

A New Test for Rational Speculative Bubbles using Forward Exchange Rates: The Case of the Interwar German Hyperinflation

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Abstract

The probabilistic structure of periodically collapsing bubbles implies different values for the slope coefficient of alternative efficient market hypothesis tests depending on whether the bubble is in an explosive regime or not. We exploit this fact and propose a new method for bubble detection. The method does not require the specification of the process followed by fundamentals, it is not affected by a possible explosive root of the determinants of the asset price, and provides a date-stamping strategy. We analyze the Reichsmark/Dollar exchange rate for the interwar German hyperinflation period and identify periods of rational exuberance.

Keywords: Rational bubble, Forward exchange rate, Explosive root, Hyperinflation.

JEL Classification: C12, C22, F31.

1 Introduction

The empirical literature on whether rational bubbles were present during the German hyperinflation period appears inconclusive (see, e.g., Burmeister and Wall, 1982, 1987; Hamilton and Whiteman, 1985; Christiano, 1987; Casella, 1989; Sargent, 1977; Taylor, 1991; Durlauf and Hooker, 1994; Engsted, 1993, 1994; Hooker, 2000). The existing literature focuses on statistical tests on the properties of the demand for money during the period up to summer 1923 when the probability of monetary reform was negligible. However, these tests depend on the validity of a variety of different hypotheses. For instance, Casella (1989) reports that the presence of bubbles can be rejected if the money supply process is assumed exogenous to the current inflation rate but not when the monetary process follows a feedback rule from inflation. While, Burmeister and Wall (1982) show that bubbles cannot be rejected if the money supply process depends on expected inflation. The assumption that the error process follows a random walk is also crucial for identification in a variety of tests (see, e.g., Christiano, 1987; Sargent, 1977; Casella, 1989).

The purpose of this paper is to propose a new test for rational speculative bubbles based on forward market data. We will set the idea out in the context of a discrete model of the foreign exchange market though it appears readily generalized to other markets involving a forward price.¹ The key intuition is that when a bubble is occurring the forward rate for period $t + n$, the spot rate at time t , and the spot outcome at $t + n$ will all embody the bubble but with

¹Evans (1986) proposes a test also employing forward rates that differs from that in this paper. He defines a speculative bubble as a subperiod during which there is a nonzero median in the distribution of the excess return to holding foreign currency when allowance is made for risk premia. Our test is conjectured solely on the basis of rational bubbles and is more specific than the test of Evans. Jarrow et al. (2011) propose a new test for bubbles based on volatility estimation techniques combined with the method of reproducing kernel Hilbert spaces. However, our sample size is too small to make inferences employing their method.

different weights. This is a consequence of the probabilistic structure of rational bubbles that pop. As a consequence we have a structure which is similar to a peso problem albeit with an explosive process. We demonstrate that when the bubble is occurring this generates a relationship between changes in (the level of) the spot rate and the forward premium (forward rate) in which the coefficient on the forward premium (forward rate) will be expected to exhibit a coefficient that can be greater than unity. Our methodology has three advantages over existing methods.² First, it is robust to the process followed by fundamentals and, under rational expectations (RE), it is free of misspecification or unobservable components issues. Second, the identification of rational bubbles is invariant to possible explosive roots in the fundamentals. Third, it provides a framework for dating bubble phenomena.

We apply our test in the interwar hyperinflation period in Germany between 24th December 1921 and 11th August 1923.³ We run rolling efficient market hypothesis (EMH) regressions employing weekly spot Reichsmark/Dollar (RM/\$) and one-month forward rates and use the Bonferroni method proposed by Cavanagh et al. (1995) for hypothesis testing in the presence of persistent regressors. Since bubbles, when occurring, imply an explosive asset price we also investigate the periods where the daily spot rate exhibited explosive behavior employing the test of Phillips et al. (2012) on a new daily series of the RM/\$ spot rate obtained from The Commercial and Financial Chronicle. The unit root test results indicate two long periods (from the middle to the end

²These methods include excess volatility tests, West's specification test, unit root and cointegration tests. See Gürkaynak (2008) for a comprehensive review of the literature on econometric tests for bubbles.

³The greater number of observations employed in our test also might mitigate against a problem outlined by Evans (1991) and West (1994) in studies employing monthly data on prices and money in the period 1921-1923. The basic problem as outlined by these authors is that, if prices and money supply are growing smoothly for a significant period of observations, standard tests may accept cointegration in money demand or the same degree of integration for the rate of inflation and rate of monetary change even in the presence of a bubble.

of 1922 and from April 1923 to the end of the sample period) of explosive behavior. However, the results based on our method suggest the exchange rate contained a rational bubble component only in the period from September to November 1922. A possible explanation for the discrepancy between the two methods is that the explosive behavior of the spot rate in some periods was due to movements in fundamentals.

