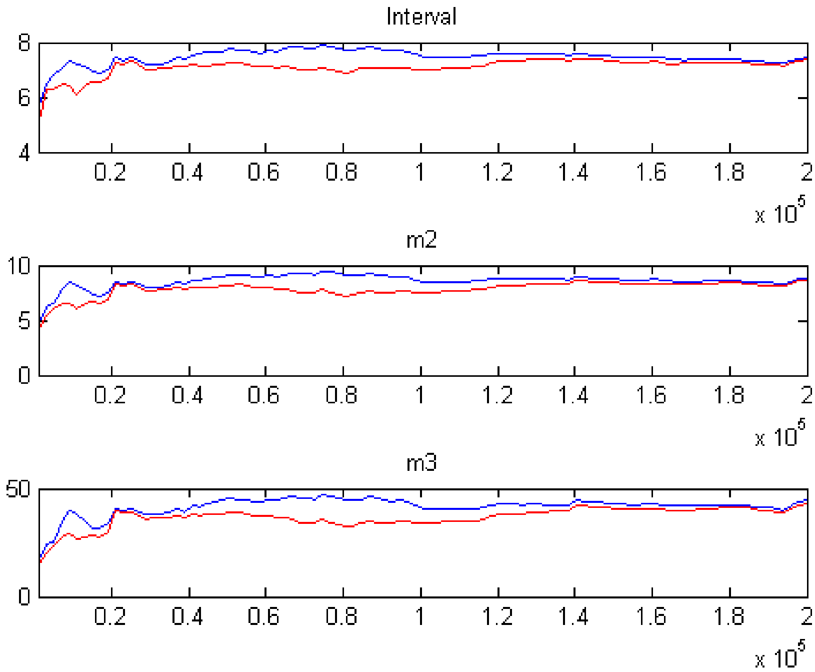


Figure 1: Multivariate Convergence Diagnostic for Germany

EEC. I use the best fitting monetary policy rule for the closed economy, and estimate domestic and foreign parameters using EEC's and German data together. Along with the estimates for the SOE Phillips curve, where I analyse the importance of the non-zero inflation part of the Phillips curve given by parameters χ_π and $\chi_{\pi^F t}$, I wish to identify what monetary policy fits the data best. Therefore, I first investigate whether the EEC central bank responds to a CPI inflation, I then show that the data suggests that PPI inflation targeting performs better. I then concentrate on understanding how important are the exchange rate movements in the simple rules for the central bank, and whether the EEC central banks systematically respond to such changes.

4.3.1 Results for Germany

In this section, I use three different simple rules for the closed economy, specified in (75), (76) and (77). I estimate each of them applying two different approaches, to assess the importance of the estimation of the non-zero steady state inflation part in the NKPC. The first approach (A1) assumes that the steady state inflation is zero, as is common in the literature, which leads to a backward looking NKPC

Parameter	Mode	S.D.	10%	Mean	90%
$\sigma\left(\varepsilon_a^*\right)$	1.0194	0.0790	0.9022	1.0321	1.1597
$\sigma\left(\varepsilon_e^*\right)$	5.5972	0.6983	3.0235	6.8599	11.2815
$\sigma\left(\varepsilon_v^*\right)$	0.3324	0.0515	0.28711	0.3526	0.4181
$\sigma\left(\varepsilon_u^*\right)$	0.5434	0.0417	0.3976	0.5278	0.6536
ρ_a^*	0.9944	0.0052	0.9866	0.9924	0.9985
ρ_e^*	0.9802	0.0064	0.9699	0.9802	0.9937
ρ_v^*	0.8417	0.0128	0.7074	0.8168	0.9313
ρ_i^*	0.9588	0.0256	0.8988	0.9418	0.9892
ϕ_π^*	1.4353	0.0316	1.3081	1.4642	1.6021
ϕ_y^*	0.0199	0.0055	0.0100	0.0232	0.0361
$\phi_{\Delta 1}^*$	0.5348	0.0195	0.2440	0.4504	0.6584
$\phi_{\Delta 2}^*$	0.3584	0.0328	0.1939	0.3845	0.5728
$\phi_{\Delta y}^*$	0.0716	0.0074	0.0084	0.0995	0.1724
χ_f^*	0.9452	0.0352	0.8041	0.8970	0.9889
χ_b^*	0.3026	0.0566	0.1288	0.2717	0.4158
κ_{mc}^*	0.4861	0.0110	0.4860	0.6213	0.7924
χ_π^*	0.2709	0.0244	0.1569	0.2248	0.2975
Π	1.0033	0.0005	1.0006	1.0045	1.0083

Table 4: Parameter Estimation Results for Germany

with $\chi_\pi^* = 0$. The second approach (A2) estimates the parameter χ_π^* as well as the steady state inflation Π. The log marginal data densities and the odds for these two specifications are portrayed in Table 3. The chains converge to the target distribution for all estimations. Figure 1 reports the convergence diagnostic for the estimation using the second approach and rule (77). There are three measures in each figure. Interval refers to an 80 % confidence interval around mean, m2 refers to the variance measure and m3 is based on the third moment of the aggregate measure. The convergence of the chains to the target distribution occurs if the between-chain measure (blue line) and the within-chain measure (red line) are

Log Data Density			Czech Rep.	Hungary	Poland
A2	CPI targeting, $\phi_S > 0$	Rule 1 (78)	-659.26	-630.38	-740.57
		Rule 2 (79)	-659.94	-633.51	-723.96
	PPI targeting, $\phi_S > 0$	Rule 1 (81)	-637.50	-603.98	-714.12
		Rule 2 (82)	-640.11	-595.45	-705.25
	PPI targeting, $\phi_S = 0$	Rule 1	-641.27	-602.56	-711.64
		Rule 2	-648.80	-594.13	-705.05
	Pure exchange rate	Rule 3 (80)	-706.85	-630.78	-842.91
A1	PPI targeting, $\phi_S > 0$	Rule 2 (82)	-646.27	-604.06	-721.94

Table 5: Marginal Data Densities under Different Approaches and Monetary Policy Rules Regimes

relatively constant and converge.

Two results emerge from the analysis of the log marginal likelihood and posterior odds. First, the estimation of the model with the second approach improves the fit to the data relatively to imposing a steady state rate of inflation that is zero. The posterior odds show that the hypothesis H0 of the steady state zero inflation can be rejected. Thus, this approach is used also for the SOE estimation in step 2. Second, the monetary policy rule (77) is clearly the best fit for the data. It follows that the more complex the rule is, the better the performance of the model. The traditional Taylor rule from (75) performs worse, whereas the "optimal" simple rule fits the data best. This evidence suggests that the central bank takes into account all the elements following from the welfare maximisation of the loss function, as derived in presence of backward looking firms. Given the log density, it is shown that including inflation change targeting improves the fit significantly.

The Bayesian estimated posterior distribution, based on the second approach and the monetary rule (77), is reported in Table 4. The table displays the mode and standard error resulting from the posterior maximisation. It also details the estimation results obtained through the Metropolis-Hastings algorithm, such as the posterior mean and the 90% posterior probability interval for both the estimated

	Rule 1			Rule 2		
	H0	H1	Post. Odds	H0	H1	Post. Odds
Czech Rep	−659.26	−637.50	0.000	−659.94	−640.11	0.000
Hungary	−630.38	−603.98	0.000	−633.51	−595.45	0.000
Poland	−740.57	−714.12	0.000	−723.96	−705.25	0.000

Table 6: Posterior Odd Test

Notes: hypothesis H0 that the central bank uses a CPI inflation targeting (78) and (79) vs hypothesis H1 that the central bank uses (81) and (82).

parameters and the standard deviation of shocks.

For all values, the highest posterior density intervals suggest that the estimated parameters are not equal to zero. Focusing on the two parameters that show how important is the non-zero steady state inflation, Table 4 shows that my estimation proposes a value around 0.2 for parameter χ_π^*, which is higher than that assumed in the prior distribution; and a value around 1.005 for the estimated trend inflation Π, implying a steady state rate of inflation of 2% percent per year. The values arerobust and lie in the confidence interval using both approaches. The estimates for the parameter χ_π^* are lower when assuming the simple Taylor rule (75) – around 0.13 for both approaches. For the remaining two other rules, the values are surprisingly stable, and lie between 0.22 and 0.26. The result for the steady state inflation Π is very similar for all three monetary policy rules in the second approach.

My estimate suggests a value of lagged inflation χ_b^* of around 0.3, in line with other empirical findings such as Galí and Gertler (1999) and Galí, Gertler, and Lopez-Salido (2001). With the exception of the cost push shock, all the autoregressive parameters for the shocks are estimated to be higher than the value of 0.8 assumed in the prior distribution. Surprisingly, the TFP shock is also very persistent, with the AR parameter around 0.99, a much higher value than the 0.83 estimated by Smets and Wouters (2003). Moreover, the monetary policy rules parameters are very robust and they all lie, independent of the estimation approach and rule, in the confidence interval given in Table 4. These parameters are all consistent with the values found in the literature.

	H0	H1	Posterior Odds
Czech Rep	-646.27	-640.11	0.002
Hungary	-604.06	-595.45	0.000
Poland	-721.94	-705.25	0.000

Table 7: Posterior Odd Test

Note: The table reports posterior odds test for EEC on the hypothesis H0: $\chi_\pi = 0$ and $\chi_{\pi^F} = 0$ against the alternative $\chi_\pi \neq 0$ and $\chi_{\pi^F} \neq 0$.

