Strategic Macroeconomic Policies in a Monetary Union

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Abstract In this paper we present an application of the dynamic tracking games framework to a monetary union. We use a small stylized nonlinear two-country macroeconomic model (MUMOD1) of a monetary union to analyse the interactions between fiscal (governments) and monetary (common central bank) policy makers, assuming different objective functions of these decision makers. Using the OPTGAME algorithm we calculate equilibrium solutions for four game strategies: one cooperative (Pareto optimal) and three non-cooperative games: the Nash game for the open-loop information pattern, the Nash game for the feedback information pattern, and the Stackelberg game for the feedback information pattern. Applying the OPTGAME algorithm to the MUMOD1 model we show how the policy makers react to demand and supply shocks according to different solution concepts. Some comments are given on possible applications to the recent sovereign debt crisis in Europe.

Keywords dynamic games; open-loop Nash equilibrium; feedback Nash equilibrium; feedback Stackelberg equilibrium; Pareto optimal solution; macroeconomics; economic dynamics; monetary union

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1 Introduction

The economic situation in the European Monetary Union (EMU) is relatively unstable nowadays due to the economic crisis of 2007-2010 and a wide range of structural problems in the affected countries. At the breakout of the last economic crisis policy makers tried to cooperate and to use coordinated countercyclical fiscal and monetary policies to reduce the negative impact of the crisis, placing great emphasis on the GDP growth rate and unemployment. Unfortunately, the public debt situation worsened dramatically and we have been facing a severe sovereign debt crisis in Europe since 2010. Today, there is no consensus among politicians on what is the best way out of the crisis. The European Monetary Union does not appear to be acting like a union of cooperating partners speaking with one voice but like a pool of independent players seeking gains for their own country only. The core of the problem seems to be a lack of agreement about objectives and strategies to pursue. This is a typical problem of dynamic strategic interaction. Hence, it is appropriate to run a study of a monetary union using concepts of dynamic game theory.

The framework of dynamic games is most suitable to describe the dynamics of a monetary union because a monetary union consists of several players with independent and different aims and instruments. Even if there are common, union-wide objectives, each of the players may assign different importance (weights) to these targets. In addition, the willingness to cooperate to achieve the common goal is country-specific as well. For these reasons it is necessary to model the conflicts ('non-cooperation') between the players. Such problems can best be modeled using the concepts and methods of dynamic game theory, which has been developed mostly by engineers and mathematicians but which has proved to be a valuable analytical tool for economists, too (see, e.g., [1]; [2]; [8]).

In this paper we present an application of the dynamic tracking game framework to a monetary union macroeconomic model. Dynamic games have been used by several authors (e.g., [7]) for modeling conflicts between monetary and fiscal policies. There is also a large body of literature on dynamic conflicts between policy makers from different countries on issues of international stabilization (e.g., [6]). Both types of conflict are present in a monetary union, because a supranational central bank interacts strategically with sovereign governments as national fiscal policy makers in the member states. Such conflicts can be analysed using either large empirical macroeconomic models or small stylized models. We follow the latter line of research and use a small stylized nonlinear two-country macroeconomic model of a monetary union (called MUMOD1) for analysing the interactions between fiscal (governments) and monetary (common central bank) policy makers, assuming different objective functions of these decision makers. Using the OPTGAME algorithm we calculate equilibrium solutions for four game strategies, one cooperative (Pareto optimal) and three non-cooperative game types: the Nash game for the open-loop information pattern, the Nash game for the feedback information pattern, and the Stackelberg game for the feedback information pattern. Applying the OPTGAME algorithm to the MUMOD1 model we show how the policy makers react optimally to demand and supply shocks. Some comments are given about possible applications to the recent sovereign debt crisis in Europe.

2 Nonlinear dynamic tracking games

The nonlinear dynamic game-theoretic problems which we consider in this paper are given in tracking form. The players are assumed to aim at minimizing quadratic deviations of the equilibrium values (according to the respective solution concept) from given target (desired) values. Thus each player minimizes an objective function J^i given by:

$$\min_{u_1^i,\ldots,u_T^i} J^i = \sum_{t=1}^{T} L_t^i(x_t, u_t^1, \ldots, u_t^N), \quad i = 1, \ldots, N, \tag{1}$$

with

$$L_t^i(x_t, u_t^1, \ldots, u_t^N) = \frac{1}{2}[X_t - \tilde{X}_t^i]' \Omega_t^i [X_t - \tilde{X}_t^i], \quad i = 1, \ldots, N. \tag{2}$$

The parameter N denotes the number of players (decision makers). T is the terminal period of the finite planing horizon, i.e. the duration of the game. X_t is an aggregated vector

$$X_t := [x_t \ \ u_t^1 \ \ u_t^2 \ \ \ldots \ \ u_t^N]', \tag{3}$$

which consists of an $(n_x \times 1)$ vector of state variables

$$x_t := [x_t^1 \ \ x_t^2 \ \ \ldots \ \ x_t^{n_x}]' \tag{4}$$

and N $(n_i \times 1)$ vectors of control variables determined by the players $i = 1, \ldots, N$:

$$\begin{aligned} u_t^1 &:= [u_t^{11} \ \ u_t^{12} \ \ \ldots \ \ u_t^{1n_1}]', \\ u_t^2 &:= [u_t^{21} \ \ u_t^{22} \ \ \ldots \ \ u_t^{2n_2}]', \\ &\ \vdots \\ u_t^N &:= [u_t^{N1} \ \ u_t^{N2} \ \ \ldots \ \ u_t^{Nn_N}]'. \end{aligned} \tag{5}$$

Thus X_t (for all $t = 1, \ldots, T$) is an r-dimensional vector, where

$$r := n_x + n_1 + n_2 + \cdots + n_N. \tag{6}$$

The desired levels of the state variables and the control variables of each player enter the quadratic objective functions (as given by equations (1) and (2)) via the terms

$$\tilde{X}_t^i := [\tilde{x}_t^i \;\; \tilde{u}_t^{i1} \;\; \tilde{u}_t^{i2} \;\; \ldots \;\; \tilde{u}_t^{iN}]'. \tag{7}$$

Each player $i = 1,...,N$ is assumed to be able to observe and monitor the control variables of the other players, i.e. deviations of other control variables can be punished in one's own objective function. For example, the central bank in a monetary union, which controls monetary policy, can also penalize 'bad' fiscal policies of member countries.