The rest of the paper is structured as follows. The next section describes the new test for rational bubbles in the context of the EMH using the forward exchange market. The test is examined for both the implied slope coefficient of the Fama regression and the regression in levels of spot on forward rates. Section 3 illustrates the empirical application using data for the German interwar hyperinflation period. The final section concludes.

2 Rational Bubbles in the Forward Exchange Market

We do not specify any particular model of exchange rate determination but rather write the log of the spot rate at time t , s_t , as the sum of the two components of its RE solution: the log of the fundamental, x_t , plus the bubble term, B_t ,⁴

$$s_t = x_t + B_t. \quad (1)$$

Assuming initially a one period horizon, the log of the forward rate at time t for delivery at time $t + 1$, $f_{t,1}$, is equal to

$$f_{t,1} = E_t(s_{t+1}) + p_t = E_t x_{t+1} + E_t B_{t+1} + p_t, \quad (2)$$

⁴This expression assumes absence of the terminality condition. For instance, in the case of the monetary model of exchange rate determination the fundamental term x_t would be the discounted value of all current and future relative money and income between the domestic and foreign countries.

where E_t is the mathematical expectation at time t , and p_t is a zero-mean covariance stationary time-varying risk premium. Rational bubbles have the property (see Diba and Grossman, 1988)

$$\begin{aligned} E_t B_{t+1} &= (1+r)B_t \\ B_{t+1} &= (1+r)B_t + v_{t+1}, \end{aligned} \tag{3}$$

where $r > 0$ is a constant derived from the structural model determining the exchange rate, and v_{t+1} is the random forecast error. We assume the simplest form of rational speculative bubbles, the type proposed by Blanchard (1979), though the analysis holds for more complicated forms such as the one proposed by Evans (1991). In his bubble process there are two regimes which occur with probabilities π and $1 - \pi$. In the first regime (A), the bubble survives with probability π and continues to grow at an expected rate $\frac{(1+r)B_t}{\pi}$, and in the second regime (C) the bubble collapses⁵

$$B_{t+1} = \begin{cases} \frac{(1+r)B_t}{\pi} + \epsilon_{t+1} & \text{in state A} \\ \epsilon_{t+1} & \text{in state C} \end{cases} \tag{4}$$

and consequently (3) holds. The difference between the actual rate of growth of the bubble term, which will show up in the spot rate, and its expected value, which is part of the forward rate, will give rise to different values of the coefficient regression in EMH tests.

2.1 Fama Regression

We first employ the regression developed by Fama (1984) for the joint hypothesis of market efficiency and RE in the case of the forward rate n periods ahead

$$s_{t+n} - f_{t,n} = \alpha + \beta_{1,n}(f_{t,n} - s_t) + u_{t+n}, \tag{5}$$

⁵The expected value of the bubble term is therefore

$$E_t(B_{t+1}) = \begin{cases} \frac{(1+r)B_t}{\pi} & \text{in state A} \\ 0 & \text{in state C} \end{cases}$$

where α , $\beta_{1,n}$ are the regression coefficients, n is the horizon of the forward rate, and u_t is the error term. It is key to our analysis to note that if there is a rational bubble at time t , and it still remains at time $t + n$, the difference between the future spot rate outcome and the current forward rate is the following

$$s_{t+n} - f_{t,n} = x_{t+n} + \frac{(1+r)^n B_t}{\pi^n} + \epsilon_{t+n} - E_t x_{t+n} - (1+r)^n B_t - p_{t,n},$$

while the forward premium is

$$f_{t,n} - s_t = E_t x_{t+n} - x_t + (1+r)^n B_t - B_t + p_{t,n}.$$

Assuming that the fundamental process, x_t , is itself not subject to a ‘peso’ process and that follows a random walk, then, under RE, the forecast error, $x_{t+n} - E_t x_{t+n} = \theta_{t+n}$, is a moving average error of order $n - 1$. In this case, the sample estimate of $\beta_{1,n}$ in (5), $\widehat{\beta}_{1,n}$, is⁶

$$\widehat{\beta}_{1,n} = \frac{-\sum_{t=1}^{T-n} p_{t,n}^2 + (1+r)^n \left(\frac{1}{\pi^n} - 1\right) [(1+r)^n - 1] \sum_{t=1}^{T-n} B_t^2}{\sum_{t=1}^{T-n} p_{t,n}^2 + [(1+r)^n - 1]^2 \sum_{t=1}^{T-n} B_t^2}. \quad (6)$$

This expression suggests that the bubble term present in the price will increase the size of the estimate of $\widehat{\beta}_{1,n}$ as long as the bubble is ongoing. For a sufficiently large sample variance of the bubble term the coefficient will approach the value $\frac{(1+r)^n \left(\frac{1}{\pi^n} - 1\right)}{[(1+r)^n - 1]} > 1$. The presence of a risk premium does not affect this result because its variance enters the numerator with a negative sign and the denominator with a positive sign which would, if present, unambiguously reduce the value of $\widehat{\beta}_{1,n}$.