4.3.2 Results for European Emerging Markets

In this section, I analyse the monetary policy rules for the SOE from Section 3.4. I use different assumptions to understand the behavior of the central banks in the EEC. The summary of the marginal data densities from the different estimations can be found in Table 5. The results of the estimations are explained below.

I report the results obtained using the second approach outlined in the previous subsection, *i.e.*, assuming that the steady state inflation differs from zero, which provides significantly better results than those delivered by the first approach.[9] It is straightforward to demonstrate that a pure exchange rate targeting policy can be rejected as the policy being implemented by at least two of the three countries, since this rule performs the worst for both Czech and Polish data. This is in line with Adolfson, Laséen, Lindé, and Villani (2008), who obtain a similar result using Swedish data.

Finally I test whether the central bank targets CPI or PPI inflation. The results of the posterior odds test, with a null hypothesis that the central bank is focusing on CPI inflation rather the PPI inflation, are displayed in Table 6. The null hypothesis can be rejected for both rules and all countries. I can thus conclude that there is a clear evidence in favor of PPI inflation targeting over CPI inflation targeting. This is in line with the theoretical literature, which shows that responding to the PPI inflation rather than the CPI delivers lower welfare losses.

I then perform the posterior odds test to show how important it is to include the non-zero component into the Phillips curve. Similar to what I did for Germany,

[9] For comparison purposes, the best fit obtained using the first method is also reported.

	H0	H1	Posterior Odds
Czech Rep	-641.27	-637.50	0.023
Hungary	-602.56	-603.98	4.161
Poland	-711.64	-714.12	11.876

Table 8: Posterior Odd Test

Note: The table reports posterior odds test for EEC on the hypothesis H0: $\phi_S = 0$ against the alternative $\phi_S \neq 0$.

I estimate the model for both rules (81) and (82). On the one hand, I assume that $\chi^{\pi} = 0$. On other hand, my estimations are obtained when assuming that $\chi^{\pi} \neq 0$. The marginal data densities displayed in Table 7 suggest that including an estimation of χ^{π} improves the fit to the data. The posterior odds ratio is zero in all cases, rejecting the null hypothesis that χ^{π} equals zero for all three countries.

Furthermore, I am interested in whether the central bank responds to changes in the exchange rate. To answer this question, I follow Lubik and Schorfeide (2007) and first estimate both rules (81) and (82) assuming that $\phi_S > 0$. Second, I estimate the same rules, but assume that the central bank is not interested in exchange rate targeting, and set $\phi_S = 0$. The null hypothesis is that the central bank does not respond to the exchange rate changes. The results for both the TR2 and TR3 rule are given in Table 8. The null hypothesis can be rejected only for the Czech Republic. This suggests that the Czech National Bank targets the exchange rate, but the Central Banks of Hungary and Poland do not.

The estimated parameters are similar for all three countries. They can be found in Tables 9 - 11. The backward looking component for producer inflation lies between 0.2 and 0.35 for all countries. Compared to Germany, the non-zero steady state inflation component is lower, but still positive and significantly different from zero. For the retailers' Phillips curve, the parameter χ^{π} is positive, whereas χ^{π}_F is slightly negative for Czech Republic and Hungary, and all are significantly different from zero. As the convergence diagnostics suggest, all the chains converge to the target distribution. For the estimation using the second approach, with reference to the Czech Republic, the convergence diagnostic is illustrated in Figure 2.[10] The prior distribution, the posterior distribution and the posterior mode of these two

[10] The remaining diagnostic illustrations are available from the author upon request.

Parameter	Mode	S.D.	10%	Mean	90%
$\sigma\left(\varepsilon_a\right)$	0.7227	0.1556	0.5380	0.7714	0.9883
$\sigma\left(\varepsilon_e\right)$	3.0276	0.4826	2.3178	3.1085	3.9074
$\sigma\left(\varepsilon_u\right)$	1.5490	0.1977	1.2970	1.6581	2.0258
$\sigma\left(\varepsilon_v\right)$	1.7389	0.2642	1.3114	1.5946	1.8639
$\sigma\left(\varepsilon_{v^F}\right)$	10.7510	3.0950	0.2225	10.6040	20.3485
$\sigma\left(\varepsilon_{rs}\right)$	4.9429	0.4970	4.2216	5.0141	5.7741
ρ_a	0.9247	0.0110	0.7732	0.8873	0.9888
ρ_e	0.8766	0.0176	0.8282	0.8763	0.9307
ρ_v	0.7025	0.0237	0.6174	0.7254	0.8318
ρ_{v^F}	0.8752	0.0225	0.7582	0.8508	0.9686
ρ_i	0.9256	0.0416	0.8079	0.8948	0.9843
ϕ_π	1.3994	0.0314	1.3291	1.4661	1.6052
ϕ_y	0.0539	0.0176	0.0229	0.0640	0.1049
$\phi_{\Delta 1}$	0.3508	0.0376	0.2291	0.3760	0.5248
$\phi_{\Delta 2}$	0.3387	0.0239	0.1735	0.3276	0.4908
$\phi_{\Delta y}$	0.1024	0.0118	0.0013	0.0476	0.0929
ϕ_S	0.1392	0.0210	0.0871	0.1440	0.2050
χ^f	0.9077	0.0364	0.6890	0.8258	0.9671
χ^b	0.3063	0.0318	0.1148	0.2788	0.4334
κ_{mc}	0.2922	0.0123	0.3357	0.4149	0.5098
χ^π	0.1036	0.0191	0.0130	0.0770	0.1472
χ_F^f	0.6355	0.1010	0.2573	0.5403	0.7991
χ_F^b	0.1928	0.0260	0.0628	0.2377	0.3873
κ_F	0.0586	0.0104	0.0202	0.0691	0.1214
χ_F^π	0.0076	0.0080	-0.0989	-0.0126	0.0709
Π	1.0041	0.0004	1.0007	1.0053	1.0097

Table 9: Parameter Estimation Results for the Czech Republic

Parameter	Mode	S.D.	10%	Mean	90%
$\sigma(\varepsilon_a)$	0.5770	0.1148	0.5589	0.7989	1.0704
$\sigma(\varepsilon_e)$	6.5825	0.5749	5.2662	6.6646	8.0768
$\sigma(\varepsilon_u)$	1.9111	0.2379	1.6213	2.0654	2.4917
$\sigma(\varepsilon_v)$	1.7238	0.2345	1.3308	1.6808	2.0113
$\sigma(\varepsilon_{v^F})$	10.5052	1.1955	0.2216	1.9184	5.2044
$\sigma(\varepsilon_{rs})$	7.0219	0.4958	5.8129	6.9522	8.0079
ρ_a	0.7869	0.0202	0.8477	0.9042	0.9626
ρ_e	0.9143	0.0089	0.8872	0.9088	0.9314
ρ_v	0.6560	0.0097	0.5739	0.6655	0.7501
ρ_{v^F}	0.8515	0.0160	0.7020	0.8312	0.9723
ρ_i	0.8868	0.0174	0.7804	0.8709	0.9634
ϕ_π	1.5100	0.0351	1.3622	1.5075	1.6407
ϕ_y	0.0404	0.0060	0.0185	0.0522	0.0854
$\phi_{\Delta 1}$	0.2777	0.0253	0.1725	0.2758	0.3749
$\phi_{\Delta 2}$	0.4318	0.0206	0.1201	0.3077	0.4223
$\phi_{\Delta y}$	0.0510	0.0095	0.0004	0.0372	0.0748
ϕ_S	0.1451	0.0107	0.0772	0.1521	0.2302
χ^f	0.8227	0.0357	0.6184	0.7971	0.9564
χ^b	0.3723	0.0468	0.1998	0.3399	0.4819
κ_{mc}	0.4320	0.0090	0.4160	0.4889	0.5653
χ^π	0.0741	0.0056	-0.0210	0.0565	0.1283
χ_F^f	0.4061	0.0358	0.0880	0.4329	0.7770
χ_F^b	0.2545	0.0164	0.0394	0.2188	0.3606
κ_F	0.0843	0.0076	0.0105	0.0493	0.0873
χ_F^π	-0.0715	0.0041	-0.1092	-0.0415	0.0235
Π	1.0097	0.0004	1.0017	1.0062	1.0102