Equation (2) contains an $(r \times r)$ penalty matrix Ω_t^i $(i = 1, \ldots, N)$, weighting the deviations of states and controls from their desired levels in any time period t $(t = 1,...,T)$. Thus the matrices

$$\Omega_t^i = \begin{bmatrix} Q_t^i & 0 & \cdots & 0 \\ 0 & R_t^{i1} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & R_t^{iN} \end{bmatrix}, \; i = 1, \ldots, N, \; t = 1, \ldots, T, \tag{8}$$

are of block-diagonal form, where the blocks Q_t^i and R_t^{ij} $(i, j = 1, \ldots, N)$ are symmetric. These blocks Q_t^i and R_t^{ij} correspond to penalty matrices for the states and the controls respectively. The matrices $Q_t^i \geq 0$ are positive semi-definite for all $i = 1, \ldots, N$; the matrices R_t^{ij} are positive semi-definite for $i \neq j$ but positive definite for $i = j$. This guarantees that the matrices $R_t^{ii} > 0$ are non-singular, a necessary requirement for the analytical tractability of the algorithm.

In a frequent special case, a discount factor α is used to calculate the penalty matrix Ω_t^i in time period t:

$$\Omega_t^i = \alpha^{t-1} \Omega_0^i, \tag{9}$$

where the initial penalty matrix Ω_0^i of player i is given.

The dynamic system, which constrains the choices of the decision makers, is given in state-space form by a first-order system of nonlinear difference equations:

$$x_t = f(x_{t-1}, x_t, u_t^1, \ldots, u_t^N, z_t), \quad x_0 = \bar{x}_0. \tag{10}$$

$\bar{x}_0$ contains the initial values of the state variables. The vector z_t contains non-controlled exogenous variables. f is a vector-valued function where f^k

$(k = 1, ..., n_x)$ denotes the kth component of f. For the algorithm, we require that the first and second derivatives of the system function f with respect to x_t, x_{t-1} and $u_t^1, ..., u_t^N$ exist and are continuous.

Equations (1), (2) and (10) define a nonlinear dynamic tracking game problem. The task, for each solution concept, is to find N trajectories of control variables u_t^i, $i = 1, ..., N$, which minimize the postulated objective functions subject to the dynamic system. In the next section, the OPTGAME3 algorithm, which is designed to solve such types of problems, is presented.

3 The OPTGAME3 algorithm

We apply the OPTGAME3 algorithm in order to solve the nonlinear dynamic tracking games as introduced in the previous section. This section briefly describes the OPTGAME3 algorithm; for more details about the solution procedures and the numerical methods used, see Blueschke et al. [3]. OPTGAME3 was programmed in C# and MATLAB. The source code of the algorithm is available from the authors on request. A very simplified structure of the OPTGAME algorithm is as follows:

Algorithm 1 Rough structure of the OPTGAME algorithm

1: initialize input parameters $x_0, (\overset{\circ}{u}{}_t^i)_{t=1}^T, (\tilde{x}_t^i)_{t=1}^T, (\tilde{u}_t^{ij})_{t=1}^T, (z_t)_{t=1}^T$ and $f(...)$
2: calculate tentative paths for states $x_t = f(x_{t-1}, x_t, u_t^1, \ldots, u_t^N, z_t)$, $t = 1, ..., T$
3: **while** the stopping criterion is not met (*nonlinearity loop*) **do**
4:     **for** T to 1 (*backward loop*) **do**
5:         linearize the system of equations: $x_t = A_t x_{t-1} + \sum_{i=1}^N B_t^i u_t^i + c_t$
6:         min J^i, get feedback matrices: G_t^i and g_t^i
7:     **end for**
8:     **for** 1 to T (*forward loop*) **do**
9:         calculate the solution: $u_t^{i*} = G_t^i x_{t-1}^* + g_t^i$ and $x_t^* = f(x_{t-1}^*, x_t, u_t^{1*}, \ldots, u_t^{N*}, z_t)$
10:     **end for**
11:     at the end of the forward loop, the solution for the current iteration of the nonlinearity loop is calculated: $(u_t^{i*}, x_t^*)_{t=1}^T$
12: **end while**
13: final solution is calculated: $(u_t^{i*})_{t=1}^T$, $(x_t^*)_{t=1}^T$, J^{i*}, J^*

The algorithm starts with the input of all required data. As indicated in step (1), tentative paths of the control variables $(\overset{\circ}{u}{}_t^i)_{t=1}^T$ are given as inputs. In order to find a tentative path for the state variables we apply an appropriate system solver like Newton-Raphson, Gauss-Seidel, Levenberg-Marquardt or Trust region in step (2). After that the nonlinearity loop can be started where we approximate the solution of the nonlinear dynamic tracking game. To this end we linearize the nonlinear system f along the tentative path determined in the previous steps. Note that we do not globally linearize the system prior to optimization but repeatedly linearize the system during the iterative optimization process. Accordingly, for each time period t we compute the reduced form of the linearization of equation (10) and approximate the nonlinear system by a linear system with time-dependent parameters in step (5).

The dynamic tracking game can then be solved for the linearized system using known optimization techniques, which results in feedback matrices G_t^i and g_t^i in step (6). These feedback matrices allow us to calculate in a forward loop the solutions (u_t^{i*} and x_t^*) of the current iteration of the nonlinearity loop and, at the end of the nonlinearity loop, the final solutions. The convergence criterion for the nonlinearity loop requires the deviations of solutions of the current from previous iterations to be smaller than a pre-specified number.

The core of the OPTGAME3 algorithm occurs in step (6) where the linearized system has to be optimized. The optimization technique for minimizing the objective functions depends on the type of the game or solution concept. The OPTGAME3 algorithm determines four game strategies: one cooperative (Pareto optimal) and three non-cooperative games: the Nash game for the open-loop information pattern, the Nash game for the feedback information pattern, and the Stackelberg game for the feedback information pattern.