It is important to note that if the fundamental process, x_t , is itself subject to a ‘peso’ process over the sample period, and it is explosive or near explosive, then the implications for the Fama regression can be identical and we are unable to discriminate between a rational speculative bubble and explosive fundamentals

⁶We further assume that the covariance between $p_{t,n}$, and B_t is zero, and also note that $E_t \theta_{t+n} B_t = 0$ due to RE assumption.

with probabilistic regimes. This is an issue in hyperinflation periods because of the probability of monetary reform. In the case of the German interwar hyperinflation, which we analyze in the next section, the possibility of an expected structural change in the money supply process has been widely studied (see, e.g., Flood and Garber, 1980, 1983; LaHaye, 1985). These studies suggest that the probability of reform was near zero until mid August 1923. Consequently, if we exclude data from mid August 1923, as has been common in other tests of bubbles (see, e.g., Casella, 1989; Hooker, 2000), we should not bias the test results.

2.2 Spot-Forward Regression

An alternative specification is to consider the regression either between the spot rate, s_t , and the forward rate, $f_{t,n}$, or between the future spot rate outcome, s_{t+n} , and the forward rate. The slope of the former yields an inconclusive result for the detection of the speculative bubble as we demonstrate in Appendix A. However, the latter specification generates an unambiguous criterion as we illustrate below. The EMH can be tested by regressing⁷ the future spot rate, s_{t+n} , on the current forward rate, $f_{t,n}$,

$$s_{t+n} = \alpha + \beta_{2,n} f_{t,n} + u_{t+n}. \quad (7)$$

In the case of an ongoing bubble, $s_{t+n} = x_{t+n} + \frac{(1+r)^n}{\pi^n} B_t + \epsilon_{t+n}$, while the forward rate remains $f_{t,n} = E_t(s_{t+n}) + p_{t,n} = E_t x_{t+n} + E_t B_{t+n} + p_{t,n} = x_t + (1+r)^n B_t + p_{t,n}$. The estimated coefficient is therefore

$$\hat{\beta}_{2,n} = \frac{\sum_{t=1}^{T-n} x_t^2 + \frac{(1+r)^{2n}}{\pi^n} \sum_{t=1}^{T-n} B_t^2}{\sum_{t=1}^{T-n} x_t^2 + (1+r)^{2n} \sum_{t=1}^{T-n} B_t^2 + \sum_{t=1}^{T-n} p_{t,n}^2}.$$

⁷In the absence of bubbles and risk premium the coefficient β in the spot-forward regression will equal one regardless of the timing of the dependent variable, s_t or s_{t+n} .

Under risk neutrality (or assuming that the sample variance of the risk premium is much smaller than the one of B_t) the bubble process would bring the value of $\widehat{\beta}_{2,n}$ above unity.⁸

One of the advantages of our method, and a difference with the unit root test methodology, is that it allows the researcher to differentiate between explosive spot (and forward) rates brought about by the process of fundamentals and by the presence of rational bubbles, assuming the series of fundamentals is available. Appendix B shows that, within this framework, the criteria used for the coefficients $\widehat{\beta}_{1,n}$ and $\widehat{\beta}_{2,n}$ are not contaminated by the presence of explosive fundamentals in the price process.

2.3 Hypothesis Testing with a Persistent Regressor

Regressions (5) and (7) might contain highly persistent even explosive variables (under the alternative of bubbles) making standard regression analysis not valid (see Cavanagh et al., 1995; Jansson and Moreira, 2006; Phillips and Magdalinos, 2008; Engsted and Nielsen, 2010). We follow Cavanagh et al. (1995) and consider the following system that represents both the Fama and spot-forward regressions⁹

$$y_{t+4} = \mu_y + \gamma x_t + u_{1,t+4}, \quad (8)$$

$$x_{t+4} = \mu_x + \rho x_{t+3} + u_{2,t+4}, \quad (9)$$

where $u_{t+4} = (u_{1,t+4}, u_{2,t+4})'$ is a martingale difference sequence with finite 4th moment and possibly non-zero correlation coefficient $\delta = \text{corr}(u_{1,t+4}, u_{2,t+4})$. Our goal is to construct a confidence interval for the slope coefficient, γ , so as to test the null hypothesis of no bubbles, $\gamma = 1$, against the alternative of

⁸Within the regression in levels, the evidence would be suggestive of bubble if the coefficient $\widehat{\beta}_{2,n}$ switches from one in a no-bubble period to above one in a bubble period. In the absence of a bubble process, risk aversion implies $\widehat{\beta}_{2,n}$ below unity for small samples.