Table 10: Parameter Estimation Results for Hungary

Parameter	Mode	S.D.	10%	Mean	90%
$\sigma(\varepsilon_a)$	1.1451	0.1256	0.8682	1.0746	1.2679
$\sigma(\varepsilon_e)$	7.0226	0.7007	5.1819	6.5509	7.8704
$\sigma(\varepsilon_u)$	1.5254	0.2327	1.3060	1.6315	1.9452
$\sigma(\varepsilon_v)$	1.3775	0.1799	1.0422	1.2815	1.5081
$\sigma(\varepsilon_{v^F})$	0.4572	0.4848	0.2284	0.8527	1.5687
$\sigma(\varepsilon_{rs})$	6.9759	0.5465	5.9946	7.0510	8.0882
ρ_a	0.9627	0.0101	0.9144	0.9471	0.9807
ρ_e	0.9128	0.0103	0.8989	0.9181	0.9394
ρ_v	0.7359	0.0133	0.4996	0.6548	0.7981
ρ_{v^F}	0.8895	0.0180	0.7818	0.8811	0.9770
ρ_i	0.8601	0.0267	0.5576	0.7165	0.8671
ϕ_π	1.5237	0.0138	1.3934	1.5019	1.6300
ϕ_y	0.0599	0.0082	0.0452	0.0866	0.1357
$\phi_{\Delta 1}$	0.3017	0.0165	0.1375	0.2259	0.3056
$\phi_{\Delta 2}$	0.3714	0.0260	0.1770	0.3229	0.4748
$\phi_{\Delta y}$	0.0939	0.0106	0.0055	0.1105	0.2116
ϕ_S	0.1120	0.0343	0.0702	0.1127	0.1551
χ^f	0.9221	0.0200	0.6402	0.7843	0.9490
χ^b	0.3732	0.0375	0.1794	0.3276	0.4820
κ_{mc}	0.4439	0.0137	0.4857	0.5697	0.6630
χ^π	0.0665	0.0109	0.0174	0.0800	0.1756
χ^f_F	0.3279	0.0369	0.3319	0.5075	0.7006
χ^b_F	0.3022	0.0486	0.4359	0.5423	0.6778
κ_F	0.0373	0.0053	0.0010	0.0106	0.0188
χ^π_F	-0.0509	0.0107	-0.0115	0.0415	0.1096
Π	1.0025	0.0006	1.0006	1.0035	1.0062

Table 11: Parameter Estimation Results for Poland

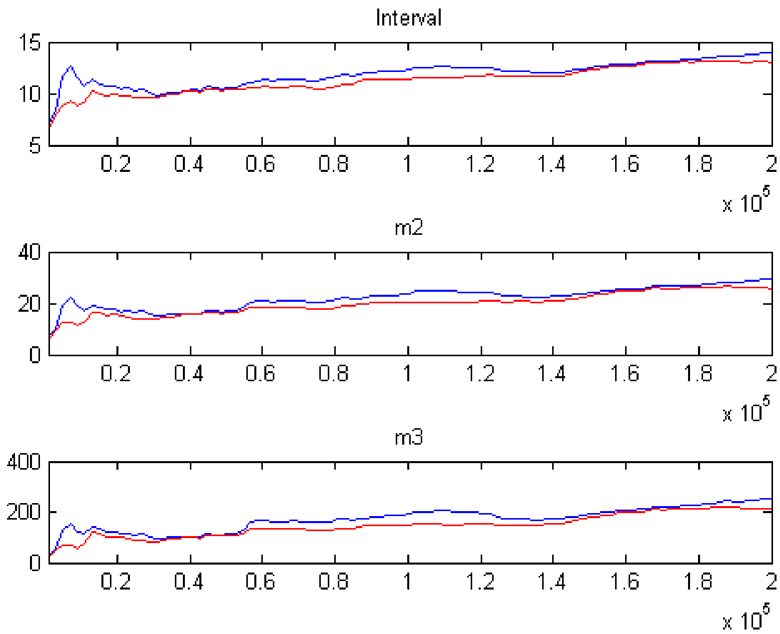


Figure 2: Multivariate Convergence Diagnostic for the Czech Republic

parameters is visually illustrated in Appendix B.

Finally, the monetary policy rule parameters are close to those reported in the literature. The central bank of all three countries respond much more actively to inflation (both to current and past changes) than to output (and its change). The estimates for exchange rate targeting in the monetary policy rule are higher, for all the three countries, than the prior values. ($\phi_S^{cz} = 0.14$, $\phi_S^{Hun} = 0.15$, $\phi_S^{Pol} = 0.11$).

Impulse Response Functions Analysis

In this section, I explain how the endogenous variables such as inflation, output, interest rate and real exchange rate respond to each structural shock over next 10 periods (*i.e.*, 2.5 years). The responses are illustrated in Figures 3-11 and, because of the similarities in the dynamic behavior of the three EEC, I only report the results of the estimates relative to the Czech Republic. In what follows, I compare the monetary policy rules in (78)-(79) and (81)-(82) to identify potential differences between CPI and PPI inflation targeting. The solid line is the median response, and the area within the dashed lines represents the 90% HPD interval.

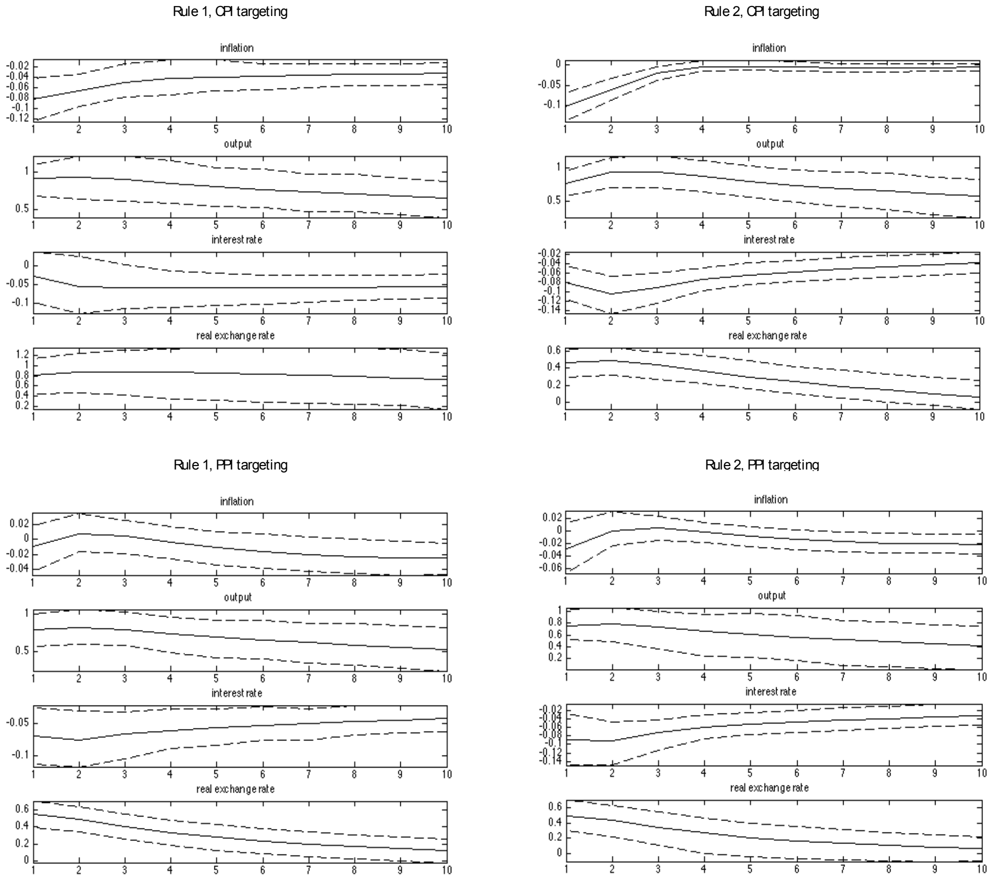


Figure 3: Impulse Responses to a Domestic TFP shock

Figure 3 displays the responses of the domestic variables to a positive domesticTFP shock. Independently on the monetary policy rule, the output reacts positively to the TFP shock, and stronger than all other variables. This result can be interpreted as follows. An increase in productivity leads to lower marginal costs, hence to a decrease in producer prices and domestic PPI inflation. Therefore, the relative prices of imported good increase, and the aggregate demand shifts towards the cheaper domestic goods. This, in turn, implies a rise in domestic aggregate output. Foreign inflation relates positively to a change in LOP gap, which is a function of the real exchange rate and the foreign (relative) price. A rise in real exchange rate leads to an increase in LOP gap, whereas a higher $\tilde{p}_t^F$ implies a lower LOP gap. As a result, the LOP gap increases less than proportionally with the shock, leading to a modest rise in imported inflation. CPI inflation, given by