Generally, open-loop Nash and Stackelberg equilibrium solutions of affine linear-quadratic games are determined using Pontryagin's maximum principle. Feedback Nash and Stackelberg equilibrium solutions are calculated using the dynamic programming (Hamilton-Jacobi-Bellman) technique. A detailed discussion on how to calculate the dynamic game solutions depending on the type of the game is given in [3]. Here we apply the algorithm to a model of a monetary union.

4 The MUMOD1 model

In this paper we use a simplified model of a monetary union called MUMOD1, which improves on the one introduced in [4] in order to derive optimal fiscal and monetary policies for the economies in a monetary union. The model is calibrated so as to deal with the problem of public debt targeting (a situation that resembles the one currently prevailing in the European Union), but no attempt is made to describe the EMU in every detail. The model builds on discrete data, which is a popular way in economics but there are similar frameworks in continuous time, see, for example, [8]. One of the most important features of our model is the fact that it allows for different kinds of exogenous shocks acting on the economies in the monetary union in an asymmetric way. Analyzing the impact of these different shocks allows us to gain insights into the dynamics of a monetary union.

In this paper we investigate three different shocks on the monetary union: a negative demand side shock and two negative supply side shocks. Before we present these three studies it is appropriate to describe the model in detail.

In the following, capital letters indicate nominal values, while lower case letters correspond to real values. Variables are denoted by Roman letters, model parameters are denoted by Greek letters. Three active policy makers are considered: the governments of the two countries responsible for decisions about fiscal policy and the common central bank of the monetary union controlling monetary policy. The two countries are labeled 1 and 2 or core and periphery respectively. MUMOD1 is a stylized model of a monetary union consisting of two homogeneous blocs of countries, which in the current European context might be identified with the stability-oriented bloc (core) and the PIIGS bloc (countries with problems due to high public debt).

The model is formulated in terms of deviations from a long-run growth path. The goods markets are modeled for each country by a short-run income-expenditure equilibrium relation (IS curve). The two countries under consideration are linked through their goods markets, namely exports and imports of goods and services. The common central bank decides on the prime rate, that is, a nominal rate of interest under its direct control (for instance, the rate at which it lends money to private banks).

Real output (or the deviation of short-run output from a long-run growth path) in country i ($i = 1,2$) at time t ($t = 1,...,T$) is determined by a reduced form demand-side equilibrium equation:

$$y_{it} = \delta_i(\pi_{jt} - \pi_{it}) - \gamma_i(r_{it} - \theta) + \rho_i y_{jt} - \beta_i \pi_{it} + \kappa_i y_{i(t-1)} - \eta_i g_{it} + zd_{it}, \qquad (11)$$

for $i \neq j$ ($i, j = 1,2$). The variable π_{it} denotes the rate of inflation in country i, r_{it} represents country i's real rate of interest and g_{it} denotes country i's real fiscal surplus (or, if negative, its fiscal deficit), measured in relation to real GDP. g_{it} in (11) is assumed to be country i's fiscal policy instrument or control variable. The natural real rate of output growth, $\theta \in [0,1]$, is assumed to be equal to the natural real rate of interest. The parameters $\delta_i, \gamma_i, \rho_i, \beta_i, \kappa_i, \eta_i$, in (11) are assumed to be positive. The variables zd_{1t} and zd_{2t} are non-controlled exogenous variables and represent demand-side shocks in the goods market.

For $t = 1,...,T$, the current real rate of interest for country i ($i = 1,2$) is given by:

$$r_{it} = I_{it} - \pi^e_{it}, \tag{12}$$

where π^e_{it} denotes the expected rate of inflation in country i and I_{it} denotes the nominal interest rate for country i, which is given by:

$$I_{it} = R_{Et} - \lambda_i g_{it} + \chi_i D_{it} + zhp_{it}, \tag{13}$$

where R_{Et} denotes the prime rate determined by the central bank of the monetary union (its control variable); $-\lambda_i$ and χ_i (λ_i and χ_i are assumed to be positive) are risk premiums for country i's fiscal deficit and public debt level. This allows for different nominal (and hence also real) rates of interest in the union in spite of a common monetary policy due to the possibility of default or similar risk of a country (a bloc of countries) with high government deficit and debt. zhp_{it} allows for exogenous shocks on the nominal rate of interest, e.g. negative after-effects of a haircut or a default (see [5] for such an analysis).

The inflation rates for each country $i = 1,2$ and $t = 1,...,T$ are determined according to an expectations-augmented Phillips curve, i.e. the actual rate of inflation depends positively on the expected rate of inflation and on the goods market excess demand (a demand-pull relation):

$$\pi_{it} = \pi^e_{it} + \xi_i y_{it} + zs_{it}, \tag{14}$$

where ξ_1 and ξ_2 are positive parameters; zs_{1t} and zs_{2t} denote non-controlled exogenous variables and represent supply-side shocks, such as oil price increases, introducing the possibility of cost-push inflation; π^e_{it} denotes the rate of inflation in country i expected to prevail during time period t, which is formed at (the end of) time period $t-1$. Inflationary expectations are formed according to the hypothesis of adaptive expectations:

$$\pi^e_{it} = \varepsilon_i \pi_{i(t-1)} + (1-\varepsilon_i)\pi^e_{i(t-1)}, \tag{15}$$

where $\varepsilon_i \in [0,1]$ are positive parameters determining the speed of adjustment of expected to actual inflation.

The average values of output and inflation in the monetary union are given by:

$$y_{Et} = \omega y_{1t} + (1-\omega) y_{2t}, \ \ \omega \in [0,1], \tag{16}$$

$$\pi_{Et} = \omega \pi_{1t} + (1-\omega) \pi_{2t}, \ \ \omega \in [0,1]. \tag{17}$$

The parameter ω expresses the weight of country 1 in the economy of the whole monetary union as defined by its output level. The same weight ω is used for calculating union-wide inflation in Eq. (17).

The government budget constraint is given as an equation for government debt of country i $(i = 1,2)$:

$$D_{it} = (1 + r_{i(t-1)})D_{i(t-1)} - g_{it} + zh_{it}, \tag{18}$$

where D_i denotes real public debt of country i measured in relation to (real) GDP. No seigniorage effects on governments' debt are assumed to be present. zh_{it} allows us to model an exogenous shock on public debt; for instance, if negative it may express default or debt relief (a haircut).