⁹We let x_t follow an AR(1) process for illustration purposes. The results hold for higher order AR processes.

an ongoing bubble, $\gamma > 1$. In this setting, the asymptotic distribution of the t -statistic for $\gamma = 1$ can be obtained by adopting a local-to-unity framework where the autoregressive coefficient is modeled as being in a $1/T$ neighborhood of unity, $\rho = 1 + c/T$, where c is a fixed constant, so that the regressor is stationary when $c < 0$, has a unit root when $c = 0$, or is explosive when $c > 0$. This distribution is non-standard and depends on the unknown degree of persistence c and the correlation coefficient δ (see, e.g., Campbell and Yogo, 2006). Although δ can be consistently estimated,¹⁰ the nuisance parameter c cannot, which creates difficulties in drawing statistical inference. We circumvent this obstacle by adopting the Bonferroni test of Cavanagh et al. (1995).

The Bonferroni procedure consists of two steps. First, we estimate a 90% confidence interval for c using the method of belts (see Stock, 1991).¹¹ Second, for each value of c in the estimated confidence interval, we construct a 90% confidence interval for γ .¹² The union of all the confidence intervals of the second stage provides a confidence interval for γ that does not depend on c . Note that because of the overlapping nature of the data in regression (8) the residuals u_1 will exhibit serial correlation. Similarly to the long-horizon regression literature, we employ Newey-West standard errors to deal with this overlap (see Torous et al., 2004).

Before we proceed to the empirical analysis, we demonstrate the applicability of the above method and the associated date-stamping strategy to simulated data. Appendix C describes the simulation exercise and the results show the way the t -statistic of regression (8) behaves according to our theoretical analysis.

¹⁰Torous et al. (2004) discuss alternative ways of estimating the long-run correlation coefficient.

¹¹Basically, the method of belts creates confidence intervals for c by inverting confidence intervals for the ADF statistic.

¹²Because Bonferroni confidence intervals can be quite conservative we use the size-adjustment suggested by Cavanagh et al. (1995).

3 Empirical Analysis

We put together a new daily series of the Reichsmark/Dollar spot rate obtained from The Commercial and Financial Chronicle (1925) for a period spanning the German hyperinflation from 24th December 1921 to 11th August 1923. The size of this novel dataset is larger from those used by previous studies that employ weekly or monthly data and hence its examination can be more informative regarding the statistical properties of the spot rate.

3.1 Unit Root Test Results

We investigate whether the spot rate displayed episodes of explosive behavior by running a unit root test recently developed by Phillips et al. (2012), the generalized sup ADF test ($GSADF$), on the daily spot rate. The appealing feature of the $GSADF$ is that it has good power properties and is consistent with multiple episodes of explosive behavior. We also employ an associated date-stamping strategy which is based on a sequence of Backward sup ADF statistics ($BSADF$). Appendix D provides the reader with the definitions of the unit root test statistics and a short summary of the date-stamping strategy.¹³

Figure 1 shows the sequence of $BSADF$ test statistics for the 494 observations of daily data. The results indicate that the spot rate did in fact exhibit explosive behavior since the estimated value for the $GSADF$ test statistic (the sup of the $BSADF$ sequence) is 4.248 which is substantially larger than the 95% critical value of 2.340. The behavior of the series, however, is not stable throughout the sample period. By comparing the $BSADF$ sequence with the corresponding 95% critical values, we observe two *long* periods of exuberance, one between July 1922 and February 1923,¹⁴ and another one between June

¹³A detailed description is out of the scope of this paper and can be found in Phillips et al. (2011) and Phillips et al. (2012).

¹⁴July 1922 is also the time when the forward premium went to a discount rather than a premium for the first time since 1921 in the weekly data of Einzig (1937). This is prior to