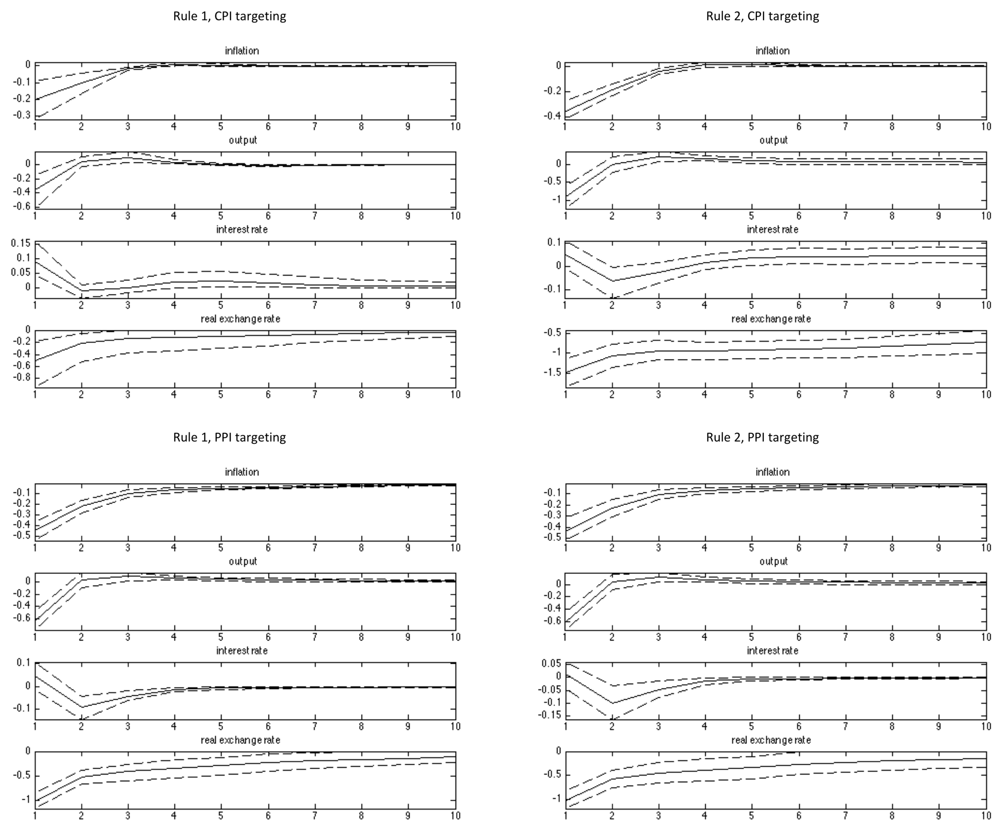


Figure 4: Impulse Responses to a Domestic Monetary Shock

the combination of domestic and foreign inflation, decreases overall since the drop in the producer prices variation entails stronger effects than the higher imported inflation.

In response to lower inflation, the central bank opts for an expansionary monetary policy, which implies a fall in the interest rate. Because the interest rate of the large economy remains constant, but the uncovered interest parity holds, the nominal and real exchange rate depreciate. These results are in line with the theoretical findings in Galí and Monacelli (2005).

It also follows from Figure 3 that the overall inflation decreases more in the case of CPI inflation targeting than with PPI inflation targeting. The reason is that when producer inflation is targeted, it fluctuates less and therefore the price for the domestic good is more stable. The decrease in PPI inflation is partly offset by the increase in imported inflation and thus, the overall inflation is less volatile than in the case of CPI targeting. It may also be noted that output and real exchange

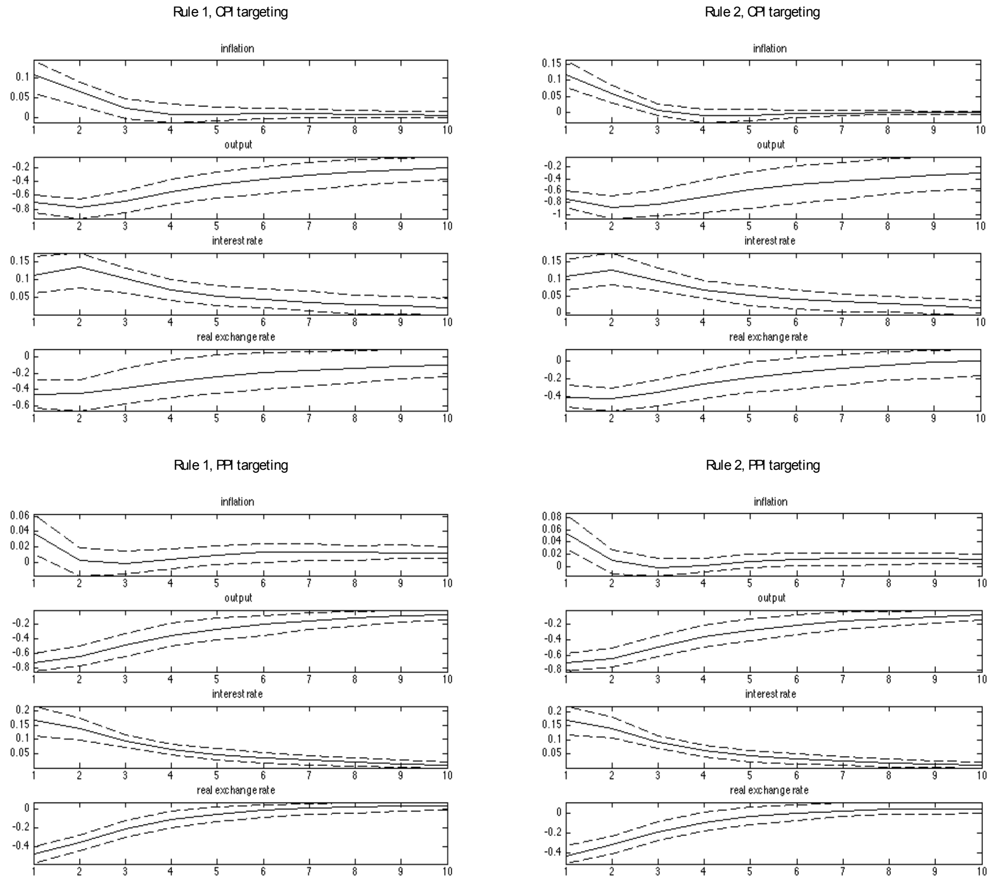


Figure 5: Impulse Responses to a Domestic Producer Cost Push Shock

rate vary only marginally, regardless the choice of the policy target.

The responses to a domestic monetary shock are presented in Figure 4. An unexpected increase in the interest rate leads to a lower aggregate output. First, a higher interest rate implies a higher return on domestic assets, and therefore makes the domestic currency more attractive. The nominal appreciation, making imports cheaper, leads to a drop in the demand for domestic goods. In turn, a downward shift in demand for domestic goods results in lower inflation and aggregate output.

Figure 5 plots the responses to a domestic cost push shock. This shock immediately increases producer inflation. The higher relative domestic price reduces the overall demand for domestic good, and therefore results in a drop in aggregate domestic output. Overall inflation also increases. Thus, the central bank reacts by raising the interest rate, which leads to an appreciation of the exchange rate and, furthermore, depresses the competitiveness of the domestic goods in the interna-

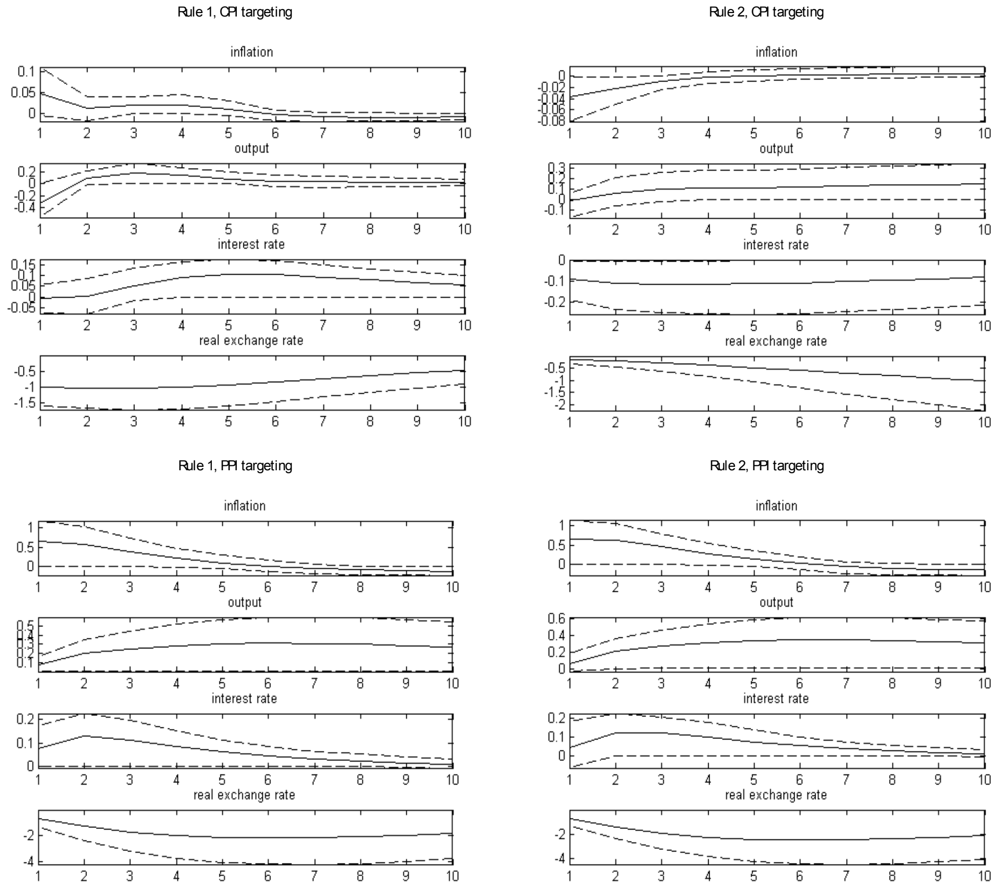


Figure 6: Impulse Responses to an Importer Cost Push Shock

tional markets. Also in the case of a cost push shock, overall inflation is less volatile if the central bank directly targets producer inflation. The initial response of the aggregate output, however, is seemingly independent of the policy rule adopted.