Both national fiscal authorities are assumed to care about stabilizing inflation (π), output (y), debt (D) and fiscal deficits of their own countries (g) at each time t. This is a policy setting which seems plausible for the actual EMU as well, with full employment (output at its potential level) and price level stability relating to country (or bloc) i's primary domestic goals, and government debt and deficit relating to its obligations according to the Treaty of the European Union. The common central bank is interested in stabilizing inflation and output in the entire monetary union, also taking into account a goal of low and stable interest rates in the union.

Equations (11)-(18) constitute a dynamic game with three players, each of them having one control variable. The model contains 14 endogenous variables and four exogenous variables and is assumed to be played over a finite time horizon. The objective functions are quadratic in the paths of deviations of state and control variables from their desired values. The game is nonlinear-quadratic and hence cannot be solved analytically but only numerically. To this end, we have to specify the parameters of the model.

The parameters of the model are specified for a slightly asymmetric monetary union; see Table 1. Here an attempt has been made to calibrate the model parameters so as to fit for the EMU. The data used for calibration include average economic indicators for the (then) 16 EMU countries from EUROSTAT up to the year 2007. Mainly based on the public finance situation, the EMU is divided into two blocs: a core (country or bloc 1) and a periphery (country or bloc 2). The first bloc has a weight of 60% in the entire economy of the monetary union (i.e. the parameter ω is equal to 0.6). The second bloc has a weight of 40% in the economy of the union; it consists of countries with higher public debt and deficits and higher interest and inflation rates on average. The weights correspond to the respective shares in EMU real GDP. For the other parameters of the model, we use values in accordance with econometric studies and plausibility considerations.

The initial values of the macroeconomic variables, which are the state variables of the dynamic game model, are presented in Table 2. The desired or ideal values assumed for the objective variables of the players are given

Table 1: Parameter values for an asymmetric monetary union, $i = 1,2$

T	θ	ω	$\delta_i, \beta_i, \eta_i, \varepsilon_i$	$\gamma_i, \rho_i, \kappa_i, \xi_i, \lambda_i$	χ_i
30	3	0.6	0.5	0.25	0.0125

in Table 3. Country 1 (the core bloc) has an initial debt level of 60% of GDP and aims to decrease this level in a linear way over time to arrive at a public debt of 50% at the end of the planning horizon. Country 2 (the periphery bloc) has an initial debt level of 80% of GDP and aims to decrease its level to 60% at the end of the planning horizon, which means that it is going to fulfil the Maastricht criterion for this economic indicator. The ideal rate of inflation is calibrated at 1.8%, which corresponds to the Eurosystem's aim of keeping inflation below, but close to, 2%. The initial values of the two blocs' government debts correspond to those at the beginning of the Great Recession, the recent financial and economic crisis. Otherwise, the initial situation is assumed to be close to equilibrium, with parameter values calibrated accordingly.

Table 2: Initial values of the two-country monetary union

$y_{i,0}$	$\pi_{i,0}$	$\pi^e_{i,0}$	$D_{1,0}$	$D_{2,0}$	$R_{E,0}$	$g_{1,0}$	$g_{2,0}$
0	2	2	60	80	3	0	0

Table 3: Target values for an asymmetric monetary union

$\tilde{y}_{it}$	$\tilde{D}_{1t}$	$\tilde{D}_{2t}$	$\tilde{\pi}_{it}$	$\tilde{\pi}_{Et}$	$\tilde{y}_{Et}$	$\tilde{g}_{it}$	$\tilde{R}_{Et}$
0	$60 \searrow 50$	$80 \searrow 60$	1.8	1.8	0	0	3

5 Effects of a negative demand-side shock

The MUMOD1 model can be used to simulate the effects of different shocks acting on the monetary union, which are reflected in the paths of the exogenous non-controlled variables, and the effects of policy reactions towards these shocks. In this section we analyse a symmetric shock which occurs on the demand side (zd_i) as given in Table 4. The numbers can best be interpreted as being measured as percentage points of real GDP.

Table 4: Negative symmetric shock on the demand side

t	1	2	3	4	5	6	$\cdots$	30
zd_1	-2	-4	-2	0	0	0	$\cdots$	0
zd_2	-2	-4	-2	0	0	0	$\cdots$	0

In the first three periods, both countries experience the same negative demand shock (zd_i) which reflects a financial and economic crisis like the one in 2007-2010. After three periods the economic environment of countries 1 and 2 stabilizes again.

Here, we investigate how the dynamics of the model and the results of the policy game (11)-(18) depend on the strategy choice of the decision makers. For this game, we calculate five different solutions: a baseline solution with the shock but with policy instruments held at pre-shock levels (zero for the fiscal balance, 3 for the central bank's interest rate), three non-cooperative game solutions and one cooperative game solution. The baseline solution does not include any policy intervention and describes a simple simulation of the dynamic system. It can be interpreted as resulting from a policy ideology of market fundamentalism prescribing non-intervention in the case of a recession.

Figures 1 - 5 show the simulation and optimization results of this experiment. Figures 1 - 2 show the results for the control variables of the players and Figures 3 - 5 show the results of selected state variables: output, inflation and public debt.