1923 and the end of the sample.¹⁵

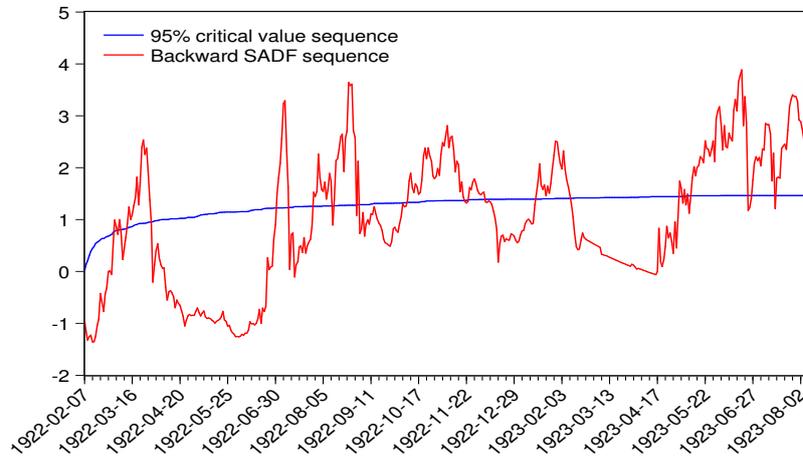


Figure 1: The Phillips et al. (2012) test for a unit root in the daily spot RM/\$. The figure displays the sequence of *BSADF* statistics (red line). The blue line shows upper critical values obtained from 2,000 Monte Carlo simulations.

The caveat of the above empirical analysis in testing for rational bubbles is that the spot (and forward) rates do contain both the fundamentals and the rational bubble terms, were the latter present in the process. In the absence of data for the fundamentals, any evidence of exuberance in the spot rate would be ambiguous about its cause and, hence, inconclusive regarding the occurrence of bubble episodes. This is especially true in the case of a hyperinflation period, the date at which agents are assumed in the extant literature to began anticipating monetary reform.

¹⁵The time interval in between those two periods, i.e., February-June 1923 coincides with an attempt from the Reichsbank to stabilize the exchange, see Graham (1930, p.86) “*The intervention of the Reichsbank did, in fact, not only keep the mark from falling farther at this time but, within a fortnight, had multiplied its exchange value two and a half times...[] For two months and a half the bank kept up the unequal struggle and maintained stability in the exchange value.*”

where explosive fundamentals should not be ruled out *a priori*.¹⁶ The periods identified as explosive in the spot rate therefore represent a necessary but not a sufficient condition for the presence of rational bubbles. Consequently, the number of “bubble” episodes identified by unit root tests is expected to be larger than or equal to the number of “bubble” episodes identified by our method.

Unfortunately, due to the fact that the forward rate is not available at the daily frequency our method cannot be applied on the dataset used for unit root testing. Instead we employ 86 weekly spot and forward exchange rates for the RM/\$ taken from Einzig (1937) for same time period.¹⁷

3.2 Fama and Spot-Forward Regression Results

We use a rolling window of 36 observations to estimate the Fama and spot-forward regressions.¹⁸ Figures 2 and 3 display the time evolution of the Newey-West t -statistics corresponding to the null hypotheses of no bubbles together with their corresponding right-tail critical values obtained from the Bonferroni procedure.

The Fama and spot-forward regression t -statistics exhibit a similar pattern. The Fama results suggest that the RM/\$ exhibited rational exuberance between September and November 1922 while the spot-forward results identify such period slightly earlier. Similarly to the unit root test results, the t -statistics in-

¹⁶This would be consistent with the fact that there were periods of time where fundamentals exhibited explosive behavior but speculative bubbles were absent. This point is made by Bresciani-Turroni (1937, p.90) in his comprehensive review of the depreciation of the mark between 1919 and 1923. “*This depreciation, however, could not go beyond certain limits if the quantity of marks had not been increased.*”

¹⁷The convention to change to a four week horizon for the forward rate occurred on the 24th December 1921, hence the start of our sample.

¹⁸Prior to estimating rolling regressions, we have run a unit root test on the Fama and Future Spot-Forward regression residual series. The null hypothesis of a unit root is rejected for both series at the 5% significance level. Note also that adopting a recursive rather than a rolling regression framework yields qualitatively similar results.

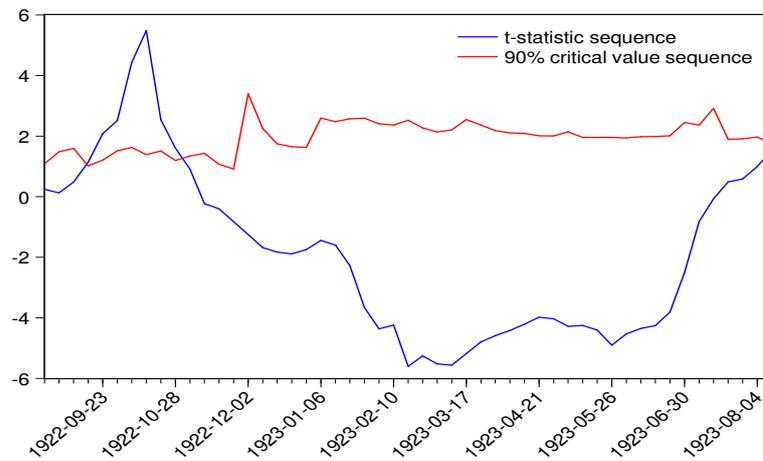


Figure 2: Rolling RM/\$ Fama regressions. The figure displays the sequence of Newey-West t-statistics (blue line) corresponding to the null hypothesis that the slope coefficient in regression (5) is unity (no bubble) based on subsamples of 36 weeks. The red line represents upper critical values obtained using the Bonferroni method described in the text.