In the presence of an importer cost push shock, the difference between the PPI inflation and CPI inflation targeting is more obvious than in the previous cases. Depending on the rule, the very dynamics of the main economic variables change. The impulse responses are illustrated in Figure 6. An importer cost push shock increases immediately the inflation of the imported goods. Thus, the price of these goods increases relative to the price of the domestically produced goods. Imports fall, and overall domestic consumption decreases, increasing the marginal utility of consumption. However, a rise in domestic production occurs, due to a higher domestic demand for domestic goods. Given the fact that risk sharing holds, the real exchange rate appreciates, which reduces competitive advantage on

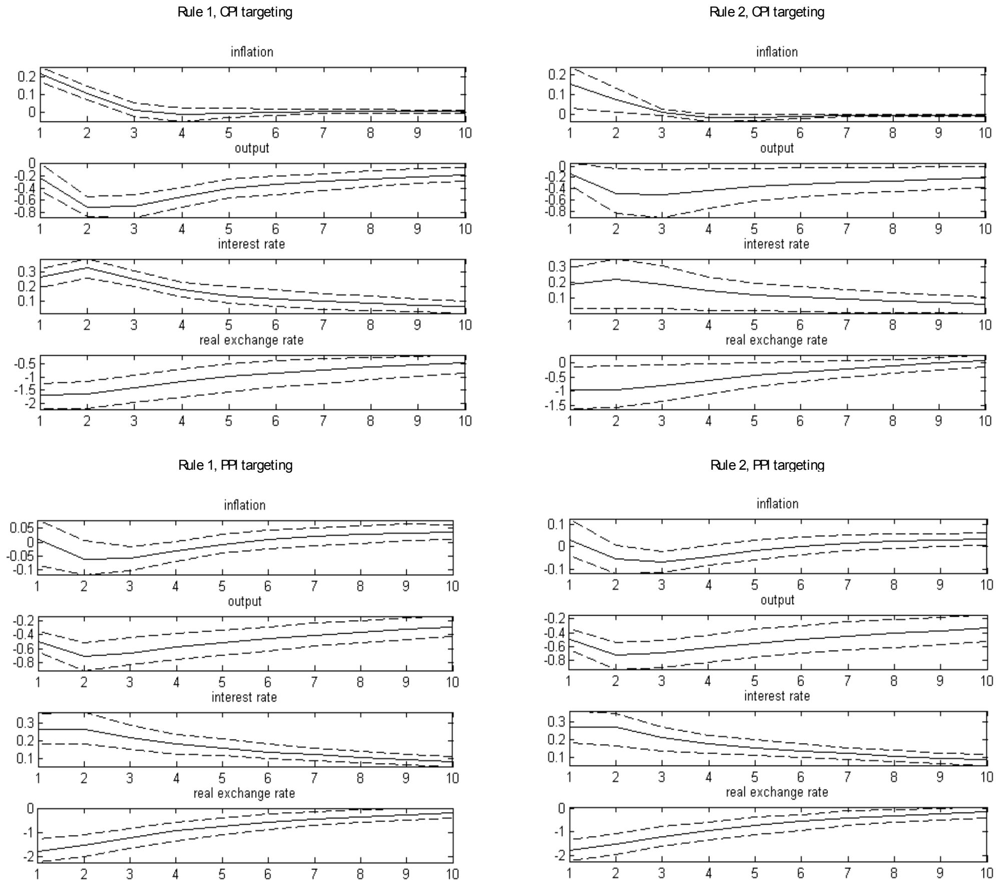


Figure 7: Impulse Responses to a Domestic Preference Shock

the international market. Therefore, the resulting effect on the domestic output is ambiguous. In the case of PPI targeting, the response of the central bank to the rising inflation is milder. As a result, output slightly increases, but this also implies a substantial rise in inflation. Under CPI targeting, the central bank intervention is stronger: this entails lower output, but inflation growth is very small.

A domestic demand shock, illustrated in Figure 7, increases overall consumption. Assuming that the risk sharing holds, the resulting decrease in the marginal utility of consumption implies an appreciation in the domestic currency, and therefore an increase in relative domestic price. As a consequence, the LOP gap decreases, and so does imported inflation. The demand for domestic goods decreases, whereas demand for foreign goods increases more than proportionally. With the increase in consumption, the marginal utility of consumption decreases and the wage increases. This is due to the fact that when agents optimise they equate

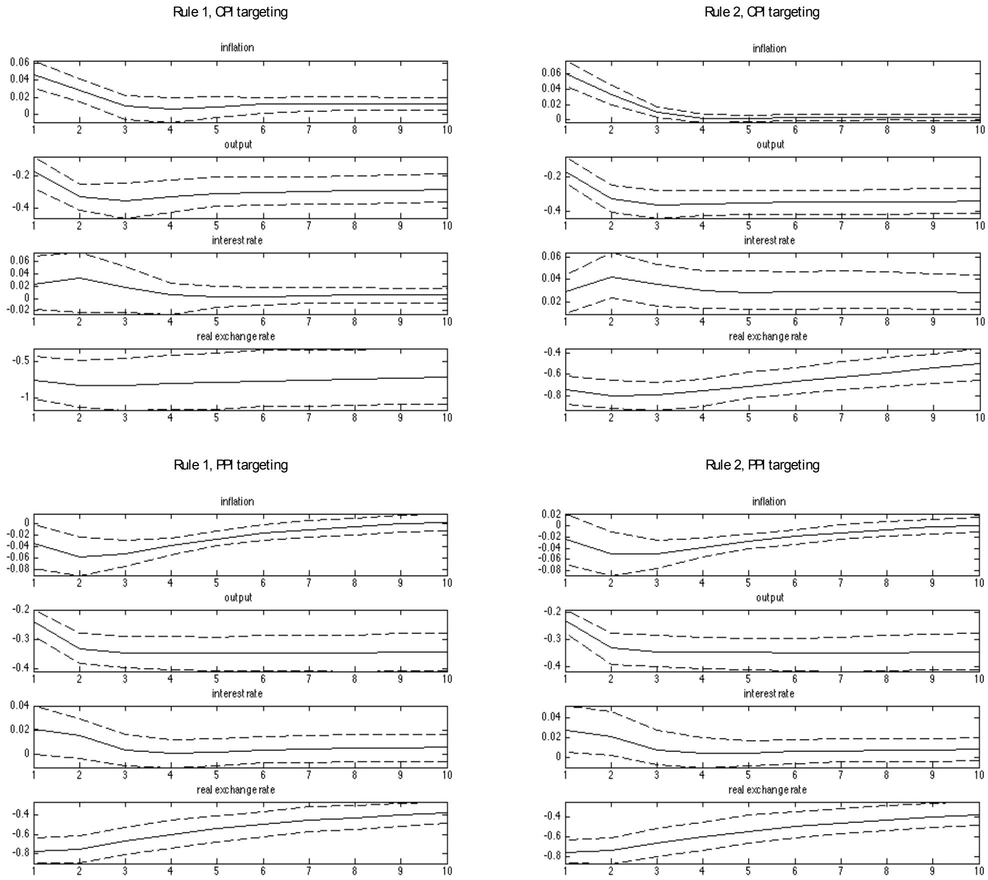


Figure 8: Impulse Responses to a Foreign TFP Shock

the ratio of marginal disutility of labour to the marginal utility of consumption and also to the real wage. This would imply that whenever consumption increases, agents tend to lower their labour supply for a given wage. Since in equilibrium labour does not decrease sufficiently to keep the ratio constant, the real wage grows. This leads to an increase in marginal costs which is partly offset due to the increase in the relative domestic price. Finally, an increase in marginal costs leads to a rise in producer inflation. The overall rate of inflation increases. Thus, the central bank tightens its policy by increasing the interest rate.

If a TFP shock hits a foreign large economy, the rate of inflation in that country falls, domestic aggregate output increases, and the central bank lowers the interest rate. The impact on the domestic variables is shown in Figure 8. The domestic currency appreciates relative to the foreign currency. The relative price for foreign good decrease, hence demand shifts toward the foreign produced goods. Foreign