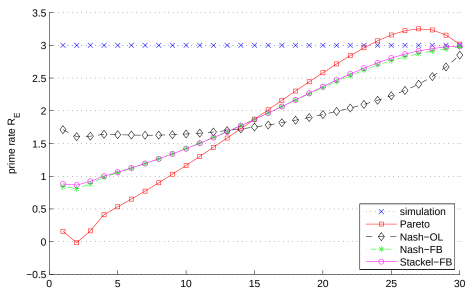

Figure 1: prime rate R_{Et} controlled by the central bank

Without policy intervention (baseline scenario, denoted by 'simulation'), both countries suffer dramatically from the economic downturn modeled by the demand-side shock in the first periods. The output of both countries

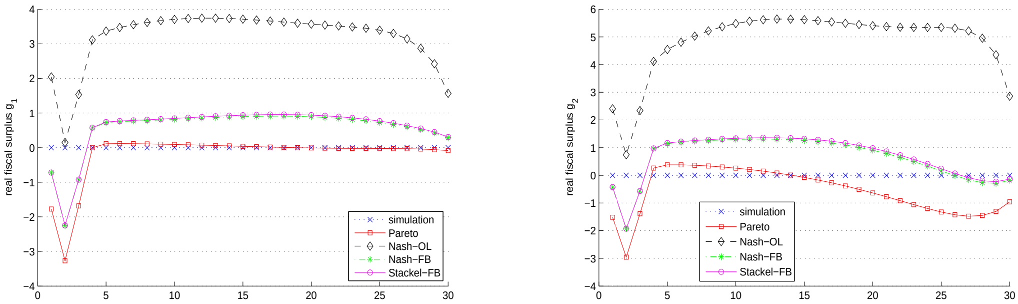

Figure 2: country i's fiscal surplus g_{it} (control variable) for $i = 1$ (core; left) and $i = 2$ (periphery; right)

drops by more than 6%, which for several European countries is a fairly good approximation of what happened in reality. This economic crisis decreases their inflation rates and starting with time period 2 creates a persisting deflation of about -0.5% to -1%. Even more dramatic is the development of public debt. Without policy intervention it increases during the whole planning horizon and arrives at levels of 240% of GDP for country 1 (or core bloc) and 390% for country 2 (or periphery bloc), which shows a need for policy actions to preserve the solvency of the governments of the monetary union.

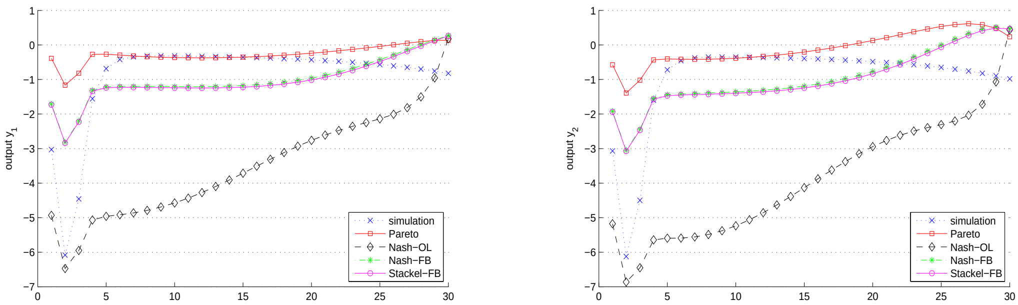

Figure 3: country i's output y_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right)

If the players (the central bank and the governments of the countries) want to react optimally to the demand-side shocks, their actions and their intensity depend on the presence or absence of cooperation. For example,

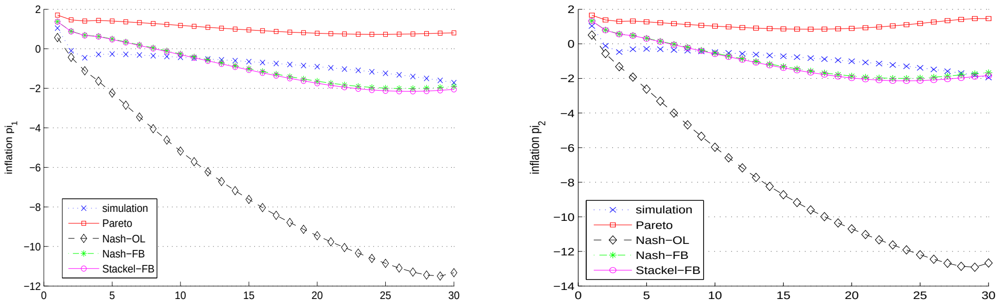

Figure 4: country i's inflation rate π_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right)

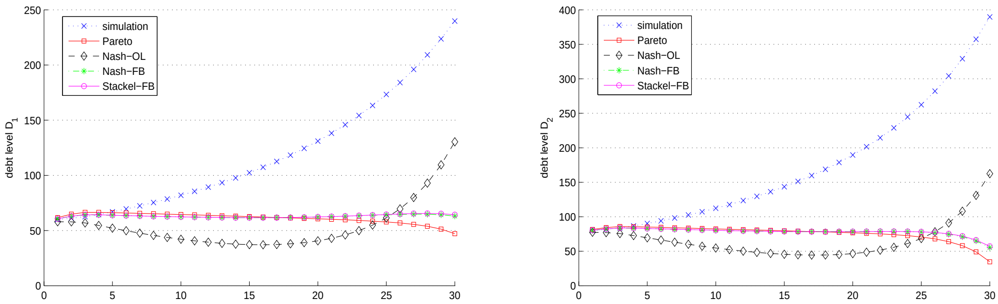

Figure 5: country i's debt level D_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right)

optimal monetary policy has to be expansionary (lowering the prime rate) in all solution concepts considered, but in the cooperative Pareto solution it is more active during the first 15 periods. The Nash open-loop solution, in contrast, is more or less constant during the whole optimization period, which causes the central bank to be less active at the beginning and relatively more active at the end of the optimization horizon.

With respect to fiscal policy, both countries are required to set expansionary actions and to create deficits in the first three periods in order to absorb the demand-side shock. After that a trade-off occurs and the governments have to take care of the financial situation and to produce primary surpluses. The only exception is the cooperative Pareto solution: cooperation between the countries and the central bank (which in this strategy runs a more active expansionary monetary policy) and the resulting moderate inflation means that the balance of public finances can be held close to zero. For country 2 it is even optimal to run a slightly expansionary fiscal policy again during the last 15 periods in the Pareto solution. Even so the countries are able to stabilize and to bring down their public debts close to the targeted values under cooperation.

The open-loop Nash solution, which assumes unilateral (not cooperating) commitment for all players, shows a bad performance. The central bank is less active than in all other solutions. The governments are forced to run restrictive fiscal policies which show that the trade-off between output and the public debt target is dominated by the latter one. The lack of cooperation between the players and the open-loop information pattern make the policy makers less flexible and as a result produce huge drops in output and an unsustainable deflation. Here both countries are trapped in a deflationary spiral, the possibility of which is frequently discussed these days for some of the European countries. An economic reason for this result is the lack of (even weak) time consistency of strategies in this solution concept, which implies very restrictive fiscal policies.