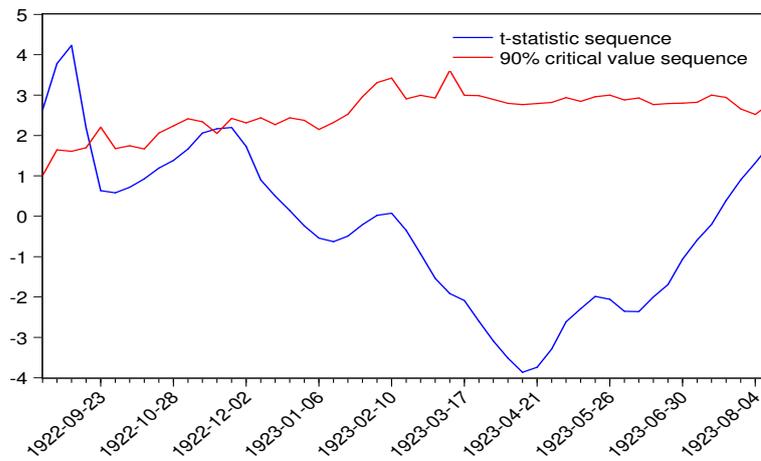


Figure 3: Rolling RM/\$ spot-forward regressions. The figure displays the sequence of Newey-West t-statistics (blue line) corresponding to the null hypothesis that the slope coefficient in regression (7) is unity (no bubble) based on subsamples of 36 weeks. The red line represents upper critical values obtained using the Bonferroni method described in the text.

crease again towards the end of the sample period but fail to exceed the critical value sequence by a small margin. These two time periods have been previously identified as consistent with strong speculative movements of the currency and deviations from fundamentals in historical reviews of the hyperinflation in Germany. Bresciani-Turroni (1937, p.102) mentions them as examples of episodes of speculation due to the quick rise and subsequent fall of the exchange rate,¹⁹ while Graham (1930) singles out the first period as representative of the ‘flight from the mark’.²⁰ Further historical evidence of the misalignment of the currency is provided by Graham (1930, p.119). The period previously identified as containing a rational bubble (around November 1922) coincides with the peak deviation of the mark from its purchasing power parity for the whole sample. This divergence between the internal value of the RM and its exchange value was also the largest of all European countries at the time.

4 Conclusions

The fact that a rational speculative bubble, which is part of the asset price, is also present in the forward rate has implications for EMH tests. We propose a method of bubble detection in the foreign exchange market that is based on the value of the slope coefficient of the Fama and spot-forward regressions. The method is valid for a variety of different data generating processes for fundamentals, including explosive behavior, and provides a date-stamping strategy.

¹⁹Bresciani-Turroni (1937, p.102) “*For some time it was foreign speculation which provoked the great fluctuations of the exchanges... [] By August 1922 the dollar rate had risen beyond 1,900 marks; but a violent reaction set in and the dollar fell to little more than 1,200 marks. On November 1st 1922, it was quoted at 4,465 marks; November 7th, 8,068; two days later, 6,711; November 21st, 6,791; and on November 28th, 8,480... [] Towards the middle of August 1923 the dollar rate rose giddily to 1, 2, 3, and 5 million marks; later falling suddenly to 3 millions.*”

²⁰Graham (1930, p.50) “*The greatest effect exerted by speculators in exchange is ordinarily supposed to have occurred after the middle of 1922...*”

We examine the Reichsmark/Dollar exchange rate from the end of 1921 until mid 1923, where the probability of monetary reform is thought to be negligible, and identify the period associated with the presence of rational bubbles.

Appendix A. Spot-forward regression

An alternative EMH test is to regress the spot rate on the forward rate, both at time t

$$s_t = \alpha + \beta_{3,n} f_{t,n} + u_t,$$

under RE and absence of peso problems in the fundamental process x_t the estimate of the slope of the forward rate is

$$\widehat{\beta}_{3,n} = \frac{\sum_{t=1}^{T-n} x_t^2 + (1+r)^n \sum_{t=1}^{T-n} B_t^2}{\sum_{t=1}^{T-n} x_t^2 + (1+r)^{2n} \sum_{t=1}^{T-n} B_t^2 + \sum_{t=1}^{T-n} p_{t,n}^2}.$$

This suggests a coefficient lower than unity ($\widehat{\beta}_{3,n} < 1$) even under risk neutrality. Another feature of this coefficient is that given that $r > 0$, $\beta_{3,n} < \beta_{3,n-i}$ for all $i \geq 0$. However, under risk aversion, the variance of the risk premium would also bring the coefficient of $\beta_{3,n}$ below unity in small samples even in the absence of bubbles, and therefore, this test is not as conclusive as the ones employing the future spot rate outcome (s_{t+n}) or the Fama regression framework.