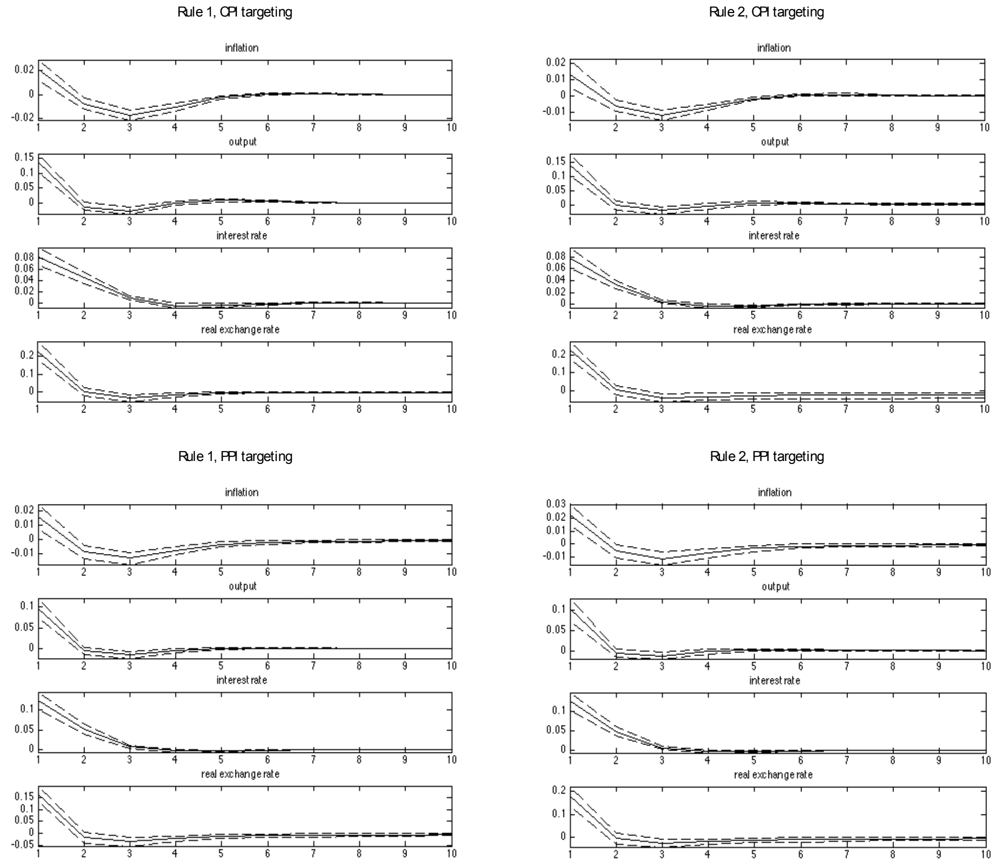


Figure 9: Impulse Responses to a Foreign Monetary Shock

inflation lowers (decrease in LOP gap) and domestic inflation rises (increase in real wage, real marginal costs). In the case of PPI targeting, the overall inflation may fall, however by CPI targeting, the CPI inflation increases initially. After the initial drop, the output decreases further as a consequence of the rise in the interest rate, having its trough in the second to third period, and afterwards returning back to its equilibrium very slowly.

A positive foreign monetary policy shock, illustrated in Figure 9, causes an immediate appreciation in the foreign currency. The domestic currency depreciates and, as a consequence, the domestic goods become cheaper relative to the foreign one. Thus, the demand for domestic good increases and so does aggregate domestic output. The overall inflation rises as well, as a consequence of an increase in domestic inflation. Therefore, the central bank opts for a contractionary monetary policy, which entails a return of the exchange rate quickly - after two periods - back to its equilibrium.

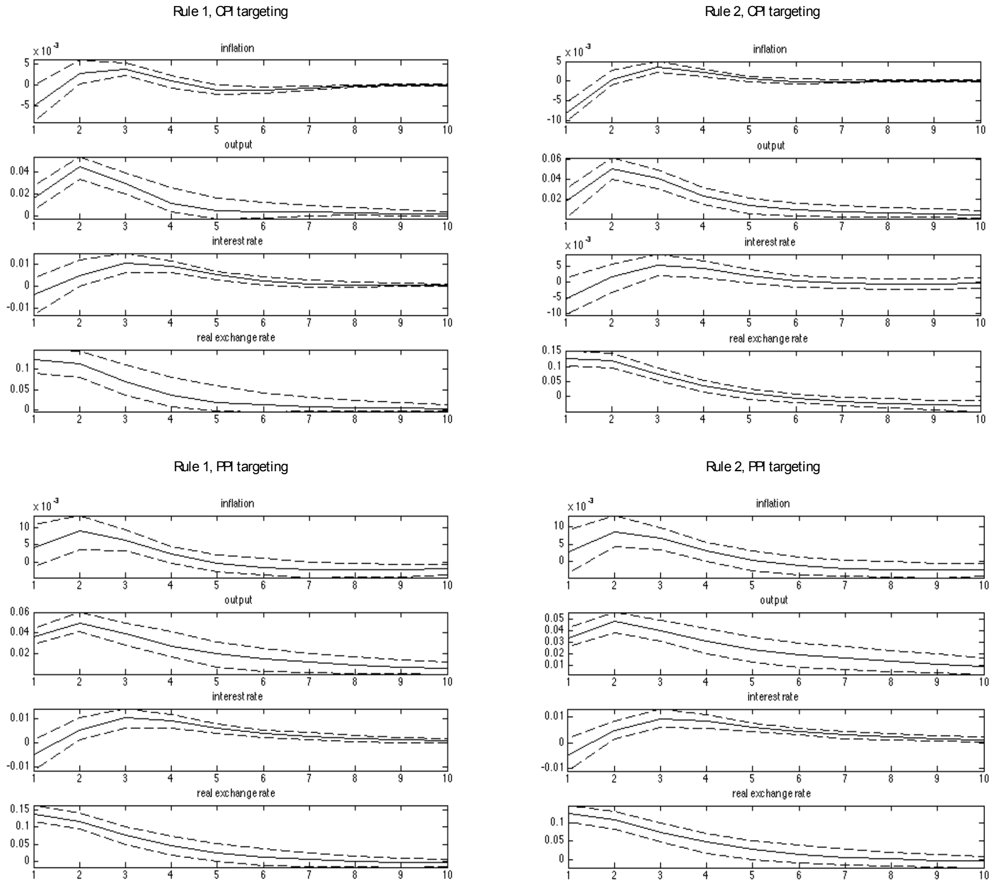


Figure 10: Impulse Responses to a Foreign Cost Push Shock

Figure 10 shows that a foreign cost push shock leads to a currency depreciation and a rise in domestic aggregate output. For the large foreign economy, the shock leads to an increase in inflation and a drop in consumption and output. The central bank increases the interest rate. As a consequence, the domestic currency depreciates and domestic goods gain a relative price advantage, which results in a demand shift toward domestic goods. Domestic aggregate output increases, but overall domestic consumption falls due to higher prices. This leads to a decrease in the real wage, and a drop in the real marginal costs. Thus, PPI inflation decreases. Overall inflation decreases if it is subject to the central bank's targeting. Nevertheless, if the central bank targets PPI inflation, the overall inflation may increase, since the rise in foreign inflation outweighs the effect of the decrease in PPI inflation. All in all, the central bank decreases the interest rate in response

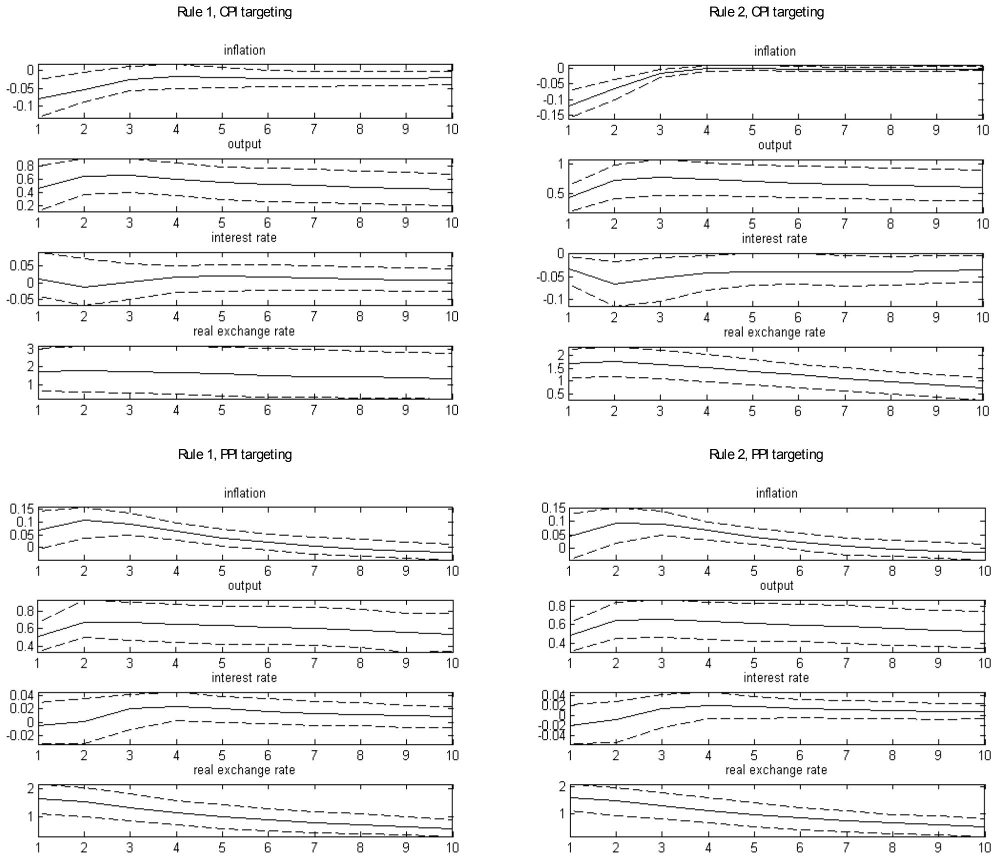


Figure 11: Impulse Responses to a Foreign Preference Shock

to a foreign cost push shock.

Similarly to a foreign cost push shock, the foreign demand shock increases domestic output. Foreign consumption initially increases, leading to a lower marginal utility of consumption in the large economy and the foreign central bank reacts with an increase in the interest rate, which has the effect of domestic currency to depreciates. The responses to the shock are presented in Figure 11. If the domestic central bank targets the PPI, overall inflation may increase. By contrast, targeting CPI leads to a drop in overall inflation. As a consequence, the latter results in an expansionary monetary policy.