The non-cooperative Nash feedback and Stackelberg feedback solutions give very similar results. In comparison to the Pareto optimal solution, the central bank acts less actively and the countries run more active fiscal policies (except during the negative demand shock). As a result, output and inflation are slightly below the values achieved in the cooperative solution, and public debt is slightly higher. Comparing these results with the ones of the Pareto solution the impact of the cooperation can be clearly observed. In the Pareto solution, the central bank cooperates and is willing to be more active in order to support the countries.

6 Effects of a persistent negative supply-side shock

In this section we analyze a symmetric shock which occurs on the supply side (zs_i) as given in Table 5.

Table 5: Negative symmetric persistent shock on the supply side

t	1	2	3	4	5	6	$\cdots$	30
zs_1	10	5	0	0	0	0	$\cdots$	0
zs_2	10	5	0	0	0	0	$\cdots$	0

We call this shock a ‘persistent’ supply-side shock because after its occurrence there is no exogenous recovery from it and the system has to adjust to the new situation endogenously. This shock could be interpreted as a simplified representation of an oil price shock leading to the worst macroeconomic scenario, stagflation. Here, in the first two periods both countries experience the same negative shock (zs_i) which directly increases the price levels and the inflation rates in the economies.

Figures 6 - 10 show the simulation and optimization results of this experiment. Figures 6 - 7 show the results for the control variables of the players and Figures 8 - 10 show the results of selected state variables: output, inflation and public debt.

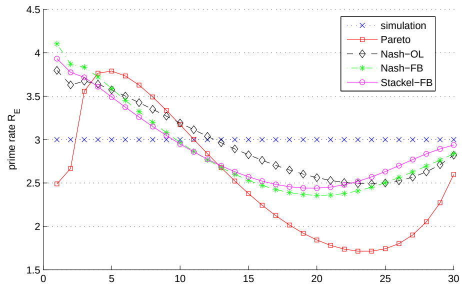

Figure 6: prime rate R_{Et} controlled by the central bank

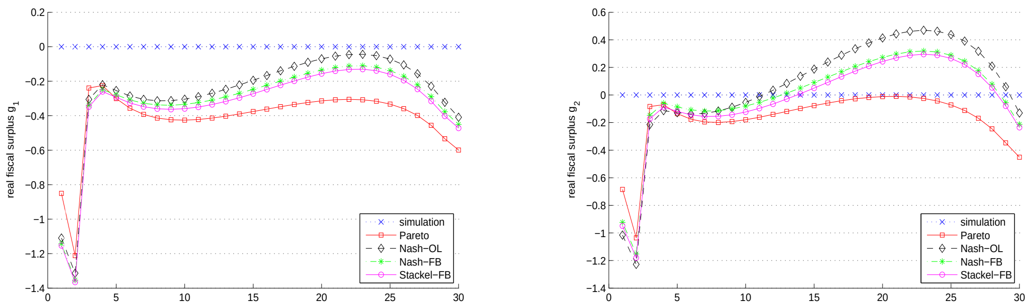

Figure 7: country i's fiscal surplus g_{it} (control variable) for $i = 1$ (core; left) and $i = 2$ (periphery; right)

Without policy intervention (baseline scenario, denoted by 'simulation'), both countries suffer dramatically from the supply-side shock especially in terms of output drop and high inflation. The output of both countries drops by more than 6% in the first two periods and improves at very slow rates so that it stays negative (i.e. below the long-run growth path) during the whole planning horizon. The inflation rates start with values of more than 10% and go back to the 'normal' values of about 2% very slowly. The one and only positive aspect of these high inflation rates is the resulting development of public debt. Except for the first three periods, where the effect of the negative deviation of output from the steady-state path outweighs the impact of the inflation-led depreciation, the public debt stays even below the targeted values.

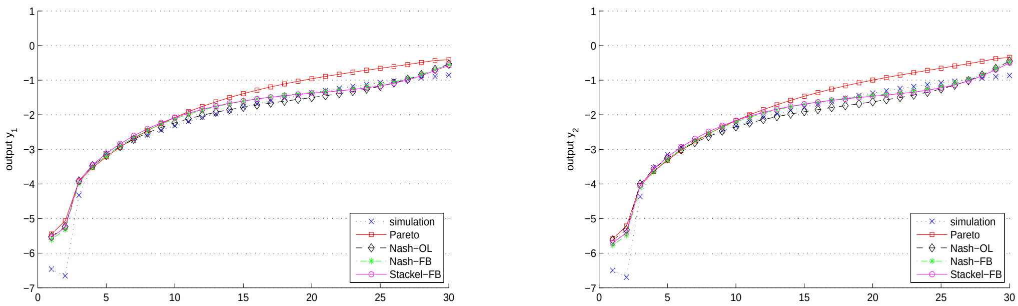

Figure 8: country i's output y_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right)

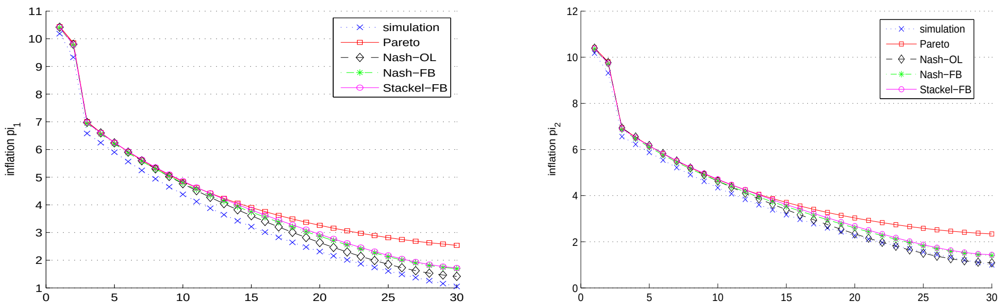

Figure 9: country i's inflation rate π_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right)

If the players want to react optimally to the supply-side shocks, again their actions and their intensity depend on the presence or absence of cooperation. The non-cooperative strategies show very similar optimal solutions. A