Appendix B. Explosive Fundamentals

Let us consider the case that the fundamentals follow an explosive process, such that $x_t = (1+k)x_{t-1} + \theta_t$, with $k > 0$. In this case, $s_{t+n} = x_{t+n} + \varepsilon_{t+n} = (1+k)^n x_t + \sum_{i=1}^n \theta_{t+i}$, while the forward rate remains $f_{t,n} = E_t(s_{t+n}) + p_{t,n} = E_t x_{t+n} + p_{t,n} = (1+k)^n x_t + p_{t,n}$. The slope of the EMH regression in this case will be the following,

$$s_{t+n} = \alpha + \beta_{4,n} f_{t,n} + u_t.$$

$$\widehat{\beta}_{4,n} = \frac{(1+k)^{2n} \sum_{t=1}^{T-n} x_t^2}{(1+k)^{2n} \sum_{t=1}^{T-n} x_t^2 + \sum_{t=1}^{T-n} p_{t,n}^2}.$$

Assuming RE, absence of peso problems in the fundamentals, and risk neutrality, the value of $\widehat{\beta}_{4,n}$ will remain at 1, while small sample estimates could

yield an estimate below unity under risk aversion. The regression in levels of contemporaneous spot and forward rate would not yield either a value above unity in the absence of rational bubbles,

$$s_t = \alpha + \beta_{5,n} f_{t,n} + u_t.$$

$$\widehat{\beta}_{5,n} = \frac{(1+k)^n \sum_{t=1}^{T-n} x_t^2}{(1+k)^{2n} \sum_{t=1}^{T-n} x_t^2 + \sum_{t=1}^{T-n} p_{t,n}^2} < 1$$

Consequently the sample estimate will be lower than unity even assuming risk neutrality. The main point is that explosive fundamentals would not bring the coefficient above unity in any of the two regressions in levels, contemporaneous or lead spot rate.

Likewise, within the Fama regression framework explosive fundamentals do not imply a value of $\widehat{\beta}_{1,n}$ above zero. In this case the value of the slope coefficient in the Fama regression (5) is

$$\widehat{\beta}_{6,n} = \frac{-\sum_{t=1}^{T-n} p_{t,n}^2}{\sum_{t=1}^{T-n} p_{t,n}^2 + [(1+k)^n - 1]^2 \sum_{t=1}^{T-n} x_t^2}.$$

Appendix C. Simulation Exercise for Regression Analysis

For the simulation exercise, we let the fundamental process x_t follow a driftless random walk, $x_t = x_{t-1} + \theta_t$ with $\theta_t \sim N(0, 100)$. The parameters of the bubbles process (4) π and r are set equal to 0.65 and 0.005, respectively, and we let the error term ϵ_t follow a normal distribution with mean 0 and variance 0.25. By using a random number generator, we simulate a single realization of 90 observations for the spot and forward rate series with a bubble occurring around the middle of the sample period, from $t = 51$ to $t = 68$. This sample size is equal to the weekly dataset used in the empirical analysis.

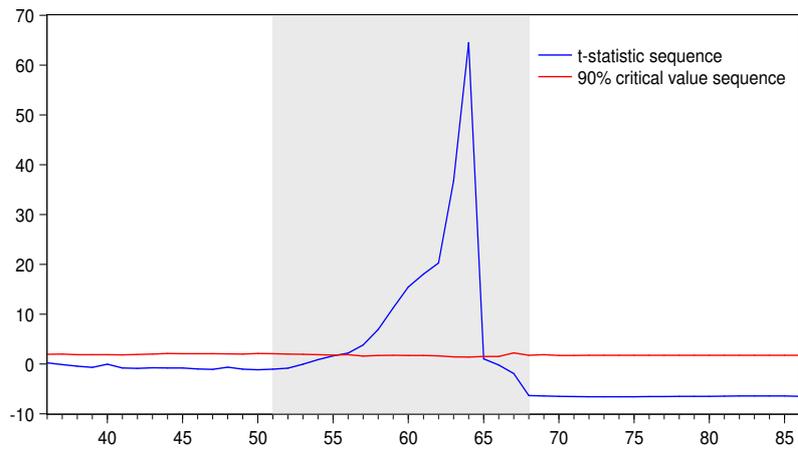


Figure 4: Rolling Fama regressions on simulated data. The figure displays the sequence of Newey-West t-statistics (blue line) corresponding to the null hypothesis that the slope coefficient in regression (5) is unity (no bubble) based on subsamples of 36 weeks. The red line represents upper critical values obtained using the Bonferroni method described in the text and the shaded area shows the bubble period.