To conclude, note that in most of the cases, targeting PPI leads to lower volatility in CPI inflation than with CPI targeting. The effect of different inflation targets on output is not that strong, hence it causes only limited changes to output. In line with the typical arguments in the theoretical literature, which maintain that PPI targeting leads to lower welfare losses, my impulse responses clearly show that such welfare gains are mainly due to the different effects on inflation generated by targeting the two alternative price indices.

5 Concluding Remarks

This work considered the characteristics and performance of simple monetary policy rules using a two-country model. First, I developed a small-scale two-country DSGE model similar to Lubik and Schorfheide (2007) with a microfounded Phillips curve, that is loglinearised around a non-zero steady state inflation. To approach the well-established empirical evidence, I assumed imperfect pass-through, home bias preferences and non-unit intratemporal elasticity of substitution between domestic and foreign goods.

I carried out Bayesian inference, using a Metropolis Hastings sampling approach, to measure the performance of this model against data of several European countries. First, using only the part of the model related to the large economy, I tested several simple nominal interest rate rules, using German data. Firstly, I demonstrated that a simple monetary policy rule mimicking an optimal rule gives the best outcome. Additionally, I showed that the estimation of the structural parameters of the model are robust to the choice of the monetary policy rule, and that the non-zero inflation part included in the Phillips curve improves the model fit significantly.

To study the model for the SOE, I used the data of EEC, namely the Czech Republic, Hungary and Poland. Using a posterior odds test, I found evidence that the central banks of all these countries target a PPI inflation instead of CPI inflation, contrary to what is usually assumed in the empirical literature. I showed that, also in the case of a SOE, the model with a non-zero steady state inflation performs substantially better. If we compare the non-zero steady state inflation component between the three EEC and Germany, we find that the magnitude is lower for the latter, though it remains positive and significantly different from zero. Further analysis about the monetary policy rules showed that a pure exchange rate target can be rejected for all three EEC, and that only the Czech Republic appears to respond to exchange rate movements.

References

Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2008): "Evaluating an estimated new Keynesian small open economy model," *Journal of Economic Dynamics and Control*, 32(8), 2690–2721.

An, S., and F. Schorfheide (2007): "Bayesian Analysis of DSGE Models," *Econometric Reviews*, 26(2-4), 113–172.

Ascari, G., and T. Ropele (2007): "Optimal monetary policy under low trend inflation," Temi di discussione (Economic working papers) 647, Bank of Italy, Economic Research Department.

Benigno, G., and P. Benigno (2003): "Designing targeting rules for international monetary policy cooperation," Working Paper Series 279, European Central Bank.

Benigno, P., and J. D. Lopez-Salido (2002): "Inflation persistence and optimal monetary policy in the Euro area," Working Paper Series 178, European Central Bank.

Blake, A., and H. Mumtaz (2012): *Applied Bayesian Econometrics for Central Bankers*. mimeo.

Brooks, S. P., and A. Gelman (1998): "General Methods for Monitoring Convergence of Iterative Simulations," *Journal of Computational and Graphical Statistics*, 7(4), 434–455.

Calvo, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 12(3), 383–398.

Cogley, T., and A. M. Sbordone (2008): "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve," *American Economic Review*, 98(5), 2101–26.

Corsetti, G., and P. Pesenti (2001): "Welfare And Macroeconomic Interdependence," *The Quarterly Journal of Economics*, 116(2), 421–445.

De Paoli, B. (2009): "Monetary policy and welfare in a small open economy," *Journal of International Economics*, 77(1), 11–22.

Devereux, M. B., and C. Engel (2003): "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility," *Review of Economic Studies*, 70(4), 765–783.

Galí, J., and M. Gertler (1999): "Inflation dynamics: A structural econometric analysis," *Journal of Monetary Economics*, 44(2), 195–222.

Galí, J., M. Gertler, and J. D. Lopez-Salido (2001): "European inflation dynamics," *European Economic Review*, 45(7), 1237–1270.

Galí, J., and T. Monacelli (2005): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72(3), 707–734.

Justiniano, A., and B. Preston (2010): "Monetary policy and uncertainty in an empirical small open-economy model," *Journal of Applied Econometrics*, 25(1), 93–128.

Linde, J. (2005): "Estimating New-Keynesian Phillips curves: A full information maximum likelihood approach," *Journal of Monetary Economics*, 52(6), 1135–1149.

Liu, P. (2006): "A Small New Keynesian Model of the New Zealand economy," Reserve Bank of New Zealand Discussion Paper Series DP2006/03, Reserve Bank of New Zealand.

Lubik, T., and F. Schorfheide (2005): "A Bayesian Look at New Open Economy Macroeconomics," Economics Working Paper Archive 521, The Johns Hopkins University,Department of Economics.

Lubik, T. A., and F. Schorfheide (2007): "Do central banks respond to exchange rate movements? A structural investigation," *Journal of Monetary Economics*, 54(4), 1069–1087.

Monacelli, T. (2003): "Monetary policy in a low pass-through environment," Working Paper Series 227, European Central Bank.

Negro, M. D., and F. Schorfheide (2009): "Inflation Dynamics in a Small Open Economy Model under Inflation Targeting: Some Evidence from Chile," in *Monetary Policy under Uncertainty and Learning*, ed. by K. Schmidt-Hebbel, C. E. Walsh, N. L. S. Editor), and K. S.-H. (Series, vol. 13 of *Central Banking, Analysis, and Economic Policies Book Series*, chap. 13, pp. 511–562. Central Bank of Chile.

Obstfeld, M., and K. Rogoff (2002): "Global Implications Of Self-Oriented National Monetary Rules," *The Quarterly Journal of Economics*, 117(2), 503–535.

Sbordone, A. M. (2002): "Prices and unit labor costs: a new test of price stickiness," *Journal of Monetary Economics*, 49(2), 265–292.

Schorfheide, F. (2008): "DSGE model-based estimation of the New Keynesian Phillips curve," *Economic Quarterly*, (Fall), 397–433.

Smets, F., and R. Wouters (2003): "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 1(5), 1123–1175.

Steinbach, M., P. Mathuloe, and B. Smit (2009): "An Open Economy New Keynesian Dsge Model Of The South African Economy," *South African Journal of Economics*, 77(2), 207–227.

Steinsson, J. (2003): "Optimal monetary policy in an economy with inflation persistence," *Journal of Monetary Economics*, 50(7), 1425–1456.

Sutherland, A. (2002): "A Simple Second-Order Solution Method for Dynamic General Equilibrium Models," Discussion Paper Series, Department of Economics 0211, Department of Economics, University of St. Andrews.

——— (2006): "The expenditure switching effect, welfare and monetary policy in a small open economy," *Journal of Economic Dynamics and Control*, 30(7), 1159–1182.

Tuesta, V., and P. Rabanal (2006): "Euro-Dollar Real Exchange Rate Dynamics in an Estimated Two-Country Model: What Is Important and What Is Not," (06/177).

Appendix A

The log-linearisation of the equations (42), (43) and (44) straightforwardly leads respectively to

$$\hat{p}_t^f = \hat{\jmath}_t - \hat{h}_t,$$

$$\hat{\jmath}_t = \left(1 - \alpha\beta\left(\Pi_H\right)^{\varepsilon}\right)\left(\hat{y}_t - \sigma\hat{c}_t + \widehat{mc}_t + \tilde{p}_{H,t} + v_t\right) + \alpha\beta\left(\Pi_H\right)^{\varepsilon}\left(\varepsilon\hat{\pi}_{H,t+1} + \hat{\jmath}_{t+1}\right),$$

$$\hat{h}_t = \left(1 - \alpha\beta\bar{\Pi}^{\varepsilon-1}\right)\left(\hat{y}_t - \sigma\hat{c}_t\right) + \left(\alpha\beta\right)\bar{\Pi}^{\varepsilon-1}\left(\varepsilon\hat{\pi}_{H,t+1} - \hat{\pi}_{t+1} + \hat{h}_{t+1}\right),$$

from which I can obtain the forward looking price in log-linearised term as

$$\begin{aligned}\hat{p}_t^f = \alpha\beta\left[\bar{\Pi}^{\varepsilon-1} - \bar{\Pi}^{\varepsilon}\right]\left(\hat{y}_t - \sigma\hat{c}_t\right) + \left(1 - \alpha\beta\left(\Pi\right)^{\varepsilon}\right)\left(\widehat{mc}_t + \tilde{p}_{H,t} + v_t\right) \\ + \alpha\beta\left\{\Pi^{\varepsilon}\varepsilon\hat{\pi}_{H,t+1} - \bar{\Pi}^{\varepsilon-1}\varepsilon\hat{\pi}_{H,t+1}\right\} \\ + \left(\alpha\beta\right)\bar{\Pi}^{\varepsilon-1}\hat{\pi}_{t+1} + \alpha\beta\left[\bar{\Pi}^{\varepsilon}\hat{\jmath}_{t+1} - \bar{\Pi}^{\varepsilon-1}\hat{h}_{t+1}\right].\end{aligned}$$