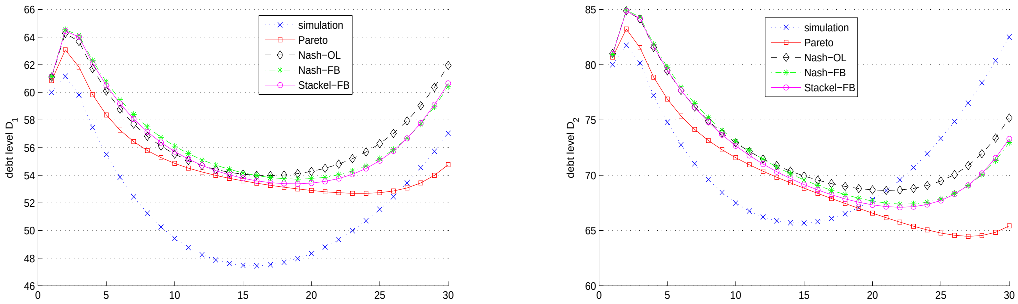

Figure 10: country i's debt level D_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right)

conflict between the central bank giving high importance to the inflation rate and the local governments caring more about GDP is well observable. The central bank reacts to the shock with a restrictive monetary policy in order to decrease the inflation rate. This restrictive monetary policy becomes less active as time goes by and after 10 to 13 periods (depending on the non-cooperative strategy played) the central bank gradually switches to an active monetary policy. On the other hand, the governments of the countries care about output and run expansionary fiscal policies. While country 1 can concentrate on the output target and therefore runs an expansionary fiscal policy over the whole optimization period, country 2 is forced to take higher public debt into account by running a slightly restrictive fiscal policy for certain periods (between periods 10 and 27). Here the trade-off between the output and public debt target is clearly visible.

From the results of the Pareto optimal solution is clear once again the benefit of the cooperation. In the first two periods, where the impact of the supply-side shock is strongest, the central bank supports the countries in reducing the drop in output by applying an active monetary policy even though the inflation rate stays high. After these two periods the central bank runs a policy similar to the non-cooperative solutions but is slightly more active. As a result the outputs of the countries in the Pareto solution are slightly above and the public debts are slightly below the ones of the non-cooperative solutions.

7 Effects of a reverse negative supply-side shock

In this section we analyze another symmetric shock which occurs on the supply side (zs_i) as given in Table 6.

Table 6: Negative symmetric reverse shock on the supply side

t	1	2	3	4	5	6	$\cdots$	30
zs_1	10	5	-5	-5	-3	-2	$\cdots$	0
zs_2	10	5	-5	-5	-3	-2	$\cdots$	0

We call this shock a ‘reverse’ supply-side shock because after its occurrence there is a smooth exogenous recovery from it. This shock could be interpreted as a temporary oil price shock with the oil price first going up and then coming back to the initial level. Such a temporary oil price shock occurred in the industrial countries in the 1980s. In the first two periods, both countries experience the same negative shock (zs_i) which directly increases the price levels in the economies and which is similar to the shock described in the previous section. After that the shock changes from the negative to a positive one during four periods.

Figures 11 - 15 show the simulation and optimization results of this experiment. Figures 11 - 12 show the results for the control variables of the players and Figures 13 - 15 show the results of selected state variables: output, inflation and public debt.

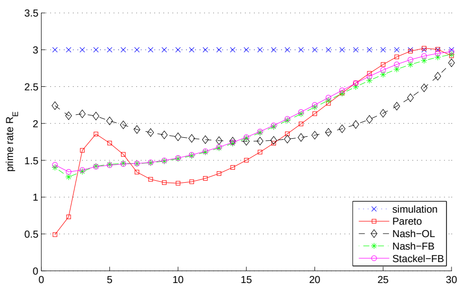

Figure 11: prime rate R_{Et} controlled by the central bank

Without policy intervention, in the first two periods, both economies show the same dynamics as in the case of a persistent supply-side shock with a

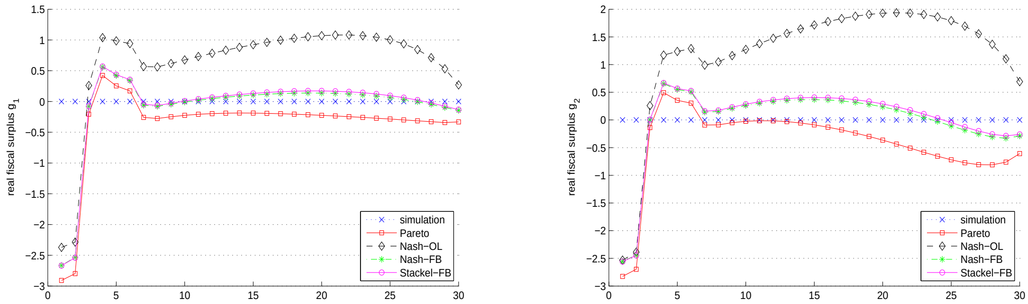

Figure 12: country i's fiscal surplus g_{it} (control variable) for $i = 1$ (core; left) and $i = 2$ (periphery; right)

drop in output by more than 6% and an increase in the inflation rate to more than 10%. In contrast to the persistent shock experiment, the reversion of the shock improves the economic situation in both countries very quickly except for the dynamics of their public debts. Now the public debt problem in the uncontrolled scenario grows to dramatic values of around 160% for the core block and 250% for the periphery block. This means that the policy actions of the players have to deal with the trade-off between the output/inflation problem in the first two periods and the public debt problem for the later periods.