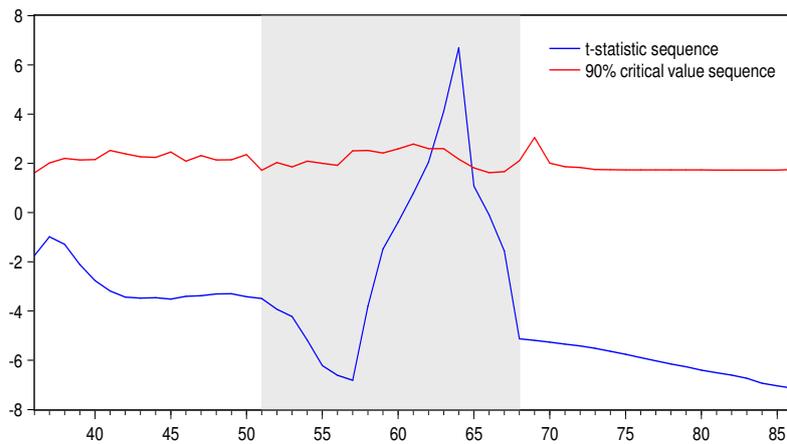


Figure 5: Rolling spot-forward regressions on simulated data. The figure displays the sequence of Newey-West t-statistics (blue line) corresponding to the null hypothesis that the slope coefficient in regression (7) is unity (no bubble) based on subsamples of 36 weeks. The red line represents upper critical values obtained using the Bonferroni method described in the text and the shaded area shows the bubble period.

Figures 4 and 5 display t -statistics corresponding to the null hypotheses that the slope coefficients in the Fama and spot-forward regressions are equal to one (the null hypotheses of no bubble). These test statistics are computed by using a rolling window of 36 observations. The red lines in the figures depict 90% critical value sequences, while, the shaded area specifies the bubble period. We observe that, in accordance with the theoretical analysis, once the bubble starts the estimated test statistics gradually increase and eventually exceed their critical values rejecting the null hypothesis of no-bubble. While, when the bubble bursts the statistics fall below the critical bound almost instantaneously.

Appendix D. Testing for Explosive Behavior

Consider the following Augmented Dickey-Fuller (ADF) regression equation

$$\Delta y_t = a_{r_1, r_2} + \beta_{r_1, r_2} y_{t-1} + \sum_i^k \psi_{r_1, r_2}^i \Delta y_{t-i} + \epsilon_t, \quad (10)$$

where $\epsilon_t \stackrel{iid}{\sim} N(0, \sigma_{r_1, r_2}^2)$, and r_1 and r_2 denote fractions of the total sample size that specify the start and the end of the (sub)sample period. We are interested in testing the null hypothesis of a unit root, $H_0 : \beta = 0$, against the alternative of explosive behavior in y_t , $H_1 : \beta > 0$. Let $ADF_{r_1}^{r_2}$ denote the test statistic corresponding to this null hypothesis. The standard ADF statistic corresponds to $r_1 = 0$ and $r_2 = 1$ and is denoted by ADF_0^1 . Although widely employed, the standard ADF test has extremely low power in detecting periodically collapsing bubbles (see Evans, 1991).

Phillips et al. (2011) propose a recursive unit root test which is based on the estimation of the ADF regression on a forward expanding sample. The sup ADF ($SADF$) test is defined by

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2},$$

where r_0 is the minimum window size, and performs well when there is a single bubble episode.

Phillips et al. (2012) extend their previous work to derive a unit root test, the Generalized *SADF* (*GSADF*), that is more powerful than the *SADF* and is consistent with multiple *bubbles*. The test and the associated date-stamping strategy are based on a Backward sup *ADF* (*BSADF*) statistic given by

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}.$$

The Generalized *SADF* statistic is computed as the sup of the *BSADF*

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1]} BSADF_{r_2}(r_0).$$

Note that the *GSADF*, contrary to the *SADF*, allows both the start and end dates to change for the computation of the test statistic.

The procedure of Phillips et al. (2012) consists, first, of testing for a unit root by comparing the $GSADF(r_0)$ to the $1 - \alpha$ critical value, where α is the nominal significance level. If the null hypothesis is rejected then periods of explosive behavior can be identified when the *BSADF* statistic is greater from the finite-sample critical value of the *SADF*. Because the distributions of both the $SADF(r_0)$ and $GSADF(r_0)$ are non-standard critical values have to be obtained through Monte Carlo simulations.

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