The log-linearisation of equations (45),(46) deliver respectively

$$\tilde{p}_t^b = \tilde{x}_{t-1} + \hat{\pi}_{H,t-1} - \hat{\pi}_t,$$

$$\hat{x}_t = \left(1 - \omega\right)\hat{p}_t^f + \omega\hat{p}_t^b.$$

The domestic price dynamics in relative terms given in equation (47) is log-linearised as

$$\hat{\pi}_t = \frac{1 - \alpha\Pi^{\varepsilon-1}}{\alpha\Pi^{\varepsilon-1}}\tilde{x}_t + \frac{\alpha\Pi^{\varepsilon-1}\tilde{p}_{H,t-1} - \tilde{p}_{H,t}}{\alpha\Pi^{\varepsilon-1}}.$$

Combining all these equations together delivers a hybrid NKPC of a form

$$\begin{aligned}\left(\alpha\bar{\Pi}^{\varepsilon-1} + \omega\left(1 - \alpha\bar{\Pi}^{\varepsilon}\left(\bar{\Pi}^{-1} - \beta\right)\right)\right)\left(\hat{\pi}_t + \Delta\tilde{p}_{H,t}\right) = \alpha\beta\bar{\Pi}^{\varepsilon}\left(\hat{\pi}_{t+1} + \Delta\tilde{p}_{H,t+1}\right) \\ + \omega\left(\hat{\pi}_{t-1} + \Delta\tilde{p}_{H,t-1}\right) + \left(1 - \alpha\Pi^{\varepsilon-1}\right)\left(1 - \omega\right)\left(1 - \alpha\beta\left(\Pi\right)^{\varepsilon}\right)\left(\widehat{mc}_t + v_t\right) \\ + \left(1 - \alpha\Pi^{\varepsilon-1}\right)\left(1 - \omega\right)\alpha\beta\left(\bar{\Pi}^{\varepsilon} - \bar{\Pi}^{\varepsilon-1}\right) \\ \cdot\left[\hat{h}_{t+1} + \left(\varepsilon - 1\right)\left(\hat{\pi}_{t+1} + \Delta\tilde{p}_{H,t+1}\right) + \left(\sigma\hat{c}_t - \hat{y}_t\right) + \Delta\tilde{p}_{H,t+1}\right]\end{aligned}$$

Using (20), which log-linearised delivers

$$\hat{\pi}_t + \Delta\tilde{p}_{H,t} = \hat{\pi}_{H,t},$$

simplifies the hybrid NKPC. After collecting the terms together and using the definition of $\hat{h}_t$ above, the Phillips curve with backward looking firms linearising around a non-zero steady state inflation yields

$$\begin{aligned}\left(\alpha\bar{\Pi}^{\varepsilon-1}+\omega\left(1-\alpha\bar{\Pi}^{\varepsilon}\left(\bar{\Pi}^{-1}-\beta\right)\right)\right)\hat{\pi}_{H,t} &= \alpha\beta\bar{\Pi}^{\varepsilon}\hat{\pi}_{H,t+1}+\omega\hat{\pi}_{H,t-1} \\ &+\left(1-\alpha\Pi^{\varepsilon-1}\right)(1-\omega)\left(1-\alpha\beta\left(\Pi\right)^{\varepsilon}\right)\left(\widehat{mc}_t+v_t\right) \\ \cdot\left(1-\alpha\Pi^{\varepsilon-1}\right)(1-\omega)\left(\bar{\Pi}^{-1}-1\right)\alpha\beta\bar{\Pi}^{\varepsilon-1} & \left[\hat{h}_{t+1}+\varepsilon\hat{\pi}_{H,t+1}-\hat{\pi}_{t+1}+\sigma\hat{c}_t-\hat{y}_t\right].\end{aligned}$$

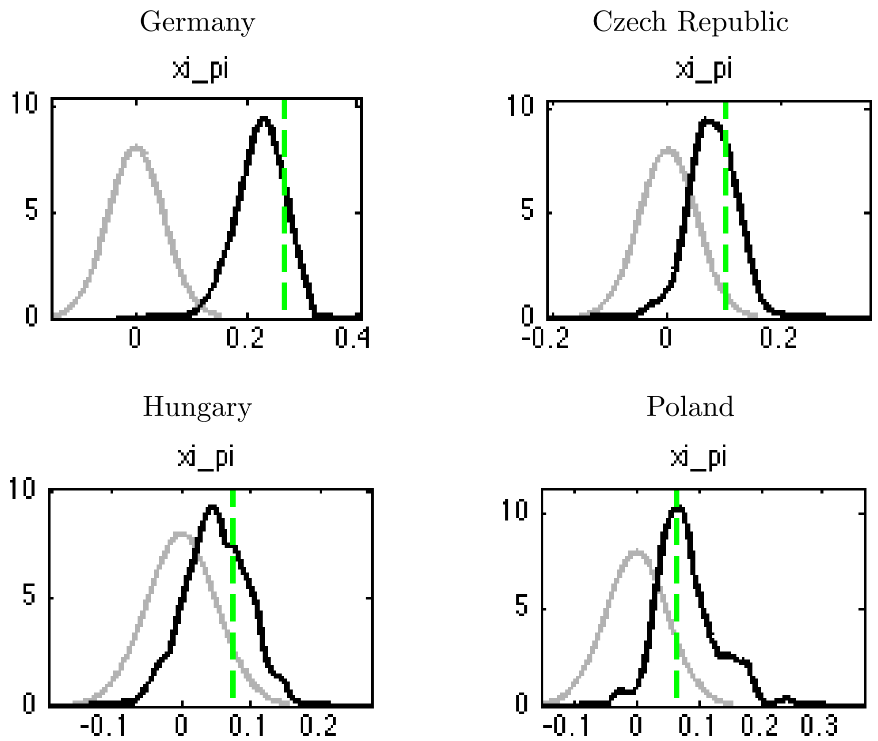
Written in terms of parameters

$$\hat{\pi}_{H,t}=\chi^f E_t\left[\hat{\pi}_{H,t+1}\right]+\chi^b\hat{\pi}_{H,t-1}+\kappa_{mc}\left(\widehat{mc}_t+v_t\right)+\chi^{\pi}\left(\hat{h}_t-\left(\hat{y}_t-\sigma\hat{c}_t\right)\right)$$

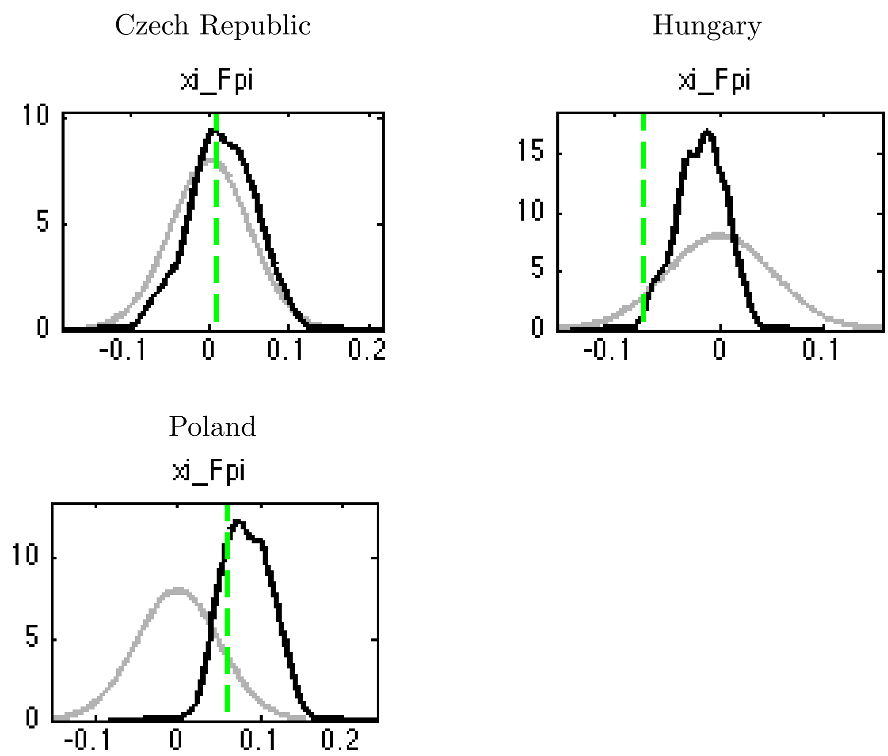
with parameters

$$\begin{aligned}\Psi &= \alpha\bar{\Pi}^{\varepsilon-1}+\omega\left(1-\alpha\bar{\Pi}^{\varepsilon}\left(\bar{\Pi}^{-1}-\beta\right)\right) \\ \chi^f &= \alpha\beta\bar{\Pi}^{\varepsilon}/\Psi, \chi^b=\omega/\Psi, \\ \kappa_{mc} &= \left(1-\alpha\Pi^{\varepsilon-1}\right)(1-\omega)\left(1-\alpha\beta\Pi^{\varepsilon}\right)/\Psi \\ \chi^{\pi} &= \left(1-\alpha\Pi^{\varepsilon-1}\right)(1-\omega)\left(\bar{\Pi}^{-1}-1\right).\end{aligned}$$

Appendix B



Prior and Posterior Distribution and Posterior Mode for the Parameter χ_π



Prior and Posterior Distribution and Posterior Mode for the Parameter χ_F^π