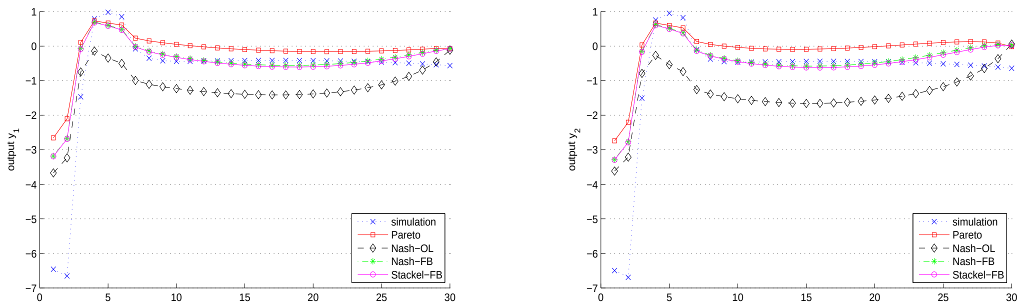

Figure 13: country i's output y_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right)

The optimal policies show more or less similar dynamics for all solution concepts. Monetary policy is expansionary during the whole optimization period, with the Pareto solution requiring it to be more active at the

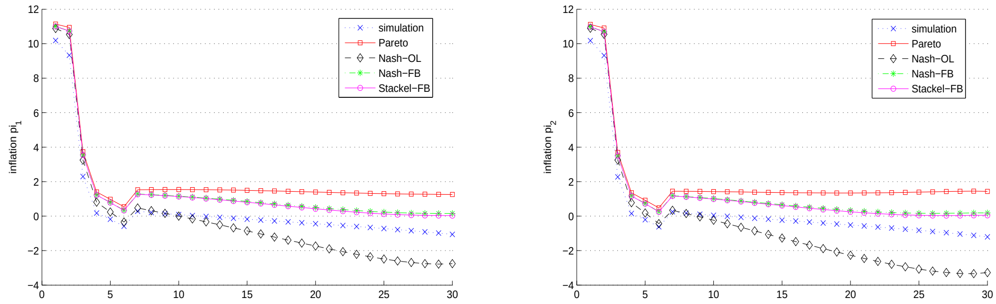

Figure 14: country i's inflation rate π_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right)

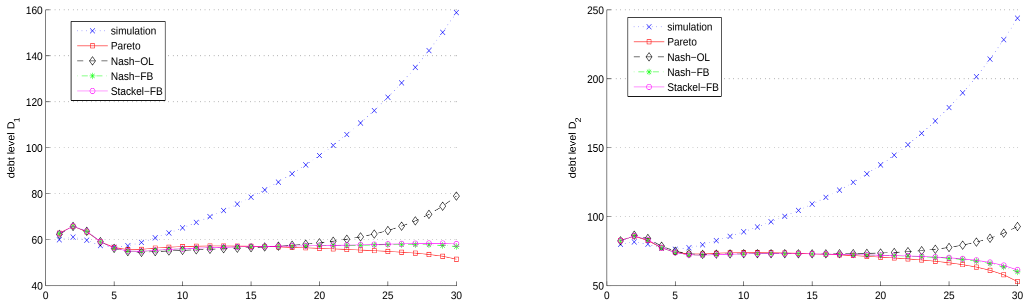

Figure 15: country i's debt level D_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right)

beginning and the Nash open-loop solution implying a nearly constant prime rate. The feedback Nash and Stackelberg solutions give results which are in between. Fiscal policy is expansionary during the first part of the supply-side shock for all strategies, requiring the governments to produce deficits in order to improve their outputs. During the second part of the supply-side shock, again all strategies require similar policies, but now restrictive ones. When the crisis runs out after six periods a slight divergence between the proposed solutions can be observed. The Nash open-loop requires restrictive fiscal policies for both countries with slightly higher surpluses for country 2. The feedback solutions (both Nash and Stackelberg) do not require active fiscal policy at all from either country. Only some minor adjustments which are less than 0.2% for country 1 and 0.5% for country 2 turn out to be optimal. And due to the cooperation between the players, in the Pareto solution both countries are able to produce some low deficits while still fulfilling the desired targets.

In the case of the output target in all game solutions, the situation is better than in the non-controlled simulation. Instead of the dramatic drop in more than 6% in the uncontrolled solution, all game solutions allow the impact of the shock to be reduced to a high degree: in the Nash open-loop solution to values between 3 and 4% and in the feedback Nash and Stackelberg solutions to around 3%. The Pareto solution gives the best performance and reduces the drop in output to values between 2 and 3%. Also for the remaining periods the Pareto solution gives the best results with output always being higher than in the other game strategies.

Regarding the inflation target all strategies show similar results during the occurrence of the crisis with the rate of inflation being more than 10% in the first two periods and decreasing quickly afterwards. After the crisis runs out the Pareto solution is able to stabilize the inflation rate around the target value of 1.8%. All other solutions produce inflation rates which lie below. In the case of the Nash open-loop solution a deflationary development can be observed.

The public debt situation can be fairly well stabilized as compared to the non-controlled simulation in all game strategies. Only in the last five periods can a slight divergence be observed. In the case of the Nash open-loop solution public debt goes up and for the other solution it goes down. This fact can be partially explained by the well-known effect of the finite horizon on the solution of optimal control problems.

8 Concluding remarks

In this paper we analysed the interactions between fiscal (governments) and monetary (common central bank) policy makers by applying a dynamic game approach to a simple macroeconomic model of a two-country monetary union. Using the OPTGAME3 algorithm, which allows us to find approximate solutions for nonlinear-quadratic dynamic tracking games, we obtained some insights into the design of economic policies facing negative shocks on the demand and the supply side. To this end we introduce three different shocks on the monetary union: a negative demand-side shock and two negative supply-side shocks, a persistent and a reverse one. The monetary union is assumed to be asymmetric in the sense of consisting of a core with less initial public debt and a periphery with higher initial public debt, which is meant to reflect the situation in the EMU.

Our results show strong trade-offs between the targets of output and public debt stabilization. Immediately at the start of the crisis nearly all results propose a countercyclical fiscal policy for the countries, with a quick switch to public debt stabilization afterwards. The 'best' results (in terms of the objective function values or losses) are achieved by the cooperative Pareto solution with a more active role played by the central bank. The trade-off between the targets price stability and output stabilization in the case of the supply shocks is less pronounced and is generally resolved in favor of output stabilization, which is due to the relatively strong reaction of output to the shock. The cooperative solution differs from the noncooperative ones more markedly in the supply-side scenarios than in the demand-side scenario. Altogether, the main policy conclusion consists in recommending coordinated fiscal and monetary policies, which may be interpreted (with caution) as recommending a fiscal pact involving governments and the common central bank.

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